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# Difference-in-Differences Techniques for Spatial Data: Local Autocorrelation and Spatial Interaction*

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**Abstract**

We consider treatment effect estimation via a difference-in-difference approach for data with local spatial interaction such that the outcome of observed units depends on their own treatment as well as on the treatment status of proximate neighbors. We show that under standard assumptions (common trend and ignorability) a straightforward spatially explicit version of the benchmark difference-in-differences regression is capable of identifying both direct and indirect treatment effects. We demonstrate the finite sample performance of our spatial estimator via Monte Carlo simulations.

**JEL-classification**
C21; C53

**Keywords**
Difference-in-differences; Monte Carlo simulation; Program evaluation; Spatial autocorrelation; Spatial interaction

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# 1 Introduction

The linear difference-in-differences (DID) model is a benchmark tool in the program evaluation literature (e.g., Ashenfelter, 1978; Ashenfelter and Card, 1985). At its core, a treatment effect is the difference between two potential outcomes, with potential outcomes being a function of treatment status (Rubin, 1974). The fundamental problem is that units are never observed in both treated and untreated states (Holland, 1986), and identification requires comparison of treated units to untreated (control) units. In the standard DID design, observations $i = (1, 2, \ldots, n)$ are observed in two time periods, $T \in \{0, 1\}$, and are grouped via $D \in \{0, 1\}$ such that $D_i = 1$ indicates treatment. Given a vector of time-varying covariates, $X_{it}$, the standard DID equation is:

$$y_{it} = \alpha_0 + \alpha_1 X_{it} + \alpha_2 D_{it} + \alpha_3 T_{it} + \alpha_4 D_{it} T_{it} + \nu_i + \varepsilon_{it}, \tag{1}$$

in which $\nu_i$ is an unobservable time-invariant individual effect and $\varepsilon_{it}$ is a mean zero error term that is uncorrelated with $D_{it}$ and $T_{it}$. It is straightforward to accommodate additional time periods. The identifying assumptions require correct linear specification of the conditional mean, a homogeneous effect of treatment, and the parallel-trends assumption that absent treatment both treated and untreated units evolve along the same temporal path. Strong or weak ignorability (unconfoundedness) is assumed as well, implying that treatment assignment is independent of the outcome, eventually conditional on $X_{it}$ and $\nu_i$. Maintaining these assumptions and suppressing the subscript, the conditional average treatment effect, $ATE(x)$ is:

$$\begin{aligned} ATE(x) \quad = \quad & \{\mathrm{E}[y|X = x, D = 1, T = 1] - \mathrm{E}[y|X = x, D = 1, T = 0]\} \\ & -\{\mathrm{E}[y|X = x, D = 0, T = 1] - \mathrm{E}[y|X = x, D = 0, T = 0]\}, \end{aligned} \tag{2}$$

or the difference in the conditional differences over time between the treated and control units. From equations (1) and (2) it follows that $ATE = \alpha_4$. Typically, equation (1) is estimated with ordinary least squares (OLS), and a rejection of $H_0 : \alpha_4 = 0$ via a $t$-test is evidence of a significant causal effect.

One requirement embedded in this well-known design is that the stable unit treatment value assumption (SUTVA) holds. The crucial relevance of this assumption has been established by Rubin (1978, 1990). Angrist et al. (1996, p. 446) point out that "SUTVA implies that potential outcomes for person $i$ are unrelated to the treatment status of other individuals." Identification of causal effects in the traditional DID setup is no longer valid in situations in which the SUTVA assumption is violated. Adjustments are clearly needed: in determining the treatment effect, it no longer suffices to only consider one's own treatment status, but the treatment status of other observation units has to be taken into account as well. Violation of the SUTVA assumption is also referred to as "interference" or "social interactions" and has been a topic of research in various disciplines, including economics (Manski, 2013). Theoretical and empirical analyses that explicitly consider the potential outcomes framework and its associated assumptions in a spatial context are still few and far between (Verbitsky-Savitz and Raudenbush, 2012; Feser, 2013; Gibbons et al., 2015).

It is obvious that in the case in which outcomes are at least in part the result of spatial interactions, the SUTVA assumption is violated. Consider the following examples. In development economics, imagine a farmer education program that has been designed to improve crop yields by enhancing farming ability through education/experience. Casual communication among farmers within a village may be sufficient to evoke indirect treatment effects, in which treated and untreated farmers benefit from treatment (knowledge) of others. In environmental economics a pollution prevention program, in which participating firms focus on implementation of sustainable management practices targeted at air-borne pollution, may affect the air quality of nearby areas, depending on prevailing wind directions. In urban economics the implementation of urban planning and building restrictions in a city will result in an interrelated web of impacts on land and housing prices across different neighborhoods, eventually even including nearby cities.

