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Penalising on the basis of the severity of the offence: A sophisticated revenue-based cartel penalty

Yannis Katsoulacos[1]
Evgenia (E.) Motchenkova[2]
David Ulph[3]

1: Athens University of Economics and Business, Greece
2: Vrije Universiteit Amsterdam, The Netherlands, TILEC, The Netherlands; Tinbergen Institute, The Netherlands
3: University of St Andrews, Scotland

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A sophisticated revenue-based cartel penalty[1]

Yannis Katsoulacos[2]
Evgenia Motchenkova[3]
David Ulph[4]

Abstract

In Katsoulacos et al. (2015) we examined the welfare properties of a number of monetary penalty regimes for tackling cartels, including revenue-based penalties, the most widely used regime. We showed that for a typical industry overcharge–based penalties welfare-dominate the others. However these penalties are subject to criticisms on the grounds of high implementation costs and lack of transparency/uncertainty. In this paper we propose a new *sophisticated revenue-based penalty regime* in which the penalty *base* is the revenue of the cartel but the penalty *rate* increases in a systematic way with the cartel overcharge. Thus, the proposed regime formalises how revenue can be used as the base while taking into account the severity of the offence. We show that this hybrid regime can replicate the desirable welfare properties of overcharge-based penalties while having relatively low levels of implementation costs and of uncertainty, concluding that the proposed penalty regime deserves very serious attention from Competition Authorities.

JEL Classification: L4 Antitrust Policy, K21 Antitrust Law, D43 Oligopoly and Other Forms of Market Imperfection.

Keywords: Antitrust Penalties, Antitrust Enforcement, Antitrust Law, Cartels.

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[2]Department of Economic Science, Athens University of Economics and Business, Patission 76, Athens 104 34, Greece. Email: ysk@hol.gr
[3]Department of Economics, Vrije Universiteit Amsterdam, TILEC and Tinbergen Institute, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands. Email: emotchenkova@feweb.vu.nl
[4] School of Economics and Finance, University of St Andrews St Andrews, KY16 9AR, Scotland. Email: du1@st-andrews.ac.uk

1. Introduction

In Katsoulacos et al. (2015) we undertook a systematic welfare comparison of a number of penalty regimes for tackling cartels: illegal-gains based penalties; the simple revenue-based penalty regime adopted by most Competition Authorities (CAs);[5] and a new penalty regime based on the cartel overcharge. We showed that for a typical industry the latter welfare dominated the other two since, by suitable choice of penalty rates, it could achieve the same level of deterrence as each of the other two, but would induce those cartels that do form to set lower prices. In this paper we extend our previous analysis in three ways: (i) we include a wider range of penalty regimes by considering also both penalties on damages and also a new penalty regime - a *sophisticated revenue-based penalty regime* - in which the penalty *base* is cartel revenue but the penalty rate applied to that base increases systematically with the cartel overcharge; (ii) we consider a wider range of criteria for assessing penalty regimes by taking account of *ease of implementation* and *transparency/certainty* – factors that weigh heavily with Competition Authorities (CAs) in deciding what penalties to use; (iii) we incorporate the requirement that these penalty regimes have to function across a range of industries and not just a typical industry. We show that:

(a) the widely-used revenue based penalty scores best in terms of both *ease of implementation* and *transparency/certainty* while the *sophisticated revenue-based penalty regime* performs reasonably well on these criteria and certainly better than the overcharge-based regime which in turn dominates both damage-based and illegal-gains based regimes;

(b) in terms of welfare the *sophisticated revenue-based penalty regime* has properties that are similar to the overcharge-based regime and definitely dominates the revenue-based regime.

As indicated, the current paper concentrates on sanctioning methods as a part of the *public enforcement* of Competition Law on cartel cases,[6] and, in particular, monetary penalties on corporations.[7] In the recent literature economists have concentrated on a

[5] See for example Bageri and Katsoulacos (2014). As noted in the ICN Report (2008) "the general view been that turnover/volume of affected commerce provides a good proxy for assessing the gravity of the behavior, both in terms of damage to consumers and illegal gain. Furthermore, such data is relatively easy to obtain" (p. 19).

[6] Public enforcement sanctioning and private damage actions serve primarily different purposes. Public enforcement's main objective is to bring cartel activity to an end and imposing sanctions for infringements which aim to punish and to deter future violations. Private damages on the other hand, focus on compensating those who have suffered harm. So, the two methods are complementary but each can contribute to the objectives of the other. Public enforcement can facilitate and stimulate private damage actions and private damage actions can contribute to deterrence and provide incentives to customers to discover and report price-fixing.

[7] The other main types of sanctions in public enforcement are: financial penalties on managers involved in price-fixing, criminal sanctions/imprisonment of individuals involved in price-fixing, debarment of individuals

comparison of the welfare properties.[8] However, other policy-relevant dimensions of the penalty regimes also have to be assessed and compared. Specifically, a complete comparison should take into account:

(i) **Ease of Implementation**.

This involves considerations relating to the administrative costs of the penalty regime – for both CAs and firms: resource costs and delay in gathering the required information and performing reliable estimates; the costs of appeals and/or a judicial review process. The number of appeals will be greater the more likely it is that the penalty regime can lead to estimation errors and/or penalty decisions can be challenged as being discriminatory.

(ii) **Transparency/Certainty**.

Penalty regimes differ in terms of how easily and accurately firms can predict the fine that they will face if they are successfully prosecuted. While in a few cases agencies adopt the view that some uncertainty can improve deterrence, when detection rates are low and the severity of penalties is constrained, this approach is recognised to have serious downsides[9], and a large number of jurisdictions (including the EC, US, Canada and Brazil) take the view that a high degree of transparency and certainty is desirable and that to achieve deterrence CAs should rely on the threat of severe penalties coupled with a significant fear of detection. Relatedly, in the interests of equality of treatment under the law, the basis on which penalties are determined – both the penalty base and penalty rate - should be as consistent as possible across cases. This criterion is closely linked to that of ease of implementation since, c*eteris* paribus, greater transparency/consistency should reduce appeals etc.

(iii) **Welfare properties**.

The traditional economics literature following from Becker (1968) focussed solely on deterrence and identified penalties based on either damages or illegal gains as being first-best optimal depending on the welfare criterion used – total welfare or consumer surplus respectively. In the next section we show that these penalties score badly in terms of ease of

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involved in price-fixing, from further employment in a position from which they could again violate antitrust laws. See for a review Katsoulacos et al. (2017).

[8] See e.g. Harrington (2004, 2005), Houba et al. (2010), Katsoulacos and Ulph (2013), Katsoulacos at al. (2015) and Bos et al. (2017) for theoretical analysis of the effects of various penalty regimes on cartel pricing and /or deterrence. The empirical analysis is provided in e.g. Levenstein and Suslow (2011, 2012, 2014), Schinkel (2007), Veljanovski (2007), Connor and Lande (2008), Allain et al. (2011), Boyer and Kotchoni (2015) or Spagnolo and Marvão (2016).

