TI 2016-087/IV
Tinbergen Institute Discussion Paper

Liquidity Runs

Rafael Matta[1]
Enrico Perotti[1,2]

[1] Faculty of Economics and Business, University of Amsterdam, The Netherlands;
[2] Tinbergen Institute, The Netherlands.

Liquidity Runs*

Rafael Matta
University of Amsterdam
matta@uva.nl

Enrico Perotti
University of Amsterdam & CEPR & Tinbergen Institute
e.c.perotti@uva.nl

This Draft: October 18, 2016

Abstract

Can the risk of losses upon premature liquidation produce bank runs? We show how a unique run equilibrium driven by asset liquidity risk arises even under minimal fundamental risk. To study the role of illiquidity we introduce realistic norms on bank default, such that mandatory stay is triggered before all illiquid assets are sold. Since illiquid assets are not available in a run, asset liquidity risk has a concave effect on run incentives, quite unlike fundamental risk. Runs are rare when asset liquidity is abundant, become more frequent as it falls and decrease again under very low asset liquidity. The socially optimal demandable debt contract limits inessential runs by targeting a high rollover yield. However, the private choice minimizes funding costs, tolerating more frequent runs when illiquid states are sufficiently rare.

Key words: liquidity risk, bank runs, global games, demandable debt, mandatory stay.

JEL classification: D8, G21

* This paper draws from the first part of a paper circulated earlier under the title "Insecure Debt". We thank Toni Ahnert, Douglas Gale, Zhiguo He, Agnese Leonello, Jean Charles Rochet and seminar participants at Bank for International Settlements, Columbia University, European Central Bank, 2015 European Economic Association Meeting, London Business School, New York Fed, New York University and the 2015 Wharton Conference on Liquidity and Financial Crises for useful comments, without implicating them for the content. Send all correspondence to Rafael Matta.

1 Introduction

The 2002–2007 credit boom was largely driven by real estate lending funded by very short term debt. The growing maturity mismatch was supported by the belief that loan securitization via asset backed securities (ABS) had made bank assets more liquid. Yet once some credit risk became apparent, ABS assets rapidly became illiquid, creating solvency concerns and ultimately propagating runs across intermediaries (Brunnermeier (2009)). Thus asset illiquidity played a critical role in propagating distress.

With hindsight, ABS prices at the peak of the crisis fell way too low relative to their ultimate performance. This is illustrated in Figure 1, which shows that market prices of tranches of high rated mortgage-backed securities first crashed then rebounded to levels close to the pre-crisis period. Only a decisive intervention by central banks avoided a large scale of fire sales that would have devastated bank balance sheets.

Figure 1: Markit CMBX Index Value of MBS Tranches with AAA Rating

This experience has led to sharper scrutiny of the degree of bank liquidity mismatch. The previous literature shows that, while maturity transformation is at heart of bank intermediation, it also creates the possibility of multiple equilibria and self-fulfilling runs (Diamond and

Dybvig (1983)). Inefficient runs may also occur in an unique equilibrium setting when there is fundamental asset risk (Goldstein and Pauzner (2005)).

In this paper we study whether there can also be a distinct effect of asset liquidity risk, defined as nonfundamental price risk (e.g. due to temporary scarsity of cash in the market). We study a simple context where all agents are risk neutral and demandable debt is efficient for contingent transaction needs (Stein (2012)) rather than for extreme liquidity demand. We establish an unique equilibrium where runs are driven by uncertainty over early liquidation value of bank assets, even if fundamental risk is arbitrarily small. This complements the result by Goldstein and Pauzner (2005) and relates it to the emerging literature on how market conditions create nonfundamental liquidity risk. There may be too little cash in the market to arbitrage mispricing, due to leverage and maturity mismatch choices by financial intermediaries (Brunnermeier and Petersen (2009) and Gromb and Vayanos (2002)). Duffie and Strulovici (2012) study how a gradual flow of arbitrage capital in a search context causes temporary trading opportunities. Finally, market participants may suddenly have limited resources due to increased adverse selection or counterparty risk (Krishnamurthy (2010) and Gorton and Ordoñez (2014)). Whatever its causes, market-wide liquidity risk may be infrequent but still cause sharp losses in a context of short term funding.

Understanding the nature of liquidity runs requires a precise characterization of how asset liquidity affects run incentives and thus an accurate process of bank default. Traditional bank run models assume that in a run all assets are sold immediately to satisfy withdrawals, so upon default all those who did not run receive nothing. In reality, a bank is declared insolvent as soon as its liquid reserves are depleted and can no longer immediately meet on-demand withdrawals. At that point a mandatory stay is triggered, interrupting asset sales and initiating an orderly liquidation process.[1] As a result of this legal provision, illiquid assets are shared also with those who did not run.

This feature contributes to a surprising concave effect of liquidity risk on run frequency,

[1]Bankruptcy law was introduced precisely to solve the externality created when creditors grab and liquidate assets in an uncoordinated fashion, destroying value.

quite unlike fundamental risk. Abundant liquid assets encourages rollover, as it supports confidence that the bank will be able to repay withdrawals. As asset liquidity falls and default risk rises, the appeal of running increases, as runners are paid out of liquid assets ahead of default. However, because of mandatory stay, the relative payoff to rollover also rises once liquid assets become very scarce, since an insolvent bank's illiquid assets are not paid out before orderly resolution. Thus, in equilibrium there is a concave, inverted U-shaped relation between asset liquidity and run frequency.

The results suggest a role for prudential regulation to limit liquidity mismatch, while current rules tend to focus on reserves. As welfare is here served by minimizing the chance of inessential runs, the model has implications on the effect of yields. A social planner would choose to offer large rents to those who do not withdraw to reduce run incentives.[2] In this setup demandable debt improves social welfare relative to autarky whenever asset are sufficiently liquid, just as in Goldstein and Pauzner (2005) banks are welfare improving when fundamental risk is low. In contrast, a profit-maximizing bank may offer a lower rollover yield in order to reduce funding costs, thus inducing more frequent runs than socially optimal.

The model adopts the framing for analysing unique run equilibria based on Goldstein and Pauzner (2005) and relies on their solution concept. Intuitively, adding interim asset liquidity risk (e.g. driven by limited cash in the market) increases the chance that depositors coordinate on a self protective run even when solvency risk is small. Some amount of fundamental risk is essential to establish an unique equilibrium result, as it defines a lower dominance region where runs are justified. Yet liquidity risk runs occur even as solvency risk becomes arbitrarily small, as long as the bank assets are sufficiently illiquid.

In a related paper (Matta and Perotti (2016)) we study how banks choose to allocate asset liquidity across lenders by their choice of secured (repo) funding. Repo debt may be designed to be absolutely safe and thus enables to reduce funding costs. However, its use shifts some

[2]This choice differs from the optimal social choice under liquidity insurance, where boosting the short term rate benefits agents hit by extreme liquidity needs. In both cases a higher short term rate results in more runs (Goldstein and Pauzner (2005)).

nonfundamental risk to unsecured lenders. In the unique equilibrium the private choice of repo funding tends to increase the chance of unsecured debt runs.

Our setting analyses the trigger point for a run on a single intermediary. Extending the analysis to more banks would better describe the dynamic of liquidity risk, as asset liquidation value is affected by each bank run. The propagation effect created by fire sales has received extensive attention in the recent literature as a major negative externality (Lorenzoni (2008) and Krishnamurthy (2010)).

2 The Basic Model

The economy lasts for three periods $t = 0, 1, 2$. It is populated by a bank and a continuum of risk neutral lenders indexed by i. The intermediary has access to a project that needs one unit of funding at $t = 0$. Agents are each endowed with one unit and are risk neutral. Their required return equals one, reflecting their alternative storage option between $t = 0$ and $t = 2$. As the mass of agents is large, perfect competition prevails.

A fraction α of lenders will face a contingent need of one unit at $t = 1$ for the purpose of transacting. If they are unable to obtain cash to transact, their alternative settlement technology implies a transaction cost τ (e.g. required to liquidate assets invested in storage). Transaction needs are identically and independently distributed and there is no aggregate liquidity demand uncertainty.

- *Project*

For each unit invested, the project generates a return of $y_t(\omega, \theta)$ at $t = 1, 2$, where $\omega \in \{H, L\}$ is aggregate state and $\theta \sim U\left(\underline{\theta}(\omega), \overline{\theta}\right)$. We interpret θ as a measure of available cash-in-the-market at the interim date. With probability λ the state is high ($\omega = H$) in which case there is no fundamental risk. With probability $1 - \lambda$ the state is low in which case there may be fundamental and asset liquidity risk. Both ω and θ are non-verifiable and realized at $t = 1$.

The overall state ω is observable by all. Only the bank observes θ, while all lenders receive an individual signal on it at $t = 1$.

If some fraction of the project is liquidated early, its rate of return is $y_1(\omega, \theta) = k + v(\theta)$. Here $k \in (\alpha, 1)$ represents the safe component (which can be securitized and used as cash) while $v(\theta) = \min\{\theta, c + 1 - k\}$ is the illiquid asset component which can be sold at a liquidation cost $c > 0$. As an alternative to early liquidation, an orderly liquidation process can be initiated at $t = 1$, in which case illiquid assets are worth $\ell > 0$ at $t = 2$. If the project is allowed to mature it generates $y_2(\omega, \theta) = r > 1$ when $\theta \geq c$ or $y_2(\omega, \theta) = 0$ when $\theta < c$.

In the high state $\underline{\theta}(H) = 1 - k + c < \overline{\theta}$ so that assets are never worth less than the initial investment. In the low state the value of illiquid assets may be as low as zero ($\underline{\theta}(L) = 0$) and there is fundamental risk for $\theta < c$. Note that as long as c is small, the project is almost always riskless even in the L state, provided it is allowed to mature. In other words, as c goes to zero the fundamental asset risk (and thus bank solvency risk) vanishes, while asset liquidation risk remains. The project payoffs are shown in Table 1.

