Interest Rate Rules, Exchange Market Pressure, and Successful Exchange Rate Management

Franc Klaassen[1,2]
Kostas Mavromatis[1]

[1] *Faculty of Economics and Business, University of Amsterdam, the Netherlands;*
[2] *Tinbergen Institute, the Netherlands.*

Interest rate rules, exchange market pressure, and successful exchange rate management

Franc Klaassen[*]

University of Amsterdam and Tinbergen Institute

&

Kostas Mavromatis

University of Amsterdam

April 28, 2016

Abstract

Central banks with an exchange rate objective set the interest rate in response to what they call "pressure." Instead, existing interest rate rules rely on the exchange rate minus its target. To stay closer to actual policy, we introduce a rule that uses exchange market pressure (EMP), the tendency of the currency to depreciate. Our rule can also explain a high interest rate even if the actual exchange rate is on target, in contrast to traditional rules. A further improvement is that the coefficient for EMP depends on the interest rate effectiveness: the rate should be used less if it is more effective. This shows how policy makers should adapt their policy in case of a structural change to avoid missing their objective. Our rule can be applied to many regimes, from the float to the fixed, and to many models, such as the New Keynesian model, as we illustrate.

Key words: DSGE, exchange market pressure, exchange rate regime, fixed exchange
            rate, monetary policy, open economy Taylor rule.

JEL classification: E43; E52; F31; F33.

[*]Corresponding author. Address: Amsterdam School of Economics, Roetersstraat 11, 1018 WB Amsterdam, The Netherlands; tel. +31-20-5254191; e-mail: f.klaassen@uva.nl.

1   Introduction

Central banks with an exchange rate objective often use the interest rate to achieve that. To set the interest rate, they consider what they call "pressure." For example, Danmarks Nationalbank (2016) writes that "in situations with upward or downward pressure on the krone, Danmarks Nationalbank unilaterally changes its interest rates in order to stabilise the krone," and the Hong Kong Monetary Authority (2009) describes its "automatic interest rate adjustment ... against downward pressure on the exchange rate." The idea is that high pressure on the currency requires a high interest rate.[1]

This paper introduces an interest rate rule to model such pressure-based policy. The rule also accounts for the effectiveness of the interest rate instrument for exchange rate management. This is important for policy makers to achieve the desired degree of forex management, and to know how to reset the interest rate in case of a structural change in the economy, such as an increase in price flexibility.

We first formalize "pressure" by the concept of exchange market pressure (EMP), which refers to the reluctance of investors to hold the domestic currency in the forex market. More formally, EMP is the relative depreciation of a currency in the absence of exchange rate policy, while keeping expectations at the levels determined by actual policy. Positive (negative) EMP means depreciation (appreciation) pressure.[2]

Then we want to account for the effectiveness of the interest rate instrument, because that may well be important for the required use of that instrument. As effectiveness depends on the structure of the economy, we exploit the information of the economic model on that by deriving the rule from the exchange rate function implied by the model. The derivation uses the insights from the EMP concept and takes the desired exchange rate regime as given.[3] It yields an interest rate rule that extends a domestically-oriented rule, such as the Taylor rule, by adding EMP in deviation from

---

[1]Also research papers refer to interest rate adjustments in response to "pressure"; see Calvo and Reinhart (2002), Jovanovic and Petreski (2014), and Ghosh et al. (2016). The latter also considers foreign exchange intervention, for which Mohanty (2013) reports that in a BIS survey among central banks almost 80% said that curbing speculative pressures on the exchange rate was the most important priority. So "pressure" also matters for this policy instrument. We focus on the interest rate, because that provides the simplest framework for introducing our idea.

[2]The EMP literature was initiated by Girton and Roper (1977) and has been further developed by Weymark (1995) and Klaassen and Jager (2011), among others. Interesting applications include Frankel and Xie (2010). In that literature, EMP not only refers to EMP itself, the function of fundamentals except the current interest rate, but also to the *measure* of EMP, which often includes the current exchange rate and interest rate. In our paper, EMP always means EMP itself, not the measure.

[3]Engel (2014) concludes that the analysis to date suggests a role for exchange rates in an optimal monetary policy rule. An alternative to a rule is to simply add an equation that pins down the exchange rate at the target level, as in Galí and Monacelli (2005) and De Paoli (2009). That can be sufficient for some analyses of the fixed rate. Our approach allows for many more regimes and yields additional insights.

the depreciation that is acceptable in the regime. The new term enters with a coefficient reflecting the interest rate effectiveness: for given pressure, the interest rate should be used less if it is more effective. The rule implements the regime exactly, because it offsets exactly the right amount of pressure.

Our foundations of using pressure and deriving the rule within the model to obtain an effectiveness-dependent coefficient differ fundamentally from the usual approach of taking an actual gap (actual minus target value) and adding it with a fixed coefficient. In particular, focusing on exchange rate management, the traditional approach has the actual exchange rate gap with an effectiveness-independent coefficient, either added to a Taylor rule, as introduced by Monacelli (2004), or to the foreign interest rate, as motivated by Benigno et al. (2007), which have both resulted in valuable insights.[4]

The different foundations lead to several improvements of our rule, namely increased realism, implementing the desired regime, and covering more exchange rate regimes. First, our rule is more realistic, in various ways. Using EMP better formalizes what central bankers actually look at, that is, pressure. Moreover, data show that exchange rates close to target can come together with substantial use of the interest rate.[5] Our rule can explain this by nonzero EMP, even if the actual (exchange rate) gap is zero. In traditional rules, a zero gap implies that the interest rate is not used.

Another sign of increased realism follows from the consequences of not using EMP. In the Monacelli rule a zero gap coefficient yields the float, and increasing it means stronger exchange rate management. The fixed rate is not covered but is the limiting case where the coefficient tends to infinity. That means the central bank sets an infinite interest rate if the exchange rate deviates from target. In contrast, EMP in our rule has a finite coefficient.

Benigno et al. focus on the fixed rate. They do not need an infinite gap coefficient. Instead, they assume the home central bank credibly forces investors to convert the foreign into the home currency if the latter depreciates beyond some value, and a similar

---

[4]The former rule has also been used by Engel and West (2005), Lubik and Schorfheide (2007), and Corsetti and Müller (2015). The latter rule has been applied by Benigno and Benigno (2008) and Born et al. (2013). Although the above papers and our work focus on the nominal exchange rate, some authors add the real rate to an interest rate rule, as in Clarida et al. (1998) and Ghosh et al. (2016).

[5]For example, consider the Annual Reports of the central banks of Denmark and Hong Kong. In February 1993 (ERM turbulence) Danmarks Nationalbank increased interest rates and succeeded in keeping the krone exchange rate stable. In 2000 (Danish referendum on euro participation) and 2008 (global financial crisis) it also defended the krone by interest rate hikes. In 2015 it set a negative interest rate to fend off appreciation pressure.
Hong Kong experienced four speculative attacks in 1997-8 (East Asian financial turmoil). In line with its automatic interest rate adjustment mechanism, the Hong Kong Monetary Authority purchased Hong Kong dollars from banks with US dollars to increase market interest rates. It also increased the savings deposit rate. The exchange rate remained stable. In 2007 (US sub-prime mortgage problems), 2008 (Lehman collapse), and 2009 the HKMA decreased interest rates to counteract appreciation pressure.

commitment is imposed on the foreign central bank if the home currency appreciates. They prove that this ensures a fixed exchange rate in equilibrium for any positive value of the gap coefficient. By using EMP, we do not need such convertibility restrictions.

The second improvement of our rule stems from the relevance of the economic structure. In case of a structural change, our rule adjusts such that the exchange rate objective is still reached exactly. In traditional rules, the coefficient for the gap does not adjust, so that the objective may be missed. For example, we can obtain the Monacelli (2004) rule as a special case of our rule and show that his gap coefficient is a reduced-form parameter determined by the interest rate effectiveness and the degree of exchange rate management. Hence, a structural change that alters the effectiveness implies a different exchange rate regime.

A third distinctive feature of our rule is that it can cover many exchange rate regimes. For example, it encompasses a full spectrum of regimes, from the float, to intermediate to the fixed exchange rate regime. This contrasts to traditional rules and reflects that our rule relies on weaker conditions. This generality combined with the fact that we have a separate parameter for the degree of exchange rate management offers a new way to estimate the de facto regime, as we will show.

Our approach is also general in the sense that it can be combined with many models. As an example, we apply it to a standard New-Keynesian open-economy model in the spirit of De Paoli (2009).

This paper connects two strands of the literature, namely the literature on interest rate rules and that on exchange market pressure. It extends not only the former, but also the latter, by deriving EMP in a modern sticky-price model, as the EMP literature typically relies on some variant of the flexible-price monetary model. This delivers insights into the sources of pressure and how structural changes will affect it. For example, more price flexibility causes more volatile pressure in our New-Keynesian model. This matters for policy makers, because periods of substantial pressure may induce them to give up a fixed exchange rate, say, and knowledge about the structure underlying EMP complements their insights from monitoring EMP, as done by the Hong Kong Monetary Authority; see He et al. (2011). So our rule is a useful tool for researchers as well as policy makers.

Finally, our approach relates to the literature on currency crises. In his seminal work, Obstfeld (1994) observes that high interest rates are used to defend against speculative pressure, and he uses that fact to motivate his model and that of many followers. Daniëls et al. (2011) explicitly model how the central bank sets the interest rate to defend, using a global games framework. We share the focus on a realistic

interest rate defense, but we do not try to explain the breakdown of a fixed exchange rate. Instead, we examine pressure arising from shocks to economic fundamentals in a not necessarily fixed regime and in a dynamic general equilibrium setting.

The structure of the rest of the paper is as follows. In Section 2 we formalize pressure. Section 3 derives our interest rate rule and exemplifies it for several regimes, and Section 4 discusses its characteristics. In Section 5 we set out the New Keynesian DSGE model and derive EMP and the interest rate rule for that model. Section 6 illustrates their characteristics using a simulation study. Section 7 concludes.

2 Formalizing pressure

As argued in the introduction, central bankers consider the pressure on their currency at the forex market when implementing exchange rate management. The literature on exchange market pressure focuses on a concept called EMP, which refers to the reluctance of investors to hold the domestic currency in the forex market. This reluctance tends to affect the exchange rate, which triggers the central bank to influence the rate. So the EMP concept seems closely related to what central bankers mean by pressure. We thus start from EMP, refine its formalization, and examine how well it formalizes their word "pressure."

We consider a two-country setting, with a domestic and a foreign country. The domestic monetary authorities, being the central bank throughout this paper, pursue some degree of exchange rate management as one of the policy goals (a perfectly free float is a valid special case), and they use the interest rate as the only instrument for that. For simplicity, official forex market intervention and capital controls are left out. Foreign authorities do not try to control the exchange rate.

2.1 The EMP concept

The idea of EMP is to split the actual (relative) depreciation of the home currency, resulting from the interplay of investors and authorities, into a part reflecting the reluctance of investors to hold the currency, called EMP, and the policy-based part, which usually intends to counteract EMP. EMP applies to any exchange rate regime and can be positive as well as negative, where the latter means there is pressure on the currency to appreciate. A convenient example concerns a fixed exchange rate that is under attack by speculators and where the attack is successfully offset by policy. Then EMP is positive, the policy-based counteracting depreciation that maintains the peg is negative and offsets EMP exactly.

More formally, exchange market pressure at time t, EMP_t, is defined as the relative depreciation of a currency in the absence of exchange rate policy, while keeping expectations at the levels determined by actual policy. One key element is the absence of exchange rate policy. We denote the interest rate rule in this situation by i_t^d, the domestically-oriented interest rate rule, such as the standard Taylor rule. Without the second key element in the EMP definition, the condition on expectations, the use of i_t^d would make EMP like the depreciation under a floating exchange rate regime. But that is not what EMP intends to capture; EMP is about the reluctance of investors to hold the currency in the *actual* regime. So expectations are kept at the level based on the actual interest rate. Also the variables that enter i_t^d are evaluated at their values under the actual exchange rate regime. Finally, the EMP definition is about the consequence of i_t^d and expectations for depreciation, which requires an exchange rate function.

