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Redistributive Consequences of Abolishing Uniform Contribution Policies in Pension Funds

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Abstract

In a pension system with uniform policies for contribution and accrual, each participant has the same contribution rate and accrual rate independent of the age at the time of payment. This is not actuarially fair because the investment horizon of young participants is longer than the investment horizon of the elderly. This paper shows the presumably unintended redistributive effects of a uniform contribution system and the consequences of switching from uniform policies to an actuarially fair system. We first analyze a stylized model with three overlapping generations to show the intuition behind these effects. Then, we quantify these effects in a more detailed model with multiple overlapping generations, realistic parameters and more detailed information on the income distribution, calibrated on the Dutch funded pension system. We first use this model to show that there is a substantial transfer of income from poor to wealthy participants under a pension scheme with uniform policies: about 10 billion euros are transferred from poor to wealthy participants under the current uniform contribution policies in the Netherlands. We then calculate the gross aggregate transition effect of abolishing the uniform policy pension for an actuarially fair system to be about 37 billion euros (or about 5% of the Dutch GDP).[1] For each cohort, the redistributive effects are less than 5% of their total pension.

Keywords: uniform policies, pension funds, transition, income inequality
JEL Codes: G23, J32

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[1]Lever et al. (2017) estimates the transition effect at 55 billion. In Section 4.4 of this paper, we discuss the four main drivers of this difference. When applying comparable parameter and model assumptions, we find a similar number.

1 Introduction

A uniform policy pension system (UPPS) consists of uniform contribution rates and uniform accrual rates, which are equal for all participants without taking into account the participant's age at the time of payment. This typically applies to DB pension schemes, such as the public sector pension plans in Australia, Canada, Germany, the Netherlands, Norway and Switzerland, the Universities Superannuation Scheme in the U.K. and the sub-national civil servants plans in the U.S. (Chen et al., 2017; OECD, 2001; Ponds et al., 2011; Westerhout et al., 2014). Under the UPPS, young people pay the same (undiscounted) as older people while getting the same pension entitlements, despite the fact that contributions of young people can render much longer, which implies higher expected cumulative investment returns. Hence, in a normal market with positive interest rates, an actuarially fair approach should for equal contributions provide a higher level of pension accrual to young people than to older people. Or equivalently, young people should pay a lower contribution than older people for the same pension rights accrual under an actuarially fair approach. Therefore, younger workers subsidize older generations under the UPPS. When young generations are old themselves, they assume that they will be subsidized by future young generations in the same manner. So a UPPS introduces a PAYG-element within the funded pension scheme, with young people paying for the elderly.

The prospect of receiving subsidies in the future to compensate the already paid subsidies is an implicit debt to the current young generations, which is rolled over to the future generations. Possibly the UPPS system was introduced to allow older people to accrue more pension rights during the early period of the funded pension scheme (second pillar). Whether intentionally or not, the first generation of older workers has gained too much and the implicit debt can be considered as the rolled-over funding gap of this initial payment to that first generation (the "first generation" problem).

Under UPPS, a young person at the beginning of his working life begins without a claim on future generations, because he/she has not paid anything yet, so has not paid too much either. However, as the participant becomes older, he/she slowly builds up that claim up to a turning point. After that, that total claim slowly declines again as the now older participant receives a subsidy from the new young working cohorts, until all claims are redeemed when the retirement age has been reached. So if that claim eventually gets extinguished, why is the UPPS nevertheless considered a problem?

Why is the UPPS a problem? First of all uniform policies are problematic because of the undesired redistribution effects that it triggers in practice. One concerns income redistribution, because income inequality between young people is smaller than between older people.[2] Highly educated young people can expect a steeper wage profile over their life than low-skilled young people. Hence, highly educated people can expect to benefit more from the subsidy when old than the less educated under the UPPS. Since the contribution rate is determined in such a way that total contributions equal the total discounted value of accrued pension rights, there is redistribution of income from the relatively poor to the relatively wealthy. Moreover, due to gender differences in life expectancy, there are transfers under the UPPS between men and women. These presumably unintended transfers between groups of participants make the pension system vulnerable (Boeijen et al., 2006).

A problem that has become more relevant recently manifests itself during labor market transitions if those transitions are from inside the coverage of the pension system to outside of it. This is in particular an important issue when transitioning from a regular labor contract incorporating pension fund contributions towards a status as self employed outside the funded pension system. Since these transitions are happening on a large scale mid career, at about the point where the contributor switches from an overpayer to an underpayer/beneficiary of the UPPS, they actually lose all the future subsidies they would have obtained from the future young generations if they would have stayed inside the system. In a way the PAYG chain is broken when transitioning into self employed status.

The same problem, albeit to a lesser extent, occurs when the working career is interrupted by periods of unemployment. Unemployment is typically followed by lower wages after reintegration, which, empirical evidence overwhelmingly shows, are not be recovered in later years. Hence, redistribution also occurs between more and less successful employees. These labor market related problems have become much more relevant due to the greatly increased labor mobility in recent years.

A final problem with the UPPS occurs when structural pension reforms are being envisaged, as is now (2017) the case in the Netherlands. One of the options is a transition to a defined contribution (DC) system, in which intergenerational contracts such as those underlying the UPPS have no place: with a pure individual or cohort specific DC system, there is no intergenerational risk-sharing at all. The question then arises how to deal with the outstanding claims of current workers on future young people.

So it is clear that the UPPS is (i) problematic in the current labor market, (ii) complicates pension reforms and (iii) causes undesirable income redistribution effects from poor to wealthy people. Hence, it should be no surprise that there is a broad support for abolishing the UPPS in the Netherlands by switching to an actuarially fair system.

Modeling the cohort specific losses of abolishing the UPPS By abolishing the UPPS and switching to an actuarially fair system, there are two most

[2]See Bonenkamp (2007) and Lever et al. (2013) for early studies on the relationship between socioeconomic status and redistribution effects in the second pillar.

likely new pension contribution systems, which are equivalent in terms of market value: (i) one with progressive contribution rates (ascending with age) and uniform accrual rates, or (ii) a system of degressive accrual rates (descending with age) and uniform contribution rates. In our modeling we will assume that increasing contribution rates / constant accrual rates will be adopted after abolishing the UPPS. However, simply closing the UPPS when switching to such a new system implies that any outstanding claims are no longer rolled over from generation to generation. Under the UPPS young people subsidize the elderly, expecting to be compensated once they are older by the then young, thereby creating an implicit debt, which is rolled over year-on-year;[3] But abolishing the UPPS just like that (without compensation to current generations in their working life), implies that those current working generations actually have to bear the full burden of this implicit debt, as they are the last in the chain. After all, they have subsidized the old in their past working years, but have not yet received all those subsidies back in the second part of their working life. Only those who have already retired at the time of the switch have no outstanding claim left since they have completed the cycle. Hence, abolishing the UPPS is a redistribution between current and future working generations. The latter benefit, because they no longer have to pay for the implicit debt. This way, future generations need to pay a lower contribution rate in order to obtain the same level of pension benefits, which we refer to as the "contribution reduction".

The question arises who is being charged for this implicit debt. There are two extreme scenarios. The current system implies that the debt is rolled over forever. The other extreme is equivalent to a *debt default*, where the entire loss is given to the current working generations. Ultimately, the allocation of the implicit debt to present and future generations is a political question. It is even possible to partly charge current retirees, even though they have already paid and received subsidies under the UPPS. However, it is easier to justify to move the loss (for a larger part) to unborn and young people who have not yet begun their working life, as these cohorts benefit from abolishing the UPPS due to the "contribution reduction". As the implicit debt is created by giving up the investment returns of young people to the elderly, it is clear that the level of claims depends on those investment returns.

Backward looking or forward looking For determining the transition effects, we can apply a *backward looking* or *forward looking* methodology. The *backward looking* methodology considers individually or group-specifically what investment return would have been achieved in the past assuming there would not have been a UPPS. This approach is an administratively complicated exercise. *Forward looking* implies that we theoretically determine how much implied debt is built

[3]Note that this rolling over of implicit debt actually is *not* a Ponzi game. Since the debt is rolled over without accruing interest, its value goes to zero in discounted value terms in the long run. Hence, rolling over the implicit debt does not imply a Ponzi-scheme (Sinn, 2000).

up by different cohorts given assumptions about market conditions, demographics and the structure of the pension scheme. In case the backward looking approach is chosen, one should correct for the pension reforms and changes in regulation in the past as well. Moreover, one can wonder why compensation for abolishing the UPPS is relevant, provided that changes in pension systems are regularly made without compensation for loss of value.

A second consideration against the backward looking approach is the complex data requirements and the practical complications that would arise if one attempts to determine to what extent individuals or groups have built up claims under the UPPS. The data required are most likely not available; there are no complete data on individual investment results nor data on whether people have been temporarily unemployed. The entire working history one would need is typically not available at an individual level.

A third problem is the fact that the employer typically also pays a part of the contribution, which most likely is based on a different approach, averaging over all employees of the firm. On average, this will not result in large differences. However, there might be large value transfers from companies with young workers to companies with older employees.

Due to all these considerations, we apply the *forward looking* methodology in this paper.

Valuation of the transition effects If the UPPS is abolished, implied future claims will expire. The basis for valuing these future claims is the market value of those commitments at the time of abolition. Defined benefit (DB) pension rights are in fact similar to a futures contracts on the investments of the pension fund. Those contracts are, in principle, also available in the capital markets outside the pension fund, which makes market valuation an objective measure for valuating the transition effects.

Particularly, this has consequences for the assumptions of the interest rate that is used in the calculation. With nominal guaranteed commitments, this needs to be the "safe market interest rate", as derived from the swap curve. Any other approach implies potentially large value transfers from young to old (at a higher interest rate than the one that follows from the swap curve) or from old to young (using a lower interest rate than the one that follows from the swap curve). One technique for the market-consistent valuation of the transition effects is the so-called Risk Neutral Pricing (RNP) approach, but there are also other techniques. Other techniques, of course, all lead to the same outcomes as RNP.

The use of RNP means that comparisons based on market value between different portfolios are corrected for differences in risk characteristics, using market prices for the relevant risks. For example, equities have a higher expected return than government bonds, but are more risky. Hence, the risk premium compensates for taking risks, based on the market prices of the corresponding risks. Therefore, the use of RNP is necessary in calculating the market consistent value of claims which are lost by abolishing the UPPS.

This paper investigates the economic features of abolishing the UPPS. Similar deterministic analyses are done by Van Ewijk (2017); Werker (2017), while Lever and Muns (2017) analyze this topic using a stochastic analysis. Our paper contributes to these studies in several ways. First, we quantify the subsidy from the poor to the wealthy which is present under the UPPS. Second, we analyze the transition effects of abolishing the UPPS in two ways: (i) analytically: by simplifying the model to three overlapping generations (OLG) we algebraically investigate the effects of the main parameters, and (ii) numerically: with multiple overlapping generations and realistic assumptions we obtain numerical outcomes from our model, which are a more realistic representation of the transition effects of abolishing the UPPS.

