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Double Limit Pricing

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Abstract

We study fossil fuel extraction by a monopolist who faces demand from a climate-aware and a climate-ignorant region. A renewable, perfect substitute for fossil energy is available at constant unit cost. The climate-aware region uses a carbon tax and a renewables subsidy as policy instruments. Due to heterogeneity in climate policies between regions, the fossil fuel price path possibly contains two limit-pricing phases. Moreover, the shape of the price path depends on the presence of arbitrators on the market. With arbitrators, the fossil price is continuous. Without arbitrators, the price jumps upward when demand from the climate-aware region drops to zero. A tightening of climate policies results in lower initial resource use. The effect on medium-run extraction and on the duration of the fossil era depends on the presence of arbitrators on the market. We numerically investigate the welfare effects of the policies of the climate-aware region. We find that the carbon tax lowers climate damage. The renewables subsidy, however, only lowers climate damage if there are arbitrators on the market.

JEL codes: Q31, Q37
Keywords: limit pricing, non-renewable resource, monopoly, carbon tax

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1 Introduction

The design of climate policies requires a good understanding of the effects of these policies on markets for fossil fuels, as 90 percent of yearly carbon emissions originates from the combustion of oil, gas, and coal (PBL, 2015). Most of globally traded fossil fuels, in particular oil, is exported by a small group of countries. OPEC, for example, owns 73 percent of the world proven reserves (EIA, 2016a). It has been shown theoretically that imperfect competition affects the time profile of the supply of non-renewable resources such as fossil fuels. Typically, monopolistic supply slows down the speed of extraction, "the monopolist is the conservationist's best friend" (cf. Dasgupta and Heal, 1979, p. 329), and results in a final limit-pricing phase during which the monopolist marginally undercuts the price of substitutes to prevent them from entering the market (cf. Hoel, 1978; Salant, l979). This paper shows that the move away from the perfectly competitive framework has even more dramatic consequences when climate policies are in place which differ between 'climate-aware' regions like the European Union and 'climate-ignorant' regions which have not yet introduced policies to reduce global warming.

We demonstrate that both the fossil extraction path and the effects of (unilateral) climate policies differ markedly from those under perfect competition. Moreover, when taking imperfectly competitive fossil fuel markets into account, the extraction path and the effects of climate policies become crucially dependent on the presence of arbitrators, whereas there is no role for them under perfect competition. Our framework of a monopolist owning a finite resource stock and exporting fossil fuels to different regions with unilateral climate policies in place enables us to characterize the deviations from the perfectly competitive equilibrium, to explore the role of arbitrators, and to investigate the effects of different types of unilateral climate policies on welfare and climate damage.

Most of the existing literature on combating climate change assumes perfectly competitive markets for fossil fuels. This is true for Integrated Assessment Models that aim at characterizing optimal climate policies (cf. Nordhaus, 2013; Golosov et al., 2014), but also for the literature on the effects of suboptimal climate policies, such as Sinn (2008, 2012) who investigates the so-called 'Green Paradox' in a single-region framework, and papers studying unilateral measures to fight global warming in multi-region models (cf. Copeland and Taylor, 2005; Eichner and Pethig, 2011; Hoel, 2011; Fischer and Salant, 2014; Ryszka and Withagen, 2014; Aichele and Felbermayr, 2015). There are only a few studies in the field of climate change economics which move away from perfect competition. Strand (2013) and Karp et al. (2016) employ a game theoretical setting in which a resource importer bloc and a resource importing fringe face a group of resource exporters. Strand (2013) compares a carbon tax and a cap-and-trade scheme in order to identify the optimal policy strategies of both players in a static environment, whereas Karp et al. (2016) study a dynamic game where the players use either taxes or quotas to exercise market power in the presence of a group of non-strategic developing countries. Kagan et al. (2015) investigate oil extraction and carbon accumulation for various production function specifications for both open- and closed-loop Nash equilibria, and compare these with the efficient and competitive outcomes. Their model is based on Liski and Tahvonen (2004) who characterize Markov perfect strategies for coalitions of resource importing and exporting countries.

These papers, however, do not account for the existence of a backstop technology, and therefore are not able to study limit-pricing strategies that fossil suppliers might pursue to prevent producers of renewable energy from entering the market. The seminal early literature investigating behaviour by monopolistic non-renewable resource suppliers (cf. Hoel, 1978; Gilbert and Goldman, 1978; Salant, l979; Stiglitz and Dasgupta, 1981; Hoel, 1983) does not pay attention to climate policies. Hassler et al. (2010) study monopolistic fossil supply in the presence of a backstop technology and climate damage caused by carbon emissions. They, however, assume that the backstop technology makes oil 'superfluous' once it arrives, implying that limit pricing does not occur. Literature on the effects of climate policy in a limit pricing framework is scarce. Jaakkola (2015) studies equilibrium climate policies in a differential game between a resource monopolist and a producer of a backstop which becomes cheaper over time due to investments, giving rise to a regime of limit-pricing behaviour with a declining price over time. In a recent paper, Andrade de Sá and Daubanes (2016) argue that demand for oil is inelastic, implying that the monopolist will choose for limit pricing throughout. As a result, carbon taxes are ineffective and backstop subsidies increase resource extraction. Our analysis is complementary to theirs. We show that, also in the case of elastic demand, limit pricing may be more important than suggested by conventional analyses of climate policy effects. Moreover, our analysis stresses the importance of arbitrators on the market for the fossil extraction path and for the effectiveness of climate policies.

We consider a monopolist that owns a finite stock of fossil fuels and faces constant unit extraction costs and the presence of a renewable perfect substitute with constant marginal production costs. Resource demand comes from a climate-aware region, which employs both a carbon tax and a backstop subsidy, and from a climate-ignorant region, which does not have any climate policies in place. To illustrate the role of arbitrators, we investigate two extreme cases: one with arbitrators on the market who can store fossil fuels without costs, implying that the monopolist is constrained to set a continuous price, and one without arbitrators, where the monopolist is free to choose a discontinuous price path.

The situation in reality lies somewhere in between these two extremes. Many nations have built strategic oil reserves and private actors have created stockpiles. It is difficult, however, to find reliable estimates of global oil inventories, as some countries, such as Russia and China, do not report their inventory levels and figures for many other countries, such as Angola, Nigeria or Brazil, are not trustworthy.[1] Furthermore, much oil is stashed in tankers, waiting off-coast for higher prices. Global crude inventories are estimated to be around 17 billion barrels in non-OECD countries and around 12 billion barrels in OECD countries (Strumpf and Friedman, 2016). With a world liquid fuel consumption at around 96.26 million barrels per day this means that there is enough crude oil to satisfy global consumption for 176 days (EIA, 2016c). The inventory of the U.S.'s 'Strategic Petroleum Reserves' (SPR) amounted to 695.1 million barrels in September 2016, corresponding to around 36 days of oil at the average US daily consumption level of 19.4 million barrels in 2015 (SPR, 2016; EIA, 2016d). Whereas the purpose of the strategic petroleum reserves in the U.S. and other countries is to stabilize supplies, there are calls for supply releases to moderate price increases (Regnier, 2007). There is a disagreement in the literature on whether the use of the reserves is an effective tool to stabilize the oil markets and on whether the existing inventory is sufficient.[2] Yet, these stockpiles might facilitate speculation: Kesicki (2010) notes that "the only way speculation can persistently influence the oil price is due to accumulation of the physical commodity." He puts forward a historical analysis which reveals that price surges are accompanied by an accumulation of crude oil in inventories. Kaufmann (2011) attributes a role to speculation in the price spike and collapse of 2007-2008 on the grounds of, amongst others, a significant increase in private US crude oil inventories since 2004. Hamilton (2009) points out that due to the price inelasticity of oil demand small increases in inventory could greatly affect the

[1] See 'The Wall Street Journal', July 2016: `http://www.wsj.com/articles/how-much-oil-is-in-storage-globally-take-a-guess-1469380040`.

[2] For a short discussion see Demirer and Kutan (2010). In their paper, they examine the informational efficiency of crude oil spot and futures markets with respect to SPR announcements. Their results suggest that the SPR program is effective in stabilizing the oil market. Following the announcements, the market adjusts prices upward (downward) after notification of inventory release (purchase of more inventories), lasting about a week following the announcement date. Yet, there are no statistically significant cumulative abnormal returns.

price. Inventories are also present in natural gas markets: working gas in storage in the U.S. amounted to 3,600 billion cubic feet in September 2016, whereas the U.S. natural gas consumption was 2,204 billion cubic feet in July 2016 (EIA, 2016b,e). Accordingly, there is enough gas in storage to ensure consumption in the U.S. for around 1.5 months. Although it remains unclear whether public and private inventories of oil and gas are sufficiently large to ensure perfect arbitrage, there is at least evidence of a certain degree of arbitrage in the oil market and of the link between oil inventories and price changes.

The results of this paper are as follows. First, we find that in the cases with and without arbitrators on the market the resource extraction paths may contain two limit pricing phases: one just before the demand from the climate-aware region vanishes due to climate policies, and one just before the depletion of the resource. The reason is that the monopolist may want to postpone the moment of losing demand from the climate aware region. Accordingly, in a world with heterogeneous climate policies, it becomes even more important to take the effects of limit pricing into account. Second, in the case without arbitrage possibilities, it is optimal for the monopolist to let the price jump upwards when demand from the regulated region drops to zero. Third, we show that the presence of arbitrators is beneficial for the climate: initial extraction is lower and the overall resource extraction phase is longer than in the case without arbitrators, reducing the present value of climate costs. Fourth, a tightening of climate policies does not result in a so-called 'Weak Green Paradox': on the contrary, initial resource consumption falls in both regimes. Climate costs might still rise as intermediate extraction goes up, and in the absence of arbitrators the overall resource extraction phase is shortened upon the introduction of a renewables subsidy. Finally, our numerical welfare analysis shows that although a renewables subsidy increases climate damages in the equilibrium without arbitrators, the presence of arbitrators reverses this outcome: with arbitrators on the market, a renewables subsidy lowers climate damage, even when cumulative fossil fuel supply remains unchanged. Furthermore, in the presence of arbitrators, the climate-aware region is consistently worse off regarding its non-green welfare because the monopolist sells more resources to the climate-ignorant region than in the absence of arbitrators.

The remainder of the paper is structured as follows. Section 2 describes the model and compares the equilibria with and without arbitrators. Section 3 examines the effects of climate policies on the time paths of fossil fuel use. Section 4 performs a welfare analysis and determines climate damage effects. Section 5 concludes and discusses our results.

2 The model

2.1 The monopolist's problem

Energy demand originates from two regions, A and B. Energy supply comes from renewable and fossil resources. Renewable energy is competitively produced in both regions at a unit cost of $b > 0$, whereas fossil fuel is supplied by a monopolist in a third region facing unit cost $k \geq 0$. We assume that these types of energy are perfect substitutes.[3] Region A conducts an active climate change policy by imposing a unit carbon tax τ on its consumers and gives a subsidy σ on the use of renewables. We assume τ and σ to be constant over time.[4] Let us define aggregate demand for fossil fuel as $q \equiv q_A + q_B$, consisting of demand from region A and demand from region B. We split up the monopolist's problem in two stages. Stage 1 starts at time zero and lasts until time T_2. During stage 1 both regions use only fossil fuel. Stage 2 starts at time T_2 and lasts until time T_4. In this stage region B still relies solely on fossil fuel, whereas region A only uses renewables. Both the switching time T_2 and the final time of fossil fuel use T_4 are optimally chosen by the monopolist for given subsidy and tax rates. There are two intermediate phases: from time zero until time $T_1 \leq T_2$ the consumer price is strictly below the price that would keep fossil fuel off the market in region A, which is $b - \sigma$, whereas from time T_2 until time $T_3 \leq T_4$ the consumer price is such that there is no fossil fuel demand from region A anymore and region B relies on fossil fuel only, at a consumer price strictly below the renewables price. The optimality of this sequence of regimes is formally demonstrated in the appendix. Intuition is given in due course. We denote the producer price in the first stage, when $t < T_2$, by $p_1(q)$, and the producer price when $t \geq T_2$, by $p_2(q)$. We need $p_1(q(t)) \leq \hat{b} \equiv b - \sigma - \tau$ in the first stage because the consumer price of fossil energy in region A, $p_1 + \tau$, should not exceed the consumer price of renewables in that region, $b - \sigma$. In the second stage the fossil price in region B should not exceed the renewables price in region B: $p_2(q) \leq b$ and the consumer price of fossil in region A, $p_2(q) + \tau$, should be prohibitively high for the consumers in that region to demand fossil fuel: $p_2(q) + \tau > b - \sigma$. Demand is illustrated in Figure 1, where we use $\hat{q}_A$ to denote demand for fossil fuel in region A if the consumer price is $b - \sigma$, or, equivalently, if the producer price is $\hat{b}$. The variable $\hat{q}_B$

[3]It has been shown by Van der Meijden and Withagen (2016) that the equilibrium with imperfect substitution converges tot the equilibrium with perfect substitution for high values of the elasticity of substitution, in a model in which demand is exerted by a single region.

