TI 2017-005/VI
Tinbergen Institute Discussion Paper

Debt Overhang, Exchange Rates and the Macroeconomics of Carry Trade

Revision: June, 2018

Sweder J.G. van Wijnbergen[1]
Egle Jakucionyte[1]

[1] Tinbergen Institute and the University of Amsterdam, the Netherlands.

Tinbergen Institute is the graduate school and research institute in economics of Erasmus University Rotterdam, the University of Amsterdam and VU University Amsterdam.

Contact: discussionpapers@tinbergen.nl

More TI discussion papers can be downloaded at http://www.tinbergen.nl

Tinbergen Institute has two locations:

Tinbergen Institute Amsterdam
Gustav Mahlerplein 117
1082 MS Amsterdam
The Netherlands
Tel.: +31(0)20 598 4580

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900

Debt Overhang, Exchange Rates and the Macroeconomics of Carry Trade

Egle Jakucionyte* and Sweder J. G. van Wijnbergen†‡

June 12, 2018

Abstract

The depreciation of the Hungarian forint in 2009 left Hungarian borrowers with a skyrocketing value of foreign currency debt. The resulting losses worsened debt overhang in debt-ridden firms and eroded bank capital. Although Hungarian banks had partially hedged exchange rate risk by extending FX-denominated loans, the ensuing debt overhang in borrowing firms exposed the banks to elevated exchange rate correlated credit risk in the run-up to an anticipated Euro adoption. This example of carry trade in emerging Europe motivates our analysis of currency mismatch losses in different sectors in the economy, and the macroconsequences of reallocating losses from the corporate to the banking sector ex post. We develop a small open economy New Keynesian DSGE model that links currency depreciation to corporate debt overhang and incorporates an active banking sector with financial frictions. The model, calibrated to the Hungarian economy, shows that after an unanticipated depreciation allocating currency mismatch losses to the banking sector generates a milder recession than if currency mismatch is placed at credit constrained firms. The government can intervene to reduce aggregate losses even further by recapitalizing banks and thus mitigating the effects of currency mismatch losses on credit supply.

Keywords: Debt overhang, foreign currency debt, leveraged banks, small open economy, Hungary;
JEL codes: E44, F41, P2

1 Introduction

In the period leading up to the great financial crisis Hungarian households and businesses exploited a favourable interest rate differential and ran up massive foreign currency debt. This carry trade was in expectation of low exchange rate volatility in the run-up to the anticipated adoption of the Euro

*Tinbergen Institute and the University of Amsterdam, the Netherlands.
†Tinbergen Institute and the University of Amsterdam, the Netherlands.
‡Corresponding author. Address: Roetersstraat 11, 1018 WB Amsterdam. Email address: s.j.g.vanwijnbergen@uva.nl (S.J.G. van Wijnbergen)

in the near future. But in the first months in 2009 the Hungarian forint lost 26% of its value against the euro and even more against the Swiss franc [1]. The clearly unanticipated sharp depreciation of the forint considerably magnified the debt-to-GDP ratio and prevalence of non-performing private loans increased sharply. Even those banks that shifted currency mismatch losses to borrowers by denominating loans in FX did not escape: while avoiding FX losses, they got increased credit risk in return. Corporate debt overhang in Hungary was as important as household debt overhang: in 2009 the share of corporate loans denominated in Swiss francs or euros was as high as the counterpart share in mortgages and amounted to more than fifty percent (Bank of Hungary (2012)). In this paper we choose to look at borrowing firms rather than indebted households to distinguish between the very different impact effects and transmission channels of non-performing corporate loans problems and the macroeconomic problems triggered by non-performing mortgages. We address household bankruptcies triggered by the deteriorating value of domestic currency in a companion paper (Jakucionyte and van Wijnbergen (2017)).

We focus on Hungary as the most pronounced case of currency carry trade via corporate loans in emerging Europe, but unhedged foreign currency borrowing in the private non-financial sector and substantial bank foreign debt were ubiquitous in the region (Bakker and Klingen (2012)). This motivates our focus on the macroeconomic implications of currency mismatch losses. In particular, what are the macroeconomic consequences of shifting exchange rate risk from borrowers to banks? Thus, besides the allocation of currency mismatch losses that reasonably resembles the Hungary's case before 2009, we also study a counterfactual case with bank lending denominated in domestic currency only. In contrast to foreign currency loans, domestic currency denomination shields domestic firms from currency mismatch and thus avoids potential debt overhang in the corporate sector, but at the expense of leaving banks with substantial funding from abroad with increased currency mismatch on their balance sheets. Resulting bank losses may impair the credit transmission channel as much as losses from non-performing loans in the former scenario. This trade-off is the topic of this paper.

We explore the macroeconomic consequences of this trade-off by developing a quantitative model with corporate debt overhang and an active banking sector facing financial frictions. We confirm that avoiding direct exposure to exchange rate fluctuations does not save banks from losses in times of domestic currency depreciation but we do show that, after an unanticipated depreciation, the economy incurs smaller aggregate losses if firms' net worth is preserved by shifting currency mismatch losses to banks. Banks are in a better position to absorb currency mismatch losses in spite of their high leverage because, in contrast to firms, they can more easily be recapitalized simply by being smaller in number. The second reason why allocating currency mismatch losses to firms generates larger real losses is that excessive corporate debt affects firms' decisions as they occur and thus inflicts output losses directly, while bank losses affect aggregate economic activity with a lag and only after a share of the effect is absorbed by bank equity.

Currency mismatch losses in Hungary

[1] By March 2009, compared to September 2008.

The currency mismatch situation in Hungary was unavoidably shaped by financial vulnerabilities developed prior to the forint depreciation. Our focus on debt overhang as triggered (or intensified) by the forint depreciation is supported by the data. In the run up to the crisis more than one half of private loans were taken in Swiss francs or euros (Bakker and Klingen (2012)). Brown and Lane (2011) and Herzberg (2010) state that foreign currency borrowing in emerging Europe was not large-scale and concentrated among exporting firms, but studies with access to firm-level data in Hungary cast doubt on the firms' ability to hedge against the currency risk: Endrész et al. (2012) find that more than 82% of firms with foreign currency debt had no foreign currency revenue from exports, the survey of 698 Hungarian firms (Bodnár (2012)) discovers that also around 80% of foreign currency borrowers did not have a natural hedge. The weaker Hungarian forint resulted in significantly more bankruptcies among firms that borrowed in Swiss francs rather than Hungarian forints (Figure 2). Vonnák (2015) confirms that currency mismatch, and not the lending practices of Hungarian banks, contributed the most to the riskiness of foreign currency borrowers.

After 2008, foreign currency borrowers in Hungary were more likely to default and reduce investment (Endrész et al. (2012)). Foreign currency borrowers were not only riskier, but, as data analysis in Endrész et al. (2012) shows, also had sizable shares in aggregate variables such as investment and debt in Hungary. We notice that at the macro level the gap between private investment and profit shares in Hungary kept increasing: after 2008 investment declined by more and took longer to recover than the measure for corporate profitability (Figure 1). Apparently, Hungarian firms were unwilling to invest retained earnings for several years which is a strong indication of worsening debt overhang. In contrast to monitoring costs based models (like Bernanke et al. (1999)), Debt Overhang based approaches can explain prolonged under-investment in the recovery environment. If firms perceive their chances to default on accumulated debt as sufficiently high, their private benefits from investing diminish (Myers (1977)). Recessions with investment falling below the socially optimal level of investment tend to be deeper and longer.

Currency mismatch both in the corporate sector and in the banking sector is at the heart of the problem. Both businesses and banks in Hungary borrowed in foreign currency (Hungarian Banking Association (2012)). The banks' currency mismatch was reinforced by tight funding links between foreign parent banks and their subsidiaries in Hungary before the crisis. Moreover, isolation of currency mismatch losses in one sector is impossible due to the credit channel as banks are the main source of credit in the economy. This is common in all of emerging Europe, where they intermediate up to 80% of total credit (World Bank (2015a); World Bank (2015b)). Passing on FX mismatch to bank borrowers would not really isolate the banks given their predominant position as providers of debt to non-financial firms: even if only borrowers would have faced currency mismatch, domestic currency depreciation would deteriorate the quality of such loans and banks would shrink credit supply anyhow faced with rising Non Performing Loans (NPL) ratio's. Damage to the credit provision channel constituted the core of the ECB critique of the early repayment scheme of foreign currency mortgages with an artificially strong exchange rate instituted by the Hungarian Government for consumer mortgages, effectively shifting losses back to the lending banks: In 2011, against the advice of the ECB (ECB (2011)), the Hungarian government adopted such a scheme to aid debt-ridden households and forced banks to take massive losses[2]. In the authorities' view, losses that extensive might have posed a real threat of interrupting credit provision in Hungary and cast doubt on saving borrowers at the expense of lenders (even when lenders are foreign-owned). Even though this policy targeted households, we take it as evidence for the importance of credit channel.

For bank losses to impair credit provision, bank funding costs and loan supply have to depend on bank performance. Indeed, banks are frequently leverage-constrained themselves during crises as their own access to funding depends on the riskiness of their balance sheets (e.g. Diamond and Rajan (2009)). The banking system in Hungary was well-capitalized in 2008 (IMF (2008)), however, liquidity shocks at the outbreak of the crisis changed the situation dramatically (Gulde and Giorgianni (2012)). The sudden dry-up of foreign funding caused a tightening of leverage constraints. To capture this channel, we introduce the second financial friction in the banking sector, namely a leverage constraint. We model it as an agency problem between banks and depositors following Gertler and Karadi (2011). The agency problem prevents banks from unlimited expansion of their balance sheets in good times. In bad times, non-performing loans in the corporate sector deplete bank equity so that the leverage constraint becomes tighter and leads to higher borrowing costs for banks. Eventually, the endogenous leverage constraint amplifies the drop in lending and economic activity. The feedback in bank lending is what makes the model structure complete and suitable to answer the research question formulated.

But what triggered the debt overhang situation to begin with? We look at the major shocks at the onset of the crisis in Hungary that could have led to domestic currency depreciation and so magnified the domestic currency value of foreign currency loans. The chronology of the pre-crisis events in emerging Europe points to external triggers instead of shocks of a local origin: despite severe domestic imbalances in emerging Europe, depreciation of local currencies followed spill-overs from the looming economic crash in advanced economies rather than happening at the same time. Based on anecdotal evidence and data (Bakker and Klingen (2012)) we choose to look at three alternative (but not mutually exclusive) potential culprits: capital outflows, a drop in world demand for domestic exports and an increase in volatility in the markets.

We feed shocks into a small open economy New Keynesian model calibrated to Hungarian data. The international trade structure embedded in the model economy is similar to the set up used in Gali and Monacelli (2005), García-Cicco et al. (2014) and Adolfson et al. (2007). But the main new feature is the introduction of explicit debt overhang on the corporate level in the manner of famous paper Merton (1974) on pricing credit risk, where he shows that limited liability essentially implies a put option written by creditors to equity holders. Myers (1977) uses this approach to explore the concept of debt overhang and its impact on investment, also a key element of our paper. So we extend the endogenous leverage constraint model of Gertler and Karadi (2011) to

[2]The estimated total bank losses from the early repayment scheme were around 1.1 billion euros or around 10% of total bank capital in Hungary (Reuters (2012); authors' calculations).

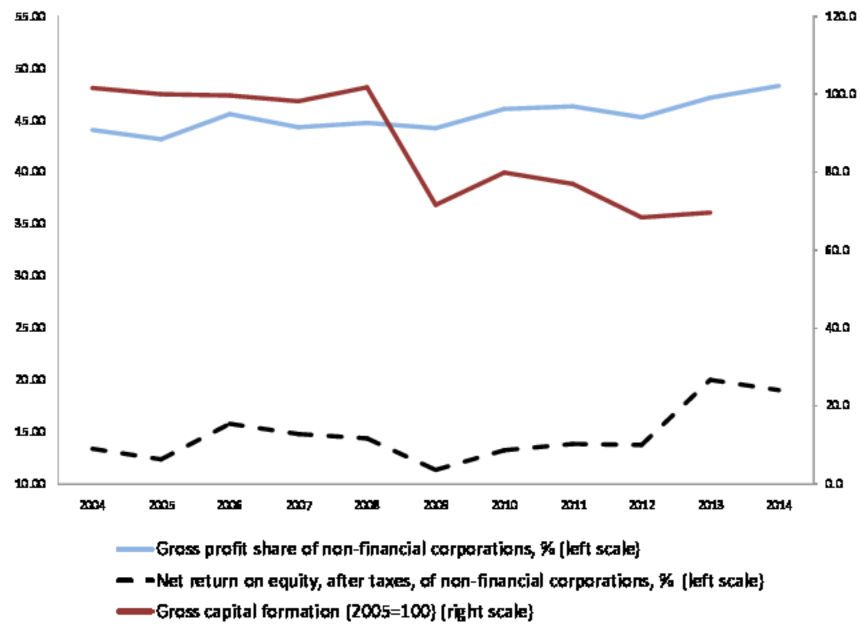


Figure 1: Profit share and private investment in Hungary.

Source: Eurostat.

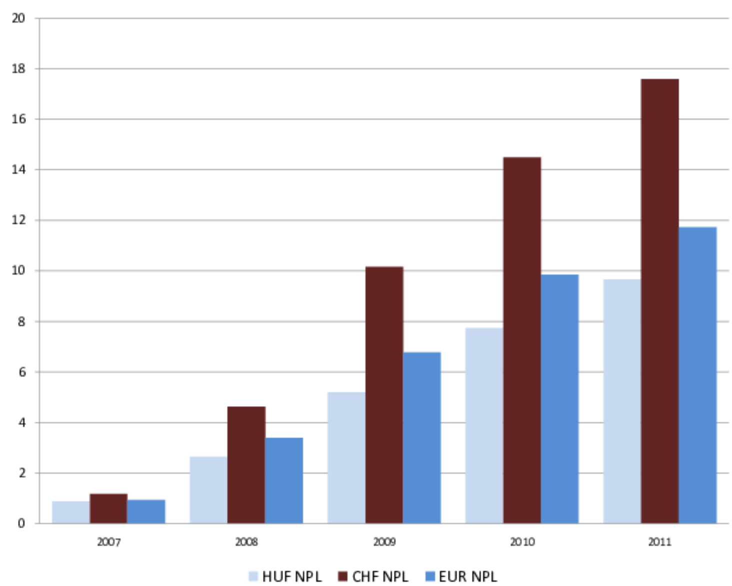


Figure 2: Ratio of non-performing corporate loans by currency in Hungary.

Source: Vonnák (2015).

include Merton (1974) like debt overhang on the corporate level with its associated moral hazard problems highlighted by Myers (1977). [3]

The Merton put option approach to financial frictions between lender and borrower leads to another novelty in the paper. Despite using first-order approximation techniques to solve the resulting DSGE, volatility does have a first order impact on model outcomes because volatility shows up in the derivatives of that Merton put with respect to corporate investment and employment, in the same way volatility has an impact on general option derivatives ("the Greeks"), so we can use our model to study the impact of volatility shocks. The volatility related put option term in the financially constrained firms' optimization problem drives a wedge between social and private benefits from investing. Besides modeling a shock to volatility of firms' future profits, we endogenize volatility by incorporating uncertainty about prices: we simulate the model going back and forth between assumed and generated volatility until the two converge, thus endogenizing the overall volatility of corporate profits [4]. The obtained volatility value contains more information about the propagation of a particular shock in our model and thus is superior to an arbitrarily calibrated value.

The debt overhang friction stems from a particular limited contractibility feature of the debt contracts in the model. Borrowing firms are subject to limited liability which skews incentives towards taking too much risk and rules out a risk-free debt contract from the menu of optimal contracts. Second, banks cannot write a contract on how the loans they extend will be used: the quantities of capital and labour are determined unilaterally by the firm after it has received the loan. In the event of adverse shocks, these frictions may create debt overhang and distort the firms' choice of capital and labour demand.

The idea that risky debt makes firm forego valuable investment opportunities of course goes back to Myers (1977). Limited liability implies that debt is risky which may incentivize a sub-optimal investment strategy. Myers (1977) does not explore how the reduced value of the firm would affect firm's borrowing costs, the idea that default risk feeds into the credit spread is formalized in Merton (1974) who derives the credit spread as reflecting the unavoidable put option on the future assets of a debtor written out by the creditor to the equity holder. Our setup incorporates both seminal ideas: if debt is high enough, firms' incentives to invest diminish and a default spread goes up reinforcing the mechanism.

Out of several explanations how debt can reinforce business cycle fluctuations, only debt overhang is suitable for our research problem. The costly state verification framework famously introduced in macroeconomics by Bernanke et al. (1999) just introduces an interest rate wedge, but because it allows lenders and borrowers to contract on investment and employment, avoids moral hazard and the associated debt overhang problems. A default wedge as in Gourio (2013) introduces corporate default effects on input providers instead of lenders and thus abstracts from the credit channel which is crucial in the Hungarian story. This paper is the first attempt to use the non-contractible investment approach to explain the role of excessive debt and foreign currency

[3] Occhino and Pescatori (2015) follow a similar approach, with one crucial difference which we come back to below.

[4] We thank Christian Stoltenberg for suggesting this numerical approach to endogenizing volatility.

debt in particular in business cycle analysis.

The structure of the paper is as follows. We discuss related literature in section 2 and the model in detail in section 3, and show simulations in section 4. We discuss the results in section 5, while section 6 concludes.

2 Related literature

There is a lengthy corporate finance literature on debt overhang that starts with the seminal paper of Myers (1977). We contribute to the literature on macroeconomic consequences of debt overhang that were first examined in Lamont (1995). He argues that debt overhang can create strategic complementarities among investments of individual agents, thus potentially leading to multiple equilibria. Philippon (2010) studies the interaction between different indebted sectors in his model economy. The paper argues that in a closed economy, bailing out banks is efficient, while bailing out insolvent households means transferring funds to households that made inefficient saving decisions and anyhow will be paid for by these same households through increased taxes. In an open economy, countries have an incentive to free ride on foreign recapitalization programs, therefore, international coordination is required. We go beyond the analysis in Philippon (2010) by explicitly introducing corporate debt overhang and its interactions with banking frictions, linking the concept of debt overhang to excessive foreign currency debt. Our set up comes closer to Gomes et al. (2016) and Occhino and Pescatori (2014), who analyze the conduct of monetary policy in an environment with nominal debt. However, they focus on the effect of unanticipated inflation, while we focus on the debt overhang situation that arizes after domestic currency depreciation.

There is a vast literature that explores foreign currency debt effects in the costly state verification framework as implemented in Bernanke and Gertler (1989) and Bernanke et al. (1999). Traditionally domestic currency depreciation invokes an expenditure switching effect that should stabilize demand for domestic goods. However, high foreign currency debt together with monitoring costs and sticky prices can potentially outweigh the expenditure switching effect and in turn make depreciations contractionary. Céspedes et al. (2004), Devereux et al. (2006), and Gertler et al. (2007) study the depreciation effects on firms in a small open economy setting. They incorporate a model of investment in which net worth affects the cost of capital and allow firms to borrow in foreign currency. They argue that even with high foreign currency debt depreciations remain expansionary. A similar model is considered in Cook (2004) where it leads to the opposite conclusion. Cook (2004) attributes this discrepancy to different assumptions made on which prices are sticky and which are flexible. If, as in Céspedes et al. (2004), input prices are sticky but output prices are not, domestic currency depreciation lowers real wages and increases revenues of firms and the depreciation remains expansionary. If, as in Cook (2004), output prices are sticky and input prices are not, revenues do not increase as fast as input costs and the depreciation can become contractionary. None of these studies explicitly considers depreciation related debt overhang and its implications for corporate employment and investment decisions and subsequently the maxcroeconomy, the key focus of our paper.

Empirical studies have established the relevance of financial frictions in explaining the macroeconomic outcomes. Without taking a stand on the prevalent financial friction, Towbin and Weber (2013) look at the data for 101 countries from 1974-2007 and show that high foreign currency debt increases the decline in investment in response to adverse external shocks. Kalemli-Özcan et al. (2015) study firm-bank-sovereign linkages in Europe to weigh the role of several financial frictions. They find that debt overhang is more important in explaining weak investment relative to explanations focusing on weak bank and other weak firm balance sheet channels. This finding supports our introduction of debt overhang and its impact on investment and employment decisions.

Another branch of the literature that we relate to is centered upon volatility shocks. A recent contribution by Christiano et al. (2014) attributes a significant share of business cycle fluctuations to idiosyncratic risk shocks fed through the time-varying idiosyncratic variance component. The variance component appears in the credit spread of entrepreneurs as in the costly state verification framework implemented in Bernanke et al. (1999). Thus the impact of the risk shock affects the credit spread rather than the default wedge in the firm's investment decision. In our setup the mechanism is different and the quantitative impact larger: volatility enters through its impact on the value of the embedded options we use to model debt overhang in line with the Merton approach to credit risk.

3 Model

Our focus is on the interaction between FX losses induced debt overhang, undercapitalized banks and corporate investment and employment decisions. To that end we introduce a Merton (1974)/Myers (1977) like debt overhang friction[5] in a model with leverage constrained banks in a small open economy context with foreign currency denominated private debt. The open sector with nominal rigidities generates realistic lending and output dynamics in the presence of foreign currency loans. We start the outline of the model by describing the more novel sections. We describe the more standard model blocks only briefly in the main text, all model details and associated derivations are in the supplementary appendix.

3.1 Financially constrained firms

Financially constrained firms live for two periods. Every period there is a new-born generation of firms and the total number of firms always constitute a continuum of mass one. In the first period firms buy two types of inputs, capital k and labour h, and have to pay for a fraction ρ in advance, which generates their demand for working capital. Production takes place in the next period.

To pay in advance, a financially constrained firm i uses two types of financing. First, it receives equity from households, $N_{i,t}^{firms}$. Second, it borrows from the bank an amount $L_{i,t}$ that consists

[5]See Occhino and Pescatori (2015) for a similar approach to corporate debt overhang, but in a closed economy model.

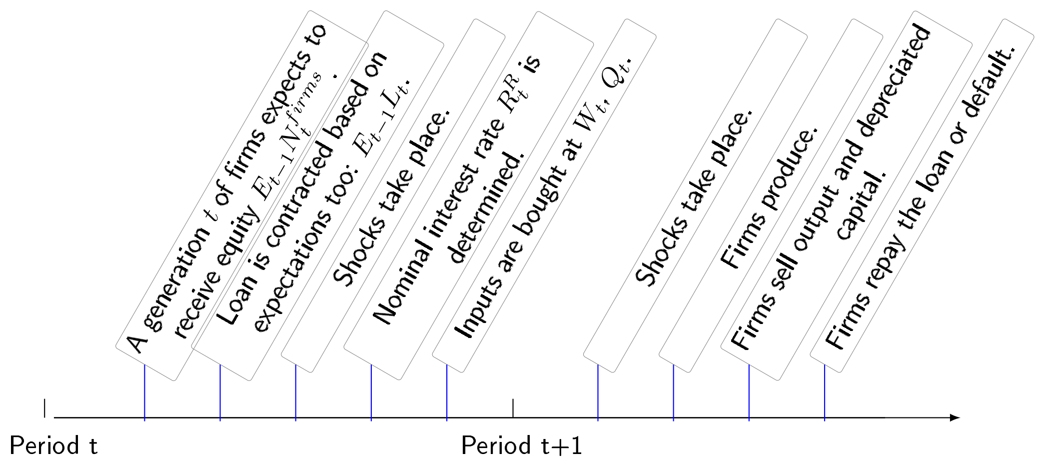


Figure 3: Timing for financially constrained firms.

of both domestic currency funds $L_{i,t}^D$ and foreign currency denominated funds $L_{i,t}^F$ such that $L_{i,t} = L_{i,t}^D + S_t L_{i,t}^F$ where S_t is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by α^F, so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

To borrow, the firm has to pledge a share κ of future revenue as collateral where $0 < \kappa \leq 1$. We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital minus the expected equity transfer from the household. It follows that in the beginning of period t the following condition holds:

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n_{i,t}^{firms}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\} \tag{1}$$

where q_t, w_t and rer_t denote the real price of capital, the real wage and the real exchange rate respectively. All three prices are expressed in units of composite goods. It follows that we define the real exchange rate as $S_t P_t^* / P_t$ where S_t is the nominal exchange rate, P_t is the price of composite goods and P_t^* defines the price level of foreign composite goods. $n_{i,t}^{firms}$ stands for the real equity transfer from the domestic household, where $n_{i,t}^{firms} \equiv N_{i,t}^{firms}/P_t$. $l_{i,t}$ stands for the size of the total loan expressed in units of composite goods and is defined as $l_{i,t} \equiv L_{i,t}/P_t$. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm's private incentives to invest in production inputs.