In the remainder of this paper, we present a discrete time OLG model in which the various economic factors involved in abolishing the UPPS can be analyzed (Section 2). In order to clarify the intuition behind these economic features, we analyze a simplified version of that model in Section 3, which involves only two working generations and one retired generation. Using this simplification, this 3-OLG model is analytically solvable. In Section 4 we consider a more realistic setting that takes into account at least forty working generations and twenty retired generations simultaneously. Section 5 summarizes the conclusions. Mathematical derivations are shown in Appendix A.

2 A discrete Time OLG Model

2.1 Model Structure

Demography There are n working generations and m retired generations. Generations become older after each period: a generation with age i becomes the generation with age $i+1$. In this model the youngest working cohort has the age $i = 1$.

There are two types of workers, distinguishing high (H) and low (L) productivity types, reflecting differences in educational achievement. High productivity workers have a steeper wage profile. Define $u_t^k = \left(u_{1,t}^k, u_{2,t}^k, \ldots, u_{n+m,t}^k\right)$ as the vector with elements $u_{i,t}^k$: the number of people from a generation with age i and type $k \in \{H, L\}$ at time t. With constant population growth g we get

$$u_{i,t+1}^k = (1+g)\, u_{i,t}^k, \forall i \in \{1, 2, \ldots, n+m\}.$$

Wage The pension base of a participant is the amount over which he/she accrues pension rights and pays contributions. In the model we refer to this as simply the wage of the participant.[4] The wage of a participant with type k age i at time t

[4]In Section 4 we calibrate the model to match the total pension base of the Dutch economy.

is defined as $w_{i,t}^k$. Pensioners have no wage, i.e. $w_{n+i,t} = 0, \forall i \in \{1, 2, \ldots, m\}$. Define $w_t^k = \left(w_{1,t}^k, w_{2,t}^k, \ldots, w_{n+m,t}^k\right)$ as the vector with wages of all age cohorts.

The wage of a cohort changes over time for two reasons: wage inflation and career development. The career development depends on the type ($k \in \{H, L\}$) and is expressed as

$$w_{i+1,t}^k = w_{i,t}^k \left(1 + c^k\right), \forall i \in \{1, 2, \ldots, n-1\}.$$

Wage inflation is expressed as

$$w_{t+1}^k (1 + \pi) = w_t^k.$$

Pension rights Pension liabilities are valued as the net present value of the pension benefits based on the current pension rights. Guarantees are valued using the risk-free nominal interest rate (r). The indexation rate z is guaranteed.[5] The price for one euro pension accrual for the cohort with age i then becomes:

$$\begin{aligned} K_i =& \begin{cases} \sum_{j=n+1-i}^{n+m-i} q^j, & i \in \{1, 2, \ldots, n\} \\ \sum_{j=0}^{n+m-i} q^j, & i \in \{n+1, n+2, \ldots, n+m\} \end{cases} \\ =& \sum_{j=\max(n+1-i,0)}^{n+m-i} q^j, \\ \text{with } q =& Q\frac{1+z}{1+r} \\ \iff K =& (K_1, K_2, \ldots, K_{n+m}). \end{aligned}$$

The variable Q scales the factors K_i for the discount rate used to calculate the pension contributions. For $Q = 1$ the price of pension accrual is actuarially fair. When there is guaranteed positive indexation ($z > 0$), but the contributions are based on a nominal funding ratio, then we have $Q = \frac{1}{1+z}$. When the contributions are not based on the risk-free rate, but expected investment returns, then we have $Q = \frac{1+r}{1+r+\mu}$, where μ denotes the risk premium of the investment portfolio. It is easy to show that for $Q = 1$ and $z = 0$ both the nominal and real funding ratio equals 100%.

The accrued pension rights of the cohort with age i are defined as $B_{i,t}$. Working cohorts accrue new pension rights with accrual rate $\rho_{i,t}^k > 0$ as a fraction of their wage. The pension rights increase with indexation and accrual as follows

$$\begin{aligned} B_{i+1,t+1}^k &= (1+z) B_{i,t}^k + \rho_{i,t}^k w_{i,t}^k, \ \forall t \text{ and } i \in \{0, 1, \ldots, n+m\} \\ \text{with } B_{0,t}^k &= 0, \ \forall t. \end{aligned}$$

A pensioner with type k and age i obtains a pension benefit at time t equal to $(1+z) B_{i,t}^k$.

[5] We consider indexation rate $z = 0$ under the benchmark parameter setting.

Uniform policy pension system (UPPS) The accrual rate is uniform, so each participant has the same accrual rate, i.e. $\rho_{i,t}^k = \rho$, so we can write ρ as a scalar instead of a vector.

The uniform contribution rate equals

$$P^U = \frac{\rho \sum_{i=1}^n \left(u_{i,t}^H w_{i,t}^H + u_{i,t}^L w_{i,t}^L\right) K_i}{PB_t}$$

and the total pension base is

$$PB_t = \sum_{i=1}^n \left(u_{i,t}^H w_{i,t}^H + u_{i,t}^L w_{i,t}^L\right).$$

In principle we assume that $u_{n,0}^H = \alpha = \left(1 - u_{n,0}^L\right)$ and $w_{1,0}^H = w_{1,0}^L = 1$, i.e. a fraction $\alpha \in [0,1]$ of the working population is of the high type and wage equals one for all participants of the youngest working cohort. Appendix A.1 shows that we can write the uniform contribution rate in this case as

$$P^U = \rho q^n \frac{\sum_{i=0}^{m-1} q^i \sum_{i=0}^{n-1} \frac{\chi_i}{q^i (1+g)^i}}{\sum_{i=0}^{n-1} \frac{\chi_i}{(1+g)^i}}$$

$$\text{with } \chi_i = \alpha \left(1 + c^H\right)^i + (1-\alpha)\left(1 + c^L\right)^i.$$

where χ_i stands for the cumulative wage increase received by cohort i averaged over the two labor types.

Pension fund The assets of the pension fund A_t evolve as

$$A_{t+1} = (1+r)\left[A_t + \rho \sum_{i=1}^n \left(u_{i,t}^H w_{i,t}^H + u_{i,t}^L w_{i,t}^L\right) K_i - \sum_{i=n+1}^{n+m} \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right)\right].$$

The first (summation) term denotes the received contributions in year t summed over all working cohorts and the second (summation) term represents the total pension benefits payed out to the retirees in year t.

The liabilities of the pension fund are equal to

$$L_t = \sum_{i=1}^{n+m} K_i \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right).$$

This can be written in recursive form as

$$L_{t+1} = (1+g)(1+\pi) L_t.$$

The funding ratio is defined as $F_t = A_t / L_t$.

2.2 Redistributive impact of abolishing uniform contribution policies: the analytics

When the UPPS is abolished and replaced by a system with actuarially fair contribution rates, the contribution rate becomes

$$P_i = \rho K_i.$$

Assuming $q = Q\frac{1+z}{1+r} < 1$ this results in contribution rates which are increasing with age.

Net value transfers The retired generations have already paid and received the subsidies under the UPPS during their working life and are thus out of the system. Working cohorts have paid subsidies, but have not yet fully received the equivalent amount in subsidies as they still have some years to go before retirement. Hence, the current working cohorts might be negatively affected by abolishing the UPPS. Against that effect is the consequence of switching to an actuarially fair system, that the contribution rate can be reduced because contributing cohorts no longer pay for the implicit debt.

Appendix A.4 shows that the net value transfer (NVT) to a generation of age j at time t is obtained as

$$\begin{aligned} NVT_{j,t} =& \Xi_j \sum_{i=\max(j-1,0)}^{n-1} \chi_i \left(\frac{(1+\pi)}{q(1+r)}\right)^i \left(q^i \frac{\sum_{l=0}^{n-1} \frac{\chi_l}{q^l(1+g)^l}}{\sum_{l=0}^{n-1} \frac{\chi_l}{(1+g)^l}} - 1\right) \\ \text{with } \Xi_j =& \rho \sum_{l=n}^{n+m-1} q^l (1+\pi)^t (1+g)^{t+n-2} \left(\frac{(1+g)(1+\pi)}{(1+r)}\right)^{1-j}. \end{aligned}$$

Abolishing UPPS obviously is a "zero-sum game", the sum of the net value transfers cancel out:

$$\sum_{j=-\infty}^{n} \left(NVT_{j,t}^{H} + NVT_{j,t}^{L}\right) = 0.$$

Setting the lower bound for j at minus infinity implies all future generations are incorporated too. This is necessary because under the UPPS the implicit debt owed to the currently young is essentially rolled over into the indefinite future (although it approaches zero in discounted value terms, so this is not a Ponzi game). When $q = 1$, i.e. $Q(1+z) = (1+r)$, we get $\left(q^i \frac{\sum_{l=0}^{n-1} \frac{\chi_l}{q^l(1+g)^l}}{\sum_{l=0}^{n-1} \frac{\chi_l}{(1+g)^l}} - 1\right) = 0$, which implies a zero net value transfer: $NWT_{j,t}^k = 0$. This is because the value of the contributions and benefits are equal with and without the UPPS. To be precise, when $Q = 1$ and $r = z$, the UPPS is equivalent to an actuarially fair system. This is not the case for $(1+r) > Q(1+z)$, because then there is a PAYG-element in the UPPS, as the young generations subsidize the old generations. According to the Aaron condition($(1+r) > (1+\pi)(1+g)$) a funded pension scheme implies a higher return than is implicit in a PAYG scheme (Aaron, 1966). However, when the total wage growth is equal to the interest rate ($(1+r) = (1+\pi)(1+g)$), then we get for future generations, i.e. $j \leq 1$, the following:

$$NVT_{j,t} = 0.$$

So when the Aaron condition holds with equality, there is no benefit for future generations by no longer having to pay for the implicit debt, since then the (implicit) return on the PAYG-element is equal to the interest rate.