[4]Constancy of the carbon tax can be motivated by constant marginal damages from carbon emissions, proportional to the use of fossil fuel (cf. Hoel, 2011). Constancy of the subsidy is more difficult to justify. But in the context of the present paper we don't wish to go into the design of a subsidy that is second-best anyway.

is defined in a similar way. Also $\hat{q} \equiv \hat{q}_A + \hat{q}_B$. The variables $\tilde{q}_A$ and $\tilde{q}_B$ represent demand in regions A and B if the consumer price is b. We assume $k < b$, which implies that at some instant of time all fossil fuel will be exhausted.[5]

We tackle the maximization problem of the monopolist by using two-stage optimal control theory (cf. Tomiyama, 1985; Makris, 2001; Valente, 2010). The idea is to first solve the problems in the two stages separately for given T_2, T_4, and $S(T_2)$:[6]

$$\Lambda_1(T_2, S_0, S(T_2)) = \max_q \int_0^{T_2} e^{-rt} \left(p_1(q(t)) - k\right) q(t) dt, \tag{1a}$$

$$\Lambda_2(T_2, T_4, S(T_2)) = \max_q \int_{T_2}^{T_4} e^{-rt} \left(p_2(q(t)) - k\right) q(t) dt, \tag{1b}$$

subject to

$$\dot{S}(t) = -q(t), \quad q(t) \geq 0, \; S(t) \geq 0, \quad S(0) = S_0, \tag{2a}$$

$$p_1(q(t)) \leq \hat{b}, \tag{2b}$$

$$p_2(q(t)) \leq b, \tag{2c}$$

$$p_2(q(t)) \geq \hat{b}. \tag{2d}$$

Subsequently, we determine the optimal T_2 and $S(T_2)$ by solving

$$\Lambda(S_0) = \max_{T_2, T_4, S(T_2)} \Lambda_1(T_2, S_0, S(T_2)) + \Lambda_2(T_2, T_4, S(T_2)). \tag{3}$$

To ensure that the second-order conditions are satisfied, we assume the net revenue, $(p_1(q) - k)q$, to be strictly concave in q for $p_1(q) < \hat{b}$, and $(p_2(q) - k)q$ to be strictly concave in q for $\hat{b} < p_2(q) < b$. The technical details of the derivations are provided in Appendix A.1. But the following gives the intuition. The Hamiltonians associated with the maximization problems (1a)-(1b) in the two stages 1 and 2 read

$$\mathcal{H}_i(q, \lambda, t) = e^{-rt}(p_i(q) - k)q - \lambda_i q, \quad i = 1, 2. \tag{4}$$

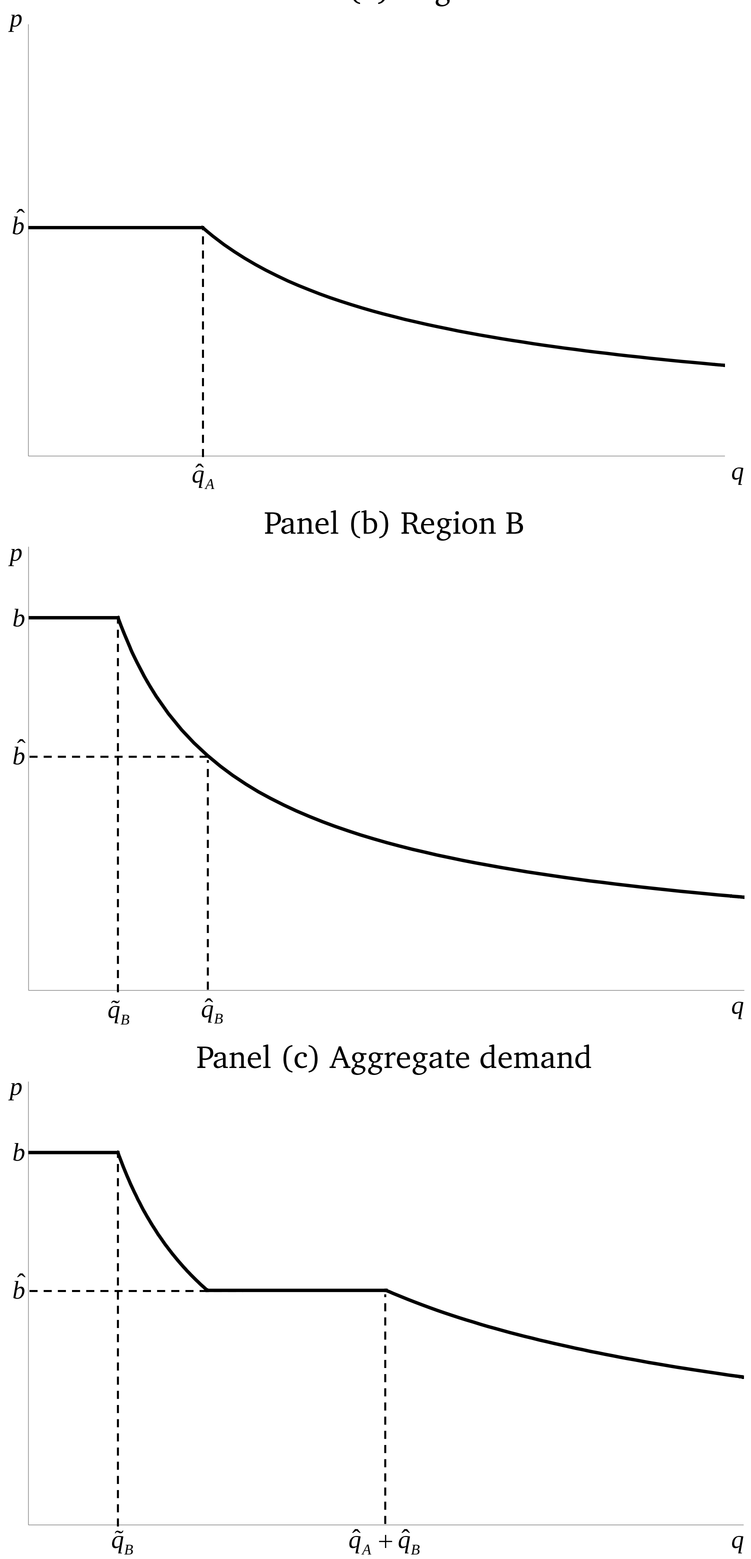
In the absence of stock-dependent extraction costs, it follows from the Hotelling rule that the shadow prices of the resource stocks in the two stages, λ_1 and λ_2, are constant. Moreover, in each stage the corresponding Lagrangian is maximized with respect to the

[5]Many of the results that we derive hold for more general cost functions, where extraction costs are not necessarily linear in extraction and may also depend on the remaining stock, in which case cumulative fossil use is determined endogenously. See Van der Meijden and Withagen (2016), who address this for a single market.

[6]Throughout, we refer to the time intervals $[0, T_2]$ and $[T_2, T_4]$ as *stages* and to subintervals within the stages, e.g., $[T_1, T_2]$ as *phases*.

Figure 1: Regional and aggregate demand

Panel (a) Region A



extraction rate, subject to the relevant constraints. The necessary first-order conditions read:

$$\lambda_1 e^{rt} = p_1'(q)q + p_1(q) - k - \mu_{11}p_1'(q), \tag{5a}$$
$$\lambda_2 e^{rt} = p_2'(q)q + p_2(q) - k - \mu_{21}p_2'(q) + \mu_{22}p_2'(q), \tag{5b}$$

where μ_{11}, μ_{21}, and μ_{22} are the non-negative Lagrange multipliers associated with the inequalities (2b)-(2d).

To determine the final time that solves problem (3), T_4, we use the condition that the Hamiltonian for the second-stage problem should equal zero at T_4, in shorthand[7]

$$\mathcal{H}_2(T_4) = 0. \tag{6}$$

The optimal switching time T_2 and the associated optimal stock $S(T_2)$ depend on what is assumed regarding arbitrage. If arbitrage is ruled out, for example if it is too costly to store fossil fuels, we know from two-stage optimal control theory that both the Hamiltonian and the shadow price must be continuous at the transition date T_2 from serving both regions to serving just region B:[8]

Lemma 1 *Suppose arbitrage is ruled out. Then the following conditions must hold:*

$$\mathcal{H}_1(T_2^-) = \mathcal{H}_2(T_2^+), \tag{7a}$$
$$\lambda_1 = \lambda_2. \tag{7b}$$

Proof. See Tomiyama (1985). □

To understand condition (7a), note that the Hamiltonian evaluated at the terminal (initial) time equals (minus) the partial derivative of the optimal value function with respect to the terminal (initial) time (cf. Theorem 3.9 in Seierstad and Sydsæter, 1987, p. 213): $\partial\Lambda/\partial T_2 = \partial\Lambda_1/\partial T_2 + \partial\Lambda_2/\partial T_2 = \mathcal{H}_1(T_2^-) - \mathcal{H}_2(T_2^+)$. Hence, if the monopolist is free to choose T_2, condition (7a) must hold in the optimum. To get the intuition behind (7b), note that the shadow price λ_1 is the present value of having one more unit of fossil fuel in the ground at time zero, whereas λ_2 is the present value of having one more unit of fossil fuel in the ground at instant of time T_2. Without arbitrage the

[7]This condition is obtained by noting that $\partial\Lambda_2(T_2, T_4, S(T_2))/\partial T_4 = \mathcal{H}_2(T_4)$ (cf. Theorem 3.9 in Seierstad and Sydsæter, 1987, p. 213).

[8]By $\mathcal{H}_1(T_2^-)$ we denote the limit of the value of the Hamiltonian in the first stage for time approaching T_2 from below, and by $\mathcal{H}_2(T_2^+)$ the value of the Hamiltonian in the second stage for time approaching T_2 from above.

monopolist can transfer the additional stock of time zero, and extraction, to T_2 without any restriction. Hence, the values must be identical in the optimum.

If arbitrage is possible, meaning fossil fuel can be bought in any amount and stored at low cost, the market price at any time, hence also at T_2, should be continuous:

$$p_1(q(T_2^-)) = p_2(q(T_2^+)) = \hat{b}. \tag{8}$$

As a result, the monopolist is no longer free to choose the duration of and the cumulative supply in the two stages, implying that (7a)-(7b) do not necessarily hold in the case with arbitrators. To understand this, note that because of condition (8) the price at T_2 is fixed at $\hat{b}$ which—given first-order condition (5b) and terminal condition (6)—poses a restriction on cumulative resource supply during the second stage: the stock at T_2, $S(T_2)$, should be larger than or equal to some threshold value (S_{02}, which we will define later), to prevent the monopolist from choosing $p(T_2^+) > \hat{b}$, which is not possible in equilibrium. Similarly, given first-order condition (5a) and cumulative supply during stage 1, condition (8) effectively imposes a lower bound on the duration of the first stage: T_2 should be larger than or equal to some threshold value (T_2^*, which is defined in Appendix A.2). The lower bounds on $S(T_2)$ and T_2 imply:[9]

Lemma 2 *Suppose arbitrage is possible. Then the following conditions must hold:*

$$\mathcal{H}_1(T_2^-) \leq \mathcal{H}_2(T_2^+), \tag{9a}$$

$$\lambda_1 \geq \lambda_2. \tag{9b}$$

Proof. See Appendix A.2. □

If in an optimum $T_2 > T_1$ then (9a) turns into an equality, which is helpful to characterize the optimum. If $T_1 = T_2$ then this equality by itself offers a useful condition.

In Appendix A.4 we derive the following relationship between the Hamiltonian at time zero and discounted profits of the monopolist, provided that (9a) holds with equality:[10]

$$\frac{\mathcal{H}_1(0)}{r} = \Lambda(S_0, b, \sigma, \tau). \tag{10}$$

[9] Appendix A.2 shows that this restriction on cumulative supply is always binding, implying strict inequality in (9b): $\lambda_1 > \lambda_2$.

[10] Whereas (10) is well known in control theory for one-stage optimal control problems (Seierstad and Sydsæter, 1987), we extend its validity for two-stage optimal control problems.