2.2 Exchange rate function

Let s_t be the (logarithm of the nominal) exchange rate at time t, which is the domestic currency price of one unit of foreign currency. The interest rate is i_t and can affect s_t in three ways. First, there is a direct effect; for example, a high i_t attracts capital and thus lowers s_t. Second, i_t matters via expectations. A high i_t could signal that many speculators attack the home currency in case of a fixed exchange rate, increasing the probability that the home currency will be devaluated, attracting other speculators, and that may cause an actual increase of s_t. The third channel is another indirect effect, but it does not involve expectations. For example, a high i_t weakens current consumption, reducing the current home price level, increasing foreign demand for home goods, and appreciating the home currency.

We take the first and third channels together and call them the contemporaneous effect of i_t on s_t. We now assume that there is some explicit function s of the interest rate i_t and other variables, collected in the vector E_t, that yields the exchange rate,

$$s_t = s\left(i_t, E_t\right), \tag{1}$$

where the i_t argument captures the interest rate impact via all contemporaneous channels, and the E_t argument picks up all other mechanisms determining the exchange rate. The symbol E_t is in italics and thus different from the operator E_t that we will use later to denote expectations conditional on the information available in period t. We will often not explicitly mention the non-expectations part of E_t.

We have thus split off all contemporaneous effects of i_t on the exchange rate and collected them in the first argument. The only other way in which i_t can affect s_t is

via the expectations variables in E_t. Hence, the separation implies that we can change i_t and compute the full impact on s_t under the condition that expectations and thus E_t remain constant. This will be crucial in the EMP formula below.

We do not impose a specific economic model on s, nor do we restrict the exchange rate determinants, because we aim for a general approach that can be used in many analyses. We also do not restrict the determinants of the expectations in E_t. The other variables in E_t may depend on expectations, predetermined, and contemporaneous variables, but the above separation implies that the latter must be independent of i_t. So contemporaneous variables such as goods prices, interest rates concerning other maturities than the one underlying i_t, national income, and fiscal policy are first cleaned for i_t by moving the i_t dependencies to the i_t argument, and then the remainder enters E_t. For example, consider good prices. The third channel mentioned above, that a high i_t lowers goods prices and appreciates the currency, is captured by the i_t argument. What remains in the E_t argument is, for example, that lower expected future income weakens current consumption, causing lower prices and appreciation, and that exogenous technological progress via lower prices causes appreciation.

2.3 Defining pressure

The definition of exchange market pressure described in Section 2.1 and expressed in the above notation is

$$EMP_t = s_t^d - s_{t-1}, \tag{2}$$

where

$$s_t^d = s\left(i_t^d, E_t\right). \tag{3}$$

So EMP_t is a function of fundamentals excluding the actual interest rate i_t.

The key part is the intermedial exchange rate s_t^d, which is the exchange rate resulting from the domestically-oriented interest rate i_t^d while keeping E_t at the actual value. So s_t^d is *not* the counterfactual exchange rate that has the alternative interest rate i_t^d and the expectations consistent with that rate. This counterfactual rate would boil down to the exchange rate under a floating exchange rate regime. Instead, by using s_t^d we do not deviate from the actual exchange rate regime, so in general s_t^d is not an equilibrium rate. In fact, all expectations throughout the paper concern the actual regime.

One can view s_t^d as a summary of forex market conditions: the more investors supply the home currency, the higher s_t^d will be. To obtain more insight, consider some examples. First, the foreign interest rate i_t^*, forex market sentiment, risk premia, and productivity shocks to the economy change E_t and thereby s_t^d. For instance, a high i_t^*

or bad sentiment cause a high s_t^d, that is, a weak intermedial value of the currency.

Another example concerns a decrease in expected inflation. By itself, this probably triggers investors to demand the currency, and this is captured by E_t in definition (3), causing a drop in s_t^d. But the drop in expected inflation may also induce the central bank to lower the interest rate if it could ignore the exchange rate objective, which would increase excess supply of the currency and thus s_t^d, and this is picked up by a decrease of i_t^d in (3).

An increase in s_t^d reflects that investors intend to sell the currency. That mimics what central bankers mean by pressure. The EMP variable relates s_t^d to the lagged rate s_{t-1} and the EMP literature calls this pressure. That is in line with the phrase "downward pressure" used by Danmarks Nationalbank (2016) and Hong Kong Monetary Authority (2009), and with He et al. (2011) from the HKMA, who write that they monitor "foreign exchange market pressure." An alternative variant would be to relate s_t^d to the objective s_t^o. Both variants contain s_t^d, the key variable. In the fixed exchange rate regime $s_{t-1} = s_t^o$, so that both variants coincide. But that no longer holds in other regimes. We choose the first variant to formalize pressure, that is,

$$\text{pressure} = EMP_t. \tag{4}$$

3 Interest rate rule

For a given exchange rate regime, the goal is to derive the home interest rate rule that yields an exchange rate consistent with that regime. We first derive the rule without imposing a specific regime, illustrating the encompassing nature of the approach. After that, we exemplify the rule for various specific regimes. We consider the short-term (nominal) market interest rate, because that is typically the focus variable in interest rate policy rules and has more and more become the target variable of central banks.

3.1 Derivation of the rule

In case of a floating exchange rate, the central bank sets the interest rate without considering the exchange rate. Hence, in that case i_t is i_t^d, the domestically-oriented rule introduced in the previous section. For any other regime, i_t differs from i_t^d, and $i_t - i_t^d$ has to capture the exchange rate objective. We thus need a rule for $i_t - i_t^d$. The idea is to link this difference to the exchange rate, and then solve for i_t to obtain the value that delivers the regime.

To quantify the impact of $i_t - i_t^d$ on the exchange rate, we start from exchange rate function (1), $s_t = s(i_t, E_t)$. Using i_t^d instead of i_t affects the exchange rate via

the first argument. However, using i_t^d may also affect expectations and thereby the exchange rate, via the second argument. This latter, indirect effect involves a switch to the floating exchange rate regime and goes against the goal of the rule that we want to derive, which is for the actual regime. We thus eliminate that indirect channel by keeping expectations constant. The other elements in the E_t vector are exogenous and predetermined variables, so they are not affected by the use of i_t^d. Hence, we keep the full E_t vector constant and take $s\left(i_t^d, E_t\right)$. Note that this equals s_t^d defined in (3).

Because s_t and s_t^d both depend on the same E_t, the difference between them is driven by the difference between i_t and i_t^d. More formally, assuming differentiability of the exchange rate function regarding the first argument and applying the mean value theorem demonstrates that the exchange rate implication of using i_t instead of i_t^d is

$$s_t = s_t^d - w_t \left(i_t - i_t^d\right), \tag{5}$$

where

$$w_t = -\frac{\partial s}{\partial i}\left(v_t\right), \tag{6}$$

and v_t is an intermediate vector on the line segment between (i_t, E_t) and $\left(i_t^d, E_t\right)$. The scalar w_t is the effectiveness of the interest rate to counteract depreciation, so it transforms the interest rate deviation $i_t - i_t^d$ into avoided depreciation units $s_t^d - s_t$. We impose $w_t \neq 0$, as using the interest rate for exchange rate purposes would otherwise be useless from the outset.[6] For a linear s function, v_t is irrelevant and w_t is constant.

Exchange rate policy typically concerns the contemporaneous exchange rate s_t. The advantage of (5) is that the policy instrument i_t is now related to s_t in a linear manner, so that we can easily solve for i_t. To implement the objective s_t^o, we propose as rule

$$i_t = i_t^d + \frac{1}{w_t}\left(s_t^d - s_t^o\right). \tag{7}$$

Indeed, substituting this rule into (5) gives

$$s_t = s_t^o. \tag{8}$$

So our rule implements the exchange rate objective exactly, by construction.

---

[6]Because the first argument of the exchange rate function includes all contemporaneous channels through which i_t affects s_t, the partial derivative w_t involves many interest rate effects. Impacts of i_t via the expectations in E_t, however, are not included. Hence, w_t is usually considered to be positive, that is, an interest rate increase appreciates the currency, so that it is effective in offsetting pressure on the currency to depreciate. Our model in Section 5 confirms this sign. In intuitive explanations below we will thus do as if w_t is positive, but we do not impose it in the derivation.

3.2 Pressure matters indeed

Our rule (7) captures that central banks look at pressure when setting their interest rate, which becomes explicit by rewriting the rule as

$$i_t = i_t^d + \frac{1}{w_t} \left(EMP_t - (s_t^o - s_{t-1}) \right). \tag{9}$$

The rule says that the central bank has to set $i_t > i_t^d$ to ward off EMP_t insofar pressure exceeds the target depreciation $s_t^o - s_{t-1}$. The magnitude of $i_t - i_t^d$ is the amount of excess pressure converted into interest rate units by dividing by the effectiveness w_t of the interest rate instrument. The dependence on EMP_t means that our rule brings together two strands of the literature, namely that on interest rate rules and EMP.

It is contemporaneous pressure that matters, not expected future pressure. This marks a difference with the inclusion of, say, expected inflation in some Taylor rules. The latter are typically used to model central bank policy to control inflation between today and a year ahead, say. Such a focus on the future is not the case in exchange rate management. The obvious example concerns the fixed rate: if today's interest rate does not offset the pressure to move away from the target today, there will be an immediate breakdown of the peg, irrespective of expected future developments. Hence, today's EMP_t is what matters for i_t.

In the derivation of our rule, we have not imposed that i_t depends on pressure. Instead, pressure has resulted in a natural way from the derivation. So our rule not only reflects but also supports that central bankers look at pressure.

3.3 The rule for specific exchange rate regimes

The derivation of our rule (7) has imposed neither a specific exchange rate regime, nor an economic model. The rule can thus be applied to many different regimes and models. The current section combines it with six exchange rate regimes, and from Section 5 onwards we illustrate the rule in a specific DSGE model.

Five exchange rate regimes are inspired by practice, namely the fixed rate, float, crawling peg without band, peg with possibly time-varying band, and a policy that moderates the rate of change (called "leaning against the wind"). IMF (2014) shows that these regimes cover the majority of the countries. Examples include Bulgaria, the United States, Nicaragua, China, and Brazil, respectively, albeit that we examine only one type of policy to implement the regime, that is, interest rate policy. The other regime is a weighted combination of the fixed and floating exchange rate regimes, which we introduce because it will be convenient in theoretical analyses.

10

Fixed exchange rate   The question at hand is what i_t the central bank should choose to make sure the exchange rate equals the target s^t. Substituting this objective into (7) gives the interest rate that hits this target by construction. In summary,

$$\text{Policy objective: } s_t^o = s^t \tag{10}$$

$$\text{Interest rate rule: } i_t = i_t^d + \frac{1}{w_t}\left(s_t^d - s^t\right). \tag{11}$$

Float   The central bank does not try to affect the exchange rate, so any tendency for the rate to move to a particular value is ignored by its interest rate policy. Therefore, the central bank sets the rate equal to the domestically-oriented value. This gives

$$\text{Policy objective: } s_t^o = s_t^d \tag{12}$$

$$\text{Interest rate rule: } i_t = i_t^d. \tag{13}$$

Weighted fixed-floating exchange rate   This regime is a weighted average of the fixed and floating exchange rate regimes, where the weight $\mu \in [0,1]$ denotes the degree of exchange rate management. This regime and the interest rule implementing it are

$$\text{Policy objective: } s_t^o = (1 - \mu)\, s_t^d + \mu s^t \tag{14}$$

$$\text{Interest rate rule: } i_t = i_t^d + \frac{1}{w_t}\mu\left(s_t^d - s^t\right). \tag{15}$$

Indeed, for $\mu = 0$ this simplifies to the two equations for the float, (12)-(13), and the higher μ, the more i_t responds to a given $s_t^d - s^t$, meaning tighter exchange rate management. For $\mu = 1$ the system boils down to (10)-(11), representing the fixed rate.

Crawling peg   The crawling peg generalizes the fixed rate by allowing for a gradual trend. Hence, the target becomes time varying, $s_t^o = s_t^t$. Similar to the previous regimes, the interest rate rule follows by substitution of this objective into (7).

Peg with band   The actual exchange rate must now lie in a band $[\underline{s}_t, \overline{s}_t]$, which may vary over time. One example is where the central bank lets the exchange rate float within the band, that is, $s_t^o = s_t^d$ if $s_t^d \in [\underline{s}_t, \overline{s}_t]$, but once the rate tends to leave the band, the central bank uses the interest rate to make sure that the exchange rate settles at the nearest boundary, so $s_t^o = \underline{s}_t$ if $s_t^d < \underline{s}_t$, and $s_t^o = \overline{s}_t$ if $s_t^d > \overline{s}_t$. This setup follows Krugman (1991), albeit that our policy variable is the interest rate instead of money supply. One could also have a one-sided band, as was the case in Switzerland until 2015, where $\underline{s}_t$ restricted the appreciation but $\overline{s}_t$ was infinite.