The amount of corporate equity available is a factor in determining the firms' demand for funds and sets its "distance to default". In bad times, a higher fraction of firms default, which decreases the total value of corporate net worth. The household pools retained earnings and distributes them to new-born firms equally. So in bad times new generations of firms receive less equity from the household, therefore to produce the same amount of goods they have to leverage up more and thus will face a higher default risk. This introduces persistence in the debt overhang shock, a dynamic mechanism that is not rpesent in Occhino and Pescatori (2014). Note that firms die after two periods and thus do not take into account profits further out in the future, which mutes the macroeconomic net worth effect to some extent. The first generation of firms that enters the scene after the shock makes its borrowing decision based on expectations about the value of its net worth, so the net worth effect materializes for future generations of firms only.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases the owner of the firm (the domestic household) steps in and transfers lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t}/P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on them as the main source of finance. These funds enter the domestic household's budget constraint as a lump-sum transfer and have no effect on either the household's or the firm's incentives.

Let the matured loan in units of composite goods be $R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right)$, where $R^R_{i,t}$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as $l^D_{i,t} \equiv L^D_{i,t}/P_t$ and $l^F_{i,t} \equiv L^F_{i,t}/P^*_t$. The contracted collateral is a fraction κ of firms' revenue from selling goods and depreciated capital in the next period, $p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}$. p^R_{t+1} stands for the price of homogenous goods, expressed in units of composite goods ($p^R_{t+1} \equiv P^R_{t+1}/P_{t+1}$). Then the decision of the financially constrained firm i born in period t whether to default or not is determined by the lower value:

$$\min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\} \tag{2}$$

where $p^R_{t+1}y^R_{i,t+1} = p^R_{t+1}A_{t+1}\theta_{i,t+1}k^{\alpha}_{i,t}h^{1-\alpha}_{i,t}$.

The firm i born in period t and endowed with corporate equity $n^{firms}_{i,t}$ maximizes profits taking the loan as given. The firm maximizes the expected sum of future revenue from selling goods and depreciated capital subtracted by the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period t also enter the maximization problem and can be summarized as the difference between the loan plus equity (both $n^{firms}_{i,t}$ and $z_{i,t}$) and working capital expenditure:

$$\max_{\{k_{i,t},h_{i,t}\}} E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_tk_{i,t}+w_th_{i,t}}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right),\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$
$$+l_{i,t}+n^{firms}_{i,t}+z_{i,t}-\rho\left(q_tk_{i,t}+w_th_{i,t}\right)$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n^{firms}_{i,t}\right\}=E_{t-1}\left\{\rho\left(q_tk_{i,t}+w_th_{i,t}\right)\right\}$$

The resulting first-order conditions are[6]:

$$k_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}}+q_{t+1}(1-\delta)-(1-\rho)\frac{q_t}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}}+q_{t+1}(1-\delta)\right)\right\}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right),\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}$$
$$+\rho q_t$$

$$h_{i,t}:\quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}-(1-\rho)\frac{w_t}{\pi_{t+1}}\right\}$$
$$-E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}\right)\right\}$$
$$=\frac{\partial cov\left(\beta\Lambda_{t,t+1},\quad \min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right),\quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}}$$
$$+\rho w_t$$

where

$$d_{2,t}\equiv\frac{E_t\ln\left(\kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)-R^R_t rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right)-E_t\ln\left(R^R_{i,t}\frac{l^D_{i,t}}{\pi_{t+1}}\right)}{\sigma_y},\qquad d_{1,t}=d_{2,t}+\sigma_y$$

[6]The derivation of the first-order conditions and the term $d_{2,t}$ in particular are provided in the supplementary appendix A1-A2.

Note the similarity to the credit risk approach pioneered by Merton (1974): because of limited liability firms effectively receive a put option from creditors, ex ante this is priced in (that is where the credit risk comes from) but because investment and employment are not contractible in the debt contract, a moral hazard problem persists. The debt overhang friction introduces an additional term in otherwise standard demand functions for capital and labour: conditions incorporate a proxy for the default probability, $(1 - \Phi(d_{1,t}))$, that reduces a marginal product of capital and a marginal product of labour. Thus in this problem the default probability is what drives the wedge between social benefits from investing and private benefits from investing. When the default probability increases, private benefits diminish and demand for labour and capital shrinks accordingly, resulting in a lower level of working capital than a socially optimal one. Under-investment in working capital has negative and prolonged implications on aggregate variables: we can distinguish between static debt overhang effects and dynamic debt overhang effects. Static debt overhang results from a decline in demand for working capital which depresses aggregate demand on impact. Dynamic debt overhang occurs because the indebted sector uses capital as input. Then sub-optimally lower demand for capital shrinks demand for investment. Lower investment today decreases capital stock available for production tomorrow which prolongs the economic recovery.

The second implication of the first-order conditions relates to the option structure as reflected by the definition of the function argument $d_{2,t}$. The default probability directly depends on a volatility term σ_y^2 which captures the variance of future profits. σ_y^2 is given by

$$var\left(\pi_{t+1}\left(\kappa\left(p_{t+1}^R y_{i,t+1}^R + q_{t+1}(1-\delta)k_{i,t}\right) - R_t^R \frac{rer_{t+1} l_{i,t}^F}{\pi_{t+1}^*}\right)\right)$$

and depends on exogenous productivity shocks, working capital and endogenous volatility of prices and exchange rate value in the domestic economy. The first-order conditions imply that increased uncertainty about of future collateral value reduces firms' chances to repay. Looming uncertainty during the latest crisis[7] highlights the importance of the volatility term in explaining borrowing conditions for firms and firms' willingness to borrow and suggests that we cannot assume constant volatility without a loss of generality. Thus we model an exogenous shock to a volatility term to simulate increased uncertainty about financially constrained firms' performance in the future as one of possible triggers of debt overhang.

Note that the default probability varies not only with stochastic components such as technology but with expected prices and exchange rates as well. This motivates our simulation exercise in which we simulate the model until the endogenously implied volatility of firms' expected collateral value converges. This exercise allows us to incorporate the second-order characteristics of the economy and obtain a better estimate for the volatility term than an arbitrary calibrated value.

In the beginning of every period, after shocks take place and a fraction of firms default, the

[7]The implied volatility indexes for both European markets and Poland rocketed in the end of 2008, see the plot in the Appendix (Figure 9). We do not have a measure for Hungary, however, the implied volatility index for Polish markets should serve as a satisfactory proxy for the markets' risk perception for the Hungarian economy.

domestic household pools the remaining net worth from non-defaulted firms into aggregate net worth by following the aggregation rule:

$$
\begin{aligned}
n_t^{firms} = \;& \omega^{firms}\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1} - (1-\rho)\frac{q_{t-1}k_{t-1} + w_{t-1}h_{t-1}}{\pi_t}\right) \\
& - \omega^{firms}\left((1-\Phi(d_{1,t-1}))\,\kappa\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1}\right) + \Phi(d_{2,t-1})R_{t-1}^R\frac{l_{t-1}^D}{\pi_t} + \Phi(d_{1,t-1})rer_t\frac{l_{t-1}^F}{\pi_t^*}\right) \\
& + \iota^{firms}\cdot n^{firms}
\end{aligned}
$$

Recall that $(1-\Phi(d_{1,t-1}))$ proxies for the default rate (by the law of large numbers this is equal to the share of defaulted firms in the economy). Then the first term on the right hand side is aggregate firms' revenue from production and selling depreciated capital minus the rest of the expenditure for working capital. The second term is the firms' aggregate expenditure for repaying loans. The difference between the two gives financially constrained firms' profits. The third term is the injection of new equity. We assume that the domestic households acts as distributor and cannot divert pooled equity funds anywhere else. Also the existing equity can be increased only by the amount $\iota^{firms}\cdot n^{firms}$ that is fixed and proportional to aggregate net worth in the steady state. Thus, this equity transfer does not depend on the household's decision. ω^{firms} is a fraction that is close but lower than unity. We assume that this parameter proxies for the equity management costs incurred by the household and use this parameter to calibrate the steady state corporate leverage to the one observed in the data.

3.2 Banks

Domestic households own all banks that operate in the domestic economy and lend to financially constrained domestic firms. We assume that there is a continuum of these banks and every period there is a probability ω that a bank continues operating. Otherwise, the net worth is transferred to the owners of the bank, domestic households.

We assume that banks give loans to firms out of accumulated equity n_t, domestic deposits d_t and foreign debt d_t^*. A fraction of banks' liabilities (foreign debt) is denominated in foreign currency which exposes banks to currency mismatch. Lending in foreign currency hedges the open currency position for banks[8]. However, shifting exchange rate risk to the credit constrained corporate sector increases the credit risk for banks. We consider two lending scenarios which have different implications for bank currency mismatch. First, banks lend in domestic currency only which creates currency mismatch on their balance sheets. The second scenario is described by bank lending in both foreign currency and domestic currency so that banks are relieved from currency mismatch. We will consider these two cases in the following discussion on shifting currency mismatch. The

[8]We calibrate the share of loans denominated in foreign currency such that banks do not have a zero open currency position in that case. This allows us to distinguish between the credit risk effects and the exchange rate risk effects.

model with loans denominated in both currencies is described here, while the model with lending in domestic currency only is described in the supplementary appendix B2.

The balance sheet constraint of a bank j, expressed in units of composite goods, is given by

$$n_{j,t} + d_{j,t} + rer_t d^*_{j,t} = l_{j,t}$$

Banks pay a nominal domestic interest rate R_t on deposits and a nominal foreign interest rate $R^*_t \xi_t$ on foreign debt. R^*_t follows a stationary AR(1) process. ξ_t denotes a premium on bank foreign debt. To ensure stationarity in the model, we assume that the premium depends on the level of foreign bank debt (as in Schmitt-Grohé and Uribe (2003)):

$$\xi_t = \exp\left(\kappa_\xi \frac{(rer_t d^*_t - rer \cdot d^*)}{rer \cdot d^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{3}$$

where ζ_t is an exogenous shock that follows a stable AR(1) process.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction λ^L of assets, but if that happens the bank goes bankrupt (i.e. cannot continue). Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is equal to or higher than the value of what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet of the bank for given amount of equity.

Loan performance directly affects bank profits, loans to domestic financially constrained firms are the only asset on the banks' balance sheet. When the default probability $(1 - \Phi(d_{2,t}))$ for financially constrained firms increases, banks expect lower returns. High corporate leverage has similar consequences as it increases the size of loans for the same level of production and reduces firms' chances to repay ceteris paribus. We define the expected return for the bank j as $R^L_{j,t}$. The definition makes use of the derivation of the expected loan payment (see the supplementary appendix A2) and in its final expression directly incorporates the default probability on corporate loans:

$$E_t\left\{\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t}\right\} \equiv E_t \min\left\{R^R_{j,t}\left(\frac{l^D_{j,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{j,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1} y^R_{j,t+1} + q_{t+1}(1-\delta)k_{j,t}\right)\right\}$$

$$\Rightarrow E_t\left\{\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t}\right\} \equiv E_t\left\{(1-\Phi(d_{1,t}))\,\kappa\left(p^R_{t+1} y^R_{j,t+1} + (1-\delta) q_{t+1} k_{j,t}\right) + \Phi(d_{2,t}) R^R_{j,t}\frac{l^D_{j,t}}{\pi_{t+1}} + \Phi(d_{1,t}) R^R_{j,t} rer_{t+1}\frac{l^F_{j,t}}{\pi^*_{t+1}}\right\} \tag{4}$$

To facilitate further discussion, we define two components of the overall bank spread (actual rate charged to borrowers minus the cost of funds to the bank). The first is the default spread, measured as the difference in the actual interest rate charged on the loan and the expected return on the loan: $E_t\left(R^R_{j,t}-R^L_{j,t}\right)/\pi_{t+1}$. The higher is the spread, the more the bank charges to compensate for the default risk. Second, there is the component of the overall bank spread that depends on the banking friction: it captures the premium that arises due to the endogenous leverage constraint. This spread is given by the difference in the expected return on the loan to financially constrained firms and the expected funding costs to the bank: $E_t\left(R^L_{j,t}/\pi_{t+1}-R^*_t\xi_t/\pi^*_{t+1}\frac{rer_{t+1}}{rer_t}\right)$. Note the role of real exchange rate changes in determining the expected costs of funding. So the overall credit spread is the sum of the default spread and the bank spread and is given by $E_t\left(R^R_{j,t}/\pi_{t+1}-R^*_t\xi_t/\pi^*_{t+1}\frac{rer_{t+1}}{rer_t}\right)$. A higher credit spread reflects tighter borrowing conditions due to either one or both of the financial frictions.

Then the optimization problem of the bank j can be written as:

$$V_{j,t}=\max_{\left\{d_{j,t},d^*_{j,t},l_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1}+\omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t}\geq\lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t}+d_{j,t}+rer_t d^*_{j,t}=l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t}=\frac{R^L_{j,t-1}}{\pi_t}l_{j,t-1}-\frac{R_{t-1}}{\pi_t}d_{j,t-1}-\frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

The first-order conditions follow:

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right)=\nu_{2,t} \tag{5}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t}\right)=\nu_{2,t} \tag{6}$$

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R^L_{j,t}}{\pi_{t+1}}\right)=\lambda^L\nu_{1,t}+\nu_{2,t} \tag{7}$$

$\nu_{1,t}$ and $\nu_{2,t}$ are the Lagrangian multiplier to the incentive constraint and the Lagrangian multiplier to the balance sheet constraint combined with the law of motion for equity, respectively.

Equations (5) and (6) govern the bank debt portfolio choice. Equation (5) presents the marginal cost to the bank from issuing one additional unit of deposits (the left hand side) in relation to the marginal benefit from increasing equity by one unit, $\nu_{2,t}$ (the right hand side). The marginal cost from issuing one additional unit of foreign bank debt is compared to the marginal benefit from increasing equity on the right hand side of equation (6) and is adjusted for changes in the exchange rate value. The structure of these choice rules suggests that in equilibrium the bank has to be indifferent between taking deposits or issuing bank debt to foreign agents.

Equation (7) presents the relation between the marginal benefit to the bank from issuing one additional unit of loans (the left hand side) and the marginal cost (the right hand side). We see that in equilibrium one additional unit of loans earns the discounted risk adjusted return on loans. Firstly, this return has to increase in the marginal cost from issuing bank debt to finance the expansion of the balance sheet, $\nu_{2,t}$. Secondly, due to the endogenous bank leverage constraint, the risk adjusted bank return on loans also increases in the share of divertable assets λ^L and the marginal loss to the bank creditor in the case of asset diversion, $\nu_{1,t}$. Both terms proxy for the marginal cost associated with the tighter incentive constraint. Moreover, the tighter leverage constraint increases the bank spread as well which translates into more credit tightening.

The first-order conditions hold together with complementary slackness conditions:

$$\nu_{1,t}: \quad \nu_{1,t}\left(V_{j,t} - \lambda^L l_{j,t}\right) = 0$$

$$\nu_{2,t}: \quad \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - rer_t l^*_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) = 0$$

The set of equilibrium conditions also includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by domestic households and is assumed to be of the size ιn. Then

$$n_t = \omega\left(\frac{R^L_{j,t-1}}{\pi_t} l_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{t-1}\right) + \iota n \tag{8}$$

3.3 Financial sector support

Financial sector support is modelled as in Kirchner and van Wijnbergen (2016) and Van der Kwaak and Van Wijnbergen (2014): we assume that the government can intervene during the crisis by injecting capital τ^{FS}_t in the banks. We assign the following rule to the recapitalization of the financial intermediary j:

$$\tau^{FI}_t = \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1}, \qquad \kappa_{FS} > 0, \quad l \geq 0$$

where $n_{j,t-1}$ is the net worth of the intermediary from the previous period. The recapitalization can be immediate ($l = 0$) or delayed ($l > 0$). The variable $shock_t$ equals the shock driving the crisis, e.g. the risk premium shock ($shock_t \equiv \xi_t$). We assume that the recapitalization is a gift from the government and does not have to be repaid (Van der Kwaak and Van Wijnbergen (2014) explore the consequences of different payback rules).

Now the bank equity increases in the equity injection from the government besides being a function of loan returns and borrowing costs:

$$n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS} \left(shock_{t-l} - shock \right) n_{j,t-1}$$

Bank's optimization problem would yield different results now. We present modified first-order conditions in the supplementary appendix B3.

3.4 Households

We assume a representative household. The household has two alternatives to invest in: make deposits d_t in a bank or buy domestic bonds issued by the government, b_t. The household supplies labour to a competitive labour market. The household has Greenwood–Hercowitz–Huffman (henceforth, GHH) preferences as in Greenwood et al. (1988), so labour supply does not depend on wealth. The household chooses a level of real consumption c_t and working hours h_t such that the following lifetime utility function is maximized:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma} \left(c_t - \frac{\chi \left(h_t \right)^{1+\varphi}}{1+\varphi} \right)^{1-\gamma} \qquad \gamma, \chi, \varphi > 0 \tag{9}$$

subject to the household's budget constraint, expressed in units of composite goods:

$$c_t + b_t + d_t = w_t h_t + \frac{R_{t-1}}{\pi_t} b_{t-1} + \frac{R_{t-1}}{\pi_t} d_{t-1} + \Pi_t - t_t \tag{10}$$

π_t denotes the composite goods price inflation. We assume that the household is indifferent between buying domestic bonds and making deposits, thus, R_t is nominal gross interest rate of both domestic bonds and deposits. The household owns all banks in the model economy and thus receives lump-sum dividends, Π_t. Taxes t_t enter the household's budget constraint in a lump-sum way as well. Lump-sum dividends from financially constrained firms are included in total dividends Π_t. Lump-sum dividends from financially constrained firms consist of firms' profits that the household receives in the beginning in the period minus the equity that the household transfers in the beginning of the period.

3.5 Production and Pricing

There are several types of firms in the domestic economy. It takes three types of firms to produce domestic aggregate inputs for composite goods. First, there are the financially constrained firms that combine purchased capital with labour and produce homogenous goods. They were analyzed in Section 3.1. Their homogenous outputs are bought by retail firms who costlessly differentiate the products bought and sell them as (local) monopolists, in Dixit and Stiglitz (1977) fashion. A similar group of firms called importers differentiate foreign (imported) goods. A composite goods producer buys the differentiated home goods and aggregates them into an aggregate domestic good y_t^H with associated price p_t^H. The same composite goods producer also buys imported differentiated goods and aggregates them into a foreign aggregate good y_t^F. The corresponding aggregate price level of foreign goods is p_t^F. All details of the derivations of the various first order conditions optimization problems can be found in the supplementary appendix D. We discuss each step in more detail below.

3.5.1 Retail firms

Homogenous goods produced by financially constrained firms are sold to domestic retail firms. A domestic retail firm j differentiates purchased inputs at no cost and sells at a monopolistic price $p_t^H(j)$. We assume that only a fraction $(1-\omega^H)$ of domestic retail firms can adjust prices every period as in Calvo (1983). The fraction ω^H of remaining firms adjust past prices by the rate π_t^{adj}. The aggregate price level that prevails in the retail sector is denoted by p_t^H. Differentiated goods from the domestic retail sector, $y_t^H(j), \quad j \in (0,1)$, are purchased by the composite goods producer.

3.5.2 Importers

Imported foreign goods undergo a differentiation process that is similar to what happens with domestic goods. The retailers differentiating foreign composite goods are called importers. Importers also exercise (local) market power and set prices in a staggered way, again as in Calvo (1983), which allows for incomplete exchange rate pass-through. Thus, $(1-\omega^F)$ of importers change their past prices to the optimal price at period t. The fraction ω^F of remaining firms adjust past prices by the rate π_t^{adj}.

3.5.3 Composite goods producer

We assume that the composite goods producer has access to an aggregation technology and can assemble differentiated goods at no cost. First, the composite goods producer assembles differentiated domestic goods $y_t^H(j)\forall j$ into domestic aggregate goods y_t^H and differentiated imported goods $y_t^F(j)\forall j$ into foreign aggregate goods y_t^F. She uses the following assembling technologies:

$$y_t^H = \left(\int_0^1 y_t^H(j)^{1-\frac{1}{\epsilon_H}}\,dj\right)^{\frac{\epsilon_H}{\epsilon_H-1}},$$

$$y_t^F = \left(\int_0^1 y_t^F(j)^{1-\frac{1}{\epsilon_F}}\,dj\right)^{\frac{\epsilon_F}{\epsilon_F-1}}$$

Then she combines domestic aggregate goods and foreign aggregate goods into composite goods y_t^C with the aggregation technology that takes the taste parameter for foreign aggregate goods η as given:

$$y_t^C \equiv \left((1-\eta)^{\frac{1}{\epsilon}}(y_t^H - ex_t)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}}(y_t^F)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{11}$$

A fraction of the total of domestic aggregate goods is used to produce composite goods and the rest is exported. We assume exports not to have imported content. ϵ stands for elasticity of substitution between domestic and foreign aggregate goods. The composite good y_t^C is sold to the domestic household, the government and capital goods producers. Its associated price is P_t.

3.5.4 Capital producers

Capital producers sell capital to financially constrained firms at the relative price q_t and buying the depreciated capital stock back next period. $q_t$is the relative price of cpaital goods in terms of the composite good P_t.To restore the depreciated capital, capital producers add composite goods (investment) i_t as additional inputs to the depreciated capital stock by using a technology subject to investment adjustment costs $\Gamma\left(\frac{i_t}{i_{t-1}}\right)$:

$$k_t = (1-\delta)k_{t-1} + \left(1 - \Gamma\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{12}$$

where adjustment costs Γ equal:

$$\Gamma\left(\frac{i_t}{i_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

3.5.5 Exporters

Perfectly competitive exporters demand ex_t units of the domestic aggregate good y_t^H, so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters.

Exports are sold at a price p_t^H/rer_t which is the price of domestic aggregate goods expressed in units of foreign composite goods, with rer_t the relative price of domestic composite goods in ternms of foreign composite goods. The foreign demand for domestic aggregate goods is price-sensitive:

$$ex_t = \eta^* \left(\frac{p_t^H}{rer_t}\right)^{-\epsilon_*} y_t^* \tag{13}$$

Consistent with the small open economy assumption, the foreign currency price of foreign composite goodsP_t^* and y_t^* are assumed to evolve exogenously.

3.6 Government

We abstract from normative analysis of government policies and take government spending as an exogenous process. We assume that to finance a stochastic stream of real government expenditure g_t and the bank recapitalization program τ_t^{FS}, the government collects lump-sum taxes t_t from the household and issues domestic bonds b_t. It has to satisfy the budget constraint (expressed in units of composite goods):

$$g_t + \tau_t^{FS} + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

We assume that taxes follow this rule:

$$t_t = t + \kappa^B (b_{t-1} - b) + \kappa^{FS} \tau_t^{FS} + e_t, \qquad 0 < \kappa^B \leq 1, \quad 0 \leq \kappa^{FS} \leq 1$$

So a fraction κ^{FS} of the recapitalization expenditure is covered by increasing the lump-sum tax and the remaining fraction $(1 - \kappa^{FS})$ is financed by issuing new government debt.

3.7 Monetary policy

The central bank conducts monetary policy using a Taylor rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R} \left(\frac{y_t^H}{\bar{y}^H}\right)^{(1-\gamma_R)\gamma_Y} \left(\frac{\pi_t^H}{\bar{\pi}^H}\right)^{(1-\gamma_R)\gamma_\pi} \exp(mp_t) \tag{14}$$

where mp_t is a monetary policy shock and the domestic aggregate goods price inflation π_t^H can be expressed as $\pi_t^H = p_t^H / p_{t-1}^H \pi_t$. $\gamma_R$is a smoothing parameter and is inversely related to the aggressivity of the monetary stance.