Our contribution is to develop a spatially explicit DID model that takes into account the possibility of spatial correlation in treatment assignment, and concurrently allows for the identification of spatial interaction in treatment responses.[1] The use of the term "spatial interaction" is deliberate. It refers to the fact that both direct and indirect treatment effects exist. The latter are sometimes referred to as "spillovers" or "network effects", resulting from contagion, displacement, communication, social comparison, or signaling (Gerber and Green, 2012). The existence of such spillovers or network effects obviously creates (additional) spatial correlation in the treatment responses. To remain agnostic to the underlying mechanisms and to avoid confusion, we simply refer to treatment effects as being either direct or indirect. We also note that the term "spatial interaction" implies that a convincing causal mechanism should be available for the causal interpretation to be viable. In other words, existing spatial patterns due to spatial sorting need to be excluded as a potential source for spatially correlated treatment effects. This is akin to the well known ignorability assumption evoked in the traditional DID setup (see Imbens and Wooldridge, 2009).

We assume treatment is binary, and restrict our focus to spatial interaction in the treatment effects that are "local", as opposed to "global". This means that the spatial effects are restricted to immediate neighbors, defined on the basis of contiguity or distance.[2] In effect, this amounts to assuming that SUTVA holds outside the bounds of immediate neighbors. In the context of the farming example one might assume that SUTVA holds *between* villages, but local spatial interaction and hence SUTVA violation is allowed for *within* villages. Generally, however, the only requirement for our approach to be valid is that SUTVA holds across some particular dimension of the sample space; it is not necessarily restricted to a discernable within–between situation. Under these general circumstances, a simple extension of the standard DID setup using spatial econometric tools allows us to estimate the $ATE$, which we decompose into a direct ("own") and indirect ("neighbor") treatment effect. Our spatial DID model is straightforward to implement, and our Monte Carlo exercises illustrate excellent finite sample performance.

## 2 Spatial Difference-in-Differences

Maintaining the notation in equation (1), imagine a data setup where we have $n$ cross-sectional, spatial observations for two time periods. These spatial observations can either represent points or areas. In the latter case, the spatial system can consist of a regular or irregular lattice structure. We operationalize local spatial interaction in outcomes using the spatial lag operator $L^s$, defined as $W_s$, where $W_s$ is a $(2n \times 2n)$ block-diagonal row-standardized spatial weights matrix containing non-zero elements for spatial units belonging to contiguity class $s$ (Anselin, 1988). The contiguity class can include immediate (or first-order) neighbors with a common border, or first- and higher-order neighbors (neighbors of the neighbors, etc.), or observational units within a certain cut-off

[1] In the spatial econometrics literature the terms "spatial correlation" and "spatial dependence" tend to be used as synonyms, although dependence is a characteristic of the joint probability density function, which can only be measured as correlation under additional assumptions such as normality, stationarity, etc. Spatial patterns on a map may be generated through an underlying dependence mechanism that may or may not be not be known, but they can also result from spatial heterogeneity (Anselin, 1988).

[2] In the global case, indirect effects propel through the entire spatial system; everybody is a neighbor of everybody else, with interactions (or correlations) subject to distance decay.

distance—as long as the number of neighbors is restricted and does not extend to the entire spatial system. Spatial interaction in outcomes is defined as $D_L T = (I + \rho L^s) D \circ T = (I + \rho W_s) D \circ T$, in which $I$ is an appropriately sized identity matrix, $\rho$ is the spatial autoregressive parameter, and $\circ$ signals element-by-element multiplication (Hadamard product). In addition we distinguish random treatment assignment $D$ from spatially correlated treatment assignment $\tilde{D}$.[3]

Four different situations are then easily defined. Equation (1) corresponds to the standard DID approach; the other three cases are:

$$
\begin{aligned}
y &= \alpha_0 + \alpha_1 \tilde{D} + \alpha_2 T + \alpha_3 \tilde{D} \circ T + \varepsilon, && (3)\\
y &= \alpha_0 + \alpha_1 D + \alpha_2 T + \alpha_3 D_L \circ T + \varepsilon, && (4)\\
y &= \alpha_0 + \alpha_1 \tilde{D} + \alpha_2 T + \alpha_3 \tilde{D}_L \circ T + \varepsilon, && (5)
\end{aligned}
$$

which are defined combining random or correlated treatment assignment with spatially isolated or spatially interacted treatment effects.

The question is whether the different constellations described above require a spatially explicit reparameterization of equation (1). It is evident that in the case of equation (3) the SUTVA assumption is not violated. Moreover, we know that even with the presence of spatially correlated exogenous covariates the unbiasedness, consistency, and efficiency properties of OLS are unaltered (Anselin, 1988). Hence, application of the standard DID approach should lead to proper identification and estimation of the average treatment effect.

The situations with spatial interaction in the responses, equations (4) and (5), are different. Obviously, the SUTVA assumption is violated and indirect effects should be explicitly modeled. One common approach is to meticulously identify all treatment and control groups and apply a difference-in-difference-in-differences technique (Imbens and Wooldridge, 2009).[4] There are several disadvantages to such an approach. First, the number of treatment and control groups becomes unwieldy and makes the approach very inefficient in small samples. Second, since there are no restrictions on the estimated parameters one may be confronted with illogical "bouncing beta's"; for instance, the situation where the treatment effect of having two treated neighbors may be greater than the treatment effect of having four treated neighbors. We therefore suggest a slightly more structured approach by explicitly modeling the spatial structure.