So, the value of the Hamiltonian equals the rent on discounted profits. This observation is useful in the analysis to follow.

2.2 Equilibrium with arbitrage

If arbitrage is possible, upward jumps in the resource price will be arbitraged away. The resource price is continuous and thus equal to $\hat{b}$ at the moment of the switch from stage 1 to stage 2. The optimal supply of fossil fuel from the perspective of the monopolist is given in the following theorem.

Theorem 1 *There exist* $0 \leq T_1 \leq T_2 \leq T_3 < T_4$ *such that*

(i) $p(t) \leq \hat{b},\ q(t) \geq \hat{q}$ *for* $0 \leq t \leq T_1$ *(phase 1),*

(ii) $p(t) = \hat{b},\ q(t) = \hat{q}$ *for* $T_1 \leq t \leq T_2$ *(phase 2),*

(iii) $\hat{b} \leq p(t) \leq b,\ q_A(t) = 0,\ q_B \geq \hat{q}_B$ *for* $T_2 \leq t \leq T_3$ *(phase 3),*

(iv) $p(t) = b, q_A(t) = 0, q_B(t) = \tilde{q}_B$ *for* $T_3 \leq t < T_4$ *(phase 4),*

(v) $S(T_4) = 0,\ q(t) = 0$ *for* $t > T_4$.

Furthermore, there exist $S_{03} \geq S_{02} > S_{01}$ *such that*

(i) $0 = T_1 = T_2 = T_3 < T_4$ *if* $S_0 \leq S_{01}$,

(ii) $0 = T_1 = T_2 < T_3 < T_4$ *if* $S_{01} < S_0 \leq S_{02}$,

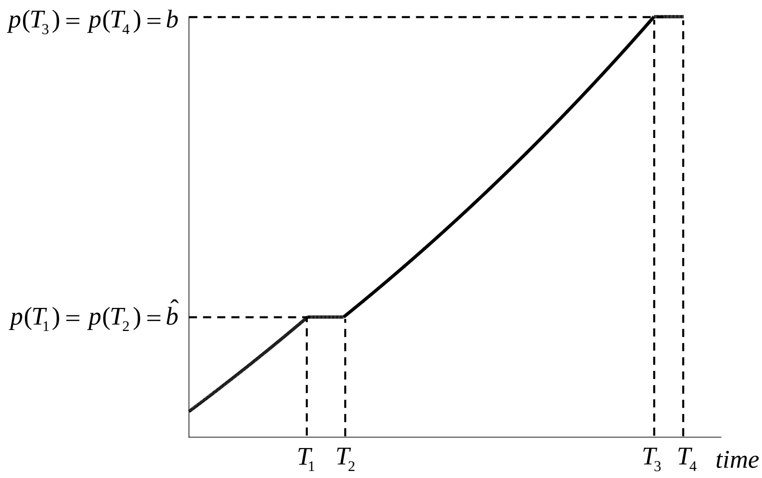
(iii) $0 = T_1 < T_2 < T_3 < T_4$ *if* $S_{02} < S_0 \leq S_{03}$,

(iv) $0 < T_1 \leq T_2 < T_3 < T_4$ *if* $S_{03} < S_0$.

Proof. See Appendix A.2. □

A typical equilibrium for a large initial stock (case (iv) in the second part of Theorem 1) and $T_2 > T_1$ is depicted in Figure 2. To get the intuition for the theorem and the figure let us first consider the case without policy instruments used by any government. This is similar to a situation in which there is a single market for fossil fuel. Van der Meijden and Withagen (2016) show (in a much more general setting) that there always exists

Figure 2: Price path with arbitrage



a final interval of time where there is limit pricing. The reason for the existence of this regime becomes clear by considering the following condition:[11]

$$e^{-rT_3}(p'(q(T_3))q(T_3) + p(q(T_3)) - k) = e^{-rT_4}(b - k). \tag{11}$$

Intuitively, the monopolist could sell its last unit of fossil at $T_3 \leq T_4$ yielding marginal profit $e^{-rT_3}(p'(q(T_3))q(T_3) + p(q(T_3)) - k)$ with $p(q(T_3)) \leq b$ (left-hand side). Alternatively, the last unit of fossil could be conserved and sold right after exhaustion of the rest of the stock, i.e. at $t = T_4^+$ against price $p = b$. As $q(T_4^+) = 0$, marginal profit would then boil down to average profit and equal $e^{-r\hat{T}_4}(b - k)$ (right-hand side). Equalizing marginal profits of both options requires $T_4 > T_3$, implying that there is always a final interval of time with limit pricing. The current value of marginal profit at T_4 when $q = 0$ is still larger than marginal profit at T_3 when $q > 0$, but the discounted value of these marginal profits is equalized by choosing an appropriate, strictly positive duration of the limit-pricing regime. It might even be the case that there is limit pricing throughout. This will occur if fossil fuel demand is inelastic (as assumed by, e.g., Andrade de Sá and Daubanes, 2016) or if the initial resource stock is small enough.[12]

These results carry over to the case of two regions. The strategy of the proof in Appendix A.2 is to construct feasible price paths for various levels of the initial resource stock and to show that these paths, and the corresponding extraction rates, satisfy all

[11]As shown in Appendix A.2, this condition is obtained by combining transversality condition $\mathcal{H}_2(T_4) = 0$ with first-order condition (5b).

[12]With inelastic demand, marginal profit at the left-hand side of (11) is negative if the price is set below the renewables cost.

necessary conditions as well as the transversality conditions, so that the paths constitute an optimum. Assuming elastic demand, we find that for an initial resource stock small enough it is optimal to have limit pricing at price b from the start. This occurs for the initial stock S_0 smaller than or equal to a critical level, denoted by S_{01}. For a larger initial stock there is scope for an initial phase with the producer price below b. If the stock is not too large, not larger than another critical level $S_{02}(> S_{01})$ the monopolist will serve only the market in the region without climate policy. The critical level S_{02} is determined as the initial stock for which the monopolist will charge an initial price exactly equal to $\hat{b}$ and, if the initial resource stock is smaller than or equal to S_{02}, the monopolist just acts as if there were only region B. With a still larger initial stock, the initial price charged in this single market would be smaller than $\hat{b}$ so that also demand from region A would be attracted. Then region A enters the picture. For initial stocks not too large, smaller than some S_{03} there may be limit pricing for a while at price $\hat{b}$. For larger initial stocks the initial price will even be below $\hat{b}$.

To understand the occurrence, or not, of an intermediate limit-pricing phase, we substitute the first-order conditions (5a)-(5b) and (8) into (9a) to get[13]

$$\eta_1(\hat{q})\left(\hat{q} - \mu_{11}(T_2^-)\right) \leq \eta_2(\hat{q}_B)\hat{q}_B, \tag{12}$$

where $\eta_i(q) \equiv -p_i'(q)q/p$ denotes the (positively defined) inverse of the producer price elasticity of demand. The left-hand side measures the increase in profits during stage 1 due to a marginal increase of T_2 (which increases the duration of stage 1). Similarly, the right-hand side measures the decrease in profits during stage 2 due to a marginal increase in T_2 (which lowers the duration of stage 2). If condition (12) holds with equality, the monopolist cannot increase his profits by reallocating time between the two stages. Consider a situation with $\mu_{11}(T_2^-) = 0$, which would correspond to an equilibrium without an intermediate limit-pricing phase (see (5a)). If the left-hand side of (12) would be higher than the right-hand side in this case, i.e., if $\eta_1(\hat{q})\hat{q} > \eta_2(\hat{q}_B)\hat{q}_B$, the monopolist could increase his profits by introducing an intermediate limit-pricing phase that extends the duration of the first stage. The duration of the limit-pricing phase depends positively on $\mu_{11}(T_2^-)$,[14] which in the optimum then is chosen to equilibrate the left- and right-hand side of (12). If, on the contrary, by imposing $\mu_{11}(T_2^-) = 0$ the left-hand side of (12) is smaller than the right-hand side, the monopolist would prefer to decrease the duration of the first stage. For given

[13]We have used $\mu_{21} = \mu_{22} = 0$ (see Appendix A.2) on the right-hand side of (12).

[14]Use first-order condition (5a) at $t = T_1$ and $t = T_2$ to get $e^{r(T_2-T_1)} = \frac{p_1'(\hat{q})\hat{q}+\hat{b}-k-\mu_{11}(T_2^-)p_1'(\hat{q})}{p_1'(\hat{q})\hat{q}+\hat{b}-k}$.

cumulative extraction during stage 1, this would necessarily require a downward shift in the price path. However, this is not allowed by restriction (8), which requires the price to equal $\hat{b}$ at T_2. Therefore, the monopolist is restricted to the equilibrium without limit pricing but with $p(T_2)$ equal to $\hat{b}$, and (12) holds with strict inequality.

2.3 Equilibrium without arbitrage

Without arbitrage, the monopolist will choose a discontinuous price path if there are policy differentials. The reason is simple and intuitive. With equal shadow prices in the two stages (according to condition (7b)) the continuity of the Hamiltonians at $\tilde{T}_2$[15] with continuous prices requires $\mathcal{H}_2(\tilde{T}_2^+) = 0$. Moreover, the Hamiltonian at time $\tilde{T}_2^+$ is proportional (with factor of proportionality $1/r$) to total profits from $\tilde{T}_2$ onwards,

$$0 < \int_{\tilde{T}_2}^{\tilde{T}_4} e^{-rt} \left(p_2(q(t)) - k\right) q(t)dt = \frac{\mathcal{H}(\tilde{T}_2) - \mathcal{H}(\tilde{T}_4)}{r} = \frac{\mathcal{H}(\tilde{T}_2)}{r}.$$

Since profits are definitely positive, the Hamiltonian at $\tilde{T}_2$ cannot be zero. Hence there is a price discontinuity. To provide further intuition for this result, consider Figure 3, which is an extended version of Figure 2 in Hoel (1984). The figure shows a discontinuous line for the marginal revenue $\pi'(q) = p'(q)q + p(q)$, corresponding to the aggregate demand function in panel (c) of Figure 1, and a flat line for the marginal cost, consisting of the sum of the current value of the scarcity rent at $t = \tilde{T}_2$, $\lambda e^{r\tilde{T}_2}$, and the marginal extraction cost, k.[16] If $p(q) < \hat{b}$ or $\hat{b} < p(q) < b$, marginal revenue at time t equals the marginal cost $\lambda e^{rt} + k$.

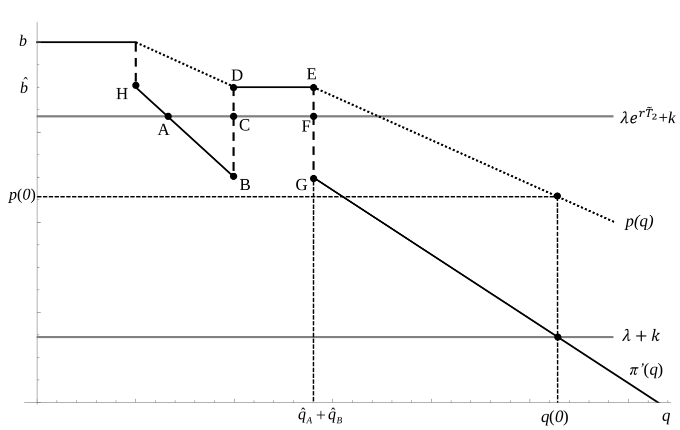
Consider first panel (a), which additionally contains a line for the inverse aggregate demand function $p(q)$ (solid flat parts, dotted downward sloping parts) and the marginal cost at $t = 0$, $\lambda + k$. We have assumed here that the initial resource stock is large enough to get marginal cost at $t = 0$ below point G in the figure. Initially, the economy is at an equilibrium where resource extraction is given by $q(0)$ and the resource price by $p(0)$. The Hamiltonian associated with the profit maximization problem has a unique maximum at $q(0)$. Over time, the scarcity rent λe^{rt} gradually increases and resource extraction goes down, giving rise to an increasing resource price. At point G, there is a discontinuity in the marginal revenue function. When the marginal cost reaches a level corresponding to this point, extraction and the resource price will continue to equal $\hat{q}_A + \hat{q}_B$ and $\hat{b}$, respectively, for a while: limit pricing. However,

[15] To distinguish the switching times from the case with arbitrators, we use tildes above the Ts.