Table 1: Project Payoffs

$t = 0$		$t = 1$		$t = 2$
-1	$\omega = H$	if held to maturity	0	r
		if early liquidation	1	0
	$\omega = L$	if held to maturity	0	$\begin{cases} r, & \text{if } \theta \geq c \\ 0, & \text{if } \theta < c \end{cases}$
		if early liquidation	$k + v(\theta) - c$	0
		orderly liquidation	k	ℓ

- *The Bankruptcy Process*

In the low state $\omega = L$ there is a chance of solvency risk as the project value may be worth less than the initial investment. To meet any repayments demanded at $t = 1$ the intermediary can use its liquid reserves k. Once these means of immediate payment are exhausted, the bank is forced to consider a fire sales of the illiquid component of its assets. If repayments are larger than the realizable liquidation proceeds, the bank is declared in default and triggers a mandatory stay on unpaid creditors.[3] This halts any further payments at $t = 1$ and enables an orderly liquidation process at $t = 2$, avoiding any fire sales and the associated cost c while ensuring that illiquid assets can be sold at $t = 2$ for ℓ. Thus an effect of mandatory stay is that all unpaid creditors are treated equally out of illiquid assets, receiving a pro rata share of the orderly liquidation value ℓ at $t = 2$.

Note that early liquidation is always inefficient since it is dominated by orderly liquidation for $\theta < c$ and by continuation for $\theta \geq c$. In contrast, orderly liquidation is efficient for $\theta < c$ (although dominated by continuation for $\theta \geq c$ and by early liquidation for $\theta \geq c + \ell$).

We focus on the realistic case when the value produced under orderly liquidation is not enough to fully repay the principal amount to all lenders ($\ell + k < 1$) and is sufficiently low relative to the asset return r. Specifically we assume:

$$-\frac{k \ln k}{(1-k)(k-\alpha)}(1-\ell-k) > r-1,$$ [4]

which can be shown to imply that the value $k - \alpha$ is not too large. In economic terms this implies bank assets are not too liquid. We also assume some bounds on the transaction cost τ:

$$\frac{r-k-\ell}{\alpha k} > \tau > r-1.$$

The lower bound implies that lenders with transaction needs prefer to receive one unit at $t = 1$ rather than the full return r at $t = 2$. The upper bound implies that the discounted return r being greater than the discounted asset value under orderly resolution, i.e., $\frac{r}{1+\alpha\tau} > k + \frac{\ell}{1+\alpha\tau}$.[5]

[3] Thus no default is triggered if early liquidation net of c enables to repay all requested repayments at $t = 1$.

[4] In the appendix we show that it implies a set $\left(\alpha, \overline{k}\right)$ of allowable k, where $\overline{k}$ is the unique solution to $-\frac{k \ln k}{(1-k)(k-\alpha)}(1-\ell-k) = r-1$ for $k > \alpha$. Note that it is weaker than $\frac{k}{k-\alpha}(1-\ell-k) > r-1$ as $-\frac{\ln k}{1-k} > 1$

[5] The discount rate reflects lenders' indifference as of $t = 0$ (hence before transaction needs are revealed) between a payment of 1 at $t = 1$ and a payment of $1 + \alpha\tau$ at $t = 2$.

Finally, we further assume that the illiquid state is not too frequent (i.e., λ is sufficiently large) to ensure that the project has positive NPV even if always liquidated under orderly resolution in the low state (assuming all lenders with transaction needs receive one unit at $t = 1$ in the high state):

$$\lambda \left[\alpha + (1-\alpha) r\right] + (1-\lambda)(k + \ell - \alpha\tau) \geq 1.$$

3 The Effect of Liquidity Risk

We follow Diamond and Dybvig (1983) in showing that demandable debt is optimal in our setting (achieves the second best) under common knowledge about θ. We then follow Goldstein and Pauzner (2005) to examine the effect of demandable debt under incomplete information.

3.1 Benchmark: Common Knowledge

We consider first the case of common knowledge about θ and examine the outcome under autarky. Each lender i faces the following maximization problem:

$$\max_{f_k \in [0,1], f_v \in [0,1], f_o \in [0,1-f_v]} U_i(f_k, f_v, f_o) \equiv f_k k + f_v (v(\theta) - c) - \mathbf{1}_i \tau + f_o \ell + \left[1 - f_k k - (f_v + f_o)(1-k)\right] y_2(\omega, \theta), \quad (1)$$

where f_k and f_v are the fractions of k and $v(\theta)$ liquidated at $t = 1$ respectively, f_o is the fraction of $v(\theta)$ orderly liquidated at $t = 2$, and $\mathbf{1}_i$ is an indicator function that equals 1 if the lender i faces transaction needs and $f_k k + f_v (v(\theta) - c) < 1$ and 0 otherwise.

Proposition 1. *Under aurtarky, lenders with transaction needs choose early liquidation at $t = 1$ for $\theta \geq c + 1 - k$, continuation for $\theta \in [c, c + 1 - k)$ and orderly liquidation for $\theta < c$. Lenders without transaction needs allow the project to mature for $\theta \geq c$ and initiate orderly liquidation for $\theta < c$.*

The resulting social welfare is

$$S_A = \lambda\left[\alpha + (1-\alpha)\, r\right] + \frac{1-\lambda}{\overline{\theta}}\left[\left(\overline{\theta} - c - 1 + k\right)(\alpha + (1-\alpha)\, r) + (1-k)\,(r - \alpha\tau) + c\,(k + \ell - \alpha\tau)\right]. \tag{2}$$

We next show that relative to the second best, the autarky solution incurs an inefficient transaction cost τ in states $\theta < c + 1 - k$.

- *Second Best*

Although the first best is not attainable when transaction needs are private information,[6] we show that a demandable debt contract achieves the second best. First we characterize the class of incentive-compatible contracts that maximize welfare conditional on $\theta < c$. Then we show that a properly designed demandable debt contract belongs to this class and also achieves maximum welfare conditional on $\theta \geq c$.

Lemma 1. *An incentive-compatible contract maximizes welfare when $\theta < c$ only if bankruptcy is implemented and all lenders are each paid one unit at $t = 1$ with probability k and $\frac{\ell}{1-k}$ at $t = 2$ with probability $1 - k$.*

Automatic stay implies the equal treatment of all unpaid lenders (with or without transaction needs), who therefore receive the same payoff under orderly resolution. As a result, an incentive compatible contract must give all lenders the same expected payment out of the reserves k (otherwise they will all claim special treatment). According to Lemma 1, this implies that lenders with transaction needs can receive one unit at $t = 1$ at most with probability k.

The following proposition shows the existence of a demandable debt contract the implements the second best. The solution belongs to the class of contracts characterized in Lemma 1 and achieves maximum welfare when $\theta \geq c$.

[6] The revelation principle requires that for a contract to achieve the first best all lenders with transaction needs are paid at least one unit at $t = 1$ while payments to other lenders must ensure they are not better off pretending to have transaction needs. However this is not feasible when $\theta < c$ since in this case the maximum payoff left after transaction needs are satisfied is $\frac{k-\alpha+\ell}{1-\alpha} < 1$. As a result, all lenders would claim to have transaction needs.

Proposition 2 (Optimality of Demandable Debt). *The second best is implemented by the following contract: (1) lenders are entitled to demand a payment of* 1 *at* $t = 1$*, with withdrawals served sequentially until the bank runs out of reserves; (2) in the absence of bankruptcy, lenders not paid at* $t = 1$ *are paid the minimum between* $d \in (1, r]$ *and the pro rata share of the surplus at* $t = 2$*,* $\min \{y_2(\omega, \theta), d\}$*, where* d *satisfies lenders' participation constraints.*

The contract described in Proposition 2 implements the following equilibria. For the *lower dominance region* $\theta < c$, it is strictly dominant for all lenders to demand payments at $t = 1$. As a result default is triggered and mandatory stay ensures efficient liquidation since the bank becomes insolvent after paying out its liquid reserves k. In the *upper dominance region* $\theta \geq c + 1 - k$ no bankruptcy occurs since reserves and illiquid assets are enough to always pay all lenders. In this case it is strictly dominant for lenders with transaction needs to withdraw at $t = 1$ and for lender without transaction needs to be paid at $t = 2$. Finally for $\theta \in [c, c + 1 - k)$ there exists an equilibrium in which only lenders with transaction needs withdraw and inefficient bankruptcy is avoided. Under this equilibrium the second best is implemented, yielding a social welfare equal to

$$\begin{aligned} S_{SB} &= \lambda [\alpha + (1 - \alpha) r] + \frac{1 - \lambda}{\bar{\theta}} \left[\left(\bar{\theta} - c \right) (\alpha + (1 - \alpha) r) + c (k + \ell - \alpha (1 - k) \tau) \right] \\ &= S_A + \frac{1 - \lambda}{\bar{\theta}} \alpha [(1 - k)(\tau - r + 1) + ck\tau] > S_A. \end{aligned} \tag{3}$$

Therefore, although the first best is not attainable with unobservable transaction needs, in the efficient equilibrium intermediaries can implement the second best through demandable debt contracts. The improvement in welfare relative to the autarkic regime arises because banks satisfy transaction needs with probability one for $\theta \in [c, c + 1 - k)$ and with probability k for $\theta < c$.

Moreover, Proposition 2 rationalizes bankruptcy rules as mandatory stay with order liquidation is efficient on the equilibrium path. Orderly liquidation is dominated by continuation for $\theta \geq c$ and by early liquidation for $\theta \geq c + \ell$, but in the equilibrium that implements the second best it is triggered if and only if $\theta < c$, that is, exactly when it is efficient.

While lenders with transaction needs always withdraw at $t = 1$, other lenders face a coordination problem when $\theta \in [c, c + 1 - k)$. There is a second equilibrium in which all lenders withdraw, resulting in inefficient bankruptcy. In this case, early liquidation is preferable to bankruptcy for $\theta - c > \ell$, although still inefficient relative to continuation. Under this second equilibrium the social welfare is given by

$$
\begin{aligned}
\underline{S} &= \lambda \left[\alpha + (1-\alpha) r\right] \\
&\quad + \frac{1-\lambda}{\overline{\theta}} \left[\left(\overline{\theta} - c - 1 + k\right)(\alpha + (1-\alpha) r) + (c + 1 - k)(k + \ell - \alpha (1-k) \tau)\right] \\
&= S_A - \frac{1-\lambda}{\overline{\theta}} \left[(1-k)(r - k - \ell - \alpha k \tau) - c \alpha k \tau\right],
\end{aligned} \tag{4}
$$

The terms inside the brackets represent the performance of demandable debt relative to the autarkic regime. The first term reflects the loss due to efficient continuation in autarky when $\theta \in [c, c + 1 - k)$.[7] The second represents the efficiency gain conditional on bankruptcy, as lenders with transaction needs are paid 1 with probability k. In this case, it is clear that demandable debt does not improve upon altarky ($\underline{S} < S_A$) if insolvency states are sufficiently infrequent (i.e., c small enough).