Without loss of generality we can describe the cases with spatial interaction in the responses by the following model:

$$
\begin{aligned}
y &= \alpha_0 + \alpha_1 D + \alpha_2 T + \alpha_3 D_L \circ T + \varepsilon,\\
&= \alpha_0 + \alpha_1 D + \alpha_2 T + \alpha_3 (I + \rho W) D \circ T + \varepsilon,\\
&= \alpha_0 + \alpha_1 D + \alpha_2 T + \alpha_3 DT + \alpha_4 W D \circ T + \varepsilon, && (6)
\end{aligned}
$$

in which $\alpha_4 = \rho\alpha_3$, and $D$ now refers to random or spatially correlated treatments. We assume $\rho \neq 0$ so that the model does not revert to the standard DID equation. Erroneously omitting the

[3] The relationship between $D$ and $\tilde{D}$ is not straightforwardly defined using the spatial lag operator, since both $D$ and $\tilde{D}$ are restricted to be binary. The dose variables simply provide information about the location of the treated points or areas, and can in practice never be observed simultaneously. Given the binary nature of the dose variable, join count statistics can be used to determine whether the dose exhibits spatial autocorrelation (Cliff and Ord, 1981).

[4] A spatial setting with local indirect effects to and from neighbors implies that, instead of defining a single treatment and a single control group, multiple treatment and control groups would have to be defined such that units with different numbers of treated neighbors would fall into different treatment and control groups. In the simple case of a $(10 \times 10)$ spatial grid and adjacency defined on the basis of sharing a border or a vertex, the number of groups based on spatially interactive responses is already 9, and this needs to be multiplied by 2 depending on whether own treatment is 0 or 1.

spatially lagged term on the right hand side renders standard DID biased and inconsistent, because of the omitted variable problem.

Given equation (6), it is straightforward to derive a conditional $ATE$ in this spatial setting as the difference between treated and control units. The treated units may be subject to indirect treatment effects caused by treated neighbors ($wd \in WD$ such that $0 < wd \leq 1$). The control group is defined as the group that is not treated at all; neither directly, nor indirectly ($WD = 0$).

$$\begin{aligned} ATE(wd) &= \{\mathrm{E}[y|D=1,T=1,WD=wd] - \mathrm{E}[y|D=1,T=0,WD=wd]\} \\ &\quad - \{\mathrm{E}[y|D=0,T=1,WD=0] - \mathrm{E}[y|D=0,T=0,WD=0]\} \\ &= \alpha_3 + \alpha_4 wd \\ &= \alpha_3(1+\rho wd). \end{aligned} \tag{7}$$

The $ATE$ is then obtained as:

$$ATE = \mathrm{E}[ATE(wd)|WD] = \alpha_3\left(1+\rho\overline{WD}\right), \tag{8}$$

where $\overline{WD}$ is the average proportion of treated neighbors, which can also be interpreted as the probability of the neighbors being treated. In many geographies and treatment settings, the probability of your neighbors being treated will asymptotically be equal to the probability of own treatment.

Several interesting observations follow from equation (8). First, in the case of spatially interactive treatment responses the $ATE$ becomes a function of the magnitude of the direct effect of treatment, the strength of the local spatial interaction, and the probability of being treated. The relevance of treatment probability in our spatial approach resembles the pivotal role of the probability of being treated in propensity score matching approaches. Second, the standard DID estimator, $ATE = \alpha_3$, is clearly biased. Third, given equation (7), one can write the $ATE$ as a (non)linear dose-response-type function of indirect treatment. One advantage of this formulation is that the $ATE$ can be evaluated at any $wd \in (0,1)$. This may give rise to interesting comparisons, for instance, the impact of (only) indirect treatment for some $0 < wd \leq 1$ can be compared to the impact of direct treatment with $wd = 0$. Finally, it is easy to see that the $ATE$ in equation (8) can be decomposed into an average *direct* treatment effect ($ADTE$), and an average *indirect* treatment effect ($AITE$).

