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Abstract

Contingent convertible capital (CoCo) is a debt instrument that converts to equity or is written off if the issuing bank fails to meet a distress threshold. The conversion increases the issuer's loss-absorption capacity, but results in wealth transfers between CoCo holders and shareholders, which may change risk-shifting incentives to shareholders. Higher risk increases the probability of CoCo conversion, while lowering the wealth transfer. We show that for Principal-Write-Down (PWD) CoCos, the net effect is to always increase risk-shifting incentives, while for equity-converting CoCos, it depends on the extent of dilution after conversion. We integrate the analysis in a game-theoretic optimal capital regulation framework and show that use of PWD or insufficiently dilutive CE CoCos requires higher capital requirements for given asset structure to offset the rising risk-shifting incentives these instruments give rise to.

1 Introduction

This paper analyzes the risk-shifting incentives that arise from letting banks issue contingent convertible capital (CoCo) to meet capital requirements set by regulators and explores the implications for the design and setting of optimal capital requirements. CoCos are hybrid instruments that are issued as debt but convert to equity or are written off if the issuing bank fails to meet a distress threshold. The threshold may be contractual, as when the bank fails to meet a preset equity ratio, or discretionary, as when regulators deem the bank to be close to an often vaguely-defined point of non-viability. CoCos are designed this way in order to relieve the issuer of the burden of raising capital in situations of financial distress (Flannery

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[2005]). As a result, CoCos have become popular with regulators because of their enhanced loss absorption capacity relative to subordinated debt. But we show in this paper that many CoCos (in fact the majority of all CoCo structures presently used) have worse risk-shifting incentives than subordinated debt, so the loss absorption capacity is bought at a price. We analytically present the link between design features and implied risk-shifting incentives compared with debt (lower capital ratios) or equity (higher quality capital) as alternative to CoCos. We then embed the analysis of risk-shifting incentives in a stylized optimal capital requirements model and show that at the social optimum the use of (insufficiently dilutive) CoCos to meet capital requirements calls for higher capital requirements as a function of their design structure. And this holds in particular for the use of Principal Write Down CoCos, irrespective of the write down percentage used. This runs counter to the current structure of the BIS standards, where capital requirement ratios are based on risk weighted assets only.

Key to our conclusions is the fact that while CoCo conversion increases the loss absorption capacity of the issuing bank, conversion may, depending on their design, also change the order of seniority. If CoCos are written off (Principal Write Down CoCos) or CoCo holders receive insufficient number of shares after conversion, CoCo holders absorb the first losses, instead of the original shareholders. This implies that at the moment of conversion, there is a wealth transfer from CoCo holders to the shareholders after a distress event has set of the conversion trigger. If CoCos are converted to equity (equity converting CoCos), CoCo holders absorb the losses together with the existing shareholders. In this case, the wealth transfer may be in favor of either the CoCo holder or the existing shareholder, depending on the terms of the conversion. These transfers do not exist when the bank uses subordinated debt. Regardless of the direction of the wealth transfer, a wedge is created relative to the use of subordinated debt, which changes the bank's risk-shifting incentives and therefore should have consequences for the design of the capital requirements regime.

Our contribution to the literature is to provide a simple theoretical model of risk-shifting in the presence of CoCos, when the conversion is automatic (based on a breach of a preset equity ratio). We use the Merton [1974] call options framework to value various types of claims on the bank's assets model (debt, CoCos, equity), enabling us to endogenize the probability of CoCo conversion in a straightforward manner, rather than treating it as exogenously given as is done in part of the literature. While a number of papers have given examples of the risk-shifting that may potentially be caused by CoCos (see for instance Berg and Kaserer [2015], Chen et al. [2017], Hilscher and Raviv [2014], Koziol and Lawrenz [2012]), the quoted papers have to rely on numerical simulation methods, which unavoidably implies that the results are conditional on particular parameter choices. Of course relying on numerical methods allows these studies to analyze much more complex models than we can in our simple but analytically tractable framework. But relying on numerical simulations instead of on a fully analytical approach may mask dependency of results on specific parameters chosen and is likely to provide less insight in the economic intuition behind the simulation results. In contrast, using our simplified call options framework yields completely analytical solutions, where the dependency of results on specific parameter values can be shown analytically. The key simplification is in the time structure: we use a simple discrete time set up where all events happen at specified times, i.e. a European option framework in discrete time. The existing asset pricing literature on CoCos uses continuous time and analyzes the conversion as a random stopping time problem, essentially replacing our European calls and puts by analogous knock in/knock out barrier option. Our approach allows for more generality, admittedly at the expense of simplification of the basic structure. This also allows us to decompose the sources of risk-shifting: an increase in the conversion probability of a given CoCo, and a decrease in the wealth transfer relative to issuing subordinated debt. We think this paper shows the benefits of complementing the largely simulation based literature by a simplified but analytically tractable approach.

In addition, our paper is the first to consider the interaction of CoCo issuance with the existing regulatory framework and to derive implications for the optimal structure and setting of capital requirements of the emergence of these new instruments. We do so by embedding the risk-shifting analysis into a game theoretic approach to the setting of optimal capital requirements, with the regulator acting as a Stackelberg leader and the banks as Stackelberg followers. Recent regulatory changes have allowed the use of CoCos to improve loss absorption capacity, but do not limit the types of CoCos that may be used to meet capital requirements. As CoCos create a wedge relative to subordinated debt because of the expected wealth transfers, replacing subordinated debt with CoCos changes the incentives of the bank. We show that this implies that, depending on their design, CoCos may foil the regulator's risk management intentions by changing the bank's risk choices - unless accompanied by an offsetting change in the capital requirements.

We apply our framework to the full range of CoCos issued so far: principal writedown (PWD) CoCos, which are not well-covered in the academic literature but widely issued (about 60% as of the first quarter of 2017), and convert-to-equity (CE) CoCos with dilutive and non-dilutive conversion ratios. While there is no question about the superiority of additional equity over subordinated debt from a regulatory perspective, the wedge brought about by the risk-shifting incentives embedded in CoCos matters in determining whether CoCos are or are not superior to subordinated debt. We find that PWD and insufficiently dilutive CE CoCos encourage banks to take riskier choices relative even to subordinated debt and thus even more relative to equity, this is because the wealth transfer that takes place when conversion occurs is always favoring the shareholders when CoCos are PWD or insufficiently dilutive CE CoCos. This reversal of seniority that takes place in the case of such insufficiently dilutive CoCos actually strengthens rather than weakens risk-shifting incentives, even when they replace subordinated debt. Obviously, allowing equity to be replaced by CoCos designed in this way makes matters much worse, to the extent that not raising the capital requirements and allowing subordinated debt to be issued to fill the gap would be a better option. But when the CoCos are of the dilutive CE variety, the risk-shifting incentive turns negative because the wealth transfer itself becomes negative - while shareholders in aggregate obtain a higher residual equity upon conversion, the old shareholders must share the total residual value (i.e. old and new claims) with the new shareholders created upon conversion. The dilutive sharing of residual equity, while not strictly equivalent to ex ante skin in the game, does make shareholders choose risk levels that make the conversion probability smaller. As a result, the risk level chosen under dilutive CE CoCos will be lower than the risk level chosen under the same amount of subordinated debt. We show that if the dilution is large enough, risk-shifting incentives can even be lower than in the case of pure equity. We then explore the consequences of these links between CoCo design and risk-shifting incentives for the optimal capital requirements structure by analyzing a Stackelberg game between the regulator as leader and the banks as followers. More equity to meet capital requirements leads to both higher loss absorption capacity and lower risk-shifting incentives, but when CoCos are used instead these two objectives are in conflict with each other: more PWD and insufficiently dilutive CoCos do increase loss absorption capacity but they simultaneously raise risk-shifting Incentives thereby opening up arbitrage opportunities (Boyson et al. [2016] make a similar point for the case of Trust Preferred Securities in the US). We show that when insufficiently dilutive CoCos and in particular PWD CoCos are used to meet capital requirements, these requirements should be raised to offset the increased risk-shifting incentives these instruments lead to.

The remainder of this paper is structured as follows. Section 2 discusses the related literature. We present the model in Section 3 and analyze risk-shifting incentives as a function of CoCo design features in Section 4. Section 5 sets up the study of optimal capital requirements and CoCos by introducing private and social costs of bankruptcy and derives implications for the optimal structure and level of capital requirements once CoCos play an important role in the banking system's capital structure. Section 6 concludes. Some useful but standard option pricing results are listed in Appendix A, while the proofs that are not in the text are presented in Appendix B.

2 Related literature

There is a small but growing body of research on the impact of CoCos on the risk-shifting incentives of banks. Koziol and Lawrenz [2012] only consider CE CoCos, and argue that risk-shifting incentives always increase relative to ordinary bonds, as long as the old equity holder gets to keep *some* shares after conversion. This strong result depends critically on their assumption that the conversion trigger coincides with the default trigger: If asset values decline enough to trigger default at a particular leverage ratio, replacing some of the debt by CoCos will leave shareholders better off: with an equal decline in asset values they are left with some claims and default is staved off, while in the straight debt case they would have lost everything. Berg and Kaserer [2015] numerically simulate the value of equity given an exogenously set mixture of debt and equity converter CoCos for four specific conversion ratios as a function of asset return variance. They argue that risk-shifting rises as wealth transfers from CoCo holders to equity holders increase, and observe that the price at which conversion takes place has a direct impact on the magnitude and even sign of these wealth transfers. They also show that several of the existing CoCos such as those issued by Lloyds and Rabobank have prices that *fall* with changes in implied asset volatility, inferring that the market recognizes the risk taken by the banks This finding points at very clear risk-shifting incentives inherent in the CoCo designs issued by those two banks. Hilscher and Raviv [2014] argue that risk-shifting incentives of banks may be mitigated by choosing the conversion ratio properly. None of these studies analyzes PWD CoCos or the interaction between CoCos as capital and optimal capital requirements. For a capital structure containing CoCos, Hilscher and Raviv [2014] derive conversion ratios at which the resulting equity vega[1] is equal to zero. Their conclusion echoes the suggestion of Calomiris and Herring [2013] on having CoCos which are sufficiently dilutive. On the other hand, Martynova and Perotti [2016] claim that both CE and PWD CoCos can mitigate risk-shifting if the trigger level is set properly. In their paper, risk-shifting takes the form of not exerting sufficient effort in monitoring the assets of the bank. However they do not consider the possibility that the bank's risk choice affects the probability of conversion and the interaction of that effect with the wealth transfer that takes place upon conversion. Accounting for the latter link is at the core of the analysis presented in this paper.

CoCo papers that are cast in an asset pricing framework come in all varieties: for example De Spiegeleer and Schoutens (2011) provide two models based on an credit derivative and equity derivative approach respectively. Authors using asset pricing models mostly do endogenize the conversion by approaching it as a stopping time problem in continuous time. Among the previously mentioned papers, Berg and Kaserer [2015] and Koziol and Lawrenz [2012] fall within this category, as do Albul et al. [2013] and in particular Chen et al. [2017]. Both Albul et al. [2013] and Chen et al. [2017] focus on CoCos that convert to equity and show that they have the potential to reduce the probability of bankruptcy since they increase loss absorption capacity. They argue that for this to happen the trigger level for CoCo conversion must be sufficiently high in order to prevent debt-induced collapse of the value of equity. However, they also show that once this has been prevented, the presence of CoCos brings about tax benefits and possibly lower rollover costs of debt due to the lower probability of bankruptcy, and argue that these features unambiguously imply that CoCos mitigate risk-shifting incentives. Albul et al. [2013] argue that debt overhang effects will cause cbanks to resist any

[1] Vega is the sensitivity of the option value with respect to the volatility of its underlying assets.

regulation imposing issuance of CoCos. Moreover, most of the structural papers only consider CoCos that convert to equity, such that there is partial dilution of existing shareholders. As a consequence, conversion in the cases they analyze always imply a loss to old shareholders. But of the more than 162.6 billion Euro face value CoCos issued by European banks as of the first quarter of 2017, substantially more than half are issued on terms that imply a wealth transfer towards equity holders once conversion takes place, a possibility that plays a substantial role in our paper. Finally none of the papers in the literature discuss the impact of swapping Cocos for straight debt or equity on the objectives regulators pursue through capital requirements and the consequences for optimal capital requirements.

3 Revisiting the call options approach to residual equity valuation

Black and Scholes [1973] and Merton [1974] have noted that the shareholders of a firm effectively hold a call option on their company's assets. The creditors of the firm have a claim over the assets to the extent of the outstanding liability, but the shareholders can obtain the full claim to the assets by paying off all outstanding liabilities. Therefore the residual claim held by the shareholders can be thought of as a call option on the firm's assets, with the outstanding liability as the strike price.

For a bank that has issued hybrid instruments such as CoCos, the valuation of its residual equity is more involved, because the change in the hybrid's "state" changes the bank's capital structure. This will in general imply a corresponding change in the valuation of the residual equity. Therefore, the valuation of residual equity involving hybrids must take the various states of the hybrid security into account.

If the probability of conversion was exogenous, valuation would be straightforward: the residual equity value of a CoCo-issuing bank can then simply be expressed as a linear combination of the residual equity values before conversion (when the CoCo is treated as debt) and after conversion (when the CoCo is either written off or is converted to equity), with the conversion probability as the weighting factor. However, CoCos convert whenever the bank encounters either an automatic or a discretionary trigger, but, either way, after "bad news" (see in particular Chan and van Wijnbergen [2015] for the signaling effect of conversions). This in turn means that expected values conditional on conversion will be different for the same asset compared to expected values conditional on no conversion having taken place, a point we explicitly incorporate in our analysis. Also, the bank's ability to choose risk levels affects the shape of the return distribution, which in turn affects the likelihood of the bank hitting the trigger value for its capital ratio. Therefore, we cannot assume that the probability of conversion is exogenous.