In summary, the preferred action of lenders without transaction needs depends on whether or not the bank is declared bankrupt. Bankruptcy occurs if and only if $\alpha + (1-\phi)(1-\alpha) > k + q\,(v\,(\theta) - c)$, where $q = 1$ for $\theta \geq c$ and $q = 0$ for $\theta < c$. Table 2 summarizes their payoffs.

Table 2: Payoffs of Lenders without Transaction Needs Conditional on $\omega = L$

	$\alpha + (1-\phi)(1-\alpha) \leq k + q\,(v\,(\theta) - c)$	$\alpha + (1-\phi)(1-\alpha) > k + q\,(v\,(\theta) - c)$
roll over	qd	$\frac{\ell}{1-k}$
withdraw	1	$\frac{k}{\alpha+(1-\phi)(1-\alpha)} + \left(1 - \frac{k}{\alpha+(1-\phi)(1-\alpha)}\right) \frac{\ell}{1-k}$

We next adopt the global game approach to obtain a unique equilibrium. Following Gold-

[7] It is positive as per our assumption that $\frac{r-k-\ell}{\alpha k} > \tau$.

stein and Pauzner (2005), we assume the bank finances the project with demandable debt and solve for the determinants of runs. In particular, this enables to pin down how the terms of demandable debt affect the probability of bankruptcy. We then solve for the optimal demandable debt contract and characterize the conditions under which it improves welfare relative to the autarkic regime.

3.2 The Unique Equilibrium under Incomplete Information

Adopting the global game approach by removing the assumption of common knowledge establishes an unique equilibrium and enables to endogenize the probability of bankruptcy (in the limit, as incomplete information goes to zero).

We now assume that while the bank observes θ, lenders receive individual noisy signals on the value of θ. Let this signal be given by

$$x_i = \theta + \sigma\eta_i, \tag{5}$$

where $\sigma > 0$ is an arbitrarily small scale parameter and η_i are i.i.d. across players and uniformly distributed over $\left[-\frac{1}{2}, \frac{1}{2}\right]$.

Once lenders receive their signal they face a complex coordination problem. Their decision to roll over now depends on both their beliefs about liquidity risk θ and on the fraction ϕ of lenders who roll over (strategic uncertainty). We will show that the unique equilibrium is in switching strategies around a common cutoff θ^*: for signals below the threshold all lenders choose to run and otherwise all choose to roll over. Uniqueness of equilibrium is established along the lines of the solution offered by Goldstein and Pauzner (2005) and Morris and Shin (2003) for global games that violate global strategic complementarity,[8] but satisfy a single crossing property. Specifically, the lenders' net rollover payoff is positive if the fraction of

[8]This arises because lenders' incentive to roll over is not monotonically increasing in the fraction of lenders who roll over.

lenders who roll over is above a certain threshold and negative otherwise.[9]

3.2.1 Equilibrium Runs

Let $\Pi(\phi, \theta)$ be the net rollover payoff relative to running. We have

$$\Pi(\phi, \theta) = \begin{cases} qd - 1, & \text{if } \alpha + (1-\phi)(1-\alpha) \leq k + q\left(v(\theta) - c\right) \\ -\frac{k}{\alpha + (1-\phi)(1-\alpha)}\left(1 - \frac{\ell}{1-k}\right), & \text{if } \alpha + (1-\phi)(1-\alpha) > k + q\left(v(\theta) - c\right) \end{cases}. \tag{6}$$

Suppose lenders follow a monotone strategy with a cutoff κ, rolling over if their signal is above κ and withdrawing if otherwise. Lender i's expectation about the fraction of rollover lenders conditional on θ is simply the probability $1 - \frac{\kappa - \theta}{\sigma}$ that any lender observes a signal above κ. This proportion is less than z if $\theta \leq \kappa - \sigma(1-z)$, assessed by each lender i under the conditional distribution of θ given his signal x_i.

As established in the literature (Morris and Shin (2003)), when $\sigma \to 0$ strategic uncertainty dominates over uncertainty about θ this probability equals z for $x_i = \kappa$. That is, the threshold type believes that the proportion of lenders that roll over follows the uniform distribution on the unit interval. The equilibrium cutoff can then be computed by the threshold type who must be indifferent between rolling over and withdrawing given his beliefs about ϕ. Formally, it is the unique θ^* such that $\int_0^1 \Pi(\phi, \theta^*)\, d\phi = 0$.

Proposition 3 (Run Cutoff)**.** *In the limit $\sigma \to 0$, the unique equilibrium at $t = 1$ has lenders following monotone strategies with threshold θ^* given by*

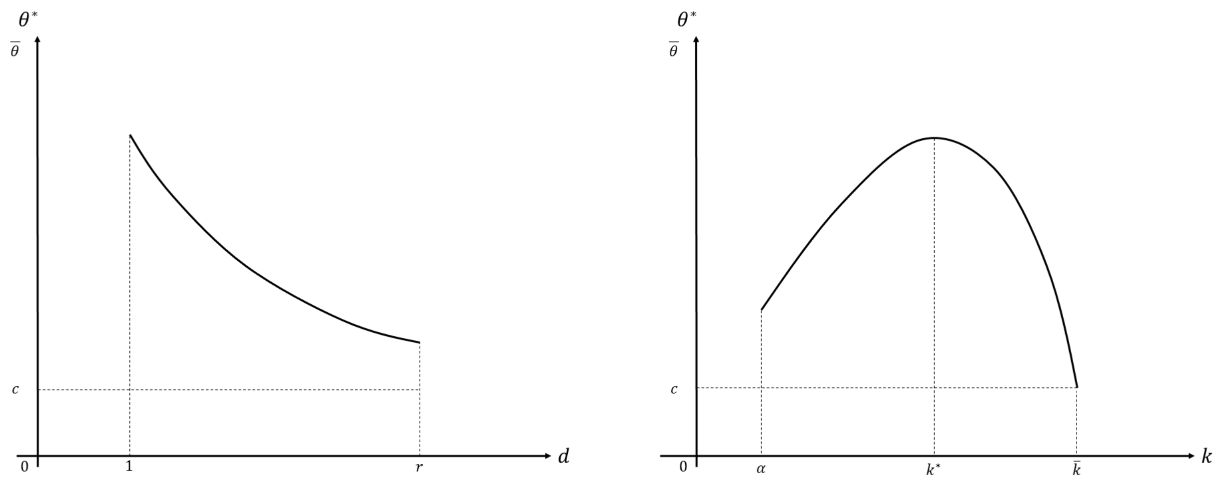
$$\theta^* = e^{\alpha \frac{d-1}{k\left(1 - \frac{\ell}{1-k}\right)} - W\left(\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)} e^{\alpha \frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}}\right)} + c - k, \tag{7}$$

where all lenders roll over if $\theta > \theta^*$ and do not roll over if $\theta < \theta^*$.[10]

[9] While the results of Goldstein and Pauzner (2005) rely on the assumption that the noise terms are uniformly distributed, in our setup any noise distribution satisfying the monotone likelihood ratio property ensures an unique equilibrium among monotonic strategies (Morris and Shin (2003)).

[10] $W(\cdot)$ is known as the Lambert W function and is the inverse function of $y = xe^x$ for $x \geq -1$.

Figure 2: Run Cutoff



(a) Rollover Reward (b) Liquid Assets

This result enables to endogenize the probability of bankruptcy.

Corollary 1 (Yield and Collateral Effects on Stability)**.** *The run threshold* θ^* *satisfies:*

(i) It is strictly decreasing and strictly convex in the roll over premium d; it is strictly concave in k, strictly decreasing or first strictly increasing then strictly decreasing.

(ii) If transaction needs are not too frequent (α *not too large), there exists* $k^* \in \left(\alpha, \overline{k}\right)$ *such that it is strictly decreasing in* k *for* $k \geq k^*$ *and strictly increasing in* k *for* $k < k^*$*.*

The comparative statics offer some intuitive insight (see Figure 2). A higher rollover premium d improves the payoff of rolling over for a given chance of default and unambiguously reduces the probability of runs.[11] However, a large rollover reward reduces the bank's payoff in all solvent states. This observation is essential to understand a bank's pricing incentives.

- *Asset Liquidity and the Frequency of Runs*

An increase in asset liquidity k has a more complex effect on the frequency of runs. There is an unambiguous negative and linear "probability" effect. Higher asset liquidity reduces the

[11]This is the specular effect of a higher short term rate in Diamond and Dybvig (1983), where it provides more liquidity insurance.

chance that the bank runs out of reserves in a run, which leads to a lower θ^*. This effect is intuitive and equivalent to having better fundamentals. But there is also a "relative payoff" effect, as less liquidity here decreases the expected payoff of both rollover and run strategies. The probability effect is dominant as assets are very liquid, so runs are less frequent. As asset liquidity declines, run incentives rise. However, because runners under mandatory stay can only be paid out of liquid assets, the relative payoff to run drops once asset liquidity becomes very scarce. This produces a hump-shaped relationship.[12]

A direct implication of Proposition 3 is:

Corollary 2. *In the limit $c \to 0$ the probability of runs is bounded away from zero ($\theta^* > 0$).*

Recall that runs are inefficient whenever $\theta \geq c$. Thus Corollary 2 shows that inefficient runs arise under asset liquidity risk even as fundamental risk becomes arbitrarily small.

3.2.2 Comparison to Pure Fundamental Risk

Our setup introduces some novel (and realistic) elements in the standard bank-run model. First we introduced asset liquidity risk as a correlated but distinct factor from fundamental risk. Next we introduced the mandatory stay provisions of the bankruptcy process, such that in default illiquid assets are also available to those that did not run.

We now compare the specific effect of both assumptions with the classic approach by Goldstein and Pauzner (2005).[13] Consider a formulation of their model with fundamental risk but no asset liquidity risk. Liquidating illiquid assets at $t = 1$ involves no fixed cost ($c = 0$) and the liquidation value equals our value under orderly resolution (ℓ). Formally, if the project is liquidated at $t = 1$ it yields $y_1(\omega, \theta) = k + v(\theta)$, where $v(\theta) = 1 - k$ for $\theta \geq 1 - k$ and $v(\theta) = \ell$ for $\theta < 1 - k$. If allowed to mature, the project generates $y_2(\omega, \theta) = r$ with probability θ and $y_2(\omega, \theta) = 0$ with probability $1 - \theta$, where θ is now uniformly distributed over $[0, 1]$. Second, bank assets are paid out subject to strict sequential service with no mandatory stay. The

[12] This effect would be even stronger if more k implied lower proceeds ℓ in the orderly liquidation process.