Leaning against the wind   The previous regimes are about the level of the exchange rate. Several central banks, however, aim at mitigating the change in the exchange rate to prevent undue fluctuations, so they counteract the difference between s_t^d and s_{t-1}. It is a leaning-against-the-wind policy, the wind being $s_t^d - s_{t-1}$. The policy is formalized by $s_t^o = s_{t-1} + (1 - \lambda)\left(s_t^d - s_{t-1}\right)$, where $\lambda \in [0, 1]$ denotes the leaning-against-the-wind intensity. This looks like the weighted fixed-floating regime, but that has as reference value the target rate s^t, while here we take s_{t-1}.

4   Characteristics of the rule and relation to the literature

The upcoming comparison between the traditional and our interest rate rule is in terms of the level s_t of the exchange rate, not the change $s_t - s_{t-1}$. The comparison for the change follows by substituting s^t below by s_{t-1} and shares the same features.

4.1   The rules to be compared

As explained in the introduction, the traditional approaches rely on the gap between the actual exchange rate and its target. The first approach, due to Monacelli (2004), adds this exchange rate gap to a standard Taylor rule, formalized by

$$i_t = i_t^d + \varphi_s\left(s_t - s^t\right), \tag{16}$$

where $\varphi_s \geq 0$.

The alternative rule, by Benigno et al. (2007), starts from the foreign interest rate instead of the Taylor rule and then adds the gap, that is,

$$i_t = i_t^* + \varphi_s^{BBG}\left(s_t - s^t\right), \tag{17}$$

where $\varphi_s^{BBG} > 0$. The rule also includes the convertibility assumptions set out in the introduction.

Our rule is generic equation (7), and imposing the exchange rate policy of interest then gives the interest rate rule for that specific exchange rate regime. In the comparison below, we ignore the time dependence of w_t and focus on

$$i_t = i_t^d + \frac{1}{w}\left(s_t^d - s_t^o\right), \tag{18}$$

where $w \neq 0$.

This description reveals that we have w and a new variable s_t^d, and we do not have

the convertibility assumptions. These differences will play a key role in explaining why our rule outperforms the traditional rules.

4.2 Encompassing multiple exchange rate regimes

To get more insight into the rules, we first study what exchange rate regimes they can generate, and how. In the Monacelli (2004) rule (16), setting $\varphi_s = 0$ yields the floating exchange rate regime, and increasing φ_s makes exchange rate management more and more tight. The fixed exchange rate is not covered but is the limiting case where $\varphi_s \to \infty$. The latter means that the central bank sets an infinite i_t if the exchange rate moves away from its target. Our rule can generate the floating, intermediate, and fixed exchange rate regimes with finite parameters.

Benigno et al. (2007) focus on the fixed rate, and they aim at implementing it without an infinite response parameter. The underlying idea of their rule is as follows. They work in a setting where uncovered interest parity (UIP) holds. The rule that sets $i_t = i_t^*$ would yield an exchange rate s_t equal to the expected rate $\mathrm{E}_t\{s_{t+1}\}$, so that unexpected shocks affecting $\mathrm{E}_t\{s_{t+1}\}$ would cause a jump in s_t, thus breaking the peg.

To avoid this, the authors propose rule (17). The central bank commits to raise i_t above i_t^* if s_t tends to exceed s^t. The authors show that without the convertibility assumptions the exchange rate would explode to plus or minus infinity with positive probability, and they prove that in equilibrium the threat of the convertibility restrictions implies that the central bank will never actually need to raise i_t. The rule effectively removes the impact of shocks on $\mathrm{E}_t\{s_{t+1}\}$ and thereby on s_t. Thus, in equilibrium $s_t = s^t$ and $i_t = i_t^*$. This holds for any $\varphi_s^{BBG} > 0$, reflecting the dominance of the convertibility assumptions. Our rule does not need the convertibility assumptions and has been derived in a setting that permits deviations from UIP.

Our rule combined with the weighted fixed-floating regime, that is, rule (15), encompasses what the two traditional rules cover. It can implement the floating and intermediate regimes, which the first traditional rule covers but not the second, as well as the fixed rate, which the second covers but not the first. So our rule is the only one that encompasses a full spectrum of exchange rate regimes (in addition to other regimes, as exemplified in Section 3.3). This is achieved by offsetting exactly the right amount of pressure.

4.3 Pressure instead of the actual gap

As argued in the introduction, central bankers with an exchange rate objective consider forex market pressure on the value of their currency. Our rule captures this idea, in

contrast to the traditional approaches, as the latter rely on the actual gap $s_t - s^t$.

The merits of using s_t^d instead of s_t can be illustrated by considering the fixed exchange rate regime, where $s_t^o = s^t = s_{t-1}$. If investors' supply weakens the currency in the sense that there is a tendency for the currency to depreciate to $s_t^d > s^t$, then our rule says that the central bank has to set $i_t > i_t^d$ to ward off the pressure. For a successful defense, the outcome is $s_t = s^t$. So our rule can explain $i_t > i_t^d$ even if the actual gap $s_t - s^t = 0$. In contrast, by relying on the actual gap, traditional approaches do as if there is no need for using the interest rate, and this holds whatever the volume of investors' supply. We view this as another indication that it is better to use pressure than the actual gap.

A standard approach in Taylor rules is to include the gap the central bank wants to close. The traditional rules follow this method, but we do not. One can, however, rewrite our rule (18) such that it also has the exchange rate gap $s_t - s^t$, as using $s_t^o = s^t$ implies

$$ i_t = i_t^d + \frac{1}{w} \left(s_t - s^t \right) + \frac{1}{w} \left(s_t^d - s_t \right). \tag{19} $$

Thus our rule introduces a new term, $s_t^d - s_t$. The comparisons in the current section illustrate the importance of this term.

4.4   Policy effectiveness and structure of the economy

The traditional rules use φ_s or φ_s^{BBG} to capture the impact of $s_t - s^t$ on i_t. This is a fixed parameter, like a standard Taylor-rule parameter. It is set independently of the structure of the economy under consideration, including the exchange rate function. To illustrate the consequences, consider an increase in financial openness that intensifies capital inflow after an interest rate increase. This makes the interest rate more effective for exchange rate purposes, so that one would expect a less aggressive interest rate response for a given $s_t - s^t$. But the traditional rules imply the same i_t as before the structural change, due to the unchanged φ_s and φ_s^{BBG}.

The consequence of keeping the same interest rate response when using the Monacelli (2004) rule is that after the structural change the actual regime is one of tighter exchange rate management, so that the central bank misses its objective (provided it successfully implemented its intended managed exchange rate in the first place). For the Benigno et al. (2007) rule, the impact of keeping φ_s^{BBG} constant is none, because the convertibility assumptions determine the outcome, irrespective of the value of the response parameter.

In our rule, the impact of a given pressure on i_t depends on w: a larger w weakens the required interest rate reaction. Recall that w reflects the elasticity of s_t with respect

to i_t, so it is the effectiveness of the interest rate instrument in achieving the exchange rate objective. This effectiveness is determined by the structure of the economy, in particular the exchange rate function. By deriving our rule in close relation with that function, we automatically account for w. Returning to the example of increased financial openness, in our framework this structural change implies a higher w, so that our rule succeeds in delivering the expected less aggressive interest rate response, thereby outperforming the traditional rules. This avoids that the intended regime is missed. The policy implication is that central bankers should account for the structure of the economy when deciding on the use of their policy instrument.

4.5 Observability

Having s_t^d in our interest rate rule implies that the computation of i_t requires knowledge of the functional form of the s-function in (1) and its determinants. In a theoretical model that is no problem. Indeed, in Section 5.5 we will calculate s_t^d for a DSGE model.

Still, traditional rule (16) has an observable variable, s_t, and that may seem an advantage over our rule, which has the unobserved s_t^d. However, this advantage is illusory. To show this, let us restrict our rule in the rest of Section 4 to the weighted fixed-floating regime. Substituting the implied rule (15) into (5) yields

$$s_t = (1 - \mu) s_t^d + \mu s^t, \tag{20}$$

which reveals that in this regime s_t is directly linked to s_t^d. We can use this relation to substitute out s_t^d from (15) and obtain

$$i_t = i_t^d + \frac{1}{w} \frac{\mu}{1 - \mu} \left(s_t - s^t \right), \tag{21}$$

provided $\mu \neq 1$. So a special case of our approach has s_t instead of s_t^d, implying that the presence of the observable s_t is no true advantage of the traditional rule. Even more so, having s_t entails the cost that i_t is no longer identified for $\mu = 1$, a cost that is avoided by using s_t^d.

4.6 Disentangling the reduced-form coefficient φ_s

Taylor-rule parameters such as φ_s in the Monacelli (2004) rule (16) are typically viewed as policy-choice parameters. However, Section 4.4 has revealed that policy effectiveness matters for the interest rate required to implement an exchange rate regime, suggesting that φ_s may be more than just a choice parameter. Indeed, representation (21) shows

that the weighted fixed-floating regime yields

$$\varphi_s = \frac{1}{w}\frac{\mu}{1-\mu},\tag{22}$$

so that both policy effectiveness w and the chosen degree of exchange rate management μ matter. A more effective interest rate (higher w) and weaker exchange rate management (lower μ) both reduce φ_s. If φ_s is instead kept constant after an increase in w, the exchange rate regime is implicitly changed into one of stronger management. Moreover, $\varphi_s/(\varphi_s+1)$ is a biased indicator of the degree of exchange rate management if $w \neq 1$. These insights from our approach improve the understanding of Taylor-rule parameters such as φ_s.

We conclude that φ_s is a reduced-form coefficient for which our approach gives the two underlying structural parameters, w and μ. Because w is determined by existing parameters only, disentangling φ_s does not increase the number of model parameters. We have simply used the structure that is already in the exchange rate function, which is not exploited in the traditional rule.

4.7 Reality check: estimating the de facto regime

Because rule (21) implies

$$\frac{1}{w}\frac{\mu}{1-\mu} = \frac{\text{stdev}\left\{i_t - i_t^d\right\}}{\text{stdev}\left\{s_t\right\}},\tag{23}$$

we can use data on $i_t - i_t^d$ and s_t to estimate the left hand side by the ratio of sample standard deviations and then, for a given w, estimate the de facto degree of exchange rate management μ. This offers a simple check of the realism of our approach.

To operationalize this, we assume that i_t^d is a linear function of domestic producer price inflation π_{Ht} with coefficient 1.5, following Monacelli (2004). One way to obtain a value of w is by specifying a model and compute it from the model parameters. We will do that in the subsequent sections. For now, we set $w = 1.55$, the value of the baseline economy in the simulation section 6.

We examine five countries, namely Australia, Canada, New Zealand, Denmark, and Hong Kong, the countries we will study in the simulation section. The first three have an official inflation targeting policy, while the latter two pursue an exchange rate target. We use 15 years of quarterly data, from 2000 through 2014.[7]

---

[7]The variables for quarter t are measured as follows. For i_t we take the three-month interbank interest rate, calculated as the period average of the daily rates in the quarter. Given period-average PPI values, we use year-on-year inflation for π_{Ht} and thus i_t^d. Then we express i_t and i_t^d at a quarterly basis; all interest rates in the paper are at this basis, so not annualized. The rate s_t is the log of the average daily price of one dollar (euro for Denmark). All data have been obtained from Datastream.
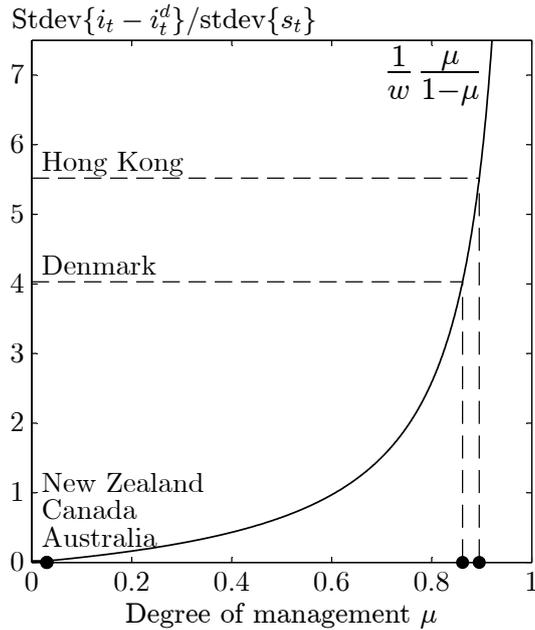
Figure 1: Estimating the degree of exchange rate management.