By expressing the bank's residual equity as a call option, and by recognizing that the probability of CoCo conversion is affected by the risk levels chosen by the bank, we are able to examine the risk-shifting incentives the CoCo-issuing bank faces. Moreover, using the method outlined above, we can examine each type of CoCo design and determine which of them provides the best and the worst incentives for risk-shifting.

3.1 Setup

CoCos have two kinds of trigger: an automatic one which occurs whenever the bank fails to meet a preset equity ratio, and a discretionary one which occurs whenever the regulator believes the bank has reached the so called point of non-viability. In this paper, we focus on the automatic type.

A model with CoCos must have at least three dates because the risk choice, the conversion itself, and the final payoffs happen at distinct dates. However, if one wants to determine the ex ante risk-shifting incentives induced by a CoCo, it is enough to know the impact of risk on the expected realizations of the asset value at the time of conversion. Therefore, while we refer to $t = 1$ and $t = 2$ events , our analysis focuses on the $t = 0$ actions.

Consider a CoCo-issuing bank. At $t = 0$, its capital structure is composed of D_d deposits, D_s CoCos, and E initial equity. We assume that the CoCo does not convert at $t = 0$. At this stage, the CoCo-issuing bank is indistinguishable from an ordinary bank with D_s subordinated debt in place of CoCos. We normalize the amounts such that $D_d + D_s + E = 1$. We take these amounts as given, because we are interested in seeing how banks choose risk for a given capital structure. Since banks face capital regulation, the bank is anyhow constrained in choosing its capital structure.

Upon obtaining these funds, the bank invests them in an asset that gives return R_2 at $t = 2$. We assume that R_2 follows a lognormal distribution with parameters $\left(\mu, \sigma^2\right)$ for the corresponding normal distribution of $\ln(R_2)$. The bank can choose the risk level σ of the assets at $t = 0$. However, once the bank has chosen σ, it cannot make changes at any later time. Call the return expected at $t = 0$ R: $R = \mathbb{E}_0(R_2)$. Also, to ensure that we analyze a pure risk effect not confounded with increases in wealth, we structure the increase in risk in such a way that $\mathbb{E}(R_2) = R$ stays unchanged (i.e. a mean-preserving spread in variance to offset the impact of Jensen's inequality). Furthermore without loss of generality we assume investors have a zero discount rate and are risk neutral; risk choices are driven by leverage, not by risk aversion. Because of these assumptions, we can interpret R as the $t = 0$ price of the risky asset.

The setup described above allows us to write the equity holder's claim as a call option on the asset return, as in Black and Scholes [1973] and Merton [1974]. We assume there is only one share, and the bank does not issue any new shares aside from those that may arise from CoCo conversion. Denote the value of the share at $t = 0$ as e_0. Thus, before conversion, the bank's residual equity may be expressed as

$$e_0 \quad = \quad C\left[R,\ D_d + D_s\right] \tag{1}$$

where $C[R,\ D]$ is a call option[2] on an asset with current price (value) R and strike price (outstanding liabilities) D in this particular instance. In all subsequent calculations, we use D to refer to a general strike price, but specify the actual level of debt (e.g. D_d or $D_d + D_s$) when appropriate.

At $t = 1$, a signal is received drawn from the same marginal distribution as R_2 but assumed correlated to R_2, i.e. the signal is informative about R_2. As a consequence the asset price changes to R_1. In principle it is possible that R_1 falls short of the total liability $D_d + D_s$. In that case the bank is in default and will be closed down. However, we only consider cases when conversion precedes default, so we assume that the $t = 1$ signal will always be high enough to preclude anticipations of insolvency.

CoCos convert at $t = 1$ when R_1 is lower than what is consistent with a preset trigger equity ratio τ. At $t = 2$, when R_2 materializes, the creditors of the bank are paid, and anything left accrues to the residual claimant, which is the equity holder of the bank. In principle it is also possible to draw conclusions from risk choices for run probabilities: for such an analysis in a global games framework, see Chan and van Wijnbergen [2015]. But in this paper we assume there is deposit insurance and therefore no risk of depositor runs in order to focus entirely on the risk-shifting implications of various CoCo designs. CoCos cannot be run on since they are non-redeemable.

3.2 The endogenous conversion probability

It is straightforward to value residual equity when D_s is subordinated debt. But when CoCos are involved, we need to consider both the change in the value of the residual equity arising from the change in the outstanding liability once conversion takes place, as well as the probability that the CoCo converts. Martynova and Perotti [2016] treat this probability as exogenous. However, this is not a desirable assumption for our purpose since the bank's choice of risk affects the distribution of the asset returns and thus unavoidably the probability of conversion. In this section, we endogenize this probability, using an analog on to the concept of distance-to-default, the distance to conversion.

The distance-to-default is a measure of the closeness of the gross asset return and the value of the outstanding liability. For lognormally distributed asset returns R and total face value of debt D, the distance-to-default d_d at $t = 0$ (hence the use of R instead of R_1) can in the context of our model be written as

$$d_d = \frac{1}{\sigma}\left[\ln\frac{R}{D} + r - \frac{\sigma^2}{2}\right] \tag{2}$$

where r is the risk-free rate. It is implicit from the use of this measure that the default event occurs when the equity ratio of the bank is 0. However, with CoCos, the relevant event is not default, but conversion. For

[2]Appendix A reproduces the basic mathematics of the Black-Scholes call options framework.

CoCos with automatic conversion, the trigger event is when the bank's equity ratio falls short of the trigger level $\tau > 0$. We therefore introduce a measure similar to distance-to-default by incorporating the trigger level τ, the distance-to-conversion d_c.[3] Automatic conversion occurs whenever

$$\frac{R-D}{R} \leq \tau \Leftrightarrow R(1-\tau) \leq D, \tag{3}$$

allowing us to write the distance-to-conversion d_c as

$$d_c = \frac{1}{\sigma}\left(\ln \frac{R\,(1-\tau)}{D} + r - \frac{\sigma^2}{2}\right). \tag{4}$$

With the assumption of lognormally distributed returns, the conversion probability then equals:

$$p^c = \Phi\left(-d_c\right) \tag{5}$$

where $\Phi\left(\cdot\right)$ is the cumulative standard normal distribution. With the conversion probability now well-defined, we can value the equity of a bank that has issued CoCos within our framework as a properly probability weighted linear combination of values of residual equity with differing amounts of outstanding liability, the approach taken in the next subsection. But first consider the properties of the relation between distance to conversion, probability of conversion, risk σ and the trigger level τ.

As d_c is a function of both τ and σ, the probability of conversion p^c must be as well. We have

$$\frac{\partial p^c}{\partial \tau} = -\phi\left(-d_c\right)\,\frac{\partial d_c}{\partial \tau} = \phi\left(-d_c\right) \times \left(\frac{1}{\sigma\left(1-\tau\right)}\right) > 0 \tag{6}$$

and

$$\frac{\partial p^c}{\partial \sigma} \quad = \quad -\phi\left(-d_c\right)\,\frac{\partial d_c}{\partial \sigma} = \phi\left(-d_c\right) \times \left(1 + \frac{d_c}{\sigma}\right) > 0 \tag{7}$$

where $\phi\left(\cdot\right)$ is the standard normal distribution. This leads to the following lemma:

Lemma 1. *The conversion probability is increasing in the risk σ taken, as well as in the trigger ratio τ .*

The intuition behind this result is relatively simple: A higher value of σ shifts more weight in the tail and so increases the conversion probability, which equals the left tail of the distribution falling below the trigger value τ. A higher trigger value reduces the distance to conversion and so also increases the probability of

[3] A similar measure has been introduced by Chan-Lau and Sy [2006], in the context of an early warning system for bank regulators.

conversion d_c. The fall in the distance-to-conversion induced by both of these factors, combined with the derivative of the cumulative standard normal distribution with respect to this parameter, deliver the lemma.

From Lemma 1 one can see that the trigger ratio τ and the risk level σ are substitutes in terms of their effect on the conversion probability. If one takes the cross partial derivative of (7) with respect to τ, one obtains

$$\frac{\partial^2 p^c}{\partial\tau\partial\sigma} = \frac{\phi\left(-d_c\right)\left(1-\tau\right)\left[\sigma d_c \frac{\partial d_c}{\partial\sigma} - 1\right]}{\sigma^2\left(1-\tau\right)^2} < 0, \tag{8}$$

which shows that the marginal conversion probability with respect to risk σ falls as the trigger ratio τ rises (see Appendix B.1 for details on this derivation). By Young's theorem, the marginal conversion probability with respect to the trigger ratio τ also falls as the risk level σ rises. This leads to following corollary:

Corollary 2. *The risk level σ and the trigger ratio τ are substitutes in terms of their effect on the conversion probability.*

Corollary 2 suggests that if the bank has a target level of the probability of conversion, it can choose lower risk levels if the trigger ratio is high enough. Similarly, if the trigger ratio is low, the bank can achieve its target by choosing higher risk levels.

3.3 Residual equity valuation with CoCos in the capital structure

In this section, we consider the valuation of residual equity when CoCos are in the capital structure. The two states (pre- and post-conversion) must be considered in the valuation. To this end, we examine how conversion alters the issuing bank's residual equity.

There are two types of CoCos that have been issued to date: principal writedown (PWD) CoCos and convert-to-equity (CE) CoCos. PWD CoCos are written off by the fraction $(1-\varphi) \in [0,\ 1]$ when the bank runs into an automatic trigger event, leaving φD_s subordinated debt on the issuing bank's balance sheet. So conversion would change the value of the bank's residual equity from $C\left[R,\ D_d + D_s\right]$ to $C\left[R,\ D_d + \varphi D_s\right]$, for any value of R, where φ represents the fraction of the CoCos that are retained on the balance sheet.[4]

CE CoCos are not written down but convert to equity at some conversion rate ψ per unit of CoCo when the issuing bank encounters an automatic trigger event.[5] So conversion would change the bank's (old) residual equity from $C\left[R,\ D_d + D_s\right]$ to $\frac{1}{1+\psi D_s}\left(C\left[R,\ D_d\right]\right)$. It is important to notice that the overall equity is now held by two equity owners, the old one and the new equity holder who has obtained his equity from the CoCo conversion. $\frac{1}{1+\psi D_s}$ measures the degree of dilution. This formula also holds for any value of R. Of course if

[4] For this reason, we refer to φ as the retention parameter.

[5] Some papers refer to the conversion price, which is the inverse of the conversion rate. That is, for conversion rate ψ the conversion price is $1/\psi$.

an adverse signal has set off an triggering event, the signal has likely affected the share price R adversely, an issue we return to below.

Both the writeoff and the equity conversion features can be accommodated by the expression in (9) to represent a general CoCo-issuing bank's residual equity after conversion.

$$\frac{C\left[R,\ D_d+\varphi D_s\right]}{1+(1-\varphi)\psi D_s} \tag{9}$$

This formula embeds all possibilities:

1. A pure CE CoCo has $\varphi = 0$, and $\psi > 0$, $\psi \in [0,\ \infty)$

2. A pure PWD CoCo with full writedown has $\varphi = 0$ and $\psi = 0$. As such, it can be seen as a limiting case of a CE CoCo with lim $\psi \to 0$; and

3. A pure PWD CoCo with partial writedown has $\varphi > 0$, $\varphi \in (0,\ 1)$ and $\psi = 0$ or

4. A CoCo with a mix of PWD and CE features has $\varphi > 0$, $\varphi \in (0,\ 1)$ and $\psi > 0$, $\psi \in [0,\ \infty)$. This kind of CoCo partially converts into shares and partially continues as subordinated debt

Note also that if the CoCo has $\psi = 0$ and $\varphi = 1$, we obtain regular subordinated debt instead of a CoCo. To date, none of the CoCos issued possess both equity conversion and principal write down characteristics at once: CoCos are either pure PWD or pure CE. Also, the two possible partial conversion CoCo types (3) and (4) are actually equivalent to a linear combination of regular subordinated debt, with weight φ, and a fully-converrted full PWD CoCo, with weight $(1-\varphi)$. So all results that we obtain for pure CE CoCos actually extend to the corresponding partial CoCo types: (1) extends to (4) and (2) extends to (3).