3 Abolishing UPPS: sensitivity analysis for two working generations and one retired generation (3-OLG)

To get to comprehensible analytical results and gain intuition, we first investigate a stylized version of the model with two working cohorts and one retired generation. Two working generations is obviously the required minimum to investigate effects of the UPPS. Hence, we assume $n = 2$ and $m = 1$. This way, the uniform contribution rate equals

$$P^U = \rho q \frac{(1+g)\,q + \chi_1}{1+g+\chi_1}.$$

The net value transfer then becomes

$$NVT^k_{2,t} = \rho q \,(1+\pi)^t \,(1+g)^t \,\frac{u^k_{2,0}\left(1+c^k\right)}{(1+g+\chi_1)}\,(q-1)$$

and for $j \leq 1$:

$$NVT^k_{j,t} = \rho q\,(1+\pi)^t\,(1+g)^t \left(\frac{(1+g)\,(1+\pi)}{(1+r)}\right)^{1-j} u^k_{2,0} \frac{\left(1+c^k\right)(1+\pi)\,(1+g) - \chi_1\,(1+r)}{(1+r)\,(1+g+\chi_1)}\,(q-1)\,.$$

When $q = 1$, i.e $(1+r) = Q\,(1+z)$, then the net value transfer equals zero for all types and all age cohorts. If $r \to \infty$, then $q \to 0$ and, hence, we obtain

$$\lim_{r\to\infty} NVT^k_{2,t} = 0.$$

In other words, the net value transfer of older generations converges to zero when the interest rate goes to infinity. Moreover, the minimum[6] is obtained at an interest rate of $r^* = 2Q\,(1+z) - 1$, with minimum $NVT^k_{2,t} = -\frac{\rho u^k_{2,0}\left(1+c^k\right)}{4(1+g+\chi_1)}$. Suppose that

[6]FOC: $\frac{\partial\left[(1+r)^{-1} - Q(1+z)(1+r)^{-2}\right]}{\partial r} = 0 \iff r = 2Q\,(1+z) - 1.$

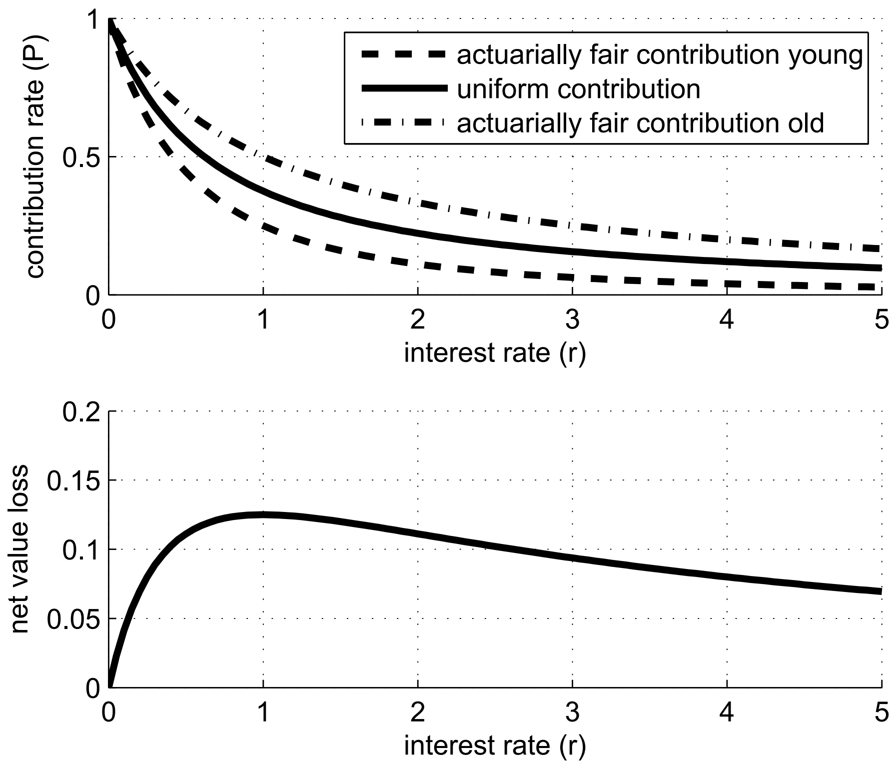

Figure 1: Contribution rates and net value loss of the oldest working cohort as a function of the interest rate $\left(\rho = Q = 1, g = z = \pi = c^H = c^L = 0\right)$

$Q = \frac{1}{1+z}$ and one period is 20 years, then people work for 40 years and are retired for 20 years. Then, the annual interest rate in this example is: $(1 + r^*)^{1/20} - 1 = 3.5\%$.

In Fig.1 the contribution rates and the net value *loss* of the older generation are presented as a function of the interest rate. The contribution rates decrease with the interest rate. For $r > r^*$ both the uniform and the actuarially fair contribution rates decrease towards zero and, hence, the differences decrease as well.

For $n = 2$ and $m = 1$ the net value transfer of generation $j \leq 1$ equals

$$NVT_{j,t} = \chi_1 \rho q \left(1 + \pi\right)^t \left(1 + g\right)^t \left(\frac{(1 + g)\,(1 + \pi)}{(1 + r)}\right)^{1-j} \frac{(1 + \pi)\,(1 + g) - (1 + r)}{(1 + r)\,(1 + g + \chi_1)} \left(q - 1\right).$$

Again this equals zero for $q = 1$ and for $(1 + r) = (1 + \pi)(1 + g)$.

The results for the current youngest generation ($j = 1$) with $t = z = g = c^H = c^L = 0$ and $\rho = Q = 1$ are presented in Fig.2 and Tab.1, where we vary both the interest rate and the wage growth. Fig.2 and Tab.1 show that the value transfer of the youngest generation is indeed zero when $r = 0$ or $r = \pi$. Moreover, the value transfer is positive (negative) when r is larger (smaller) than π.

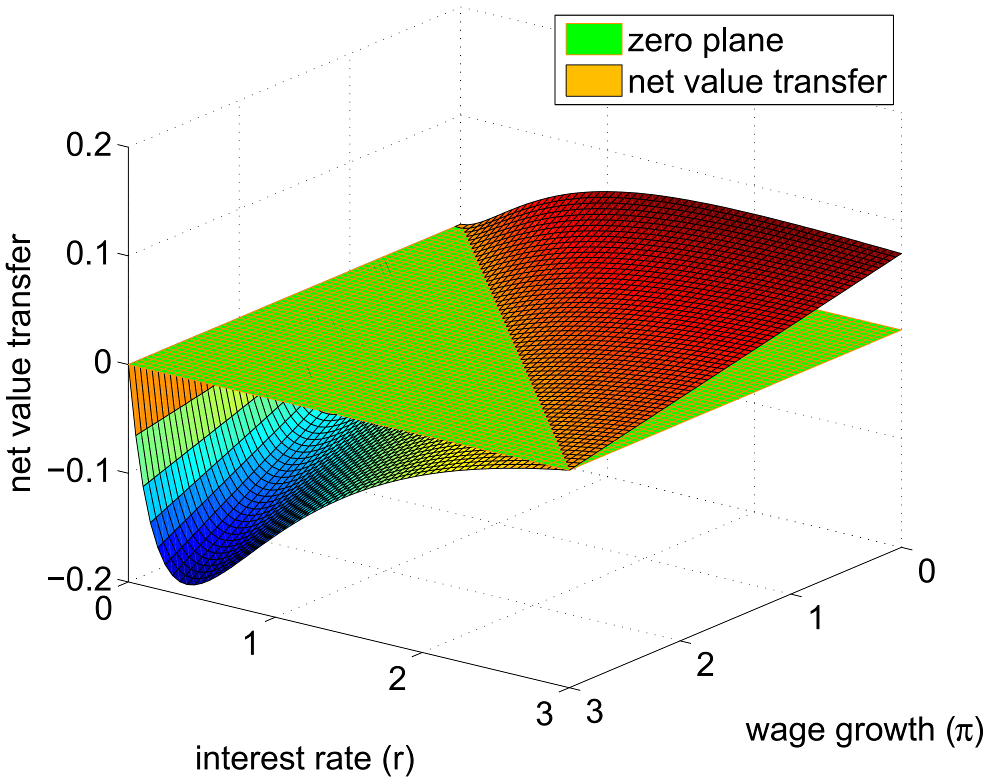

Figure 2: Net value transfer of the youngest generation as a function of the interest rate (r) and wage growth (π) $\left(Q = \rho = 1, z = g = c^H = c^L = 0\right)$

Table 1: Net value transfer of the youngest generation as a function of the interest rate (r) and wage growth (π) $\left(Q = \rho = 1, z = g = c^H = c^L = 0\right)$

Net value transfer youngest generation	$\pi = 0$	$\pi = 1$	$\pi = 2$	$\pi = 3$
$r = 0$	0	0	0	0
$r = 1$	0.063	0	-0.063	-0.125
$r = 2$	0.074	0.037	0	-0.037
$r = 3$	0.070	0.047	0.023	0

3.1 The UPPS and implicit subsidies from poor to rich

Using the same stylized model we next analyze the implicit redistribution from poor to rich in the UPPS. Under the UPPS the youngest working cohort pays a larger contribution rate than what is actuarially fair:

$$\left(P^U - P_1\right)\left(u^H_{1,t}w^H_{1,t} + u^L_{1,t}w^L_{1,t}\right) = \frac{\rho q \chi_1}{1 + g + \chi_1}\left(u^H_{1,t}w^H_{1,t} + u^L_{1,t}w^L_{1,t}\right)(1 - q) > 0.$$

The old working cohort receives this back as a subsidy. The *group* with the high type obtains a fraction $\frac{\alpha\left(1+c^H\right)}{\chi_1}$, while the low type group gets the complement fraction $\frac{(1-\alpha)\left(1+c^L\right)}{\chi_1}$, so an individual high type participant gets $\frac{\left(1+c^H\right)}{\chi_1}$, while the low type participant gets $\frac{\left(1+c^L\right)}{\chi_1}$. Since the contributions of the two types are the same (period one wages are identical across types) and $c_H > c_L$, the UPPS implies a subsidy from poor to rich. This is sometimes called "perverse solidarity", since people who are more highly educated (and in practice healthier) profit more (Bovenberg et al., 2006; Chen and Beetsma, 2015; Sutrisna, 2010). In this paper we refer to the "perverse subsidy", as the subsidy from poor to rich the UPPS leads to.

So compensating the old working cohorts for the excess payments in their first working period implies that wealthy participants obtain more compensation than poor participants, while they have paid the same subsidy in the past, since we assumed that wage when young is equal for both types $\left(w^H_{1,t} = w^L_{1,t}\right)$. The difference in received subsidy between a low type and high type participant of the old working cohort equals $TTE_t\frac{c^H - c^L}{\chi_1} > 0$, where $TTE_t = -NWV_{2,t}$ denotes the total transition effect.

In one period the subsidy from young to old is at least $TTE_t\frac{\left(1+c^L\right)}{\chi_1}$, while the wealthy participants get an additional subsidy of $TTE_t\frac{c^H - c^L}{\chi_1}$. Tab.2 summarizes the different subsidies under the UPPS.

Suppose the high type group is half of the working population, i.e. $\alpha = 0.5$, and the high type has double the wage of a low type in his second working period

Table 2: Subsidies under the UPPS

Total subsidy from all young to wealthy old	$-NVT_{2,t}^{H}=\frac{\rho q(1+\pi)^{t}(1+g)^{t}}{(1+g+\chi_1)}(1-q)\,\alpha\left(1+c^{H}\right)$
Total subsidy from all young to poor old	$-NVT_{2,t}^{L}=\frac{\rho q(1+\pi)^{t}(1+g)^{t}}{(1+g+\chi_1)}(1-q)(1-\alpha)\left(1+c^{L}\right)$
Total subsidy from young to old	$TTE_t=-NVT_{2,t}=-NVT_{2,t}^{H}-NVT_{2,t}^{L}$
Perverse subsidy to a wealthy participant	$\frac{-NWV_{2,t}^{H}}{\alpha}-\frac{-NWV_{2,t}^{L}}{1-\alpha}=\left(c^{H}-c^{L}\right)\frac{TTE_t}{\chi_1}$
Perverse subsidy to all wealthy participants	$\alpha\left(\frac{-NWV_{2,t}^{H}}{\alpha}-\frac{-NWV_{2,t}^{L}}{1-\alpha}\right)=\alpha\left(c^{H}-c^{L}\right)\frac{TTE_t}{\chi_1}$

($c^H = 2$ en $c^L = 1$), then the subsidy from the young to the poor old is $\frac{4}{5}TTE_t$, while the wealthy participants get an additional subsidy of $\frac{2}{5}TTE_t$. This example illustrates that one-third of the total transition effect for a rich participant consists of the perverse subsidy.