[9] Thus, it may lead to under deterrence when lower penalties are mistakenly anticipated by potential offenders or over deterrence when innocent agreements are deterred by overestimating fines. Further, and very importantly in practice, the less *discretion* an agency has (limiting uncertainty) the less the degree of litigation on the amount of the fine by companies fined and the lower the risk of been accused of discrimination and public criticism of subjectivity and arbitrariness. See for details on this ICN Report (2008).

implementation and transparency/consistency, which may be why they are rarely if ever used.[10] The more recent literature has therefore focused on comparing penalty regimes in a second-best world, where it is assumed that, as is true in practice, there are a variety of factors such as bankruptcy considerations which mean that penalties cannot be set so as to deter all or even most cartels.[11] So, a proper second-best welfare comparison has to take into account the effects of a given penalty regime on both deterrence and on the price set by those cartels that do form. The most extensive and rigorous recent comparison of the welfare properties of the penalty regimes described below is contained in Katsoulacos et al. (2015).

Clearly a penalty regime is better than another one if it easier to implement, has greater transparency/certainty and has a superior welfare impact. This does not hold for the range of regimes considered here and regimes that are superior in terms of their welfare properties are not superior (and may be inferior) in terms of the other assessment criteria.

Our main contribution in this paper is to demonstrate that a sophisticated revenue-based regime on the one hand avoids the serious problems of implementability and uncertainty of penalty regimes with good welfare properties (such as the overcharge-based and damage-based regimes), while at the same time being superior in its welfare impact (both in terms of its price and its deterrence effects) to the only regime that scores well in implementability and uncertainty (the simple revenue-based regime).[12]

The structure of the paper is as follows. Section 2 provides a brief descriptions and preliminary comparison of the main penalty regimes including the one proposed in this paper in terms of the three assessment criteria above. Section 3 provides a detailed analysis of the price effects and deterrence properties of the sophisticated revenue-based penalty regime and demonstrates its superiority relative to the simple revenue-based regime, having similar properties to the overcharge-based regime. Section 4 concludes.

2. Brief review and preliminary comparisons of monetary penalty regimes

In order to understand what is involved in calculating each of the alternative penalties we define them here with reference to the Figure 1 below in which, for simplicity, we consider

[10] Many countries explicitly provide in their statues for the imposition of penalties based on illegal gains. Indeed 9 out of the 17 countries that participated in the ICN survey in 2008 do so, including US and China (see ICN Report 2008, p.19). Penalties based on illegal gains can either take the place of revenue-based penalties (as in US) or they can constitute an additional penalty that is combined with the revenue-based penalty in order to reach the overall penalty figure imposed on law violators (as in China). However, illegal gains-based penalties are rarely implemented – see ICN Report 2008. In a private communication, Greg Werden confirms that in US a penalty based on illegal gains has only been imposed once in the USA.

[11] Difficulties of first-best solutions in practice were discussed in e.g. Bos and Schinkel (2006), Buccirossi and Spagnolo (2007), Harrington (2010), Katsoulacos and Ulph (2013) and Houba et al. (2017).

[12] The sophisticated revenue-based regime is superior to illegal gains regime in terms of all assessment criteria.

the market for a homogeneous product with linear demand and constant marginal cost $c > 0$. The (potentially) imperfectly competitive "but-for" price and output are $\left(p^B, Q^B\right)$, $p^B \geq c$ and $\left(p^C, Q^C\right)$, $p^C > p^B$ represent the cartel price and output. If the "but-for" situation were one of perfect competition we would have $p^B = c$. $R^C = p^C Q^C$ is the cartel revenue.

1. Damages-based penalties were proposed in the seminal article of Becker (1968) examining first-best optimal penalties – under the assumption that the objective of the enforcing agency is to maximise (total) social welfare. In Figure 1 they are given by the area *A*+*D*.

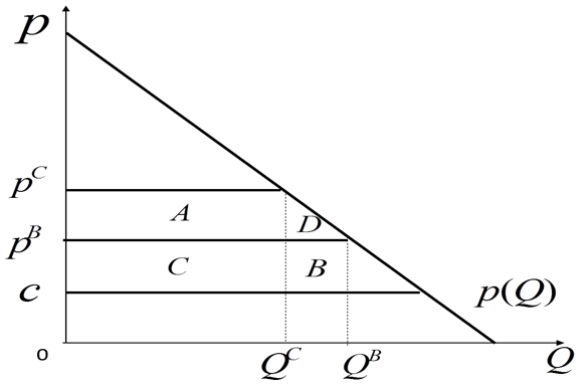

Figure 1

2. Illegal gains (or profit)-based penalties were early identified for their welfare properties, their adoption being proposed most forcefully by Landes (1983) when the objective is to deter conduct that does not generate any efficiencies (such as price fixing) in order to avoid the reduction in consumer surplus that results from such conduct. Illegal gains are defined as cartel's profits over and above the counterfactual level of profits so in Figure 1 they are given by the area *A* - *B*.[13]
3. Revenue–based penalties are the penalties most often adopted and implemented by CAs throughout the world. In Figure 1 they are given by $\rho_R R^C = \rho_R (p^C Q^C)$ where ρ_R is the baseline penalty rate. While the actual penalty rate is often varied by CAs depending on a number of mitigating and aggravating circumstances, we note that these often relate either to different types of infringement or to other behaviours that firms displayed in the course of the investigation. Consequently the baseline penalty rate applied in pure cartel cases as considered here can be treated as constant across different cartels cases. It is such a

[13] In the special case where the counterfactual price is the marginal cost (competitive price), the illegal gains are the same as the cartel profits.

constant penalty rate that the empirically-based literature on appropriate cartel penalties – e.g. Katsoulacos and Ulph (2013), Connor and Lande (2008) – seeks to determine.

4. Overcharge-based penalties are given by $\rho_O \theta p^B Q^B$ where $\theta = (p^C - p^B)/p^B$ is the proportional overcharge and $\rho_O > 0$ the penalty rate that is applied to the "but-for" revenue $R^B = p^B Q^B$, and is constant across cartel cases.
5. Sophisticated revenue-based penalties use as base the revenue of the cartel but the penalty rate depends on (and increases with) the cartel overcharge rate. It is given by $\rho_{SR}(\theta) R^C$ where ρ_{SR} is the penalty rate (written as a function of the overcharge). Once again we take it that this function is constant across cartel cases.

Table 1 shows the information required for calculating the above penalties. It is categorised as Observable (O) or Unobservable (U) and in accordance with the difficulty in getting the information, as H: High, M: Medium and L: Low.

Table 1: Information required for the calculation of alternative penalties

Penalty / Information Required	Revenue-based $\rho^R p^C Q^C$	Sophisticated revenue-based $\rho^{SR}(\theta) p^C Q^C$	Overcharge-based $\rho^O (p^C - p^B) Q^B$	Illegal gains-based $A - B$ in Fig.1	Damages-based $A+D$ in Fig.1
Turnover $p^C Q^C$ (O; L)	X	X			
Cartel volume of sales, Q^C (O; L)				X	X
Counterfactual price and, so, Overcharge $\theta = (p^C - p^B)/p^B$ (U; M)		X	X	X	X
Counterfactual volume of sales Q^B (U; H)			X	X	X
Cost Information (c) (U; H)				X	
Information about Demand Structure (U; H)					X

Implementation and Transparency

From Table 1 the following comments can be made regarding the ease of implementation and transparency properties of the various penalty regimes.