Denote by e_{coco} the value of a general CoCo-issuing bank's residual equity at $t = 0$. As previously mentioned, the $t = 0$ value of residual equity of a bank with CoCos in the capital structure can be written as a linear combination of the pre-conversion state and the post-conversion state, with the probability of conversion p^c as the weighting factor. We need some more definitions before we can implement this approach. Since at $t = 0$ it is not known whether the CoCo will be triggered but it is known that if it does the $t = 1$ signal must have been sufficiently bad for the trigger to go off, the conditional expectation of $t = 2$ asset value must be different in the pre-conversion and the post-conversion states. Define R^+ as the expected value of R conditional on there NOT having been a conversion: $R^+ = \mathbb{E}\left(R_2|\text{Without conversion at } t=1\right)$. Similarly we can define R^- as the expected value of R conditional on there having been a conversion: $R^- = \mathbb{E}\left(R_2|\text{With conversion at } t=1\right)$. Clearly $R = \mathbb{E}\left(R_2\right) = (1-p^c)R^+ + p^cR^-$. We can rewrite this expression as (10) which will be useful in the following sections.

$$\left(R - R^+\right) = p^c\left(R^- - R^+\right). \tag{10}$$

With this expression, we may write the CoCo-issuing bank's residual equity as:

$$
\begin{aligned}
e_{coco} &= p^c \, \frac{C\left[R^-,\ D_d+\varphi D_s\right]}{1+\psi D_s} + (1-p^c)\ C\left[R^+,\ D_d+D_s\right] && (11)\\
&= C\left[R^+,\ D_d+D_s\right] + p^c\left(\frac{C\left[R^-,\ D_d+\varphi D_s\right]}{1+\psi D_s} - C\left[R^+,\ D_d+D_s\right]\right)\\
&= C\left[R^+,\ D_d+D_s\right] + p^c\left(\frac{C\left[R^-,\ D_d+\varphi D_s\right]}{1+\psi D_s} - C\left[R^-,\ D_d+D_s\right]\right)\\
&\quad +p^c\left(C\left[R^-,\ D_d+D_s\right] - C\left[R^+,\ D_d+D_s\right]\right)\\
&\approx C\left[R,\ D_d+D_s\right] - \triangle(R-R^+) - p^c\triangle(R^+-R^-) + p^cW\\
&= C\left[R,\ D_d+D_s\right] - p^c\triangle(R^- - R^+) - p^c\triangle(R^+-R^-) + p^cW\\
&= C\left[R,\ D_d+D_s\right] + p^cW\\
e_{coco} &= e_0 + p^cW
\end{aligned}
$$

using (10) going from line 4 to line 5 of (11), and W is the wealth transfer as defined below.

$$
W = \frac{C\left[R^-,\ D_d+\varphi D_s\right]}{1+\psi D_s} - C\left[R^-,\ D_d+D_s\right] \tag{12}
$$

$\triangle$ is the option delta, the option derivative with respect to R.[6] Thus, the ex ante value of residual equity of a CoCo-issuing bank can be expressed as the value of a bank's residual equity as if it had issued subordinated debt instead of a CoCo, e_0, plus an expected value of the wealth transfer term p^cW.

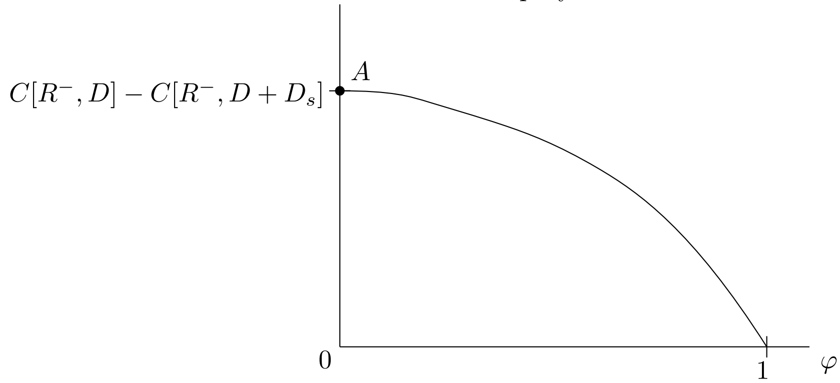
The expected wealth transfer may be positive or negative, depending on the values of ψ and φ. But for a PWD CoCo the expected wealth transfer p^cW_{pwd} is:

$$
p^cW_{pwd} = p^c\left(C\left[R^-,\ D_d+\varphi D_s\right] - C\left[R^-,\ D_d+D_s\right]\right), \tag{13}
$$

(13) is always positive because the lower implied strike price after conversion $(D_d+\varphi D_s)$ increases the value of the call option held by the bank's shareholder. Thus, the difference between $C\left[R^-,\ D_d+\varphi D_s\right]$ and $C\left[R^-,\ D_d+D_s\right]$ is always larger than 0, and increases as φ moves from 1 to 0, as the CoCo moves more to a full write down PWD CoCo. Figure 1 illustrates the change in the wealth transfer from the point of view of the bank shareholder. At Point A in the Figure, when $\varphi=0$, the wealth transfer from the CoCo holder to the existing shareholder is at its highest value. This is because at $\varphi=0$ nothing is left for the CoCo holder.

[6] In deriving (11) we make one approximation: we assume $\triangle$ to be constant over the domain of R, i.e. the second derivative $\triangle_R \approx 0$.

Figure 1: Wealth transfers from CoCo holders to equity holders for various levels of φ

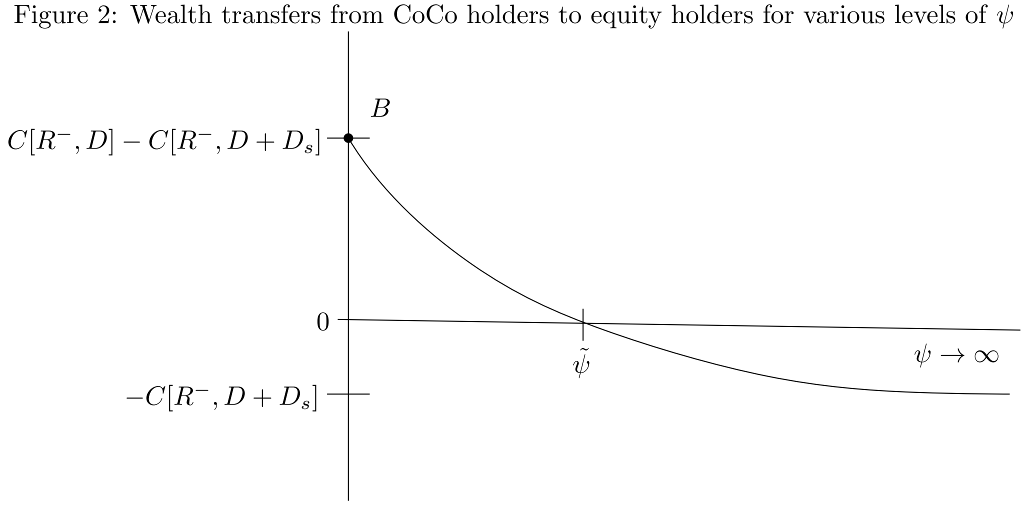


For a pure CE CoCo, the expected wealth transfer $p^c W_{ce}$ is

$$p^c W_{ce} = p^c \left(\frac{C\left[R^-,\ D_d\right]}{1 + \psi D_s} - C\left[R^-,\ D_d + D_s\right] \right), \tag{14}$$

which may be positive or negative over the the range of ψ, which is the entire positive segment of the real axis. Figure 2 illustrates this, again from the point of view of the original equity holder.

Figure 2: Wealth transfers from CoCo holders to equity holders for various levels of ψ



At Point B in Figure 2, when $\psi = 0$, the wealth transfer reaches its highest value. At this value of ψ, the CE CoCo is equivalent to a full PWD CoCo. However, as $\psi \to \infty$ the CoCo holder completely dilutes the original shareholder, at $\psi = \infty$ the original shareholder is wiped out altogether after a conversion. So then the wealth transfer is from the original shareholder to the CoCo holder. As the wealth transfer term W_{ce} is continuous in ψ, there must exist a value of ψ that sets the wealth transfer of a CE CoCo exactly equal to

0. Call this value $\bar{\psi}$; $\bar{\psi}$ solves $W_{ce}(\psi) = 0$. This expression allows for an explicit solution for $\bar{\psi}$:

$$\bar{\psi} = \frac{1}{D_s}\left(\frac{C\left[R^-,\ D_d\right]}{C\left[R^-,\ D_d + D_s\right]} - 1\right), \tag{15}$$

which is positive, since $C\left[R^-,\ D_d\right] > C\left[R^-,\ D_d + D_s\right]$. At $\bar{\psi}$, the number of new shares $\bar{\psi}D_s$ valued at the pre-conversion value of $C\left[R^-,\ D_d + D_s\right]$ is just equal to the difference in the values of residual equity pre- and post-conversion: $C\left[R^-,\ D_d\right] - C\left[R^-,\ D_d + D_s\right]$.[7] We refer to ψ as the dilution parameter: any value of $\psi < \bar{\psi}$ leads to a wealth transfer from the CoCo holder to the shareholder. And any value of $\psi > \bar{\psi}$ leads to a wealth transfer from the shareholder to the CoCo holder. Only at $\psi = \bar{\psi}$ is there a neutral conversion in the sense of a zero wealth transfer after a conversion. Note that $\psi > 0$ since ${\partial C}/{\partial D} > 0$. This will become relevant in the next section.

4 The risk-shifting incentives induced by CoCos

In the previous section, we have shown that conversion of a PWD CoCos always involves a positive wealth transfer from the CoCo holder to the shareholders, but that both the direction and the magnitude of CE CoCo wealth transfers upon conversion vary with the dilution parameter ψ. Since risk levels of the bank's assets influence the probability of conversion, one should expect these wealth transfers to have an impact on the risk choices banks make given their capital structure, so that is the choice we turn to now. In this section we analyze how the *marginal* incentives to take on additional risk change when a bank replaces a given amount of subordinated debt D_s by an equal face value amount of CoCos, and how this change in incentives depend on the design features of the CoCo issued. To that end we take the derivative of the expected wealth transfers with respect to σ. This is because the expected wealth transfer measures the impact on residual equity of replacing a given amount of subordinated debt with an equivalent amount of CoCos. In effect, we are looking at the differential effect of CoCos on a bank's risk-making decisions, with subordinated debt as the benchmark. Initially, in Section 4.1 we derive the expression for risk-shifting incentives from the expected wealth transfer arising from CoCo conversion. We focus on CE CoCos first as we can show that everything else is a special case of such a CoCo. In Section 4.2, we focus on the impact of different parameters: the dilution parameter ψ, the retention parameter φ and the trigger level τ on the risk-shifting incentives. Focusing on CE CoCos first allows us to work through the rest of the types of CoCos in an organized manner.

[7] Calomiris and Herring [2013] has a similar discussion and the recommendation to use a conversion price closely related to our definition of $\bar{\psi}$. Also, this price is critical according to Sundaresan and Wang [2015] if multiple equilibria are to be avoided in the case of market-based (share price) conversion triggers.

4.1 The risk-shifting incentive defined

Consider a pure CE CoCo as described in Section 3. We write the value of residual equity of a bank that has replaced D_s subordinated debt by issuing a CoCo as

$$e_{ce} \quad = \quad e_0 + p^c \left(\frac{C\,[R^-,\ D_d]}{1+\psi D_s} - C\left[R^-,\ D_d + D_s\right] \right). \tag{16}$$

The differential valuation effect of issuing a CE CoCo to retire the same amount of subordinated debt is given by the expected wealth transfer term $p^c W_{ce}$:

$$p^c W_{ce} = e_{ce} - e_0 = p^c \left(\frac{C\,[R^-,\ D_d]}{1+\psi D_s} - C\left[R^-,\ D_d + D_s\right] \right). \tag{17}$$

Define now the risk-shifting incentive of such a bank as RSI_{ce}. This term is the derivative of $p^c W_{ce}$ with respect to σ, as shown in (18):

$$RSI_{ce} \quad = \quad \underbrace{\frac{\partial p^c}{\partial \sigma} \left(\frac{C\,[R^-,\ D_d]}{1+\psi D_s} - C\left[R^-,\ D_d + D_s\right] \right)}_{CF_{ce}} + \underbrace{p^c \left(\frac{V\,[R^-,\ D_d]}{1+\psi D_s} - V\left[R^-,\ D_d + D_s\right] \right)}_{WF_{ce}}, \tag{18}$$

where we have used the vega notation to denote the derivative of a call with respect to σ. RSI_{ce} has two components, the conversion probability factor (CF_{ce}) and the wealth transfer factor (WF_{ce}). CF_{ce} represents the increase in the probability of conversion as risk increases, holding the wealth transfer constant. WF_{ce} represents the change in size of the wealth transfer as risk increases, holding the conversion probability constant.

4.2 Effect of design parameters on risk-shifting incentives

4.2.1 Risk-shifting incentives as a function of the dilution parameter ψ

Consider first the conversion probability factor CF_{ce}, reproduced in (19):

$$CF_{ce} = \frac{\partial p^c}{\partial \sigma} \left(\frac{C\,[R^-,\ D_d]}{1+\psi D_s} - C\left[R^-,\ D_d + D_s\right] \right). \tag{19}$$

CF_{ce} also has two components, the derivative of the conversion probability with respect to σ, and the wealth transfer itself. From Lemma 1, we know that $\frac{\partial p^c}{\partial \sigma} > 0$. The sign of CF_{ce} then depends on the sign of the wealth transfer. For CE CoCos, this is determined by the expression derived in (15), which is the ψ that sets wealth transfers equal to zero. For CE CoCos with $\psi < \bar{\psi}$, the wealth transfer to equity holders is always positive. On the other hand, for CE CoCos with $\psi > \bar{\psi}$, the wealth transfer is always negative. Therefore,

$CF_{ce} < 0$ only if $\psi > \bar{\psi}$, and the other way around. This also means that $CF_{ce} < 0$ when $\psi \to \infty$, as this value of ψ is greater than $\bar{\psi}$.