The estimates of $\text{stdev}\{i_t - i_t^d\}/\text{stdev}\{s_t\}$ are 0.02, 0.02, 0.02, 4.03, and 5.52 for the respective countries. Figure 1 illustrates the implied μ. For Australia, Canada, New Zealand the estimated μ is 0.03, which is consistent with their inflation targeting policies. For Denmark we obtain 0.86, and for Hong Kong 0.89, meaning that their regimes can be characterised as an about 90% fixed and 10% floating exchange rate regime. In our view this is plausible given their strong exchange rate targeting.[8] We conclude from this simple analysis that our pressure-based interest rate rule can deliver useful insights into structural parameters such as μ, which are not identified in traditional rules.

5   Illustration in a log-linearized DSGE model

In Section 3 we have derived our interest rate rule (7) and shown that it guarantees an exchange rate that is consistent with the chosen exchange rate regime. The approach is general in the sense that it can be applied to many models. The current section presents one specific model to illustrate the rule. That is, the model will specify the

---

[8]The value of w that underlies the μ estimates is based on the core parameter values in Table 1. These are estimates. To quantify the reliability of w, we use the information on the posterior distributions of the core parameters, as reported by Justiniano and Preston (2010), to estimate the posterior distribution of w. The resulting 95% credible interval for w is [1.38, 1.98]. The implied intervals for μ are [0.03, 0.04] for Australia, Canada, and New Zealand, [0.85, 0.89] for Denmark, and [0.88, 0.92] for Hong Kong. These are narrow, so that we simply focus on the point estimates.

s-function (1), which yields expressions for w_t and s_t^d, and together with central bank choices for the domestically-oriented interest rate rule i_t^d and the exchange rate regime in s_t^o, all elements of our rule are known.

We take a two-country rational expectations New Keynesian model where the home country is a small open economy, in the spirit of De Paoli (2009). Many other elements and derivations will be standard, as described by Galí (2008), and Galí and Monacelli (2005). We aim for simplicity.

5.1   The model

Appendix A specifies the model and derives the zero-inflation efficient steady state. We log-linearize the model around that steady state and use the log-linearized version from now on. The relevant equations are (24)-(34), which we describe below, and they are derived in the Web Appendix B. Lowercase Latin letters denote the logarithm of variables, except for the interest rate, and an asterisk refers to the foreign country or currency. Table 1 in Section 6 defines all parameters and gives their ranges.

$$\text{Labor supply} \quad : \quad \gamma \ell_t + \sigma c_t = w_t - p_t \tag{24}$$

$$\text{Consumption Euler} \quad : \quad \sigma c_t = \sigma \mathrm{E}_t \left\{ c_{t+1} \right\} - (i_t - \mathrm{E}_t \left\{ \pi_{t+1} \right\} - \delta) \tag{25}$$

$$\text{Calvo-based pricing} \quad : \quad \pi_{Ht} = \beta \mathrm{E}_t \left\{ \pi_{H,t+1} \right\} + \kappa_{mc}(mc_{Ht} - \log\left(1 - 1/\theta\right)) \tag{26}$$

$$\text{Output gap drives costs} \quad : \quad mc_{Ht} - \log\left(1 - 1/\theta\right) = (\sigma_\nu + \gamma)\, y_t^g \tag{27}$$

$$\text{International risk sharing} \quad : \quad c_t - c_t^* = 1/\sigma \cdot q_t \tag{28}$$

$$\text{Labor market equilibrium} \quad : \quad \ell_t = y_t - a_t \tag{29}$$

$$\text{Law of one price} \quad : \quad p_{Ft} = p_{Ft}^* + s_t \tag{30}$$

$$\text{Goods market equilibrium} \quad : \quad y_t = \nu c_t + (1 - \nu)\, c_t^* + \eta \left(1 - \nu^2\right) (p_{Ft} - p_{Ht}) \tag{31}$$

$$\text{Goods market eq. abroad} \quad : \quad y_t^* = c_t^* \tag{32}$$

$$\text{CPI} \quad : \quad p_t = \nu p_{Ht} + (1 - \nu)\, p_{Ft} \tag{33}$$

$$\text{CPI abroad} \quad : \quad p_t^* = p_{Ft}^*. \tag{34}$$

The world is populated with a continuum of households, where the population in the home country H lies in the segment $[0, n)$, while that of the rest of the world F is in $[n, 1]$. Households live forever and have identical preferences, both within and across countries. They derive utility from the consumption of domestic and foreign goods, with home bias in preferences, and disutility from supplying labor to firms. They live in cashless economies, and capital markets are complete, both domestically and internationally, with frictionless trade in assets.

Households maximize expected lifetime utility, where expectations E_t are conditional on the information available in period t. Optimization yields labor supply equation (24) and consumption Euler equation (25), where ℓ_t is labor supply in period t, c_t is consumption, w_t is the wage rate, p_t is the consumer price index (CPI), i_t is the interest rate set by the central bank, and $\pi_t = p_t - p_{t-1}$ is CPI inflation.

Firms specialize in the production of one firm-specific good. Domestic firms produce the varieties in $[0, n)$ and foreign firms those in $[n, 1]$. Each firm uses labor supplied by the households, and a linear technology.

The firm sells its good under monopolistic competition. It sells at home and abroad without trade frictions. Prices are set in the producer's currency, and they are sticky a la Calvo (1983). Hence, the producer price index (PPI) p_{Ht} depends on its lag and the price chosen by firms that are allowed to reset the price. Profit maximization by firms then yields PPI inflation $\pi_{Ht} = p_{Ht} - p_{H,t-1}$ based on (26), showing the importance of real marginal cost mc_{Ht}, which enters the formula in deviation from its steady-state value.

Equilibrium concerns three markets. First, the asset market is in equilibrium if the perfect international risk sharing relation (28) holds, given symmetric initial conditions, where $q_t = s_t + p_t^* - p_t$ is the real exchange rate. Second, labor market equilibrium is given by (29), where y_t is domestic output, and a_t is labor productivity, which is common across home firms and evolves exogenously according to some stationary stochastic process. Third, for the goods market free international trade implies the law of one price, so that import price index p_{Ft} follows from (30), where p_{Ft}^* is the foreign PPI in foreign currency. The goods market also clears for all varieties.

To mimic that the domestic country is small, we now take the limit $n \to 0$. That gives goods market clearing at home (31) and abroad (32). The former captures that higher prices for imports relative to domestically produced goods (higher terms of trade $p_{Ft} - p_{Ht}$) cause substitution towards domestic goods, stimulating domestic production. The limiting case also implies that home CPI in (33) follows from home PPI and the import price index, and that foreign CPI p_t^* is simply the foreign PPI, as (34) shows. Finally, the above results imply that real marginal cost is driven by the output gap y_t^g, as formalized by (27).

5.2 NKPC and IS

As in typical log-linearized New Keynesian models, our model exhibits a convenient recursive representation, consisting of the New Keynesian Phillips curve (NKPC) and the IS equation, as derived in Web Appendix B. The NKPC shows that domestic

producer price inflation π_{Ht} is governed by current and expected future output gaps. The IS equation says that the output gap follows from the current and expected future real interest rate in deviation from its natural counterpart.

In our model, we obtain

$$\text{NKPC:} \qquad \pi_{Ht} = \beta E_t \{\pi_{H,t+1}\} + \kappa_y y_t^g \tag{35}$$

$$\text{IS:} \qquad y_t^g = E_t \{y_{t+1}^g\} - \frac{1}{\sigma_\nu} (i_t - E_t \{\pi_{H,t+1}\} - r_t^n), \tag{36}$$

and the natural rate of interest is

$$r_t^n = \delta + \sigma_\nu \frac{1+\gamma}{\sigma_\nu + \gamma} E_t \{\Delta a_{t+1}\} + \sigma_\nu \frac{(1-\nu^2)(\sigma\eta - 1)\gamma}{\sigma_\nu + \gamma} E_t \{\Delta y_{t+1}^*\}. \tag{37}$$

For the foreign country a similar derivation of its producer price inflation π_{Ft}^* and output gap y_t^{g*} applies, though it is slightly simpler due to the lack of influence from the home country. The resulting equations mimic those of the closed economy, as expected:

$$\text{NKPC*:} \qquad \pi_{Ft}^* = \beta E_t \{\pi_{F,t+1}^*\} + \kappa_y^* y_t^{g*} \tag{38}$$

$$\text{IS*:} \qquad y_t^{g*} = E_t \{y_{t+1}^{g*}\} - \frac{1}{\sigma} (i_t^* - E_t \{\pi_{F,t+1}^*\} - r_t^{*n}), \tag{39}$$

where

$$r_t^{*n} = \delta + \sigma \frac{1+\gamma}{\sigma + \gamma} E_t \{\Delta a_{t+1}^*\}. \tag{40}$$

5.3 Exchange rate function

So far, we have derived the non-policy block of the economy. To close the model, we have to determine the interest rate rule that implements a chosen exchange rate objective. Following the structure of Sections 2-3, we first compute the exchange rate function implied by the model, and in the following sections we derive pressure and use that to derive the interest rate rule.

The arguments of exchange rate function (1) are i_t and the vector E_t. The i_t argument captures the interest rate impact on the exchange rate via all contemporaneous channels, while E_t accounts for everything else, that is, the impact of the interest rate via expectations, the impact of expectations themselves, and the influence of predetermined and exogenous variables, as discussed in Section 2.2. So we first derive the equilibrium exchange rate in this (i_t, E_t)-form.

The exchange rate s_t follows from risk-sharing relation (28), given domestic and

foreign consumer price indices and consumption levels:

$$s_t \quad = \quad p_t - p_t^* + \sigma \left(c_t - c_t^* \right) \tag{41}$$

$$= \quad p_{Ht} - p_{Ft}^* + \frac{1}{\nu} \sigma \left(c_t - c_t^* \right), \tag{42}$$

where the second line uses that p_t depends on s_t, formalized by (33) and (30). Producer prices p_{Ht} and p_{Ft}^* come from profit maximization by firms. Consumption c_t and c_t^* follow from household optimization. So s_t brings about (42) and thus clears the asset market, which makes the equation a natural starting point.

The first determinant of s_t is the price difference $p_{Ht} - p_{Ft}^*$. The firms set p_{Ht} as described by the NKPC. Substituting IS yields

$$\begin{cases} p_{Ht} &= p_{H,t-1} + \beta \mathrm{E}_t \left\{ \pi_{H,t+1} \right\} + \kappa_y \left[\mathrm{E}_t \left\{ y_{t+1}^g \right\} - \frac{1}{\sigma_\nu} \left(i_t - \mathrm{E}_t \left\{ \pi_{H,t+1} \right\} - r_t^n \right) \right] \\ p_{Ft}^* &= p_{F,t-1}^* + \beta \mathrm{E}_t \left\{ \pi_{F,t+1}^* \right\} + \kappa_y^* \left[\mathrm{E}_t \left\{ y_{t+1}^{g*} \right\} - \frac{1}{\sigma} \left(i_t^* - \mathrm{E}_t \left\{ \pi_{F,t+1}^* \right\} - r_t^{*n} \right) \right]. \end{cases} \tag{43}$$

The predetermined $p_{H,t-1}$, r_t^n, and all foreign variables are not affected by i_t and are thus part of the E_t vector. Because y_{t+1}^g does not involve contemporaneous variables, $\mathrm{E}_t \left\{ y_{t+1}^g \right\}$ is part of E_t as well. As the NKPC (35) implies that $\mathrm{E}_t \left\{ \pi_{H,t+1} \right\} = \beta \mathrm{E}_t \left\{ \pi_{H,t+2} \right\} + \kappa_y \mathrm{E}_t \left\{ y_{t+1}^g \right\}$, also $\mathrm{E}_t \left\{ \pi_{H,t+1} \right\}$ is fully driven by expectations of future variables, so that there is no contemporaneous effect of i_t, making $\mathrm{E}_t \left\{ \pi_{H,t+1} \right\}$ part of E_t. Therefore, the i_t-term in (43) captures all contemporaneous impacts of the interest rate on p_{Ht}, so the expressions for both p_{Ht} and p_{Ft}^* are in (i_t, E_t)-form.