Damages-based penalties are very difficult to estimate accurately since, in addition to unobservable counterfactual prices and volumes of sales, they require information about the demand structure in order to calculate the area D. Thus, such penalties have very significant

implementation problems and a low degree of transparency raising significantly the probability of been challenged for being false or discriminatory. For these reasons they very rarely form the basis of antitrust sanctioning in practice.

Illegal gains-based penalties are also very difficult to estimate accurately in most cases and their estimation is likely to be subject to quite significant errors also due to the need to estimate costs as well counterfactual prices and volumes of sales. Thus such penalties also have significant implementation problems and can create a low degree of transparency. Nevertheless, as already noted, because they are thought to have good deterrence properties they are sometimes included in the range of penalty structures that might be used, though they are very rarely implemented in practice.

Simple revenue-based penalties, which are currently widely applied, owe their popularity to the fact that they score high in terms of ease of implementation and also high transparency (low uncertainty). They only require information about turnover, which is public.

Overcharge-based penalties require estimates of the price overcharge and, more significantly, the counterfactual volume of sales. While, as discussed below, establishing the overcharge is nowadays not too problematic, there are less well established techniques for establishing the counterfactual volume of sales, so this regime scores also low in implementation and transparency.

Sophisticated revenue-based penalties require for their calculation the cartel revenue and also estimates of the price overcharge, but do not require information about counterfactual volumes of sales, and so certainly score higher than the Overcharge-based penalties in terms of implementation and transparency. However, although there are implementation and transparency concerns arising from the need to calculate the price overcharge, for the following two reasons we think that these are often exaggerated.

(a) The overcharge arising in cartel cases has been routinely estimated for many years in private damage claims. These claims have been a very important feature of the North America jurisdictions, have been introduced in EU competition policy since 2014, and are gradually becoming popular in the EU countries too. As discussed in Brander and Ross (2017), there are now a range of well-tried and well-understood methodologies (of varying degrees of sophistication) for estimating the overcharge and so, as the two prominent authors in this area wrote recently "Overall, we feel that a great deal of progress in damage estimation and related topics has been made in the past two decades. In addition, data availability has significantly improved and computing power has

increased greatly. Therefore, good estimates of damages from price-fixing and related anticompetitive practices can often be obtained".[14]

(b) It is sometimes argued that having to calculate the overcharge in order to take it into account in setting monetary penalties imposes an excessive burden on CAs. As the argument goes, in private damages claims the estimation is undertaken by those claiming damages and the Courts just have to balance the evidence presented and choose between these and the counter estimates made by the defendants. However, this is certainly not a strong argument. If the sophisticated revenue-based regime is adopted then there is nothing to stop the CAs requesting the parties (defendants and plaintiffs) to make available their estimates of the price overcharge (with detailed justification) along with the other documents that they are asked to produce during the investigative procedure. Indeed, such a request, if mandatory, would likely have beneficial welfare effects since it will increase the costs to cartel offenders of been detected – having to try to show low overcharge rates before this is required for dealing with private damage claims, and will incentivise plaintiffs not to make false claims of law violation.

The following Result summarizes the above discussion:

Result 1: Sophisticated revenue-based penalties are superior in terms of ease of implementation and transparency to overcharge-based, illegal gains-based and damages-based penalties. They do not perform as well in terms of these criteria as simple revenue-based penalties.

Welfare and Overall Comparisons

As pointed out above, both damages-based and illegal-gains based penalties have argued to be first-best optimal regimes depending on the welfare criterion used – total welfare or consumer surplus respectively.[15] However, given their very poor implementation and transparency properties we will not consider their welfare properties in any further detail.

Turning to the second-best policies discussed above - simple revenue-based penalties, sophisticated revenue-based penalties and overcharge-based penalties - we show in the next section that both sophisticated revenue-based penalties and overcharge-based penalties

[14] Brander and Ross (2017). See also Brander and Ross (2006).

[15] The presumption is that if illegal-gains are fully removed through the penalty regime then all cartels will be deterred. However if, for a variety of reasons, not all cartels are deterred then, as we showed in Katsoulacos et.al (2015), illegal-gains based penalties have rather poor welfare properties, since those cartels that do form will set the monopoly overcharge.

welfare dominate simple revenue-based penalties. Drawing the various criteria together produces the following table of results.

Table 2: Summary of Performance on All Assessment Criteria

Assessment criterion / Penalty Regime	Ease of Implementation	Transparency/ Certainty	Welfare
Damages-based	Low	Low	Strong
Illegal gains-based	Low	Low	Moderate
Overcharge-based	Low	Low	Strong
Revenue-based	High	High	Poor
Sophisticated revenue-based	Moderate to High	Moderate to High	Strong

So we have the following:

Result 2 Taking all assessment criteria into account, a Sophisticated Revenue-Based penalty regime dominates both an overcharge-based regime and an illegal-gains based regime.

In the next section we formally establish the welfare properties of the three second-best regimes.

3. Analysis of the Welfare Properties of Second-Best Regimes

As indicated above, in this section we undertake a systematic comparison of the welfare properties of a sophisticated revenue-based regime with those of an overcharge regime and a simple revenue-based regime. In doing this we extend in an important way the analysis of the Katsoulacos et. al. (2015) paper in which the analysis concentrated on a single "typical" industry. Here the analysis takes into account that penalties will have to apply to cartels arising in a range of different industries and that, in order to satisfy the principle of non-discrimination or equality of treatment, the penalty *rates* (by which we mean the baseline rates before account is taking of various aggravating and mitigating circumstances) should not vary across industries.

As we will show, this implies that the degree of deterrence under a simple revenue-based regime varies across industries in a way that is inversely related to the (monopoly) overcharge in that industry, while under both an overcharge regime and a suitably designed sophisticated revenue-based penalty regime the degree of deterrence will be constant across

industries. This implies that for the overcharge-based regime we can no longer make the same claims in relation to deterrence relative to the simple revenue-based regime, as we did in Katsoulacos et. al. (2015). Instead, we show that both it and a suitably designed sophisticated revenue-based can achieve the same constant degree of deterrence across all industries and that this can equal the degree of deterrence achieved by a standard revenue-based penalty regime in the average industry. But then both an overcharge regime and a suitably designed sophisticated revenue-based penalty regime achieve **greater** deterrence than a standard revenue-based penalty regime in industries with above average monopoly overcharges. So they achieve greater deterrence where it matters most. Thus, by explicitly recognizing that penalties have to be applied across a range of different industries (in terms of the overcharge), we are here making a different and more powerful claim for the deterrence properties of the overcharge-based regime than in our previous paper, which explains why we have to include a systematic examination of its properties in this section and cannot simply refer back to the analysis of this regime that we undertook in our previous paper.