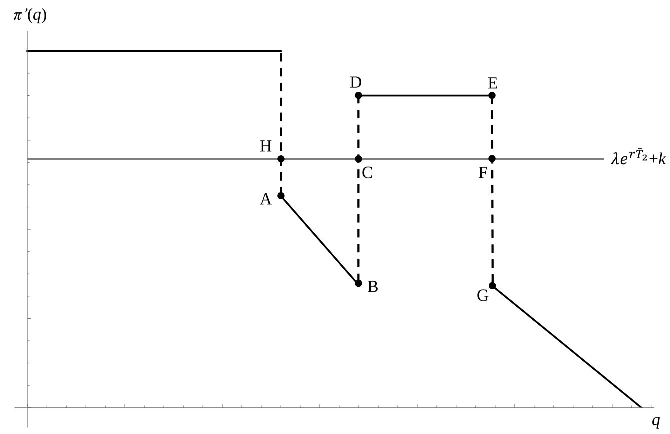
[16] We have used (7b) to write $\lambda \equiv \lambda_1 = \lambda_2$. Note that λ is still to be determined.

Figure 3: Stage switch - four scenarios

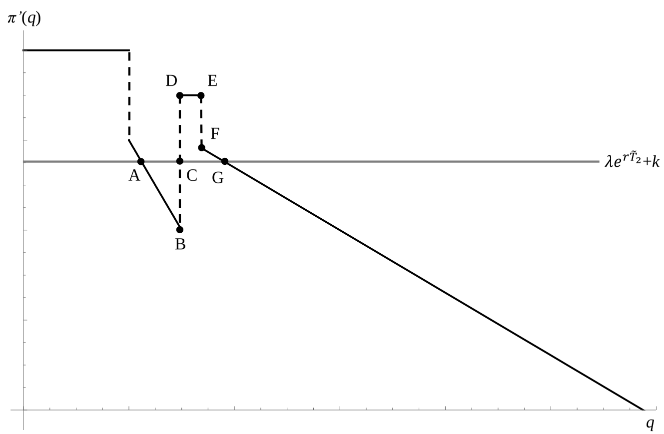
(a) Limit pricing, small price jump



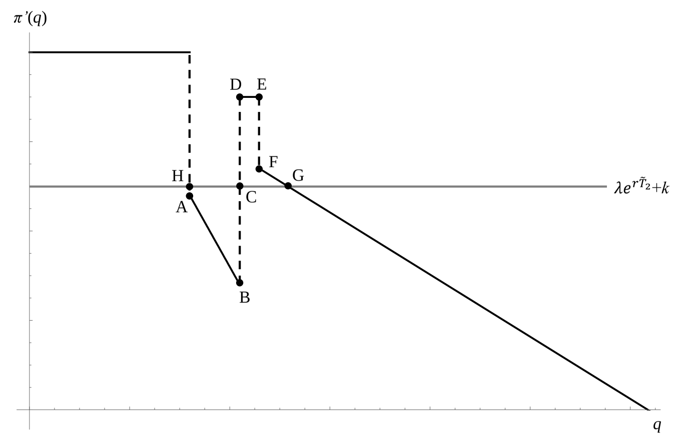
(b) Limit pricing, large price jump



(c) No limit pricing, small price jump



(d) No limit pricing, large price jump



Notes: The discontinuous line represents marginal revenue $\pi'(q) = p'(q)q + p(q)$ and the flat line gives marginal cost at $t = \tilde{T}_2$. Panel (a) shows the case with limit pricing and $p_2(\tilde{T}_2^+) < b$, panel (b) presents the case with limit pricing and $p_2(\tilde{T}_2^+) = b$, panel (c) depicts the scenario without limit pricing and with $p_2(\tilde{T}_2^+) < b$, panel (d) illustrates the case without limit pricing and with $p_2(\tilde{T}_2^+) = b$.

whenever the marginal cost is at a level in between points B and D, there exists a second intersection point of the marginal revenue and marginal cost lines, e.g. at point A, implying that the Hamiltonian has another local maximum. The change in profits when the monopolist would move from point A to point F in the figure is given by rectangle CDEF (where marginal revenue is above marginal cost), minus triangle ABC (where marginal revenue is below marginal cost). Hence, as long as the surface given by the triangle ABC is smaller than the surface within the rectangle CDEF, the global maximum is still located at $\hat{q}_A+\hat{q}_B$. At time $t=\tilde{T}_2$ both areas have exactly the same size. Given that the scarcity rent keeps on rising, the optimal point will jump from F to A at $\tilde{T}_2$: extraction jumps down and the resource price jumps up. After the switch, resource extraction will gradually decline while the flat marginal cost line increases until point H is reached, when another limit-pricing phase starts until the stock is exhausted.

In panel (b), the global maximum is still located at $\hat{q}_A+\hat{q}_B$ when the marginal cost reaches the level corresponding with point A. At $t=\tilde{T}_2$, the area ABCH equals CDEF, implying that the price jumps immediately from one limit-pricing regime with $p=\hat{b}$ to the other limit-pricing regime with $p=b$, which will last until the stock is exhausted. In panel (c), the regime switch will take place when area ABC equals CDEFG, which occurs before point F is reached. Hence, there will be no intermediate phase of limit pricing: the price jumps from $p<\hat{b}$ towards $p\in(\hat{b},b)$. Finally, in panel (d) there is again no intermediate limit-pricing phase: the price will jump from $p<\hat{b}$ towards $p=b$ at $t=\tilde{T}_2$, when area ABCH equals CDEFG.

The figure has shown all the possible stage switch scenarios. All of them feature an upward jump in the resource price. The upper two panels are characterized by an intermediate limit-pricing regime with $p=\hat{b}$. Moreover, the two left panels feature an increasing resource price after the stage switch, whereas the two right panels show situations in which the economy jumps to a limit-pricing regime with $p=b$.

The optimal program for the monopolist is summarized in Theorem 2.

Theorem 2 *There exist* $0\leq\tilde{T}_1\leq\tilde{T}_2\leq\tilde{T}_3<\tilde{T}_4$ *such that*

(i) $p(t)\leq\hat{b},\, q(t)\geq\hat{q}$ *for* $0\leq t\leq\tilde{T}_1$ *(phase 1),*

(ii) $p(t)=\hat{b},\ q(t)=\hat{q}$ *for* $\tilde{T}_1\leq t\leq\tilde{T}_2$ *(phase 2),*

(iii) $\hat{b}\leq p(t)\leq b,\ q_A(t)=0,\ q_B\geq\hat{q}_B$ *for* $\tilde{T}_2\leq t\leq\tilde{T}_3$ *(phase 3),*

(iv) $p(t)=b, q_A(t)=0, q_B(t)=\tilde{q}_B$ *for* $\tilde{T}_3\leq t<\tilde{T}_4$ *(phase 4),*

(v) $S(\tilde{T}_4)=0,\ q(t)=0$ *for* $t>\tilde{T}_4$.

Furthermore, there exist $S_{03} \geq S_{02} > S_{01}$ *such that*

(i) $0 = \tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 < \tilde{T}_4$ *if* $\tilde{S}_0 \leq \tilde{S}_{01}$*,*

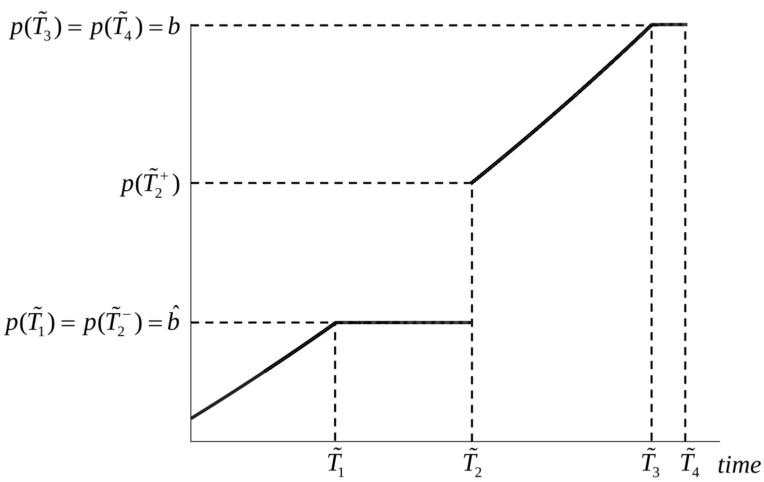
(ii) $0 = \tilde{T}_1 = \tilde{T}_2 < \tilde{T}_3 < \tilde{T}_4$ *or* $0 = \tilde{T}_1 < \tilde{T}_2 = \tilde{T}_3 < \tilde{T}_4$ *if* $\tilde{S}_{01} < \tilde{S}_0 \leq \tilde{S}_{02}$*,*

(iii) $0 = \tilde{T}_1 < \tilde{T}_2 \leq \tilde{T}_3 < \tilde{T}_4$ *if* $\tilde{S}_{02} < \tilde{S}_0 \leq \tilde{S}_{03}$*,*

(iv) $0 < \tilde{T}_1 \leq \tilde{T}_2 \leq \tilde{T}_3 < \tilde{T}_4$ *if* $\tilde{S}_{03} < \tilde{S}_0$.

Proof. The proof can be found in Appendix A.3. □

Figure 4: Price path without arbitrage



Panel (a) of Figure 3 shows the scenario in which all phases exist: $0 < \tilde{T}_1 < \tilde{T}_2 < \tilde{T}_3 < \tilde{T}_4$. The corresponding time profile is depicted in Figure 4. In panel (b) of Figure 3, phase 3 is degenerate: $\tilde{T}_2 = \tilde{T}_3 < \tilde{T}_4$. The monopolist switches from the first limit-pricing phase immediately to the second limit-pricing phase, without an intermediate phase of price increase. Panel (c) shows the case in which phase 2 is degenerate: $\tilde{T}_1 = \tilde{T}_2 < \tilde{T}_3 < \tilde{T}_4$. The first limit-pricing phase drops out and the price increases from $p < \hat{b}$ to a price above the limit price, $p > \hat{b}$. A larger jump from $p < \hat{b}$ to the second limit price is also possible, as in panel (d), where both the second and the third phase are degenerate: $\tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 < \tilde{T}_4$. The occurrence of these 'degenerate' equilibria depends on the functional form of the marginal profit functions and therefore on the aggregate demand functions. Hence, even large initial resource stocks do not guarantee a 'non-degenerate' equilibrium comparable to the one depicted in Figure 4 and in panel (a) of Figure 3.

2.4 Comparison of equilibria

The presence of arbitrators affects the resource extraction path considerably. Proposition 1 deals with the effect on initial extraction and on the length of the period of time over which extraction of fossil fuel takes place.

Proposition 1 *Suppose $S_0 > \max\{S_{03}, \tilde{S}_{03}\}$ (implying that $p(0) < \hat{b}$ in the equilibria with and without arbitrage).*

(i) Initial resource extraction is lower in the presence of arbitrators.

(ii) It takes longer to deplete the non-renewable resource in the presence of arbitrators, i.e., $T_4 > \tilde{T}_4$.

Proof. Part (i) follows from the fact that arbitrage has a negative impact on the monopolist's profits, which are proportional to the Hamiltonian at time zero, according to (10). If $p(0) < \hat{b}$ we have $\mathcal{H}_1(0) = -p_1'(q(0))q^2(0)$ from (4) and (5a). Hence, the decline in profits causes a fall in $q(0)$.

Part (ii) follows by noting that a lower $q(0)$ implies a higher $p(0)$ in the case with arbitrage. Given that the duration of the final limit-pricing regime in the case without arbitrage is smaller than or equal to the duration of the final limit-pricing regime in the case with arbitrage (see Appendices A.2-A.3), a longer overall extraction period in the case without arbitrage would imply that p would be lower (and thus demand higher) throughout, while it takes longer to deplete the initial stock. This violates the resource constraint. $\square$

To demonstrate the effect of the presence of arbitrators on the entire price and extraction paths, we provide an example with the following HARA utility function:

$$U^i\left(\frac{q_i + x_i}{n_i}\right) = \frac{1-\varphi}{\varphi}\left[\left(\frac{\psi\left(\frac{q_i+x_i}{n_i}\right)}{1-\varphi} + \chi\right)^{\varphi} - \chi^{\varphi}\right], \quad i = A, B, \tag{13}$$

where x_i denotes consumption of renewables and n_i the population size in region i, which we use as a pivotal parameter. Accordingly, demand for fossil fuels in region A and B is given by, respectively

$$q_A = \begin{cases} n_A \frac{1-\varphi}{\psi}\left[\left(\frac{p+\tau}{\psi}\right)^{\frac{1}{\varphi-1}} - \chi\right] & \text{if } p \leq \hat{b} \\ 0 & \text{if } p > \hat{b} \end{cases}, \tag{14a}$$

$$q_B = \begin{cases} n_B \frac{1-\varphi}{\psi} \left[\left(\frac{p}{\psi} \right)^{\frac{1}{\varphi-1}} - \chi \right] & \text{if } p \leq b \\ 0 & \text{if } p > b \end{cases} . \tag{14b}$$

Our benchmark parametrization is shown in Table 1. Figure 5 shows the equilibria with (dashed gray lines) and without (solid black lines) arbitrators. Panel (a) contains time profiles for the price, and panel (b) for extraction. Both panels also depict the equilibrium under perfect competition (dotted gray lines). In our numerical example, the difference in initial extraction between the cases with and without arbitrators is small, but depletion occurs about a decade later in the presence of arbitrators. The intermediate limit-pricing phase of the first stage in the equilibrium with arbitrators is relatively short, and the switch to the second stage during which the monopolist only supplies to region B occurs sooner. The reason is that arbitrators effectively force the monopolist to sell a larger share of its stock to region B, because it is not possible to let the price jump upwards at the time of the switch. In the perfectly competitive equilibrium, initial extraction is larger and depletion occurs sooner than in both monopolistic equilibria, as shown by the dotted gray line. The perfectly competitive equilibrium does not feature limit-pricing phases.