The $ADTE$ is defined as the difference between treated and control units, both differenced over time:

$$\begin{aligned} ADTE &= \{\mathrm{E}[y|D=1,T=1,WD=0] - \mathrm{E}[y|D=1,T=0,WD=0]\} \\ &\quad - \{\mathrm{E}[y|D=0,T=1,WD=0] - \mathrm{E}[y|D=0,T=0,WD=0]\} \\ &= \alpha_3, \end{aligned} \tag{9}$$

where the $ADTE$ is identified by setting indirect treatment $WD$ to zero. The conditional $AITE$ is defined as the difference between indirectly treated units and control units, both differenced over time:[5]

$$\begin{aligned} AITE(wd) &= \{\mathrm{E}[y|D=0,T=1,WD=wd] - \mathrm{E}[y|D=0,T=0,WD=wd]\} \\ &\quad - \{\mathrm{E}[y|D=0,T=1,WD=0] - \mathrm{E}[y|D=0,T=0,WD=0]\} \\ &= \alpha_4 wd \end{aligned} \tag{10}$$

[5] It is easy to show that the $AITE$ is identical for treated and untreated units.

which yields $AITE = \alpha_4 \overline{WD}$. It is important to notice that in all cases, the necessary control group consists of units that are neither treated directly nor indirectly.

## 3 Monte Carlo Simulations

Our spatially explicit DID setup is a straightforward extension of the benchmark DID model, and is easily estimated using standard regression and matrix multiplication tools. To provide a brief illustration of the performance of our approach, we consider Monte Carlo simulations for the following data generating process (DGP):

$$y_{it} = \alpha_0 + \alpha_1 X_{it} + \alpha_2 D_{it} + \alpha_3 T_{it} + \alpha_4 R_{it} + \varepsilon_{it}, \tag{11}$$

with $R_{it}$ for the responses, and all other notation as before. We generate $\varepsilon_{it} \sim \mathcal{N}(0,1)$. Key aspects of our DGP are as follows:

| | |
|---|---|
| (a) $\dot{X}_{it} = a\dot{X}_{it-1} + \nu_i$ | common trend, |
| (b) $X = (I + \ell W)\dot{X}$ | random or autocorrelated sorting, |
| (c) $D_t = D_{t-1} \sim \text{Bern}(n, \Phi(X_{t-1}))$ | random or autocorrelated binary treatment, |
| (d) $R = (I + \rho W) D \circ T$ | autocorrelated response due to interaction. |

First, we draw $\dot{X}_{it-1}$ from a standard normal and calculate $\dot{X}_{it}$ using a common trend for the treatment and control group with $a = 1.02$ and $\nu_i \sim \mathcal{N}(0, (a-1)^2)$. Second, we stack these variables in $X$ and potentially allow this conditioning variable to be spatially autocorrelated through a given autoregressive parameter $\ell$, and a row-standardized block-diagonal matrix with first-order queen weights on a regular lattice. The parameter $\ell$ is not recovered in the estimation, but this conditioning variable allows us to incorporate an initial situation of spatial sorting into the model. Third, the sorting variable is clearly correlated with the binary treatments defined in (c) above, which are generated using a Bernoulli distribution for the normally distributed treatment probabilities $\Phi(X_{t-1})$. As a result, we can investigate the impact of violating the ignorability assumption, because $D_{it}$ and $y_{it}$ are only conditionally independent, which implies that $X_{it}$ should be included in the DID model to maintain unconfoundedness. Finally, the responses are potentially autocorrelated due to spatial interaction. The simulations are conducted with 1,000 replications, maintaining $\alpha_k = 1, \forall k$, and varying $\ell = \{0, 0.9\}$, $\rho = \{0, 0.50, 0.90\}$ and $n = \{100, 900, 2500\}$.[6]

We basically compare two different DID estimators: the traditional DID estimator from equation (1), and our suggested spatial DID estimator given in equation (6). These are labeled DID and SDID, respectively. In addition we investigate the same estimators, but we add the potentially spatially correlated conditioning variable $X$ to the estimated equation; these estimators are labeled DIDX and SDIDX, respectively. Table 1 concisely presents the simulation results in terms of bias and root mean squared error (RMSE) of the $ATE$ estimator provided in equation (8).

Several interesting conclusions emerge from Table 1. In the situation with random treatment assignment and no spatial interaction in outcomes, the performance of all estimators is satisfactory. In particular, DIDX achieves the smallest bias and the lowest RMSE because in this case DIDX is perfectly specified. This superior performance of DIDX is reinforced if we allow treatment assignment to be spatially correlated. However, in that case our suggested SDID estimator is severely biased, because it overestimates the $ATE$ due to the violation of the ignorability assumption. That is, the SDID estimator incorrectly interprets the spatial sorting in $X$ to be spatial interaction in

[6] A more extensive set of simulation results is presented in the Appendix.