3.8 Market clearing

The domestic household, the government and capital producers buy composite goods. Remember that exports did not consist of composite goods but of home goods only. So the supply of composite goods y_t^C has to satisfy the aggregate demand of these domestic agents:

$$y_t^C = c_t + i_t + g_t \tag{15}$$

3.9 The current account and its components

The trade balance expressed in units of composite goods is given by:

$$tb_t = p_t^H ex_t - m_t$$

where m_t denotes the value of imports and can be expressed as $m_t \equiv rer_t D_t^F y_t^F$ (see the supplementary appendix J for details).

So the current account is given by the sum of real trade balance and real net income from abroad. In units of composite goods the current account is given by:

$$ca_t = tb_t + ni_t \tag{16}$$

The domestic household owns banks that issue foreign debt d_t^*. Banks are the only agents to borrow from abroad. Also, we assume that nobody in the domestic economy lends to foreign agents. As a result, real net income from abroad is negative and equal to minus payments of bank foreign debt. It follows that

$$ca_t = tb_t - \left(R_{t-1}^* \xi_{t-1} - 1\right) rer_t \frac{d_{t-1}^*}{\pi_t^*}$$

In equilibrium the current account has to equal the capital account balance which is given by the change in bank foreign debt. The equilibrium condition is follows, expressing the change in foreign debt in units of composite goods as well to get:

$$tb_t - \left(R_{t-1}^* \xi_{t-1} - 1\right) rer_t \frac{d_{t-1}^*}{\pi_t^*} = -\left(rer_t d_t^* - rer_t \frac{d_{t-1}^*}{\pi_t^*}\right)$$

4 Preliminaries to analyzing the model

4.1 Calibration

To employ the theoretical model for empirical simulation, all parameters are calibrated to Hungarian data. We list calibrated parameter values and targeted steady state values in Table 2 in the Appendix. Parameters that are endogenously determined in steady state are $\beta, \chi, \eta^*, \kappa, \omega$ and π^*. χ is chosen such that average working hours in the steady is 0.3 as it is common in the literature. η^* is chosen such that the ratio between the steady state foreign output and the domestic output is equal to the share of the Hungarian GDP in the EU GDP, namely 0.007. π^* follows from satisfying the UIP condition in the steady state given the foreign nominal interest rate of 4.5 p.p. in annual terms. The most important ones of the rest of endogenously determined and calibrated parameters are discussed below.

The financial frictions we introduce bring a few additional parameters to calibrate. The debt overhang friction depends on the corporate default rate value in the steady state, $1 - \Phi(d_2)$. Due to *de facto* non-existent corporate bond market in Hungary, we choose to calibrate the steady state default probability to an average default frequency of corporate loans in Hungary over the period 2002-2007 as reported by the Bank of Hungary. This makes $1 - \Phi(d_2) \approx 0.03$. We choose the bankruptcy loss parameter κ such that the steady state default probability in the model matches the data counterpart. The banking friction relies on the fraction of capital that can be diverted, λ^L, the proportional transfer to the new bankers entering the market each period, ι, and bank leverage in the steady state. We calibrate ι to 0.002 following the original paper of Gertler and Karadi (2011). Bank leverage matches the average bank leverage in the OECD data for year 2007. We make an adjustment to the average bank leverage of 8.6 in Hungary as reported by Bank of

Hungary: we adjust for the average fraction of loans in total assets and get $8.6 \cdot 0.65 \approx 5.6$. The remaining parameter, λ^L, is chosen such that the lending spread in the steady state match the observed difference between nominal corporate loan interest rate and nominal corporate deposit rate in Hungary in 2001:Q1-2008:Q3 (data from the Bank of Hungary). Our computations yield an annual lending spread of 2.7 p.p. It follows that $\lambda^L = 0.45$.

We calibrate the share of foreign currency loans in total corporate loans to 0.6 to match the aggregate share of FX corporate loans in Hungary in 2007-2008 (Krekó et al. (2010)). For the model with loans of hybrid denomination we calibrate the steady state trade balance such that bank liabilities denominated in foreign currency would match foreign currency loans exactly.

We have also calibrated several steady state values using data from the Eurostat online database. The steady state annual inflation in Hungary over the period 2001:Q1-2008:Q3 was 5.9 p.p., we choose the discount factor β such that the steady state inflation in the model matches the data counterpart. The ratio of government spending to GDP, s^g, is set to 0.22. The ratio of imported goods in domestic consumption is computed in the following way. We take the share of imports to GDP in Hungary (72.7 percent) over the period 2002:Q1-2008:Q4 and adjust it given the average import share in the Hungarian exports (56 percent; OECD (2017)). Since in our model exports are assumed to be of domestic origin entirely, we lower the observed import share in GDP by the amount of imports used in export production and get that the import share in domestic demand should constitute around 37 percent in our model. Thus we calibrate η to 0.37 to achieve the desired steady state share. For simplicity we set the steady state level of the nominal exchange rate to unity.

We pick autocorrlelation coefficients for shocks from Jakab and Kónya (2016), except the autocorrelation coefficient for the world demand shock. We set it to 0.95 which is high given the estimates in the literature, however, our two-period setup demands high persistence of this shock to get realistic responses to the world demand shock. If the shock is not persistent enough, firms set their investment level based on price expectations rather than the drop in demand. However, if the demand shift is persistent enough, firms take the decline in output into account too, leading to realistic model responses. This puzzle is unlikely to occur in setups with infinitely-living firms which would explain low estimates in the literature.

4.2 Endogenizing volatility

As we pointed out in the discussion of the financially constrained firms' optimization problem, our model is capable of studying volatility effects. Besides modeling a shock to volatility of firms' future profits, we can endogenize the volatility term by incorporating uncertainty about prices. We obtain the endogenized volatility value for future profits of financially constrained firms by simulating the theoretical model iteratively until the value for σ converges. In this section we explain why the obtained volatility value is a better choice than an arbitrary calibrated value. We shortly describe the simulation procedure as well.

The frst order conditions that govern financially constrained firms' behavior contain a proxy for the default probability. The default probability depends not only on expected values of future revenue and liabilities but on variances of those future revenue and liabilities as well. And because prices are endogenous, the default probability is not only affected by stochastic components such as technology but also by (the volatility of) production prices and exchange rates as well. Therefore, we cannot postulate the variance of future output or future liabilities as an exogenous process dependent on technology and current state variables only. The variance of endogenous variables is unknown, but we can obtain an estimate from simulated series. In the supplementary appendix A2 we derive the precise expression we need to be able to compute the default probability and simulate the model:

$$\sigma_{y,t+1}^2 = var\left(ln\left(\pi_{t+1}\left(\kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right)\right)\right)$$

To simulate the model we need a numerical value for $\sigma_{y,t+1} = \sqrt{\sigma_{y,t+1}^2}$. We assume $\sigma_{y,t+1}$ to be constant ($\sigma_{y,t+1} = \sigma_y$).

To find a value for $\hat{\sigma}_y$ as close to the true value as possible we follow several steps:

1. Set a threshold level for convergence of the calibrated $\hat{\sigma}_y$ to the value of $\tilde{\sigma}_y$ that follows from the simulated time series generated by the model.

2. Choose an initial value for $\hat{\sigma}_y$.

3. Simulate the model with the chosen value for $\hat{\sigma}_y$.

4. Compute volatility of $\bar{y}_{t+1}$ from simulated time series and denote it by $\tilde{\sigma}_y^2$.

5. Compute the difference between the chosen value $\hat{\sigma}_y$ and the simulated value $\tilde{\sigma}_y$. If the difference is larger than the threshold value, set $\hat{\sigma}_y = \tilde{\sigma}_y$ and repeat steps 3-5.

Converged values are presented in Table 1. We obtain estimates of the profits volatility value by simulating series of shocks, namely a productivity shock z_t, a risk premium shock ζ_t, an idiosyncratic volatility shock σ_t and a world demand shock y_t^*. Autocorrelation coefficients are presented in Table 2. Standard errors of these shocks are 0.01 for all shocks except the risk premium that is 0.02. In the model with the endogenous leverage constraint we set the standard deviation of the risk premium shock to 0.012 because otherwise the model becomes unstable. Nevertheless, the volatility estimates from the models without the endogenous leverage constraint are comparable to each other. The estimates from the models with the endogenous leverage constraint can also be compared to other estimates from the models with the endogenous leverage constraint.

Debt denomination	Banking friction	Value
FX & domestic currency	No	0.165
Domestic currency	No	0.037
FX & domestic currency	Endogenous leverage constraint	0.243
Domestic currency	Endogenous leverage constraint	0.090

Table 1: Simulated standard deviations of expected profits for firms (σ_y)

5 Results

We now dissect the interaction of financial distress in the firms' sector and losses in the banking sector. We begin by discussing the debt overhang friction in the firms' sector and its consequences in the periods of unanticipated depreciation. Next we add the banking friction to the setup to see how leverage-constrained banks can amplify the shocks even further. The relative importance of the frictions is analyzed by comparing two scenarios of allocating currency mismatch losses. Given the large foreign bank funding flows into emerging Europe prior to end of 2008, we assume that domestic banks issue debt denominated in foreign currency which creates currency mismatch unless banks match foreign currency liabilities with loans issued in foreign currency. Therefore, banks shift currency mismatch to domestic borrowers by lending in foreign currency. We compare the model economy with bank lending in domestic currency and bank lending in both foreign currency and domestic currency to explore which currency mismatch situation generates larger macroeconomic losses.

More plots for every shock discussed in the following section can be found in the appendix. Here we present graphs with the most important variables only.

5.1 Debt overhang in the financially constrained firms' sector

In this section we do not introduce balance sheet constrained banks yet, we will do that in the sections that follow. Borrowing in foreign currency makes domestic financially constrained firms prone to debt overhang whenever there is an unanticipated depreciation of the domestic currency. If the expected value of debt indeed exceeds the expected collateral value, the indebted firm faces a higher chance of losing its collateral (future revenue) to creditors. The firm's marginal benefits from investing diminish. In the setting with non-contractible investment, the rising possibility of default is enough to create a slump in output by decreasing investment. But before exploring the consequences of depreciation, we first consider the shocks that may have triggered the domestic currency depreciation to begin with and thereby increased default probabilities. In our small open economy setup we consider in turn a country risk premium shock, a negative world demand shock and a shock to the exogenous component of the volatility of profits generated in the financially constrained firms' sector.

Regardless of the denomination of corporate debt, the listed shocks are expected to bring an economic downturn by either dampening aggregate demand or supply. Accumulated foreign currency debt makes the corporate default probability depend not only on the aggregate level of economic activity but on the degree of currency mismatch as well. Thus, whenever the domestic currency depreciates, foreign currency debt opens an additional contractionary channel that operates through even higher default probabilities and thus more intense debt overhang in the financially constrained firms' sector.

Potential drivers of an exchange rate crash: risk premium shocks

The simulation results confirm our hypothesis that debt overhang amplifies adverse effects on aggregate variables more if firms have their debt denominated in foreign currency rather than in domestic currency. In Figure 4 capital outflows, which are triggered by a shock to the country risk premium on bank foreign debt, lead to a sell-off of domestic currency and trigger a depreciation. To mute rising domestic inflation, the central bank responds by raising the domestic nominal interest rate and thereby adds to the recessionary pressure. The rise of the policy rate in particular reduces consumption as the depreciation and associated relative price changes trigger the substitution effect. Higher interest rates translate into higher borrowing costs, reducing investment too. Currency mismatch for firms makes the recession deeper as investment in working capital decreases not only due to lower aggregate demand but also due to debt burden weighing on firms' marginal benefits from investing. We observe that, if financially constrained firms borrow in foreign currency, a repayment probability is substantially lower, an interest rate on their loans rises higher and they post lower demand for labour and capital goods. Note that domestic currency depreciation not only distorts decisions ex ante, but deprives firms of available funds ex post: lower firms' profits result in lower corporate worth and thus higher dependence on external funds which come at a now higher default spread.

Potential drivers of an exchange rate crash:a negative shock to export demand

In Figure 5 below, we show the (model) response to a decline in world demand for domestic export. The exchange rate depreciates, which has a positive impact on competitiveness. This effect is reinforced because domestic prices decline in response to the drop in external demand: the deflation increases external competitiveness and so helps also to (partially) offset the negative quantity impact of the shock. Firms that borrowed in domestic currency are not adversely affected by the depreciation and as a consequence their repayment probability does not decrease. After the initial shock, the recovery of competitiveness makes firms expect higher output again, and again higher inflation tomorrow compared to today. This improves their probability of repayment and lowers their funding costs. Firms that borrowed in foreign currency, however, face a higher debt burden after the depreciation , so after the negative demand shock and the depreciation induced increase in their debt ratio their expected repayment probability goes down and their funding costs go up commensurately. The increase in debt overhang in turn slows down their investment and lowers their demand for labor, all of which becomes clear by comparing the two lines in Figure 5. As a consequence the recessionary impact of the negative world demand shock is substantially higher in the case of FX denominated corporate loans.

Figure 4: Country risk premium shock of 5 p.p. in the model without leverage-constrained banks.

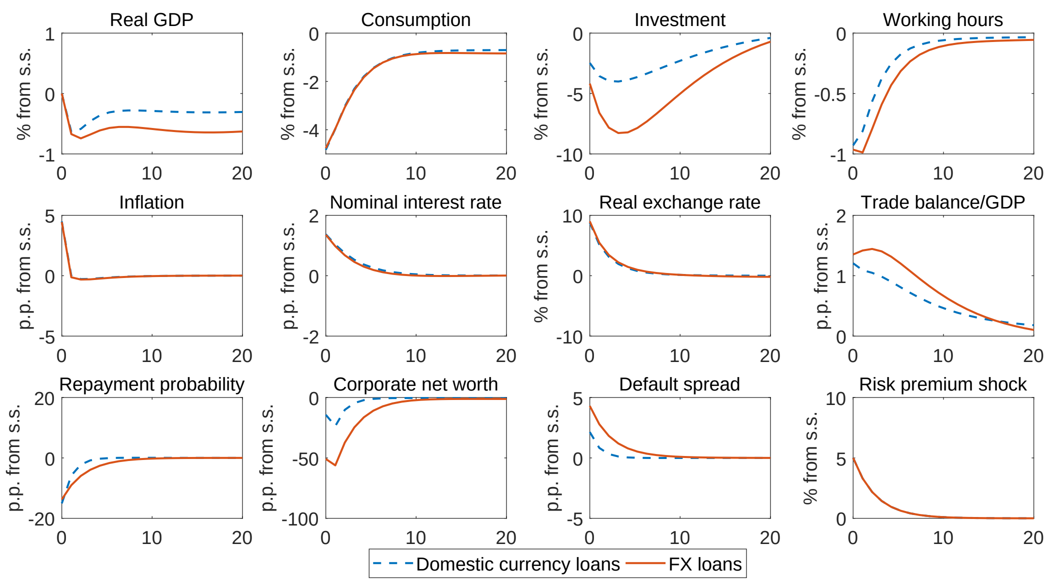
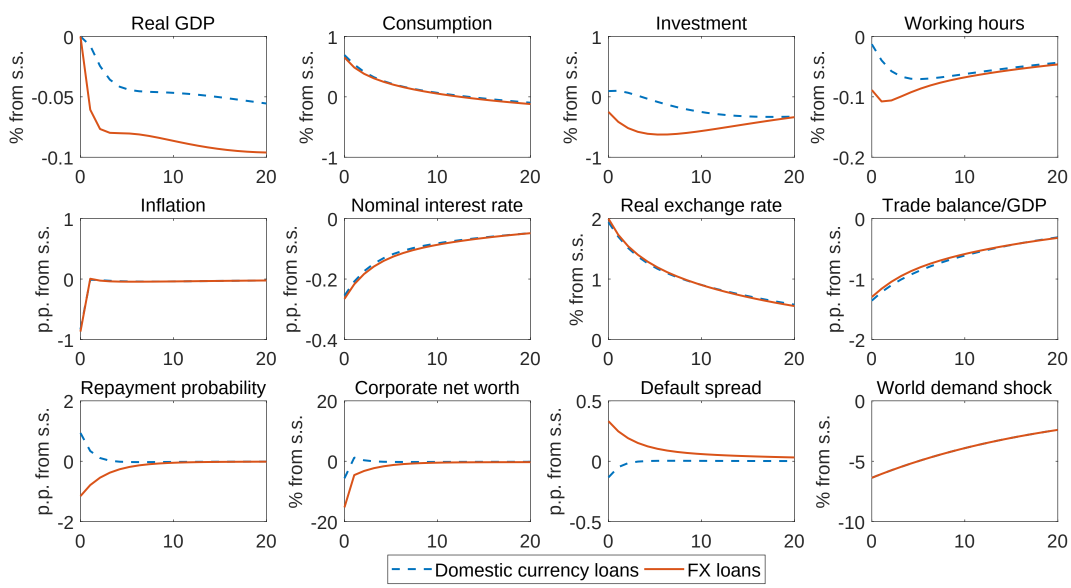


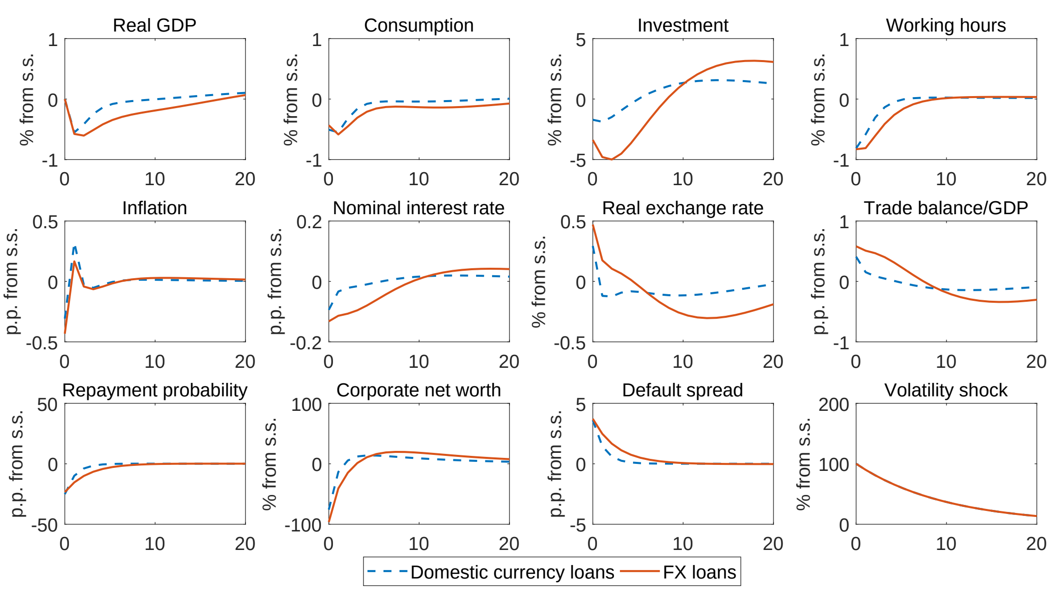
Figure 5: World demand shock of -6.4% in the model without leverage-constrained banks.



Potential drivers of an exchange rate crash: volatility shocks

In contrast to the two shocks considered so far, the volatility shock primarily affects not only the demand side but over time also the supply side of the economy by making the firms' future profits more uncertain with direct consequences both on investment (future supply) and on labor demand, impacting current supply. This has a direct effect on the default probability as the higher uncertainty magnifies the expected distance between the collateral value and the debt value. Then, for any debt burden and any productivity level, firms face lower chances to repay their debt and lenders respond by raising interest rates on corporate loans. Figure 6 depicts how in this case debt overhang weighs on the firms' incentives to invest and in turn the economy falls into a recession. The increased uncertainty of firms' future profits has an indirect effect on household consumption by lowering income: firms post lower demand for labour and wages decrease. The substitution effect stimulates consumption as the central bank copes with the slump and the corresponding deflation by cutting the policy rate, however, this effect appears to be small. Overall, the volatility shock generates responses of relatively large magnitude, changes in investment are particularly large. Initially, foreign currency debt generates more contraction than accumulated domestic currency debt, however, after two periods the real exchange rate depreciation in the former cases subsides and depreciation-driven debt overhang loses its influence completely. The difference between the case with borrower currency mismatch and without it is small. The volatility shock directly hits firms' chances to repay and the depreciation effect apparently becomes of second order. In other words, the magnitude of the change in the default probability overshadows the risk related to the increased value of foreign currency debt.

Figure 6: Volatility shock of 100% in the model without leverage-constrained banks.



To sum up, capital outflows can trigger domestic currency depreciation which in turn has a contractionary impact when there is currency mismatch in the corporate sector. Compared to firms borrowing in domestic currency only, depreciation lands firms indebted in foreign currency in a more severe debt overhang situation. Under-investment and a deeper fall in output then follow. The debt overhang angle is a new argument in the contractionary devaluation debate. We should mention that some characteristics of our model lead to smaller impact of the real depreciation than one should expect in practice. First the short term nature of debt; this makes debt overhang fade away much more quickly than would occur if we had assumed long term debt commitments. Second, the timing of the firm's optimization problem is such that firms learn about their net worth value after the borrowing amount has been decided on. Therefore, even though domestic currency depreciation triggers more defaults because of the reduced corporate net worth (Figure 5), the feedback of the shock through the corporate net worth channel comes with a delay. Third, firms die after two periods and so do not have to take into account future profits after that second period which mutes the net worth effect to some extent as well.

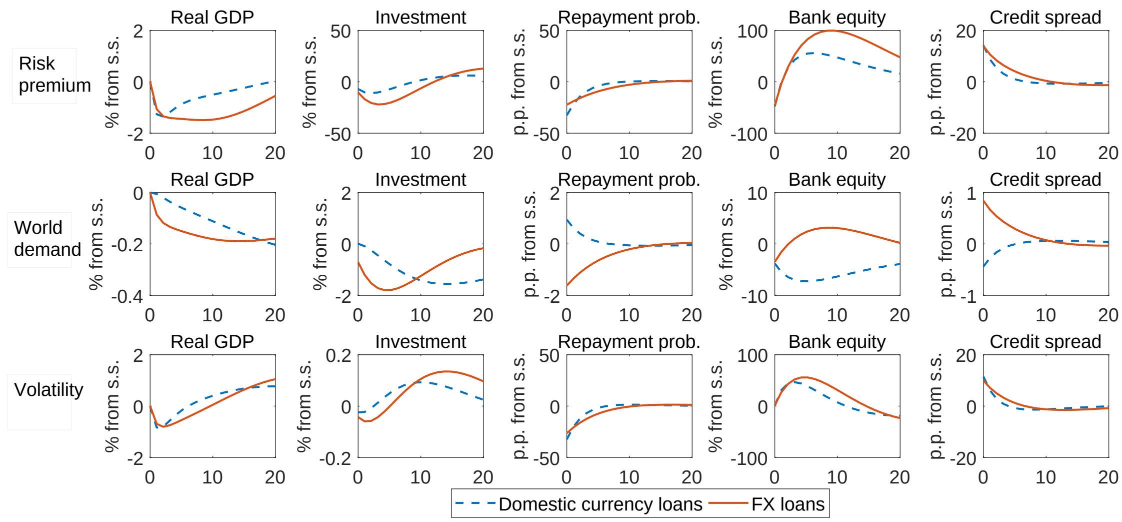
5.2 Leverage-constrained banks and FX losses

Consider now leverage constrained banks. The agency problem between banks and depositors generates an endogenous credit spread which tightens or improves borrowing conditions for banks depending on bank leverage. The credit spread moves countercyclically: in bad times non-performing loans deplete bank capital, bank leverage goes up and exacerbates the agency problem, increasing bank borrowing costs. Financial distress in the banking sector translates into worse borrowing conditions for the borrowing firms. Therefore, in bad times the binding bank leverage constraint amplifies initial losses in the economy.

Currency mismatch triggers bank losses and thus also exacerbates the agency problem for banks. When banks face currency mismatch, domestic currency depreciation has an immediate negative effect on bank capital and thus on leverage and bank borrowing costs. Lending in foreign currency insulates the bank balance sheet from exchange rate risk but leads to higher credit risk. So when domestic currency depreciation creates losses for domestic firms that borrowed in foreign currency, the ratio non-performing loans increases and bank profits decline anyhow, once again worsening the bank agency problem. Therefore, currency mismatch for both banks and financially constrained firms will affect the bank agency problem, FX denomination of bank loans does not provide the banks with an effective hedge.