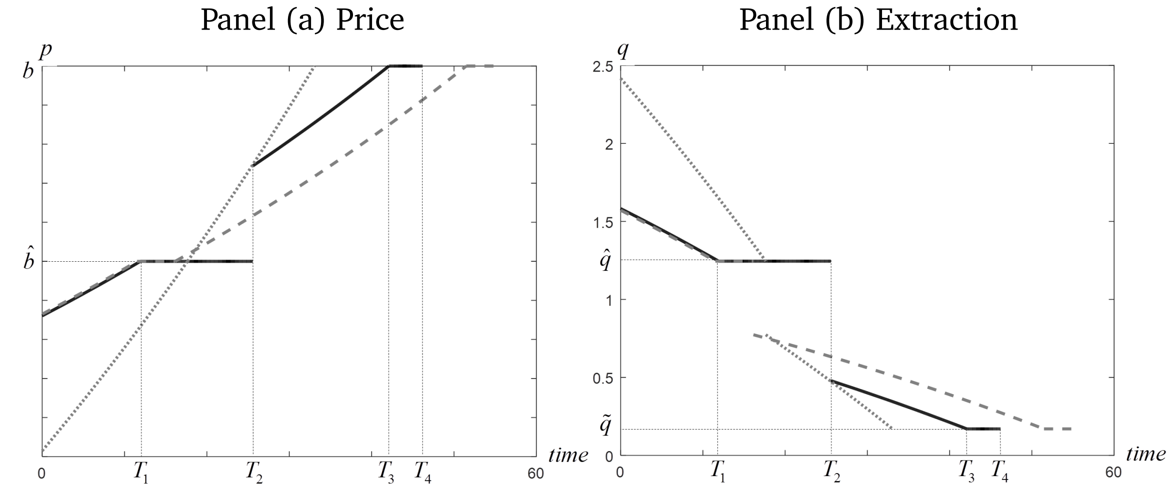
HARA parameter	φ	2
HARA parameter	ψ	0.91
HARA parameter	χ	9.1
Backstop production cost	b	8
Extraction cost	k	1
Discount rate	r	0.01
Initial resource stock	S_0	40
Size region A	n_A	0.5
Size region B	n_B	0.5
Climate damage parameter	δ	0.01144

Table 1: Benchmark parametrization

3 Policy analysis

In the present framework several policy relevant issues can be addressed. In this section we consider the question how the introduction or tightening of climate change policies in one region, whereas the other region stays inactive, affects supply of fossil fuel. The next section studies the welfare effects in this context. We first investigate what happens with initial fossil supply, $q(0)$, and with the time it takes to deplete the entire fossil fuel stock, T_4, starting from arbitrary initial carbon tax and renewable subsidy

Figure 5: Comparison of equilibria



Notes: The solid black (dashed gray) lines correspond to the equilibrium without (with) arbitrage. The dotted gray lines correspond to the equilibrium under perfect competition. Parameter values are $\varphi = 2$, $\psi = 0.91$, $\chi = 9.1$, $b = 8$, $k = 1$; $r = 0.01$, $S_0 = 40$, $\sigma = 0.5$, $\tau = 0.5$, and $n_A = n_B = 0.5$.

rates in region A.

Proposition 2 *Suppose* $S_0 > \max\{S_{03}, \tilde{S}_{03}\}$ *(implying that* $p(0) < \hat{b}$ *in the equilibria with and without arbitrage) and that the Hamiltonian is continuous (i.e.,* (9a) *holds with equality).*

(i) An increase in the backstop subsidy or an increase in the carbon tax lowers initial resource extraction in both equilibria.

(ii) An increase in the backstop subsidy lowers the time of exhaustion T_4 *in the equilibrium without arbitrators.*

Proof. Part (i). An increase in σ or τ makes the constraints that the monopolist faces more stringent. Hence $d\Lambda(S_0, b, \sigma, \tau)/d\sigma < 0$ and $d\Lambda(S_0, b, \sigma, \tau)/d\tau < 0$, which from (10) gives $d\mathcal{H}_1(0)/d\sigma < 0$ and $d\mathcal{H}_1(0)/d\tau < 0$. Moreover, from the strict concavity of $(p_1(q) - k)q$ in q, $\mathcal{H}_1(0) = -p_1'(q(0))q^2(0)$ implies $d\mathcal{H}_1(0)/dq(0) > 0$. Therefore, we get $dq(0)/d\sigma < 0$ and $dq(0)/d\tau < 0$.

Part (ii). First-order condition (5a) with $\lambda = \lambda_1 = \lambda_2$ and $\mu_{11}(0) = 0$ gives $\lambda(0) = p_1'(q(0))q(0) + p_1(q(0); \tau) - k$, which implies $d\lambda/dq(0) = [2p_1'(q(0)) + q(0)p_1''(q(0))] - d\tau/dq(0)$, the first term of which is negative due to strict concavity of $(p_1(q) - k)q$ in q. Using this result together with the end condition $\lambda = (b - k)e^{-rT_4}$, and $dq(0)/d\sigma < 0$ from part (i), while keeping $d\tau = 0$, we find $dT_4/db > 0$ and $dT_4/d\sigma < 0$. $\square$

In the remainder of this section, we interpret these findings and show how the introduction of a climate change policy in one region, whereas the other region stays inactive, affects fossil fuel supply. So, we now suppose that initially regions A and B are identical in policy terms, or $\tau = \sigma = 0$, and that region A introduces a subsidy on renewables, $\sigma > 0$, or a carbon tax, $\tau > 0$. Because an intermediate limit-pricing regime may appear once climate policies are in place, and because the monopolist responds differently to climate policies depending on the presence of arbitrators, it is interesting to show how the entire price and extraction paths are affected by the carbon tax and the renewables subsidy in the situations with and without arbitrators on the market. Figure 6 compares the equilibrium without climate policies (solid black line) to a regime with a carbon tax (dashed line) and a renewables subsidy (dotted line) in our numerical example with HARA utility. Panel (a) depicts the case with and panel (b) without arbitrators. The superscripts τ and σ attached to the different regime switching times refer to the cases with a carbon tax and a renewables subsidy, respectively.

In line with Proposition 2, Figure 6 shows that initial extraction goes down upon the introduction of the carbon tax and the renewables subsidy in both equilibria. In the equilibrium without arbitrators, the subsidy speeds up depletion (cf. panel (b) of Figure 6, in line with Proposition 2 (ii)), whereas it postpones depletion in the presence of arbitrators (cf. panel (a)). The reason is that in the equilibrium with arbitrators, a subsidy lowers $\hat{b}$ and thus forces the monopolist to sell a larger share of its fossil reserves during the second stage, when demand from the climate-aware region has vanished. In the equilibrium without arbitrators, however, the monopolist can let the price jump upward at the moment of the switch, which lowers cumulative fossil extraction during the second stage.[17] In our numerical example, the carbon tax postpones depletion in both equilibria. Moreover, it induces the monopolist to only perform an intermediate limit-pricing strategy when there are no arbitrators on the market, from $\tilde{T}_1^\tau$ until $\tilde{T}_2^\tau$ in panel (b).

Intuitively, in the presence of arbitrators the carbon tax forces the monopolist to sell more during the second stage. Furthermore, the tax lowers the profitability of fossil extraction during the first stage relative to the second stage. The monopolist responds by by reducing the duration of the intermediate limit-pricing phase to shorten the first stage. If the tax becomes large enough, the intermediate limit-pricing phase disappears altogether. In case of a renewables subsidy, however, both equilibria feature

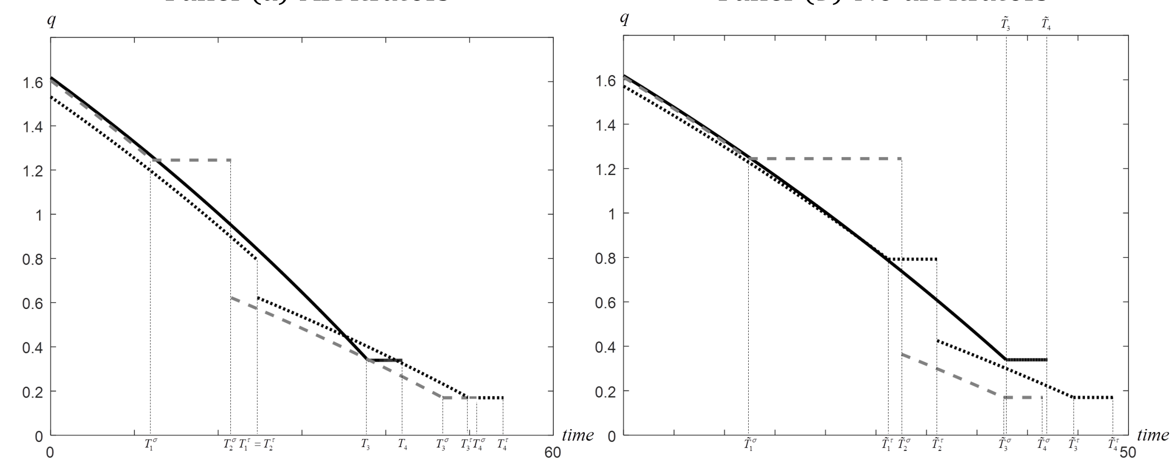
[17]Technically, in the equilibrium without arbitrators we have $\lambda_1 = \lambda_2$ and λ_1 increases due to the subsidy. It then follows from $\lambda_1 e^{rT_4} = b - k$ that T_4 must go down. In the equilibrium with arbitrators, however, $\lambda_1 > \lambda_2$, implying that this reasoning does not hold.

an intermediate limit-pricing phase.[18]

Figure 6: Effect of climate policies on extraction paths

Panel (a) Arbitrators

Panel (b) No arbitrators



Notes: The solid black lines correspond to the equilibrium with $\sigma = \tau = 0$. The dashed line represents the case with $\sigma = 0.75$ and $\tau = 0$. The dotted line represents the case with $\sigma = 0$ and $\tau = 0.75$. Parameter values are $\varphi = 2$, $\psi = 0.91$, $\chi = 9.1$, $b = 8$, $k = 1$; $r = 0.01$, $S_0 = 40$, and $n_A = n_B = 0.5$. Superscripts τ and σ refer to the cases with a carbon tax and a renewables subsidy, respectively.

4 Welfare analysis

The asymmetric effects of climate policies on the extraction path across the two equilibria have consequences for welfare as well. In this section, we perform a numerical welfare analysis in our example with HARA utility. For the damage function we take $D(E) = \delta E$. We choose the damage parameter δ such that climate damages in region A correspond to 50 US$ per ton carbon. The associated Pigouvian tax rate is 0.572.[19] Furthermore, we assume quasi-linear preferences so that total welfare in region A is

[18]Technically, the carbon tax increases the term $\eta_2(\hat{q}_B)\hat{q}_B$ on the right-hand side of condition (12) by more than the term $\eta_1(\hat{q})\hat{q}$ on the left-hand side, as final demand in the climate-aware region, $\hat{q}_A$, remains unaffected, whereas $\hat{q}_B$ goes up. Therefore, $\mu_{11}(T_2^-)$ must fall. If the tax is large enough, the limit-pricing phase might disappear (i.e., $\mu_{11} = 0$), as in our numerical example. The subsidy, however, both increases $\hat{q}_A$ and $\hat{q}_B$, which attenuates the effect on the duration of the intermediate limit-pricing phase.