**Table 1:** Bias and RMSE for different $ATE$ estimators.

| $\ell$ | $\rho$ | ATE estimator | Bias | | | RMSE | | |
|---|---|---|---|---|---|---|---|---|
| | | | $n = 100$ | $n = 900$ | $n = 2500$ | $n = 100$ | $n = 900$ | $n = 2500$ |
| 0.0 | 0.0 | DID | 0.0350 | 0.0214 | 0.0249 | 0.2207 | 0.0770 | 0.0489 |
| | | DIDX | 0.0121 | -0.0012 | 0.0023 | 0.2196 | 0.0745 | 0.0445 |
| | | SDID | 0.0342 | 0.0207 | 0.0306 | 0.3455 | 0.1198 | 0.0775 |
| | | SDIDX | 0.0069 | -0.0062 | 0.0078 | 0.3033 | 0.1014 | 0.0625 |
| | 0.5 | DID | -0.2227 | -0.2254 | -0.2281 | 0.2969 | 0.2263 | 0.2281 |
| | | DIDX | -0.2455 | -0.2480 | -0.2507 | 0.3108 | 0.2484 | 0.2507 |
| | | SDID | 0.0078 | 0.0273 | 0.0215 | 0.3694 | 0.1229 | 0.0738 |
| | | SDIDX | -0.0042 | 0.0037 | -0.0008 | 0.3162 | 0.1046 | 0.0613 |
| | 0.9 | DID | -0.4420 | -0.4270 | -0.4277 | 0.4573 | 0.4270 | 0.4277 |
| | | DIDX | -0.4647 | -0.4495 | -0.4503 | 0.4773 | 0.4495 | 0.4503 |
| | | SDID | 0.0151 | 0.0281 | 0.0249 | 0.3714 | 0.1230 | 0.0752 |
| | | SDIDX | -0.0002 | 0.0051 | 0.0018 | 0.3126 | 0.1043 | 0.0624 |
| 0.9 | 0.0 | DID | 0.0289 | 0.0205 | 0.0283 | 0.2214 | 0.0766 | 0.0505 |
| | | DIDX | 0.0043 | -0.0040 | 0.0040 | 0.2204 | 0.0744 | 0.0449 |
| | | SDID | 0.5589 | 0.5491 | 0.5607 | 0.5876 | 0.5491 | 0.5607 |
| | | SDIDX | 0.0000 | -0.0059 | 0.0071 | 0.2818 | 0.0932 | 0.0559 |
| | 0.5 | DID | -0.1807 | -0.1832 | -0.1840 | 0.2819 | 0.1847 | 0.1840 |
| | | DIDX | -0.2055 | -0.2080 | -0.2087 | 0.2936 | 0.2110 | 0.2087 |
| | | SDID | 0.5513 | 0.5596 | 0.5560 | 0.5923 | 0.5547 | 0.5560 |
| | | SDIDX | -0.0072 | 0.0031 | -0.0009 | 0.2901 | 0.0740 | 0.0559 |
| | 0.9 | DID | -0.3614 | -0.3500 | -0.3510 | 0.3931 | 0.3500 | 0.3510 |
| | | DIDX | -0.3868 | -0.3750 | -0.3760 | 0.4125 | 0.3750 | 0.3760 |
| | | SDID | 0.5562 | 0.5605 | 0.5573 | 0.5856 | 0.5605 | 0.5573 |
| | | SDIDX | 0.0016 | 0.0050 | 0.0014 | 0.2835 | 0.0900 | 0.0573 |

the outcome. In the case where there is spatial interaction, DID and DIDX are obviously biased and inconsistent. They underestimate the $ATE$, because the SUTVA assumption is violated and these estimators do not account for the positive spatial interaction. In the case where there is both spatial sorting and spatial interaction, the SDID estimator is biased and inconsistent, because even although it accounts for the spatial interaction, it fails to recognize the violation of the ignorability assumption. The effect of spatial sorting is picked up as a treatment effect, which it is not. Overall, the situation of treatment assignment being spatially correlated rather than random does not affect the different estimators much, except for the SDID estimator that erroneously attributes spatial sorting to spatial interaction. The performance of the SDIDX estimator is superior, meaning it is unbiased and consistent in all situations encompassed by our Monte Carlo design.

## 4 Conclusion

We develop a difference-in-differences estimator for data that exhibit local spatial interaction. As long as the spatial interaction is local, it is possible to define a proper control group of observations that are untreated, neither directly nor indirectly. We use this control group, as well as the assumption of structured spatial interactions to identify the average treatment effect that depends on a treated unit's own treatment as well as the share of proximate neighbors that also receive treatment. This $ATE$ parameter can be decomposed into an average direct and indirect treatment effect. A logical next step will be to consider global spatial correlation and interaction in order to see which changes to the standard approach are needed to develop an appropriate spatial difference-in-difference estimator for the global case.

For the local case developed here, we consider an array of different spatial and a-spatial scenarios, and show via Monte Carlo simulations that our spatial DID estimator performs well in finite samples. The simulations do highlight that it is imperative to condition on pre-existing spatially correlated sorting, especially if the sorting is correlated with treatment assignment. The spatial DID estimator should always be implemented with conditioning variables that capture pre-existing observed and unobserved differences across space. We emphasize that our SDIDX estimator draws on a simple spatial correlation mechanism developed in the spatial econometrics literature, and is straightforward for practitioners to apply using a variety of available software.