3.1 Model Setup

The model is the repeated game model of cartel formation and pricing behaviour employed in Katsoulacos et al. (2015). We consider an economy comprising a range of types of industry, in each of which there is a homogeneous product produced by a number of firms. Firms have the same constant unit costs of production. For a typical industry let $c > 0$ denote the common unit costs of production and let demand be given by the downward-sloping demand function $Q(p)$ with associated elasticity $\eta(p) = -\dfrac{pQ'(p)}{Q(p)} > 0$. We assume that for all types of industry:

$$\eta(p) \text{ is non-decreasing in } p \text{ and } \exists \tilde{p} \geq 0 \text{ s.t. } \eta(p) > 1 \; \forall p > \tilde{p}. \qquad (1)$$

In what follows an *industry type* is characterised by $c, Q(.)$ $\left(\text{and hence } \eta(.)\right)$.

We assume that the form of competition in each industry is Bertrand competition. So the "but-for" price, output, revenue and profit - denoted respectively by p^B, Q^B, R^B and π^B - are given by: $p^B = c, Q^B = Q(c), R^B = cQ(c)$ and $\pi^B = 0$. Also, for a cartel to be able to effectively raise price above the "but-for" level all firms in an industry have to join the cartel.

If a cartel forms and sets a price $p > c$ then the percentage overcharge is $\theta = \left(p - c\right)/c$. So the price is given by $p = c(1+\theta)$. For any given overcharge set by a cartel the associated industry operating profits and revenue will be:

$$\pi(\theta) = c\theta Q\big(c(1+\theta)\big) \text{ and } R(\theta) = c(1+\theta)Q\big(c(1+\theta)\big). \quad (2)$$

There is a Competition Authority (CA) that investigates, discovers, prosecutes and penalises cartels. As explained above, we focus on the following three penalty regimes.

(a) *Revenue-Based Penalty Regime*, R. Here the penalty base is cartel revenue and there is penalty rate, $\rho^R > 0$ applied to that base, where in the interests of transparency this penalty rate is the same across all industries. So the financial penalty imposed under this regime will be

$$F_R(\theta) = \rho_R c(1+\theta)Q\big(c(1+\theta)\big). \quad (3)$$

(b) *Sophisticated Revenue-Based Penalty Regime*, SR. Here the penalty base is once again cartel revenue, but now the penalty rate applied to that base is a non-decreasing function of the cartel overcharge $\rho_{SR}(\theta) > 0$, where, once again, in the interests of transparency/consistency the function $\rho_{SR}(\theta)$ is the same across all industries. So the financial penalty imposed under this regime will be:

$$F_{SR}(\theta) = \rho_{SR}(\theta)c(1+\theta)Q\big(c(1+\theta)\big). \quad (4)$$

(c) *Overcharge-Based Penalty Regime,* O. Here, as defined in Katsoulacos et al. (2015), the penalty base is the overcharge multiplied by counterfactual revenue, and there is a penalty rate $\rho_O > 0$ applied to that base, which, once again, is the same across all industries. So the financial penalty imposed under this regime will be:

$$F_O(\theta) = \rho_O \theta c Q\big(c\big). \quad (5)$$

Notice that under all regimes the penalty paid will vary with the cartel overcharge, which can either be because of the design of the regime or because of the way in which the penalty base varies with the overcharge – a relationship that will vary across industry types.

Let β, $0 \le \beta < 1$ denote the probability that in each period a cartel is detected, successfully prosecuted and penalised according to one of penalty schedules specified above. We recognise that there could be a variety of channels through which this probability might increase with the cartel overcharge. However, the precise relationship between β and θ is likely to vary from case to case[16] in a way which neither economists nor CAs can predict in advance, and, since both the absolute and relative welfare properties of different penalty regimes could vary depending on this precise functional relationship, this makes it difficult to produce recommendations about the properties and relative merits of different penalty

[16] For example, this can be related to individual firms' perceptions about how increase in prices may increase the suspicion of competition authorities and, eventually, also the likelihood of investigation and conviction.

regimes. So we follow the existing literature on both the type and level of penalties and assume that β is independent of θ, and, moreover, its value is common knowledge. In addition, as in KMU (2015), we assume that $\beta\rho_r < 1$, for $r = R, O$. The properties of the function $\rho_{SR}(\theta)$ will be explored systematically in the next section.

As in Katsoulacos et al. (2015) - and also, for example, Motta and Polo (2003) and Chen and Rey (2013) - we continue to assume the cartel re-establishes following a successful prosecution.[17] Given this and our other assumptions, it follows that the expected present value of profits for a single firm that is a member of a cartel in a given industry that has set an overcharge θ and faces the penalty regime $r \in \{R, SR, O\}$ is given by

$$V_r(\theta) = \frac{\pi(\theta) - \beta F_r(\theta)}{n(1-\delta)} \tag{6}$$

where, $n \geq 2$ is the number of firms in the industry and $\delta, 0 < \delta < 1$ is the discount factor. As in Katsoulacos et al. (2015), $\Delta \equiv n(1-\delta)$ denotes the *intrinsic difficulty* of holding the cartel together. For any given *industry type* $\{c, Q(.)\}$ there is continuum of possible industries $\{c, Q(.), \Delta\}$, where Δ is uniformly distributed on [0,1].

Following standard grim-trigger strategy profile firms collude on cartel overcharge, θ, in the first period and continue setting θ as long as no firm defects. If a firm defects from the cartel it sets an overcharge below the cartel overcharge, and, for a single period takes the entire industry profits Any deviation by any firm leads to competition at price *c*, for ever more. We also assume that defecting firm is immune from future prosecution by the CA.[18] Then since the overcharge set by a cartel could be above the monopoly overcharge,

[17] In related work we have assumed that collusive activity can re-emerge following successful prosecution. This produces more complex formulae for *V*(.) but does not affect the main qualitative results of the paper, so we stick with the simpler assumption. More specifically, one can assume that, after detection, there is a constant probability γ, $0 \leq \gamma \leq 1$ that the cartel will continue in existence after detection. In this case one simply replaces the term $\Delta = n(1-\delta)$ that appears in our analysis with the more general expression $\Delta_\gamma = n(1-\delta)\left[1 + \frac{\beta\delta(1-\gamma)}{1-\delta}\right]$. Note that with this generalization we can perform similar analysis but with more general expression for maximum critical level of difficulty of holding the cartel together. Then the unconstrained cartel overcharges under different penalty regimes will not be affected by this change, while the maximum critical difficulty will go down under all relevant regimes, so that the relative performance of various penalty regimes will be unaffected. So, for the issues with which we are dealing, nothing of substance is affected by this more general set up.