The problem is created because wage inequality is low when young (zero in our example) and high when old while the uniform contribution rate is the same for all participants, independent of type. An alternative would be to differentiate between type, by considering a uniform contribution rate for wealthy and poor separately:

$$P^{U,k}=\frac{\rho\sum_{i=1}^{n}\left(u_{i,t}^{k}w_{i,t}^{k}K_i\right)}{\sum_{i=1}^{n}\left(u_{i,t}^{k}w_{i,t}^{k}\right)}=\rho q\frac{(1+g)\,q+\left(1+c^{k}\right)}{(1+g)+(1+c^{k})}.$$

This way, the contribution rate remains independent of age. This alternative system favors the poor participants compared to UPPS, since there is no perverse subsidy. We can analyze the perverse subsidy further by comparing the UPPS with this alternative system. Appendix A.5.1 shows that the group of type k from generation j obtains the following gain (G) when switching from the UPPS to this alternative system:

$$G_{2,t}^{k}=\rho q u_{2,0}^{k}(1+\pi)^{t}(1+g)^{t}\left(1+c^{k}\right)\left(\frac{(1+g)\,q+\chi_1}{1+g+\chi_1}-\frac{(1+g)\,q+\left(1+c^{k}\right)}{1+g+1+c^{k}}\right)$$

and for $j \leq 1$:

$$G_{j,t}^{k}=\rho q u_{2,0}^{k}(1+\pi)^{t}(1+g)^{t}\left(\frac{(1+g)(1+\pi)}{(1+r)}\right)^{1-j} * \left((1+g)+\frac{(1+g)(1+\pi)\left(1+c^{k}\right)}{(1+r)}\right)\left(\frac{(1+g)\,q+\chi_1}{1+g+\chi_1}-\frac{(1+g)\,q+\left(1+c^{k}\right)}{1+g+1+c^{k}}\right).$$

When the Aaron condition holds, i.e. $((1+r)>(1+g)(1+\pi))$, the sum of these gains is zero:

$$\begin{aligned}\sum_{j=-\infty}^{2} G_{j,t}^{H}+G_{j,t}^{L} &=0\\ \iff \frac{(1+r)}{(1+r)-(1+\pi)(1+g)} &=\sum_{j=0}^{\infty}\left(\frac{(1+g)(1+\pi)}{(1+r)}\right)^{j}\\ \iff (1+g)(1+\pi)<(1+r)\,.\end{aligned}$$

From the above equations we observe that when there is no difference between poor and wealthy, i.e. $\alpha=0$, $\alpha=1$ or $c^H=c^L$, then we have $G_{j,t}^{k}=0,\forall j$. Again this also holds for $q=1$. In all other cases there is a perverse subsidy under the UPPS from one type to another.

4 n working generations and m retired generations (multi-OLG): the UPPS in the Netherlands

The 3-OLG model (with $n=2$ and $m=1$) provided insights into the determinants of the transition effects, but is too simplified to provide quantitatively realistic estimates.We therefore switch from our simplified model to a more realistic OLG model , with $n=40$ working cohorts and $m=20$ retired cohorts. Note that in this model we give an alternative interpretation to age: the employees have the age of 1 to 40 and retirement starts at the age of 41. A more realistic interpretation is simply obtained by adding 25 years to these ages. We refer to the latter as the "real age".

4.1 Calibration

We calibrate the model on the Dutch economy, as the UPPS applies to the major part of the Dutch second pillar and there is broad support for replacing the UPPS by an actuarially fair system of contributions, so analyzing the associated transition effects is of substantial policy interest. Moreover, the recent coalition agreement of the Dutch government announced that it would elaborate on these plans. The Dutch second pillar is large, with total assets about twice the size of the Dutch GDP. Since at the time of writing the nominal funding ratios of Dutch pension funds are around 100%, we assume that the indexation rate is zero ($z=0$). Fig.3 shows the risk-free yield curve based on the risk-free swap rate at the end of September 2017. Our model is a stylized version, because we do not distinguish between discount factors for liabilities with different maturities. In other words, we take a flat risk-free yield curve. Specifically, we assume that in our benchmark parameter setting the risk-free interest rate is $r=1.0\%$. As shown in Fig.3, this corresponds to the risk-free swap rate with maturity of 11 years.

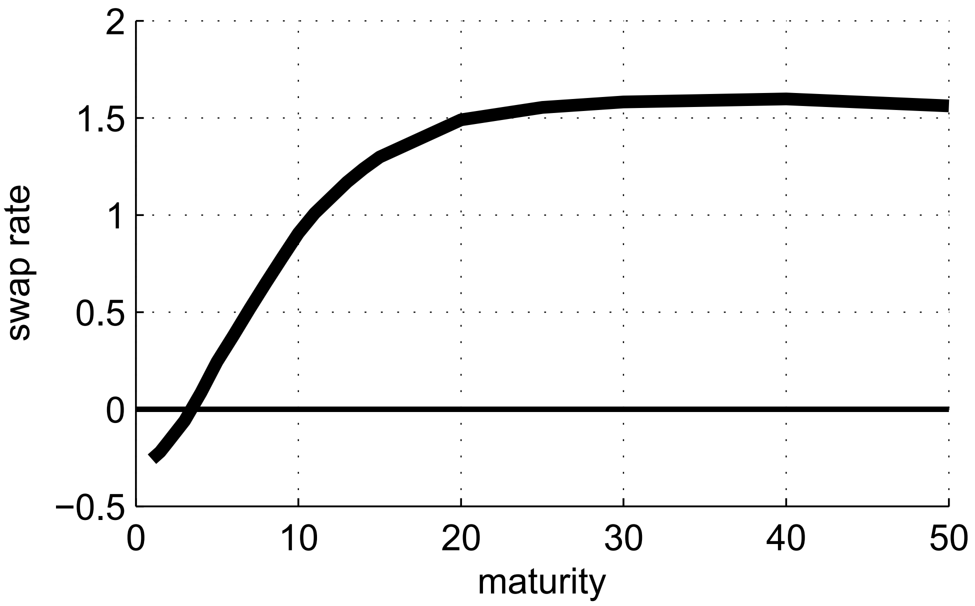

Figure 3: Risk-free yield curve based on the risk-free swap rate at September 30, 2017. Source: DNB (2017)

We assume that the accrual rate equals $\rho = 1.829\%$.[7] We calibrate the model to match the pension base of 112 billion euros.[8] We assume that the initial funding ratio is $F_0 = 100\%$, the wage growth factor is $\pi = 1\%$ and population growth is $g = 0\%$.[9] The career development is $c^H = c^L = 0.5\%$ under our benchmark parameter setting. Finally, we assume that the discount rate for determining the price of pension accrual is actuarially fair, i.e. $Q = \frac{1}{1+z} = 1$. Tab.3 provides an overview of the parameter settings.

4.2 Transition effects of abolishing UPPS

Similar to the figures and table in Section 3, Fig.4, Fig.5 and Tab.4 show the net value *loss*es and gains for the various generations. A negative cohort number refers to an cohort that is as yet unborn. The numbers are generated based on the parameter Set 1 from Tab.3, where we have 60 overlapping generations. Note that the shapes of the graphs are quite similar to the ones in Section 3 with only three overlapping generations.

Fig.6 shows the net value transfer per cohort as a function of age corresponding

[7]The weighted average of the accrual rates of Dutch pension funds in 2016 equals 1.829%.

[8]According to the DNB E-line annual reports 2016 for Dutch pension funds the pension base is 112 billion euros. Including Dutch collective pension arrangements with insurance companies which also apply a UPPS would increase the pension base to 126 billion euros. The Dutch GDP over 2016 is about 700 billion euros (CBS, 2017).

[9]Labor supply projections indicate a growth rate close to zero (Euwals and den Ouden, 2014).

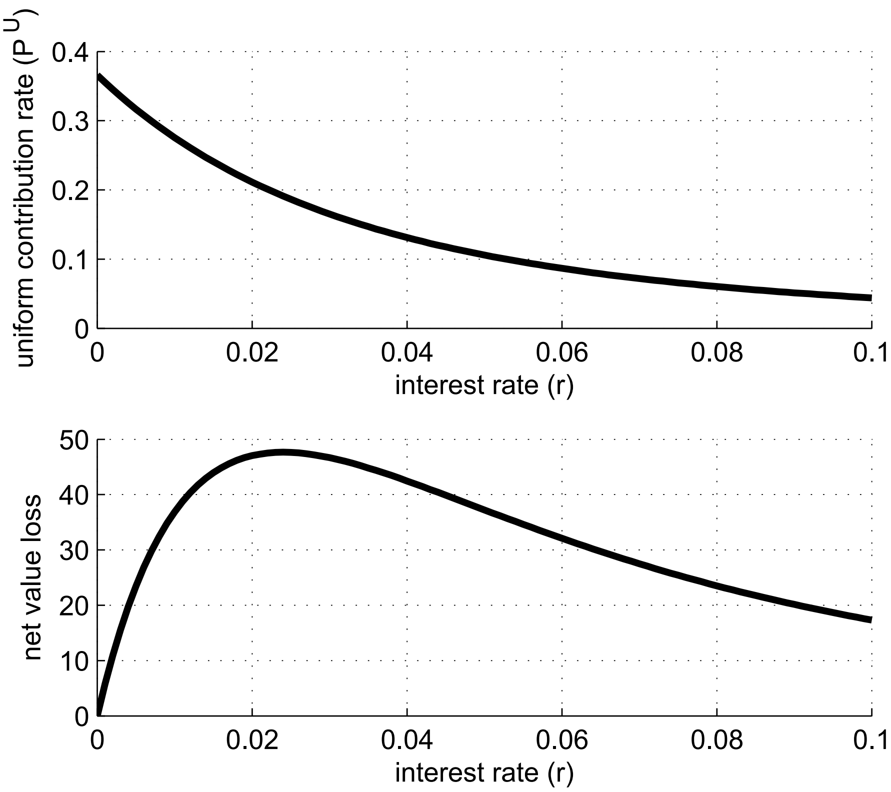

Figure 4: The contribution rate and the net value loss in billion euros for unfortunate generations as a function of the interest rate Tab.3 Set 1 provides the underlying parameter assumptions.

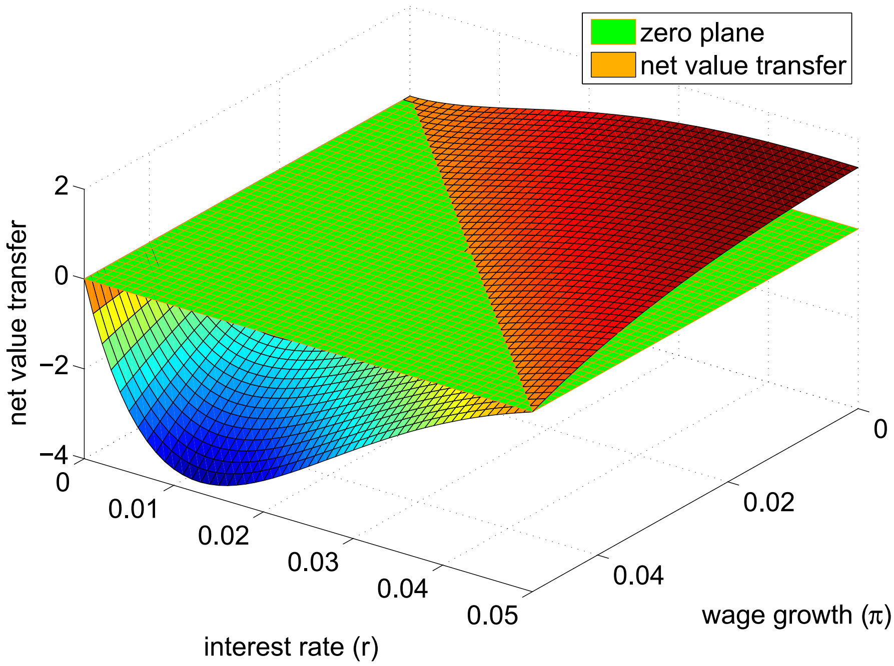

Figure 5: Net value transfer of the youngest generation in billion euros as a function of the interest rate (r) and wage growth (π) Tab.3 Set 1 provides the underlying parameter assumptions.