[13] While in Goldstein and Pauzner (2005) demandable debt improves liquidity risk sharing, in our model it reduce contingent transaction costs.

Table 3: Payoffs of Lenders Conditional on $\omega = L$ with Sequential Service

		$1-\phi \leq \ell + k$	$1-\phi > \ell + k$
sequential service	roll over	θd	0
	withdraw	1	$\frac{\ell+k}{1-\phi}$
mandatory stay	roll over	θd	$\frac{\ell}{1-k}$
	withdraw	1	$\frac{k}{1-\phi} + \left(1 - \frac{k}{1-\phi}\right)\frac{\ell}{1-k}$

bank pays 1 to withdrawal tenders until both liquid (k) and illiquid (ℓ) assets are exhausted. Without loss of generality our analysis here sets $\alpha = 0$, such that lenders face no transaction needs.

After solving for the unique equilibrium result in this benchmark model we re-introduce mandatory stay. The payoffs are shown in Table 3. In the next step we compare the new outcome with our results under asset liquidity risk.

Proposition 4. *The unique run equilibrium under the proposed reformulation has the following features in terms of liquid assets k:*

(i) the run cutoff with sequential service θ_S^* is strictly convex and strictly decreasing in k;

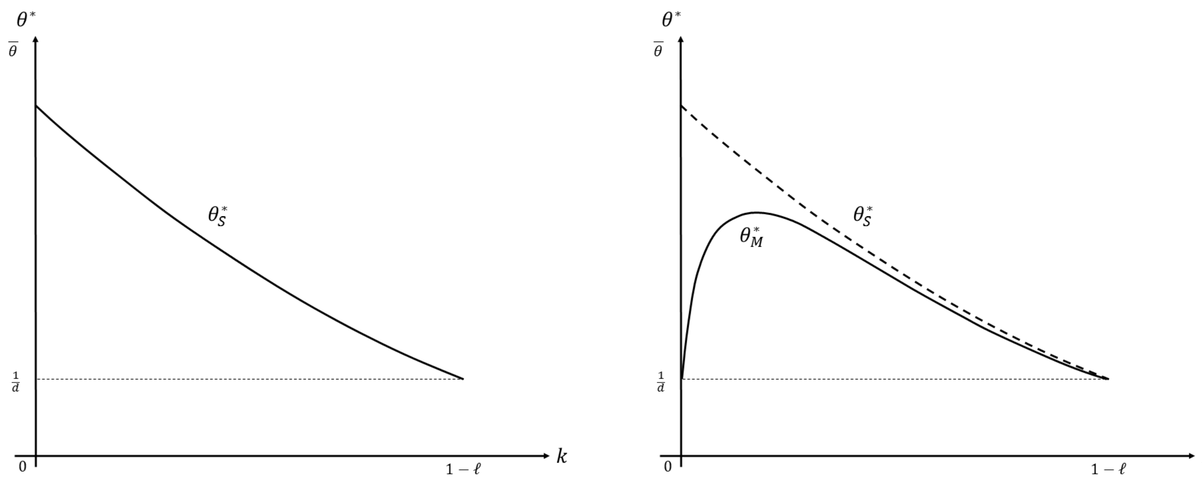
(ii) the run cutoff with mandatory stay $\theta_M^* < \theta_S^*$ is strictly quasi-concave, first strictly increasing then strictly decreasing in k.

Figure 3 shows the run cutoff as a function of asset liquidity under pure fundamental risk. Figure 3a maps the run threshold under sequential service as in Goldstein and Pauzner (2005), while Figure 3b depicts the run threshold under mandatory stay.

Several conclusions can be draw from Figure 3. First, without mandatory stay the incidence of runs is monotonic and always decreasing in asset liquidity k. This reflects the direct effect of greater asset liquidity, namely a lower asset risk. Second, introducing mandatory stay produces a first increasing, then decreasing quasiconcave run threshold in terms of asset liquidity k.[14]

[14] In this case we can establish that the threshold is neither concave nor convex.

Figure 3: Run Cutoff



(a) Sequential Service (b) Mandatory Stay

Third, runs are less frequent under mandatory stay. Note that the last result validates the efficiency of the mandatory stay provision. Intuitively the bankruptcy provisions subtracts asset from early withdrawals and reinforces the incentive to roll over.

In summary, we can show that the concavity of the run threshold arise from the combination of asset liquidity risk and the associated bankruptcy procedure.

We next endogenize the private and social planner choice of funding.

3.3 Pricing Demandable Debt

This section examines the bank's initial choice of the yield d promised to demandable debt when rolled over till $t = 2$. To focus on runs driven by asset liquidity risk, we henceforth take $c \to 0$. Because the project has positive NPV for any funding choice we can focus on the stability tradeoff, excluding other effects of its financing structure.

The ex ante expected payoff of lenders as a function of its face value d is

$$V_L(d) = \lambda\left[(1-\alpha)d + \alpha\right] + (1-\lambda)\left\{\frac{\overline{\theta} - \theta^*(d)}{\overline{\theta}}\left[(1-\alpha)d + \alpha\right] + \frac{\theta^*(d)}{\overline{\theta}}\left[k + \ell - \alpha(1-k)\tau\right]\right\}.$$

The bank's expected payoff can be written as the return of the project of a solvent bank $\alpha +$

$(1-\alpha)\,r$ minus financing costs $V_L(d)$ and the expected deadweight loss in bankruptcy $DW(d)$,

$$V_B(d) = \lambda\left[(1-\alpha)\,r - (1-\alpha)\,d\right] + (1-\lambda)\left(\frac{\overline{\theta} - \theta^*(d)}{\overline{\theta}}\right)\left[(1-\alpha)\,r - (1-\alpha)\,d\right] \tag{8}$$
$$= \alpha + (1-\alpha)\,r - V_L(d) - DW(d)\,,$$

where

$$DW(d) = (1-\lambda)\,\frac{\theta^*(d)}{\overline{\theta}}\left\{\alpha + (1-\alpha)\,r - \left[k + \ell - \alpha\,(1-k)\,\tau\right]\right\}. \tag{9}$$

3.3.1 Socially Optimal Pricing

Next we characterize the socially optimal pricing of the demandable debt contract. The social planner chooses the face value d to maximizes the aggregate payoff subject to the participation constraint of the bank and its lenders:

$$\max_{d}\; r - DW(d) \tag{10}$$

subject to

$$V_B(d) \geq 0,\; V_L(d) \geq 1.$$

In other words, the optimal financing policy minimizes the chance of runs (a pure deadweight loss) subject to agents' participation constraints. It is immediately intuitive that since $-DW(d)$ is increasing in d, the social planner would increase d as much as possible.

Increasing d above lenders' breakeven level implies lenders receive a "rollover rent"above their participation threshold. The maximum rollover rent is reached when the bank's participation constraint is binding at $d = r$, as all asset value is promised to depositors rolling over. Proposition 5 characterizes the socially optimal financing policy.

Proposition 5 (Optimal Funding). *Let d^o be the optimal solution to (10). The socially optimal financing contract requires the bank to offer the maximum possible rollover rent ($d^o = r$).*

We can now compare the welfare achieved by the social planner under equilibrium uniqueness, S_o, relative to that obtained under the second best, S_{SB}, when $c \to 0$:

$$\begin{aligned} S_o &= \lambda\left[\alpha + (1-\alpha) r\right] + \frac{1-\lambda}{\overline{\theta}}\left[\left(\overline{\theta} - \theta^{\star}\left(d^o\right)\right)\left(\alpha + (1-\alpha) r\right) + \theta^{\star}\left(d^o\right)\left(k + \ell - \alpha\left(1-k\right)\tau\right)\right] \\ &= S_{SB} - DW\left(d^o\right). \end{aligned}$$

The welfare loss resulting from the socially optimal demandable debt contract is due to inefficient bankruptcy when $\theta \in [0, \theta^{\star})$. Proposition 6 characterizes the conditions under which it improves welfare relative to autarky.

Proposition 6 (When Can Banks Create Value?)**.** *The deadweight loss $DW\left(d^o\right)$ associated with the socially optimal demandable debt is strictly quasi-concave in asset liquidity k, either strictly decreasing, or first strictly increasing then strictly decreasing. Thus banks offering the socially optimal demandable debt contract are more likely to improve welfare at either high or low levels of asset liquidity. Banks improve welfare if their asset liquidity is sufficiently large.*

Recall that autarky creates unnecessary liquidation losses for agents with transaction needs, while banks create losses from miscoordination in some solvent states. Thus banks outperform the autarkic regime when the probability of liquidity runs is lowest, namely when asset liquidity k is either high or low. Banks need to hold enough liquid assets to meet the minimum expected withdrawal α. Thus a sufficient condition for banks to improve welfare upon autarky is that their asset liquidity is large enough.

3.3.2 Private Pricing

Having solved for the social optimal choice of demandable debt, we turn to examine the private choice. The bank's problem is to choose the rollover reward d that maximizes its payoff subject to the participation constraint:

$$\max_{d} V_B(d) \tag{11}$$

subject to

$$V_L(d) \geq 1.$$

In making this choice the bank trades off the cost of financing d against the expected deadweight loss from runs.

Proposition 7 (Private Inefficiency)**.** *Let d^* be the optimal solution to (11). The probability of bankruptcy under the socially optimal funding structure is always lower than under the private funding choice: $\theta^*(d^o) < \theta^*(d^*)$.*

While the social planner minimizes the probability of runs by choosing the maximum feasible rollover value $d^o = r$, the private choice of d^* is lower than the social optimum value, leading to a higher threshold $\theta^*(d^*) > \theta^*(d^o)$ and thus more frequent runs. As a result, the private choice of demandable debt is less likely to improve welfare relative to autarky.

Proposition 8 characterizes the optimal private funding choice.

Proposition 8 (Private Pricing)**.** *The bank's financing policy is characterized as follows:*

(i) The privately optimal choice of d^* either holds lenders to their participation constraint, or leads to a positive rollover rent characterized by $-\frac{\partial DW(d^*)}{\partial d} = \frac{\partial V_L(d^*)}{\partial d}$.

(ii) There exists a cutoff $\lambda_1 \in [0,1)$ such that, if $\lambda > \lambda_1$, the bank offers no rollover rents to its lenders.