[18] Note the opposite assumption would not affect the main qualitative results of the paper. Allowing for the possibility of prosecuting price-deviating firms does not affect the collusive value in (6). Hence, the unconstrained cartel overcharges under different penalty regimes will not be affected by this change. So the main results about welfare advantages of the sophisticated revenue based regime compared to simple revenue-based structure will not change. On the other hand, the cartel stability condition in (8) will be relaxed, and the degree of deterrence in section 3.3 will be affected but in the same direction for all the penalty regimes.

$\theta^M = \arg\max \pi(\theta)$ a defecting firm trying to make the maximum profits in the single period will set the monopoly overcharge whenever the cartel overcharge is above the monopoly overcharge, but will set an overcharge just a fraction below the cartel overcharge whenever this is at or below the monopoly overcharge, thereby capturing the entire cartel profits. So defection profits are

$$\pi^d(\theta) = \begin{cases} \pi(\theta^M), & \theta > \theta^M \\ \pi(\theta), & \theta \le \theta^M \end{cases} . \tag{7}$$

For a cartel to be stable it has to satisfy the cartel stability condition:

$$V_r(\theta) \ge \pi^d(\theta). \tag{8}$$

Although we recognise that, by the Folk Theorem, a range of possible cartel overcharges could potentially be equilibria, we follow the existing literature on the design of cartel policy and assume that the overcharge set by a cartel facing penalty regime, *r,* is that which maximises $V_r(\theta)$ subject to $\theta \ge 0$ and the stability condition (8). We denote this by θ_r^C. There are two cases to consider. If the stability condition does not bite then:

$$\theta_r^C = \hat{\theta}_r^C = \arg\max \left[\pi(\theta) - \beta F_r(\theta) \right] \tag{9}$$

and is independent of Δ (though it depends on the *industry type*). On the other hand if the stability condition bites then θ_r^C is the solution to

$$\pi(\theta) = \beta F_r(\theta) + \Delta \pi^d(\theta), \tag{10}$$

and so is a function of Δ (as well as the *industry type*).

Finally, we let $\bar{\Delta}_r$ be the maximum critical value of Δ such that, under penalty regime *r*, either the stability condition or the non-negative overcharge constraint bites[19]. Clearly if there were no Competition Authority - and so $\beta = 0$ - then a cartel would always set the monopoly overcharge and the maximum critical value of Δ would be 1. Whereas, once there is an active competition authority enforcing penalties on non-defecting cartel members we must have $\bar{\Delta}_r < 1$.[20] So we can define the *degree of deterrence* achieved by penalty regime *r*, D_r, as the fraction of industries in which cartels would have formed in the

[19] The maximum critical level of difficulty, Δ, is the direct analogue of the minimum critical discount rate used in much of the literature.

[20] This is why we have confined attention to values of $\Delta \le 1$. The assumption of a uniform distribution just makes it easier to translate statements about $\bar{\Delta}_r$ into statements about proportion of industries where cartels are deterred.

absence of a Competition Authority in which they do not form given the presence of a Competition Authority operating penalty regime *r*. Formally:

$$D_r = 1 - \overline{\Delta}_r \ . \qquad (11)$$

Having set out the framework, we now investigate how both the cartel overcharge and the degree of deterrence vary depending on which of the three penalty regimes set out above is employed by the Competition Authority. Our focus will be on showing that the sophisticated revenue-based penalty regime outperforms the widely used standard revenue-based penalty regime in terms of both price and deterrence. As such it has similar welfare properties to an overcharge-based regime.

3.2 Cartel Pricing

As discussed above there are potentially two types of solution – those where the stability constraint (8) does not bite (i.e. unconstrained pricing solutions) and those in which it does (i.e. constrained pricing).

3.2.1 Unconstrained Pricing Solutions

We start by re-stating a result established in Katsoulacos et al. (2015)[21] namely that, in every type of industry the overcharge set by a cartel under a simple revenue-based penalty regime, R is above the monopoly overcharge; while that under an overcharge-based regime, O, is below the monopoly overcharge. Formally we have:

Proposition 1: *For every type of industry,* $\hat{\theta}_R^C > \theta^M > \hat{\theta}_O^C$.

Proof: See Appendix 2

The intuition is as follows. If there were no Competition Authority the cartel would set the monopoly overcharge, which is characterised by marginal revenue equal positive marginal cost. When there is a Competition Authority then net profits are the operating profits minus expected financial penalty, so under any penalty regime cartels will set the overcharge taking into account how it affects both operating profits $\pi(\theta)$ and the financial penalty they will incur if prosecuted, $F_r(\theta)$. Now, under a simple revenue-based penalty regime the cartel can reduce the penalty base by cutting output and hence raising the overcharge, so the cartel overcharge will be above the monopoly overcharge; whereas under

[21] One reason for repeating the proposition here is that we offer a new method of proof which we exploit in the in our analysis of the pricing properties of a Sophisticated Revenue-Based regime in Proposition 2 below.

an overcharge-based regime the penalty is an increasing function of the overcharge so the cartel will have an incentive to set an overcharge below the monopoly overcharge.

We turn now to a *sophisticated* revenue-based penalty regime under which the penalty rate applied to revenue varies with the overcharge according to a strictly increasing function $\rho_{SR}(\theta)$ which, in the interests of legal certainty (equality of treatment under the law), is the same across all industries. We then have:

Proposition 2: *If the penalty rate function under a sophisticated revenue-based penalty regime satisfies the condition:*

$$\frac{\rho'_{SR}(\theta)}{\rho_{SR}(\theta)} > \frac{1}{\theta(1+\theta)} \quad \forall \theta \geq 0 \tag{12}$$

then in every type of industry $\hat{\theta}^{C}_{SR} < \theta^{M}$.

Proof: See Appendix 2

The intuition is straightforward. For the reasons given above, under any regime in which cartel revenue forms the base of the penalty there will be powerful incentives to raise the overcharge in order to reduce the base. This can be countered if the penalty rate imposed on the base rises sufficiently fast with the overcharge. Proposition 2 makes precise just how fast the penalty rate has to rise to ensure that the cartel overcharge is below the monopoly overcharge. There are two related points to notice.

First, from (5) and (2) we can express the financial penalty under an overcharge-based regime as a fraction of revenue so we can think of an overcharge-based regime as a form of sophisticated revenue-based penalty regime in which the penalty-rate function takes the form:

$$\rho_{SRO}(\theta) \equiv \frac{\rho_O \theta Q(c)}{(1+\theta)Q\big(c(1+\theta)\big)}, \tag{13}$$

which implies

$$\frac{\rho'_{SRO}(\theta)}{\rho_{SRO}(\theta)} = \frac{1}{\theta(1+\theta)} + \frac{\eta\big(c(1+\theta)\big)}{1+\theta} > \frac{1}{\theta(1+\theta)}. \tag{14}$$

This provides an alternative as to why an overcharge-based penalty regime "works" – i.e. generates cartel overcharges below the monopoly overcharge – because it is effectively equivalent to a sophisticated revenue-based penalty regime that satisfies (12) and so for which Proposition 2 applies. However the penalty rate function defined by (13) does not satisfy our other stipulation that it should not depend on industry characteristics.