[19]Take OPEC reserves equal to 150 billion ton carbon (Heede and Oreskes, 2016) and an oil price equal to 606.5 US$ per ton carbon (Golosov et al., 2014). In our benchmark equilibrium with $S_0 = 40$, $\rho = 0.01$, and $\tau = \sigma = 0$ we get $p(0) = 6.9407$. Together, these numbers imply that 50 US$ per ton carbon correspond to $n_A\delta/\rho = 0.572$ units of the numeraire per unit of the resource, yielding $\delta = 0.01144$ for $n_A = 0.5$.

defined by

$$W^A = \int_0^\infty e^{-rt} \left(n_A U^A \left(\frac{q_A(t) + x_A(t)}{n_A} \right) - b x_A(t) - p(q_A(t)) q_A(t) \right) dt - \int_0^\infty e^{-rt} n_A D(E(t)) dt, \tag{15}$$

where $E(t) = E_0 + \int_0^t (q_A(s) + q_B(s)) ds$. Fossil fuel demand is given by (14a) and demand for renewables follows from $dU^A(x_A)/dx_A = b - \sigma$ if $x_A > 0$.

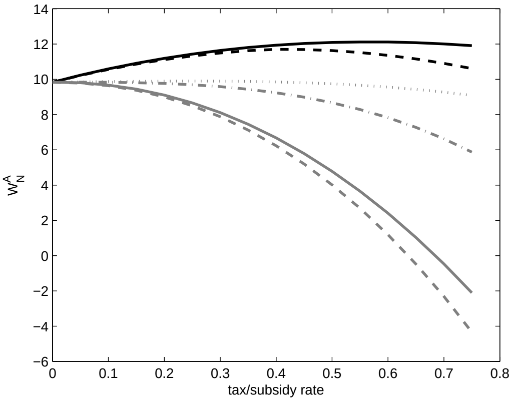
Given the distortions due to monopolistic fossil fuel supply and the climate externality, the equilibrium without a carbon tax and a renewables subsidy is clearly second-best. Furthermore, the policy instruments do not only affect efficiency by changing the timing of fossil supply, but also the distribution of welfare between regions A and B and the monopolist by changing the scarcity rent. Figure 7 shows the effect of a carbon tax and a renewables subsidy on different welfare components: non-green welfare, W_N^A (the first integral in (15)), climate damage, Z (the second integral), and total welfare, W^A, which is the difference between the two. The black lines represent the effect of a carbon tax, whereas the gray lines show the effect of a renewables subsidy. Panels (a), (b), and (c) depict the case of monopolistic fossil fuel supply, where the solid (dashed) lines correspond to the case without (with) arbitrators on the market. Panels (d), (e), and (f) exhibit the situation under perfectly competitive fossil fuel supply.

Panel (a) shows that the renewables subsidy lowers non-green welfare in both equilibria, whereas a not too high carbon tax is beneficial for non-green welfare. The difference between the instruments in terms of welfare effects is largely due to the fact that the renewables subsidy distorts energy use after depletion of the fossil stock, because we have assumed that the subsidy remains in place forever. The dotted (dash-dotted) lines show that the effect of a subsidy that is unexpectedly and permanently removed after depletion of the fossil reserve in the case without (with) arbitrators is less harmful for non-green welfare.

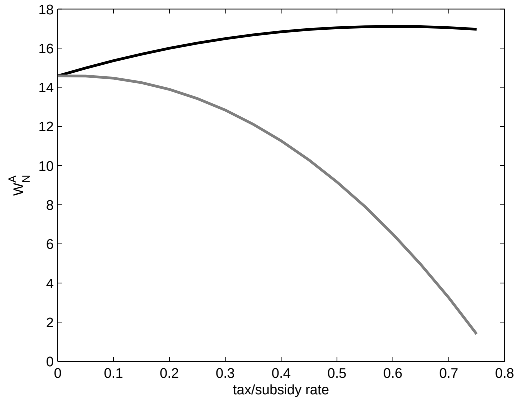
It can be seen from the results in panel (b) that the introduction of a carbon tax lowers climate damage in both equilibria. In contrast, the subsidy aggravates climate damage in the equilibrium without arbitrators due to front-loading of fossil supply, a so called Green Paradox effect materializes (cf. Sinn, 2008, 2012). When there are arbitrators on the market, however, the subsidy lowers climate damage. The reason is that the presence of arbitrators forces the monopolist to sell a larger share of their reserves in the second stage, when fossil demand from the climate-aware region A has dropped to zero. Accordingly, the switch from supplying both regions to only supplying

Figure 7: Welfare effects of a renewable subsidy and a carbon tax

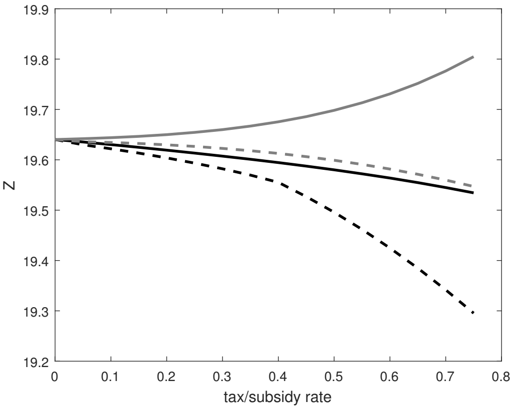
(a) Non-green welfare, monopoly



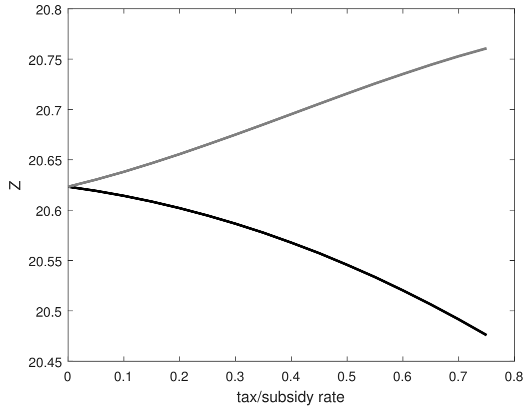
(d) Non-green welfare, perfect competition



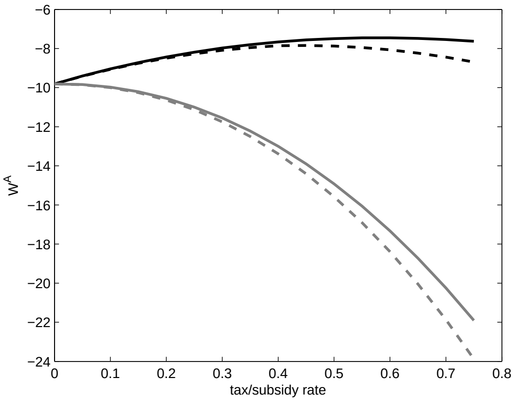
(b) Climate damage, monopoly



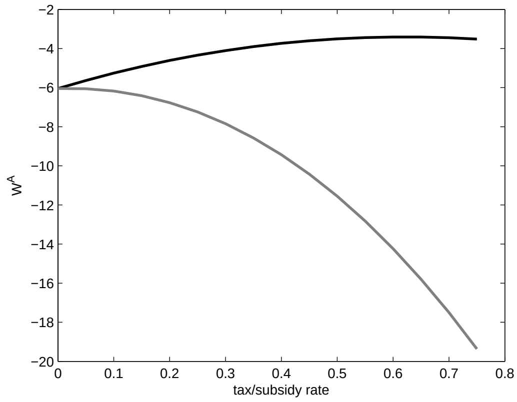
(e) Climate damage, perfect competition



(c) Total welfare, monopoly



(f) Total welfare, perfect competition



Notes: The solid (dashed) line represents the case without (with) arbitrators. The black lines correspond to the scenarios with a carbon tax. The gray lines correspond to the scenarios with a renewables subsidy. In panel (a), the dotted (dash-dotted) lines represent the situation of a subsidy that is unexpectedly removed at the moment of depletion of the fossil stock in the case without (with) arbitrators. Parameter values are $\delta = 0.01144$, $\varphi = 2$, $\psi = 0.91$, $\chi = 9.1$, $b = 8$, $k = 1$; $r = 0.01$, $S_0 = 40$, and $n_A = n_B = 0.5$.

region B occurs sooner, which implies that fossil supply is spread out over a longer time horizon, as shown in Figure 6.

Panel (c) shows that, on balance, the unilateral introduction of a not too high carbon tax increases welfare, whereas a renewables subsidy is detrimental for welfare in region A. The right column of Figure 7 exhibits the results of the welfare analysis when the market for fossil fuels would be perfectly competitive. Qualitatively, the effects of renewables subsidies and carbon taxes are comparable to the monopolistic case without arbitrators. The welfare level, however, differs from the monopolistic case. Non-green welfare is larger under perfect competition (panel (d)). Climate damage, however, is larger as well (panel (e)). In the specific example at hand, this implies that aggregate welfare is higher under perfect competition, as shown in panel (f).

The most outstanding result of our welfare analysis is that although a renewables subsidy actually increases climate damages under perfect competition and monopoly without arbitrators (due to a Green Paradox effect), the presence of arbitrators reverses this outcome: with arbitrators on the market, a renewables subsidy lowers climate damage, even when cumulative fossil fuel supply remains unchanged.

5 Conclusion

This paper offers a full characterization of the equilibrium in a resource extraction framework with monopolistic supply of fossil fuel and multiple heterogeneous regions with differential climate policies. The framework gives rise to a two-stage optimal control problem for the monopolist. It has been shown that with differential climate change policies two stages appear in the equilibrium: A first stage, in which both regional markets, i.e. the markets in the regions with and without climate policies, are served (at least if the resource stock is large enough). This initial stage is followed by a stage in which only the region without climate policies in place is supplied with fossil fuel. The latter stage always has a final phase with limit pricing, whereas the former stage may not entail a limit-pricing phase. In the absence of arbitrage, there is an upward price jump at the transition point from the first to the second stage.

Our results are complementary to those of Andrade de Sá and Daubanes (2016). They argue that in case of inelastic demand, oil suppliers choose for limit pricing throughout, which restrains the effectiveness of climate policies such as carbon taxation and renewables subsidies. We show that, also in the case of elastic demand, limit pricing may be more important than suggested by conventional analyses of climate policy effects. The reason is that heterogeneous climate policies may cause an additional, intermediate limit pricing phase. Moreover, we have emphasized the importance of arbitrators on the market for the fossil extraction path and for the effectiveness of climate policies.

With the possibility of arbitrage, the monopolist is unable to let the price jump at the transition to supplying fossil fuel only to the region without climate policies in place. Hence, the transition takes place earlier, at the cost of a shorter first phase of limit pricing, which also implies, however, a longer time to fully deplete the fossil fuel stock. Our numerical welfare analysis suggests that a subsidy for renewables increases climate damage in the case without arbitrators, due to a Green Paradox effect. In the case with arbitrators, on the contrary, the renewables subsidy lowers climate damage, because the monopolist is forced to sell a larger share of his fossil reserve during the second stage, when demand from the policy active region has vanished. This result is relevant for policy makers.

Another policy relevant issue regards the social welfare effects in regions that consider to take unilateral action against climate change. Upon the introduction or tightening of climate policies, the monopolist shifts its supply to the unregulated region such that the regulated region switches earlier to backstop use. This (intertemporal) carbon leakage effect lowers non-green welfare in the regulated region. We see that a carbon tax policy may still increase social welfare in these regions, in particular when arbitrage is not feasible. On the contrary, a subsidy for renewables is detrimental to welfare in these regions. The conclusion that a carbon tax performs better than a subsidy is maintained even if the subsidy is (unexpectedly) reduced to zero as soon as all fossil fuel is depleted. Finally, maybe surprisingly, arbitrage has a negative but not a major impact on total social welfare in the region that takes unilateral action. These results are obtained for specific welfare functions but it is to be expected that at least the superiority of taxation remains valid in more general settings.

Although we have constrained ourselves to studying the case of a pure monopoly, which is not the most accurate representation of the real world, the occurrence of limit-pricing in the model indicates that backstop investments (or subsidies for renewables, as introduced formally in this model) lead to lower initial fossil fuel supply. This is the opposite of what is found in case of perfect competition. Hence, we can exclude the occurrence of a Weak Green Paradox as a consequence of climate policy. This concept, however, is not of much use for judging the desirability of climate policies in our framework. The reason is that due to the existence of the limit-pricing phases, the resource extraction and price paths before and after the policy changes cross several times.