Table 3: Parameter settings

Description	Symbol	Set 1	Set 2	Set 3
Working cohorts	n	40	40	40
Retired cohorts	m	20	20	20
Interest rate	r	1%	**1.5%**	1%
Indexation rate	z	0%	0%	0%
Wage inflation	π	0.5%	**1%**	0.5%
Population growth	g	0%	0%	0%
Fraction population type H	α	100%	100%	**50%**
Career development type H	c^H	0.5%	0.5%	**1%**
Career development type L	c^L	0.5%	0.5%	**0%**
Pension base (in billion euros)	PB_t	112	112	112
Accrual rate	ρ	1.829%	1.829%	1.829%
Transition effect (in billion euros)		36.90	47.99	36.93

Table 4: Net value transfer of the youngest generation in billion euros as function of the interest rate (r) and wage growth (π) Tab.3 Set 1 provides the underlying parameter assumptions.

Net value transfer youngest generation	$\pi = 0.00$	$\pi = 0.01$	$\pi = 0.02$	$\pi = 0.05$
$r = 0.00$	0	0	0	0
$r = 0.01$	0.332	0	-0.489	-3.639
$r = 0.02$	0.824	0.496	0	-3.303
$r = 0.05$	1.369	1.297	1.160	0

to parameter Set 1. The age of 0 represents a generation which enters the labor market next year. Negative values for age represent future generations. The sum of all net value transfers equals zero. This indicates that abolishing the UPPS is indeed a zero-sum game.

Fig.6 shows that most currently working cohorts lose value by abolishing the UPPS, while future cohorts and some of the youngest currently working cohorts benefit because they escape financing the implicit PAYG element debt. We define the sum of the net value losses as the transition effect. The last row of Tab.3 indicates the transition effects for three different parameter settings, which result in estimates ranging from 37 and 48 billion euros, i.e. 5% to 7% of the Dutch GDP.

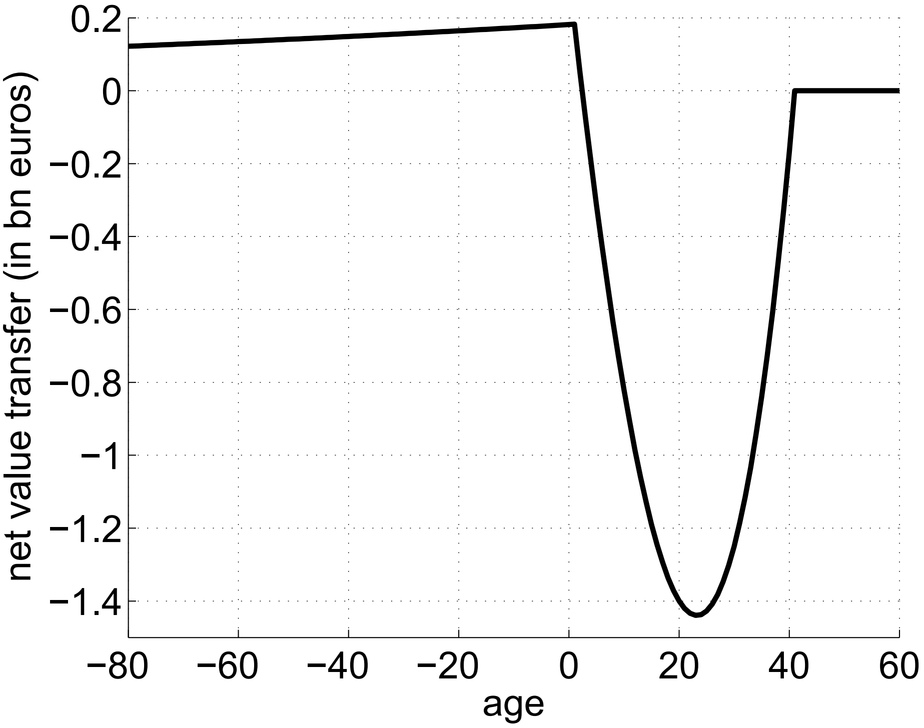

Figure 6: Net value transfer per cohort in billion euros as a function of age Tab.3 Set 1 provides the underlying parameter assumptions. *Note that in this model we give an alternative interpretation to age: the employees have the age of* 1 *to* 40 *and retirement starts at the age of* 41. *A more realistic interpretation is simply obtained by adding 25 years to these ages.*

4.3 Wage growth

Deelen (2012) shows that wage has a quadratic relation towards tenure. In line with his results we now apply the following model for career development:

$$f^i(s) = b_0 + sb_1 + s^2 b_2,\ i \in \{H, L\}.$$

Hence, the wage for a participant with type i and age s at time t equals $w^i_{s,t} = f^i(s)(1+\pi)^t$.

If we do not distinguish between types, i.e. $\alpha = 1$, then using data on the aggregate wage profile for the Dutch labor market over 2014 (CBS, 2017) we estimate the following coefficients:

$$\begin{aligned}\hat{b}_0 &= 19.380,\\ \hat{b}_1 &= 2.501,\\ \hat{b}_2 &= -0.052.\end{aligned}$$

From the forecast table of the Dutch Actuarial Society (Actuarieel Genootschap, 2014) we determine the mortality rates of someone with real age of 55 in 2017, where we take the average of men and women. Similarly we use the mortality rates to obtain an age distribution for the entire population. The annuity factors of the vector K now become

$$K_i = \sum_{j=\max(n+1-i,0)}^{T-i} \frac{p_{i+j}}{p_i} q^j,$$

where p_i is the probability that someone with real age of 25 year becomes at least $25+i$. The parameter T is the final age, which we set equal to 100, i.e. nobody gets older than 125 year in terms of real age.[10]

The working population and the pension base as a function of age are shown in Fig.7. The pension base is obtained by deducting the franchise of 13,000 euros from the gross wage.[11] Close to retirement less people have a full-time job, which reduces the pension base further. Modeling this income distribution, this demography and and the parameter settings of Set 1 and Set 2 from Tab.3 result in a transition effect of, respectively, **36.72 and 44.50 billion euros**.

Fig.8 shows the net value transfer under the more realistic working population and pension base. This time, the vertical axis presents the net value transfer per cohort in terms of percentage change of the total pension value. Fig.8 shows that this net value transfer of a future generation is +0.63%, while the maximum loss

[10] Since $p_i < 0.1\%, \forall i \geq 83$, the probability that somebody with real age of 25 becomes at least $25 + 83 = 108$ years old in terms of real age is less than 0.1%. Hence, the results are not affected by choosing a larger value for the parameter T.

[11] The franchise is the level of income over which no pension is accrued in the second pillar since it is covered under the national first pillar PAYG system (the "AOW").

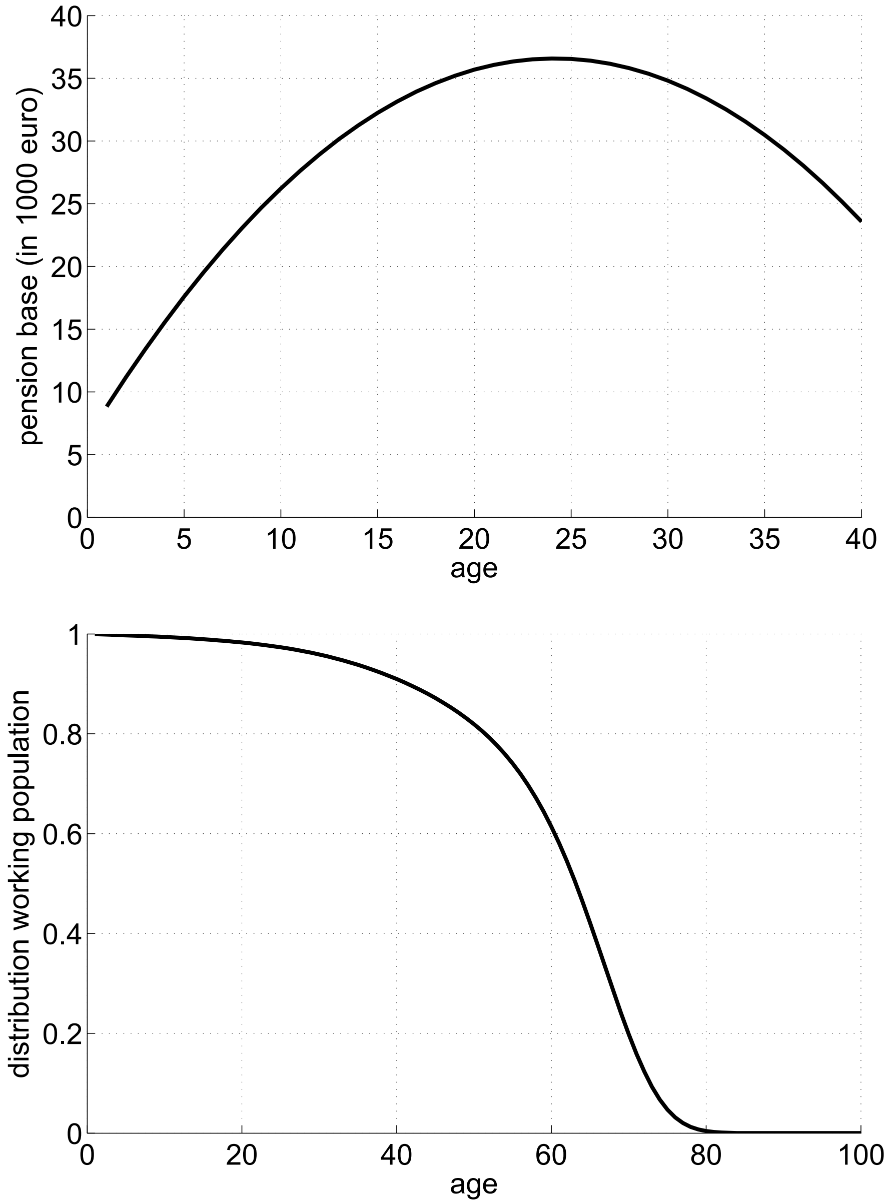

Figure 7: Pension base over life and working population. *Note that in this model we give an alternative interpretation to age: the employees have the age of* 1 *to* 40 *and retirement starts at the age of* 41*. A more realistic interpretation is simply obtained by adding 25 years to these ages.*