The second exchange rate determinant is the consumption difference $c_t - c_t^*$. It follows from the home Euler equation (25) and its foreign equivalent:

$$\sigma \left(c_t - c_t^* \right) = \sigma \left(\mathrm{E}_t c_{t+1} - \mathrm{E}_t c_{t+1}^* \right) - \left(i_t - i_t^* \right) + \left(\mathrm{E}_t \pi_{t+1} - \mathrm{E}_t \pi_{t+1}^* \right). \tag{44}$$

The foreign variables are again part of the E_t vector. Because $\mathrm{E}_t \left\{ c_{t+1} \right\}$ is determined by expectations of future variables, there is no contemporaneous effect of i_t, so that $\mathrm{E}_t \left\{ c_{t+1} \right\}$ is part of E_t. For $\mathrm{E}_t \left\{ \pi_{t+1} \right\}$ it is important to realize that households base their consumption decision on $\mathrm{E}_t \left\{ \pi_{t+1} \right\}$ as a whole, not on just the p_t part within it. Hence, i_t can only affect c_t via $\mathrm{E}_t \left\{ \pi_{t+1} \right\}$ if the latter changes, so that $\mathrm{E}_t \left\{ \pi_{t+1} \right\}$ does not contain a contemporaneous channel and is thus part of E_t. In total, (44) is in the (i_t, E_t)-form that we are after.

The exchange rate process implied by the model is thus

$$s_t = -w i_t + v' E_t, \tag{45}$$

where

$$w = \frac{\kappa_y}{\sigma_\nu} + \frac{1}{\nu} \tag{46}$$

and

$$v = \begin{bmatrix} \frac{\kappa_y^*}{\sigma} + \frac{1}{\nu} \\ 1 \\ \beta \\ \kappa_y \\ \kappa_y - \kappa_y^* \\ \frac{\kappa_y}{\sigma_\nu} \\ \frac{\kappa_y}{\sigma_\nu} - \frac{\kappa_y^*}{\sigma} \\ \frac{\sigma}{\nu} \\ \frac{1}{\nu} \end{bmatrix} \text{ and } E_t = \begin{bmatrix} i_t^* \\ s_{t-1} - tot_{t-1} \\ E_t\{\pi_{H,t+1}\} - E_t\{\pi_{F,t+1}^*\} \\ E_t\{y_{t+1}^g\} - E_t\{y_{t+1}^{g*}\} \\ E_t\{y_{t+1}^{g*}\} \\ E_t\{\pi_{H,t+1}\} + r_t^n - \left(E_t\{\pi_{F,t+1}^*\} + r_t^{*n}\right) \\ E_t\{\pi_{F,t+1}^*\} + r_t^{*n} \\ E_t c_{t+1} - E_t c_{t+1}^* \\ E_t \pi_{t+1} - E_t \pi_{t+1}^* \end{bmatrix}. \tag{47}$$

Equation (45) illustrates exchange rate function (1) and highlights the special role of i_t. Formula (46) is the model-implied version of (6), so it specifies the effectiveness of the interest rate to counteract depreciation while keeping E_t constant. It is constant over time, that is, $w_t = w$. The parameter ranges in Table 1 imply that $w > 0$, so that an interest rate increase strenghtens the home currency. The w parameter is a function of the structural parameters of the model, so the policy effectiveness is fully determined by the structure of the economy.

The expression for E_t in (47) discloses what else matters for the exchange rate according to the model, namely expectations, and other variables insofar as they are not affected by i_t, that is, i_t^*, r_t^n, r_t^{*n}, and the predetermined $s_{t-1} - tot_{t-1}$. Because s_{t-1} has a unit coefficient in v, one could also write (45) in terms of Δs_t and the adjusted E_t would then consist of stationary variables only. However, that does not imply that s_t is non-stationary, because to implement an exchange rate regime the interest rate i_t may depend on the exchange rate level so as to counteract deviations from target, and that can result in a stationary s_t, similar to an error-correction specification.

5.4   Policy effectiveness

An important feature of the model is that the interest rate i_t affects the exchange rate s_t, and the effectivenss w of i_t in doing so is one of the key novelties of our interest rate rule. The way i_t affects s_t, so the channels underlying w, becomes clear from

differentiating exchange rate determination formula (41),

$$w = \frac{-1}{1 - \frac{\partial p_t}{\partial s_t}} \left(\frac{\partial p_t}{\partial i_t} + \frac{\partial \sigma c_t}{\partial i_t} \right), \tag{48}$$

where

$$\frac{\partial p_t}{\partial i_t} = \frac{\partial p_t}{\partial p_{Ht}} \frac{\partial p_{Ht}}{\partial mc_{Ht}} \frac{\partial mc_{Ht}}{\partial y_t^g} \frac{\partial y_t^g}{\partial i_t}. \tag{49}$$

Increasing i_t has a direct impact on households in two ways. The first leads to $\frac{\partial p_t}{\partial i_t} + \frac{\partial \sigma c_t}{\partial i_t}$ in (48), which captures the following mechanisms. The higher i_t makes current consumption more expensive relative to future consumption, inducing households to reduce c_t. This is quantified by $\partial \sigma c_t / \partial i_t = -1$, based on (44).

The reduced demand for home-produced goods lowers the output gap ($\partial y_t^g / \partial i_t = -1/\sigma_\nu$), so marginal costs shrink ($\partial mc_{Ht} / \partial y_t^g = \sigma_\nu + \gamma$), inducing firms to lower their prices ($\partial p_{Ht} / \partial mc_{Ht} = \kappa_{mc}$), as formalized by (43). Consumer prices then drop due to home bias in consumption ($\partial p_t / \partial p_{Ht} = \nu$). Equation (49) combines these effects, explaining the total price change of $\partial p_t / \partial i_t = -\nu \kappa_{mc} (\sigma_\nu + \gamma) / \sigma_\nu$.

The second effect of the higher i_t on households concerns the asset market, where equilibrium is disturbed. Risk-sharing formula (41) and the decrease in c_t and p_t imply that s_t decreases. The latter is by assumption. We have not explicitly modeled how exactly the exchange rate equilibrates the asset market, but a possible mechanism would be that, due to the cheaper home bond, households substitute from foreign to home bonds, thereby demanding home currency until its appreciation has equilibrated the market. Whatever the equilibrating mechanism, the appreciation decreases import prices and thus consumer prices by $\partial p_t / \partial s_t = 1 - \nu$, so that s_t has to decrease even further to restore equilibrium.

This breakdown also demonstrates how the economic structure affects the effectiveness of interest rate policy. For example, $\partial \kappa_{mc} / \partial \omega < 0$ implies that less price stickiness (lower ω) increases w, so that increasing price flexibility enhances policy effectiveness, facilitating the defense of a peg for a given amount of pressure. Moreover, $\partial \sigma_\nu / \partial \eta < 0$ means that if home and foreign goods are closer substitutes (higher η), the response of the output gap to the interest rate hike intensifies while marginal cost responds less to each unit of output gap change. The first effect dominates, because a higher η increases w. So closer substitutability facilitates the defense of a peg against a given pressure.

5.5 Pressure

The key part of pressure EMP_t is the intermedial exchange rate s_t^d. It follows directly from definition (3) and the model-implied s function (45), so that

$$s_t^d = -wi_t^d + v'E_t. \tag{50}$$

This yields

$$EMP_t = -wi_t^d + v'E_t - s_{t-1}. \tag{51}$$

So the model pins down how EMP_t depends on exchange rate fundamentals.

The EMP_t expression represents a contribution of our paper to the literature on exchange market pressure, addressed in Section 2.1. It is a modern version of the EMP expressions in that literature, which typically rely on some variant of the flexible-price monetary model. In the latter model a higher interest rate depreciates the currency. This has the uncomfortable implication that a low instead of high interest rate is needed to ward off speculation against the currency. The implication is driven by the fact that the monetary model assumes flexible goods prices. Although this assumption is fine for a long-run analysis, exchange market pressure is analyzed as a short-run phenomenon. In the short run price flexibility is limited, and this is accounted for in our model. Indeed, in our model a higher interest rate appreciates instead of depreciates the currency and thus wards off depreciation pressure. So our model is a useful contribution to the EMP literature as well.

To reveal the sources of pressure according to the model, rewrite (51) as

$$EMP_t = w\left(i_t^* - i_t^d\right) + (v_1 - w)\,i_t^* + v_2'E_{2t} - s_{t-1}, \tag{52}$$

where v_1 is the top element of the v-vector in (47), v_2 is the rest of v, and likewise i_t^* is the top element of E_t, and E_{2t} is the rest. The sources of pressure are thus i_t^*, the drivers of i_t^d, and the variables in E_{2t} except for s_{t-1}.

5.6 Interest rate rule

The interest rate rule that, within the above model, implements a given exchange rate regime, starts from our generic rule (7). The ingredients s_t^o and i_t^d depend on choices made by the central bank. For s_t^o we consider the weighted fixed-floating regime (14), which is simple and encompasses a full spectrum of regimes, from floating ($\mu = 0$) to fixed at a specific target value s^t ($\mu = 1$). Both μ and s^t are policy choices by the central bank. This implies that the rule specializes to (15).

For the domestically-oriented rate i_t^d we simply take

$$i_t^d = \delta + \varphi_\pi \pi_{Ht}, \tag{53}$$

though one could also use a Taylor rule with CPI inflation and output gap.

The interest rate rule results by substituting (50) for s_t^d in rule (15), giving

$$i_t = (1 - \mu) \, i_t^d + \mu \frac{1}{w} \left(v' E_t - s^t \right), \tag{54}$$

where i_t^d is given by (53) and w by (46). This rule guarantees that the exchange rate regime is implemented at every t, that is, $s_t = s_t^o$.

The proposed interest rate is a weighted average of i_t^d and $\frac{1}{w} \left(v' E_t - s^t \right)$, where the weight is the degree of exchange rate management μ. In case of a floating exchange rate ($\mu = 0$) the interest rate is simply the domestically-oriented rule i_t^d, as usual.

On the other extreme, to implement a fixed rate ($\mu = 1$) the central bank cannot pursue i_t^d at all. The latter drops out of (54), bringing the analysis automatically in line with the well-known incompatible trinity. For example, a one percentage point lower i_t^d by itself motivates an equally lower i_t, but implementing that would cause a w %-points higher s_t, which would have to be offset by a one %-point higher i_t to maintain the peg, making i_t^d on balance irrelevant for the actual interest rate. Instead of looking at i_t^d, the central bank should focus on $v' E_t - s^t$. If market sentiment in $v' E_t$ tends to move the exchange rate away from the target, the excess change $v' E_t - s^t$, converted into interest rate units by dividing by w, pins down the interest rate.

Rule (54) illustrates the improvements over the Monacelli (2004) rule (16) analyzed in Section 4: our rule covers the fixed exchange rate regime, uses pressure instead of s_t, accounts for the policy effectiveness w, and disentangles φ_s into w and μ. A new improvement is that our rule automatically accounts for the incompatible trinity.

The Benigno et al. (2007) rule (17) concerns the fixed exchange rate, and for that case our rule becomes

$$i_t = \frac{1}{w} \left(v' E_t - s^t \right) \tag{55}$$

$$= i_t^* + \frac{1}{w} \left[(v_1 - w) \, i_t^* + v_2' E_{2t} - s^t \right]. \tag{56}$$

Comparing this with their rule again shows that our rule has pressure instead of s_t, uses a specific model-driven parameter w instead of the undetermined φ_s^{BBG}, and does not rely on convertibility assumptions. A common feature is that also in our rule following i_t^* is important for implementing the peg.

6 Simulations from the model

In Section 4 we have discussed the characteristics of our interest rate rule without imposing a specific model. To illustrate some of these characteristics, we now simulate from the model just developed. The main insights from these simulations are not specific to the model, parameter values, or draws of the shocks.

6.1 Model calibration, solution, and simulation

One period in the model is one quarter. For the simulations we assume that (the log of) home labor productivity a_t follows an AR(1) process with autoregressive coefficient ρ_a and that the i.i.d. shock involved has mean zero and standard deviation σ_a. The same holds for foreign productivity a_t^*. The foreign central bank follows the rule

$$i_t^* = \delta + \varphi_\pi \pi_t^* + \varepsilon_{it}^*, \tag{57}$$

where ε_{it}^* is an i.i.d. monetary policy shock with zero mean and standard deviation σ_i^*.