In other words, the private choice of the rollover yield d balances lower funding costs against a higher deadweight loss. When illiquidity is not too frequent, runs are rare so the private choice of demandable debt chooses a rollover rate that leaves no rents to depositors. This results reflects the critical tension in financial regulation. Relying on inexpensive short term debt maximizes bank profits in solvent states, but increases the chance of nonfundamental bank insolvency and costly early liquidation.

4 Conclusion

While the bank run literature has naturally focused on fundamental asset risk, the recent crisis has highlighted how asset illiquidity plays a critical role in triggering as well as propagating distress. Next to excessive credit risk, intermediaries had built an extreme imbalance in liquidity transformation that massively amplified the effect of shocks. This insight motivates a careful examination of run incentives about asset liquidity risk, under a precise description of a bank default process.

We are able to establish the existence of an unique run threshold equilibrium even when the fundamental value of bank assets are almost certainly safe. Our contribution relates to recent advances in our understanding of how funding decisions and market conditions affect intermediaries' access to liquidity and contribute to risk creation even when unrelated to solvency issues. Following the Diamond and Dybvig (1983) and Goldstein and Pauzver (2005) approach, we offer a simple rationale for demandable debt and study the unique equilibrium under incomplete information. To focus on asset liquidity, demandable debt is here justified by contingent payment needs rather than extreme liquidity insurance. We also carefully describe the allocation of liquidity risk inherent in run incentives by a precise characterization of the process of bank default. While existing models assume that withdrawals are met by asset sales until no assets are left, in reality less liquid assets cannot be sold immediately without huge losses. To avoid a hasty and inefficient termination of real projects, bankruptcy law forces an automatic stay on all lenders once the borrower runs out of liquid assets. Remaining assets are then sold under orderly resolution, limiting fire sales of very illiquid assets. We are able to show that this arrangement is indeed efficient. In addition, it matches the reality of bank bankruptcy. Many billions in assets were left in the Lehman bankruptcy process after its default. Our approach distinguishes among the fire sale price of assets, their orderly liquidation value and their present value upon continuation, a distinction issue that has become quite topical since the crisis.

This precise allocation of liquidity risk produces a most surprising result. Unlike the case of pure fundamental risk, the chance of a bank run is not monotonic in asset liquidity risk, quite unlike the monotonic effect of fundamental asset risk. Future research needs to establish the implications of this result and its implications for the study of liquidity regulation, a theme long neglected and only recently restored in banking legislation.

In a related paper (Matta and Perotti (2016)) we study how banks may choose to ex ante allocate asset liquidity across lenders by their choice of secured (repo) funding. Repo debt may be designed to be absolutely safe and thus enables to reduce funding costs, but its use shifts some nonfundamental risk to unsecured lenders. In the unique run equilibrium, the private choice of repo funding tends to increase the chance of inefficient runs.

Finally, there may be more applications to our realistic description of interim asset values whenever market resources available for arbitraging mispricing are scarce.

Appendix

Proof of Proposition 1. Using the Principle of Iterated Suprema, (2) can be rewritten as:

$$\max_{f_k\in[0,1]}\max_{f_v\in[0,1]}\max_{f_o\in[0,1-f_v]} U_i\left(f_k,f_v,f_o\right)=f_k k\left(1-y_2\left(\omega,\theta\right)\right)+f_v\left[v\left(\theta\right)-c-\left(1-k\right)y_2\left(\omega,\theta\right)\right]-\mathbf{1}_i\tau$$
$$+f_o\left[\ell-\left(1-k\right)y_2\left(\omega,\theta\right)\right]+y_2\left(\omega,\theta\right).$$

If $\theta < c$, then $y_2(\omega,\theta)=0$ and $U_i(\cdot,\cdot,f_o)$ is strictly increasing in f_o, which implies $f_o = 1-f_v$. Since $k+v(\theta)-c<1$, for all f_k and f_v it holds that $\mathbf{1}_i=1$ if i faces transaction needs and $\mathbf{1}_i=0$ if otherwise. Thus, $U_i(\cdot,f_v,1-f_v)$ is strictly decreasing in f_v for all i such that $f_v=0$. Finally, $U_i(f_k,0,1)$ is strictly increasing in k for all i, implying $f_k=1$.

If $\theta\in[c,c+1-k)$, then $y_2(\omega,\theta)=r$ and $U_i(\cdot,\cdot,f_o)$ is strictly decreasing in f_o, which implies $f_o=0$. Since $k+v(\theta)-c<1$, for all f_k and f_v it holds that $\mathbf{1}_i=1$ if i faces transaction needs and $\mathbf{1}_i=0$ if otherwise. Thus, $U_i(\cdot,f_v,0)$ is strictly decreasing in f_v for all i such that $f_v=0$. Finally, $U_i(f_k,0,0)$ is strictly decreasing in k, resulting in $f_k=0$.

If $\theta\geq c+1-k$, then $y_2(\omega,\theta)=r$ and $U_i(\cdot,\cdot,f_o)$ is strictly decreasing in f_o, which implies $f_o=0$. If i does not face transaction needs, $\mathbf{1}_i=0$ for all f_k and f_v such that $U_i(\cdot,f_v,0)$ is strictly decreasing in f_v, implying $f_v=0$. In this case, $U_i(f_k,0,0)$ is strictly decreasing in k, which results in $f_k=0$. If i faces transaction needs, $\mathbf{1}_i=1$ for $f_k<1$ or $f_v<1$; for $f_k=f_v=1$, $\mathbf{1}_i=0$. Therefore, we have the following: (i) for $f_k<1$ it follows that $U_i(\cdot,f_v,0)$ is strictly decreasing in f_v, implying $f_v=0$; (ii) for $f_k=1$ we have $U_i(1,f_v,0)$ strictly decreasing in f_v for $f_v<1$, which implies $f_v=1$ since $U_i(1,1,0)-\max\limits_{f_v\in[0,1)}U_i(1,f_v,0)=U_i(1,1,0)-U_i(1,0,0)=\tau-(1-k)(r-1)>0$. Lastly, for $f_k<1$ we have $U_i(f_k,0,0)$ strictly decreasing in f_k, such that $U_i(1,1,0)=1>\max\limits_{f_k\in[0,1)}U_i(f_k,0,0)=U_i(0,0,0)=r-\tau$, which implies $f_k=1$. □

Proof of Lemma 1. We search for incentive-compatible contracts that maximize welfare conditional on both bankruptcy and $\theta<c$. The optimal incentive-compatible contracts that induce continuation conditional on $\theta<c$ are characterized by setting $\ell=0$, from which it trivially follows that they implement a strictly lower level of welfare. Let (w_τ^t,w^t) for $t=1,2$ be the contractual repayments to lenders with and without transaction needs, respectively. Denote π_τ and π the probabilities that lenders with and without transaction needs receive w_τ^1 and w^1

out of k. The optimal contract maximizes the aggregate payoff when $\theta < c$ if it solves

$$\max_{w_\tau^1, w^1 \geq 0, \pi_\tau, \pi \in [0,1]} k + \ell - \alpha \left[\pi_\tau \mathbf{1}_{\{w_\tau^1 < 1\}} + (1 - \pi_\tau) \tau \right] \tag{A.1}$$

subject to

$$\begin{aligned}
&\alpha \pi_\tau w_\tau^1 + (1-\alpha) \pi w^1 = k, \\
&\pi w^1 + (1-\pi) \tfrac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi} \geq \pi_\tau w_\tau^1 + (1-\pi_\tau) \tfrac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi}, \\
&\pi_\tau \left(w_\tau^1 - \mathbf{1}_{\{w_\tau^1<1\}} \tau \right) + (1-\pi_\tau) \left(\tfrac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi} - \tau \right) \\
&\qquad\qquad \geq \pi \left(w^1 - \mathbf{1}_{\{w^1<1\}} \tau \right) + (1-\pi) \left(\tfrac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi} - \tau \right),
\end{aligned}$$

where the equality is the resource constraint and the two inequalities are the incentive compatibility constraints.

The proof consists of a series of claims. Our first claim is that a solution must have $w_\tau^1 \geq 1$. Suppose not, i.e., $w_\tau^1 < 1$ such that the overall payoff equals $k + \ell - \tau$. Then a choice of $w_\tau^{1\prime} = w^{1\prime} = 1$ and $\pi_\tau' = \pi' = k$ satisfy all the constraints and gives a higher aggregate payoff: $k + \ell - \alpha(1-k)\tau > k + \ell - \tau$. This contradicts $w_\tau^1 < 1$ being optimal.

Our second claim is that a solution must also have either $w^1 \geq 1$ or $\pi w^1 \geq \pi_\tau w_\tau^1$. Suppose not, i.e., $w^1 < 1$ and $\pi w^1 < \pi_\tau w_\tau^1$. The former inequality along with the result in the previous paragraph implies $w^1 < 1 \leq w_\tau^1$. The latter inequality along with the first incentive constraint implies $\pi < \pi_\tau$. Moreover, we must have $\frac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi} < 1$; otherwise the first and second incentive constraints imply that the expected payoffs of lenders without and with transaction needs are greater than 1 and $1 - (1-\pi_\tau)\tau$ respectively, resulting in an aggregate payoff that exceeds its maximum of $k + \ell - \alpha(1-\pi_\tau)\tau$. The combination of these results yields

$$\begin{aligned}
\pi w^1 + (1-\pi) \frac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi} &< \pi w_\tau^1 + (1-\pi) \frac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi} \\
&< \pi_\tau w_\tau^1 + (1-\pi_\tau) \frac{\ell}{1-\alpha\pi_\tau-(1-\alpha)\pi},
\end{aligned}$$

which contradicts the first incentive constraint.

Our third claim is that, when $\pi w^1 \geq \pi_\tau w_\tau^1$, welfare is maximized for $w_\tau^1 = w^1 = 1$ and $\pi_\tau = \pi = k$. If $\pi w^1 \geq \pi_\tau w_\tau^1$, then we must have $\pi_\tau w_\tau^1 \leq k$; otherwise $\alpha \pi_\tau w_\tau^1 + (1-\alpha)\pi w^1 > \alpha k + (1-\alpha) k = k$, which contradicts the resource constraint. Since $w_\tau^1 \geq 1$, the maximum welfare is achieved for $w_\tau^1 = 1$ and $\pi_\tau = k$, which yields the result.