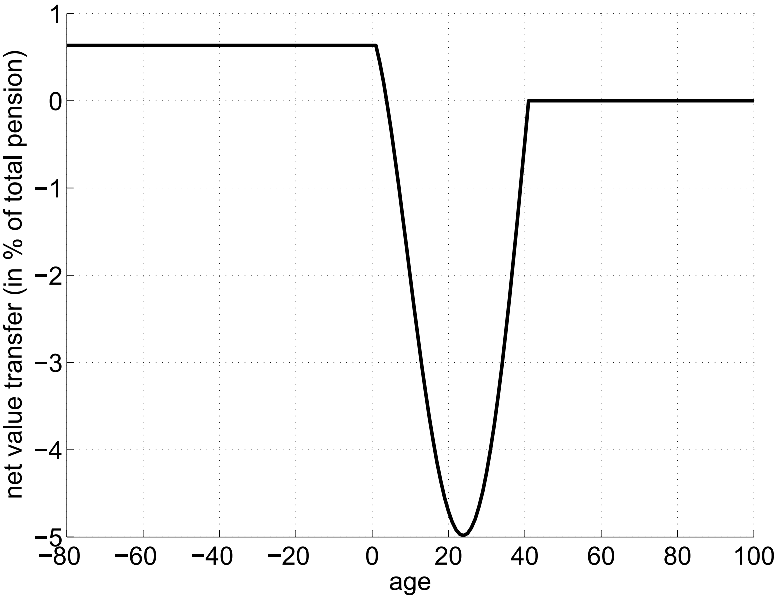

Figure 8: Net value transfer per cohort as percentage of total pension value as a function of age see Tab.3 Set 1 for the underlying parameter settings and Section 4.3 for the underlying working population and pension base over life. *Note that in this model we give an alternative interpretation to age: the employees have the age of* 1 *to* 40 *and retirement starts at the age of* 41. *A more realistic interpretation is simply obtained by adding 25 years to these ages.*

(-4.98%) is obtained by the cohort which is currently 24 years old (i.e. real age 49). Hence, the net value transfers are within the range of ±5% of total pension value for each cohort.

4.4 Sensitivity analysis

We again take Set 1 from Tab.3 for the benchmark parameter settings. We do not apply the parameters c^H and c^L yet, using instead the aggregate working population and pension base we described in Section 4.3. For the sensitivity analysis we vary one parameter at the time and show the corresponding changes in the transition effect in Fig.9. From these graphs we can conclude that the transition effect is quite sensitive to several parameter assumptions. The transition effect is largest for an interest rate around 2.1%. For lower interest rates, the transition effect is increasing with the interest rate, while for larger interest rates ($r > 2.1\%$) the transition effect decreases with r. The reason is that for high interest rates, the uniform and the actuarially fair contribution rates decrease towards zero and, hence, the differences decrease as well. Wage growth increases the transition effect, while population growth decreases the transition effect. The number of years working increases the transition effect for low n, because the difference between the actuarially fair contribution rates and the uniform contribution rate become larger with more working years. However, for large n, people have a short retirement period, as mortality rates remain equal, such that the number of working years decrease the transition effect. The latter effect dominates at $n \geq 42$ (i.e. real age 67). The pension base and the accrual rate are both directly proportional to the transition effect.

Fig.9 shows that the transition effect strongly depends on the parameter settings chosen. Lever et al. (2017) present a transition effect of 55 billion euros. There are several explanations for this difference. Here we discuss the four main differences in parameter and modeling assumptions. First, we do not model the survivor's pension, as the UPPS does not fully apply to this pension in the Netherlands, while Lever et al. (2017) assume a top-up of 0.25%-point for the accrual rate to compensate for this. Second, we apply a pension base of 112 billion euros for pension funds, while Lever et al. (2017) take 160 billion euros; their number is higher because they include pension arrangements with insurance companies and because they assume a total pension base growth between 2016-2020 of 13.6%. We use 2016 as a base year and we exclude insurance companies from our analysis since the UPPS does not apply to insurance company based pensions. Third, Lever et al. (2017) apply a yield curve which increases up to almost 1.5% for long maturities, while we simplify the analysis by taking a flat yield curve which equals 1%. Fourth, the outcomes of Lever et al. (2017) are based on a stochastic approach, which also takes pension cuts and indexation of pension rights into account, while we apply a deterministic setting with nominal (guaranteed) pension rights. The first and the second difference in assumptions, i.e. the larger accrual rate and pension base, result in a larger transition effect. It is not a priori obvious whether the difference in yield curve increases or decreases the transition effect. The stochastic analysis most likely results in a lower transition effect as, in the Netherlands, the pension cuts in response to low funding ratios are in principle unlimited while adjustments upwards (once funding ratios are high) are limited to what is necessary for indexation to the wage- or price inflation. This would imply lower results under a stochastic estimation.

4.5 Subsidy from poor to rich

So far we did not distinguish between poor and rich participants in this section. To make that distinction we use the wage profiles for $f^H(s)$ and $f^L(s)$ using data of wages in the Netherlands from CBS (2017). Tab.5 shows the estimated coefficients for $\alpha = 1$, $\alpha = 0.35$ and $\alpha = 0.1$. The pension base profiles are shown in Fig.10. These are the gross wages after deducting the franchise. Moreover, note in the right panel of Fig.10 that we limit the pension base to 100,000 euros minus the franchise for the rich, as this is nowadays the fiscal maximum for pension accrual in the Netherlands.

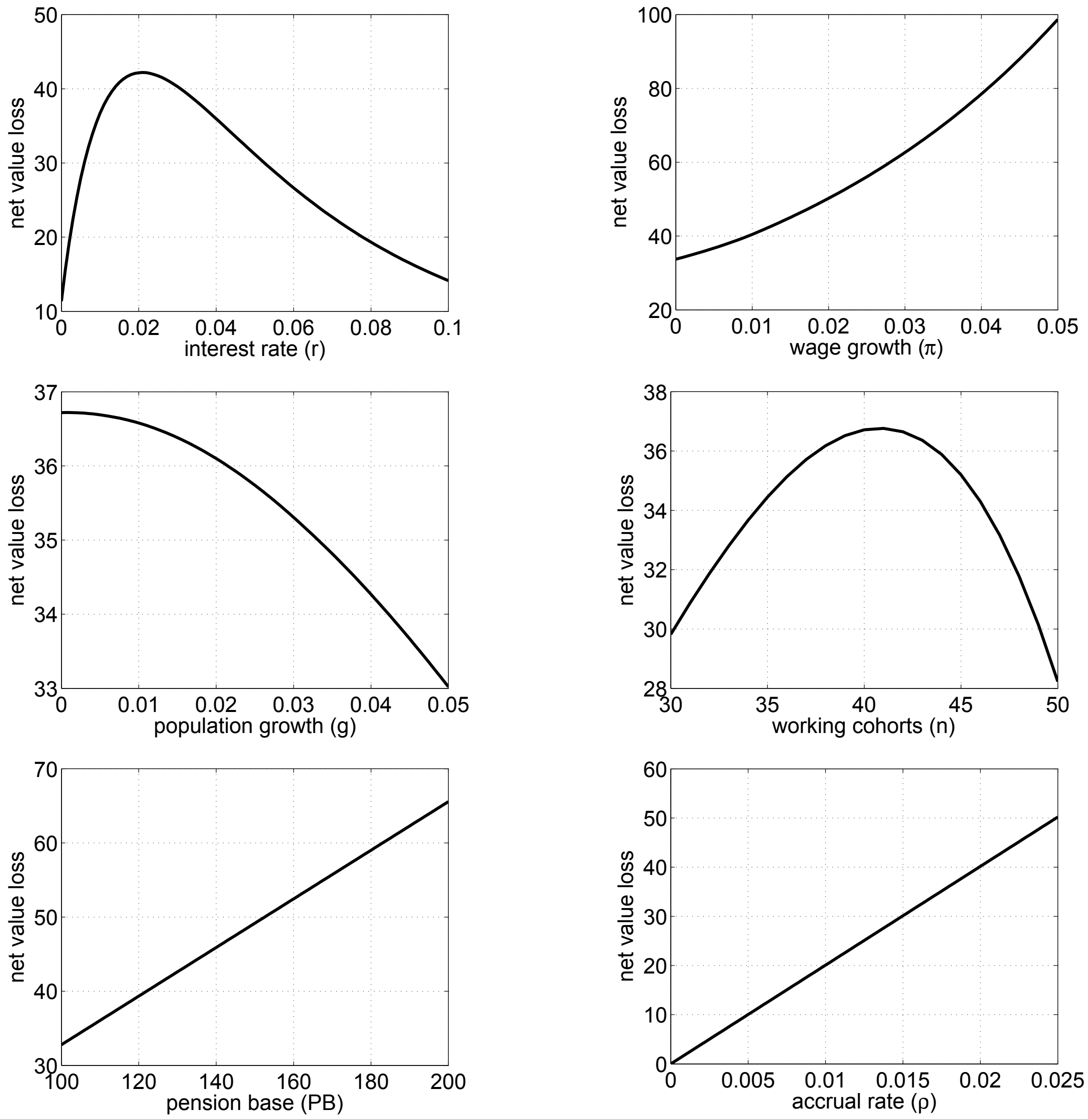

Figure 9: Sensitivity analysis of the transition effect in billion euros

Table 5: Estimated coefficients for several wage profiles see Tab.3 Set 1 for the underlying benchmark parameter settings

Fraction type H	$\alpha = 1$		$\alpha = 0.35$		$\alpha = 0.1$	
Type	L	H	L	H	L	H
$\hat{b}_0$	-	19.378	10.935	35.609	15.776	50.440
$\hat{b}_1$	-	2.501	1.628	4.021	2.100	6.172
$\hat{b}_2$	-	-0.052	-0.037	-0.078	-0.044	-0.089
Transition effect (in bn euros)						
Total	36.72		37.02		37.67	
Per type	-	36.72	6.48	30.54	11.73	25.94
Per 1% population	-	0.3672	0.0997	0.8727	0.2882	1.1734
Perverse subsidy (in bn euros)	-	-	-10.06	10.06	-10.73	10.73

Empirically, we find that when we take the wage profile of the 10% wealthiest participants (i.e. $\alpha = 0.1$), the differences in wage profile are larger between poor and rich than what we get when we take the wealthiest 35% of the population (i.e. $\alpha = 0.35$). When taking into account that the wealthy group has a steeper wage profile, the aggregate transition effect remains about 37 billion: the perverse subsidy mechanism redistributes but does not lead to any significant change in the overall size of the transition effect.

With a steeper wage profile, old participants gain more from the subsidies from young to old under the UPPS. Hence, the transition effect is larger for wealthy participants. The second-to-last row of Tab.5 shows that the transition effect of a rich participant (type H) is indeed substantially larger than for a poor participant (type L). The poor group has a relatively flat wage profile and, therefore, a lower transition effect. On the other hand, wealthy participants typically paid more subsidy to old cohorts when they were young themselves under the UPPS.