We set the target $s^t = 0$. All parameter values are based on Justiniano and Preston (2010). Table 1 presents them, and its note provides further motivation.

We solve the model numerically using the Sims (2002) algorithm. The solution can be cast as a reduced-form VAR model of the 21×1-vector

$$[c_t, \mathrm{E}_t c_{t+1}, c_t^*, \mathrm{E}_t c_{t+1}^*, \pi_{Ht}, \mathrm{E}_t \pi_{H,t+1}, \pi_t, \mathrm{E}_t \pi_{t+1}, \pi_t^*, \mathrm{E}_t \pi_{t+1}^*, ...$$
$$y_t, y_t^*, i_t, i_t^*, s_t, \mathrm{E}_t s_{t+1}, q_t, tot_t, EMP_t, a_t, a_t^*]'.$$

We focus on unique stationary solutions, abstracting thus from sunspot equilibria. Determinacy is achieved in all exchange rate regimes. The initial value of each variable is its steady-state value, and the log exchange rate is initialized at zero.

The three shocks are drawn from the normal distribution. We draw them for 60 periods (15 years), from which we compute the paths of the variables of interest. For ease of comparison we keep the realized shocks the same across the plotted paths.

6.2 Encompassing multiple regimes

One advantage of our rule (54) over traditional rules is that it can be applied to many exchange rate regimes, as explained in Section 4.2. To obtain a first insight into the performance of our rule, we simulate paths for variables in three different regimes, namely the float ($\mu = 0$), an intermediate regime ($\mu = 0.5$), and the peg ($\mu = 1$). A representative set of paths is depicted in Figure 2. The variables $EMP_t, i_t, i_t^d, i_t^*, s_t$ in

Table 1: Model parameters

Par.	Range	Value	Description
Core parameters			
β	$(0,1)$	0.99	subjective discount factor
γ	>0	1.17	inverse of Frisch elasticity of labor supply
σ	>0	1.20	inverse of elasticity of intertemporal substitution for consumption
η	>0	0.68	elasticity of subst. between home & foreign goods
θ	>1	8.00	elasticity of subst. between varieties produced within a country
ω	$(0,1)$	0.72	Calvo fraction of firms not allowed to change prices (stickiness)
n	$(0,1)$	$\to 0$	size of the home economy
ν	$(0,1]$	0.75	home bias in preferences
φ_π	≥ 0	2.06	inflation impact on interest rate in Taylor rule
μ	$[0,1]$	—	degree of exchange rate management
Additional parameters for simulation			
ρ_a	$(-1,1)$	0.81	AR(1) coefficient in labor productivity process
σ_a	≥ 0	0.52	standard deviation of labor productivity shock (in %)
σ_i^*	≥ 0	0.12	standard deviation of foreign monetary policy shock (in %)
Derived parameters			
δ	$(0,1)$	0.01	$=-\log(\beta)$: subjective discount rate
α	$[0,1]$	$\to .75$	$=1-(1-n)(1-\nu)$: share of home goods in home consumption
α^*	$[0,1]$	$\to 0$	$=n(1-\nu)$: share of home goods in foreign consumption
τ	$[0,1]$	0.13	$=1-\frac{\theta-1}{\theta}$: employment subsidy
σ_ν	>0	1.31	$=\frac{\sigma}{(1-\nu^2)\sigma\eta+\nu^2}$: inverse of interest rate impact in IS curve (36)
κ_{mc}	>0	0.11	$=\frac{(1-\omega)(1-\omega\beta)}{\omega}$: marginal cost impact on PPI inflation in (26)
κ_y	>0	0.28	$=\kappa_{mc}(\sigma_\nu+\gamma)$: output gap impact in home Phillips curve (35)
κ_y^*	>0	0.26	$=\kappa_{mc}^*(\sigma+\gamma)$: output gap impact in foreign Phillips curve (38)
w	>1	1.55	$=\frac{\kappa_y}{\sigma_\nu}+\frac{1}{\nu}$: effectiveness of i_t to counteract depreciation

Foreign parameters $\beta^*, \gamma^*, \sigma^*, \theta^*, \omega^*, \varphi_\pi^*, \rho_a^*, \sigma_a^*, \delta^*, \tau^*$, and κ_{mc}^* equal their home counterparts.
The values of the core and additional parameters for simulation have been taken from Justiniano and Preston (2010). The authors estimate a small open-economy model for three countries vis-à-vis the United States, namely for Australia, Canada, and New Zealand, using Bayesian techniques, though they calibrate the values for β, θ, and ν. We take the average of their three posterior medians.
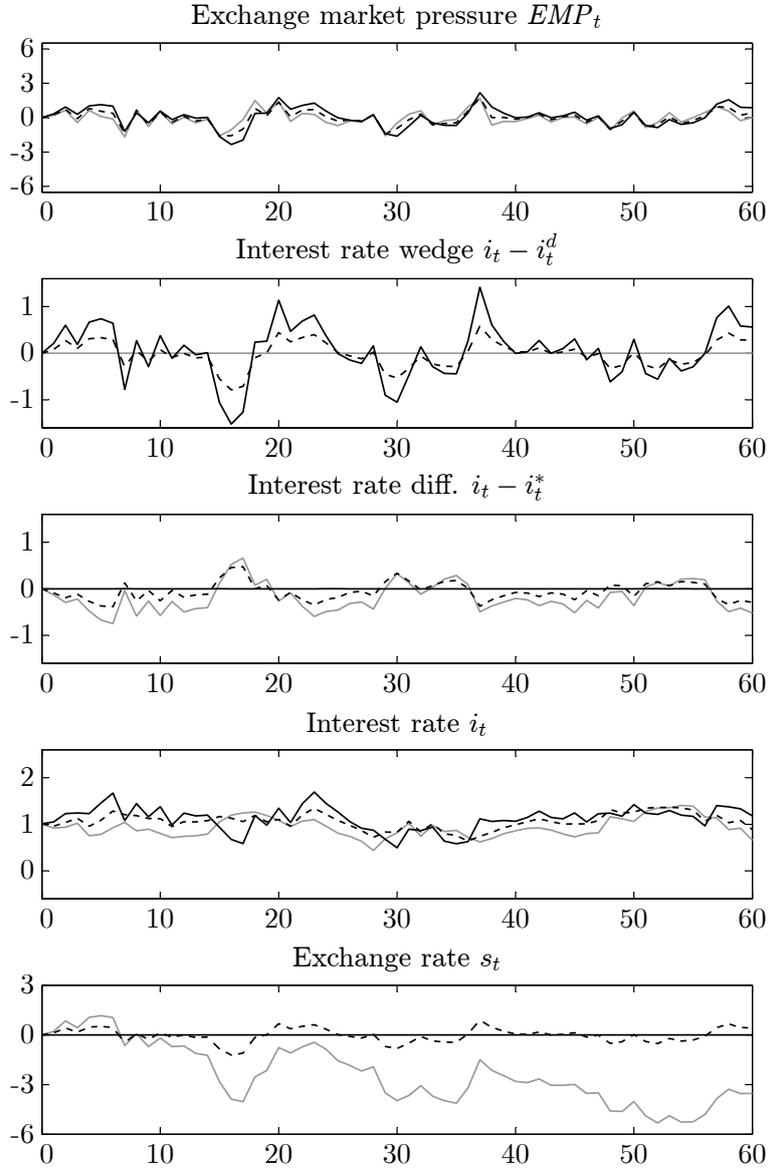
the graphs are all in percentage terms.

Figure 2: Our rule (54) in various exchange rate regimes: from float ($\mu = 0$, grey) to intermediate ($\mu = 0.5$, dashed) to fixed ($\mu = 1$, black).

Under the float ($\mu = 0$), the interest rate i_t equals the domestically-oriented rate i_t^d, visualized by the horizontal line in the second panel. As i_t^d is driven by inflation, the interest rate does not stabilize the exchange rate s_t, which is consistent with the fact that the grey line in the bottom panel does not revert to zero.

The stronger the exchange rate management, the more the central bank has to account for exchange rate fundamentals when setting the interest rate, making i_t^d less

and E_t in our rule more relevant. The dashed line in the bottom panel visualizes that $\mu = 0.5$ already stabilizes the exchange rate here considerably. The line also suggests that the weighted fixed-floating regime can be a practical linear approximation of various other exchange rate policies, such as the peg with band.

For the fixed exchange rate regime ($\mu = 1$), only E_t matters, and the shocks that hit the economy each and every period have to be offset by interest rate policy. The black line in the middle panel visualizes that $i_t = i_t^*$ in equilibrium, which follows from the fact that our rule implies $s_t = s^t$ by construction and that the model contains UIP, by virtue of (28). The bottom panel shows that our rule yields an exchange rate that stays on target continuously. Clearly, the rule succeeds in offsetting the influence of shocks on s_t by exactly the required amount. This is not surprising, because the rule has been developed such that it implements the exchange rate regime by construction.

6.3 Pressure

One key novelty of our rule is that it depends on pressure, as discussed in Section 4.3. Let us focus on the fixed exchange rate regime. Despite the constancy of the exchange rate, the top panel of Figure 2 reveals that the shocks cause periods of noticeable pressure EMP_t on the peg. The sources of pressure follow from EMP expression (52). This can be simplified for the fixed exchange rate, where our rule collapses to (55), so that combining the equilibrium outcome $i_t = i_t^*$ with (56) and (52) yields

$$EMP_t = w \left(i_t^* - i_t^d \right). \tag{58}$$

Hence, i_t^* and i_t^d are the sources of pressure that matter here on balance. For example, at $t = 23$ the high i_t^* is the main cause of the depreciation pressure (positive EMP_t), while at $t = 37$ the economy experiences serious deflation and thus a low i_t^d, causing again depreciation pressure. In periods of large positive or negative pressure, the central bank has to accept an interest rate that differs substantially from the domestically-oriented rate, which in practice may induce policy makers to give up the peg.

6.4 Structural change, policy effectiveness, and missing the peg

A distinctive feature of our rule compared to the traditional rules is that the economic structure matters, via w, as explained in Section 4.4. We now study the impact of a structural change for the fixed exchange rate regime ($\mu = 1$) and the consequences if policy makers fail to account for this. We focus on the degree of price stickiness ω. We reduce it (only in this section) from 0.72 to $\widetilde{\omega} = 0.59$, the lower end of the average 90%

credible interval reported by Justiniano and Preston (2010), so that home producer prices become more flexible.[9]

The policy effectiveness parameter increases from 1.55 to $\widetilde{w} = 1.88$, so an increase in i_t now causes a sharper drop in home producer prices and thereby a larger appreciation. This increased interest rate effectiveness affects our rule, as it prescribes a less aggressive i_t to maintain the peg for given pressure. This is a plausible novelty of our rule.

The actual use of i_t not only depends on its effectiveness, but also on pressure EMP_t. More price flexibility affects EMP_t in two ways, as reflected by (58). First, it increases the exchange rate consequences $\widetilde{w}$ of a given wedge $i_t^* - i_t^d$. Second, it makes PPI inflation π_{Ht} and thus i_t^d more volatile. In total, comparing the black line in the top panel of Figure 3 to that in Figure 2 demonstrates that here more price flexibility creates more volatile pressure. Such consequences of a structural change for pressure are important for policy makers to be aware of.
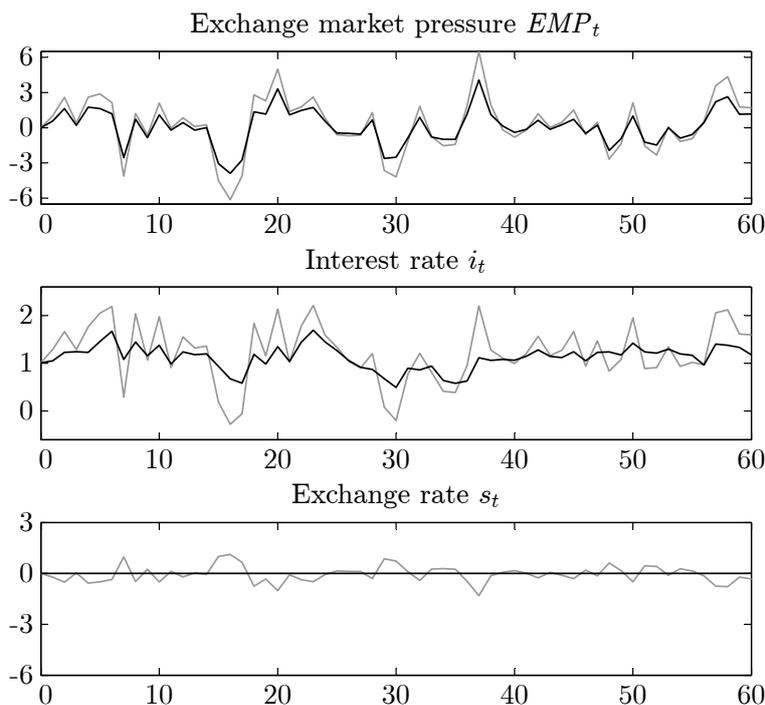

Figure 3: Impact of increased price flexibility (lower ω), depending on whether the fixed exchange rate policy accounts for it (black) or not (grey).