But what about the wealth transfer factor WF_{ce}? This equals:

$$WF_{ce} = p^c \frac{\partial}{\partial \sigma} \left(\frac{C\left[R^-,\ D_d\right]}{1+\psi D_s} - C\left[R^-,\ D_d + D_s\right] \right). \tag{20}$$

WF_{ce} represents the impact of the risk level on the value of the wealth transfer itself, for a given probability of conversion. From the above section, we know that the value of the wealth transfer varies as ψ moves from 0 to ∞. When $\psi = 0$, WF_{ce} reduces to $WF_{ce} = p^c \frac{\partial}{\partial \sigma}\left(C\left[R^-,\ D_d\right] - C\left[R^-,\ D_d + D_s\right]\right)$ and we can express it as the difference between the vegas[8] of two call options that differ only in the strike price. That is,

$$WF_{ce}(\psi = 0) = p^c \left(V\left[R^-,\ D_d\right] - V\left[R^-,\ D_d + D_s\right] \right) \tag{21}$$

where $V[\cdot]$ is the call option vega. As $V[\cdot]$ is continuously differentiable, we can use the mean value theorem to rewrite (21) as:

$$WF_{ce}(\psi = 0) = -p^c D_s\ V_D\left[R^-,\ D'\right] < 0, \tag{22}$$

where $D' \in [D_d,\ D_d + D_s]$ and V_D is the derivative of vega with respect to the strike price D, which we show in Appendix A.4 to be always positive, hence the inequality at the end of (22).

On the other hand, when $\psi \to \infty$, we have

$$WF_{ce}(\psi \to \infty) = -p^c V\left[R^-,\ D_d + D_s\right] < 0$$

as well, since $V[\cdot] > 0$. Therefore, WF_{ce} is negative given any value of risk and leverage. The intuition behind this is that a conversion increases the issuing bank's skin in the game, making risk-shifting less attractive. So we are left with a potential ambiguity: an increase in the risk σ increases RSI_{ce} through the conversion factor, and decreases RSI_{ce} through the wealth transfer factor. To arrive at an unambiguous conclusion regarding the net effect of CF_{ce} and WF_{ce}, we first analyze the impact of changes in the dilution parameter ψ to RSI_{ce} as a whole.

Note that ψ has no impact on the probability of conversion for given σ, since it does not influence the distance to conversion. So the derivative of RSI_{ce} with respect to ψ simply equals:

$$\frac{\partial RSI_{ce}}{\partial \psi} = -\frac{D_s}{(1+\psi D_s)^2} \left(\frac{\partial p^c}{\partial \sigma} C\left[R^-,\ D_d\right] + p^c V\left[R^-,\ D_d\right] \right) < 0. \tag{23}$$

[8] Vega is the derivative of a call option with respect to σ.

When $\psi = 0$, we can write RSI_{ce} as follows, which has an ambiguious sign, using results from the previous section:

$$RSI_{ce}\left(\psi=0\right)=\underbrace{\frac{\partial p^{c}}{\partial\sigma}\left(C\left[R^{-},\ D_{d}\right]-C\left[R^{-},\ D_{d}+D_{s}\right]\right)}_{CF_{ce}>0}+\underbrace{p^{c}\left(V\left[R^{-},\ D_{d}\right]-V\left[R^{-},\ D_{d}+D_{s}\right]\right)}_{WF_{ce}<0}. \quad (24)$$

Also, when $\psi \to \infty$, we would have, at the limit,

$$RSI_{ce}\left(\psi\to\infty\right) = \underbrace{\frac{\partial p^{c}}{\partial\sigma}\left(-C\left[R^{-},\ D_{d}+D_{s}\right]\right)}_{CF_{ce}<0}+\underbrace{p^{c}\left(-V\left[R^{-},\ D_{d}+D_{s}\right]\right)<0}_{WF_{ce}<0}. \quad (25)$$

When $\psi \to \infty$, a CE CoCo unambiguously leads to lower risk-shifting incentives compared to having subordinated debt. This is because conversion leaves nothing for the original shareholder, so both CF_{ce} and WF_{ce} are negative, making for an overall negative impact on risk-shifting.

The above analysis leads to the natural question of whether there is a value for the dilution parameter for which risk-shifting incentives are exactly equal to zero. Call this value $\tilde{\psi}$. We obtain this value by setting (18) to 0 and solving for ψ. [9] The resulting expression for $\tilde{\psi}$ is

$$\tilde{\psi}=\frac{1}{D_{s}}\left(\frac{\frac{\partial p^{c}}{\partial\sigma}C\left[R^{-},\ D_{d}\right]+p^{c}V\left[R^{-},\ D_{d}\right]}{\frac{\partial p^{c}}{\partial\sigma}C\left[R^{-},\ D_{d}+D_{s}\right]+p^{c}V\left[R^{-},\ D_{d}+D_{s}\right]}-1\right). \quad (26)$$

From the definition of $\tilde{\psi}$ it follows that for any $\psi \in [0,\ \tilde{\psi})$ risk-shifting incentives are positive (i.e. worse than in the alternative capital structure with subordinated debt instead of CoCos), but for all $\psi \in [\tilde{\psi},\ \infty)$ the risk-shifting incentives are negative. So if the CE CoCo is sufficiently dilutive (meaning $\psi > \tilde{\psi}$), the CoCo replacing subordinated debt will actually improve (i.e. reduce) risk-shifting incentives:

What remains to be seen is whether $\tilde{\psi} > 0$, or, in words, whether sufficiently dilutive CoCos actually exist. With some manipulation we can rewrite $\tilde{\psi}$ in terms of $\bar{\psi}$ as follows, which means that indeed, $\tilde{\psi} > 0$ since $\bar{\psi} > 0$:

[9] This question is also posed in Hilscher and Raviv [2014], who find the conversion ratio that achieves zero vega. However they only consider the wealth transfer and the leverage channels, not the endogeneity of the conversion probability to the risk choice, which plays a crucial role in our analysis.

$$
\begin{aligned}
\tilde{\psi} &= \left(\bar{\psi} + \frac{\frac{V[R^-,\ D_d]}{C[R^-,\ D_d]} - \frac{V[R^-,\ D_d+D_s]}{C[R^-,\ D_d+D_s]}}{p^c/p^c_\sigma + V\left[R^-,\ D_d+D_s\right]/C\left[R^-,\ D_d+D_s\right]} \right) \\
&= \bar{\psi} + \frac{V\left[R^-,\ D_d\right]/C\left[R^-,\ D_d\right]}{p^c/p^c_\sigma + V\left[R^-,\ D_d+D_s\right]/C\left[R^-,\ D_d+D_s\right]} \left[1 - \frac{V\left[R^-,\ D_d+D_s\right]/V\left[R^-,\ D_d+D_s\right]}{C\left[R^-,\ D_d+D_s\right]/C\left[R^-,\ D_d\right]} \right] \quad (27) \\
&= \bar{\psi} + \frac{1}{D_s} \frac{V\left[R^-,\ D_d\right]/C\left[R^-,\ D_d\right]}{p^c/p^c_\sigma + V\left[R^-,\ D_d+D_s\right]/C\left[R^-,\ D_d+D_s\right]} \left[1 - \frac{V_D\left[R^-,\ D_d+D_s\right]D'}{C_D\left[R^-,\ D_d+D_s\right]D''} \right],
\end{aligned}
$$

where D' and D'' are both within the interval $[D_d,\ D_d+D_s]$. The expression beside $\bar{\psi}$ is positive because $V_D > 0 > C_D$. This is actually a strong result. First of all it shows that insufficiently dilutive CoCos do exist, i.e. all CoCos with dilution parameters lower than $\tilde{\psi}$, which is a non-empty set since $\tilde{\psi} > 0$.

Lemma 3. *There exists a value $\tilde{\psi} > 0$ for the dilution parameter value ψ such that any CE CoCo with $\psi \in [0,\ \tilde{\psi})$ is insufficiently dilutive, and any CE CoCo with $\psi \in [\tilde{\psi},\ \infty)$ is sufficiently dilutive.*

Lemma 3 allows us to state a proposition regarding the risk-shifting incentives of CE CoCos relative to subordinated debt.

Proposition 4. *For any risk level σ and leverage D, replacing subordinated debt by a CE CoCo will lead to lower risk-shifting incentives for the old equity holder if the dilution parameter ψ is larger than $\tilde{\psi}$.*

Note that full writedown PWD CoCos are a special case of insufficiently dilutive CE CoCos: they induce the worst risk-shifting incentives out of all kinds of CoCos. That is, $\psi = 0$ is the minimum value within the interval $[0,\ \tilde{\psi})$. Therefore, the results from the insufficiently dilutive CE CoCos also carry over to the full writedown PWD CoCos as well. It is worth noting that full writedown PWD CoCos create the largest wealth transfer from CoCo holders to existing equityholders out of all the CoCos. The following proposition follows directly:

Proposition 5. *For any risk level σ and leverage D , replacing subordinated debt by a full write down PWD CoCo ($\psi = \varphi = 0$) will always lead to higher risk-shifting incentives*

This result takes on special relevance given the fact that more than 50% of all CoCos issued are in fact PWD CoCos. The higher loss absorption capacity that is bought by issuing all these CoCos may thus in fact increase rather than reduce the fragility of the financial system.

In fact (27) implies that at the zero wealth transfer point $\bar{\psi}$ risk-shifting incentives are also positive since $\bar{\psi} < \tilde{\psi}$. This is an intuitive result: we have seen that a higher risk makes for a smaller wealth transfer (albeit a more likely one), so the wealth transfer becomes negative if σ increases at the zero wealth transfer point, which benefits the old shareholder. So the dilution parameter should be set at a higher level than the one required to just effect a zero net wealth transfer when conversion takes place:

Proposition 6. *CE CoCos that induce just zero wealth transfers ($\psi = \bar{\psi}$) are insufficiently dilutive, and thus induce positive risk-shifting incentives for the issuing bank.*

4.2.2 Risk-shifting incentives as a function of the retention parameter φ

We have shown that the risk-shifting incentives for complete write down PWD CoCos are positive, but that proof only applied to the borderline case of retention parameter $\varphi = 0$, i.e. for complete write down PWD CoCos. But what is the impact on marginal incentives of the retention parameter φ itself? Intuitively one would surmise that partial PWD CoCos also increase risk-shifting incentives, but can we show that? In this section we show that the answer is yes.

To see this, note that a partial write down of a CoCo with retention parameter φ yields the identical outcome upon conversion that one gets from a a linear combination of φ subordinated debt and $(1-\varphi)$ of a full write down PWD Coco:

$$\begin{aligned} C\left[R^{-},\ D_d + \varphi D_s\right] &= C\left[R^{-},\ D_d\right] + \varphi\left(C\left[R^{-},\ D_d + D_s\right] - C\left[R^{-},\ D_d\right]\right) \\ &= (1-\varphi)\, C\left[R^{-},\ D_d\right] + \varphi\, C\left[R^{-},\ D_d + D_s\right] \end{aligned} \tag{28}$$

which means that the expected wealth transfer from such a CoCo will be

$$\begin{aligned} p^c W_{(\psi=0,\varphi>0)} &= p^c\left[(1-\varphi)\, C\left[R^{-},\ D_d\right] + \varphi\, C\left[R^{-},\ D_d + D_s\right] - C\left[R^{-},\ D_d + D_s\right]\right] \\ &= p^c\left[(1-\varphi)\, C\left[R^{-},\ D_d\right] - (1-\varphi)\, C\left[R^{-},\ D_d + D_s\right]\right] \\ &= (1-\varphi)\, p^c\left[C\left[R^{-},\ D_d\right] - C\left[R^{-},\ D_d + D_s\right]\right] \end{aligned} \tag{29}$$

which is merely $1-\varphi$ of the expected wealth transfers from full PWD CoCos. As such, it is easy to see that Proposition 5 holds, scaled by $1-\varphi$.

Corollary 7. *For any risk level σ and leverage D, replacing subordinated debt by a partial write down PWD CoCo ($\varphi > 0$, $\psi = 0$) will always lead to higher risk-shifting incentives. The risk-shifting incentives fall as φ rises, and disappears at $\varphi = 1$.*

The result is intuitive. We have shown that the RSI of full PWD CoCos ($\psi = \varphi = 0$) is positive. Note as well that for $\varphi = 1$ and $\psi = 0$, the CoCo is in fact subordinated debt, so $\varphi = 1$ implies that nothing changes at conversion. Therefore by construction $RSI(\varphi = 1)$, which measures the risk-shifting incentive with respect to what it is under subordinated debt instead of a CoCo, must be zero, so we have that $RSI(\varphi = 0) > RSI(\varphi = 1) = 0$. In Appendix B.2 we show that RSI($\Psi = 0, \varphi > 0$) > 0 follows, confirming Corollary 7.