Our fourth claim is that, when $w^1 \geq 1$, welfare is also maximized for $w_\tau^1 = w^1 = 1$ and $\pi_\tau = \pi = k$. The first order necessary conditions for minimizing $-\pi_\tau$ subject to the constraints in (A.1) — after solving the equality constraint for w^1 so as to eliminate it from the incentive compatibility constraints — are given by the constraints in (A.1) in addition to

$$\pi_\tau : -1 - \gamma\alpha w_\tau^1 - \mu_1 \left[\frac{\ell(1-\pi)}{(1-\alpha\pi_\tau-(1-\alpha)\pi)^2} - \frac{w_\tau^1}{1-\alpha} \right] - \mu_2 \left[\tau - \frac{\ell(1-\pi)}{(1-\alpha\pi_\tau-(1-\alpha)\pi)^2} + \frac{w_\tau^1}{1-\alpha} \right] - \mu_5 + \mu_6 = 0, \tag{A.2}$$

$$\pi : -\gamma(1-\alpha)w^1 - \mu_1 \left[-\frac{\ell(1-\pi_\tau)}{(1-\alpha\pi_\tau-(1-\alpha)\pi)^2} \right] - \mu_2 \left[-\tau + \frac{\ell(1-\pi_\tau)}{(1-\alpha\pi_\tau-(1-\alpha)\pi)^2} \right] - \mu_7 + \mu_8 = 0, \tag{A.3}$$

$$w_\tau^1 : -\gamma\alpha\pi_\tau - \mu_1 \left[-\frac{\pi_\tau}{1-\alpha} \right] - \mu_2 \left[\frac{\pi_\tau}{1-\alpha} \right] - \mu_3 = 0, \tag{A.4}$$

$$w^1 : -\gamma(1-\alpha)\pi - \mu_4 = 0, \tag{A.5}$$

$$\mu_1 \left[\frac{\ell(\pi_\tau-\pi)}{1-\alpha\pi_\tau-(1-\alpha)\pi} - \frac{\pi_\tau w_\tau^1 - k}{1-\alpha} \right] = 0,\ \mu_1 \geq 0, \tag{A.6}$$

$$\mu_2 \left[\tau(\pi_\tau-\pi) - \frac{\ell(\pi_\tau-\pi)}{1-\alpha\pi_\tau-(1-\alpha)\pi} + \frac{\pi_\tau w_\tau^1 - k}{1-\alpha} \right] = 0,\ \mu_2 \geq 0, \tag{A.7}$$

$$\mu_3 \left[w_\tau^1 - 1 \right] = 0,\ \mu_3 \geq 0, \tag{A.8}$$

$$\mu_4 \left[w^1 - 1 \right] = 0,\ \mu_4 \geq 0 \tag{A.9}$$

$$\mu_5 \pi_\tau = 0,\ \mu_5 \geq 0 \tag{A.10}$$

$$\mu_6 \left[-\pi_\tau + 1 \right] = 0,\ \mu_6 \geq 0 \tag{A.11}$$

$$\mu_7 \pi = 0,\ \mu_7 \geq 0 \tag{A.12}$$

$$\mu_8 \left[-\pi + 1 \right] = 0,\ \mu_8 \geq 0, \tag{A.13}$$

where γ and μ_j for $j = 1, ..., 8$ are the multipliers for the equality and inequality constraints, respectively. Note that $\pi > 0$, which implies $\mu_7 = 0$; otherwise the equality constraint in (A.1) implies $\pi_\tau w_\tau^1 = \frac{k}{\alpha} > 1$, which in turn implies the first incentive compatibility constraint in (A.1) is violated (i.e., the term inside the brackets in (A.6) is negative). As a result, (A.5) implies $\gamma = \mu_4 = 0$. Next, note that $\pi_\tau < 1$; otherwise the term inside the brackets in (A.6) equals $\frac{\ell+k-w_\tau^1}{1-\alpha} < 0$, violating the first incentive compatibility constraint in (A.1). This implies $\mu_6 = 0$. By (A.3), it must be that $\mu_2 > 0$; otherwise $\mu_1 = \mu_8 = 0$, such that by (A.2) we have $-1 - \mu_5 = 0$, which is a contradiction. Finally, if $\pi_\tau > 0$, we must have $\mu_1 > 0$, since otherwise the left-hand side of by (A.4) is negative, yielding a contradiction. In this case, both μ_1 and μ_2

are positive, such that (A.6) and (A.7) imply $\pi_\tau = \pi = \frac{k}{w_\tau^1}$. In addition, we must have $\mu_3 > 0$, which implies $w_\tau^1 = 1$ and $\pi_\tau = \pi = k$; otherwise (A.4) implies $\mu_1 = \mu_2$, which by (A.3) implies $\mu_2 \tau = 0$, a contradiction. The resource constraint in (A.1) then implies $w^1 = 1$. □

Proof of Proposition 2. For $\theta < c$, it is strictly dominant for all lenders to demand payments at $t = 1$, in which case bankruptcy results. To see this note that, in the event of bankruptcy, lenders without transaction needs that withdraw receive an expected payment of at least $k + (1-k)\frac{\ell}{1-k}$, while those that do not receive $\frac{\ell}{1-k}$ with certainty. In the absence of bankruptcy, those that demand payments at $t = 1$ are paid one unit, while the others receive 0 at $t = 2$. Demanding payments at $t = 1$ is even more attractive for lenders with transaction needs as it allows them to avoid the transaction cost τ with certainty in the absence of bankruptcy and at least with probability k in the event of bankruptcy.

For $\theta \geq c + 1 - k$, no bankruptcy occurs since the reserves and illiquid assets are enough to pay all lenders. In this case, it is strictly dominant for lenders without transaction needs to wait and receive $\min\{r, d\} > 1$ at $t = 2$. It is also strictly dominant for lenders with transaction needs to withdraw at $t = 1$, since a payment of one unit at $t = 1$ is greater than $r - \tau$ — this follows from $\tau > r - 1$ — their maximum payoff if they wait until $t = 2$.

For $\theta \in [c, c + 1 - k)$, it is strictly dominant for lenders with transaction needs to withdraw at $t = 1$. In the event of bankruptcy, lenders with transaction needs that demand payments at $t = 1$ receive an expected payment of at least $k + (1-k)\left(\frac{\ell}{1-k} - \tau\right)$, while those that do not receive $\frac{\ell}{1-k} - \tau$ with certainty. In the absence of bankruptcy, those that demand payments at $t = 1$ are paid one unit, while those that do not receive at most $r - \tau < 1$. For lenders without transaction needs, however, their preference depends on the actions played by other lenders. There exists an equilibrium in which only lenders with transaction needs withdraw. Under these strategies, bankruptcy does not occur. Lenders without transaction needs prefer their payment of $d > 1$ at $t = 2$ to demanding a payment of one unit at $t = 1$. This establishes existence. However, there is another equilibrium that results in inefficient bankruptcy. If all lenders demand payment at $t = 1$, then bankruptcy results. In this case, lenders without transaction needs prefer their expected payoff of $k + (1-k)\frac{\ell}{1-k}$ than to not demand payment at $t = 1$ and receive $\frac{\ell}{1-k}$ with certainty at $t = 2$. □

Proof of Proposition 3. Goldstein and Pauzner (2000) and Morris and Shin (2003) prove this result for a general class of global games, including those where θ is drawn from a uniform distribution on $\left[\underline{\theta}, \overline{\theta}\right]$, the noise terms η_i are i.i.d. across players and drawn from a uniform distribution on $\left[-\frac{1}{2}, \frac{1}{2}\right]$ and that satisfy the following additional conditions: (i) for each θ, there exists $\phi^* \in \mathbb{R} \cup \{-\infty, \infty\}$ such that $\Pi(\phi, \theta) > 0$ if $\phi > \phi^*$ and $\Pi(\phi, \theta) < 0$ if $\phi < \phi^*$; (ii) $\Pi(\phi, \theta)$ is nondecreasing in θ; (iii) there exists a unique θ^* that satisfies $\int_0^1 \Pi(\phi, \theta^*)\, d\phi = 0$; (iv) there exists $\overline{D}$ and $\underline{D}$ with $\sigma < \min\left\{\overline{\theta} - \overline{D}, \underline{D} - \underline{\theta}\right\}$ and $\epsilon > 0$ such that $\Pi(\phi, \theta) \leq -\epsilon$ for all $\phi \in [0, 1]$ and $\theta \leq \underline{D}$ and $\Pi(\phi, \theta) > \epsilon$ for all $\phi \in [0, 1]$ and $\theta \geq \overline{D}$; and (v) continuity of $\int_0^1 w(\phi)\,\Pi(\phi,x)\,d\phi$ with respect to signal x and density w. Except for (iii), $\Pi(\phi,\theta)$ clearly satisfies (i), (ii), (iv) and (v).

We now show that (iii) is also satisfied. Since $\Pi(\phi,\theta) < 0$ for all d and $\theta < c$, then if θ^* exists it must be that $\theta^* \geq c$. Therefore, condition (iii) is satisfied if there exists a unique $\theta^* \geq c$ such that $\int_0^1 \Pi(\phi,\theta^*)\,d\phi = 0$, that is,

$$\Delta(\theta^*;d) \equiv k\left(1-\frac{\ell}{1-k}\right)\ln(\theta^*-c+k)+(\theta^*-c+k-\alpha)(d-1)=0. \tag{A.14}$$

Moreover, since $\Delta(\theta;d)$ is strictly increasing in θ, we must show that $\Delta(c;d) \leq 0$ for all d (otherwise for some d we have $\Delta(\theta;d) \geq \Delta(c;d) > 0$ for all $\theta \geq c$ and no θ^* would satisfy $\Delta(\theta^*;d) = 0$).

We have that (a) $\Delta(c;d)$ is strictly increasing in d, (b) d is bounded by r (in which case the bank's participation constraint binds) and

$$\text{(c) } \Delta(c;r) = k\left(1-\frac{\ell}{1-k}\right)\ln\frac{k}{e^{-\frac{\frac{k-\alpha}{k}(r-1)}{1-\frac{\ell}{1-k}}}} \leq (<)\, 0 \text{ if } e^{-\frac{\frac{k-\alpha}{k}(r-1)}{1-\frac{\ell}{1-k}}} \geq (>)\, k.$$

Thus, our assumption that $-\frac{k\ln k}{(1-k)(k-\alpha)}(1-k-\ell) > r-1$ implies that $\Delta(c;d) \leq \Delta(c;r) < 0$ for all d. Moreover, $\Delta(c+1-k;d) > 0$ for all d such that there exists $\theta^* \in (c, c+1-k)$ satisfying $\Delta(\theta^*;d) = 0$. Finally, there is a unique such θ^* as $\Delta(\theta;d)$ is strictly increasing in θ.