The equilibrium value of i_t remains i_t^*, so the higher policy effectiveness and the increased volatility of pressure cancel out. The black line in the middle panel of Figure

---

[9]This implies $\widetilde{\kappa}_{mc} = 0.29$ $\widetilde{\kappa}_y = 0.72$, and $\widetilde{w} = 1.88$. The other parameters in Table 1 do not change. In particular, ω^* is unchanged; changing it would have virtually no impact on the results relevant here.

3 thus still equals i_t^*, and the bottom panel demonstrates that the adjusted rule keeps on implementing the fixed exchange rate.

The outcome changes if the central bank does not adjust the rule in response to the reduced price stickiness, that is, it uses w instead of $\widetilde{w}$ in the rule. As the grey lines in Figure 3 reveal, EMP_t and i_t become more volatile, and the peg is missed. The intuition is as follows. The central bank no longer weakens the interest rate response to a given EMP_t. This causes an overreaction of i_t. It turns out that EMP_t becomes also more volatile, and this increased pressure further intensifies the interest rate response. This causes i_t to deviate from i_t^*. If $i_t > i_t^*$, the home currency appreciates, so that the target is missed.

The policy recommendation is that central bankers should account for the effectiveness of their policy instrument, and thus for the economic structure, when determining its use. This is not accounted for in traditional interest rate rules. Our rule provides a solution, and the simulation study demonstrates that it matters for realizing the policy objective.

7 Conclusion

Central bankers look at pressure when setting the interest rate for forex management, and the magnitude of the interest rate required to implement a regime is likely to depend on the effectivess of the interest rate. This paper has derived a new interest rate rule for both aspects: the interest rate depends on pressure, defined as EMP, and the more effective the rate, the less it should be used to offset a given pressure. Both features are novel in the literature. They also explain why our rule is more realistic than the traditional rules and covers more exchange rate regimes. Our rule implements the intended regime exactly, because it offsets the right amount of pressure.

As a simple reality check, we have used the rule to calculate for some countries their de facto degrees of exchange rate management. This is based on the weighted fixed-floating regime, with weight μ on the fixed regime, which seems a practical linear approximation of various actual exchange rate policies, such as the peg with band. For the inflation targeters Australia, Canada, New Zealand, we find $\mu = 0.03$, and for the exchange rate targeters Denmark and Hong Kong we obtain 0.86 and 0.89, respectively, which we consider plausible.

The rule can be applied to many economic models. We have illustrated it in a standard two-country New Keynesian model for a small open economy. The model determines how EMP depends on exchange rate fundamentals. The coefficient of EMP in the interest rate rule depends on the structure of the economy, such as the Calvo

degree of price stickiness.

Our approach is not only useful for researchers, but it also helps policy makers. One policy recommendation concerns how central bankers should account for the economic structure when setting policy. For example, reduced price stickiness makes the interest rate instrument more effective, and ignoring this in the rule implies that the intended exchange rate regime is missed. Our rule adjusts to the structural change and thus still implements the regime.

A second policy relevance is that our approach delivers insights into the sources of exchange market pressure and how a structural change affects it. This matters, as periods of large pressure may in practice induce policy makers to give up a peg.

The general applicability of our approach, both regarding regimes and models, can facilitate studies on optimal exchange rate management (further eased by our new structural parameter μ), research on models with risk premia, and so on. In another paper we apply our idea to formalize foreign exchange interventions by the central bank. Such a model would then facilitate studies on emerging markets where central banks use forex intervention to pursue leaning-against-the-wind exchange rate management. This is left for future research.

Appendix

A DSGE model

A.1 Households

The world is populated with a continuum of households, where the population in the home country H lies in the segment $[0, n)$, while that of the rest of the world F is in $[n, 1]$. Domestic households maximize expected lifetime utility

$$\max \mathrm{E}_t \sum_{k=0}^{\infty} \beta^k \left(\frac{C_{t+k}^{1-\sigma}}{1-\sigma} - \frac{L_{t+k}^{1+\gamma}}{1+\gamma} \right), \tag{59}$$

subject to a budget constraint (specified later), by choosing a path $\{C_{t+k}, L_{t+k}\}_{k=0,\dots,\infty}$, where C_{t+k} is household consumption and L_{t+k} is labor supply at time $t+k$.

Consider period t.[10] Consumption enters the domestic household's utility as an index C_t, which is the CES aggregate of the indices of domestic consumption of home

---

[10]Results for $t+k \geq t$ follow by substituting t by $t+k$, while keeping expectations conditional on t.

and foreign (imported) goods, C_{Ht} and C_{Ft}, respectively:

$$C_t = \left(\alpha^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + (1-\alpha)^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}. \tag{60}$$

The parameter α, determining the preference for home-produced goods in consumption, increases with the size of the home country, n, and with home bias ν. We model $1 - \alpha = (1-n)(1-\nu)$. Hence $\nu > 0$ means that domestic households consume fewer foreign-produced goods than the size of the foreign country implies, reflecting home bias.[11]

The index of domestic consumption of home goods, C_{Ht}, is the CES aggregate of the consumption of all varieties produced in country H. These are varieties $j \in [0, n)$. The index of domestic consumption of foreign goods, C_{Ft}, is a similar CES aggregate, but concerning all varieties produced in F, which are $j \in [n, 1]$. Domestic consumption of variety j is denoted by $C_t(j)$. In formula,

$$\begin{cases} C_{Ht} = & \left[\left(\frac{1}{n} \right)^{\frac{1}{\theta}} \int_0^n C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}} \\ C_{Ft} = & \left[\left(\frac{1}{1-n} \right)^{\frac{1}{\theta}} \int_n^1 C_t(j)^{\frac{\theta-1}{\theta}} dj \right]^{\frac{\theta}{\theta-1}}. \end{cases} \tag{61}$$

So θ concerns the substitutability between varieties produced within a country, whereas η in (60) is about the substitution between home and foreign goods.

As usual, utility maximization requires that within period t households maximize C_t for a given expenditure on home and foreign indices and they maximize C_{Ht} (C_{Ft}) for a given level of expenditure on home (foreign) varieties. Let $P_t(j)$ denote the price of variety j in domestic currency. The resulting demand function for each variety is

$$C_t(j) = \begin{cases} \alpha & \left(\frac{P_t(j)}{P_{Ht}} \right)^{-\theta} \left(\frac{P_{Ht}}{P_t} \right)^{-\eta} C_t, & \text{for home} \quad \text{varieties } j \in [0, n) \\ (1-\alpha) & \left(\frac{P_t(j)}{P_{Ft}} \right)^{-\theta} \left(\frac{P_{Ft}}{P_t} \right)^{-\eta} C_t, & \text{for foreign varieties } j \in [n, 1], \end{cases} \tag{62}$$

where

$$\begin{cases} P_{Ht} = & \left[\frac{1}{n} \int_0^n P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \\ P_{Ft} = & \left[\frac{1}{1-n} \int_n^1 P_t(j)^{1-\theta} dj \right]^{\frac{1}{1-\theta}} \end{cases} \tag{63}$$

are the home producer price index and the foreign producer price index expressed in

---

[11] Because foreign households have identical preferences, their consumption index C_t^* equals the right hand side of (60) with α substituted by α^*, C_{Ht} by C_{Ht}^*, and C_{Ft} by C_{Ft}^*. Moreover, $\alpha^* = n(1-\nu)$.

domestic currency, respectively, and

$$P_t = \left(\alpha P_{Ht}^{1-\eta} + (1 - \alpha) P_{Ft}^{1-\eta} \right)^{\frac{1}{1-\eta}} \tag{64}$$

is the consumer price index in the home country. This implies that total consumption expenditure by domestic households is $P_t C_t$.[12]

We can now specify the period budget constraint

$$P_t C_t + \mathrm{E}_t \left\{ \Lambda_{t,t+1} S_{t+1} B_{t+1} \right\} \leq W_t L_t + S_t B_t + \Pi_t - T_t, \tag{65}$$

where we rule out Ponzi schemes. Here B_t is the value in foreign currency of a portfolio of a full set of state-contingent assets held at the beginning of period t, reflecting our complete markets assumption, $S_t = \exp(s_t)$ is the nominal exchange rate in level form, $\Lambda_{t,t+1}$ is the stochastic discount factor making $\mathrm{E}_t \left\{ \Lambda_{t,t+1} S_{t+1} B_{t+1} \right\}$ the home-currency value at time t of the portfolio that yields a payoff in $t+1$, W_t is the nominal wage, Π_t is nominal firm profits transfered to households, and T_t is lump-sum taxes.

As usual, the first-order conditions consist of the optimality condition regarding the intratemporal consumption-leisure trade off

$$C_t^\sigma L_t^\gamma = \frac{W_t}{P_t} \tag{66}$$

and the intertemporal optimality relation linking the stochastic discount factor to the intertemporal marginal rate of substitution in Euler equation

$$\Lambda_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}}, \tag{67}$$

for all possible states of nature at times t and $t+1$. Note that $\mathrm{E}_t \Lambda_{t,t+1}$ is the value of a portfolio that yields one unit of the domestic currency in $t+1$ (mimicing a riskless domestic bond), so that the interest rate is $i_t = -\log(\mathrm{E}_t \left\{ \Lambda_{t,t+1} \right\})$. Given i_t, prices, and the budget constraint, the (expectational) Euler equation determines C_t.

A.2 Firms

---

[12]Similar expressions hold for the foreign country, for both demand and prices. Foreign demand follows from (62) by substituting α, C, and the four P symbols by α^*, C^*, and P^*, respectively. The home producer price index in foreign currency P_{Ht}^* and the foreign producer price index (in foreign currency) P_{Ft}^* follow from the right hand sides of (63) by substituting $P_t(j)$ by $P_t^*(j)$. The foreign consumer price index P_t^* (in foreign currency) equals the right hand side of (64) with α substituted by α^*, P_{Ht} by P_{Ht}^*, and P_{Ft} by P_{Ft}^*.

Firms use labor supplied by the households and a linear technology. Hence, output of the domestic firm that produces variety j is

$$Y_t(j) = A_t L_t(j), \tag{68}$$

where A_t is labor productivity, which is common across firms (within a country). Because of a labor subsidy τ, financed by taxes T_t, marginal cost is

$$MC_t = (1 - \tau) W_t / A_t, \tag{69}$$

which is independent of output and thus common across firms. The firm sells its good in a monopolistically competitive market with free international trade. Profits are

$$\Pi_t(j) = (P_t(j) - MC_t) Y_t(j). \tag{70}$$

The firm sets the price in a sticky fashion a la Calvo (1983). That is, each date with probability ω the firm is not allowed to change its price. When the firm is allowed to set a new price $P_t^{opt}(j)$, it will do so optimally, that is, by maximizing the current market value of the profits resulting while that price remains in place. Suppose the new price holds until $t + k \geq t$. Let $Y_{t+k|t}(j)$ denote total demand $C_{t+k}(j) + \frac{1-n}{n} C_{t+k}^*(j)$ evaluated at $P_t^{opt}(j)$. The firm's objective function is therefore

$$\max \sum_{k=0}^{\infty} \omega^k \mathrm{E}_t \left\{ \Lambda_{t,t+k} \left(P_t^{opt}(j) - MC_{t+k} \right) Y_{t+k|t}(j) \right\}. \tag{71}$$

To derive the first-order condition, first note that (62) and its foreign counterpart imply that $\partial Y_{t+k|t}(j) / \partial P_t^{opt}(j) = -\theta Y_{t+k|t}(j) / P_t^{opt}(j)$. Moreover, other home firms face the same optimization problem, so that all domestic firms will choose the same new price $P_{Ht}^{opt} = P_t^{opt}(j)$. The price can be solved from the first-order condition

$$\sum_{k=0}^{\infty} \omega^k \mathrm{E}_t \left\{ \Lambda_{t,t+k} Y_{t+k|t} \left[P_{Ht}^{opt} - \frac{\theta}{\theta - 1} MC_{t+k} \right] \right\} = 0. \tag{72}$$

A.3   Equilibrium

World equilibrium requires that asset, labor, and goods markets are in equilibrium. Consider period t.