Our study exhibits some limitations. We do not derive optimal policies and assume constancy over time of the policy instruments. Moreover, it would be interesting to allow for differences in climate policies between countries within the policy-active world, which would give rise to the existence of additional limit-pricing regimes. Furthermore, we assume that the monopolist is not able to use price discrimination. This is a valid assumption for the oil market, for instance, since oil can be easily shipped and is traded globally. Yet, the assumption might not hold in the case of gas, which is traded mostly regionally or by bilateral trading agreements. Additionally, we do not consider strategic behavior on the part of the importing and exporting regions. This is an interesting and promising direction to extend the paper. Also, the markets for fossil fuels are not purely monopolistic. Research should be extended so as to include oligopoly or cartel-fringe market structures, which might answer questions related to the sequence of fuel extraction and the conditions under which simultaneous limit-pricing will take place (cf. Benchekroun et al., 2009, 2010). The model would also gain in value by allowing for technological progress in the backstop technology, for R&D expenditures on developing better backstop technologies, which would allow for decreasing fossil prices and increasing energy use during limit-pricing phases (cf. Jaakkola, 2015) and partial exhaustion if the marginal costs of the backstop technology rapidly fall below the marginal extraction cost of fossil fuels (cf. Fischer and Salant, 2012). Finally, it would be interesting to allow for set-up costs of renewables (like windmills).

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A Appendix

A.1 Two-stage optimal control

The necessary conditions for the solution of the monopolist's problem are derived from the theory on two-stage optimal control problem, as described in, e.g., Tomiyama (1985). The Hamiltonians $\mathcal{H}_i$ associated with the first stage ($i = 1$) and second stage ($i = 2$) of the optimal control problem for the monopolist are given by:

$$\mathcal{H}_i = e^{-rt}(p_i(q) - k)q - \lambda_i q, \quad i = 1, 2, \tag{A.1}$$

where λ_i denotes the shadow price of the resource stock in stage i. The corresponding Lagrangians read

$$\mathcal{L}_1 = e^{-rt}(p_1(q) - k)q - \lambda_1 q + e^{-rt}\mu_{11}(\hat{b} - p_1(q)), \tag{A.2a}$$

$$\mathcal{L}_2 = e^{-rt}(p_2(q) - k)q - \lambda_2 q + e^{-rt}[\mu_{21}(b - p_2(q)) + \mu_{22}(p_2(q) - \hat{b})], \tag{A.2b}$$

where the μ_{ij}'s are Lagrange multipliers associated with the inequality constraints (2b)-(2d). The following complementary slackness conditions hold (we omit the time argument when there is no danger of confusion):

$$\mu_{11}(\hat{b} - p_1(q)) = 0, \ \mu_{11} \geq 0, \tag{A.3a}$$

$$\mu_{21}(b - p_2(q)) = 0, \ \mu_{21} \geq 0, \tag{A.3b}$$

$$\mu_{22}(p_2(q) - \hat{b}) = 0, \ \mu_{22} \geq 0, \tag{A.3c}$$

which require that the μ_{ij}s equal zero as long as the corresponding restrictions on the price are non-binding. The necessary first-order conditions with respect to resource extraction read:

$$\lambda_1 e^{rt} = p_1'(q)q + p_1(q) - k - \mu_{11}p_1'(q), \tag{A.4a}$$

$$\lambda_2 e^{rt} = p_2'(q)q + p_2(q) - k - \mu_{21}p_2'(q) + \mu_{22}p_2'(q). \tag{A.4b}$$

The first-order conditions with respect to the resource stock requires the shadow price of the resource to be constant over time:

$$\dot{\lambda}_i = -\frac{\partial \mathcal{H}_i}{\partial S} = 0. \tag{A.5}$$

The optimality conditions can be used to find the solution to the monopolist's problem for given T_2, T_4, and $S(T_2)$. The following transversality condition holds since the optimal stopping time T_4 is endogenous (cf. Seierstad and Sydsæter, 1987, p. 213):

$$\mathcal{H}_2(T_4) = 0, \tag{A.6}$$

where $\mathcal{H}_2(T_4)$ is shorthand for the Hamiltonian evaluated at the instant of time T_4. So

$$e^{-rT_4}(p_2(q(T_4)) - k)q(T_4) - \lambda_2 q(T_4) = 0. \tag{A.7}$$

By combining (A.4b) and (A.7) we find $\mu_{21}(T_4) - \mu_{22}(T_4) = q(T_4) > 0$, which gives $\mu_{21}(T_4) > 0$ and thus $\mu_{22}(T_4) = 0$ and $p(T_4) = b$. Hence, there always exists a final non-degenerate interval of time with limit pricing at b. As a result, (A.6) implies

$$e^{-rT_4}(b - k) = \lambda_2. \tag{A.8}$$

More necessary conditions are needed to characterize the optimum. These depend on what is assumed with regard to arbitrage. We will analyze the two cases separately and prove Theorems 1 and 2.

A.2 Equilibrium with arbitrators

If arbitrage is possible at negligible cost the price path must be continuous over the entire program, in particular at the transition, if any, from stage 1 to stage 2, as required by condition (8). Our strategy to prove Theorem 1 is to define critical initial fossil fuel stocks that warrant a specific optimal extraction path. If a program satisfies the necessary conditions and the transversality conditions, it is the unique optimum due to our assumption of strict concavity of the profit function.

We have shown already that there is a final stage with limit pricing at b. We suppose that climate policies are in place: $\sigma > 0$ and/or $\tau > 0$. Consider the final phase, with $T_3 \leq t \leq T_4$, and define $T_4 - T_3$ and S_0 by

$$e^{rT_4 - rT_3} = \frac{b - k}{p_2'(\tilde{q}_B)\tilde{q}_B + b - k}, \text{ and } S_{01} = (T_4 - T_3)\tilde{q}_B. \tag{A.9}$$

The claim is that if $S_0 \leq S_{01}$, it is optimal to have limit pricing at a price b from the start, until full exhaustion. To prove this, we check all the necessary conditions for this program. Set $T_1 = T_2 = T_3 = 0$. Define $\hat{T}_4$ by $\hat{T}_4 = S_0/\tilde{q}_B$. For $0 \leq t \leq \hat{T}_4$ take

$\mu_{22}(t) = 0$, $q(t) = \tilde{q}_B$. Moreover, using (A.9),

$$p_2'(\tilde{q}_B)\tilde{q}_B + b - k = (b-k)e^{-rT_4} \leq (b-k)e^{-r\hat{T}_4}.$$

since $\hat{T}_4 \leq T_4$ defined above because $S_0 \leq S_{01}$ by assumption. Note that $p_2(\tilde{q}_B) = b$ and use first-order condition (A.4b) and transversality condition (A.6) to get

$$e^{rt}\mu_{21}(t)p_2'(\tilde{q}_B) = p_2'(\tilde{q}_B)\tilde{q}_B + b - k - (b-k)e^{r(t-\hat{T}_4)} < 0,$$

for all $t > 0$. Hence $\mu_{21}(t) > 0$ for $0 < t \leq \hat{T}_4$. Therefore, all necessary conditions and the transversality condition are satisfied so that the program proposed is optimal.

Now consider the phase with $T_2 \leq t \leq T_3$. Define $T_3 - T_2$ and S_{02} by

$$e^{rT_3 - rT_2} = \frac{p_2'(\tilde{q}_B)\tilde{q}_B + b - k}{p_2'(\hat{q}_B)\hat{q}_B + \hat{b} - k} \text{ and } S_{02} = \int_{T_2}^{T_3} g_2(\lambda_2 e^{rt})dt + S_{01}, \tag{A.10}$$

with $\lambda_2 e^{rT_2} = p_2'(\hat{q}_B)\hat{q}_B + \hat{b} - k$ and where $g_2(\lambda_2 e^{rt})$ is the solution for $q(t)$ from (A.4b) with $\mu_{12}(t) = \mu_{22}(t) = 0$ ($T_2 \leq t \leq T_3$). The claim is that if $S_0 = S_{02} > S_{01}$ it is optimal to set the initial price equal to $\hat{b}$, let the price increase up to b and have limit pricing at price b thereafter, hence serving region B's market only. We show that this program satisfies all the necessary conditions as well as the transversality condition and is therefore optimal. Set $T_2 = 0$. Time T_3 follows from (A.10) with $T_2 = 0$. For $0 \leq t \leq T_3$ use first-order condition (A.4b) with $\mu_{12}(t) = \mu_{22}(t) = 0$ ($T_2 \leq t \leq T_3$) to get

$$(p_2'(\hat{q}_B)\hat{q}_B + \hat{b} - k)e^{rt} = p_2'(q(t))q(t) + p_2(q(t)) - k. \tag{A.11}$$

Hence, q is decreasing along the interval, from the assumption that $(p_2(q)-k)q$ is strictly concave, and therefore p is increasing, so that $\hat{b} < p(t) < b$ in the interior of the interval and the assumption $\mu_{12}(t) = \mu_{22}(t) = 0$ is warranted. Once the price b is reached at T_3 the resource stock equals S_{01}. From then on it is optimal to have limit pricing, as in the previous case. This proves the claim. If $S_{02} > S_0 > S_{01}$ then it is clearly optimal to set $T_2 = 0$ and $q(0) < \hat{q}_B$.

For the phase with $T_1 \leq t \leq T_2$ we distinguish between two different cases: $T_2 > T_1$ and $T_2 = T_1$. From Lemma 2 and the surrounding discussion, we have the following optimality condition:

$$\mathcal{H}_1(T_2^-) \leq \mathcal{H}_2(T_2^+), \ \left(\mathcal{H}_1(T_2^-) - \mathcal{H}_2(T_2^+)\right)(T_2 - T_1) = 0, \tag{A.12}$$

which requires the Hamiltonian to be continuous at $t = T_2$ if $T_2 > T_1$.

First, consider the case with $p_2'(\hat{q}_B)(\hat{q}_B)^2 - p_1'(\hat{q})(\hat{q})^2 > 0$, which implies

$$\frac{p_2'(\hat{q}_B)\hat{q}_B^2 + (\hat{b} - k)\hat{q}}{p_1'(\hat{q})\hat{q}^2 + (\hat{b} - k)\hat{q}} > 1. \tag{A.13}$$

Define $T_2 - T_1$ and S_{03} by

$$e^{rT_2 - rT_1} = \frac{p_2'(\hat{q}_B)\hat{q}_B^2 + (\hat{b} - k)\hat{q}}{p_1'(\hat{q})\hat{q}^2 + (\hat{b} - k)\hat{q}}, \text{ and } S_{03} = (T_2 - T_1)\hat{q} + S_{02}. \tag{A.14}$$

The claim is that if $S_{03} = S_0$ it is optimal to start from time zero on with limit pricing at $\hat{b}$ until the stock has reached S_{02}, from where on the previously derived optimal path is followed. To prove the claim we construct a path that satisfies all the necessary conditions and the transversality condition. Take $T_1 = 0$. Time T_2 follows from (A.14). Take $q_A(t) = \hat{q}_A$ and $q_B(t) = \hat{q}_B$ for $0 \le t \le T_2$. At T_2 the stock has reached S_{02} and the path derived previously is optimal from then on. Moreover, the price is continuous at T_2. Take λ_1 and λ_2 such that

$$e^{-rT_2}(\hat{b} - k)\hat{q} - \lambda_1\hat{q} = e^{-rT_2}(\hat{b} - k)\hat{q}_B - \lambda_2\hat{q}_B, \text{ and } \lambda_2 e^{rT_2} = p_2'(\hat{q}_B)\hat{q}_B + \hat{b} - k. \tag{A.15}$$

Then (A.12) is satisfied. It follows from (A.14) that $\lambda_1 = p_1'(\hat{q})\hat{q} + (\hat{b} - k)$. Hence, (A.4a) is satisfied with $\mu_{11}(0) = 0$. We still need to check the non-negativity of the multiplier μ_{11} for $0 < t \le T_2$. From (A.4a), we have

$$e^{rt}\mu_{11}(t)p_1'(\hat{q}) = (p_1'(\hat{q})\hat{q} + \hat{b} - k)(1 - e^{rt}) < 0 \text{ for } t > 0. \tag{A.16}$$

Hence $\mu_{11}(t) \ge 0$ for $0 < t \le T_2$. If $S_{03} > S_0 > S_{02}$ then it is clearly optimal to start with limit pricing at price $\hat{b}$ from the outset.

Second, consider the case with $p_2'(\hat{q}_B)(\hat{q}_B)^2 - p_1'(\hat{q})(\hat{q})^2 \le 0$. Define $T_1 = T_2$, which implies that (A.12) is satisfied.[20] With $T_1 = T_2$ there is no intermediate limit-pricing, i.e., $S_{03} = S_{02}$, so that with $S_0 = S_{02} = S_{03}$ it is optimal to set the initial price equal to $\hat{b}$, let the price increase up to b and have limit pricing at price b thereafter, hence serving region B's market only, as shown before.