4.2.3 Impact of changing the trigger levelτ on risk-shifting incentives

Consider next the impact of the trigger level τ on the risk-shifting incentives. Clearly the trigger level itself has no impact on the transfer that takes place when the CoCo is triggered, which means the effect is solely through the probability of conversion, not the wealth transfer. As before, the risk-shifting incentive is calculated by taking the derivative of the expected wealth transfer $p^c W$ with respect to σ, as shown in (30).

$$RSI = \frac{\partial p^c W}{\partial \sigma} = \frac{\partial p^c}{\partial \sigma} W + p^c \frac{\partial W}{\partial \sigma} \tag{30}$$

Since $\frac{\partial W}{\partial \tau} = 0$,differentiating the risk-shifting incentive with respect to τ leads to the following expression:

$$\frac{\partial RSI}{\partial \tau} = \frac{\partial^2 p^c}{\partial \sigma \partial \tau} W + \frac{\partial p^c}{\partial \tau} \frac{\partial W}{\partial \sigma}. \tag{31}$$

From Lemma 1, $\frac{\partial p^c}{\partial \tau} > 0$ while $\frac{\partial^2 p^c}{\partial \tau \partial \sigma} < 0$ follows from Corollary 2. The net effect must take the wealth transfers into consideration. For PWD and insufficiently dilutive CE CoCos, the wealth transfer is always positive, while the marginal effect of risk on the wealth transfer is negative. So raising the trigger level τ always reduces the risk-shifting incentives embedded in those CoCo designs.[10] This is a possible way of mitigating the ill effects of CoCos that were designed to favor the original shareholders. As for dilutive CE CoCos, the fact that $\frac{\partial^2 p^c}{\partial \sigma \partial \tau} < 0$ interacts with the negativity of the wealth transfer, such that the net effect is more ambiguous but in that case the CoCo will always be an improvement over subordinated debt from the point of risk-shifting incentives anyhow, making the impact of the trigger level less relevant.

Proposition 8. *For PWD and insufficiently dilutive CE CoCos, the risk-shifting incentive is decreasing in the trigger ratio τ. For sufficiently dilutive CE CoCos, the impact of τ depends on the size of the wealth transfer.*

This result supports the Basel III requirement of a trigger level of at least 5.125% or higher for a CoCo to qualify as Additional Tier 1 capital.

[10] Martynova and Perotti [2016] also find that increasing the trigger level induces the banks to exert more effort in order to stave off conversion. This is consistent with our result that risk-shifting incentives decline as the trigger level rises.

5 Socially and privately optimal risk choices and the structure of capital regulation

The goal of banking regulation is to protect the system from default externalities, and by extension, prevent the use of taxpayer money for bailout purposes. If there are private costs of default and and social costs that exceed the private costs, private choices on risk levels and capital structure will not be socially optimal.[11] With the menu of securities banks can issue restricted to debt and equity, the analysis is relatively straightforward. Kashyap and Stein [2004] for example show that private capital structure choices will involve excessive leverage when social costs of default exceed private costs, and derive optimal capital requirements that resemble what is in fact implemented in the current Basel III framework for capital regulation. In addition, they argue that the current Basel framework is excessively pro-cyclical and argue for the use of different risk curves in different phases of the business cycle. The reason why the resulting structure in Kashyap and Stein [2004] is relatively simple, with capital requirements exclusively dependent on asset side characteristics, is that with just debt and equity the regulator's dual objectives are in fact aligned. A regulator wants to make sure there is enough loss absorption capacity for when things go wrong (ex post) and wants to mitigate risk-shifting incentives to reduce the probability that they go wrong (ex ante incentives). Higher equity requirements reduce both problems: more capital implies higher loss absorption capacity but also decrease ex ante risk-shifting incentives. But this alignment of objectives may break down when CoCos are introduced in the menu of choices. CoCos of course increase loss absorption capacity but we have shown in the previous sections that they may in fact increase rather than mitigate risk-shifting incentives, thereby potentially confronting the regulator with a difficult tradeoff between his two objectives. In line with this observation, Boyson et al. [2016] show that low franchise value US banks use trust preferred securities to increase their risk exposure in spite of capital regulation and leverage constraints designed to rein in that very exposure.In this section we use a simple framework similar to theirs to derive the consequences for the structure of capital regulation of allowing CoCos to be counted as capital.

The starting point is that regulators cannot dictate the risk choice banks make but can only impose leverage limits (See Boyson et al. [2016] for a similar approach.[12]) We model the analysis as a Stackelberg game, where banks choose their portfolio risk subject to regulatory capital requirements, and, in line with the Stackelberg structure, the regulator hits a target level of default probability by choosing those capital requirements knowing the risk choice banks will then make. In the next section we derive what is in effect the

[11] There is of course a large literature on this point. Kashyap and Stein [2004] provide a particularly clear and simple exposition. VanHoose [2007] is a very informative survey on bank behavior and capital regulation.

[12] The published version of Boyson et al. [2016] omits the theory model for which we refer the reader to the working paper version NBER WP 19984

bank's reaction curve RC, its risk choice conditional given that it knows the regulator will choose a leverage level D (Section 5.1). We then show how that choice is affected if subordinated debt is replaced by CoCos of different design structures and trigger levels (Section 5.2). In Section 5.3 we show how the regulator, acting as a Stackelberg leader, chooses maximum leverage given the bank's reaction curve RC and show how that choice is (or rather should be) affected by the introduction of CoCos.

5.1 The bank's objective function for given leverage D

Expected default costs have two components: the actual costs of bankruptcy once it occurs, and the probability that default occurs. The bankruptcy costs may be reputational or legal in nature, and distinct from social costs such as contagion effects on other banks, or the social costs of taxpayer-funded bailouts. Call them X. We keep these costs exogenous to our analysis, in line with our use of a partial equilibrium framework. We do postulate that the public costs X_s exceed private costs X_p: $X_s > X_p$. Expected costs of default then equal $p_c X_i$, with $i \in (s,\ p)$. The probability of default p_d can be calculated in the same way as p_c, the probability of conversion, was calculated. We again use the concept of a distance-to-default measure as defined in (2) where we show it to be a function of both the variance σ^2 and leverage D. Trigger levels are set higher than Tier 1 capital requirements under Basel III rules, in line with the intention to let them act as bail-in instruments rather than bail-out instruments. As a consequence, the probability of default is distinct from and higher than the probability of conversion. That does not exclude that for a sufficiently low draw of R_1 at $t = 1$ there is a significant probability that both events will take place at $t = 2$, like happened during the Banco Popular collapse in Portugal (R. Smith [2017]). The literature on CoCos has paid more attention to the probability of default than to the probability of conversion, perhaps due to the emphasis on the loss-absorption capacity of CoCos. But Chen et al. [2017] and Hilscher and Raviv [2014] do endogenize the probability of conversion, in the same way we define and analyze it; both papers use structural credit risk models to analyze CoCos, like we do, with richer dynamics although with exclusive focus on CE CoCos. More importantly, neither they nor others in the literature on CoCos consider the interaction between risk choices and the bank's capital structure that is the main focus of this section and the next.

Let X_p represent the bank's private costs of default, and let p^d represent the bank's probability of default. The expected cost of default then equal $p^d X_p$.

$$\begin{aligned} p^d\left(\sigma^2,\ D\right) &\approx p^d\left(\bar{\sigma}^2,\ \bar{D}\right) + \frac{\partial p^d}{\partial \sigma^2}\left(\bar{\sigma}^2,\ \bar{D}\right)\sigma^2 + \frac{\partial p^d}{\partial D}\left(\bar{\sigma}^2,\ \bar{D}\right) D \\ &= \frac{1}{2}\sigma^2 b + cD. \end{aligned} \tag{32}$$

In what follows we will ignore higher order derivatives of p^c for analytical tractability. Expected (private) default costs become:

$$p^d(\sigma,\ D)\,X_p = \left(\frac{1}{2}\sigma^2 b + cD\right)X_p. \tag{33}$$

This parameterization reflects that a higher risk choice and a higher leverage ratio make default more likely.

The bank then chooses σ^2 to maximize the value of its residual equity net of expected default costs given the expected return R and given leverage D:

$$\max\ C\left[R,\ D\right] - p^d X_p = \max\ C\left[R,\ D\right] - \left(\frac{1}{2}\sigma^2 b + cD\right)X_p. \tag{34}$$

The bank maximizes (34) by choosing σ. In line with the Stackelberg structure of the game between bank and regulator that we adopt in the next section, we assume that the bank optimizes for given level of leverage D (i.e. we work conditional on a given capital structure): the bank anticipates that the regulator will mandate a maximum leverage ratio (minimum capital ratio) and optimizes given that capital requirement For a given D, the first-order conditions associated with (34) is

$$V\left[R,\ D|\sigma\right] = \sigma^* b X_p, \tag{35}$$

where the notation $V\left[R,\ D|\sigma\right]$means that the function $V\left[R,\ D\right]$ is evaluated at $\sigma = \sigma^*$. So we get:

$$\sigma^* = \frac{V\left[R,\ D|\sigma^*\right]}{bX_p}$$

The risk choice comes down to a trade off between higher option value when σ goes up $(V > 0)$ against a higher probability of default when σ rises $(b \geq 0)$.

5.2 Privately optimal bank risk levels under different capital structures

5.2.1 Subordinated Debt vs. Additional Equity

To set the stage, consider first the benchmark case where the bank's capital structure has D_d deposits, and $D_s + E$ initial equity at $t = 0$. Given this capital structure, the bank essentially holds a call option on the asset return R at a strike price of D_d, leading to an objective function of the form

$$\max C\left[R,\ D_d\right] - \left(\frac{1}{2}\sigma^2 b + cD_d\right)X_p \tag{36}$$

and the first-order condition

$$\sigma_e^* = \frac{V[R,\ D_d|\sigma_e^*]}{bX_p} \tag{37}$$

where σ_e^* represents the optimal risk level given the all equity financing of $R - D_d$ (the only liability is from deposits).

But when the bank's capital structure at $t = 0$ consists of D_d deposits, D_s subordinated debt, and only E initial equity, the strike price is $D_d + D_s$, leading to the objective function

$$\max\ C[R,\ D_d + D_s] - \left(\frac{1}{2}\sigma^2 b + c(D_d + D_s)\right) X_p. \tag{38}$$

The solution for the corresponding first order condition is σ_s^*, which is the optimal risk level with the higher debt level $D_d + D_s$:

$$\begin{aligned}
\sigma_s^* &= \frac{V[R,\ D_d + D_s|\sigma_s^*]}{bX_p} \\
&= \frac{1}{bX_p}\left[V[R,\ D_d|\sigma_e^*] + (V_\sigma|\sigma_e^*)(\sigma_d^* - \sigma_e^*) + (V_D|\sigma_e^*)\ D_s\right] \\
&= \sigma_e^* + \frac{(V_D|\sigma_e^*)\ D_s}{bX_p - (V_\sigma|\sigma_e^*)} > \sigma_e^*
\end{aligned} \tag{39}$$

This is true for positive bankrupcty costs bX_p and for negative V_σ. However, we show in Appendix A.3 that V_σ is always negative whenever $d_2 < 0 < d_1$, a condition that is more likely to hold for poorly capitalized banks (low ratios of asset value R to outstanding liabilities D) and for high volatility. Figure 3 illustrates the case:

Figure 3: Optimal Risk Choice of Banks when D_s is Additional Equity versus Subordinated Debt

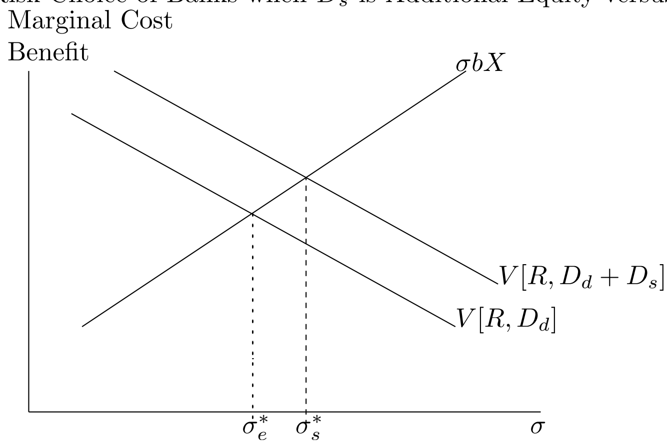


Figure 3 shows that the vega of a bank with D_s additional equity intersects the marginal cost line σbX_p

at a smaller value of σ compared to the vega of a bank with D_s subordinated debt. That σ_s^* is higher than σ_e^* reflects the higher risk-shifting incentives from issuing D_s subordinated debt relative to issuing the same amount of additional equity.

Proposition 9. *The optimal amount of risk σ_s^* that a bank takes with D_s subordinated debt is higher than the optimal amount of risk σ_e^* if the bank has issued D_s additional equity instead.*

This result is intuitive: the bank has more skin-in-the-game when it has issued more equity so it chooses lower risk levels.