The uniform contribution rate is the same for all participants, independent of age or type. As we did before we can alternatively differentiate between type, by considering a uniform contribution rate for wealthy and poor separately:

$$P^{U,k} = \frac{\rho \sum_{i=1}^{n} \left(u_{i,t}^{k} w_{i,t}^{k} K_i\right)}{\sum_{i=1}^{n} \left(u_{i,t}^{k} w_{i,t}^{k}\right)}.$$

This way, the contribution rate remains independent of age. This alternative system is more favorable to the poor participants, since there is no perverse subsidy. We can compare the UPPS with this alternative system in order to quantify the perverse subsidy under the UPPS. That procedure leads to estimates of total perverse subsidies for $\alpha = 0.35$ and $\alpha = 0.1$ of, respectively, 10.06 and 10.73 billion euros (see the last row of Tab.5).

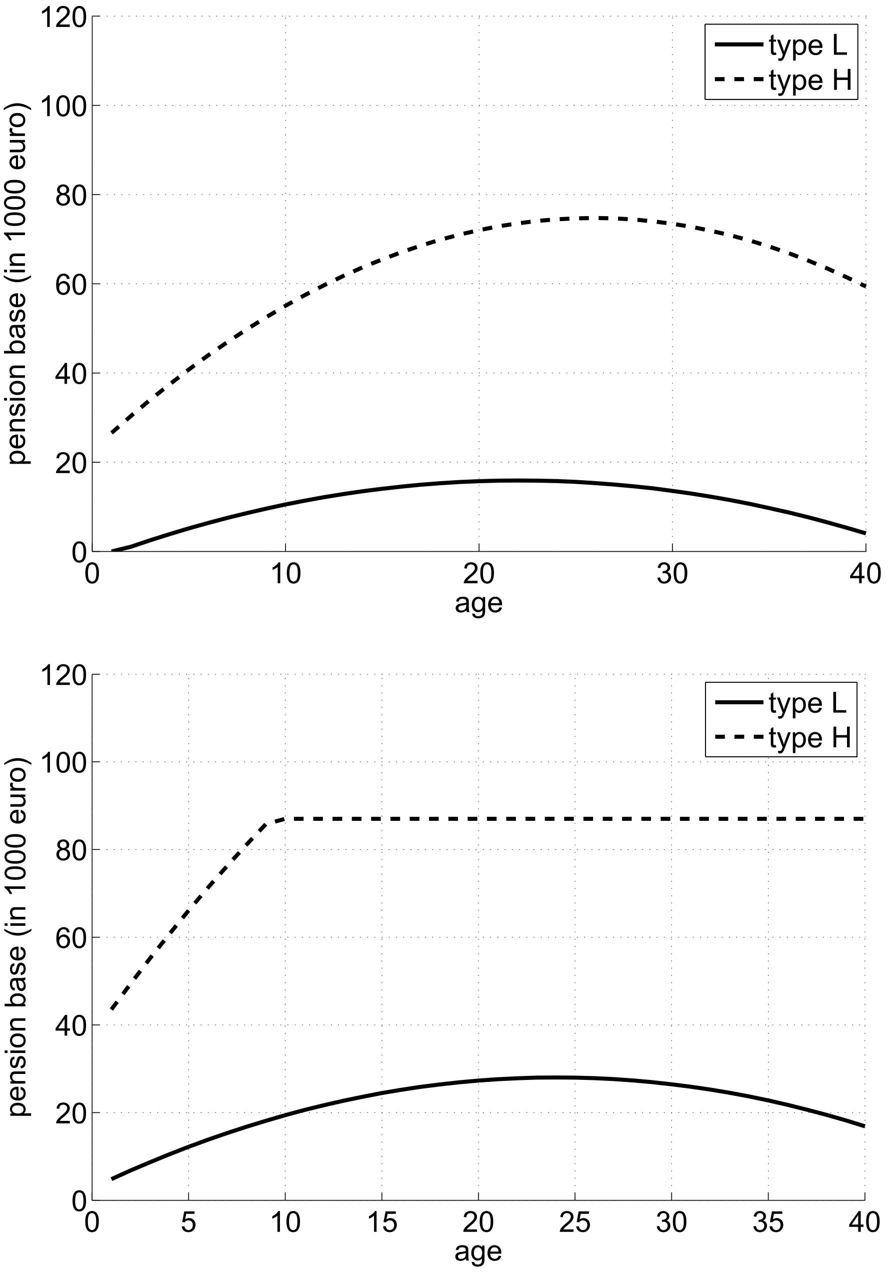

Figure 10: Pension base as a function of age for poor and wealthy participants. In the upper (lower) panel the fraction of wealthy participants is 35% (10%). *Note that in this model we give an alternative interpretation to age: the employees have the age of* 1 *to* 40 *and retirement starts at the age of* 41*. A more realistic interpretation is simply obtained by adding 25 years to these ages.*

These results indicate that there are substantial value transfers under the UPPS from poor to rich, which is in itself already a reason to abolish the UPPS and switch to an actuarially fair system. Moreover, this perverse subsidy in the UPPS can arguably be construed as an argument for not fully compensating for all the negative transition effects that are triggered by abolishing the UPPS.

5 Conclusion

By abolishing the UPPS most current working cohorts will miss out on subsidies they were entitled to since they have paid similar subsidies to old cohorts in their earlier working years. However, some young generations and all future generations benefit from abolishing the UPPS due to contribution reduction. This paper shows the net value transfers for different generations and for different parameter assumptions. We refer to the transition effect as the total transfer loss of generations by abolishing the UPPS. From our benchmark estimate we obtain a transition effect of about 37 billion. However, arguably more important than this aggregate number is our conclusion that for each cohort the transition effect as a percentage of their pension value lies within the range of $\pm 5\%$. Admittedly we also have shown that these numbers are quite sensitive to the parameter assumptions, so other parameter choices result in different estimates of the size of the transition effect. We show that the pension base and the accrual rate are both directly proportional to the transition effect; higher wage growth increases the transition effect, while (labor) population growth decreases the transition effect. The transition effect is largest for an interest rate slightly above 2%. Below this limit, the transition effect strongly increases with the interest rate, while for higher interest rates the transition effect gradually and slowly decreases with the interest rate. The reason is that for high interest rates, the uniform and the actuarially fair contribution rates decrease towards zero and, hence, the differences decrease as well. Finally, we have shown that there are substantial value transfers from poor to wealthy participants under the UPPS, which in itself is already a strong reason for abolishing the UPPS. The total perverse subsidy from poor to rich embedded in the current Dutch second pillar pension system is about 10 billion euros, which is about 1.5% of the Dutch GDP. This value transfer from poor to wealthy will continue when the UPPS is not abolished. The fact that the current system embeds a subsidy from poor to rich is arguably a reason for not (fully) compensating for the transition effect upon abolishing the UPPS.

The analysis in this paper can usefully be extended into several directions. First, we apply a deterministic model, but using a stochastic model would allow for the conditional indexation of pension rights and the probability of cutting pensions in calculating the transition effect, although we do not expect the results to change significantly once a stochastic model is used. Second, we assume that the term structure is flat, but in reality there is a yield curve, which means that a different discount rate should be used for different maturities. Finally, the estimation for the perverse subsidy from poor to rich can be improved in two ways. First the social economic differences between participants within one pension fund are in practice smaller than differences between all participants in the system as a whole. For example, some pension funds have participants from one type of profession only and average life-expectancy certainly varies across professional groups. By modeling one pension fund with participants equal to the total labor force, i.e. with both poor and wealthy participants, the perverse subsidy may well be overestimated. But second and likely to be of more quantitative significance, poor participants have a much lower life expectancy than rich participants. Not taking into account differences in life expectancy between poor and rich participants leads one to underestimate the perverse subsidy within the UPPS.

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A Mathematical derivations

A.1 Uniform contribution rate

The pension base can be rewritten as follows:

$$\begin{aligned}
PB_t =& \sum_{i=1}^{n} \left(u_{i,t}^{H} w_{i,t}^{H} + u_{i,t}^{L} w_{i,t}^{L}\right) \\
=& (1+\pi)^t (1+g)^t \sum_{i=1}^{n} \left(u_{i,0}^{H} w_{i,0}^{H} + u_{i,0}^{L} w_{i,0}^{L}\right) \\
=& (1+\pi)^t (1+g)^{n+t} \sum_{i=1}^{n} \left[\alpha (1+g)^{-i} w_{i,0}^{H} + (1-\alpha)(1+g)^{-i} w_{i,0}^{L}\right] \\
=& (1+\pi)^t (1+g)^{n+t} \sum_{i=1}^{n} \left[\alpha (1+g)^{-i} \left(1+c^H\right)^{i-1} + (1-\alpha)(1+g)^{-i} \left(1+c^L\right)^{i-1}\right] \\
=& (1+\pi)^t (1+g)^{n-1+t} \sum_{i=0}^{n-1} \left[\frac{\alpha \left(1+c^H\right)^{i} + (1-\alpha)\left(1+c^L\right)^{i}}{(1+g)^i}\right] \\
=& (1+\pi)^t (1+g)^{n-1+t} \sum_{i=0}^{n-1} \frac{\chi_i}{(1+g)^i}
\end{aligned}$$

with $\chi_i = \alpha\left(1+c^H\right)^i + (1-\alpha)\left(1+c^L\right)^i$

and, hence, the uniform contribution rate can be rewritten as

$$\begin{aligned}
P^U =& \frac{\rho \sum_{i=1}^{n} \left(u_{i,t}^{H} w_{i,t}^{H} + u_{i,t}^{L} w_{i,t}^{L}\right) K_i}{PB_t} \\
=& \frac{\rho \sum_{i=1}^{n} \left(u_{i,t}^{H} w_{i,t}^{H} + u_{i,t}^{L} w_{i,t}^{L}\right) K_i}{\sum_{i=1}^{n} \left(u_{i,t}^{H} w_{i,t}^{H} + u_{i,t}^{L} w_{i,t}^{L}\right)} \\
=& \rho q^{n+1} \frac{(1+\pi)^t (1+g)^t \sum_{i=0}^{m-1} q^i \sum_{i=1}^{n} \left[q^{-i}\left(u_{i,0}^{H} w_{i,0}^{H} + u_{i,0}^{L} w_{i,0}^{L}\right)\right]}{(1+\pi)^t (1+g)^{n-1+t} \sum_{i=0}^{n-1} \frac{\chi_i}{(1+g)^i}} \\
=& \rho q^{n} \frac{\sum_{i=0}^{m-1} q^i \sum_{i=0}^{n-1} \frac{\chi_i}{q^i (1+g)^i}}{\sum_{i=0}^{n-1} \frac{\chi_i}{(1+g)^i}}.
\end{aligned}$$

A.2 Pension liabilities

We can write L_t as

$$\begin{aligned}
L_t =& \sum_{i=1}^{n} K_i \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) + \sum_{i=n+1}^{m} K_i \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) \\
=& \sum_{i=1}^{n} \sum_{j=n+1-i}^{n+m-i} q^j \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) + \sum_{i=n+1}^{m} \sum_{j=0}^{n+m-i} q^j \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) \\
=& \sum_{i=1}^{n} \left[(1+g)^{n-i} \left(\alpha B_{i,t}^H + (1-\alpha) B_{i,t}^L\right) \sum_{j=n+1-i}^{n+m-i} q^j \right] \\
&+ \sum_{i=n+1}^{m} (1+g)^{n-i} \left(\alpha B_{i,t}^H + (1-\alpha) B_{i,t}^L\right) \sum_{j=0}^{n+m-i} q^j \\
\text{with } B_{i,t}^k =& \rho \left(1+c^k\right)^{i-1} \sum_{j=1}^{i} \left(\frac{(1+z)}{(1+\pi)\left(1+c^k\right)} \right)^{j-1} \quad \text{for } i \leq n \\
=& B_{n,t}^k \left(\frac{1+z}{1+\pi} \right)^{i-n} \quad \text{for } i > n.
\end{aligned}$$

The first and the second summation sign on the right hand side of L_t are the pension rights of all working cohorts and all retired cohorts, respectively.