Second, precisely because the right hand side of (12) does not depend on industry characteristics we can always pick penalty rate functions that are free from industry characteristics and yet, if they satisfy (12) we can be confident that they have the desirable

property of driving the cartel overcharge below the monopoly overcharge in every industry. Obviously, infinitely many functions can satisfy condition (12), but in the interests of our other criteria of implementability and legal certainty (equality of treatment before the law) we choose the simplest possible functional form which is the linear function

$$\rho_{SR}(\theta) = \tilde{\rho}_{SR}\theta, \quad \tilde{\rho}_{SR} > 0, \tag{15}$$

which one can easily verify satisfies (12). Consequently, in what follows we will confine attention to the class of *Linear Sophisticated* Revenue-Based Penalty Regimes in which the penalty-rate function is given by (15). Consistent with the assumption made above for the Revenue-Based penalty regime we assume that $\beta\tilde{\rho}_{SR} < 1$.

3.2.2 Constrained Pricing Solutions

By substituting in turn (3)-(5) into (8) we obtain the cartel stability condition under all three penalty regimes, and can consider to what extent this constrains the cartel overcharge.

Under a standard Revenue-Based Penalty Regime a cartel sets a price above the monopoly price in which case the defection profits are just the monopoly profits. But since these are independent of the cartel overcharge, the stability condition does not constrain the cartel overcharge which should then be set so as to maximise $V_R(\theta)$ and so the cartel overcharge will be $\hat{\theta}_R^C$. Consequently the cartel stability condition is purely a constraint on Δ which takes the form:

$$\Delta \leq \frac{Y(\beta\rho_R)}{Y(0)} < 1 \ , \tag{16}$$

where $Y(z) \equiv \underset{\theta}{MAX}\, \pi(\theta) - zR(\theta)$ and, by the Envelope Theorem is a strictly decreasing function of z – and so explains the final inequality in (16).

Under a *Linear Sophisticated* Revenue-Based Penalty Regime the cartel price is below the monopoly price and so (8) becomes:

$$V(\theta) = \frac{c\theta Q\big(c(1+\theta)\big) - \beta\tilde{\rho}_{SR}\theta c(1+\theta)Q\big(c(1+\theta)\big)}{\Delta} \geq c\theta Q\big(c(1+\theta)\big) = \pi^d(\theta).$$

which implies

$$\theta \leq \frac{(1-\beta\tilde{\rho}_{SR}) - \Delta}{\beta\tilde{\rho}_{SR}}. \tag{17}$$

This upper bound on θ is a linear decreasing function of Δ taking the value zero when $\Delta = 1-\beta\tilde{\rho}_{SR} < 1$. So there are certainly values of $\Delta \approx 1-\beta\tilde{\rho}_{SR}$ for which the upper bound in (17) lies below the unconstrained overcharge $\hat{\theta}_{SR}^C$ and so the cartel stability condition bites

and constrains the overcharge that the cartel can set. But then the overall cartel overcharge under a *Linear Sophisticated* Revenue-Based penalty regime is:

$$\theta_{SR}^{C} = MIN\left[\hat{\theta}_{SR}^{C}, \frac{(1-\beta\tilde{\rho}_{SR})-\Delta}{\beta\tilde{\rho}_{SR}}\right], \quad 0 \le \Delta \le 1-\beta\tilde{\rho}_{SR} < 1. \tag{18}$$

Finally, it is straightforward to show that under an Overcharge-Based Penalty Regime the cartel stability condition is

$$\frac{Q\big(c(1+\theta)\big)}{Q(c)} \ge \frac{\beta\rho_O}{1-\Delta}. \tag{19}$$

Since the left hand side of this expression is less than 1 for any positive value of θ while the right hand side takes the value 1 when $\Delta = 1-\beta\rho_O$ it follows that there will be a range of values of Δ for which the stability condition bites and constrains the cartel overcharge. So under this penalty regime the cartel overcharge is

$$\theta_O^C = MIN\left[\hat{\theta}_O^C, \psi_O(\Delta)\right] \tag{20}$$

where $\psi_O(\Delta)$ is the constrained overcharge defined implicitly by the equation

$$\frac{Q\left[c\left(1+\psi_O(\Delta)\right)\right]}{Q(c)} \equiv \frac{\beta\rho_O}{1-\Delta} \tag{21}$$

and is a strictly decreasing function of Δ which tends to zero as $\Delta \to 1-\beta\rho_O$. At this level of generality it is impossible to say more about the precise shape of this function.

So the cartel stability condition constrains the overcharge that cartels set for both the *Linear Sophisticated* Penalty Regime and the Overcharge-Based Penalty Regime. However at this level of generality it is impossible to determine whether the price – either unconstrained or constrained - is higher under one regime than the other.[22] So in the left panel of Figure 2 below we show the cartel overcharge under a *Linear Sophisticated* Revenue-Based Penalty Regime (solid line) in comparison with that under a Revenue-Based Penalty Regime (dashed line). While the right panel of Figure 2 illustrates the same comparison for the case of an Overcharge-Based Penalty Regime (dotted line).

[22]Numerical analysis shows that under standard linear demand both outcomes are possible. The comparison depends on the shape and the structure of the demand function and on the parameters of the penalty functions.

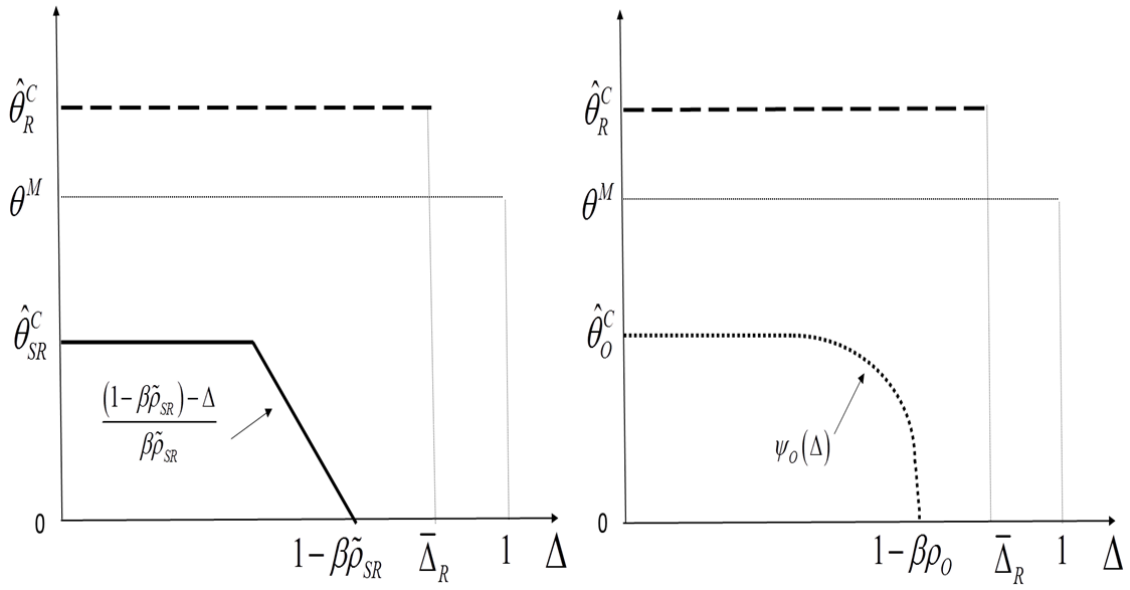

Figure 2: Comparison of Cartel Overcharge Under *Linear Sophisticated* Revenue-Based Penalty Regime (solid line), Cartel Overcharge Under standard Revenue-Based Regime (dashed line) and Cartel Overcharge Under Overcharge-Based Penalty Regime (dotted line)