In this section we show that when the model is extended by including an endogenously determined bank leverage constraint, aggregate losses are still smaller when corporate loans are denominated in domestic currency than when a share of corporate debt is foreign currency loans. From a macro point of view the outcome when impact losses stay in the banking system is preferable. We explore the role of bank leverage constraint in more detail by once again analyzing the economy's response to respectively a risk premium shock, a negative shock to global demand for the country's export goods and to a volatility shock. Figure 7 presents outcomes of these shocks depending on whether currency mismatch losses are placed in the banking sector (firms borrow in domestic currency) or shifted to firms (firms borrow in foreign currency). Firms with foreign currency debt

Figure 7: IRFs in the model with leverage-constrained banks.



experience higher default probability because the shocks trigger domestic currency depreciation and increase the debt value, not only lead to a loss in aggregate demand. Figure 7 shows that after the shocks firms indebted in foreign currency become more likely to default and worsened debt overhang distorts their investment decision, so that investment decreases more. Non-performing foreign currency loans have a small effect on bank equity. On the contrary, when firms borrow in domestic currency, the domestic currency depreciation has a persistent negative effect on bank equity. Bank currency mismatch losses translate into persistent real losses for two reasons. First, banks cut lending to all firms rather than just troubled firms which constrains economic activity severely. Second, since banks accumulate equity out of retained earnings, even temporary bank losses can have a persistent effect on borrowing conditions in the economy. Therefore, as expected, currency mismatch in the banking sector has a stronger effect on bank equity. Is the effect on bank equity sufficient to make shifting currency mismatch losses for banks more recessionary than currency mismatch losses for firms? The answer is no. Figure 7 plots output outcomes for all the considered cases with the endogenous bank leverage constraints. We see that in all cases foreign currency loans deepen the recession. However, only in the case of the capital outflow, the difference between output outcomes is somewhat larger.

Why does shifting currency mismatch losses to banks cause smaller output losses? Banks are in a better position to absorb currency mismatch losses because, in contrast to firms, they do not internalize the impact of their actions on their own default risk. We assume that the bank default probability is exogenous and thus it neither varies with the aggregate conditions nor increases distortions for bank lending decisions. Consequently, unexpected bank losses affect borrowing conditions for firms and thus aggregate economic activity less than losses in the firms' sector. We make this assumption to reflect the fact that banks often expect to be rescued by the government or by their (foreign) parent bank, while the number of firms is too large to expect a masssive bail out program or sufficient financial support to prevent them from going bankrupt. We do not explicitly incorporate bank expectations to be rescued here but just impose exogenous bank default: in our setup banks expect no feedback from their actions on their own default probability. The second reason why allocating currency mismatch losses to firms generates larger real losses is that firms burdened with debt decrease aggregate output and demand directly, while banks affect aggregate economic activity with a lag and only after part of the losses have been absorbed by banks through reduced equity.

We should note that the assumption of the financially constrained firms' exit after two-periods makes the effect of the debt overhang friction weaker. In contrast, banks do incorporate their full future net worth dynamics in their optimization problem, so bank losses have a more prolonged effect on the economy. Nevertheless we find that leaving the impact of currency mismatch in the banks instead of passing it on to firms is likely to result in lower macroeconomic losses.

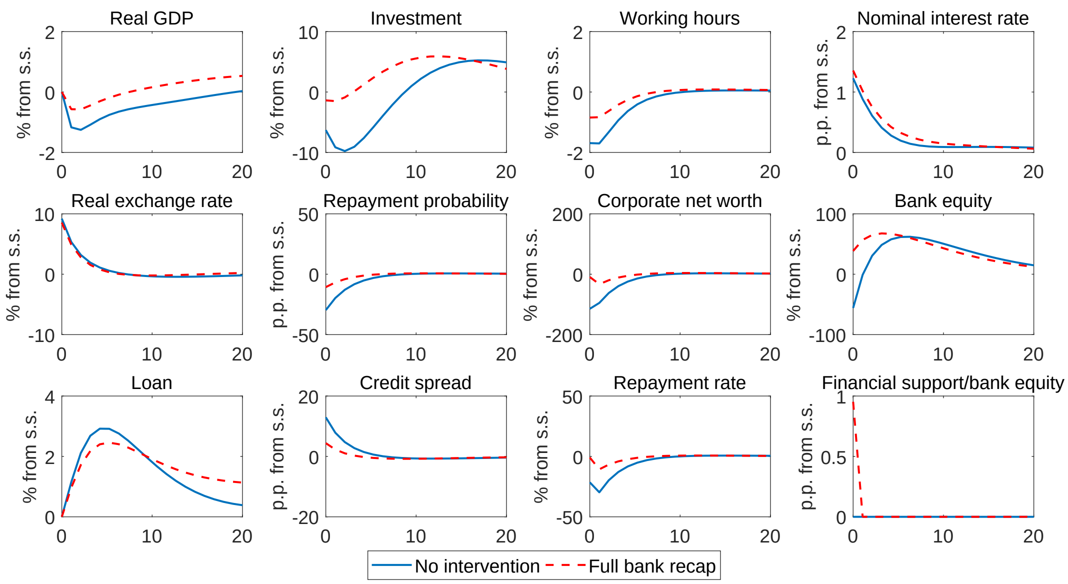
5.3 Bank recapitalization

We saw in the previous sections that shifting currency mismatch losses to banks reduces corporate debt overhang and leads to less recessionary macroeconomic outcomes after an exchange rate collapse. A logical policy response is then obviously to recapitalize banks so as to further minimize negative amplification effects. In this section we look at exactly that scenario: banks are absorbing the FX losses while financially constrained firms are allowed to avoid currency mismatch.

Financial sector support is modeled as a one-time transfer to banks. Consider the case of the capital outflows/foreign risk premium shock which generated the largest economic downturn in the series of our experiments. Figure 8 shows how full recapitalization of the banking sector after the increase in the country risk premium immediately relaxes the endogenous bank leverage constraint and improves bank borrowing conditions. The credit spread increases by less on impact which immediately leads to a less restricted bank credit supply to firms. Consequently, the economy undergoes a smaller recession than comes out without a recapitalization. The somewhat counterintuitive difference in loan responses can be explained by the dynamics of bank equity. In both cases bank equity recovers and even grows after a couple of periods and thus the endogenous leverage constrained is relaxed as time goes by, easing lending to new firms. Also higher loans do not imply higher firm activities: firms hire less and invest less in response to the shock in both cases, but use loans to compensate for negative equity transfers from households (see the dynamics of corporate net worth). Bank recapitalization brings a smaller recession, raising firm profits and corporate equity transfer for new firms, which allows firms to borrow slightly less in the second quarter after the shock. In the long run firms borrow more, if banks receive an equity injection.

To sum up, the experiments show that allocating FX losses to the banking sector and if necessary recapitalizing that sector is an effective way of mitigating the recessionary impact of a FX crisis in

Figure 8: No government intervention vs. bank recapitalization.



the presence of corporate and banking sector foreign debt. Currency mismatch in the banking sector can be efficiently alleviated ex post, something that is much harder to do for FX mismatch induced corporate debt overhang because there are many more firms than banks. We should make the qualifying remark that in the case of a highly indebted sovereign and extensive commercial bank holdings of sovereign debt, the effectiveness of recaps may be undermined by the negative price effect triggered by the increase in sovereign debt that comes with a recap exercise. See Van der Kwaak and Van Wijnbergen (2014) who elaborate on this so called doomloop.

6 Conclusions

Hungary saw its exchange rate collapse after the Great Financial Crisis started in Western Europe. After the Lehman collapse and the deepening of the crisis in Western Europe, Hungary's external enviromment became much more hostile: export demand declined, risk premia on Hungary's foreign loans increased, as did overall volatility. This experience has led to fierce debates as to how the ensuing FX losses should be allocated. In this paper we have first traced the macroeconomic consequences of these various foreign shocks for macroeconomic performance and in particular their impact on the exchange rate and the ensuing debt overhang situation in the presence of FX denominated external debt. To this end we have constructed a small open economy DSGE model with as main innovative feature an explicit incorporation of debt overhang in line with the in finance by now traditional Merton credit risk model (Merton (1974)). Under this approach, which has strong support in the finance literature, investment (and in our case employment) are not contractible ex ante when bank loans are decided upon. This approach is different from the ones made in the canonical model of frictions between a lender-bank and its corporate borrowers, the model first proposed by Bernanke et al. (1999). This difference in assumptions has substantial consequences for the macroeconomic impact of financial shocks because it leads to ex ante moral hazard problems and as a consequence a larger impact of financial shocks on investment and in our case employment. The Merton model has found widespread empirical support in the finance literature and helps, we find, in understanding the severity of the macroeconomic consequences of depreciation induced debt overhang problems. We subsequently use the model developed and presented here to discuss this particular set of questions: from a macroeconomic perspective, will the economy be better off shifting the FX losses to the banking system or should firms absorb the losses themselves.

The implementation of the option based Merton credit risk model also leads to another innovative feature of our model: volatility shocks have a first order impact on outcomes in spite of our use of linear approximations in solving the DSGE model. The reason for that is that the first order conditions of the firm's optimization problem involve derivatives of the options implicit in the Merton credit risk approach and these derivatives depend on the variance of the relevant cash flows. We propose and use an iterative procedure to analyse the consequences of volatility shocks to exogenous components of the endogenous volatility processes in this model.

We first use the model to analyse the various shocks emanating from the deterioration in Hungary's external environment and compare the outcomes that obtain under different banking frictions. In particluar, we simulate a drop in external demand for Hungarian exports, a shock raising the country risk premium Hungarian borrowers need to pay to foreign lenders and a shock increasing the exogenous component of the stochastic cash flow volatility process. All three took place in practice. Under all three shocks, the exchange rate depreciates, with debt overhang as a consequence for those actors that had debt denominated in foreign currency. For all shocks and the various frictions under which we simulate their impact, we alternatively simulate what happens when banks take on the FX risk by denominating their loans in local currency, and as an alternative scenario the consequences when banks have passed on FX risk by denominating their corporate loans in FX too. In all cases we highlight a new channel through which depreciation has a contractionary impact. By increasing or triggering debt overhang in sectors with currency mismatch on their balance sheet, the negative macroeconomic impact of external shocks is amplified when they trigger a depreciation. Corporate debt overhang plays a crucial role in this mechanism.

In a second set of simulations we trace the impact of similar external shocks but now placing the capital losses after the depreciation in the banking sector; by assuming that the banks denominated thier FX financed business loans in local currency. With this setup a direct debt overhang effect after the depreciation of the exchange rate is avoided in the corporate sector but the corporate sector cannot escape the indirect negative macroeconomic effects that are once again triggered by a depreciation. Now bank capital is eroded and, faced with an endogenously generated leverage constraint, banks raise lending rates and reduce loan volumes, in this way amplifying the recessionary impact of the external shocks.This explains our general finding that concentrating FX losses in banks leads to smaller overall negative macroeconomic impact of external shocks.

This second set of experiments naturally leads to our third set of simulations: if concentrating FX losses in banks leads to smaller but still significant negative amplification effects because of the eroded bank capital base FX losses will trigger, a bank recapitalization is a natural policy response. Indeed we show that adding a bank recapitalization largely offsets the negative effects of debt overhang in banks after FX losses have eroded their capital base.

Overall the model runs lend support to at least elements of the approach followed in Hungary: shift FX losses to banks by converting corporate loans at pre-crisis exchange rates and recapitalize banks if necessary to reduce or even offset completely any negative amplification effects through tighter financial constraints triggered by depreciation of the currency.

Our argument would be strengthened if we would introduce longer lived firms with long term debt since the debt overhang problems would be magnified in this more realistic capital structure environment. An additional argument is more practical: of course firms could be recapitalized as well as banks but doing so is vastly more difficult given that there are many more firms than banks and legal and institutional frameworks to recapitalize firms are non-existant and may even run into state aid prohibitions.

References

ADOLFSON, M., S. LASÉEN, J. LINDÉ, AND M. VILLANI (2007): "Bayesian estimation of an open economy DSGE model with incomplete pass-through," *Journal of International Economics*, 72, 481–511.

BAKKER, M. B. B. AND M. C. KLINGEN (2012): *How Emerging Europe Came Through the 2008/09 Crisis: An Account by the Staff of the IMF's European Department*, International Monetary Fund.

BANK OF HUNGARY (2012): "Report on Financial Stability, November 2012," .

BERNANKE, B. AND M. GERTLER (1989): "Agency costs, net worth, and business fluctuations," *The American Economic Review*, 14–31.

BERNANKE, B. S., M. GERTLER, AND S. GILCHRIST (1999): "The financial accelerator in a quantitative business cycle framework," *Handbook of macroeconomics*, 1, 1341–1393.

BODNÁR, K. (2012): "Exchange rate exposure of Hungarian enterprises – results of a survey," *MNB Working Papers No. 2009/80*.

BROWN, M. AND P. R. LANE (2011): "Debt Overhang in Emerging Europe?" *World Bank Policy Research Working Paper No. 5784*.

CALVO, G. A. (1983): "Staggered prices in a utility-maximizing framework," *Journal of monetary Economics*, 12, 383–398.

CÉSPEDES, L. F., R. CHANG, AND A. VELASCO (2004): "Balance Sheets and Exchange Rate Policy," *American Economic Review*, 94, 1183–1193.

CHRISTIANO, L. J., R. MOTTO, AND M. ROSTAGNO (2014): "Risk shocks," *The American Economic Review*, 104, 27–65.

COOK, D. (2004): "Monetary policy in emerging markets: Can liability dollarization explain contractionary devaluations?" *Journal of Monetary Economics*, 51, 1155–1181.

DEVEREUX, M. B., P. R. LANE, AND J. XU (2006): "Exchange rates and monetary policy in emerging market economies," *The Economic Journal*, 116, 478–506.

DIAMOND, D. W. AND R. G. RAJAN (2009): "The Credit Crisis: Conjectures about Causes and Remedies," *The American Economic Review*, 99, 606–610.

DIXIT, A. K. AND J. E. STIGLITZ (1977): "Monopolistic competition and optimum product diversity," *The American Economic Review*, 67, 297–308.

ECB (2011): "Opinion on foreign currency mortgages and residential property loan agreements (CON/2011/87), 4.11.2011," .

ENDRÉSZ, M., G. GYÖNGYÖSI, AND P. HARASZTOSI (2012): "Currency mismatch and the sub-prime crisis: firm-level stylised facts from Hungary," *MNB Working Papers, No. 2012/8*.

GALI, J. AND T. MONACELLI (2005): "Monetary policy and exchange rate volatility in a small open economy," *The Review of Economic Studies*, 72, 707–734.

GARCÍA-CICCO, J., M. KIRCHNER, AND S. JUSTEL (2014): "Financial Frictions and the Transmission of Foreign Shocks in Chile," *Central Bank of Chile Working paper series No. 772*.

GERTLER, M., S. GILCHRIST, AND F. M. NATALUCCI (2007): "External constraints on monetary policy and the financial accelerator," *Journal of Money, Credit and Banking*, 39, 295–330.

GERTLER, M. AND P. KARADI (2011): "A model of unconventional monetary policy," *Journal of monetary Economics*, 58, 17–34.

GOMES, J., U. JERMANN, AND L. SCHMID (2016): "Sticky Leverage," *American Economic Review*, 106, 3800–3828.

GOURIO, F. (2013): "Financial distress and endogenous uncertainty," in *2013 Meeting Papers*, Society for Economic Dynamics, 108.

GREENWOOD, J., Z. HERCOWITZ, AND G. W. HUFFMAN (1988): "Investment, capacity utilization, and the real business cycle," *The American Economic Review*, 402–417.

GULDE, A. AND L. GIORGIANNI (2012): "Republic of Hungary: Staff Report for the 2011 Article IV Consultation and Second Post-Program Monitoring Discussions," *IMF Country Report No. 12/13*.

HERZBERG, V. (2010): "Assessing the Risk of Private Sector Debt Overhang in the Baltic Countries," *IMF Working paper series No. 10/250*.

HUNGARIAN BANKING ASSOCIATION (2012): "Report on 2011 activities of the Hungarian banking association," *Budapest, March*.

IMF (2008): "Hungary: 2008 Article IV Consultation—Staff Report," *Country Report No. 08/313*.

JAKAB, Z. AND I. KÓNYA (2016): "An Open Economy DSGE Model with Search-and-Matching Frictions: The Case of Hungary," *Emerging Markets Finance and Trade*, 52, 1606–1626.

JAKAB, Z. M. AND B. VILÁGI (2008): "An estimated DSGE model of the Hungarian economy," *MNB Working Papers No. 2008/9*.

Jakucionyte, E. and S. J. van Wijnbergen (2017): "Foreign Currency Carry Trade Gone Wrong: Corporate and Consumer Losses in Emerging Europe," *Unpublished manuscript*.

Kalemli-Özcan, S., L. Laeven, and D. Moreno (2015): "Debt Overhang in Europe: Evidence from Firm-Bank-Sovereign Linkages," *University of Maryland*.

Kirchner, M. and S. van Wijnbergen (2016): "Fiscal deficits, financial fragility, and the effectiveness of government policies," *Journal of Monetary Economics*, 80, 51–68.

Krekó, J., M. Endrész, et al. (2010): "The role of foreign currency lending in the impact of the exchange rate on the real economy," *MNB Bulletin (discontinued)*, 5, 29–38.

Lamont, O. (1995): "Corporate-debt overhang and macroeconomic expectations," *The American Economic Review*, 1106–1117.

Merton, R. C. (1974): "On the pricing of corporate debt: The risk structure of interest rates," *The Journal of finance*, 29, 449–470.

Myers, S. C. (1977): "Determinants of corporate borrowing," *Journal of financial economics*, 5, 147–175.

Occhino, F. and A. Pescatori (2014): "Leverage, investment, and optimal monetary policy," *The BE Journal of Macroeconomics*, 14, 511–531.

——— (2015): "Debt overhang in a business cycle model," *European Economic Review*, 73, 58–84.

OECD (2017): "OECD data. Series: Domestic value added in gross exports. Import content of exports," *At: https://data.oecd.org/trade/domestic-value-added-in-gross-exports.htm*.

Philippon, T. (2010): "Debt overhang and recapitalization in closed and open economies," *IMF Economic Review*, 58, 157–178.

Reuters (2012): "Hungarian banks see 20 pct of FX borrowers opting for early repayment, January 3, 2012," *Available at http://uk.reuters.com/article/hungary-banks-idUKL6E8C316K20120103.*

Schmitt-Grohé, S. and M. Uribe (2003): "Closing small open economy models," *Journal of international Economics*, 61, 163–185.

Towbin, P. and S. Weber (2013): "Limits of floating exchange rates: The role of foreign currency debt and import structure," *Journal of Development Economics*, 101, 179–194.

Van der Kwaak, C. and S. Van Wijnbergen (2014): "Financial fragility, sovereign default risk and the limits to commercial bank bail-outs," *Journal of Economic Dynamics and Control*, 43, 218–240.

Vonnák, D. (2015): "Decomposing the riskiness of corporate foreign currency lending: The case of Hungary," *IEHAS Discussion Papers*.

World Bank (2015a): "World Bank Databank. Domestic credit to private sector (https://data.worldbank.org/indicator/FS.AST.PRVT.GD.ZS".

——— (2015b): "World Bank Databank. Domestic credit to private sector by banks (https://data.worldbank.org/indicator/FD.AST.PRVT.GD.ZS".

Appendix

Tables and figures

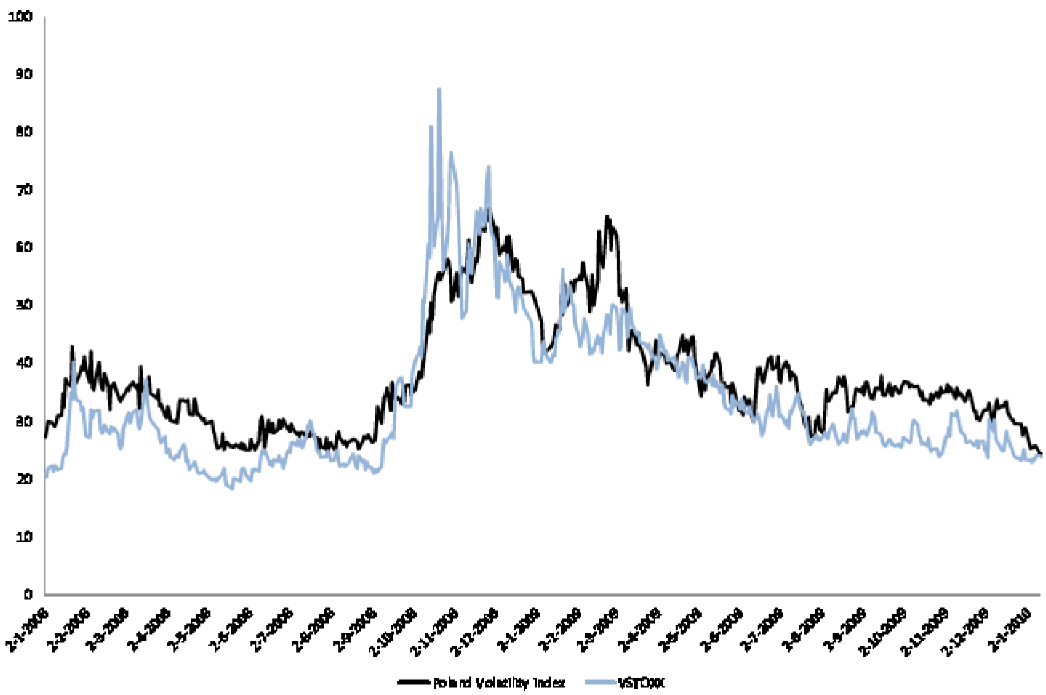


Figure 9: Implied volatility indexes.

Sources: EURO STOXX 50 Volatility Indices database and the courtesy of the blog 'Volatility Futures & Options', available at http://onlyvix.blogspot.nl/2013/03/polands-volatility-index.html.

Parameter	Description	Value	Source
	Calibrated parameters		
β	Household's discount factor	0.9970	to match $\pi = 1.059$
γ	Coefficient in GHH preferences	1.6	Jakab and Világi (2008)
ϕ	Labour supply elasticity	8	Jakab and Világi (2008)
α	Capital share in production	0.34	calibrated
δ	Capital depreciation rate	0.025	Jakab and Világi (2008)
ϵ	E.o.S. between domestic and imported goods	1.5	Gali and Monacelli (2005)
ϵ^H	E.o.S. between varieties of domestic goods	6	Jakab and Világi (2008)
ϵ^F	E.o.S. between varieties of imported goods	6	Jakab and Világi (2008)
ϵ^*	E.o.S. for exports	1.5	Gali and Monacelli (2005)
θ^H	Calvo parameter, domestic goods	0.75	Gali and Monacelli (2005)
θ^F	Calvo parameter, imported goods	0.75	calibrated
η	Share of x^F in y^C	0.37	to match average imports share of 37%
η^*	Share of ex in y^*	0.0033	calibrated
κ	Investment adjustment cost parameter	13	Jakab and Világi (2008)
κ_b	Tax feedback parameter for government debt	0.05	calibrated
z	Technology in SS	1	calibrated
π	Inflation in SS	1.059	average in the data in annual terms
p^H	Relative price of x^H in SS	1	calibrated
n	Working hours in SS	0.3	calibrated
S	Nominal exchange rate in SS	1	calibrated
y^*	Total foreign output in SS	104	calibrated
R	Risk-free rate in SS	1.073	average in the data in annual terms
R^*	Foreign interest rate in SS	1.045	calibrated
s^g	Gov. consumption/ GDP in SS	0.22	average in the data
π^*	Foreign inflation rate	1	from RER definition in SS
ξ	Risk premium on international bonds in SS	1.01	calibrated
κ_ξ	Elasticity of country risk to net asset position	0.001	Jakab and Világi (2008)
ζ	Exogenous shock to the bond premium in SS	1	calibrated
ρ_R	Interest rate smoothing	0.766	Jakab and Világi (2008)
α_π	Interest policy rule (inflation)	1.375	Jakab and Világi (2008)
α_y	Interest policy rule (output)	0.2	calibrated
ρ_σ	Volatility shock autoregr. coeff.	0.9	Occhino and Pescatori (2015)
ρ_{y^*}	World demand shock autoregr. coeff.	0.95	calibrated
ρ_ζ	Risk premium shock autoregr. coeff.	0.66	Jakab and Kónya (2016)
	Financially constrained firms' parameters		
$1-\Phi(d_2)$	Corporate default rate in SS	0.03	average in the data
ρ	Fraction of working capital to be paid in advance	0.8	-
α^F	Share of FX loans	0.6	average in the data
ι^{firms}	Proportional transfer to the entering firms	0.002	calibrated
lev^{firms}	Bank leverage in SS	3.3	average in the data
	Banking sector parameters		
λ^L	Fraction of capital that can be diverted	0.45	calibrated
ι	Proportional transfer to the entering bankers	0.002	Gertler and Karadi (2011)
lev	Bank leverage in SS	5.6	average in the data

Table 2: Parameters

Figure 10: Country risk premium shock in the model without leverage-constrained banks.