For the derivation of the cutoff θ^*, note that (A.14) can be rewritten as

$$\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}e^{\alpha\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}} = \left[\alpha\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)} - \ln(\theta^*-c+k)\right]e^{\alpha\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}-\ln(\theta^*-c+k)}. \tag{A.15}$$

Let $W(\cdot)$ be the inverse function of $y = xe^x$ for $x \geq -1$ (the Lambert W function), that is, $x = W(y)$. Along with (A.15) it implies $\theta^* = e^{\alpha\frac{d-1}{k(1-\frac{\ell}{1-k})}-W\left(\frac{d-1}{k\left(1-\frac{\ell}{1-k}\right)}e^{\alpha\frac{d-1}{k(1-\frac{\ell}{1-k})}}\right)} + c - k.$ □

Proof of Corollary 1. First, implicitly differentiating $y = W(y)\,e^{W(y)}$ results in

$$W' = \frac{W}{(W+1)\,y} = \frac{e^{-W}}{1+W} > 0.$$

This, along with the definitions $x(d,k) \equiv \alpha\frac{d-1}{k(1-\frac{\ell}{1-k})}$ and $z(x(d,k)) \equiv x - W\left(\frac{x}{\alpha}e^x\right)$, allow us to compute

$$\begin{aligned}\frac{\partial\theta^*}{\partial d} &= e^z\frac{\partial z}{\partial d}\\ &= e^z\frac{\alpha}{k(1-\frac{\ell}{1-k})}\left[1-W'\left(\frac{x}{\alpha}e^x+\frac{e^x}{\alpha}\right)\right]\\ &= e^z\frac{\alpha}{k(1-\frac{\ell}{1-k})}\frac{x-W}{(W+1)\,x}<0,\end{aligned}$$

where the inequality follows from the fact that $x-W<0$, which is implied by (A.15). We further have that

$$\begin{aligned}\frac{\partial^2\theta^*}{\partial d^2} &= e^z\left(\frac{\partial z}{\partial d}\right)^2+e^z\frac{\partial^2 z}{\partial d^2}\\ &= e^z\left(\frac{\partial z}{\partial d}\right)^2+e^z\left[\frac{\alpha}{k(1-\frac{\ell}{1-k})}\right]^2\left[\frac{x-W}{(W+1)^2x^2}-(x-W)\frac{W'x\left(\frac{x}{\alpha}e^x+\frac{e^x}{\alpha}\right)+W+1}{(W+1)^2x^2}\right]\\ &= e^z\left(\frac{\partial z}{\partial d}\right)^2+e^z\left[\frac{\alpha}{k(1-\frac{\ell}{1-k})}\right]^2\left[\frac{W\,(W-x)\,(W+x+2)}{(W+1)^3x^2}\right]>0.\end{aligned}$$

Finally, let us rewrite $\theta^*=\alpha\frac{W}{x}+c-k$. Differentiating with respect to k gives

$$\frac{\partial\theta^*}{\partial k}=-\frac{1}{(d-1)}\underbrace{\left[1-\frac{\ell}{(1-k)^2}\right]}_{A}\underbrace{\frac{W\,(x-W)}{W+1}}_{B}-1,$$

with

$$\begin{aligned}\frac{\partial A}{\partial k} &= -\frac{2\ell}{(1-k)^3}<0,\\ \frac{\partial B}{\partial k} &= -\alpha\frac{d-1}{k^2(1-\frac{\ell}{1-k})^2}\frac{(W+x+2)}{(W+1)^2x}AB.\end{aligned}$$

If $(1-k)^2\leq\ell$, then $A\leq 0$ such that B is decreasing, which implies $\frac{\partial\theta^*}{\partial k}$ is strictly decreasing. If $(1-k)^2>\ell$, then $A>0$ such that B is strictly increasing, which implies $\frac{\partial\theta^*}{\partial k}$ is strictly decreasing. Therefore, $\frac{\partial^2\theta^*}{\partial k^2}<0$, which implies that θ^* is strictly increasing, strictly decreasing, or first strictly increasing then strictly decreasing. The first possibility is ruled out as $\frac{\partial\theta^*}{\partial k}<0$ for $(1-k)^2\leq\ell$. Moreover, $\lim_{\alpha\to 0}\lim_{k\to\alpha}\frac{\partial\theta^*}{\partial k}=+\infty$. Therefore, if α is not too large, there exists

$k^* \in \left(\alpha, 1-\sqrt{\ell}\right)$ such that $\frac{\partial \theta^*}{\partial k} = 0$, with $\frac{\partial \theta^*}{\partial k} > 0$ for $k < k^*$ and $\frac{\partial \theta^*}{\partial k} < 0$ for $k > k^*$. □

Proof of Corollary 2. This result is shown in the proof of Proposition 2. □

Proof of Proposition 4. It can be easily verified that conditions (i) through (v) in the proof of Proposition 2 are satisfied, yielding the uniqueness result.

The cutoff θ_S^* such that $\int_0^1 \Pi\left(\phi, \theta_S^*\right) d\phi = 0$ is given by

$$\theta_S^* = \frac{1 - \ln\left(\ell + k\right)}{d}, \tag{A.16}$$

with $\frac{\partial \theta_S^*}{\partial k} = -\frac{1}{d(\ell+k)} < 0$ and $\frac{\partial^2 \theta_S^*}{\partial k^2} = \frac{1}{d(\ell+k)^2} > 0$.

The cutoff θ_M^* such that $\int_0^1 \Pi\left(\phi, \theta_M^*\right) d\phi = 0$ is given by

$$\theta_M^* = \frac{1 - \frac{k}{\ell+k}\left(1 - \frac{\ell}{1-k}\right)\ln\left(\ell+k\right)}{d} < \theta_S^*, \tag{A.17}$$

with

$$\frac{\partial \theta_M^*}{\partial k} = -\frac{1}{d\left(1-k\right)\left(\ell+k\right)^2}\left[\frac{\ell\left(1-\ell-2k\right)\ln\left(\ell+k\right)}{1-k} + k\left(1-\ell-k\right)\right]. \tag{A.18}$$

We now show that $-\frac{\partial \theta_M^*}{\partial k}$ is a strict single crossing function, which is equivalent to $-\theta_M^*$ being strictly quasi-convex, which in turn is equivalent to θ_M^* being strictly quasi-concave. The sum of two strict single crossing functions is also a strict single crossing function if and only if they satisfy strict signed-ratio monotonicity (Quah and Strulovici, 2012). The functions $f\left(k\right)$ and $g\left(k\right)$ satisfy strict signed-ratio monotonicity if whenever $f\left(k\right) > 0$ and $g\left(k\right) < 0$, $\frac{-g(k)}{f(k)}$ is strictly decreasing and whenever $f\left(k\right) < 0$ and $g\left(k\right) > 0$, $\frac{-f(k)}{g(k)}$ is strictly decreasing.

Write $f\left(k\right) = k\left(1-\ell-k\right)$ and $g\left(k\right) = \frac{(1-\ell-2k)\ln(\ell+k)}{\ell^{-1}(1-k)}$. Note that $f\left(k\right)$ is a strict single crossing function since it is always positive. $g\left(k\right)$ is also a strict single crossing function as it is the sum of two single crossing functions $h\left(k\right) = \frac{(1-\ell)\ln(\ell+k)}{\ell^{-1}(1-k)} < 0$ and $z\left(k\right) = \frac{-2k\ln(\ell+k)}{\ell^{-1}(1-k)} > 0$ that satisfy strict signed-ratio monotonicity, since $-\frac{h(k)}{z(k)} = \frac{1-\ell}{2k}$ is strictly decreasing. Moreover, $-\frac{g(k)}{f(k)}$ is strictly decreasing whenever $g\left(k\right) < 0$, which is true if and only if $1-\ell-2k > 0$. This follows because the numerator is decreasing, $-g' = \frac{\ell+1}{\ell^{-1}(1-k)^2}\ln\left(\ell+k\right) - \frac{1-\ell-2k}{\ell^{-1}(1-k)(\ell+k)} < 0$, while the denominator is increasing, $f' = 1-\ell-2k > 0$. Therefore, θ_M^* is strictly quasi-concave.

Finally, since θ_M^* is strictly quasi-concave, it is strictly decreasing, or strictly increasing, or strictly increasing then strictly decreasing. The first two are ruled out since $\lim_{k\to 0} \frac{\partial \theta_M^*}{\partial k} = -\frac{(1-\ell)\ln \ell}{\ell} > 0$ and $\lim_{k\to\frac{1-\ell}{2}} \frac{\partial \theta_M^*}{\partial k} = -\frac{4(1-\ell)^2}{d(1+\ell)^3} < 0$. □

Proof of Proposition 5. The aggregate payoff $r - DW(d)$ is clearly increasing in d. The bank's payoff is strictly concave in d as

$$\frac{\overline{\theta}}{1-\alpha}\frac{\partial^2 V_B(d)}{\partial d^2} = 2(1-\lambda)\frac{\partial \theta^*}{\partial d} - (1-\lambda)(r-d)\frac{\partial^2 \theta^*}{\partial d^2} < 0,$$

which in turn implies $V_B(d)$ is either (1) decreasing or (2) increasing and then decreasing since

$$\frac{\overline{\theta}}{1-\alpha}\frac{\partial V_B(d)}{\partial d} = -\left[\overline{\theta} - (1-\lambda)\theta^*\right] - (1-\lambda)(r-d)\frac{\partial \theta^*}{\partial d}$$

is negative for $d = r$. If $\frac{\partial V_B(d)}{\partial d} \leq 0$ for all d, then $V_B(d)$ is monotone decreasing. If for some d' we have $\frac{\partial V_B(d')}{\partial d} > 0$, then there exists $d'' \in (d', r)$ such that $\frac{\partial V_B(d'')}{\partial d} = 0$. Since $V_B(d)$ is strictly concave in d, $\frac{\partial V_B(d)}{\partial d} > 0$ for $d < d''$ and $\frac{\partial V_B(d)}{\partial d} < 0$ for $d > d''$. Moreover, the bank's participation constraint binds when $d = r$, which implies $V_B(d) < 0$ for all $d > r$. Therefore, the social planner chooses the maximum feasible rollover rent $d^o = r$. □