3.3 Deterrence

From the analysis in the previous sub-section we see that the maximum critical level of difficulty of holding a cartel together in the various regimes, $\bar{\Delta}_r$, is determined as pure upper bound constraint on Δ in the case of a standard Revenue-Based Regime. While for both the *Linear Sophisticated* Revenue-Based Regime and the Overcharge-Based Regime it is the value at which the constrained overcharge is driven to zero.

From (16), (18) and (21) we immediately have:

$$(i)\ \bar{\Delta}_R = \frac{Y(\beta\rho_R)}{Y(0)} < 1; \quad (ii)\bar{\Delta}_{SR} = 1-\beta\tilde{\rho}_{SR} < 1; \quad (iii)\bar{\Delta}_O = 1-\beta\rho_O < 1. \tag{22}$$

Consequently from (11) the degree of deterrence achieved by each of the penalty regimes is:

$$(i)\ D_R = 1-\frac{Y(\beta\rho_R)}{Y(0)}; \quad (ii)D_{SR} = \beta\tilde{\rho}_{SR}; \quad (iii)\, D_O = \beta\rho_O. \tag{23}$$

Analysis of the degrees of deterrence in (23) gives rise to the following proposition:

Proposition 3

(i) *The degree of deterrence achieved by a Linear Sophisticated Revenue-Based Penalty Regime is the same across all industry types and is equal to the toughness of the regime – i.e. the probability of successful prosecution multiplied by* $\tilde{\rho}_{SR}$ *the constant rate at which the penalty rate is increased with the overcharge*;

(ii) *Similarly the degree of deterrence achieved by an Overcharge-Based Regime is also the same across all types of industry and is equal to the toughness of this regime – i.e. the probability of successful prosecution multiplied by the penalty rate.*

(iii) *Indeed if* $\rho_O = \tilde{\rho}_{SR}$ *then the common degree of deterrence achieved by each of these regimes will be the same*.

(iv) *However the degree of deterrence produced by a Revenue-Based Regime will vary across different types of industry.*

Corollary *The degree of deterrence achieved under a Revenue-Based Regime varies across industries in a way that is inversely related to the monopoly overcharge in each industry.*

Proof: See Appendix 2

So, in addition to its poor pricing properties, another disadvantage of the conventional Revenue-Based penalty regime is that, even though the same penalty rate applies across all industries, it creates variable deterrence across industries and indeed deters least heavily in those industries where the monopoly overcharge is greatest.

Since both the *Linear Sophisticated* Revenue-Based regime and the Overcharge-Based Regime achieve the same degree of deterrence across all industries we cannot get exact deterrence equivalence industry by industry between each of these regimes and the Revenue-Based regime. Instead all we can get is a comparison of the deterrence obtained by each of these regimes with that obtained "on average" under a standard *Simple* Revenue-Based penalty regime – more precisely with the deterrence achieved by such a regime in the industry with the average monopoly overcharge. That is suppose that under a *Linear Sophisticated* Revenue-Based regime we choose the constant rate at which, the penalty rate will increase with the overcharge, $\tilde{\rho}_{SR}$ such that

$$\tilde{\rho}_{SR} = \rho_O = \rho_R \left(\frac{1+\bar{\theta}^M}{\bar{\theta}^M} \right) > \rho_R . \qquad (24)$$

Then we have:

Proposition 4: *In comparison with the standard revenue-based penalty regime, both a* ***linear*** *sophisticated revenue-based and an overcharge-based regime can achieve the same degree of deterrence for industries with the average monopoly overcharge and* ***higher*** *degrees of deterrence in those industries with above-average monopoly overcharges.*

So, in terms of the degree of deterrence, both the Linear Sophisticated Revenue-Based and the Overcharge-Based regime work better where it matters most.

To see the potential implications of this for the value of $\tilde{\rho}_{SR}$ implied by (24) and for the levels of penalties to which it gives rise, we first need to determine some estimates of the average monopoly overcharge, $\overline{\theta}^{M}$. We start from studies of the average cartel overcharge, $\overline{\theta}^{C}$. A meta-analysis in Connor and Bolotova (2006) suggests a value of $\overline{\theta}^{C} = 0.31$. On the other hand, a more recent study by Boyer and Kotchoni (2015), which corrects for various biases in Connor and Bolotova (2006), gives figures of 13.6% and 17.5% depending on the sample used. So we set a High estimate of $\overline{\theta}_{H}^{C} = 0.3$ and a Low estimate of $\overline{\theta}_{L}^{C} = 0.15$. If these are cartel overcharges emerging under widely used *simple* revenue-based regimes then, from Proposition 1, the average monopoly overcharge will be lower.

So let us assume that associated High and Low estimates of this are, respectively $\overline{\theta}_{H}^{M} = 0.25$ and $\overline{\theta}_{L}^{M} = 0.125$. The typical penalty rate used in *simple* revenue-based penalty regimes is $\rho_{R} = 0.1$. So plugging these figures into (24) the associated figures for $\tilde{\rho}_{SR}$ would be 0.5 and 0.9 respectively.

If we calculate the actual penalty rate that would be charged on cartels setting what one took to be the average cartel overcharge, then, if one thought that the average overcharge was $\overline{\theta}_{H}^{C} = 0.3$ the penalty rate that would be applied to the cartel's revenue under a sophisticated revenue-based penalty regime would be 15%, whereas if one thought that the average overcharge was $\overline{\theta}_{L}^{C} = 0.15$ then, under a sophisticated revenue-based penalty regime, any cartel setting such an average overcharge would face a penalty equal to 13.5% of its revenue.

Given the linear nature of our proposed sophisticated revenue-based penalty regime the penalty rates that would be applied to cartels setting overcharges that were a factor f of the average cartel overcharge – i.e. for which $\theta = f\overline{\theta}^{C}$ would be just 0.15f and 0.135f respectively. So the penalties applied to cartels setting overcharges that were 3 or 4 times the average would be facing penalties of around 50% - the sort of figure proposed in a number of papers by Connor and Lande – e.g. Connor and Lande (2008).

In summary, the main conclusion from these calculations is that the precise penalties that would be imposed under the linear sophisticated revenue-based penalty regime that we propose are not very sensitive to the underlying estimate of the monopoly overcharge that is assumed. However penalties do rise quite sharply with the overcharge that is actually set by cartels.