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## Appendix: Detailed simulation results

The simulation results presented in the main text have demonstrated the finite sample performance of our proposed estimator in each of the settings outlined in equations (1) and (3)–(5) for several representative values of key parameters in our Monte Carlo design. In this appendix, we provide a complete set of results for all of the simulations we conducted. In all cases, our DGP is defined as in equation (11), except that here we consider a wider set of combinations of $\ell = \{0, 0.5, 0.9\}$, $\rho = \{0, 0.25, 0.50, 0.75, 0.90, 1, 2, 3, 4, 5\}$, and $n = \{100, 225, 400, 900, 1600, 2500\}$ corresponding to square regular lattice grids of size $\{10, 15, 20, 30, 40, 50\}$. We note that under our assumed restriction that spatial interaction is local, $\rho$ is unconstrained. In addition to the $ATE$, we also report the bias and RMSE for the $ADTE$ and $AITE$ for the SDID and SDIDX estimators.

The simulation results are reported in Table A1–A8. The caption of each table indicates whether the numbers reported correspond to the bias or the RMSE, and provide the value for $\ell$ and the range of values for $\rho$. All tables consider the full range of $n$, report the $ATE$ for all four estimators, and report the $ADTE$ and $AITE$ for our two proposed spatial estimators.

Our extended simulation results largely mimic the results summarized in the main text. An additional insight gleaned from these extended simulation results is the performance of the SDID estimator when there is spatial sorting via $\ell > 0$. As shown previously, the SDID estimator erroneously picks up spatial sorting as if it is spatial interaction; hence the $ATE$ is biased. From the extended simulation results we can see that both the $ATE$ and $AITE$ are indeed biased, however the $ADTE$ for this estimator is not (see, for instance, Table A3). It is clear that the bias in the $ATE$ in this scenario comes from bias in the $AITE$.

It is also apparent that the biases shown across the various misspecified models increases with the magnitude of $\ell$ and $\rho$. For instance, the bias in the SDID estimator when $\ell > 0$ increases as $\ell$ increases; or, the bias in DID and DIDX induced by $\rho$ increases as $\rho$ increases. The poor performance of DID and DIDX is particularly apparent for $\rho \geq 1$, as shown in Tables A7 and A8. It is worth pointing out that large values of $\rho$ seem less plausible in an empirical setting. Consider, for example, the case in which 50 percent of the neighbors are treated, but the observational unit itself is not. Then, a value of $\rho = 2$ implies that the indirect interaction with one's neighbors is equivalent to direct treatment of the unit of observation. Clearly, in some empirical cases, and for some values of $\rho$ and $WD$, this may be plausible. However, we suspect that in general, the value of $\rho$ will not be so large as to imply that indirect treatment has a substantially larger impact than direct treatment.