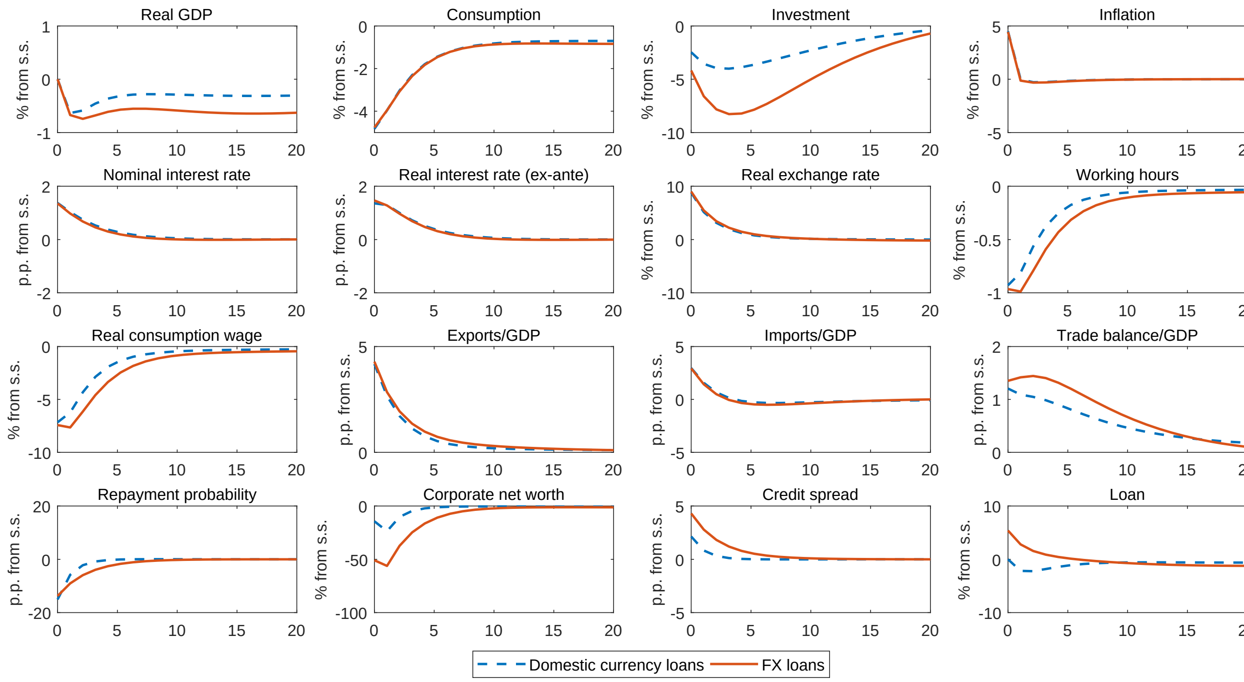


Figure 11: World demand shock in the model without leverage-constrained banks.

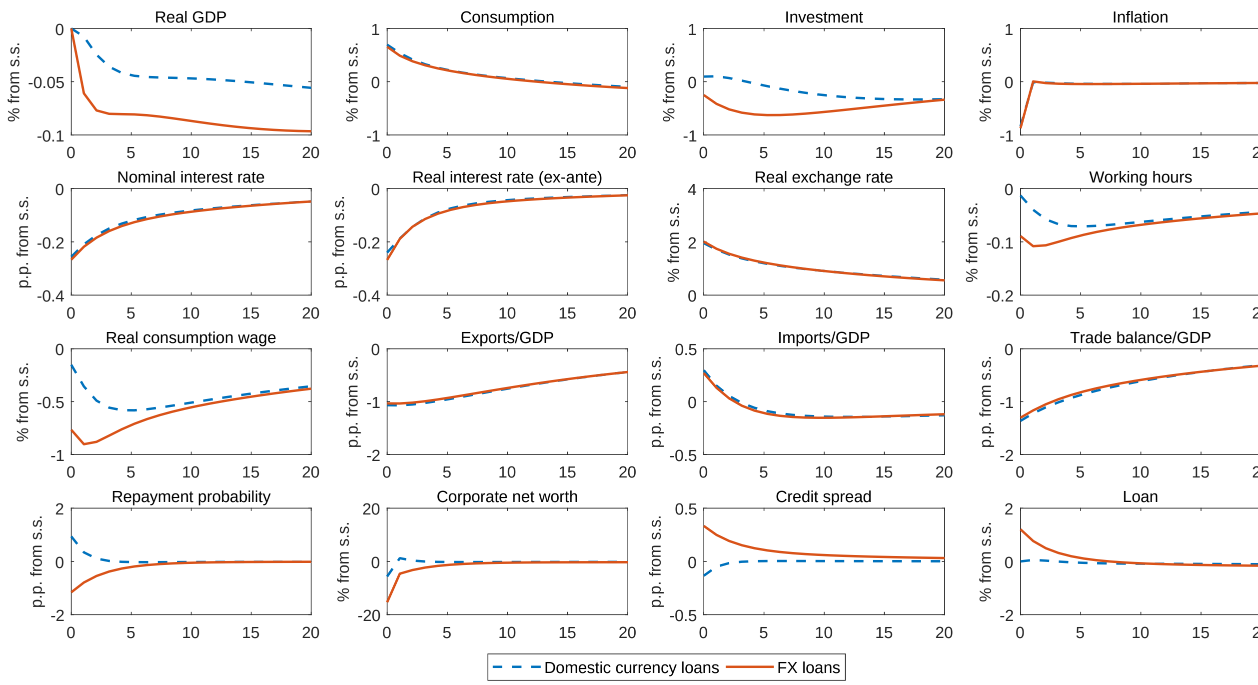


Figure 12: Volatility shock in the model without leverage-constrained banks.

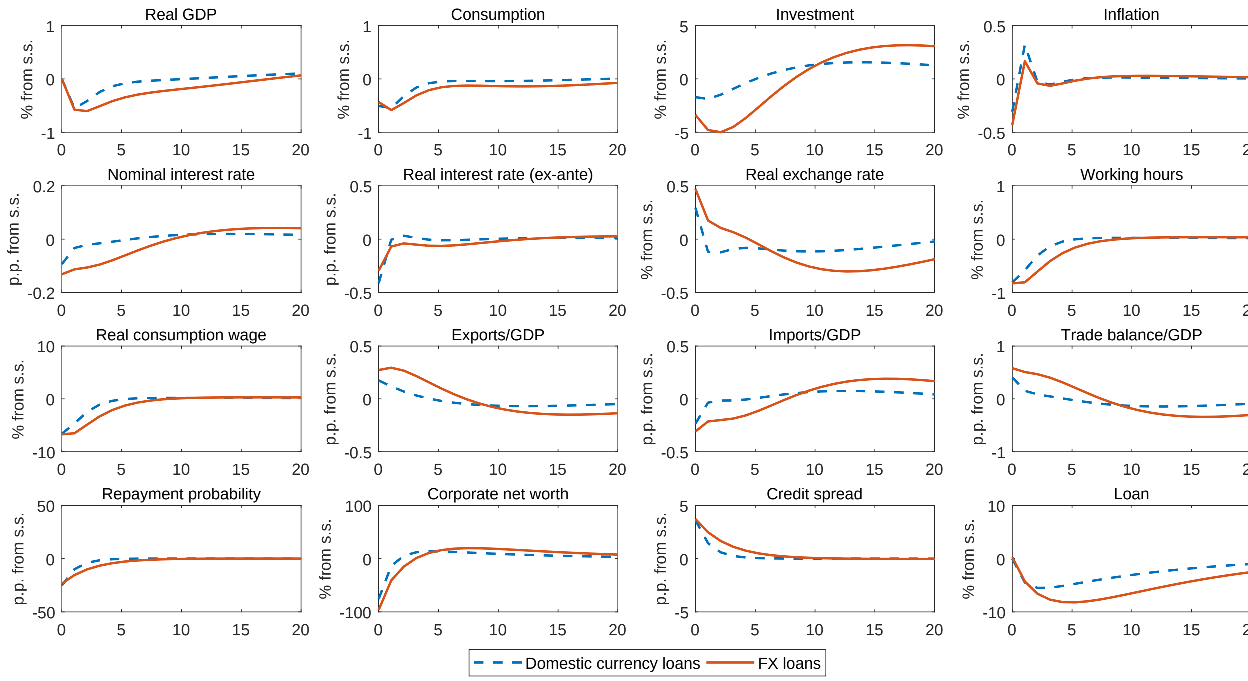


Figure 13: Country risk premium shock in the model with leverage-constrained banks.

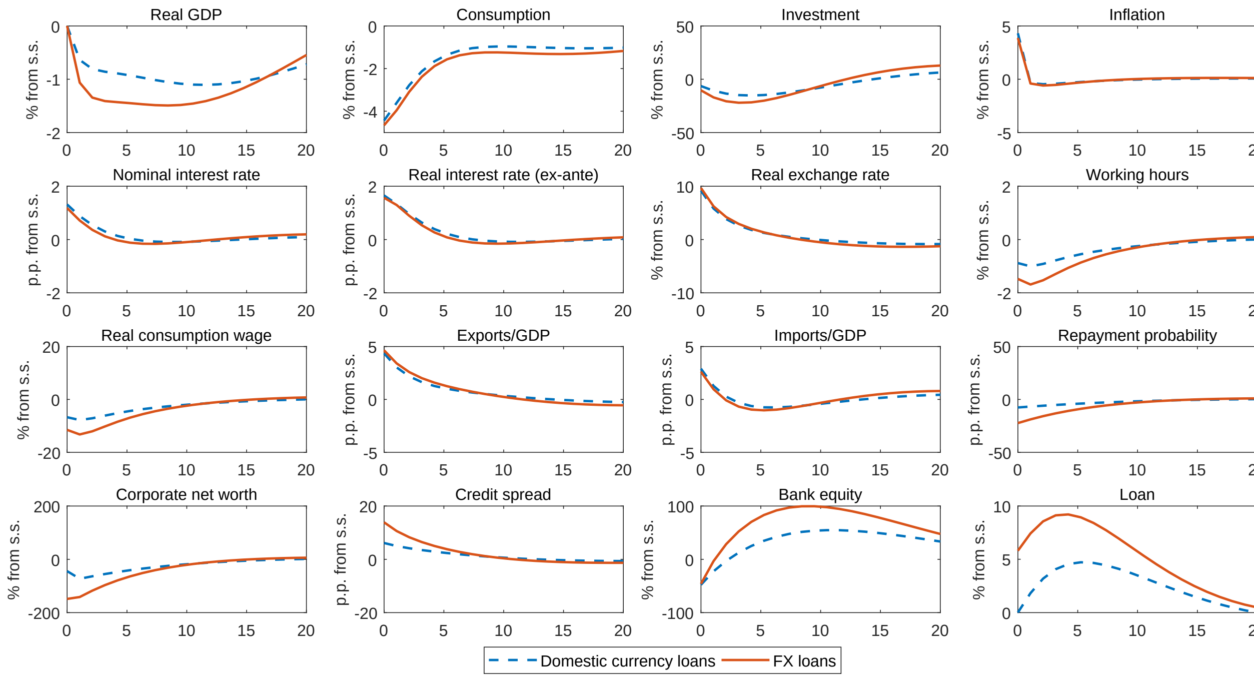


Figure 14: World demand shock in the model with leverage-constrained banks.

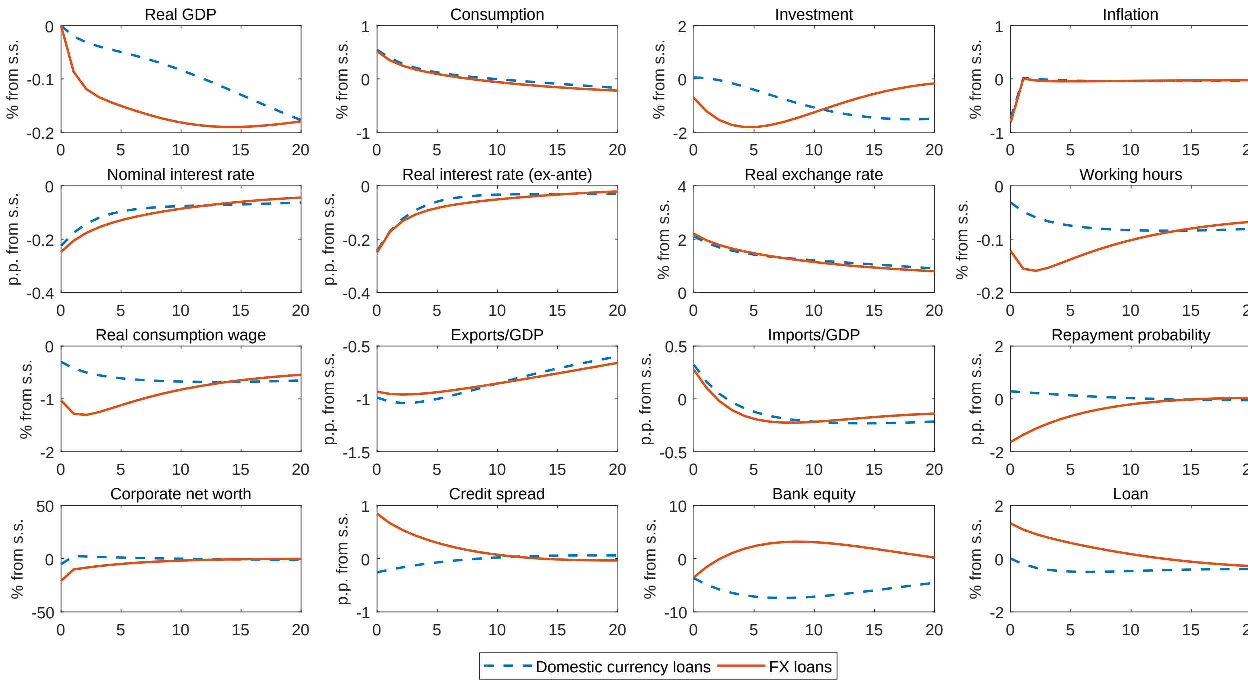
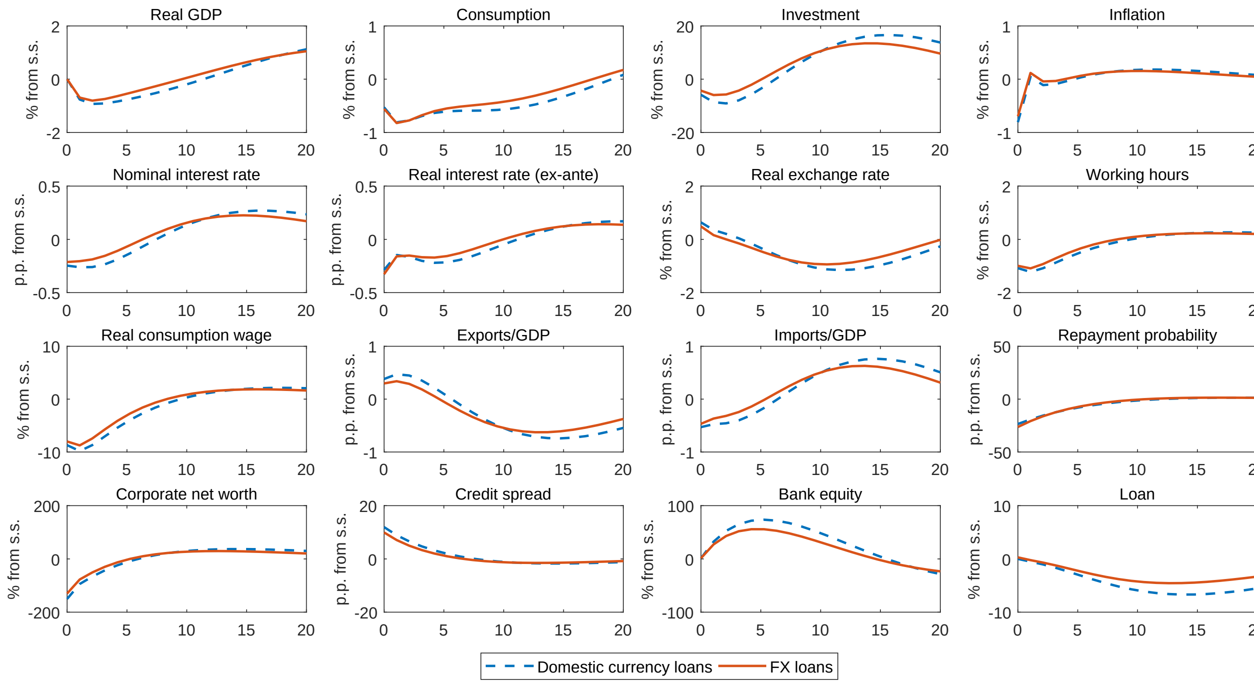


Figure 15: Volatility shock in the model with leverage-constrained banks.



Mathematical derivations

A: Financially constrained firms

A1: Solving the financially constrained firms' profit maximization problem with FX loans

Financially constrained firms live for two periods. Every period there is a new-born generation of firms and the total number of firms always constitute a continuum of mass one. In the first period firms buy two types of inputs, capital k and labour h, and have to pay for a fraction ρ in advance, which generates their demand for working capital. Production takes place in the next period.

To pay in advance, a financially constrained firm i uses two types of financing. First, it receives equity from households, $N_{i,t}^{firms}$. Second, it borrows from the bank an amount $L_{i,t}$ that consists of both domestic currency funds $L_{i,t}^{D}$ and foreign currency denominated funds $L_{i,t}^{F}$ such that $L_{i,t} = L_{i,t}^{D} + S_t L_{i,t}^{F}$ where S_t is the nominal exchange rate. We assume that the share of foreign currency denominated funds is fixed and denoted by α^F, so that the firm can choose the size of the total loan but not the denomination structure. This assumption allows us to calibrate the open position of banks and is innocuous enough, since we study the consequences of foreign currency borrowing rather than the choice of the borrowing currency.

To borrow, the firm has to pledge a share κ of future revenue as collateral where $0 < \kappa \leq 1$. We assume that the firm decides how much to borrow before shocks arrive and the prices of production inputs are revealed. Then the demanded size of the loan is equal to the expected expenditure for working capital minus the expected equity transfer from the household. It follows that in the beginning of period t the following condition holds:

$$E_{t-1}\left\{L_{i,t}\right\} + E_{t-1}\left\{N_{i,t}^{firms}\right\} = E_{t-1}\left\{\rho\left(Q_t k_{i,t} + W_t h_{i,t}\right)\right\} \tag{17}$$

Or, in units of composite goods associated with price P_t,

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n_{i,t}^{firms}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\} \tag{18}$$

q_t, w_t and rer_t denote the real price of capital, the real wage and the real exchange rate respectively. We express all three prices are expressed in units of composite goods. It follows that we define q_t as Q_t/P_t, w_t as W_t/P_t and the real exchange rate as $S_t P_t^*/P_t$ where S_t is the nominal exchange rate, P_t is the price of composite goods and P_t^* defines the price level of foreign composite goods. $n_{i,t}^{firms}$ stands for the equity transfer from the domestic household, where $n_{i,t}^{firms} \equiv N_{i,t}^{firms}/P_t$. $l_{i,t}$ stands for the size of the total loan expressed in units of composite goods and is defined as $l_{i,t} \equiv L_{i,t}/P_t$. After the loan is taken, shocks materialize, however, the predetermined size of the loan creates the debt overhang effect by distorting firm's private incentives to invest in production inputs.

Because of the timing of new information, the actual demand for working capital by the firm will in most cases not equal the loan amount received. We assume that in such cases the owner of the firm (the domestic household) steps in and transfers lump-sum funds $Z_{i,t}$ (where $z_{i,t} \equiv Z_{i,t}/P_t$) to cover the difference. Importantly, these funds constitute residual funding and firms cannot rely on them as the main source of finance. These funds enter the domestic household's budget constraint as a lump-sum transfer and have no effect on either the household's or the firm's incentives.

Let the matured loan in units of composite goods be $R_{i,t}^{R}\left(\frac{l_{i,t}^{D}}{\pi_{t+1}} + rer_{t+1}\frac{l_{i,t}^{F}}{\pi_{t+1}^{*}}\right)$, where $R_{i,t}^{R}$ is the nominal gross interest rate on the loan. The bank sets interest rates on loans after the shocks take place, therefore, the loan rate adjusts to clear the loan market. We define real loans in different currencies as $l^D_{i,t} \equiv L^D_{i,t}/P_t$ and $l^F_{i,t} \equiv L^F_{i,t}/P^*_t$. The contracted collateral is a fraction κ of firms' revenue from selling goods and depreciated capital in the next period, $p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}$. p^R_{t+1} stands for the price of homogenous goods, expressed in units of composite goods ($p^R_{t+1} \equiv P^R_{t+1}/P_{t+1}$). Then the decision of the financially constrained firm i born in period t whether to default or not is determined by the lower value:

$$\min\left\{R^R_{i,t}\left(L^D_{i,t}+S_{t+1}L^F_{i,t}\right), \quad \kappa\left(P^R_{t+1}y^R_{i,t+1}+Q_{t+1}(1-\delta)k_{i,t}\right)\right\} \tag{19}$$

Deflating by P_{t+1} gives the expression in units of composite goods:

$$\min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\} \tag{20}$$

where $p^R_{t+1}y^R_{i,t+1} = p^R_{t+1}A_{t+1}\theta_{i,t+1}k^\alpha_{i,t}h^{1-\alpha}_{i,t}$.

The firm i born in period t and endowed with corporate equity $N^{firms}_{i,t}$ maximizes profits taking the loan as given. The firm maximizes expected profits given by future revenue from selling goods and depreciated capital minus the second fraction of working capital expenditure together with expenses related to the debt payment. Financial flows received in period t also enter the maximization problem and can be summarized as the difference between the loan plus equity and working capital expenditure:

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\frac{\left\{P^R_{t+1}y^R_{i,t+1}+Q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\left(Q_tk_{i,t}+W_th_{i,t}\right)\right\}}{P_{t+1}} \\ & -E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R^R_{i,t}\left(L^D_{i,t}+S_{t+1}L^F_{i,t}\right)}{P_{t+1}}, \quad \frac{\kappa\left(P^R_{t+1}y^R_{i,t+1}+Q_{t+1}(1-\delta)k_{i,t}\right)}{P_{t+1}}\right\} \\ & +\frac{L_{i,t}+N^{firms}_{i,t}+Z_{i,t}}{P_t}-\frac{\rho\left(Q_tk_{i,t}+W_th_{i,t}\right)}{P_t}\end{aligned}$$

s.t.

$$\frac{E_{t-1}\left\{L_{i,t}+N^{firms}_{i,t}\right\}}{P_t}=\frac{E_{t-1}\left\{\rho\left(Q_tk_{i,t}+W_th_{i,t}\right)\right\}}{P_t}$$

Using the previously introduced definitions yields

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}-(1-\rho)\frac{q_tk_{i,t}+w_th_{i,t}}{\pi_{t+1}}\right\} \\ & -E_t\beta\Lambda_{t,t+1}\min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}}+rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{i,t+1}+q_{t+1}(1-\delta)k_{i,t}\right)\right\} \\ & +l_{i,t}+n^{firms}_{i,t}+z_{i,t}-\rho\left(q_tk_{i,t}+w_th_{i,t}\right)\end{aligned}$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\}+E_{t-1}\left\{n^{firms}_{i,t}\right\}=E_{t-1}\left\{\rho\left(q_tk_{i,t}+w_th_{i,t}\right)\right\}$$

The resulting first-order conditions are:

$$k_{i,t}: \quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) - (1-\rho)\frac{q_t}{\pi_{t+1}}\right\}$$
$$- E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta)\right)\right\}$$
$$= \frac{\partial cov\left(\beta\Lambda_{t,t+1}, \quad \min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}}$$
$$+ \rho q_t$$

$$h_{i,t}: \quad E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} - (1-\rho)\frac{w_t}{\pi_{t+1}}\right\}$$
$$- E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}\right)\right\}$$
$$= \frac{\partial cov\left(\beta\Lambda_{t,t+1}, \quad \min\left\{R^R_{i,t}\left(\frac{l^D_{i,t}}{\pi_{t+1}} + rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right), \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}}$$
$$+ \rho w_t$$

where

$$d_{2,t} \equiv \frac{E_t \ln\left(\kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right) - R^R_t rer_{t+1}\frac{l^F_{i,t}}{\pi^*_{t+1}}\right) - E_t \ln\left(R^R_{i,t}\frac{l^D_{i,t}}{\pi_{t+1}}\right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y$$

The derivation of $d_{2,t}$ is given in the next subsection and results for the first-order conditions are given by equations (A2.1) and (A2.2).