A.3.1 Asset market

As for the home country, market completeness implies there is also a unique stochastic discount factor for foreign-currency payoffs, which is $\Lambda^*_{t,t+1} = \beta \left(C^*_{t+1}/C^*_t \right)^{-\sigma} P^*_t/P^*_{t+1}$ for all possible states of nature at times t and $t+1$. Given free international trade in assets, arbitrage yields the asset market equilibrium relation

$$\Lambda_{t,t+1} = \Lambda^*_{t,t+1} \frac{S_t}{S_{t+1}}, \tag{73}$$

which is a stochastic version of uncovered interest parity.

Put differently, substituting the expressions for $\Lambda_{t,t+1}$ and $\Lambda^*_{t,t+1}$ shows that the model has the familiar perfect risk sharing relation between home and foreign households[13]

$$C_t = C^*_t Q_t^{1/\sigma}, \tag{74}$$

where

$$Q_t = S_t P^*_t / P_t \tag{75}$$

is the real exchange rate.

A.3.2 Labor market

Labor market equilibrium at home and abroad requires

$$\begin{cases} L_t &= \frac{1}{n} \int_0^n L_t\left(j\right) dj \\ L^*_t &= \frac{1}{1-n} \int_n^1 L^*_t\left(j\right) dj. \end{cases} \tag{76}$$

A.3.3 Goods market

Goods market equilibrium consists of two parts. First, frictionless trade results in the law of one price. So for each variety $j \in [0,1]$ the price set by the producer in its currency implies that the price in the foreign currency fulfills

$$P_t\left(j\right) = S_t P^*_t\left(j\right). \tag{77}$$

For the producer price indices this yields $P_{Ht} = P^*_{Ht} S_t$ and $P_{Ft} = P^*_{Ft} S_t$. Still, home bias implies $\alpha > \alpha^*$, so that in general for the consumer price index $P_t \neq P^*_t S_t$, meaning a deviation from purchasing power parity.

---

[13]We assume symmetric initial conditions. Without this, the right-hand side of the equation should be extended by a constant factor. That would end up as an additive constant in the exchange rate equation of interest (45) and be irrelevant for our analysis. So the symmetry assumption is innocuous.

The second part of goods market equilibrium is the markets for all varieties clear:

$$
\begin{cases}
Y_t(j) &= C_t(j) + \frac{1-n}{n}C_t^*(j), & \text{for home \quad varieties} \\
Y_t^*(j) &= \frac{n}{1-n}C_t(j) + C_t^*(j), & \text{for foreign varieties.}
\end{cases}
\tag{78}
$$

For the home-varieties line, substitute the top demand function of (62) for $C_t(j)$ and its foreign counterpart (as explained in footnote 12) for $C_t^*(j)$. For the foreign-varieties line, we do the same, but now using the bottom demand function of (62). This yields

$$
\begin{cases}
Y_t(j) &= \left(\frac{P_t(j)}{P_{Ht}}\right)^{-\theta}\left(\frac{P_{Ht}}{P_t}\right)^{-\eta}\left[\alpha C_t + \frac{1-n}{n}\alpha^*\left(\frac{1}{Q_t}\right)^{-\eta}C_t^*\right], & \text{for home \quad v.} \\
Y_t^*(j) &= \left(\frac{P_t^*(j)}{P_{Ft}^*}\right)^{-\theta}\left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta}\left[\frac{n}{1-n}(1-\alpha)Q_t^{-\eta}C_t + (1-\alpha^*)C_t^*\right], & \text{for foreign v.}
\end{cases}
\tag{79}
$$

Substituting these into the definitions of aggregate output

$$
\begin{cases}
Y_t &= \left[\frac{1}{n}\int_0^n Y_t(j)^{\frac{\theta-1}{\theta}}dj\right]^{\frac{\theta}{\theta-1}} \\
Y_t^* &= \left[\frac{1}{1-n}\int_n^1 Y_t^*(j)^{\frac{\theta-1}{\theta}}dj\right]^{\frac{\theta}{\theta-1}}
\end{cases}
\tag{80}
$$

gives the following expressions for aggregate output:

$$
\begin{cases}
Y_t &= \left(\frac{P_{Ht}}{P_t}\right)^{-\eta}\left[\alpha C_t + \frac{\alpha^*(1-n)}{n}\left(\frac{1}{Q_t}\right)^{-\eta}C_t^*\right] \\
Y_t^* &= \left(\frac{P_{Ft}^*}{P_t^*}\right)^{-\eta}\left[\frac{n}{1-n}(1-\alpha)Q_t^{-\eta}C_t + (1-\alpha^*)C_t^*\right].
\end{cases}
\tag{81}
$$

A.4 Taking the limit $n \to 0$ to obtain the small economy

To mimic the small open economy we take the limit $n \to 0$. This implies $\alpha \to \nu$ and $\alpha^* \to 0$. The limiting CPIs resulting from (64) become

$$
\begin{cases}
P_t = \left(\nu P_{Ht}^{1-\eta} + (1-\nu)P_{Ft}^{1-\eta}\right)^{\frac{1}{1-\eta}} \\
P_t^* = P_{Ft}^*,
\end{cases}
\tag{82}
$$

and the limiting values of aggregate output in (81) are

$$
\begin{cases}
Y_t = \left(\frac{P_{Ht}}{P_t}\right)^{-\eta}\left[\nu C_t + (1-\nu)\left(\frac{1}{Q_t}\right)^{-\eta}C_t^*\right] \\
Y_t^* = C_t^*.
\end{cases}
\tag{83}
$$

A.5 Steady state

Here we compute the symmetric zero-inflation efficient steady state of the model. All variables refer to the values in that steady state. Similar values apply to the foreign

country, unless explicitly stated otherwise.

Given that all shocks are set to zero, productivity is constant over time, denoted by A. We assume constant wage W, and our symmetry assumption means that $A = A^*$ and $W = W^*$. Marginal cost becomes $MC = (1 - \tau) W/A$. The firm's first-order condition (72) then yields that each firm chooses price $P_H^{opt} = W/A$, so that the labor subsidy τ as given in Table 1 renders the steady state efficient. The producer price index is thus also constant $P_H = W/A$, as is real marginal cost $MC/P_H = (\theta - 1)/\theta$. Similarly, the foreign producer price index is $P_F^* = W^*/A^*$. For simplicity, we assume that these indices are equal when expressed in the same currency, that is, $P_H = SP_F^* = P_F$. So the consumer price index $P = P_H$, showing that the steady state exhibits zero inflation. As $P^* = P_F^*$, the real exchange rate $Q = 1$, so that PPP holds in the steady state.

International risk sharing (74) then implies $C = C^*$, and goods market equilibrium (83) yields $Y = C$. Given constancy of C, Euler equation (67) shows that the stochastic discount factor is $\Lambda = \beta$. Because of the consumption-leisure condition (66) and $P = W/A$, we obtain $L = (Y^{-\sigma} A)^{1/\gamma}$. Because all firms j charge the same price, output per firm is the same across varieties and equal to Y. Labor market equilibrium (76) then yields $L = Y/A$. Therefore, $Y = A^{(1+\gamma)/(\sigma+\gamma)}$.

References

Benigno, G., Benigno, P., 2008. Exchange rate determination under interest rate rules. Journal of International Money and Finance 27 (6), 971–993.

Benigno, G., Benigno, P., Ghironi, F., 2007. Interest rate rules for fixed exchange rate regimes. Journal of Economic Dynamics and Control 31 (7), 2196–2211.

Born, B., Juessen, F., Müller, G. J., 2013. Exchange rate regimes and fiscal multipliers. Journal of Economic Dynamics and Control 37 (2), 446–465.

Calvo, G. A., 1983. Staggered prices in a utility-maximizing framework. Journal of Monetary Economics 12 (3), 383–398.

Calvo, G. A., Reinhart, C. M., 2002. Fear of floating. The Quarterly Journal of Economics 117 (2), pp. 379–408.

Clarida, R., Galí, J., Gertler, M., 1998. Monetary policy rules in practice Some international evidence. European Economic Review 42 (6), 1033–1067.

Corsetti, G., Müller, G. J., 2015. Fiscal Multipliers: Lessons from the Great Recession for Small Open Economies. Report to the Swedish Fiscal Policy Council 2015/2, Swedish Fiscal Policy Council.

Daniëls, T. R., Jager, H., Klaassen, F., 2011. Currency crises with the threat of an interest rate defence. Journal of International Economics 85 (1), 14–24.

Danmarks Nationalbank, 2016. Monetary and exchange-rate policy. `www.nationalbanken.dk/en/monetarypolicy/implementation`, accessed: April 26, 2016.

De Paoli, B., 2009. Monetary policy and welfare in a small open economy. Journal of International Economics 77 (1), 11–22.

Engel, C., 2014. Exchange rate stabilization and welfare. Annual Review of Economics 6 (1), 155–177.

Engel, C., West, K. D., 2005. Exchange Rates and Fundamentals. Journal of Political Economy 113 (3), 485–517.

Frankel, J., Xie, D., 2010. Estimation of De Facto Flexibility Parameter and Basket Weights in Evolving Exchange Rate Regimes. American Economic Review 100 (2), 568–572.

Galí, J., 2008. Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework. Princeton University Press.

Galí, J., Monacelli, T., 2005. Monetary Policy and Exchange Rate Volatility in a Small Open Economy. Review of Economic Studies 72 (3), 707–734.

Ghosh, A. R., Ostry, J. D., Chamon, M., 2016. Two targets, two instruments: Monetary and exchange rate policies in emerging market economies. Journal of International Money and Finance 60, 172–196.

Girton, L., Roper, D., 1977. A Monetary Model of Exchange Market Pressure Applied to the Postwar Canadian Experience. American Economic Review 67 (4), 537–48.

He, D., Ng, P., Zhang, W., 2011. How do we monitor fund flows and foreign exchange market pressures in Hong Kong? HKMA Research Letters 1, Hong Kong Monetary Authority.

Hong Kong Monetary Authority, 2009. Annual Report. Hong Kong.

IMF, 2014. Annual Report on Exchange Arrangements and Exchange Restrictions. Washington.

Jovanovic, B., Petreski, M., 2014. Monetary policy, exchange rates and labor unions in SEE and the CIS during the financial crisis. Economic Systems 38 (3), 309–332.

Justiniano, A., Preston, B., 2010. Monetary policy and uncertainty in an empirical small open-economy model. Journal of Applied Econometrics 25 (1), 93–128.

Klaassen, F., Jager, H., 2011. Definition-consistent measurement of exchange market pressure. Journal of International Money and Finance 30 (1), 74–95.

Krugman, P. R., 1991. Target zones and exchange rate dynamics. The Quarterly Journal of Economics 106 (3), 669–682.

Lubik, T. A., Schorfheide, F., 2007. Do central banks respond to exchange rate movements? A structural investigation. Journal of Monetary Economics 54 (4), 1069–1087.

Mohanty, M., 2013. Market volatility and foreign exchange intervention in EMEs: what has changed? An overview. In: Bank for International Settlements (Ed.), BIS Papers. Vol. 73. Bank for International Settlements, pp. 1–10.

Monacelli, T., 2004. Into the Mussa puzzle: monetary policy regimes and the real exchange rate in a small open economy. Journal of International Economics 62 (1), 191–217.

Obstfeld, M., 1994. The logic of currency crises. Cahiers Économiques et Monétaires 43, 189–213.

Sims, C. A., 2002. Solving Linear Rational Expectations Models. Computational Economics 20 (1-2), 1–20.

Weymark, D. N., 1995. Estimating exchange market pressure and the degree of exchange market intervention for canada. Journal of International Economics 39 (3-4), 273 – 295.