5.2.2 PWD and CE CoCos instead of subordinated debt

When a bank issues D_s CoCos in place of the same amount of subordinated debt, the bank's objective function becomes

$$\max\ C\left[R,\ D_d + D_s\right] + p^c W - \left(\frac{1}{2}\sigma^2 b + c\left(D_d + D_s\right)\right) X_p \tag{40}$$

which is similar to (38) but with the expected wealth transfer term $p^c W$ added in. The accompanying first order condition is

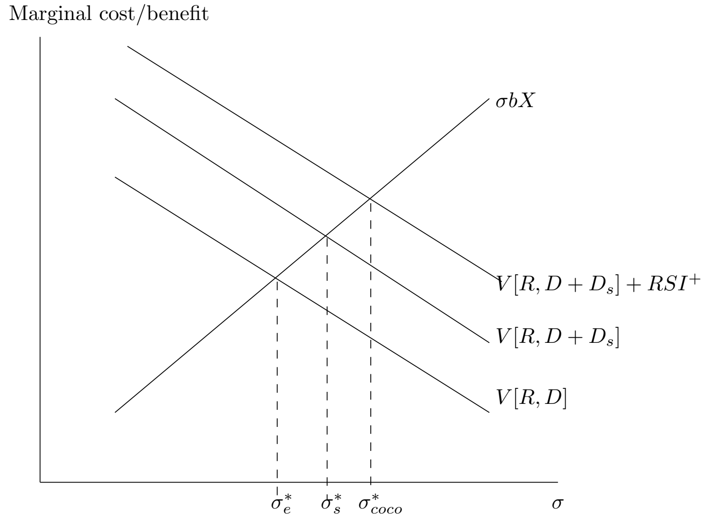
$$V\left[R,\ D_d + D_s\right] + RSI = \sigma_{coco}^* b X_p, \tag{41}$$

where RSI is the risk-shifting incentive (defined in Section 4) arising from the expected wealth transfer $p^c W$ and σ_{coco} is the optimal risk choice once subordinated debt D_s has been replaced by an equivalent amount of CoCos. It follows that the relative risk-shifting impact of swapping subordinated debt for a CE CoCo will depend on the dilution parameter ψ, since we have also seen in Section 4 that RSI depends on ψ. The sign and magnitude of RSI determines how much the bank's behavior will change relative to the subordinated debt case, and, as shown in Section 4, that sign depends on whether ψ is larger or smaller than the value at which $RSI = 0$, $\tilde{\psi}$. That is,

$$\sigma_{coco}^* > \sigma_s^* \Leftrightarrow \psi > \tilde{\psi}. \tag{42}$$

We have shown in Section 4 that PWD CoCos and insufficiently dilutive CE CoCos have positive risk-shifting incentives, while sufficiently dilutive CE CoCos have negative risk-shifting incentives. Therefore, for PWD CoCos and insufficiently dilutive CE CoCos, $V\left[R,\ D_d + D_s\right] + RSI$ must lie above that of $V\left[R,\ D_d + D_s\right]$. Similarly, $V\left[R,\ D_d + D_s\right] + RSI$ must lie below $V\left[R,\ D_d + D_s\right]$ for dilutive CE CoCos. Figure 4 below illustrates the first example, for insufficiently dilutive CoCos ($\psi < \tilde{\psi}$), and by extension, all PWD CoCos.

Figure 4: Optimal risk choices of banks with CoCos instead of subordinated debt: $\psi < \tilde{\psi}$



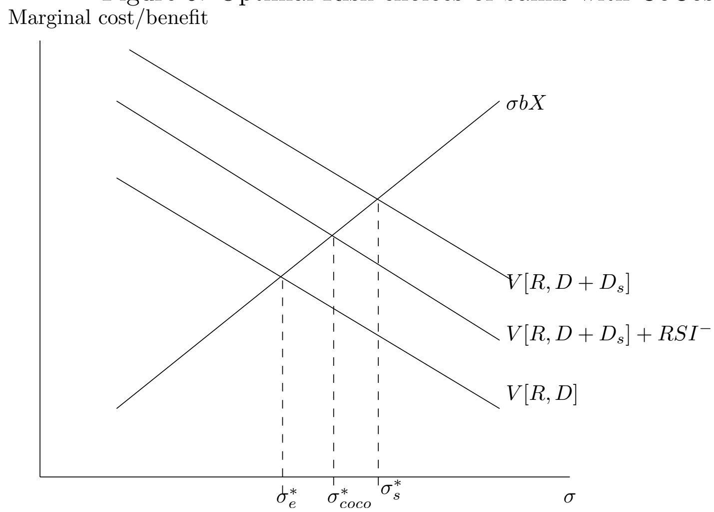
A priori this result holds for full write down PWD CoCos since they correspond to $\psi = 0 < \tilde{\psi}$, so we obtain the following proposition:

Proposition 10. *The optimal amount of risk that a bank takes with D_s worth of either PWD CoCos or insufficiently dilutive CE CoCos is higher than the optimal amount of risk if the bank has issued D_s subordinated debt.*

It is true that PWD CoCos improve loss absorption after conversion, and therefore meet the criteria for inclusion in Additional Tier 1 capital. However, as they elicit positive risk-shifting incentives before conversion, their use may make it more likely that the loss absorption capacity will be necessary in the future. Both Proposition 10 and Corollary 7 shows that RSI turns positive for PWD CoCos. On the other hand, insufficiently dilutive CoCos have a slight advantage over PWD CoCos in that they force the old shareholder to relinquish part of the residual equity resulting from a conversion, but under Proposition 4, if $\psi > \tilde{\psi}$, the old shareholder still finds it more attractive to make conversion more likely instead.

Figure 5 below depicts the other case, where CoCos *are* dilutive, i.e. $\psi > \tilde{\psi}$.

Figure 5: Optimal Risk choices of banks with CoCos instead of subordinated debt: $\psi > \tilde{\psi}$



RSI^+ and RSI^- indicate that the risk-shifting incentives are positive and negative, respectively.

Likewise, under Proposition 4, it is clear that sufficiently dilutive CE CoCos induce lower risk choices than the same amount of subordinated debt. As such, their inclusion as Additional Tier 1 capital is an improvement, but as they do not constitute skin in the game ex ante, they are still different from equity. Nonetheless, the threat of dilution effectively deters risk-shifting. We can summarize the results in the following proposition:

Proposition 11. *The privately optimal amount of risk that a bank takes with D_s sufficiently dilutive CE CoCos is lower than the (privately) optimal amount of risk if the bank would have issued D_s subordinated debt instead.*

5.2.3 Dilutive CoCos versus equity instead of subordinated debt

Thus far we have proven two sets of results, $\sigma_{ce}^* < \sigma_s^*$ with dilutive CE CoCos, and $\sigma_{ce}^* > \sigma_s^*$ otherwise. But can we determine how CE CoCos compare with straight equity in terms of risk choice? Post-conversion, dilutive CoCos and straight equity provide the same loss absorption capacity. But before conversion, the threat of a forthcoming dilution leads to less risk-shifting in order to stave off dilution in the case of dilutive CoCos. Against that effect is the fact that before conversion equity implies more skin in the game which leads to lower risk choices before conversion for the same amount of additional equity. Is it possible that the fear of dilution dominates the more skin in the game factor, which would cause such a CoCo to have even better characteristics than pure equity? Does there exist a dilution parameter high enough to trigger better risk-shifting incentives for CE CoCos relative to additional equity?

Recall from (37) that when D_s is equity, the strike price is D_d, so the first order condition is $V\left[R,\ D_d|\sigma_e^*\right] = \sigma_e^* bX_p$. From (41) we get for a CoCo instead of equity: $V\left[R,\ D_d + D_s|\sigma_{coco}^*\right] + RSI = \sigma_{coco}^* bX_p$.

If we decompose $V\left[R,\ D_d + D_s|\sigma_{coco}^*\right]$ in terms of σ_e and $V\left[R,\ D_d\right]$, we can rewrite the first order condition of a CE CoCo as

$$\begin{aligned} V\left[R,\ D_d|\sigma_e^*\right] + V_\sigma(\sigma_{coco}^* - \sigma_e^*) + (V_D|D_d)\ D_s + RSI &= (\sigma_{coco}^* - \sigma_e^*)\, bX_p + \sigma_e^* bX_p \\ \sigma_{coco}^* &= \sigma_e^* + \frac{(V_D|D_d)\ D_s + RSI}{bX_p - (V_\sigma|\sigma_e^*)} \end{aligned} \tag{43}$$

Thus, any ψ that sets $(V_D|D_d)\ D_s + RSI \geq 0$ makes the risk-shifting incentive of D_s CE CoCo smaller than or equal to the risk-shifting incentive for D_s additional equity, for equal loss absorption capacity after conversion.[13] Call this value ψ_{eq}. In particular, the condition for a CoCo to induce lower risk-shifting incentives than straight equity becomes:

$$\psi \geq \psi_{eq} = \frac{1}{D_s}\left(\frac{p^c V\left[R^-,\ D_d\right] + \frac{\partial p^c}{\partial \sigma} C\left[R^-,\ D_d\right]}{p^c V\left[R^-,\ D_d + D_s\right] + \frac{\partial p^c}{\partial \sigma} C\left[R^-,\ D_d + D_s\right] - \left(\frac{R^- \phi(d_1)}{D}\right)\left(\frac{d_1}{\sigma}\right) D_s} - 1\right). \tag{44}$$

Note that $\psi_{eq} > \tilde{\psi}$ (i.e. it is larger than the dilution parameter at which $RSI = 0$) because:

$$\frac{\partial p^c}{\partial \sigma} C\left[R^-,\ D_d + D_s\right] + p^c V\left[R^-,\ D_d + D_s\right] > \frac{\partial p^c}{\partial \sigma} C\left[R^-,\ D_d + D_s\right] + p^c V\left[R^-,\ D_d + D_s\right] - \frac{R^-}{D}\frac{d_1}{\sigma}\phi\left(d_1\right) D_s,$$

which obtains from the fact that at $\psi = \tilde{\psi}$, $RSI = 0$ and since RSI is decreasing in ψ, it must be that $\psi_{eq} > \bar{\psi}$.

This means that if the conversion ratio ψ of CE CoCos are super-dilutive (i.e. when $\psi \in [\psi_{eq},\ \infty)$), they are from a regulatory point of view even better than straight equity in terms of risk-shifting incentives.The following proposition thus holds:

Proposition 12. For $\psi \in \left[0,\ \tilde{\psi}\right]$, *we have* $\sigma_e^* < \sigma_s^* < \sigma_{ce}^*$ *For* $\psi \in \left[\tilde{\psi},\ \psi_{eq}\right]$ *we have* $\sigma_e^* < \sigma_{ce}^* < \sigma_s^* < \sigma_{pwd}^*$. *Finally, for* $\psi \in [\psi_{eq},\ \infty]$, *we get a strong result:* $\sigma_{ce}^* < \sigma_e^* < \sigma_s^* < \sigma_{pwd}^*$.

So when the CoCo is super-dilutive (i.e. $\psi > \psi_{eq}$), D_s CE CoCos provide lower risk-shifting incentive compared even to the same amount of additional straight equity, for equal loss absorption capacity. And even when they are not super-dilutive but still provide at least a zero wealth transfer to the old shareholder, they still perform better than either subordinated debt or PWD CoCos, in that they provide less risk-shifting incentives for the same loss absorption capacity as subordinated debt would. But if the CoCos are not dilutive

[13] Again assuming that V_σ is small enough to not reverse the sign of the denominator. This will certainly not happen whenever $d_2 < 0 < d_1$.

at all, they are worse than subordinated debt in that they provide even worse risk-shifting incentives for equal loss absorption capacity. In that case they should not be part of Additional Tier 1 capital.

5.2.4 Risk-shifting incentives and the trigger level for conversion

In the previous sections, we have already seen that an increase in τ reduces the distance-to-conversion, thereby increasing the conversion probability. However, it does not play a role in the probability of default. To see this, consider again the first order condition for a general CoCo, as in (41) relative to the one for subordinated debt, as in ((39)). This results in the following optimal risk choice:

$$\sigma^*_{coco} = \sigma^*_s + \frac{RSI}{bX_p - (V_\sigma|\sigma^*_s)} > \sigma^*_s \tag{45}$$

τ only plays a role in RSI. Therefore, taking the derivative of σ^*_{coco} with respect to τ is equivalent to looking at the sign of RSI's derivative with respect to τ:

$$\frac{\partial \sigma^*_{coco}}{\partial \tau} = \frac{1}{bX_p - (V_\sigma|\sigma^*_s)} \frac{\partial RSI}{\partial \tau}. \tag{46}$$

We already know from Corollary 8 that $\frac{\partial RSI}{\partial \tau} < 0$ for PWD and insufficiently dilutive CE CoCos, while the sign is ambiguous for dilutive CE CoCos. Therefore, holding everything else constant, an increase in the trigger ratio causes a decrease in the risk-shifting incentives of a bank that has issued either PWD or insufficiently dilutive CE CoCos.

Proposition 13. *Taking the probability of default into consideration, a bank that has issued PWD or insufficiently dilutive CE CoCos will lower its risk-shifting in response to a higher trigger ratio for conversion.*

5.3 Risk Choices, CoCos and capital regulation

Recent regulatory changes pushed CoCos to the frontline. From Basel III, CoCos now form part of Additional Tier 1 and Tier 2 capital for bank. This means that CoCos will comprise at most 3.5% out of the 8.0% minimum total capital. These regulations imply that CoCos will form a substantial portion of a bank's balance sheet in the near future, replacing subordinated debt to a large extent. However, as we have seen in the previous section, the replacement of subordinated debt with CoCos have implications on a bank's risk choices because of the expected wealth transfers.

In the previous subsections we analyzed the banks' risk choices given imposed capital requirements, as the first step in our Stackelberg set up of the game between a regulator and the banks it supervises. The next step is to analyze the regulator's choice of leverage given the reaction curves of the commercial banks. Since the regulator cannot control the banks' risk choices directly, we assume, in line with Boyson et al. [2016], that the regulator targets a desired default probability which it achieves by setting capital requirements assuming banks follow their reaction curve in response. Because the regulator is assured that the bank will comply with its mandates, we can indeed model the situation as a Stackelberg game: the regulator sets the target probability level knowing the bank's objective function, letting the bank react to those requirements by choosing its desired risk levels.

5.3.1 Setup

We have previously mentioned that the bank's expected costs of default are a function of both risk σ and leverage D, as in (33). This implies that for a target probability of default $\overline{p_d}$, there is a tradeoff between risk and leverage. The probability of default was defined in (32), and reproduced here.