4. Conclusions

We conclude that sophisticated revenue-based penalties in which the penalty rate that is applied to revenue rises linearly with the level of overcharge, according to a pre-announced formula, will welfare-dominate the currently widely used simple revenue-based penalties in terms of both the prices that they induce cartels to set and the levels of deterrence achieved. Moreover, as discussed above, they are relatively easy to implement and do not give rise to any significant transparency/uncertainty concerns: specifically, implementability and transparency concerns do *not* seems so serious as to outweigh the welfare advantages of the sophisticated revenue-based penalties relative to the simple revenue-based penalties. Thus, we recommend that it is the former penalties that should be used by CAs.

Appendix: Proofs of Propositions

Proof of Proposition 1: It is a standard result that the monopoly overcharge is the solution to the equation

$$\eta\left[c\left(1+\theta\right)\right]=\frac{1+\theta}{\theta} \tag{A.1}$$

Insert (3), and (5) from the text into the maximand in (9), differentiate, set the derivative to zero and re-arrange and we find that: $\hat{\theta}_R^C$, $\hat{\theta}_O^C$ are, respectively, solutions to the equations:

$$\eta\left[c\left(1+\theta\right)\right]=\left[\frac{1-\beta\rho_R}{\frac{\theta}{1+\theta}-\beta\rho_R}\right]\equiv\varphi_R(\theta) \tag{A.2}$$

and

$$\eta\left[c\left(1+\theta\right)\right]=\frac{1+\theta}{\theta}\left[1-\frac{\beta\rho_O Q(c)}{Q\left(c(1+\theta)\right)}\right]\equiv\varphi_O(\theta) \tag{A.3}$$

It is readily verified that in each case the expression on the RHS of the equation is a decreasing function of θ[23], while, from (1) in the text, the common term on the LHS is a strictly increasing function of θ. Moreover since clearly:

$$\left[\frac{1-\beta\rho_R}{\frac{\theta}{1+\theta}-\beta\rho_R}\right]>\frac{1+\theta}{\theta}>\frac{1+\theta}{\theta}\left[1-\frac{\beta\rho_O Q(c)}{Q\left(c(1+\theta)\right)}\right], \tag{A.4}$$

the proposition is established. Figure 3 below illustrates the proof.

[23] Indeed notice that $\varphi_O(\theta)=0$ when $Q\left(c(1+\theta)\right)=\beta\rho_O Q(c)$

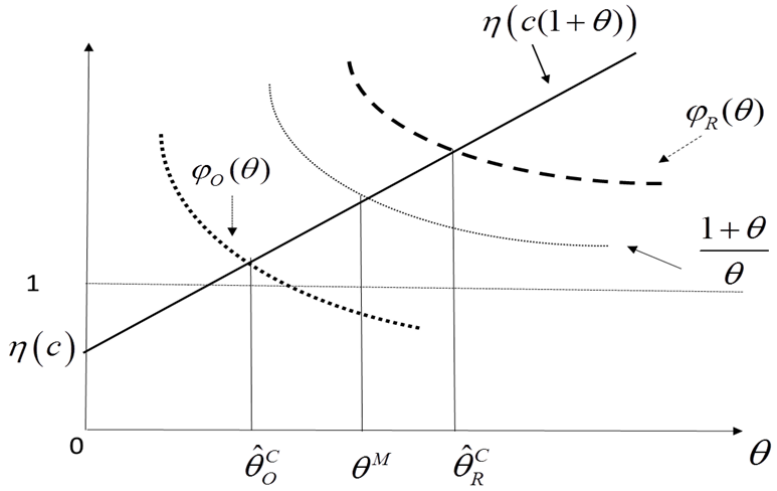

Figure 3: Unconstrained Cartel Overcharges for simple Revenue-based penalty regime and for Overcharge-based regime

Proof of Proposition 2: Insert (4) into the maximand in (9), differentiate, set the derivative to zero and re-arrange and we find that: $\hat{\theta}_{R1}^C$ is the solution to the equation:

$$\eta\left(c(1+\theta)\right) = \frac{1-\beta\rho(\theta)-\beta(1+\theta)\rho'_{SR}(\theta)}{\frac{\theta}{1+\theta}-\beta\rho_{SR}(\theta)} \equiv \varphi_{SR}(\theta) \qquad \text{(A.5)}$$

But then, if (12) holds, $\varphi_{SR}(\theta) < \dfrac{1-\beta\rho_{SR}(\theta)\left(1+\frac{1}{\theta}\right)}{\frac{\theta}{(1+\theta)}-\beta\rho_{SR}(\theta)} = \dfrac{1+\theta}{\theta}$ and the Proposition is proved.

Figure 4 illustrates the proof.

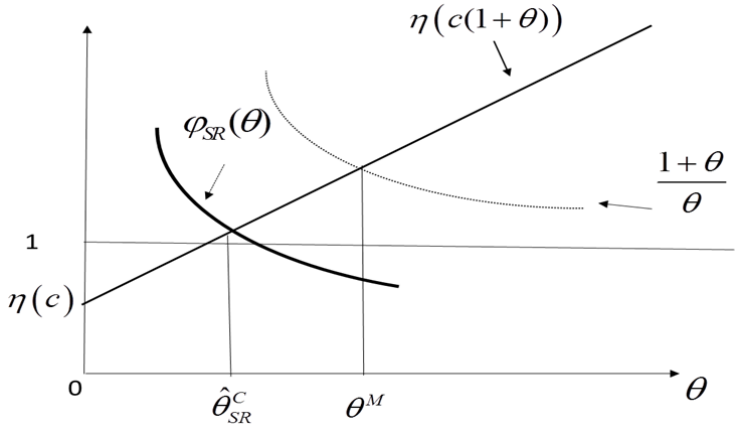

Figure 4: Unconstrained Cartel Overcharge for Sophisticated Revenue-Based Penalty

Proof of Corollary to Proposition 3: Take a first-order Taylor approximation to $Y\left(\beta\rho_{R0}\right)$ around 0 and, bearing in mind that (i) by the Envelope Theorem $Y'(z)=-R\left(\hat{\theta}(z)\right)$ where $\hat{\theta}(z)$ is the overcharge that maximises $Y\left(z\right)$; (ii) when $z=0$ $\hat{\theta}(0)=\theta^M$ we have $Y\left(\beta\rho_R\right)\approx Y(0)-\beta\rho_R R(\theta^M)=c\theta^M Q\left(c(1+\theta^M)\right)-\beta\rho_R c\left(1+\theta^M\right)Q\left(c(1+\theta^M)\right)$.

So $D_R=1-\dfrac{Y\left(\beta\rho_R\right)}{Y(0)}=1-\left\{1-\beta\rho_R\left[\dfrac{c(1+\theta^M)Q\left[c\left(1+\theta^M\right)\right]}{c\theta^M Q\left[c\left(1+\theta^M\right)\right]}\right]\right\}=\beta\rho_R\dfrac{(1+\theta^M)}{\theta^M}$,

which proves the result.

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