A.3 Funding ratio development

The change of the funding ratio per period is

$$
\begin{aligned}
\Delta F_{t+1} =& F_{t+1} - F_t \\
=& \frac{(1+r)\left[A_t + \rho \sum_{i=1}^{n} \left(u_{i,t}^H w_{i,t}^H + u_{i,t}^L w_{i,t}^L\right) K_i\right]}{(1+g)(1+\pi) L_t} \\
&- \frac{(1+r) \sum_{i=n+1}^{n+m} \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right)}{(1+g)(1+\pi) L_t} - \frac{A_t}{L_t} \\
=& \frac{(1+r)\rho K' \left(u_t^H \circ w_t^H + u_t^L \circ w_t^L\right)}{(1+g)(1+\pi) L_t} \\
&- \frac{(1+r) \sum_{i=n+1}^{n+m} \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right)}{(1+g)(1+\pi) L_t} \\
&+ \frac{[(1+r) - (1+g)(1+\pi)] F_t}{(1+g)(1+\pi)} \\
\iff (1+g)(1+\pi) L_t dF_{t+1} =& \sum_{i=1}^{n} K_i \rho (1+r) \left(u_{i,t}^H w_{i,t}^H + u_{i,t}^L w_{i,t}^L\right) \\
&+ \sum_{i=1}^{n} K_i F_t [(1+r) - (1+g)(1+\pi)] \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) \\
&+ \sum_{i=n+1}^{m} \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) [(1+r) - (1+g)(1+\pi)] F_t K_i \\
&- \sum_{i=n+1}^{m} \left(u_{i,t}^H B_{i,t}^H + u_{i,t}^L B_{i,t}^L\right) (1+r).
\end{aligned}
$$

A.4 Net value transfer

The net value transfer of a group of generation j with type k is equal to

$$
\begin{aligned}
NVT_{j,t}^k =& u_{j,t}^k \sum_{i=\max(j,1)}^{n} \frac{w_{i,t+i-j}^k \left(P^U - P_i\right)}{(1+r)^{i-j}} \\
=& \frac{\Xi_j u_{n,0}^k}{\rho \sum_{l=n}^{n+m-1} q^l} \sum_{i=\max(j,1)}^{n} \left(\frac{(1+\pi)\left(1+c^k\right)}{(1+r)}\right)^{i-1} \left(P^U - P_i\right) \\
=& \Xi_j u_{n,0}^k \sum_{i=\max(j-1,0)}^{n-1} \left(\frac{(1+\pi)\left(1+c^k\right)}{q(1+r)}\right)^{i} \left(q^i \frac{\sum_{l=0}^{n-1} \frac{\chi_l}{q^l (1+g)^l}}{\sum_{l=0}^{n-1} \frac{\chi_l}{(1+g)^l}} - 1\right) \\
\text{with } \Xi_j =& \rho \sum_{l=n}^{n+m-1} q^l (1+\pi)^t (1+g)^{t+n-2} \left(\frac{(1+g)(1+\pi)}{(1+r)}\right)^{1-j}.
\end{aligned}
$$

For the entire generation, the net value transfer is equal to

$$\begin{aligned}NVT_{j,t} =& NVT^H_{j,t} + NVT^L_{j,t}\\ =&\Xi_j \sum_{i=\max(j-1,0)}^{n-1} \chi_i \left(\frac{(1+\pi)}{q(1+r)}\right)^i \left(q^i \frac{\sum_{l=0}^{n-1}\frac{\chi_l}{q^l(1+g)^l}}{\sum_{l=0}^{n-1}\frac{\chi_l}{(1+g)^l}} - 1\right).\end{aligned}$$

A.5 3-OLG model

A.5.1 Subsidy from poor to wealthy

The group with type k from generation j has the following gain under the alternative UPPS where we distinguish between different types:

$$\begin{aligned}G^k_{2,t} =& u^k_{2,t} w^k_{2,t} \left(P^U - P^{U,k}\right)\\ =& u^k_{2,0}(1+\pi)^t (1+g)^t \left(1+c^k\right)\left(P^U - P^{U,k}\right)\\ =& \rho q u^k_{2,0}(1+\pi)^t(1+g)^t\left(1+c^k\right)\left(\frac{(1+g)q+\chi_1}{1+g+\chi_1} - \frac{(1+g)q+\left(1+c^k\right)}{1+g+1+c^k}\right)\end{aligned}$$

and for $j \leq 1$:

$$\begin{aligned}G^k_{j,t} =& u^k_{j,t} \sum_{i=\max(j,1)}^{2} \frac{w^k_{i,t+i-j}\left(P^U - P^{U,k}\right)}{(1+r)^{i-j}}\\ =& (1+\pi)^t(1+g)^t u^k_{j,0} \frac{(1+\pi)^{1-j}}{(1+r)^{1-j}}\left(1+\frac{(1+\pi)}{(1+r)}\left(1+c^k\right)\right)\left(P^U - P^{U,k}\right)\\ =& \rho q u^k_{2,0}(1+\pi)^t(1+g)^t\left(\frac{(1+g)(1+\pi)}{(1+r)}\right)^{1-j}\\ &* \left((1+g)+\frac{(1+g)(1+\pi)\left(1+c^k\right)}{(1+r)}\right)\left(\frac{(1+g)q+\chi_1}{1+g+\chi_1} - \frac{(1+g)q+\left(1+c^k\right)}{1+g+1+c^k}\right).\end{aligned}$$

For an entire generation, by taking the sum of the two types, we get:

$$\begin{aligned}G_{2,t} =& G^H_{2,t} + G^L_{2,t}\\ =& \rho q u^H_{2,0}(1+\pi)^t(1+g)^t\left(1+c^H\right)\left(\frac{(1+g)q+\chi_1}{1+g+\chi_1} - \frac{(1+g)q+\left(1+c^H\right)}{1+g+1+c^H}\right)\\ &+ \rho q u^L_{2,0}(1+\pi)^t(1+g)^t\left(1+c^L\right)\left(\frac{(1+g)q+\chi_1}{1+g+\chi_1} - \frac{(1+g)q+\left(1+c^L\right)}{1+g+1+c^L}\right)\\ =& \rho q (1+\pi)^t (1+g)^t\\ &* \left(\chi_1\frac{(1+g)q+\chi_1}{1+g+\chi_1} - \alpha\left(1+c^H\right)\frac{(1+g)q+\left(1+c^H\right)}{1+g+1+c^H} - (1-\alpha)\left(1+c^L\right)\frac{(1+g)q+\left(1+c^L\right)}{1+g+1+c^L}\right)\\ =& \rho q (1+\pi)^t(1+g)^t(1+g)^2\frac{\left[\chi_1\chi_1 - (1-\alpha)\left(1+c^L\right)\left(1+c^L\right) - \alpha\left(1+c^H\right)\left(1+c^H\right)\right]}{(1+g+\chi_1)\left(1+g+1+c^H\right)\left(1+g+1+c^L\right)}(1-q)\\ =& \rho q (1+\pi)^t(1+g)^{t+2}\frac{\alpha(1-\alpha)\left(c^H-c^L\right)^2(q-1)}{(1+g+\chi_1)\left(1+g+1+c^H\right)\left(1+g+1+c^L\right)}\end{aligned}$$

and for $j \leq 1$:

$$G_{j,t} = \rho q (1+\pi)^t (1+g)^{t+2} \left(\frac{(1+g)(1+\pi)}{(1+r)}\right)^{1-j} \frac{\left[1-\frac{(1+\pi)(1+g)}{(1+r)}\right](1-\alpha)\alpha\left(c^H-c^L\right)^2(1-q)}{(1+g+\chi_1)(1+g+1+c^H)(1+g+1+c^L)}.$$

A.5.2 Funding ratio development

Without policy instruments the funding ratio is non-stationary:

$$\begin{aligned}
(1+g)(1+\pi)L_t\Delta F_{t+1} =& q^2\rho(1+r)(1+g)\\
&+q\left\{\rho(1+r)\chi_1 + F_t\left[(1+r)-(1+g)(1+\pi)\right]\left(\alpha B^H_{2,t}+(1-\alpha)B^L_{2,t}\right)\right\}\\
&+\frac{1}{1+g}\left(\alpha B^H_{3,t}+(1-\alpha)B^L_{3,t}\right)\left\{\left[(1+r)-(1+g)(1+\pi)\right]F_t-(1+r)\right\}\\
\iff \frac{(1+g)(1+\pi)L_t}{\rho}\Delta F_{t+1} =& q^2(1+r)(1+g)\\
&+q\left\{(1+r)\chi_1 + F_t\left[(1+r)-(1+g)(1+\pi)\right]\left(\chi_1+\frac{1+z}{1+\pi}\right)\right\}\\
&+\frac{1}{1+g}\left(\frac{1+z}{1+\pi}\right)\left\{F_t\left[(1+r)-(1+g)(1+\pi)\right]-(1+r)\right\}\left(\chi_1+\frac{1+z}{1+\pi}\right)\\
=& q^2(1+r)(1+g)+q(1+r)\chi_1-\frac{(1+r)(1+z)}{(1+g)(1+\pi)}\left(\chi_1+\frac{1+z}{1+\pi}\right)\\
&+F_t\left[(1+r)-(1+g)(1+\pi)\right]\left(q+\frac{1+z}{(1+g)(1+\pi)}\right)\left(\chi_1+\frac{1+z}{1+\pi}\right)\\
\text{with } B^k_{3,t} =& B^k_{2,t}\left(\frac{1+z}{1+\pi}\right)\\
=& \rho\frac{(1+\pi)\left(1+c^k\right)+(1+z)}{(1+\pi)}\left(\frac{1+z}{1+\pi}\right).
\end{aligned}$$

For $Q=1$, we have

$$\begin{aligned}
\Delta F_{t+1} =& \frac{\rho(1+z)\left[(1+g)(1+\pi)-(1+r)\right]}{(1+r)(1+g)^2(1+\pi)^2 L_t}\\
&*\left\{(1-F_t)\left(\frac{1+z}{1+\pi}+\chi_1\right)\left[(1+g)(1+\pi)+(1+r)\right]-(1+g)(1+\pi)\chi_1\right\}
\end{aligned}$$

which is equal to zero when $(1+r)=(1+g)(1+\pi)$ or when the funding ratio is equal to

$$\frac{(1+z)(1+g)(1+\pi)+(1+r)\left[(1+\pi)\chi_1+(1+z)\right]}{\left[(1+r)+(1+g)(1+\pi)\right]\left[(1+\pi)\chi_1+(1+z)\right]}.$$

Hence, we need to apply policy instruments in order to make the funding ratio stationary. For example, it is common to have an indexation rate which depends on the funding ratio. In other words, this implies that the indexation ratio becomes a function of the funding ratio, e.g. $z(F_t)$.