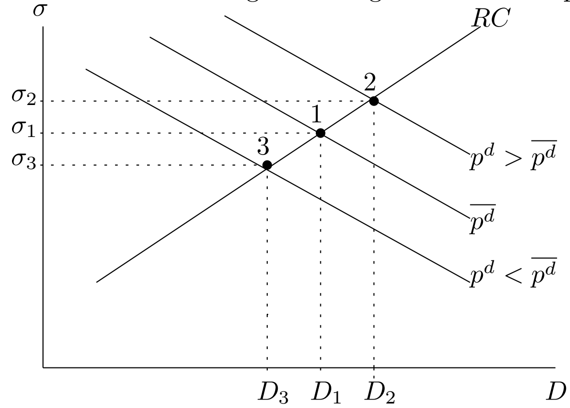
$$p^d = \frac{1}{2}\sigma^2 b + cD. \tag{47}$$

The regulator sets a target level of this probability, call that probability $\overline{p^d}$, very much in the way capital requirements and risk weights are determined under the successive Basel regimes. It follows from (47) that there is a tradeoff between risk σ and leverage D for a constant p^d: For a bank to comply with $\overline{p^d}$, any increase in σ must be compensated by a decrease in D and vice versa. By totally differentiating (47) and setting it to 0, we obtain the following negative slope:

$$\begin{aligned} 0 &= \sigma b \; d\sigma + c \; dD \\ \left.\frac{d\sigma}{dD}\right|_{\overline{p^d}} &= -\frac{c}{\sigma b}. \end{aligned} \tag{48}$$

The downward sloping line labeled $\overline{p^d}$ in Figure 6 illustrates the tradeoff between risk and leverage that this choice of a given default probability implies. Given $\overline{p^d}$, a bank can choose a higher σ if leverage D is lower and still maintain the probability of default at $\overline{p^d}$. A higher target default probability corresponds to an upward shift in the downward sloping line in Figure 6 and a lower probability a corresponding shift downwards of the $\overline{p^d}$ line.

Figure 6: A bank's Reaction Curve RC against a regulator's chosen probability of default



We turn now to the bank's reaction function. In Section 5.1 we have shown that there is a positive relationship between a bank's leverage and choice of risk levels, the bank's risk-shifting incentives increase with leverage. We can draw a reaction curve (RC) that shows the bank's best risk choice as leverage changes. RC can be interpreted as the reaction of the Stackelberg follower. The bank's first-order condition under a capital structure with D total debt is set forth in (35). By totally differentiating the bank's first-order condition, we obtain the condition that the bank must obey if it wants to maximize the value of its residual equity:

$$\begin{aligned} 0 &= V_D\left[R,\ D\right]\ dD + \left(-bX_p\right)\ d\sigma \\ \left.\frac{d\sigma}{dD}\right|_{RC} &= \frac{V_D\left[R,\ D\right]}{bX_p}, \end{aligned} \tag{49}$$

which is positive. See RC in Figure 6. The representation is very much simplified: we draw the curves as linear for presentational purposes only.

The regulator will set set capital requirements (leverage) D in addition to $\overline{p^d}$, which when combined with the bank's reaction curve, leads a bank to choose a particular level of σ. At issue then is how the Stackelberg leader (regulator) picks the right point off that curve by imposing that capital requirements, or equivalently in our set up, the maximum amount of leverage D. To a regulator, there is a tradeoff between risk and leverage if it wants to hit its target probability if default. Figure 6 shows how the system works. Imposing a maximum leverage D_3 implies that the chosen variance can be σ_2, if the target is $\overline{p^d}$. However, to a bank, risk and leverage reinforce each other, as reflected in the slope of the reaction curve. So for given requirement D, say a maximum leverage of D_3, it will choose a lower level of risk,: σ_3 in Figure 6. But this implies that the bank takes too little risk relative to that which is considered optimal by the regulator, as Point 3 lies on $p^d < \overline{p^d}$. Similarly, if the regulator imposes a maximum leverage D_2, the optimal risk from her viewpoint is $\sigma_3$if it wants to stay at the $\overline{p^d}$ line. But at D_2 the bank's RC implies that it will then choose σ_2, which is now too much risk compared to what the regulator deems optimal: Point 2 lies on $p^d < \overline{p^d}$. Only if the regulator imposes leverage D_1 will the bank choose a risk level σ_1 that is compatible with the $\overline{p^d}$ specified by the regulator, at the intersection of the $\overline{p^d}$ and RC lines:. So point 1 is the equilibrium solution to the Stackelberg game between the regulator and the bank. This example shows how the regulator keeps the bank's reaction curve in mind when setting capital requirements as its part in the Stackelberg game against the bank.

5.3.2 Replacing subordinated debt with CoCos

So we have seen that the regulator can achieve its desired level of default probabilities by choosing the maximum leverage ratio taking into account the bank's reaction curve, banks may undermine that target by introducing CoCos. To see that consider how the interaction between risk choice and leverage changes when CoCos that can be bailed in, written down, or converted to equity are introduced on the liability side. The ability to eliminate all or part of D_s changes a bank's reaction curve, meriting further attention. Consider now what happens when, possibly in response to the recent change in capital standards, subordinated debt is replaced by CoCos. In Section 5.2, we have shown that CoCos have risk-shifting incentives which differ from subordinated debt, because of the expected wealth transfers. Therefore, a CoCo-issuing bank's first order condition for a given debt D should also take the risk-shifting incentives into account stemming from the presence of CoCos on its balance sheet. The FOC then becomes:

$$V\left[R,\ D\right] + RSI = \sigma b X_p. \tag{50}$$

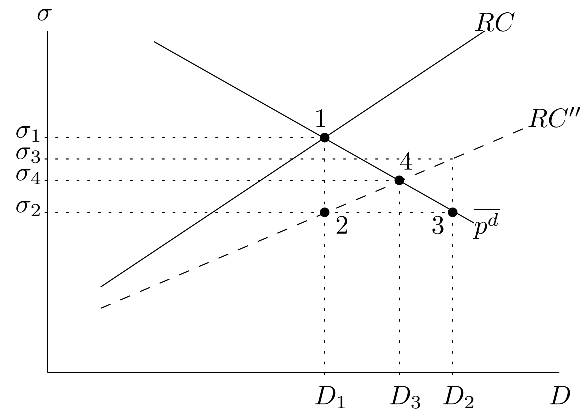
Clearly replacing subordinated debt by CoCos for example necessarily alters the reaction curve of a bank because of the additional RSI term, which involves both σ and D as well. If we totally differentiate RSI with respect to both parameters, we obtain

$$\begin{aligned} 0 &= \frac{\partial RSI}{\partial \sigma}\, d\sigma + \frac{\partial RSI}{\partial D}\, dD \\ \frac{d\sigma}{dD} &= -\frac{\frac{\partial RSI}{\partial D}}{\frac{\partial RSI}{\partial \sigma}}. \end{aligned} \tag{51}$$

For a CoCo with positive RSI (such as PWD and insufficiently dilutive CE CoCos), (51) is positive, because the risk-shifting incentive is increasing in leverage (less skin in the game implies higher gambling incentives) and decreasing in risk (diminishing marginal returns). Of course, for a CoCo with negative RSI (dilutive CE CoCos), (51) is negative.

Consider first PWD and insufficiently dilutive CE CoCos. Let RC' denote the reaction curve drawn using (50). Since the risk-shifting incentive is positive, the reaction curve RC' must lie above that of RC. Figure 7 represents the change simply as an upward twist in the slope.

Figure 7: Upward rotation of the Reaction Curve corresponding to replacing subordinated debt by risk-inducing CoCos



So suppose that the regulator has chosen the probability of default $\overline{p^d}$ and has imposed leverage D_1 on the banks, i.e. Point 1 in Figure 7, as in the benchmark case. Then, suppose for the sake of increasing loss absorption capacity, D_s subordinated debt is completely replaced with either a PWD or a insufficiently dilutive CE CoCo. This change causes the reaction curve to twist up from RC to RC'. As the bank did not change its leverage ratio, it still has D_1 leverage, but because of the potential wealth transfer brought about by the change from subordinated debt to equity, the risk incentives are higher: the bank's position is now at Point 2, where leverage is at D_1 but risk choice is at $\sigma_2 > \sigma_1$. What should the regulator do in this situation? At Point 2, the risk level σ_2 and leverage D_1 combination implies a probability of default which is higher than $\overline{p^d}$. To get back at $\overline{p^d}$ for risk level σ_2, she should impose higher capital requirements (lower leverage) D_2, as indicated in Figure 7 . But raising capital requirements by an additional $D_1 - D_2$ in turn leads to a lower risk choice of σ_3, which now implies a probability of default below $\overline{p^d}$, and so on. The new set of equilibrium values is at Point 4, with a higher risk choice than at Point 1 but a correspondingly larger loss absorption capacity because of the associated higher capital requirement.

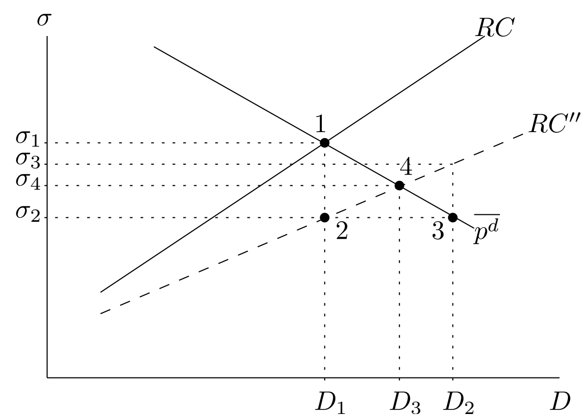
Proposition 14. *When PWD and insufficiently dilutive CE CoCos are used by banks in their capital structure in place of subordinated debt, regulators should increase capital requirements if they want banks to choose risk levels that are consistent with the regulators' own preference/stipulated default probability.*

The issuance of PWD and insufficiently dilutive CE CoCos to fulfill TLAC requirements causes banks to choose higher risk levels than would obtain if straight equity or even subordinated debt would have been chosen, and the regulator should impose correspondingly higher capital requirements. PWD and insufficiently dilutive CE CoCos therefore are poor substitutes for equity for compliance with TLAC requirements.

So given that subordinated debt only qualifies as Tier 2 capital under Basel III, it is arguable that PWD CoCos should not have been included as Additional Tier 1 equity regardless of the trigger level, because PWD CoCos lead to higher risk-shifting incentives. As conversion of a writedown CoCo wipes out a junior creditor, it allows the shareholder/manager to jump the seniority ladder. Therefore, they will not act in a safer manner even when compared with the case where these instruments are subordinated debt instead. Much of the CoCos issued between 2013 to 2015 have done just that, replace expiring subordinated debt.

The situation is better when dilutive CE CoCos are considered, because the movement of the expected wealth transfer is away from the shareholder to the CoCo holder. Relative to subordinated debt, the same amount of CoCos have an additional term, RSI. The RSI for CE CoCos fall as the dilution parameter ψ increases, and are negative for $\psi > \tilde{\psi}$. Therefore, combining (50) and (51) for a negative value of RSI, the RC twists downwards to some RC'' instead of upwards. Figure 8 shows this other case.

Figure 8: Downward rotation of the bank's Reaction Curve corresponding to replacing subordinated debt by dilutive CoCos



As with the other case, suppose that the regulator has chosen the probability of default $\overline{p^d}$ and has imposed leverage D_1 on the banks, i.e. Point 1 in Figure 8, as in the benchmark case. Then, suppose for the sake of increasing loss absorption capacity, D_s subordinated debt is completely replaced with a dilutive CoCo. This change causes the reaction curve to twist down from RC to RC''. The fall in the reaction curve for a given leverage D_1 actually causes the bank's risk choice to fall from σ_1 to σ_2, in contrast to if the reaction curve twists upwards. To reach Point 4 in Figure 8, the regulator actually has to lower capital requirements to induce banks to take the optimal level of risk given RC'' and $\overline{p^d}$, which is σ_4. Seen this way, dilutive CoCos are a legitimate component of Additional Tier 1 capital, because they induce banks to choose lower risk levels for a given leverage D.

Proposition 15. *When dilutive CE CoCos are used by banks in their capital structure instead of subordinated* *debt, regulators may decrease capital requirements if they want banks to choose risk levels that are consistent with the regulator's stipulated default probability.*

What Propositions 14 and 15 come down to is that just taking the asset side into account in setting capital requirements leaves banks arbitrage opportunities by using CoCos tailored to their own preferences. In particular using PWD or insufficiently dilutive CE CoCos to fulfill T1 capital requirements will lead to higher risk-shifting incentives than the use of equity, and in the case of PWDs or more generally insufficiently dilutive CoCos ($\psi < \tilde{\psi}$) even to higher risk-shifting requirements than the use of subordinated debt will lead to. As a consequence, setting risk weights for different asset categories is no longer enough once CoCos are used on the liability side to achieve consistency between the implied risk-shifting incentives and the regulator's required (maximum) default probability.

6 Conclusion

CoCos have become popular among banks since the emergence of Basel III and the Total Loss Absorption Capacity (TLAC) Standard proposed by the Financial Stability Board. The reason is that CoCo conversion enhances loss absorption capacity by reducing the bank's leverage but act as debt before they are converted. However, an unintended consequence of this feature is that a wealth transfer occurs between the CoCo holders and the original shareholders when the conversion takes place. Depending on its sign and magnitude that wealth transfer may encourage the issuing bank to make conversion more likely by taking on more risk. So contrary to the case where only equity is used to meet capital requirements, once we have CoCos on the liability side of the balance sheet, there is a potential conflict between the Loss Absorption and Risk Mitigation objectives a regulator has, depending on the design features of the CoCo used. In this paper, we have analyzed the implications of these wealth transfers on the issuing bank's risk-shifting incentives and the ensuing impact on capital requirements if regulators are still planning to use those capital requirements to achieve a for them acceptable default probability of the regulated bank.