The first-order conditions hold together with the ex ante budget constraint:

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n^{firms}_{i,t}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\}$$

In the beginning of the next period, after shocks take place and a fraction of firms default, the domestic household pools the remaining net worth from non-defaulted firms into aggregate net worth by the following aggregation rule:

$$\begin{aligned} n^{firms}_t = \; & \omega^{firms}\left(p^R_t y^R_t + q_t(1-\delta)k_{t-1} - (1-\rho)\frac{q_{t-1}k_{t-1} + w_{t-1}h_{t-1}}{\pi_t}\right) \\ & -\omega^{firms}\left(\left(1-\Phi(d_{1,t-1})\right)\kappa\left(p^R_t y^R_t + q_t(1-\delta)k_{t-1}\right) + \Phi(d_{2,t-1})R^R_{t-1}\frac{l^D_{t-1}}{\pi_t} + \Phi(d_{1,t-1})rer_t\frac{l^F_{t-1}}{\pi^*_t}\right) \\ & +\iota^{firms}\cdot n^{firms} \end{aligned}$$

Recall that $(1-\Phi(d_{1,t-1}))$ proxies for the default rate (by the law of large numbers this is equal to the share of defaulted firms in the economy). Then the first term on the right hand side is aggregate firms' revenue from production and selling depreciated capital minus the rest of the expenditure for working capital. The second term is the firms' aggregate expenditure for repaying loans. The difference between the two gives financially constrained firms' profits. The third term is the injection of new equity. We assume that the domestic household acts as distributor and cannot divert pooled equity funds anywhere else. Also the existing equity can be increased only by the amount $\iota^{firms} \cdot n^{firms}$ that is fixed and proportional to aggregate net worth in the steady state. Thus, this equity transfer does not depend on the household's decision. ω^{firms} is a fraction that is close but lower than unity. We assume that this parameter proxies for the equity management costs incurred by the household and use this parameter to calibrate the steady state corporate leverage to the one observed in the data.

A2: Derivation of the default probability

We need to compute the expected value of the firm's payment function (we abstract from indices i for the sake of brevity):

$$E_t \min \left\{ R_t^R \left(\frac{l_t^D}{\pi_{t+1}} + rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*} \right), \quad \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t \right) \right\}$$

To simplify, we re-order the terms in the following way:

$$E_t \min \left\{ R_t^R \frac{l_t^D}{\pi_{t+1}}, \quad \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t \right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*} \right\} + E_t R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*}$$

Further we focus on the first term only, since it defines the default decision and contains all necessary prices too:

$$E_t \min \left\{ R_t^R \frac{l_t^D}{\pi_{t+1}}, \quad \kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t \right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*} \right\}$$

Define $\bar{y}_{t+1} \equiv \pi_{t+1} \left(\kappa \left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t \right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*} \right)$, where

$$\bar{y}_{t+1} \sim \text{log-normal} \left(\mu_{\bar{y}_{t+1}}, \sigma_y^2 \right)$$

Then the modified minimum function can be re-written as

$$E_t \min \left\{ R_t^R l_t^D, \quad \bar{y}_{t+1} \right\}$$

Further,

$$\begin{aligned}
E_t \min\left\{R_t^R l_t^D, \quad \bar{y}_{t+1}\right\} &= R_t^R l_t^D Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right) + \left(1 - Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right)\right) E_t\left(\bar{y}_{t+1} \mid \bar{y}_{t+1} < R_t^R l_t^D\right) \\
&= R_t^R l_t^D Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right) + \left(1 - Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right)\right) \int_0^{R_t^R l_t^D} \frac{\bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right)}{1 - Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right)} \\
&= R_t^R l_t^D Pr\left(R_t^R l_t^D < \bar{y}_{t+1}\right) + \int_0^{R_t^R l_t^D} \bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \int_{R_t^R l_t^D}^{\infty} dF\left(\bar{y}_{t+1}\right) + \int_0^{R_t^R l_t^D} \bar{y}_{t+1} dF\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \int_{R_t^R l_t^D}^{\infty} \frac{1}{\bar{y}_{t+1}\sigma_y\sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right)-\mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&+ \int_0^{R_t^R l_t^D} \frac{\bar{y}_{t+1}}{\bar{y}_{t+1}\sigma_y\sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right)-\mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \Phi\left(\frac{ln\left(\bar{y}_{t+1}\right) - \mu_y}{\sigma_y}\right) |_{R_t^R l_t^D}^{\infty} + \int_0^{R_t^R l_t^D} \frac{1}{\sigma_y\sqrt{2\pi}} e^{\frac{-\left(ln\left(\bar{y}_{t+1}\right)-\mu_y\right)^2}{2\sigma_y^2}} d\left(\bar{y}_{t+1}\right) \\
&= R_t^R l_t^D \left(1 - \Phi\left(\frac{ln\left(R_t^R l_t^D\right) - \mu_y}{\sigma_y}\right)\right) - \frac{1}{2} e^{\mu_y + \frac{\sigma_y^2}{2}} erf\left(\frac{-ln\left(\bar{y}_{t+1}\right) + \mu_y + \sigma_y^2}{\sqrt{2}\sigma_y}\right) |_0^{R_t^R l_t^D} \\
&= R_t^R l_t^D \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y}\right) + \frac{1}{2} E_t(\bar{y}_{t+1}) \left(erf\left(\frac{ln\left(R_t^R l_t^D\right) - \mu_y - \sigma_y^2}{\sqrt{2}\sigma_y}\right) + 1\right) \\
&= R_t^R l_t^D \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y}\right) + E_t(\bar{y}_{t+1}) \Phi\left(\frac{ln\left(R_t^R l_t^D\right) - \mu_y - \sigma_y^2}{\sigma_y}\right) \\
&= R_t^R l_t^D \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y}\right) + E_t(\bar{y}_{t+1}) \left(1 - \Phi\left(\frac{\mu_y - ln\left(R_t^R l_t^D\right)}{\sigma_y} + \sigma_y\right)\right)
\end{aligned}$$

The expression can be simplified as

$$E_t \min\left\{R_t^R l_t^D, \quad \bar{y}_{t+1}\right\} = \left(1 - \Phi(d_{1,t})\right) E_t\left(\bar{y}_{t+1}\right) + \Phi(d_{2,t}) R_t^R l_t^D$$

where

$$d_{2,t} \equiv \frac{\mu_y - \ln\left(R_t^R l_t^D\right)}{\sigma_y}, \qquad d_{1,t} \equiv d_{2,t} + \sigma_y$$

where

$$\mu_y \equiv E_t \ln\left(\bar{y}_{t+1}\right)$$

or

$$d_{2,t} \equiv \frac{E_t \ln\left(\bar{y}_{t+1}/\pi_{t+1}\right) - \ln\left(R_t^R/\pi_{t+1} l_t^D\right)}{\sigma_y}, \qquad d_{1,t} \equiv d_{2,t} + \sigma_y$$

Recall that $\bar{y}_{t+1} \equiv \pi_{t+1}\left(\kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*}\right)$ so it can be substituted back to get complete expressions. Then $\sigma_y^2 = var\left(\bar{y}_{t+1}\right) = var\left(\pi_{t+1}\left(\kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1} \frac{l_t^F}{\pi_{t+1}^*}\right)\right)$.

To solve for the first-order conditions, we differentiate the expected loan payment w.r.t. k_t:

$$\begin{aligned}\frac{\partial E_t \min\left\{R_t^R l_t^D, \quad \bar{y}_{t+1}\right\}}{\partial k_t} &= \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t \bar{y}_{t+1}}{\partial k_t} \\ &- \quad E_t \bar{y}_{t+1}\frac{\partial \Phi(d_{1,t})}{\partial d_{1,t}}\frac{\partial d_{1,t}}{\partial k_t} + R_t^R l_t^D \frac{\partial \Phi(d_{2,t})}{\partial d_{2,t}}\frac{\partial d_{2,t}}{\partial k_t} \\ &= \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t \bar{y}_{t+1}}{\partial k_t}\end{aligned}$$

where the proof of the last expression comes from by using $\frac{\partial d_{1,t}}{\partial k_t} = \frac{\partial d_{2,t}}{\partial k_t}$ and computing the following:

$$\begin{aligned} & -E_t\left(\bar{y}_{t+1}\right)\Phi'(d_{1,t}) + R_t^R l_t^D \Phi'(d_{2,t}) \\ = \quad & -e^{\ln(E_t\bar{y}_{t+1})}\Phi'(d_{1,t}) + e^{\ln(R_t^R l_t^D)}\Phi'(d_{2,t}) \\ = \quad & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{1,t}^2} + e^{\ln(R_t^R l_t^D)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2} \\ = \quad & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}\left(d_{2,t}^2+2d_{2,t}\sigma_y+\sigma_y^2\right)} + e^{\ln(R_t^R l_t^D)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2} \\ = \quad & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2}e^{-\left(d_{2,t}\sigma_y+\frac{1}{2}\sigma_y^2\right)} + e^{\ln(R_t^R l_t^D)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2} \\ = \quad & -e^{\ln(E_t\bar{y}_{t+1})}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2}e^{-E_t\left(\ln\bar{y}_{t+1}\right)-\ln(R_t^R l_t^D)+\frac{1}{2}\sigma_y^2} + e^{\ln(R_t^R l_t^D)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2} \\ = \quad & \frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2}\left[-e^{\ln(E_t\bar{y}_{t+1})}e^{-\left(\ln(E_t\bar{y}_{t+1})-\frac{1}{2}\sigma_y^2-\ln(R_t^R l_t^D)+\frac{1}{2}\sigma_y^2\right)} + e^{\ln(R_t^R l_t^D)}\right] \\ = \quad & -\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2}e^{\ln(R_t^R l_t^D)} + e^{\ln(R_t^R l_t^D)}\frac{1}{\sqrt{2\pi}}e^{-\frac{1}{2}d_{2,t}^2} \\ = \quad & 0,\end{aligned}$$

where such expressions are used as

$$E_t \ln\left(\bar{y}_{t+1}\right) = \ln\left(E_t\bar{y}_{t+1}\right) - \frac{1}{2}\sigma_y^2$$

and the definition of the variable $d_{1,t}$. Substituting a definition for $\bar{y}_{t+1}$ back gives

$$\frac{\partial E_t \min\left\{\frac{R_t^R}{\pi_{t+1}}l_t^D, \quad \kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}\right\}}{\partial k_t} = \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t \kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right)}{\partial k_t} \tag{A2.1}$$

Similarly it can be showed that

$$\frac{\partial E_t \min\left\{\frac{R_t^R}{\pi_{t+1}}l_t^D, \quad \kappa\left(p_{t+1}^R y_{t+1}^R + q_{t+1}(1-\delta)k_t\right) - R_t^R rer_{t+1}\frac{l_t^F}{\pi_{t+1}^*}\right\}}{\partial h_t} = \left(1-\Phi(d_{1,t})\right)\frac{\partial E_t \kappa\left(p_{t+1}^R y_{t+1}^R\right)}{\partial h_t} \tag{A2.2}$$

A3: Solving the financially constrained firms' profit maximization problem with domestic currency loans

Now the matured loan in units of composite goods is $R^R_{i,t}\frac{L_{i,t}}{P_{t+1}} \equiv R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}$. The loan is denominated in domestic currency and $R^R_{i,t}$ is the nominal gross interest rate on the loan. The contracted collateral is a fraction κ of firms' revenue from selling goods and depreciated capital in the next period. In units of composite goods the contracted collateral can be expressed as $p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}$. Then the decision of the financially constrained firm i born in period t whether to default or not is determined by the lower value:

$$\min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}$$

As previously, $p^R_{t+1}y^R_{i,t+1} = p^R_{t+1}A_{t+1}\theta_{i,t+1}k^{\alpha}_{i,t}h^{1-\alpha}_{i,t}$, $p^R_{t+1} \equiv P^R_{t+1}/P_{t+1}$ and $q_{t+1} \equiv Q_{t+1}/P_{t+1}$.

Financial flows received in period t also enter the maximization problem and can be summarized as the difference between the loan plus equity (both $N^{firms}_{i,t}$ and $Z_{i,t}$) and working capital expenditure expressed in units of composite goods:

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{\frac{P^R_{t+1}y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t} - (1-\rho)\left(Q_tk_{i,t} + W_th_{i,t}\right)}{P_{t+1}}\right\} \\ & - E_t\beta\Lambda_{t,t+1}\min\left\{\frac{R^R_{i,t}L_{i,t}}{P_{t+1}}, \quad \frac{\kappa\left(P^R_{t+1}y^R_{i,t+1} + Q_{t+1}(1-\delta)k_{i,t}\right)}{P_{t+1}}\right\} \\ & + \frac{L_{i,t} + N^{firms}_{i,t} + Z_{i,t}}{P_t} - \frac{\rho\left(Q_tk_{i,t} + W_th_{i,t}\right)}{P_t}\end{aligned}$$

s.t.

$$\frac{E_{t-1}\left\{L_{i,t} + N^{firms}_{i,t}\right\}}{P_t} = \frac{E_{t-1}\left\{\rho\left(Q_tk_{i,t} + W_th_{i,t}\right)\right\}}{P_t}$$

Using the previously introduced definitions yields

$$\begin{aligned}\max_{\{k_{i,t},h_{i,t}\}} & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t} - (1-\rho)\frac{q_tk_{i,t} + w_th_{i,t}}{\pi_{t+1}}\right\} \\ & - E_t\beta\Lambda_{t,t+1}\min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\} \\ & + l_{i,t} + n^{firms}_{i,t} + z_{i,t} - \rho\left(q_tk_{i,t} + w_th_{i,t}\right)\end{aligned}$$

s.t.

$$E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n^{firms}_{i,t}\right\} = E_{t-1}\left\{\rho\left(q_tk_{i,t} + w_th_{i,t}\right)\right\}$$

The resulting first-order conditions are:

$$
\begin{aligned}
k_{i,t}: \quad & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta) - (1-\rho)\frac{q_t}{\pi_{t+1}}\right\} \\
& - E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial k_{i,t}} + q_{t+1}(1-\delta)\right)\right\} \\
& = \frac{\partial cov\left(\beta\Lambda_{t,t+1}, \quad \min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial k_{i,t}} \\
& + \rho q_t
\end{aligned}
$$

$$
\begin{aligned}
h_{i,t}: \quad & E_t\beta\Lambda_{t,t+1}\left\{p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}} - (1-\rho)\frac{w_t}{\pi_{t+1}}\right\} \\
& - E_t\beta\Lambda_{t,t+1}\left\{(1-\Phi(d_{1,t}))\kappa\left(p^R_{t+1}\frac{\partial y^R_{i,t+1}}{\partial h_{i,t}}\right)\right\} \\
& = \frac{\partial cov\left(\beta\Lambda_{t,t+1}, \quad \min\left\{R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}, \quad \kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right\}\right)}{\partial h_{i,t}} \\
& + \rho w_t
\end{aligned}
$$

where

$$
d_{2,t} \equiv \frac{E_t \ln\left(\kappa\left(p^R_{t+1}y^R_{i,t+1} + q_{t+1}(1-\delta)k_{i,t}\right)\right) - E_t \ln\left(R^R_{i,t}\frac{l_{i,t}}{\pi_{t+1}}\right)}{\sigma_y}, \qquad d_{1,t} = d_{2,t} + \sigma_y
$$

and $\sigma_y^2 = var\left(\pi_{t+1}\kappa\left(p^R_{t+1}y^R_{t+1} + q_{t+1}(1-\delta)k_t\right)\right)$.

The first-order conditions hold together with the ex ante budget constraint:

$$
E_{t-1}\left\{l_{i,t}\right\} + E_{t-1}\left\{n^{firms}_{i,t}\right\} = E_{t-1}\left\{\rho\left(q_t k_{i,t} + w_t h_{i,t}\right)\right\}
$$

B: Solving the banks' optimization problem

B1: Lending in foreign currency and domestic currency with a fixed denomination structure

The domestic household owns all banks that operate in the domestic economy and lend to financially constrained firms. We assume that there is a continuum of these banks and every period there is a probability ω that a bank continues operating. Otherwise, the net worth is transferred to the owner of the bank, the domestic household. We assume that banks give loans out of accumulated equity N_t, deposits D_t and foreign debt D^*_t. The balance sheet constraint of a bank j, expressed in units of composite goods, is given by

$$
\frac{N_{j,t} + D_{j,t} + S_t D^*_{j,t}}{P_t} = \frac{L_{j,t}}{P_t}
$$

$L_{j,t}$ consists of both domestic currency funds $L^D_{j,t}$ and foreign currency denominated funds $L^F_{j,t}$ such that $L_{j,t} = L^D_{j,t} + S_t L^F_{j,t}$ where S_t is the nominal exchange rate.

Banks pay a nominal domestic interest rate R_t on deposits and a nominal foreign interest rate $R_t^* \xi_t$ on foreign debt. R_t^* follows a stationary AR(1) process. ξ_t denotes a premium on bank foreign debt. To ensure stationarity in the model, we assume that the premium depends on the level of bank foreign debt (as in Schmitt-Grohé and Uribe, 2003):

$$\xi_t = \exp\left(\kappa_\xi \frac{(S_t D_t^* - S \cdot D^*)}{S \cdot D^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{21}$$

where ζ_t is an exogenous shock that follows a stable AR(1) process.

Banks are subject to an agency problem as in Gertler and Karadi (2011). At the end of every period, bankers can divert a fraction λ^L of assets, but if that happens the bank goes bankrupt (i.e. cannot continue). Creditors take this possibility into account and lend only up to the point where the continuation value of the bank is equal to or higher than the value of what can be diverted. This condition acts as an incentive constraint for the bank and eventually limits expansion of the balance sheet of the bank for given amount of equity.

The only asset on the banks' balance sheet is loans to financially constrained firms, thus, the expected nominal return of the bank j is defined as $R_{j,t}^L$ and given by:

$$E_t\left\{R_{j,t}^L L_{j,t}\right\} \equiv E_t \min\left\{R_{j,t}^R\left(L_{j,t}^D + S_{t+1}L_{j,t}^F\right), \quad \kappa\left(P_{t+1}^R y_{j,t+1}^R + Q_{t+1}(1-\delta)k_{j,t}\right)\right\}$$

Or, units of composite goods,

$$E_t\left\{\frac{R_{j,t}^L}{\pi_{t+1}} l_{j,t}\right\} \equiv E_t \min\left\{R_{j,t}^R\left(\frac{l_{j,t}^D}{\pi_{t+1}} + rer_{t+1}\frac{l_{j,t}^F}{\pi_{t+1}^*}\right), \quad \kappa\left(p_{t+1}^R y_{j,t+1}^R + q_{t+1}(1-\delta)k_{j,t}\right)\right\}$$

$$\Rightarrow E_t\left\{\frac{R_{j,t}^L}{\pi_{t+1}} l_{j,t}\right\} \equiv E_t\left\{(1-\Phi(d_{1,t}))\,\kappa\left(p_{t+1}^R y_{j,t+1}^R + (1-\delta)q_{t+1}k_{j,t}\right) + \Phi(d_{2,t})R_{j,t}^R\frac{l_{j,t}^D}{\pi_{t+1}} + \Phi(d_{1,t})R_{j,t}^R rer_{t+1}\frac{l_{j,t}^F}{\pi_{t+1}^*}\right\} \tag{22}$$

Then the optimization problem of the bank j can be written as:

$$V_{j,t} = \max_{\left\{D_{j,t}, D_{j,t}^*, L_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)\frac{N_{j,t+1}}{P_{t+1}} + \omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L \frac{L_{j,t}}{P_t}, \qquad \textit{(Incentive constraint)}$$

$$\frac{N_{j,t} + D_{j,t} + S_t D_{j,t}^*}{P_t} = \frac{L_{j,t}}{P_t}, \qquad \textit{(Balance sheet constraint)}$$

$$\frac{N_{j,t}}{P_t} = \frac{R_{j,t-1}^L}{P_t}L_{j,t-1} - \frac{R_{t-1}}{P_t}D_{j,t-1} - \frac{R_{t-1}^*\xi_{t-1}}{P_t}S_t D_{j,t-1}^* \qquad \textit{(LoM of net worth)}$$

We define $rer_t \equiv P_t^* S_t / P_t$, $d_{j,t}^* \equiv D_{j,t}^*/P_t^*$, $d_{j,t} \equiv D_{j,t}/P_t$, $l_{j,t} \equiv L_{j,t}/P_t$, and $n_{j,t} \equiv N_{j,t}/P_t$. It follows that

$$V_{j,t} = \max_{\left\{d_{j,t}, d_{j,t}^*, l_{j,t}\right\}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1} + \omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t} + d_{j,t} + rer_t d^*_{j,t} = l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} \qquad \textit{(LoM of net worth)}$$

Lagrangian of the problem can be formulated as:

$$\begin{aligned} L = \quad & (1+\nu_{1,t}) E_t \beta \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} - \frac{R_t}{\pi_{t+1}} d_{j,t} - \frac{R^*_t \xi_t}{\pi^*_{t+1}} rer_{t+1} d^*_{j,t} \right) + \omega V_{j,t+1} \right\} \\ & - \nu_{1,t} \lambda^L l_{j,t} \\ & + \nu_{2,t} \left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t} \right) \end{aligned}$$

This gives the first-order conditions:

$$l_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R^L_{j,t}}{\pi_{t+1}} \right) + \omega \frac{\partial V(.)}{\partial l_{j,t}} \right\} = \lambda^L \nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R_t}{\pi_{t+1}} \right) - \omega \frac{\partial V(.)}{\partial d_{j,t}} \right\} = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) \left(\frac{R^*_t \xi_t}{\pi^*_{t+1}} rer_{t+1} \right) - \omega \frac{\partial V(.)}{\partial d^*_{j,t}} \right\} = \nu_{2,t} rer_t$$

with complementary slackness conditions:

$$\nu_{1,t}: \quad \nu_{1,t} \left(V_{j,t} - \lambda^L l_{j,t} \right) = 0$$

$$\nu_{2,t}: \quad \nu_{2,t} \left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t} \right) = 0$$

Further, the first-order conditions can be expressed as

$$l_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R^L_{j,t}}{\pi_{t+1}} \right) = \lambda^L \nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R_t}{\pi_{t+1}} \right) = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t}) \beta E_t \Lambda_{t,t+1} \left\{ (1-\omega) + \omega \nu_{2,t+1} \right\} \left(\frac{R^*_t \xi_t}{\pi^*_{t+1}} \frac{rer_{t+1}}{rer_t} \right) = \nu_{2,t}$$

Besides these first-order conditions, the set of equilibrium conditions includes the law of motion for aggregate net worth of banks and the bank incentive constraint. First, we formulate the law of motion for aggregate net worth. We assume that aggregate net worth consists of the net worth of non-bankrupted banks and the new worth of new banks. The new equity is injected by the domestic household and is assumed to be of the size ιn. Then

$$n_t = \omega \left(\frac{R^L_{j,t-1}}{\pi_t} l_{t-1} - \frac{R_{t-1}}{\pi_t} d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{t-1} \right) + \iota n$$

To include the incentive constraint in the equilibrium conditions, we have to redefine it by using the value of marginal utility from increasing assets by one unit and the value of marginal disutility from increasing debt by one unit. It follows from the previously derived results that the value of the bank j can also be defined as:

$$\begin{aligned} V_{j,t} &= \left(\lambda^L \tfrac{\nu_{1,t}}{1+\nu_{1,t}} + \tfrac{\nu_{2,t}}{1+\nu_{1,t}} \right) l_{j,t} - \tfrac{\nu_{2,t}}{1+\nu_{1,t}} d_{j,t} - \tfrac{\nu_{2,t}}{1+\nu_{1,t}} rer_t d^*_{j,t} \\ &= \tfrac{\nu_{2,t}}{1+\nu_{1,t}} \left(l_{j,t} - d_{j,t} - rer_t d^*_{j,t} \right) + \lambda^L \tfrac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t} \end{aligned}$$

$$\Rightarrow \quad V_{j,t} = \frac{\nu_{2,t}}{1+\nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t}$$

Then we can modify the incentive constraint as

$$\frac{\nu_{2,t}}{1+\nu_{1,t}} n_{j,t} + \lambda^L \frac{\nu_{1,t}}{1+\nu_{1,t}} l_{j,t} \geq \lambda^L l_{j,t}$$

$$\Rightarrow \quad \nu_{2,t} n_{j,t} \geq \lambda^L l_{j,t}$$

B2: Lending in domestic currency only

Now the only asset on the banks' balance sheet is domestic currency loans extended to financially constrained firms, thus, the expected nominal return of the bank j is defined as $R^L_{j,t}$ and given by:

$$E_t \left\{ R^L_{j,t} L_{j,t} \right\} \equiv E_t \min \left\{ R^R_{j,t} L_{j,t}, \quad \kappa \left(P^R_{t+1} y^R_{j,t+1} + Q_{t+1}(1-\delta) k_{j,t} \right) \right\}$$

Or, in units of composite goods,

$$E_t \left\{ \frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \min \left\{ R^R_{j,t} \frac{l_{j,t}}{\pi_{t+1}}, \quad \kappa \left(p^R_{t+1} y^R_{j,t+1} + q_{t+1}(1-\delta) k_{j,t} \right) \right\}$$

$$\Rightarrow E_t \left\{ \frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} \right\} \equiv E_t \left\{ (1 - \Phi(d_{1,t}))\, \kappa \left(p^R_{t+1} y^R_{j,t+1} + (1-\delta) q_{t+1} k_{j,t} \right) + \Phi(d_{2,t}) R^R_{j,t} \frac{l_{j,t}}{\pi_{t+1}} \right\} \tag{23}$$

The rest of derivations for the bank's optimization problem remain the same.