**Table A1:** Bias for different $ATE$ estimators with $0 \leq \rho < 1$ and $\ell = 0$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 0.00 | DID ATE | 0.0350 | 0.0254 | 0.0159 | 0.0214 | 0.0248 | 0.0249 |
| | DIDX ATE | 0.0121 | 0.0030 | -0.0066 | -0.0012 | 0.0022 | 0.0023 |
| | S-DID ADTE | 0.0369 | 0.0249 | 0.0159 | 0.0213 | 0.0248 | 0.0249 |
| | S-DID AITE | -0.0026 | 0.0000 | -0.0017 | -0.0006 | -0.0008 | 0.0057 |
| | S-DID ATE | 0.0342 | 0.0249 | 0.0142 | 0.0207 | 0.0239 | 0.0306 |
| | S-DIDX ADTE | 0.0122 | 0.0025 | -0.0066 | -0.0012 | 0.0022 | 0.0023 |
| | S-DIDX AITE | -0.0053 | -0.0026 | -0.0047 | -0.0050 | -0.0011 | 0.0055 |
| | S-DIDX ATE | 0.0069 | -0.0001 | -0.0113 | -0.0062 | 0.0011 | 0.0078 |
| 0.10 | DID ATE | -0.0269 | -0.0278 | -0.0321 | -0.0241 | -0.0280 | -0.0255 |
| | DIDX ATE | -0.0494 | -0.0505 | -0.0547 | -0.0468 | -0.0506 | -0.0481 |
| | S-DID ADTE | 0.0238 | 0.0223 | 0.0182 | 0.0260 | 0.0220 | 0.0246 |
| | S-DID AITE | 0.0042 | 0.0026 | -0.0086 | -0.0024 | 0.0011 | -0.0013 |
| | S-DID ATE | 0.0280 | 0.0249 | 0.0095 | 0.0236 | 0.0231 | 0.0233 |
| | S-DIDX ADTE | 0.0013 | 0.0002 | -0.0043 | 0.0032 | -0.0006 | 0.0020 |
| | S-DIDX AITE | 0.0033 | -0.0006 | -0.0021 | 0.0002 | 0.0004 | 0.0011 |
| | S-DIDX ATE | 0.0046 | -0.0004 | -0.0064 | 0.0033 | -0.0002 | 0.0031 |
| 0.25 | DID ATE | -0.1020 | -0.1084 | -0.1055 | -0.1015 | -0.1065 | -0.1033 |
| | DIDX ATE | -0.1246 | -0.1308 | -0.1280 | -0.1241 | -0.1290 | -0.1259 |
| | S-DID ADTE | 0.0244 | 0.0176 | 0.0211 | 0.0239 | 0.0186 | 0.0219 |
| | S-DID AITE | -0.0233 | 0.0140 | -0.0063 | -0.0013 | 0.0012 | 0.0009 |
| | S-DID ATE | 0.0011 | 0.0317 | 0.0148 | 0.0226 | 0.0198 | 0.0228 |
| | S-DIDX ADTE | 0.0031 | -0.0047 | -0.0015 | 0.0012 | -0.0039 | -0.0007 |
| | S-DIDX AITE | -0.0101 | 0.0118 | -0.0042 | 0.0003 | 0.0019 | -0.0011 |
| | S-DIDX ATE | -0.0070 | 0.0071 | -0.0057 | 0.0016 | -0.0020 | -0.0018 |
| 0.50 | DID ATE | -0.2227 | -0.2248 | -0.2246 | -0.2254 | -0.2314 | -0.2281 |
| | DIDX ATE | -0.2455 | -0.2474 | -0.2472 | -0.2480 | -0.2541 | -0.2507 |
| | S-DID ADTE | 0.0288 | 0.0273 | 0.0267 | 0.0251 | 0.0192 | 0.0219 |
| | S-DID AITE | -0.0210 | -0.0077 | 0.0042 | 0.0022 | 0.0027 | -0.0004 |
| | S-DID ATE | 0.0078 | 0.0196 | 0.0309 | 0.0273 | 0.0219 | 0.0215 |
| | S-DIDX ADTE | 0.0075 | 0.0047 | 0.0041 | 0.0026 | -0.0035 | -0.0007 |
| | S-DIDX AITE | -0.0117 | -0.0062 | 0.0062 | 0.0011 | 0.0025 | -0.0001 |
| | S-DIDX ATE | -0.0042 | -0.0015 | 0.0102 | 0.0037 | -0.0010 | -0.0008 |
| 0.75 | DID ATE | -0.3498 | -0.3615 | -0.3517 | -0.3521 | -0.3500 | -0.3558 |
| | DIDX ATE | -0.3722 | -0.3839 | -0.3744 | -0.3746 | -0.3724 | -0.3783 |
| | S-DID ADTE | 0.0277 | 0.0147 | 0.0267 | 0.0235 | 0.0251 | 0.0200 |
| | S-DID AITE | 0.0005 | 0.0002 | -0.0006 | -0.0009 | -0.0017 | 0.0009 |
| | S-DID ATE | 0.0282 | 0.0149 | 0.0261 | 0.0226 | 0.0235 | 0.0209 |
| | S-DIDX ADTE | 0.0083 | -0.0084 | 0.0038 | 0.0010 | 0.0027 | -0.0026 |
| | S-DIDX AITE | 0.0017 | 0.0002 | 0.0008 | -0.0008 | -0.0003 | -0.0004 |
| | S-DIDX ATE | 0.0100 | -0.0081 | 0.0046 | 0.0002 | 0.0024 | -0.0030 |
| 0.90 | DID ATE | -0.4420 | -0.4318 | -0.4299 | -0.4270 | -0.4272 | -0.4277 |
| | DIDX ATE | -0.4647 | -0.4544 | -0.4524 | -0.4495 | -0.4497 | -0.4503 |
| | S-DID ADTE | 0.0164 | 0.0241 | 0.0208 | 0.0237 | 0.0237 | 0.0230 |
| | S-DID AITE | -0.0013 | -0.0022 | 0.0039 | 0.0044 | 0.0052 | 0.0018 |
| | S-DID ATE | 0.0151 | 0.0219 | 0.0247 | 0.0281 | 0.0289 | 0.0249 |
| | S-DIDX ADTE | -0.0061 | 0.0009 | -0.0019 | 0.0013 | 0.0012 | 0.0005 |
| | S-DIDX AITE | 0.0059 | -0.0025 | -0.0060 | 0.0038 | 0.0058 | 0.0012 |
| | S-DIDX ATE | -0.0002 | -0.0016 | -0.0079 | 0.0051 | 0.0070 | 0.0018 |