By writing the issuing bank's residual equity as a linear combination of the pre-and post-conversion states, with the probability of conversion as the weighting factor, we were able to express the residual equity as one of a bank that has issued subordinated debt, plus an expected wealth transfer. The expected wealth transfer is the product of the wealth transfer and the conversion probability. While the literature has paid attention to the wealth transfer, it has largely taken the conversion probability as exogenous. We have endogenized this probability, as we recognize that it is influenced by a bank's risk choices.

We show that the strength of the risk-shifting incentives is strongly influenced by CoCo design. As insufficiently dilutive CE CoCos always transfer wealth to equity holders upon conversion, the risk-shifting incentive is positive. On the other hand, sufficiently dilutive CE CoCos transfer wealth from equity holders to CoCo holders upon conversion and so act as an extra deterrent to risk-shifting: the threat of dilution results in negative risk-shifting incentives relative to subordinated debt. The risk-shifting incentives act as a wedge in a bank's optimization problem, such that the optimal risk choice is different from that under the same amount of subordinated debt. For PWD CoCos and insufficiently dilutive CE CoCos, the risk choices are higher than under the same amount of subordinated debt, while for dilutive CE CoCos, they will be lower.

We have obtained particularly strong results for PWD CoCos. First of all full write down PWD CoCos are the limiting case of CE CoCos as the dilution parameter (the number of shares created upon conversion) approaches zero. So full write down PWD CoCos always induce worse risk-shifting incentives even when compared with subordinated debt, and obviously even more so when compared to the use of straight equity. And second, we have shown that this results extends to also to incomplete write down CoCos (with retention parameter $\varphi > 0$), which is not a trivial result as they cannot be obtained as a limiting case of a CE CoCo.

These results naturally lead to further questions concerning capital requirements. An important implication of our results is that the interaction between capital requirements and asset-side portfolio risk must be reconsidered whenever amendments are made to existing policies. If CoCos are to continue to play an important role in the capital structure of banks, the level of capital requirements should also depend on how they are met. In that vein we have shown that some of the disadvantages of insufficiently dilutive CoCos can be offset by raising the bar higher: if inappropriate CoCo design increases risk-shifting incentives, that effect can be counteracted by requiring more skin in the game, i.e. by setting the requirement ratios higher than they are set for the case of pure equity or sufficiently dilutive CoCos.

These results are important in setting regulations. Basel III and the TLAC Standard were written with the focus on increasing loss absorption capacity of the financial system. To a substantial extent, this loss absorption capacity is being filled by CoCos, potentially completely for meeting TLAC requirements and up to one quarter of the total for T1 capital requirements. But to achieve a more robust financial system, it is not enough to only consider loss absorption capacity. We must also consider regulation that prevents banks from choosing excessively risky actions in the first place, as the designers of Basel II fully realized when introducing risk weights. Capital regulation is also meant to force banks to put more skin in the game in order to reduce risk-shifting incentives, not just to increase loss absorption capacity for given risk levels. We embedded our analysis of risk-shifting incetives and CoCo design structure into a game theoretic analysis of optima capital requirements, with the regulator as Stackelberg leader and banks as Stackelberg followers. Since CoCos are hybrids of debt and equity, the risk levels they induce will generally be different from the ones induced by the use of debt and equity only for the same capital requirements. And as we have shown, not all CoCos are created equal - some have higher risk-shifting incentives than others. We find that at the very least, the type of CoCo that is allowed to fill in Additional Tier 1 capital requirements should be restricted to equity converters, and among those only CE CoCos which are sufficiently dilutive. Alternatively, if PWD or more generally insufficiently dilutive CoCos are allowed, their use should lead to correspondingly higher capital requirements. Only then can regulators achieve consistency between the risk-shifting incentives embedded in the banks' balance sheet structure and the regulator's target probability of default of the regulated bank.

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A Some standard option pricing results: the call option function and its derivatives

Consider a firm with asset value $\mathcal{R}$ and outstanding liabilities D.[14] The value of its equity is akin to that of a call option with expected return $\mathcal{R}$and strike price D. We write $C[\mathcal{R},\ D]$, where the full expression is

$$\begin{aligned} C[\mathcal{R},\ D] &= \exp(-r)\left[\mathcal{R}\exp(r)\,\Phi(d_1) - D\Phi(d_2)\right] \\ &= \mathcal{R}\Phi(d_1) - \exp(-r)\,D\Phi(d_2) \end{aligned}$$

where r is the risk-free rate, $\Phi(\cdot)$ is the cumulative density function of the standard normal distribution, $d_1 = \frac{1}{\sigma}\left[\ln\frac{\mathcal{R}}{D} + r + \frac{1}{2}\sigma^2\right]$ and $d_2 = \frac{1}{\sigma}\left[\ln\frac{\mathcal{R}}{D} + r - \frac{1}{2}\sigma^2\right]$. We use the following first and second-order partial derivatives of $C[\mathcal{R},\ D]$ in the chapter.

[14] In this section, we use $\mathcal{R}$ (calligraphic font) to denote a general asset value, and to distinguish from R (regular font) that is used in the main text, which has the specific meaning of the expected value of the firm's assets.

A.1 Vega

Vega is the sensitivity of the option value with respect to the volatility of its underlying assets. It is calculated by taking the derivative of the call option with respect to volatility σ. For any asset value $\mathcal{R}$ and outstanding liability D:

$$V[\mathcal{R},\ D] \quad = \quad \frac{\partial C[\mathcal{R},\ D]}{\partial \sigma} = \mathcal{R}\phi(d_1) > 0$$

where $\phi(\cdot)$ is the probability density function of the standard normal distribution.

A.2 C_D: The derivative of the call option with respect to the strike price D

For any asset value $\mathcal{R}$ and outstanding liability D, the derivative of the call option with respect to D is

$$C_D = \frac{\partial C[\mathcal{R},\ D]}{\partial D} = -\exp(-r)\,\Phi(d_2) < 0$$

A.3 V_σ: The second-order derivative of $C[\mathcal{R},\ D]$ with respect to σ [15]

The second-order derivative of $C[\mathcal{R},\ D]$ with respect to σ is the first-order derivative of vega with respect to σ. We refer to this as V_σ in the text.

$$\begin{aligned} V_\sigma &= \frac{\partial^2 C[\mathcal{R},\ D]}{\partial \sigma^2} = \frac{\partial V[\mathcal{R},\ D]}{\partial \sigma} \\ &= \mathcal{R}\phi'(d_1)\frac{\partial d_1}{\partial \sigma} \\ &= V[\mathcal{R},\ D]\left(\frac{d_1 d_2}{\sigma}\right) \end{aligned}$$

Note that for a positive distance-to-default d_2 we get that $V_\sigma > 0$. V_σ is smaller the smaller the product $d_1 d_2$. And in particular we get$V_\sigma < 0$ if and only if the following inequalities hold:

$$-\frac{\sigma^2}{2} < \log\left(\frac{R}{D}\right) < \frac{\sigma^2}{2} \tag{52}$$

since then $d_1 d_2 < 0$.We get $V_\sigma > 0$ again for very negative values for $d_2 < d_1 < 0$, a case we will not consider in this paper. Note that (52) is more likely to hold for weakly capitalized banks (low R/D) and for high volatility σ^2.

[15]This quantity is known in the option pricing literature as a second order "Greek" and is referred to as "Vomma" (although there is of course no such letter in the Greek alphabet).

A.4 V_D: The cross-order partial derivative of $C\left[\mathcal{R},\ D\right]$ with respect to σ and D

The cross-order partial derivative of $C\left[R,\ D\right]$ with respect to σ and D is also the first-order derivative of vega with respect to the strike price D. We refer to this shorthand as V_D in the main text. For $d_1 > d_2 > 0$, we have

$$V_D = \frac{\partial^2 C\left[\mathcal{R},\ D\right]}{\partial\sigma\partial D} = \frac{\partial V\left[\mathcal{R},\ D\right]}{\partial D} = \mathcal{R}\phi'\left(d_1\right)\frac{\partial d_1}{\partial D} = -\mathcal{R}\phi\left(d_1\right)d_1\left(-\frac{1}{\sigma D}\right) = \frac{\mathcal{R}}{D}\ \phi\left(d_1\right)\ \frac{d_1}{\sigma} > 0$$

B Proofs for various results in the paper

B.1 Proof that $\frac{\partial^2 p^c}{\partial\tau\partial\sigma} < 0$

$$\begin{aligned}
\frac{\partial^2 p^c}{\partial\tau\partial\sigma} &= \frac{\partial}{\partial\sigma}\left(\frac{\partial p^c}{\partial\tau}\right) \\
&= \frac{\partial}{\partial\sigma}\left(\phi\left(-d_c\right)\frac{1}{\sigma\left(1-\tau\right)}\right) \\
&= \sigma\left(1-\tau\right)\phi'\left(-d_c\right)\left(-\frac{\partial d_c}{\partial\sigma}\right) - \phi\left(-d_c\right)\left(1-\tau\right) \\
&= \frac{\sigma\left(1-\tau\right)\phi\left(-d_c\right)d_c\left(\frac{\partial d_c}{\partial\sigma}\right) - \phi\left(-d_c\right)\left(1-\tau\right)}{\sigma^2\left(1-\tau\right)^2} \\
&= \frac{\phi\left(-d_c\right)\left(1-\tau\right)\left[\sigma d_c\frac{\partial d_c}{\partial\sigma} - 1\right]}{\sigma^2\left(1-\tau\right)^2} \\
&= \frac{\phi\left(-d_c\right)\left(1-\tau\right)\left[-\sigma d_c\left(1+\frac{d_c}{\sigma}\right) - 1\right]}{\sigma^2\left(1-\tau\right)^2} \\
&< 0
\end{aligned}$$

where we use :

$$\begin{aligned}
\sigma d_c\frac{\partial d_c}{\partial\sigma} &= \sigma d_c\left[\frac{\sigma\left(-\sigma\right) - d_c\sigma}{\sigma^2}\right] \\
&= -\sigma d_c\left(1+\frac{d_c}{\sigma}\right)
\end{aligned}$$

B.2 Impact of φ on the risk-shifting incentives of PWD CoCos.

Since $C\left[R^{-},\ D_d+D_s\right]$ and $V\left[R^{-},\ D_d+D_s\right]$ are not functions of φ, we may express**??** as

$$\begin{aligned}
\frac{\partial RSI_{pwd}}{\partial\varphi} &= \underbrace{\frac{\partial p^c}{\partial\sigma}\frac{\partial C\left[R^{-},\ D_d+\varphi D_s\right]}{\partial\varphi}}_{\partial CF_{pwd}/\partial\varphi}+\underbrace{p^c\frac{\partial V\left[R^{-},\ D_d+\varphi D_s\right]}{\partial\varphi}}_{\partial WF_{pwd}/\partial\varphi}\\
&= \underbrace{-\frac{\partial p^c}{\partial\sigma}\exp\left(-r\right)\Phi\left(d_2^*\right)D_s}_{\partial CF_{pwd}/\partial\varphi}+\underbrace{p^c\frac{R^{-}\phi\left(d_1^*\right)D_s}{D_d+\varphi D_s}\frac{d_1}{\sigma}}_{\partial WF_{pwd}/\partial\varphi}\\
&= \underbrace{-\frac{\partial p^c}{\partial\sigma}\exp\left(-r\right)\Phi\left(d_2^*\right)D_s}_{\partial CF_{pwd}/\partial\varphi}+\underbrace{p^c V_D^*\frac{D_s}{D_d+\varphi D_s}}_{\partial WF_{pwd}/\partial\varphi}
\end{aligned}$$

Line 2 follows from the standard expression of the derivative of a call with respect to the strike price using the chain rule to get the derivative with respect to φ (the strike price is $D_d+\varphi D_s$).The notations d_1^* and d_2^* indicate that the functions d_1 and d_2 were evaluated at strike price $D_d+\varphi D_s$ instead of a generic strike price D.

Consider the equation RSI(φ) = 0.We know that $RSI(1)=0$. If$RSI(\varphi)=0$,for a $\varphi<1$, it follows that $RSI(\varphi)-RSI(1)=0$. This implies:

$$p_\sigma^c\frac{\overbrace{(W(1)-W(\varphi))}}{A}+p^c\frac{\overbrace{(W_\sigma(1)-W_\sigma(\varphi))}}{B}\quad\doteq\quad 0$$

Note that $W(1)=0$ and $W(\varphi)<0$, so $A>0$. And $p_\sigma^c>0$ too. Consider next (B). Since W(1) is always zero for all values of its arguments, $W_\sigma=0$ too. So if we establish that $W_\sigma(\varphi)<0$,the equality cannot hold and multiple solutions to RSI(φ) = 0are ruled out.

$$\begin{aligned}
W_\sigma(\varphi) &= V(\cdot,\ D_d+D_s)-V(\cdot,\ D_d+\varphi D_s)\\
&= -V_D D'\\
&< 0
\end{aligned}$$

for $\varphi<1$and $D_d+\varphi D_s<D'<D_d+\varphi D$. This establishes that the equality cannot hold, i.e. assuming multiple solutions leads to a contradiction so multiple solutions for $RSI(\varphi)=0$ can be ruled out.