B3: Financial sector support

This segment of the model closely follows Kirchner and van Wijnbergen (2011). We assume that the government can intervene during the crisis by injecting capital τ^{FS}_t to the financial sector. We assign the following rule to the recap of the financial intermediary j:

$$\tau_t^{FI} = \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1}, \qquad \kappa_{FS} > 0, \quad l \geq 0$$

where $n_{j,t-1}$ is the net worth of the intermediary from the previous period. The recap can be immediate ($l = 0$) or delayed ($l > 0$). We introduce a new variable $shock_t$ that coincides with the variable driving the crisis, e.g. the risk premium shock ($shock_t \equiv \xi_t$). We assume that the recap is a gift from the government and does not have to be repaid.

Then the optimization problem of the financial intermediary j as defined in subsection B1 can be modified to

$$V_{j,t} = \max_{l_{j,t}, d_{j,t}, d^*_{j,t}} \quad E_t\left[\beta\Lambda_{t,t+1}\left\{(1-\omega)n_{j,t+1} + \omega V_{j,t+1}\right\}\right]$$

s.t.

$$V_{j,t} \geq \lambda^L l_{j,t}, \qquad \textit{(Incentive constraint)}$$

$$n_{j,t} + d_{j,t} + rer_t d^*_{j,t} = l_{j,t}, \qquad \textit{(Balance sheet constraint)}$$

$$n_{j,t} = \frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1} \quad \textit{(LoM of net worth)}$$

Lagrangian of the problem can be formulated as:

$$\begin{aligned} L = \quad & (1+\nu_{1,t})E_t\beta\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}} l_{j,t} - \frac{R_t}{\pi_{t+1}} d_{j,t} - \frac{R^*_t\xi_t}{\pi^*_{t+1}} rer_{t+1} d^*_{j,t} + \kappa_{FS}\left(shock_{t-l+1} - shock\right) n_{j,t}\right) + \omega V_{j,t+1}\right\} \\ & - \nu_{1,t}\lambda^L l_{j,t} \\ & + \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) \end{aligned}$$

This gives the first-order conditions:

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^L_{j,t}}{\pi_{t+1}} + \kappa_{FS}\left(shock_{t-l+1} - shock\right)\right) + \omega\frac{\partial V(.)}{\partial l_{j,t}}\right\} = \lambda^L\nu_{1,t} + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R_t}{\pi_{t+1}} + \kappa_{FS}\left(shock_{t-l+1} - shock\right)\right) - \omega\frac{\partial V(.)}{\partial d_{j,t}}\right\} = \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)\left(\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t} + \kappa_{FS}\left(shock_{t-l+1} - shock\right)\right) - \omega\frac{\partial V(.)}{\partial d^*_{j,t}}\right\} = \nu_{2,t}$$

with complementary slackness conditions:

$$\nu_{1,t}: \nu_{1,t}\left(V_{j,t} - \lambda^L l_{j,t}\right) = 0$$

$$\nu_{2,t}: \nu_{2,t}\left(\frac{R^L_{j,t-1}}{\pi_t} l_{j,t-1} - \frac{R_{t-1}}{\pi_t} d_{j,t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t} rer_t d^*_{j,t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right) n_{j,t-1} - l_{j,t} + d_{j,t} + rer_t d^*_{j,t}\right) = 0$$

Further, the first-order conditions can be expressed as

$$l_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\frac{R^L_{j,t}}{\pi_{t+1}} + (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}(1-\omega)\kappa_{FS}\left(shock_{t-l+1} - shock\right)$$
$$= \nu_{1,t}\lambda^L + \nu_{2,t}$$

$$d_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\frac{R_t}{\pi_{t+1}} + (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}(1-\omega)\kappa_{FS}\left(shock_{t-l+1} - shock\right)$$
$$= \nu_{2,t}$$

$$d^*_{j,t}: \quad (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\frac{R^*_t\xi_t}{\pi^*_{t+1}}\frac{rer_{t+1}}{rer_t} + (1+\nu_{1,t})\beta E_t \Lambda_{t,t+1}(1-\omega)\kappa_{FS}\left(shock_{t-l+1} - shock\right)$$
$$= \nu_{2,t}$$

Aggregate net worth evolves as

$$n_t = \omega\left[\frac{R^L_{j,t-1}}{\pi_t}l_{t-1} - \frac{R_{t-1}}{\pi_t}d_{t-1} - \frac{R^*_{t-1}\xi_{t-1}}{\pi^*_t}rer_t d^*_{t-1} + \kappa_{FS}\left(shock_{t-l} - shock\right)n_{t-1}\right] + \iota n$$

C: Household's problem

We assume a representative household. The household has two alternatives to invest in: make deposits D_t in a bank or buy bonds issued by the government, B_t. The household supplies labour to a competitive labour market. The household has Greenwood–Hercowitz–Huffman (henceforth, GHH) preferences as in Greenwood et al. (1988), so labour supply does not depend on wealth. The household chooses a level of real consumption c_t and working hours h_t such that the following lifetime utility function is maximized:

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{1-\gamma}\left(c_t - \frac{\chi\left(h_t\right)^{1+\varphi}}{1+\varphi}\right)^{1-\gamma} \qquad \gamma, \chi, \varphi > 0 \tag{24}$$

subject to the household's budget constraint:

$$C_t + B_t + D_t = W_t h_t + R_{t-1}B_{t-1} + R_{t-1}D_{t-1} + P_t\Pi_t - T_t$$

The budget constraint, expressed in units of composite goods, is given by

$$c_t + b_t + d_t = w_t h_t + \frac{R_{t-1}}{\pi_t}b_{t-1} + \frac{R_{t-1}}{\pi_t}d_{t-1} + \Pi_t - t_t \tag{25}$$

π_t denotes the composite goods price inflation, $c_t \equiv C_t/P_t$, $w_t \equiv W_t/P_t$, $b_t \equiv B_t/P_t$, $d_t \equiv D_t/P_t$, $t_t \equiv T_t/P_t$. We assume that the household is indifferent between buying bonds and making deposits, thus, R_t is nominal gross interest rate of both bonds and deposits. The household owns all banks in the model economy and thus receives lump-sum dividends, Π_t. Taxes t_t enter the household's budget constraint in a lump-sum way as well. Lump-sum dividends from financially constrained firms are included in total dividends Π_t. Lump-sum dividends from financially constrained firms consist of firms' profits that the household receives in the beginning in the period minus the equity that the household transfers to the firms in the beginning of the period (in response to liquidity shortage, if there is any):

$$\Pi_t^{firms} = \omega^{firms}\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1} - (1-\rho)\frac{q_{t-1}k_{t-1}+w_{t-1}h_{t-1}}{\pi_t}\right)$$
$$-\omega^{firms}\left(\kappa\left(1-\Phi(d_{1,t-1})\right)\left(p_t^R y_t^R + q_t(1-\delta)k_{t-1}\right) + \Phi(d_{2,t-1})R_{t-1}^R\frac{l_{t-1}^D}{\pi_t} + \Phi(d_{1,t-1})rer_t\frac{l_{t-1}^F}{\pi_t^*}\right)$$
$$-n_t^{firms} - z_t$$
$$= -\iota^{firms}\cdot n^{firms} - z_t$$

The final result follows from the definition of aggregate corporate net worth given in the financially constrained firms' problem in section A1.

The household's optimization problem gives first-order conditions:

$$\lambda_t = \left(c_t - \frac{\chi\left(h_t\right)^{1+\varphi}}{1+\varphi}\right)^{-\gamma}$$

$$w_t = \chi\left(h_t\right)^{\varphi}$$

$$E_t\beta\Lambda_{t,t+1}\frac{R_t}{\pi_{t+1}} = 1$$

We denote $\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t}$ where λ_t is the Lagrangian multiplier to the household's budget constraint.

D: Production and Pricing

There are several types of firms in the domestic economy. It takes three types of firms to produce domestic aggregate inputs for composite goods. First, there are the financially constrained firms that combine purchased capital with labour and produce homogenous goods. They were analyzed in Section 3.1. Their homogenous outputs are bought by retail firms who costlessly differentiate the products bought and sell them as (local) monopolists, in Dixit-Stiglitz (1977) fashion. A similar group of firms called importers differentiate foreign (imported) goods. A composite goods producer buys the differentiated home goods and aggregates them into an aggregate domestic good y_t^H with associated price p_t^H. The same composite goods producer also buys imported differentiated goods and aggregates them into a foreign aggregate good y_t^F. The corresponding aggregate price level of foreign goods is p_t^F. All details of the derivations of the various first order conditions optimization problems can be found in the supplementary appendix D. We discuss each step in more detail below.

The structure of the production sector is exhibited in Figure 16.

D1: Retail firms

Homogenous goods produced by financially constrained firms are sold to domestic retail firms. We assume that there is a continuum of domestic retail firms. A domestic retail firm j differentiates purchased inputs at p_t^R and sells at a monopolistic price $p_t^H(j)$. Differentiated goods from the domestic retail sector, $y_t^H(j), j \in (0,1)$, are purchased by the composite goods producer.

Retail firms are subject to sticky prices as in Calvo (1983), so every period $(1-\omega^H)$ of them adjust prices to the optimal reset price $P_t^{\#}(j)$. Then the profit of a retail firm j that is allowed to adjust its price in period t

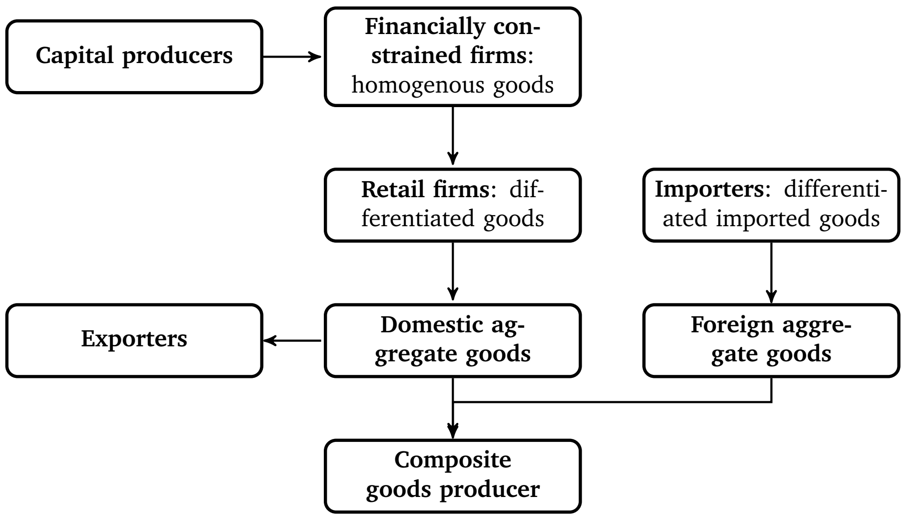


Figure 16: Structure of the production sector.

is thus given by $\left(P_t^{\#}(j) - P_t^R\right) y_t^H(j)$. The rest of retail firms adjust their past prices by the rate $\pi_t^{adj} = \pi$.

Then the aggregate price level of retail goods P_t^H is defined as

$$P_t^H = \left((1-\omega^H)\left(P_t^{\#}\right)^{1-\epsilon_H} + \omega^H\left(P_{t-1}^H \pi_t^{adj}\right)^{1-\epsilon_H}\right)^{1/(1-\epsilon_H)}$$

Define

$$\tilde{p}_t^H \equiv \frac{P_t^{\#}}{P_t^H} \tag{B.1}$$

It follows that

$$\Rightarrow \quad 1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H\left(\frac{P_{t-1}^H \pi_t^{adj}}{P_t^H}\right)^{1-\epsilon_H}$$

Re-writing in terms of relative prices with respect to the price level of composite goods P_t such that $p_t^H \equiv P_t^H/P_t$ gives

$$\Rightarrow \quad 1 = (1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H} + \omega^H\left(\frac{p_{t-1}^H \pi_t^{adj}}{\pi_t p_t^H}\right)^{1-\epsilon_H}$$

As a result, a retail firm j solves the optimization problem how to set the optimal price $P_t^{\#}(j)$ conditional on not changing it in the future that be be formalized as:

$$\max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty}\left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \frac{\left(P_t^{\#}(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right) - P_{t+s}^R\right)}{P_{t+s}} y_{t+s}^H(j)$$

s.t. demand for retail goods (equation (27))

$$y_t^H(j) = \left(\frac{P_t^{\#}(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} y_t^H$$

Define $p_t^R \equiv \frac{P_t^R}{P_t}$:

$$\max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#}(j)}{P_{t+s}} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right) - p_{t+s}^R \right) y_{t+s}^H(j)$$

s.t. demand for retail goods

$$y_t^H(j) = \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^H} \right)^{-\epsilon_H} y_t^H$$

$$\Rightarrow \max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}} \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H - p_{t+s}^R \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H \right)$$

$$\Rightarrow \max_{P_t^{\#}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{1-\epsilon_H} y_{t+s}^H - p_{t+s}^R \left(\frac{P_t^{\#}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_{t+s}^H} \right)^{-\epsilon_H} y_{t+s}^H \right)$$

We take a derivative w.r.t. $P_t^{\#}(j)$ and rearrange terms:

$$E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left((1-\epsilon_H) p_{t+s}^H \left(P_t^{\#}(j)\right)^{-\epsilon_H} \left(\frac{\prod_{j=1}^{j=s} \pi_{t+j}^{adj}}{P_{t+s}^H} \right)^{1-\epsilon_H} - \epsilon_H p_{t+s}^R \left(P_t^{\#}(j)\right)^{-\epsilon_H-1} \left(\frac{\prod_{j=1}^{j=s} \pi_{t+j}^{adj}}{P_{t+s}^H} \right)^{-\epsilon_H} \right) y_{t+s}^H$$

$$\Rightarrow E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left((1-\epsilon_H) p_{t+s}^H P_t^{\#}(j) \left(P_{t+s}^H\right)^{\epsilon_H-1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} - \epsilon_H p_{t+s}^R \left(P_{t+s}^H\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} \right) y_{t+s}^H = 0$$

$$\Rightarrow P_t^{\#}(j) = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(P_{t+s}^H\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} y_{t+s}^H \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(P_{t+s}^H\right)^{\epsilon_H-1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} y_{t+s}^H \right)} = 0$$

$$\Rightarrow \frac{P_t^{\#}(j)}{P_t^H} = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_H} y_{t+s}^H \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_{t+s}^H}{P_t^H} \right)^{\epsilon_H-1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_H} y_{t+s}^H \right)}$$

Since $\tilde{p}_t^H \equiv P_t^{\#}/P_t^H$,

$$\tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)^{-\epsilon_H} y_{t+s}^H\right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{P_{t+s}^H}{P_t^H}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)^{1-\epsilon_H} y_{t+s}^H\right)}$$

$$\Rightarrow \tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^R \left(\frac{p_{t+s}^H \pi_{t+s}}{p_t^H}\right)^{\epsilon_H} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)^{-\epsilon_H} y_{t+s}^H\right)}{E_t \sum_{s=0}^{\infty} \left(\omega^H\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^H \left(\frac{p_{t+s}^H \pi_{t+s}}{p_t^H}\right)^{\epsilon_H - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)^{1-\epsilon_H} y_{t+s}^H\right)}$$

$$\Rightarrow \tilde{p}_t^H = \frac{\epsilon_H}{(\epsilon_H - 1)} \frac{F_{1,t}^H}{F_{2,t}^H}$$

where

$$F_{1,t}^H = p_t^R y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}\right)^{\epsilon_H} F_{1,t+1}^H$$

and

$$F_{2,t}^H = p_t^H y_t^H + E_t \omega^H \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^H \pi_{t+1}}{p_t^H \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}\right)^{\epsilon_H - 1} F_{2,t+1}^H$$

D2: Importers

We assume that there is a continuum of monopolistically competitive importers. They buy a variety j of foreign goods $y_t^F(j)$ at price P_t^* and sell it to the composite goods producer at a nominal price $P_t^F(j)$, expressed in domestic currency.

Every period there is a fraction $(1 - \omega^F)$ of importers who can adjust their prices, in Calvo (1983) fashion. The set of importers who can adjust the price choose it such that their profits are maximized. The rest of importers adjust their past prices by the rate $\pi_t^{adj} = \pi$. As a result, an importer j solves the optimization problem how to set the optimal price $P_t^{\#F}(j)$ conditional on not changing it in the future:

$$\max_{P_t^{\#F}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \frac{\left(P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right) - S_{t+s} P_{t+s}^*\right)}{P_{t+s}} y_{t+s}^F(j) \tag{26}$$

s.t.

$$y_t^F(j) = \eta \left(\frac{P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_t^F}\right)^{-\epsilon} y_t^F$$

Since $rer_t \equiv S_t P_t^* / P_t$,

$$\max_{P_t^{\#F}(j)} E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(\frac{P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj}\right)}{P_{t+s}} - rer_{t+s}\right) y_{t+s}^F(j)$$

s.t.

$$y_t^F(j) = \eta \left(\frac{P_t^{\#F}(j) \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)}{P_t^F} \right)^{-\epsilon} y_t^F$$

In analogy to the problem of retail firms, we maximize expected profits and rearrange terms. Since all importers who can adjust their price set the same optimal price, $P_t^{\#F}(j) = P_t^{\#F} \forall j$. After introducing a variable $\tilde{p}_t^F$, which is defined as

$$\tilde{p}_t^F \equiv P_t^{\#F} / P_t^F, \tag{B.2}$$

we can show that the optimal price-setting equation follows as

$$\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(rer_{t+s} \left(\frac{p_{t+s}^F \pi_{t+s}}{p_t^F} \right)^{\epsilon_F} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{-\epsilon_F} y_{t+s}^F \right)}{E_t \sum_{s=0}^{\infty} \left(\omega^F\right)^s \beta^s \Lambda_{t,t+s} \left(p_{t+s}^F \left(\frac{p_{t+s}^F \pi_{t+s}}{p_t^F} \right)^{\epsilon_F - 1} \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)^{1-\epsilon_F} y_{t+s}^F \right)}$$

$$\Rightarrow \tilde{p}_t^F = \frac{\epsilon_F}{(\epsilon_F - 1)} \frac{F_{1,t}^F}{F_{2,t}^F}$$

where

$$F_{1,t}^F = rer_t y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F} F_{1,t+1}^F$$

and

$$F_{2,t}^F = p_t^F y_t^F + E_t \omega^F \beta \Lambda_{t,t+1} \left(\frac{p_{t+1}^F \pi_{t+1}}{p_t^F \left(\prod_{j=1}^{j=s} \pi_{t+j}^{adj} \right)} \right)^{\epsilon_F - 1} F_{2,t+1}^F$$

Deriving an aggregate price level of imported goods' produces the following expression: $1 = (1 - \omega^F) \left(\tilde{p}_t^F \right)^{1-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F \pi_t^{adj}}{p_t^F \pi_t} \right)^{1-\epsilon_F}$.

D3: Price dispersion

We define the price dispersion for retail goods as

$$D_t^H \equiv \int_0^1 \left(\frac{P_t^H(j)}{P_t^H} \right)^{-\epsilon_H} dj$$

$(1-\omega^H)$ of firms update prices to the same optimal price $P_t^{\#}$ and ω^H of firms adjust the last period's price with the adjustment term π_t^{adj}. This gives

$$\begin{aligned}
D_t^H &= \int_0^{1-\omega^H} \left(\frac{P_t^{\#}}{P_t^H}\right)^{-\epsilon_H} dj + \int_{1-\omega^H}^1 \left(\frac{P_{t-1}^H(j)\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} dj \\
&= \int_0^{1-\omega^H} \left(\frac{P_t^{\#}}{P_t^H}\right)^{-\epsilon_H} dj + \int_{1-\omega^H}^1 \left(\frac{P_{t-1}^H(j)}{P_{t-1}^H}\right)^{-\epsilon_H} \left(\frac{P_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} dj \\
&= (1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \int_{1-\omega^H}^1 D_{t-1}^H \left(\frac{P_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{P_t^H}\right)^{-\epsilon_H} dj \\
&= (1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H} + \omega^H \left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{\pi_t p_t^H}\right)^{-\epsilon_H} D_{t-1}^H
\end{aligned}$$

In analogy, the price dispersion of importers' goods is given by

$$D_t^F \equiv \int_0^1 \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} dj$$

and it follows a rule

$$D_t^F = (1-\omega^F)\left(\tilde{p}_t^F\right)^{-\epsilon_F} + \omega^F \left(\frac{p_{t-1}^F\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{\pi_t p_t^F}\right)^{-\epsilon_F} D_{t-1}^F$$

D4: Composite goods producer

The composite goods producer combines domestic aggregate goods and foreign aggregate goods into composite goods and sells them to the household, the government and capital goods producers. We define the supply of composite goods as y_t^C. Its associated price is P_t. The demanded amount of production inputs, namely, domestic aggregate goods and foreign aggregate goods, is denoted as x_t^H and x_t^F respectively.