Proof of Proposition 6. First, we show that $DW(d^o)$ is strictly quasi-concave. Differentiating $-DW(d^o)$ with respect to k gives

$$-\frac{\partial\, DW(d^o)}{\partial k} = \frac{1-\lambda}{\overline{\theta}}\left\{-\frac{\partial \theta^*(d^o)}{\partial k}\left[\alpha + (1-\alpha)r - k - \ell + \alpha(1-k)\tau\right] + \theta^*(d^o)(1+\alpha\tau)\right\}.$$

Write $f(k) = -\frac{\partial \theta^*(d^o)}{\partial k}\left[\alpha + (1-\alpha)r - k - \ell + \alpha(1-k)\tau\right]$ and $g(k) = \theta^*(d^o)(1+\alpha\tau)$. From Corollary 1 we know that $f(k)$ is a strict single crossing function since it is either strictly decreasing, or $f(k) < 0$ for $k < k^*$, $f(k) = 0$ for $k = k^*$ and $f(k) > 0$ for $k > k^*$. $g(k)$ is also a strict single crossing function since as it is always positive. Moreover, $\frac{-f(k)}{g(k)}$ is strictly decreasing whenever $f(k) < 0$, since in this case $\frac{\partial \theta^*(d^o)}{\partial k} > 0$ which, along with the strict concavity of $\theta^*(d^o)$, implies that the numerator is strictly decreasing while the denominator is strictly increasing. Therefore, $f(k)$ and $g(k)$ satisfy strict signed ratio monotonicity (see the proof of Proposition 4 for details), which implies $f(k) + g(k)$ is a strict single crossing function. This, in turn, implies that $-DW(d^o)$ is strictly quasi-convex, which is equivalent to $DW(d^o)$ being strictly quasi-concave. It follows that $DW(d^o)$ is strictly increasing, strictly decreasing, or first strictly increasing then strictly decreasing. The first possibility is ruled out as $\frac{\partial\, DW(d^o)}{\partial k} < 0$ for $k > k^*$.

Now we show that $DW(d^o)$ is close to 0 for k sufficiently large, in which case S_o is close to S_{SB}. We do so in two steps. First, we show that our assumption $0 > y(k) \equiv \frac{k \ln k}{(1-k)}(1 - \ell - k) + (k-\alpha)(r-1)$ implies that the set of allowable k is given by $\left(k, \overline{k}\right)$. Second, we show that $DW(d^o)$ approaches 0 as k approaches $\overline{k}$.

Note that $y' = \left[\frac{(1-k)^2-\ell}{(1-k)^2}\right] \ln k + \frac{r(1-k)-\ell}{1-k}$. Write $f(k) = \left[\frac{(1-k)^2-\ell}{(1-k)^2}\right] \ln k$ and $g(k) = \frac{r(1-k)-\ell}{1-k}$. $f(k)$ is a strict single crossing function since it negative for $k < 1 - \sqrt{\ell}$, equal to zero for $k = 1-\sqrt{\ell}$ and strictly positive for $k > 1-\sqrt{\ell}$. $g(k)$ is also a strict single crossing function since it is always positive. Moreover, $\frac{-f(k)}{g(k)} = -\frac{\ln k}{1-k}\frac{(1-k)^2-\ell}{r(1-k)-\ell}$ is strictly decreasing whenever $f(k) < 0$, since in this case $(1-k)^2 - \ell > 0$, the derivative of $\frac{\ln k}{1-k}$ with respect to k is $\frac{1-k+k\ln k}{k(1-k)^2} > 0$ (the inequality follows from a simple application of the Mean Value Theorem) and the derivative of $\frac{(1-k)^2-\ell}{r(1-k)-\ell}$ with respect to k equals $-\frac{r(1-k)^2-2\ell(1-k)+r\ell}{[r(1-k)-\ell]^2} < -\frac{(1-k-\ell)^2+\ell(r-\ell)}{[r(1-k)-\ell]^2} < 0$. Therefore, $f(k)$ and $g(k)$ satisfy strict signed ratio monotonicity (see the proof of Proposition 4 for details), which implies y' is a strict single crossing function. As a result, y is strictly quasi-convex, which is equivalent to being strictly decreasing, strictly increasing, or first strictly increasing then strictly decreasing. The first possibility is ruled out since $y' > 0$ for $k < 1 - \sqrt{\ell}$. Moreover, $y < 0$ when k is close enough to α and positive when it near $1-\ell$, which implies that y crosses 0 exactly once and from below. Therefore, the set of allowable k is $\alpha < k < \overline{k}$, with the supremum $\overline{k}$ such that $y\left(\overline{k}\right) = 0$.

Next, from (A.14) we have that $\Delta\left(c; r, \overline{k}\right) = 0$. Write $z(k,r)$ and $h(\theta^*(k); r, k)$ as the left- and right-hand sides of (A.15), respectively. We have that $z(r,k) = h(\theta^*(k); r, k) < h(c; r, k)$, where the inequality results from $h(\theta^*(k); r, k)$ being strictly decreasing in $\theta^*(k) > c$. Since (A.15) is equivalent to (A.14) and $\Delta\left(c; r, \overline{k}\right) = 0$, it follows that $\lim_{k\to\overline{k}} h(c; r, k) = \lim_{k\to\overline{k}} z(r,k)$. Thus, from the Squeeze Theorem we have that $\lim_{k\to\overline{k}} \theta^*(k) = c$. Since we take c to be arbitrarily small and $S_{SB} > S_A$, we have $S_o > S_A$ for k sufficiently close to $\overline{k}$. □

Proof of Proposition 7. Suppose that $\theta^o(d^o) \geq \theta^*(d^*)$. Since we assume the project has positive NPV, the bank's payoff under (11) is greater than zero. But then a contract with d marginally greater than d^* satisfies both participation constraints in (10) and results in $\theta^o(d^o) \geq \theta^*(d^*) > \theta^*(d)$. But this contradicts d^o being a solution to (10). □

Proof of Proposition 8. The first order necessary conditions are

$$-\frac{\partial DW(d)}{\partial d} = \frac{\partial V_L(d)}{\partial d}(1-\mu), \tag{A.19}$$

$$\mu\left[V_L(d) - 1\right] = 0, \tag{A.20}$$

$$V_L(d) \geq 1, \tag{A.21}$$

$$\mu \geq 0. \tag{A.22}$$

Since $V_B(d)$ is strictly concave (see Proof of Proposition 2), any d satisfying the first order conditions is a global maximizer, which shows (i).

For (ii), note that

$$\frac{\partial DW(d)}{\partial d} = \frac{(1-\lambda)}{\bar{\theta}}\frac{\partial \theta^*(d)}{\partial d}\left[\alpha + (1-\alpha) r - k - \ell + \alpha (1-k)\tau\right], \tag{A.23}$$

$$\frac{\partial V_L(d)}{\partial d} = 1 - \alpha - \frac{(1-\lambda)}{\bar{\theta}}\left\{(1-\alpha)\theta^*(d) + \frac{\partial \theta^*(d)}{\partial d}\left[\alpha + (1-\alpha) d - k - \ell + \alpha(1-k)\tau\right]\right\}. \tag{A.24}$$

Consider $\mu = 0$. As λ gets close to 1, the left- and right-hand sides of (A.19) approach 0 ((A.23) approximates 0) and $1-\alpha$ ((A.24) converges to $1-\alpha$), respectively. Since the derivative of the right-hand side of (A.19) with respect to λ is greater than that of the left-hand side, which is negative, there are only two possibilities: either the left-hand side of (A.19) is smaller than the right-hand side for all $\lambda \geq \lambda_1 = 0$, or there exists $\lambda(d) \in (0,1)$ such that the left-hand side of (A.19) is smaller than the right-hand side if $\lambda > \lambda(d)$ and at least as great if otherwise. If the former is true for all d, then (A.19) can only be satified if $\mu > 0$. Suppose there exists d such that the latter is true and denote Y the set of all such d. If $\lambda > \lambda_1 = \sup\{\lambda(d) : d \in Y\}$, then (A.19) can only be satified if $\mu > 0$. Combining these two possibilities we deduct that there exists a cutoff $\lambda_1 \in [0,1)$ such $\mu > 0$ if $\lambda > \lambda_1$, which in turn implies that $V_L(d) - 1 = 0$ (from (A.20)). □

References

[1] Brunnermeier, M., 2009, "Deciphering the Liquidity and Credit Crunch 2007–2008," *Journal of Economic Perspectives* 23, 77–100.

[2] Brunnermeier, M. and L. Petersen, 2009, "Market Liquidity and Funding Liquidity," *Review of Financial Studies* 22, 2201–2238.

[3] Diamond, D. and P. Dybvig, 1983, "Bank Runs, Deposit Insurance and Liquidity," *Journal of Political Econommy* 91, 401–419.

[4] Duffie, D. and B. Strulovici, 2012, "Capital Mobility and Asset Pricing," *Econometrica* 80, 2469–2509.

[5] Goldstein, I. and A. Pauzner, 2005, "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance* 60, 1293–1328.

[6] Gorton, G. and G. Ordoñez, 2014, "Collateral Crises," *American Economic Review* 104, 343–78.

[7] Gromb, D. and D. Vayanos, 2002, "Equilibrium and Welfare in Markets with Financially Constrained Arbitrageurs," *Journal of Financial Economics* 66, 361–407.

[8] Krishnamurthy, A., 2010, "Amplification Mechanisms in Liquidity Crises," *American Economic Journal: Macroeconomics* 2, 1–30.

[9] Lorenzoni, G., 2008, "Inefficient Credit Booms," *Review of Economic Studies* 75, 809–833.

[10] Matta, R. and E. Perotti, 2016, "Insecure Debt," University of Amsterdam Working Paper.

[11] Morris, S. and H. Shin, 2003, "Global games: Theory and applications," in Dewatripont, M., Hansen, M. and S. Turnovsky (Eds), *Advances in economics and econometrics (proceedings of the Eighth World Congress of the Econometric Society)*, Cambridge University Press.

[12] Quah, J. and B. Strulovici, 2012, "Aggregating the Single Crossing Property, " *Econometrica* 80, 2333–2348.

[13] Stein, J., 2012, "Monetary Policy as Financial-Stability Regulation," *Quarterly Journal of Economics* 127, 57–95.