Note that in both cases, (A.12) implies $((\hat{b} - k)e^{-rT_2} - \lambda_2)(\hat{q} - \hat{q}_B) \le (\lambda_1 - \lambda_2)\hat{q}$, which from $\hat{q} > \hat{q}_B$ and (A.2b) with $\mu_{21}(T_2) = \mu_{22}(T_2)$ imposed yields $\lambda_1 > \lambda_2$.

[20] If $p_2'(\hat{q}_B)(\hat{q}_B)^2 - p_1'(\hat{q})(\hat{q})^2 < 0$, restriction $p(q(T_2^-) = \hat{b}$ implies that the monopolist is forced to choose T_2 equal to the T_2 it would choose if $p_2'(\hat{q}_B)(\hat{q}_B)^2 - p_1'(\hat{q})(\hat{q})^2 = 0$, say T_2^* . Hence, the monopolist effectively faces the constraint $T_2 \ge T_2^*$, implying $\mathcal{H}_1(T_2^-) \le \mathcal{H}_2(T_2^+)$.

Finally, consider the phase with $0 \le t \le T_1$. Define T_1 and S_0 by

$$e^{rT_1} = \frac{p_1'(\tilde{q})\tilde{q} + p_1(q) - k}{\lambda_1} \text{ and } S_0 = \int_0^{T_1} g_1(\lambda_1 e^{-rt})dt + S_{03}, \tag{A.17}$$

where $g_1(\lambda_1 e^{rt})$ is the solution for $q(t)$ from (A.4a) with $\mu_{11}(t) = 0$ $(t \le T_1)$.

The claim is that if $S_0 > S_{03}$ it is optimal to set the initial price below $\hat{b}$ and let the price increase up to $\hat{b}$. Time T_1 follows from (A.17). For $0 \le t \le T_1$ use first-order condition (A.4a) with $\mu_{11}(t) = 0$ $(t \le T_1)$ to get

$$(p_1'(\hat{q})\hat{q} + \hat{b} - k)e^{rt} = p_1'(q(t))q(t) + p_1(q(t)) - k. \tag{A.18}$$

Hence, q is decreasing along the interval, from the assumption that $(p_1(q)-k)q$ is strictly concave, and therefore p is increasing, so that $p(t) < \hat{b}$ in the interior of the interval and the assumption $\mu_{11}(t) = 0$ is warranted. Once the price $\hat{b}$ is reached at T_1 the resource stock equals S_{03} From then on it is optimal to have either limit pricing for a while at price $\hat{b}$ or to let the price increase over time up to b, as shown before.

This proves Theorem 1.

A.3 Equilibrium without arbitrators

If arbitrage is not possible, the monopolist is free to choose $S(\tilde{T}_2)$ and $\tilde{T}_2$, implying that the Hamiltonians and the shadow prices in the two stages coincide in the optimum, as stated in Lemma 1. Theorem 2 can be proved along the same lines as Theorem 1. Here we will focus on the main differences: a jump in the price of fossil fuel at $\tilde{T}_2$ (if $\tilde{T}_2 > 0$) and a potential shorter maximum duration of the final limit-pricing phase, because $\mu_{21}(\tilde{T}_2^+)$ is no longer necessarily equal to zero.

Lemma A.1 *Provided that $\tilde{T}_2 > 0$, the resource price jumps up at $\tilde{T}_2$.*

Proof. Suppose, on the contrary, that the price is continuous at $\tilde{T}_2$. Then $p(\tilde{T}_2^-) = p(\tilde{T}_2^+) = \hat{b}$. Since $q(\tilde{T}_2^-) = \hat{q}_A + \hat{q}_B$ and $q(\tilde{T}_2^+) = \hat{q}_B > 0$, it follows from (A.6) and (7b) in (7a) that $\mathcal{H}_1(\tilde{T}_2) = \mathcal{H}_2(\tilde{T}_2) = 0$. However, substitution of (A.4b) into (A.1) gives

$$\mathcal{H}_2(\tilde{T}_2) = -\left\{p_2'(q(\tilde{T}_2^+))q(\tilde{T}_2^+) + \mu_{22}(\tilde{T}_2^+)p_2'(q(\tilde{T}_2^+))\right\} q(\tilde{T}_2^+) > 0,$$

where we have used $p_2(q(\tilde{T}_2^+)) = \hat{b} < b$, implying that $\mu_{21}(\tilde{T}_2^+) = 0$. So, we have reached a contradiction. Hence, the price must jump up at $t = \tilde{T}_2$. $\square$

The size of the jump and the conditions under which an intermediate limit-pricing phase before the jump takes place can also be determined analytically by combining the first-order conditions (A.4a)-(A.4b) with the two matching conditions (7a) and (7b), yielding the result in the following lemma.

Lemma A.2 *Provided that $\tilde{T}_2 > 0$, the jumps in resource extraction and the resource price, and the values of the Lagrange multipliers at the stage switch satisfy:*[21]

$$p_1[q(\tilde{T}_2^-)] - p_2[q(\tilde{T}_2^+)] = p_2'[q(\tilde{T}_2^+)]\left[q(\tilde{T}_2^-) - q(\tilde{T}_2^+)\right]\frac{q(\tilde{T}_2^+) - \mu_{21}(\tilde{T}_2^+)}{q(\tilde{T}_2^-)}, \tag{A.19a}$$

$$\begin{aligned}\mu_{11}(\tilde{T}_2^-)p_1'[q(\tilde{T}_2^-)] &= p_1[q(\tilde{T}_2^-)] + p_1'[q(\tilde{T}_2^-)]q(\tilde{T}_2^-) \\ &\quad - \left(p_2[q(\tilde{T}_2^+)] + p_2'[q(\tilde{T}_2^+)]q(\tilde{T}_2^+)\right) + \mu_{21}(\tilde{T}_2^+)p_2'[q(\tilde{T}_2^+)].\end{aligned} \tag{A.19b}$$

Proof. First, note that $\mu_{22}(\tilde{T}_2^+) = 0$ due to the result in Lemma A.1. Second, substitution of (A.4b) into (A.1) and subsequently using (7a) gives (A.19a). Third, combining (A.4a)-(A.4b) gives (A.19b). □

Condition (A.19a) relates the jump in the resource price to the jump in demand, whereas condition (A.19b) determines the value of the Lagrange multiplier μ_{11} at the end of the first stage. The existence of a limit-pricing phase before $\tilde{T}_2$ requires a positive $\mu_{11}(\tilde{T}_2^-)$ (from the complementary slackness condition (A.3a)) and therefore a negative right-hand side. Although the second row of (A.19b) is always negative, the marginal revenue showing up in the first row should not be too high in order for the right-hand side to be negative on balance. In terms of panels (a) and (b) of Figure 3 in the main text, point G should not be located too high (like points F are in panels (c) and (d)).

Considering the duration of the final limit-pricing phase ($\tilde{T}_3 \leq t \leq \tilde{T}_4$), define $\tilde{T}_4 - \tilde{T}_3$ and $\tilde{S}_0$ by

$$e^{r\tilde{T}_4 - r\tilde{T}_3} = \frac{b-k}{p_2'(\tilde{q}_B)\tilde{q}_B + b - k - \mu_{21}(\tilde{T}_3)p_2'\hat{q}_B}, \text{ and } \tilde{S}_{01} = (\tilde{T}_4 - \tilde{T}_3)\tilde{q}_B. \tag{A.20}$$

The claim is that if $\tilde{S}_0 \leq \tilde{S}_{01}$, it is optimal to have limit pricing at a price b from the start, until full exhaustion. To prove this, we check all the necessary conditions for this program. Set $\tilde{T}_1 = \tilde{T}_2 = \tilde{T}_3 = 0$. Define $\hat{T}_4$ by $\hat{T}_4 = \tilde{S}_0/\tilde{q}_B$. For $0 \leq t \leq \hat{T}_4$ take

[21] Together with the complementary slackness conditions (A.3a) and (A.3b), equations (A.19a)-(A.19b) can be used to solve for, $q(\tilde{T}_2^-)$, $\mu_{11}(\tilde{T}_2^-)$, $q(\tilde{T}_2^+)$, and $\mu_{21}(\tilde{T}_2^+)$.

$\mu_{22}(t) = 0$, $q(t) = \tilde{q}_B$. Moreover, using (A.20),

$$p_2'(\tilde{q}_B)\tilde{q}_B + b - k - \mu_{21}(0)p_2'\hat{q}_B = (b-k)e^{-r\tilde{T}_4} \leq (b-k)e^{-r\hat{T}_4}.$$

since $\hat{T}_4 \leq \tilde{T}_4$ defined above because $\tilde{S}_0 \leq \tilde{S}_{01}$ by assumption. Note that $p_2(\tilde{q}_B) = b$ and use first-order condition (A.4b) and transversality condition (A.6) to get

$$e^{rt}\mu_{21}(t)p_2'(\tilde{q}_B) = p_2'(\tilde{q}_B)\tilde{q}_B + b - k - (b-k)e^{r(t-\hat{T}_4)} < 0,$$

for all $t > 0$. Hence $\mu_{21}(t) \geq 0$ for $0 < t \leq \hat{T}_4$. Therefore, all necessary conditions and the transversality condition are satisfied so that the program proposed is optimal.

Note that if $\mu_{21}(\tilde{T}_3) > 0$, we get $\tilde{T}_4 - \tilde{T}_3 < T_4 - T_3$ (see (A.9) and (A.20)), implying that the duration of the final limit-pricing phase is shorter than in the case with arbitrators. Furthermore, $\mu_{21}(\tilde{T}_3) > 0$ also gives $\tilde{T}_3 = \tilde{T}_2$, which is not possible in the equilibrium with arbitrators (provided that $T_2 > 0$).

The remaining critical initial fossil fuel stocks $\tilde{S}_{02}$ and $\tilde{S}_{03}$ that warrant a specific optimal extraction path can be defined along the lines of the proof of Theorem 1. If a program satisfies the necessary conditions and the transversality conditions, it is the unique optimum due to our assumption of strict concavity of the profit function.

A.4 Discounted profits

The results so far can be used to derive equation (10) in the main text:

Lemma A.3 *Suppose the Hamiltonian is continuous, then the relationship between the Hamiltonian at time zero and discounted profits of the monopolist is given by:*

$$\frac{\mathcal{H}_1(0)}{r} = \Lambda(S_0, b, \sigma, \tau). \tag{A.21}$$

Proof. The time derivative of the Hamiltonian $\mathcal{H}_1$ in (A.1) is given by

$$\dot{\mathcal{H}}_1 = \frac{\partial \mathcal{H}_1}{\partial S}\dot{S} + \frac{\partial \mathcal{H}_1}{\partial \lambda_1}\dot{\lambda}_1 + \frac{\partial \mathcal{H}_1}{\partial q}\dot{q} + \frac{\partial \mathcal{H}_1}{\partial t} = \mu_{11}(t)p_1'(q(t))\dot{q} + \frac{\partial \mathcal{H}_1}{\partial t} = \frac{\partial \mathcal{H}_1}{\partial t}, \tag{A.22a}$$

where the second equality is obtained by substituting $\partial\mathcal{H}_1/\partial S = -\dot{\lambda}_1$ and $\partial\mathcal{H}_1/\partial\lambda_1 = \dot{S}$, and the third equality uses (A.3a) and (A.4a) (with either $\dot{q} = 0$ or $\mu_{11} = 0$). Similarly, by using $\partial\mathcal{H}_2/\partial S = -\dot{\lambda}_2$ and $\partial\mathcal{H}_2/\partial\lambda_2 = \dot{S}$, (A.4b), and (A.3b)-(A.3c) we obtain

$$\dot{\mathcal{H}}_2 = (\mu_{22}(t) - \mu_{21}(t))\, p_2'(q(t))\dot{q} + \frac{\partial \mathcal{H}_2}{\partial t} = \frac{\partial \mathcal{H}_2}{\partial t}. \tag{A.22b}$$

Integration of (A.22a)-(A.22b) over t gives

$$\int_0^{T_2} e^{-rt}(p_1(q) - k)q dt = \frac{\mathcal{H}_1(0) - \mathcal{H}_1(T_2)}{r}, \tag{A.23a}$$

$$\int_{T_2}^{T_4} e^{-rt}(p_2(q) - k)q dt = \frac{\mathcal{H}_2(T_2) - \mathcal{H}_2(T_4)}{r}. \tag{A.23b}$$

Combining (A.23a)-(A.23b) while using (A.6) and continuity of the Hamiltonian gives (A.21). □