Domestic aggregate goods. Domestic aggregate goods y_t^H result from assembling retailers' production $y_t^H(j)$ for $j \in [0,1]$, each bought at price $P_t^H(j)$, expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of retail goods be $P_t^H \equiv \left(\int_0^1 \left(P_t^H(j)\right)^{1-\epsilon_H} dj\right)^{1/(1-\epsilon_H)}$, expressed in domestic currency. Then it follows that the demand for retail goods is given as a solution to the problem

$$\max_{y_t^H(j)} \left\{P_t^H y_t^H - \int_0^1 P_t^H(j) y_t^H(j) dj\right\}$$

subject to the assembling technology

$$y_t^H = \left(\int_0^1 y_t^H(j)^{1-\frac{1}{\epsilon_H}} dj\right)^{\frac{\epsilon_H}{\epsilon_H - 1}}$$

and to the market clearing constraint that says that domestic aggregate goods are used as input by the composite goods producer and face foreign demand ex_t:

$$y_t^H = x_t^H + ex_t$$

As a result, optimal demand for retail goods of variety j is given by

$$y_t^H(j) = \left(\frac{P_t^H(j)}{P_t^H}\right)^{-\epsilon_H} y_t^H \tag{27}$$

Foreign aggregate goods. Foreign aggregate goods y_t^F result from assembling importers' production $y_t^F(j)$ for $j \in [0,1]$, each bought at price $P_t^F(j)$, expressed in domestic currency, and with no additional costs incurred. Let the aggregate price level of importers' goods be $P_t^F \equiv \left(\int_0^1 \left(P_t^F(j)\right)^{1-\epsilon_F} dj\right)^{1/(1-\epsilon_F)}$, expressed in domestic currency. Then it follows that the demand for importers' goods is given as a solution to the problem

$$\max_{y_t^F(j)} \left\{P_t^F y_t^F - \int_0^1 P_t^F(j) y_t^F(j) dj\right\}$$

subject to the assembling technology

$$y_t^F = \left(\int_0^1 y_t^F(j)^{1-\frac{1}{\epsilon_F}} dj\right)^{\frac{\epsilon_F}{\epsilon_F - 1}}$$

and to the market clearing constraint that says that all foreign aggregate goods are used to satisfy the demand of the composite goods producer:

$$y_t^F = x_t^F$$

As a result, optimal demand for importers' production of variety j is given by

$$y_t^F(j) = \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} y_t^F \tag{28}$$

and demand for foreign aggregate goods clears $x_t^F = y_t^F$.

Composite goods. Given inputs x_t^H and x_t^F, composite goods are assembled with the aggregation technology

$$y_t^C \equiv \left((1-\eta)^{\frac{1}{\epsilon}} (x_t^H)^{\frac{\epsilon-1}{\epsilon}} + \eta^{\frac{1}{\epsilon}} (x_t^F)^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{29}$$

where ϵ stands for elasticity of substitution between domestically produced goods and imported goods. A parameter η proxies for openness of the home economy.

The composite goods producer operates in a perfectly competitive market, so she maximizes profits $P_t y_t^C - P_t^H x_t^H - P_t^F x_t^F$ subject to the technology (29). This boils down to two demand conditions:

$$x_t^H = (1-\eta)\left(\frac{P_t^H}{P_t}\right)^{-\epsilon} y_t^C$$

and

$$x_t^F = \eta \left(\frac{P_t^F}{P_t}\right)^{-\epsilon} y_t^C$$

Further, we introduce relative prices $p_t^H \equiv P_t^H/P_t$ and $p_t^F \equiv P_t^F/P_t$ and get

$$x_t^H = (1-\eta)\left(p_t^H\right)^{-\epsilon} y_t^C \tag{30}$$

and

$$x_t^F = \eta\left(p_t^F\right)^{-\epsilon} y_t^C \tag{31}$$

D5: Capital producers

Capital producers participate in the domestic economy by selling capital to financially constrained firms at the real competitive price q_t and buying the depreciated capital stock back next period. To restore the depreciated capital, capital producers add composite goods (investment) i_t as additional inputs to the depreciated capital stock by using the technology subject to investment adjustment costs $\Gamma\left(\frac{i_t}{i_{t-1}}\right)$:

$$k_t = (1-\delta)k_{t-1} + \left(1 - \Gamma\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{32}$$

where adjustment costs Γ equal:

$$\Gamma\left(\frac{i_t}{i_{t-1}}\right) = \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2$$

Capital producers maximize profits, expressed in units of composite goods, subject to the production technology by choosing an optimal level of investment:

$$\max_{i_t} \beta E_t \Lambda_{t,t+1}\left\{(1-\rho)\frac{q_t}{\pi_{t+1}}k_t\right\} + \rho q_t k_t - q_t(1-\delta)k_{t-1} - i_t \tag{33}$$

s.t.

$$k_t = (1-\delta)k_{t-1} + \left(1 - \Gamma\left(\frac{i_t}{i_{t-1}}\right)\right) i_t \tag{34}$$

The optimization problem takes into account the share of capital purchases paid immediately ρ as opposed to the share of the payment $(1-\rho)$ delayed to the next period which makes it slightly different from a standard optimization problem solved by competitive capital producers.

Optimizing gives the demand function for investment:

$$\frac{1}{q_t} = \rho\left(1 - \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2\right) - \rho\gamma\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}} + \rho\gamma\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{35}$$

$$+(1-\rho)\beta E_t\Lambda_{t,t+1}\frac{q_{t+1}}{q_t}\left(1 - \frac{\gamma}{2}\left(\frac{i_t}{i_{t-1}} - 1\right)^2 - \gamma\left(\frac{i_t}{i_{t-1}} - 1\right)\frac{i_t}{i_{t-1}}\right) + (1-\rho)\gamma\beta^2 E_t\Lambda_{t,t+2}\frac{q_{t+1}}{q_t}\left(\frac{i_{t+1}}{i_t} - 1\right)\left(\frac{i_{t+1}}{i_t}\right)^2 \tag{36}$$

D6: Exporters

We assume that perfectly competitive exporters demand ex_t units of the domestic aggregate good y_t^H, so the supply of the assembled production of domestic retailers has to satisfy both the demand of the composite goods producer and the demand of exporters. Exported goods consist of the domestic aggregate, so they do not use imported inputs.

We abstract from modelling trade barriers. Hence, the rest of the world demands ex_t units of domestic aggregate goods at a price $P_t^{H*} = P_t^H/S_t$, which is the price of domestic aggregate goods expressed in units of foreign composite goods. We assume that all economies in the world are identical and their demand for domestic aggregate goods can be aggregated and expressed relative to world output y_t^*. The foreign demand for domestic aggregate goods is price-sensitive:

$$ex_t = \eta^* \left(\frac{p_t^H}{rer_t} \right)^{-\epsilon_*} y_t^* \tag{37}$$

Consistent with the small open economy assumption, P_t^* and y_t^* are assumed to evolve exogenously. η^* is the foreign households' taste parameter for domestic aggregate goods. ϵ^* defines the elasticity of substitution between domestic aggregate goods and goods produced in other economies.

E: Government

The government collects lump-sum taxes T_t from the household and issues domestic bonds B_t to finance a stochastic stream of nominal government expenditure, G_t, and the bank recap $P_t \tau_t^{FS}$. Therefore, it satisfies the budget constraint:

$$G_t + P_t \tau_t^{FS} + R_{t-1} B_{t-1} = T_t + B_t$$

Given $g_t \equiv G_t/P_t$, $b_t \equiv B_t/P_t$ and $t_t \equiv T_t/P_t$, the budget constraint can be expressed in units of composite goods as

$$g_t + \tau_t^{FS} + \frac{R_{t-1}}{\pi_t} b_{t-1} = t_t + b_t$$

Taxes in units of composite goods follow this tax rule:

$$t_t = t + \kappa^B (b_{t-1} - b) + \kappa^{FS} \tau_t^{FS} + e_t, \qquad 0 < \kappa^B \leq 1, \quad 0 \leq \kappa^{FS} \leq 1$$

The rule tells that a share κ^{FS} of the recap expenditure is covered by increasing the lump-sum tax and the rest (a share $(1 - \kappa^{FS})$) is financed with new government debt.

F: Central bank

The central bank conducts monetary policy by following the Taylor rule:

$$\frac{R_t}{\bar{R}} = \left(\frac{R_{t-1}}{\bar{R}} \right)^{\gamma_R} \left(\frac{y_t^H}{\bar{y^H}} \right)^{(1-\gamma_R)\gamma_Y} \left(\frac{\pi_t^H}{\bar{\pi^H}} \right)^{(1-\gamma_R)\gamma_\pi} \exp(mp_t) \tag{38}$$

mp_t is a monetary policy shock and the domestic aggregate goods price inflation π_t^H can be expressed as $\pi_t^H = p_t^H / p_{t-1}^H \pi_t$.

G: Market clearing

The domestic household, the government and capital producers buy composite goods. Therefore, the supply of composite goods y_t^C has to satisfy the aggregate demand of domestic agents:

$$y_t^C = c_t + i_t + g_t \tag{39}$$

H: Current account and its components

First, we derive an expression for aggregate nominal imports M_t in units of domestic currency.

We aggregate importers' demand for foreign composite $y_t^F(j)\forall j \in (0,1)$ that is priced at P_t^* and use the nominal exchange rate S_t to convert to domestic currency:

$$M_t = \int_0^1 S_t P_t^* y_t^F(j) dj$$

Further we use the derived demand function (28) to get

$$M_t = \int_0^1 S_t P_t^* \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} y_t^F$$

Define the price dispersion of importers' goods as $D_t^F \equiv \int_0^1 \left(\frac{P_t^F(j)}{P_t^F}\right)^{-\epsilon_F} dj$ (more details on the price dispersion are in subsection D3). Then

$$M_t = S_t P_t^* D_t^F y_t^F \tag{40}$$

which in units of composite goods is given by

$$m_t \equiv \frac{M_t}{P_t} = rer_t D_t^F y_t^F \tag{41}$$

Second, we define nominal exports EX_t, expressed in units of domestic currency. Since exports are purchased at the price P_t^{H*}, expressed in foreign currency, nominal exports EX_t, expressed in units of domestic currency, is given by

$$EX_t = S_t P_t^{H*} ex_t = P_t^H ex_t \tag{42}$$

Finally, the trade balance TB_t evolves as

$$TB_t = EX_t - M_t$$

Recall definitions for nominal exports and nominal imports in units of domestic currency (equations (42) and (40)). Then the trade balance in units of composite goods can be expressed as

$$tb_t \equiv \frac{TB_t}{P_t} = \frac{P_t^H ex_t}{P_t} - \frac{S_t P_t^* D_t^F y_t^F}{P_t}$$

$$\Rightarrow \qquad tb_t = p_t^H ex_t - rer_t D_t^F y_t^F$$

Since $m_t \equiv rer_t D_t^F y_t^F$,

$$tb_t = p_t^H ex_t - m_t$$

A current account is given by the sum of nominal trade balance and nominal net income from abroad. The domestic household owns banks that borrow from the foreign household, so, as a result, net income from abroad is negative and equal to minus payments on bank foreign debt:

$$CA_t = TB_t - \left(R_{t-1}^* \xi_{t-1} - 1\right) S_t D_{t-1}^*$$

Further, we express the current account in units of composite goods as ca_t ($ca_t \equiv CA_t/P_t$):

$$ca_t = tb_t - (R^*_{t-1}\xi_{t-1} - 1)\,\frac{S_t D^*_{t-1}}{P_t}$$

$$\Rightarrow \quad ca_t = tb_t - (R^*_{t-1}\xi_{t-1} - 1)\,rer_t\frac{d^*_{t-1}}{\pi^*_t} \tag{43}$$

In equilibrium the current account has to equal the capital account balance CP_t. In our case the capital account balance is given by the change in stocks of bank foreign debt:

$$CP_t = -\left(S_t D^*_t - S_t D^*_{t-1}\right)$$

We express the capital account balance in units of composite goods as cp_t ($cp_t \equiv CP_t/P_t$):

$$cp_t = -\left(rer_t d^*_t - rer_t\frac{d^*_{t-1}}{\pi^*_t}\right)$$

Then, next to the current account definition (43), we impose an additional restriction that enters the set of equilibrium equations:

$$ca_t = -\left(rer_t d^*_t - rer_t\frac{d^*_{t-1}}{\pi^*_t}\right) \tag{44}$$

I: Equilibrium equations of the model with foreign currency debt and leverage-constrained banks

The model is described by 48 endogenous variables:

$$\Big\{\lambda_t, c_t, h_t, w_t, R_t, d_{1,t}, d_{2,t}, R^R_t, l_t, l^D_t, l^F_t, n^{firms}_t, \pi_t, \Lambda_{t,t+1}, p^R_t, k_t, i_t, q_t, p^H_t, \tilde{p}^H_t, D^H_t, y^H_t, x^H_t, F^H_{1,t},$$
$$F^H_{2,t}, y^C_t, p^F_t, y^F_t, x^F_t, m_t, ex_t, \tilde{p}^F_t, D^F_t, F^F_{1,t}, F^F_{2,t}, R^L_t, d^*_t, d_t, n_t, \nu_{1,t}, \nu_{2,t}, t_t, b_t, rer_t, S_t, tb_t, ca_t, \xi_t\Big\}$$

They are given by 48 equilibrium equations below.

Households

$$\lambda_t = \left(c_t - \frac{\chi\,(h_t)^{1+\varphi}}{1+\varphi}\right)^{-\gamma} \tag{I.1}$$

$$w_t = \chi\,(h_t)^{\varphi} \tag{I.2}$$

$$\Lambda_{t,t+1} \equiv \frac{\lambda_{t+1}}{\lambda_t} \tag{I.3}$$

$$E_t\beta\Lambda_{t,t+1}\frac{R_t}{\pi_{t+1}} = 1 \tag{I.4}$$

Financially constrained firms

$$E_t\beta\Lambda_{t,t+1}\left\{(1-(1-\Phi(d_{1,t}))\kappa)\left(\alpha p^R_{t+1}A_{t+1}k_t^{\alpha-1}h_t^{1-\alpha} + q_{t+1}(1-\delta)\right) - (1-\rho)\frac{q_t}{\pi_{t+1}}\right\} = \rho q_t \tag{I.5}$$

$$E_t\beta\Lambda_{t,t+1}\left\{(1-(1-\Phi(d_{1,t}))\kappa)\,(1-\alpha)p^R_{t+1}A_{t+1}k_t^{\alpha}h_t^{-\alpha} - (1-\rho)\frac{w_t}{\pi_{t+1}}\right\} = \rho w_t \tag{I.6}$$

$$E_{t-1}\left\{l_t\right\}+E_{t-1}\left\{n_t^{firms}\right\}=E_{t-1}\left\{\rho\left(q_tk_t+w_th_t\right)\right\} \tag{I.7}$$

$$d_{2,t}\equiv\frac{E_t\ln\left(\kappa\left(p_{t+1}^Ry_{i,t+1}^R+q_{t+1}(1-\delta)k_{i,t}\right)-R_t^Rrer_{t+1}\frac{l_{i,t}^F}{\pi_{t+1}^*}\right)-E_t\ln\left(R_{i,t}^R\frac{l_{i,t}^D}{\pi_{t+1}}\right)}{\sigma_y} \tag{I.8}$$

$$d_{1,t}\equiv d_{2,t}+\sigma_y \tag{I.9}$$

$$\begin{aligned}n_t^{firms}=&\omega^{firms}\left(p_t^Ry_t^R+q_t(1-\delta)k_{t-1}-(1-\rho)\frac{q_{t-1}k_{t-1}+w_{t-1}h_{t-1}}{\pi_t}\right)\\&-\omega^{firms}\left((1-\Phi(d_{1,t-1}))\,\kappa\left(p_t^Ry_t^R+q_t(1-\delta)k_{t-1}\right)+\Phi(d_{2,t-1})R_{t-1}^R\frac{l_{t-1}^D}{\pi_t}+\Phi(d_{1,t-1})rer_t\frac{l_{t-1}^F}{\pi_t^*}\right)\\&+\iota^{firms}\cdot n^{firms}\end{aligned} \tag{I.10}$$

$$l_t=l_t^D+rer_tl_t^F \tag{I.11}$$

$$l_t^D=(1-\alpha^F)l_t \tag{I.12}$$

Capital producers

$$k_t=(1-\delta)k_{t-1}+\left(1-\Gamma\left(\frac{i_t}{i_{t-1}}\right)\right)i_t \tag{I.13}$$

$$\begin{aligned}\frac{1}{q_t}=&\rho\left(1-\tfrac{\gamma}{2}\left(\tfrac{i_t}{i_{t-1}}-1\right)^2\right)-\rho\gamma\left(\tfrac{i_t}{i_{t-1}}-1\right)\tfrac{i_t}{i_{t-1}}+\rho\gamma\beta E_t\Lambda_{t,t+1}\tfrac{q_{t+1}}{q_t}\left(\tfrac{i_{t+1}}{i_t}-1\right)\left(\tfrac{i_{t+1}}{i_t}\right)^2\\&+(1-\rho)\gamma\beta E_t\Lambda_{t,t+1}\tfrac{q_{t+1}}{q_t}\left(1-\tfrac{\gamma}{2}\left(\tfrac{i_t}{i_{t-1}}-1\right)^2-\gamma\left(\tfrac{i_t}{i_{t-1}}-1\right)\tfrac{i_t}{i_{t-1}}\right)+(1-\rho)\gamma\beta^2E_t\Lambda_{t,t+2}\tfrac{q_{t+1}}{q_t}\left(\tfrac{i_{t+1}}{i_t}-1\right)\left(\tfrac{i_{t+1}}{i_t}\right)^2\end{aligned}$$

Retail firms

$$1=(1-\omega^H)\left(\tilde{p}_t^H\right)^{1-\epsilon_H}+\omega^H\left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^H\pi_t}\right)^{1-\epsilon_H} \tag{I.15}$$

$$D_t^H=(1-\omega^H)\left(\tilde{p}_t^H\right)^{-\epsilon_H}+\omega^H\left(\frac{p_{t-1}^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_t^H\pi_t}\right)^{-\epsilon_H}D_{t-1}^H \tag{I.16}$$

$$\tilde{p}_t^H=\frac{\epsilon_H}{(\epsilon_H-1)}\frac{F_{1,t}^H}{F_{2,t}^H} \tag{I.17}$$

$$F_{1,t}^H=p_t^Ry_t^H+E_t\omega^H\beta\Lambda_{t,t+1}\left(\frac{p_{t+1}^H\pi_{t+1}}{p_t^H\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_H}F_{1,t+1}^H \tag{I.18}$$

$$F_{2,t}^{H}=p_{t}^{H}y_{t}^{H}+E_{t}\omega^{H}\beta\Lambda_{t,t+1}\left(\frac{p_{t+1}^{H}\pi_{t+1}}{p_{t}^{H}\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_{H}-1}F_{2,t+1}^{H} \tag{I.19}$$

$$D_{t}^{H}y_{t}^{H}=A_{t}\theta_{t}F\left(k_{t-1},n_{t-1}\right) \tag{I.20}$$

Composite goods producer

$$y_{t}^{C}\equiv\left((1-\eta)^{\frac{1}{\epsilon}}(x_{t}^{H})^{\frac{\epsilon-1}{\epsilon}}+\eta^{\frac{1}{\epsilon}}(x_{t}^{F})^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}} \tag{I.21}$$

$$x_{t}^{H}=(1-\eta)\left(p_{t}^{H}\right)^{-\epsilon}y_{t}^{C} \tag{I.22}$$

$$x_{t}^{F}=\eta\left(p_{t}^{F}\right)^{-\epsilon}y_{t}^{C} \tag{I.23}$$

Exporters

$$ex_{t}=\eta^{*}\left(\frac{p_{t}^{H}}{rer_{t}}\right)^{-\epsilon_{*}}y_{t}^{*} \tag{I.24}$$

Definition of the real exchange rate

$$\frac{rer_{t}}{rer_{t-1}}=\frac{S_{t}}{S_{t-1}}\frac{\pi_{t}^{*}}{\pi_{t}} \tag{I.25}$$

Importers

$$1=(1-\omega^{F})\left(\tilde{p}_{t}^{F}\right)^{1-\epsilon_{F}}+\omega^{F}\left(\frac{p_{t-1}^{F}\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{p_{t}^{F}\pi_{t}}\right)^{1-\epsilon_{F}} \tag{I.26}$$

$$D_{t}^{F}=(1-\omega^{F})\left(\tilde{p}_{t}^{F}\right)^{-\epsilon_{F}}+\omega^{F}\left(\frac{p_{t-1}^{F}\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}{\pi_{t}p_{t}^{F}}\right)^{-\epsilon_{F}}D_{t-1}^{F} \tag{I.27}$$

$$\tilde{p}_{t}^{F}=\frac{\epsilon_{F}}{(\epsilon_{F}-1)}\frac{F_{1,t}^{F}}{F_{2,t}^{F}} \tag{I.28}$$

$$F_{1,t}^{F}=rer_{t}y_{t}^{F}+E_{t}\omega^{F}\beta\Lambda_{t,t+1}\left(\frac{p_{t+1}^{F}\pi_{t+1}}{p_{t}^{F}\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_{F}}F_{1,t+1}^{F} \tag{I.29}$$

$$F_{2,t}^{F}=p_{t}^{F}y_{t}^{F}+E_{t}\omega^{F}\beta\Lambda_{t,t+1}\left(\frac{p_{t+1}^{F}\pi_{t+1}}{p_{t}^{F}\left(\prod_{j=1}^{j=s}\pi_{t+j}^{adj}\right)}\right)^{\epsilon_{F}-1}F_{2,t+1}^{F} \tag{I.30}$$

$$m_{t}=rer_{t}D_{t}^{F}y_{t}^{F} \tag{I.31}$$

Banks

$$E_{t}\left\{\frac{R_{t}^{L}}{\pi_{t+1}}l_{t}\right\}\equiv E_{t}\left\{(1-\Phi(d_{1,t}))\,\kappa\left(p_{t+1}^{R}y_{t+1}^{R}+(1-\delta)q_{t+1}k_{t}\right)+\Phi(d_{2,t})R_{t}^{R}\frac{l_{t}^{D}}{\pi_{t+1}}+\Phi(d_{1,t})R_{t}^{R}rer_{t+1}\frac{l_{t}^{F}}{\pi_{t+1}^{*}}\right\} \tag{I.32}$$

$$(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t^L}{\pi_{t+1}}\right)=\lambda^L\nu_{1,t}+\nu_{2,t} \tag{I.33}$$

$$(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t}{\pi_{t+1}}\right)=\nu_{2,t} \tag{I.34}$$

$$(1+\nu_{1,t})\beta E_t\Lambda_{t,t+1}\left\{(1-\omega)+\omega\nu_{2,t+1}\right\}\left(\frac{R_t^*\xi_t}{\pi_{t+1}^*}\frac{rer_{t+1}}{rer_t}\right)=\nu_{2,t} \tag{I.35}$$

$$n_t=\omega\left(\frac{R_{j,t-1}^L}{\pi_t}l_{t-1}-\frac{R_{t-1}}{\pi_t}d_{t-1}-\frac{R_{t-1}^*\xi_{t-1}}{\pi_t^*}rer_t d_{t-1}^*\right)+\iota n \tag{I.36}$$

$$\nu_{2,t}n_t\geq\lambda^L l_t \tag{I.37}$$

$$n_t+d_t+rer_t d_t^*=l_t \tag{I.38}$$

Monetary policy

$$\frac{R_t}{\bar{R}}=\left(\frac{R_{t-1}}{\bar{R}}\right)^{\gamma_R}\left(\frac{y_t^H}{\bar{y^H}}\right)^{(1-\gamma_R)\gamma_Y}\left(\frac{p_t^H/p_{t-1}^H\pi_t}{\bar{\pi}}\right)^{(1-\gamma_R)\gamma_\pi}\exp(mp_t) \tag{I.39}$$

Government

$$g_t+\frac{R_{t-1}}{\pi_t}b_{t-1}=t_t+b_t \tag{I.40}$$

$$t_t=\bar{t}+\kappa_b\left(b_{t-1}-\bar{b}\right)+\tau_t \tag{I.41}$$

Aggregate demand of domestic agents has to equal aggregate supply of composite goods

$$y_t^C=c_t+i_t+g_t \tag{I.42}$$

Aggregate demand for domestic aggregate goods and demand for exports clears with production of domestic aggregate goods

$$y_t^H=x_t^H+ex_t \tag{I.43}$$

Aggregate domestic demand for foreign aggregate goods clears with imports

$$y_t^F=x_t^F \tag{I.44}$$

Trade balance

$$tb_t=p_t^H ex_t-m_t \tag{I.45}$$

Current account

$$ca_t=tb_t-\left(R_{t-1}^*\xi_{t-1}-1\right)rer_t\frac{d_{t-1}^*}{\pi_t^*} \tag{I.46}$$

$$ca_t = -\left(rer_t d_t^* - rer_t \frac{d_{t-1}^*}{\pi_t^*}\right) \tag{I.47}$$

$$\xi_t = \exp\left(\phi \frac{(rer_t d_t^* - rer \cdot d^*)}{rer \cdot d^*} + \frac{\zeta_t - \zeta}{\zeta}\right) \tag{I.48}$$

There are 10 exogenous variables:

$$\{A_t, \theta_t, \pi_t^*, R_t^*, \zeta_t, y_t^*, mp_t, g_t, \tau_t\}$$

document
document