**Table A2:** RMSE for different $ATE$ estimators with $0 \leq \rho < 1$ and $\ell = 0$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 0.00 | DID ATE | 0.2207 | 0.1552 | 0.1151 | 0.0770 | 0.0584 | 0.0489 |
| | DIDX ATE | 0.2196 | 0.1533 | 0.1141 | 0.0745 | 0.0545 | 0.0445 |
| | S-DID ADTE | 0.2216 | 0.1565 | 0.1156 | 0.0771 | 0.0584 | 0.0489 |
| | S-DID AITE | 0.2688 | 0.1937 | 0.1437 | 0.0941 | 0.0728 | 0.0577 |
| | S-DID ATE | 0.3455 | 0.2530 | 0.1874 | 0.1198 | 0.0928 | 0.0775 |
| | S-DIDX ADTE | 0.2205 | 0.1542 | 0.1147 | 0.0746 | 0.0546 | 0.0445 |
| | S-DIDX AITE | 0.2097 | 0.1450 | 0.1095 | 0.0706 | 0.0535 | 0.0438 |
| | S-DIDX ATE | 0.3033 | 0.2169 | 0.1604 | 0.1014 | 0.0763 | 0.0625 |
| 0.10 | DID ATE | 0.2243 | 0.1461 | 0.1123 | 0.0756 | 0.0610 | 0.0505 |
| | DIDX ATE | 0.2276 | 0.1495 | 0.1179 | 0.0836 | 0.0703 | 0.0614 |
| | S-DID ADTE | 0.2239 | 0.1477 | 0.1107 | 0.0755 | 0.0591 | 0.0501 |
| | S-DID AITE | 0.2816 | 0.1884 | 0.1382 | 0.0960 | 0.0692 | 0.0575 |
| | S-DID ATE | 0.3725 | 0.2454 | 0.1783 | 0.1237 | 0.0885 | 0.0788 |
| | S-DIDX ADTE | 0.2235 | 0.1458 | 0.1095 | 0.0728 | 0.0563 | 0.0464 |
| | S-DIDX AITE | 0.2164 | 0.1411 | 0.1068 | 0.0714 | 0.0534 | 0.0455 |
| | S-DIDX ATE | 0.3259 | 0.2053 | 0.1557 | 0.1037 | 0.0760 | 0.0661 |
| 0.25 | DID ATE | 0.2416 | 0.1756 | 0.1440 | 0.1141 | 0.1109 | 0.1053 |
| | DIDX ATE | 0.2484 | 0.1859 | 0.1569 | 0.1315 | 0.1312 | 0.1268 |
| | S-DID ADTE | 0.2314 | 0.1521 | 0.1124 | 0.0760 | 0.0582 | 0.0478 |
| | S-DID AITE | 0.2836 | 0.1884 | 0.1385 | 0.0931 | 0.0696 | 0.0583 |
| | S-DID ATE | 0.3679 | 0.2463 | 0.1771 | 0.1209 | 0.0911 | 0.0732 |
| | S-DIDX ADTE | 0.2289 | 0.1517 | 0.1113 | 0.0738 | 0.0562 | 0.0448 |
| | S-DIDX AITE | 0.2152 | 0.1418 | 0.1099 | 0.0729 | 0.0544 | 0.0439 |
| | S-DIDX ATE | 0.3158 | 0.2088 | 0.1545 | 0.1051 | 0.0790 | 0.0612 |
| 0.50 | DID ATE | 0.2969 | 0.2439 | 0.2323 | 0.2263 | 0.2314 | 0.2281 |
| | DIDX ATE | 0.3108 | 0.2619 | 0.2530 | 0.2484 | 0.2541 | 0.2507 |
| | S-DID ADTE | 0.2347 | 0.1499 | 0.1138 | 0.0775 | 0.0599 | 0.0479 |
| | S-DID AITE | 0.2699 | 0.1887 | 0.1410 | 0.0950 | 0.0701 | 0.0569 |
| | S-DID ATE | 0.3694 | 0.2422 | 0.1838 | 0.1229 | 0.0931 | 0.0738 |
| | S-DIDX ADTE | 0.2322 | 0.1475 | 0.1123 | 0.0750 | 0.0580 | 0.0452 |
| | S-DIDX AITE | 0.2081 | 0.1470 | 0.1060 | 0.0744 | 0.0556 | 0.0431 |
| | S-DIDX ATE | 0.3162 | 0.2135 | 0.1589 | 0.1046 | 0.0823 | 0.0613 |
| 0.75 | DID ATE | 0.3790 | 0.3657 | 0.3526 | 0.3521 | 0.3500 | 0.3558 |
| | DIDX ATE | 0.3966 | 0.3872 | 0.3749 | 0.3746 | 0.3724 | 0.3783 |
| | S-DID ADTE | 0.2277 | 0.1428 | 0.1136 | 0.0761 | 0.0596 | 0.0476 |
| | S-DID AITE | 0.2806 | 0.1838 | 0.1424 | 0.0947 | 0.0720 | 0.0595 |
| | S-DID ATE | 0.3618 | 0.2348 | 0.1816 | 0.1215 | 0.0905 | 0.0764 |
| | S-DIDX ADTE | 0.2269 | 0.1414 | 0.1114 | 0.0738 | 0.0560 | 0.0446 |
| | S-DIDX AITE | 0.2106 | 0.1420 | 0.1095 | 0.0740 | 0.0548 | 0.0448 |
| | S-DIDX ATE | 0.3034 | 0.2027 | 0.1560 | 0.1039 | 0.0789 | 0.0639 |
| 0.90 | DID ATE | 0.4573 | 0.4334 | 0.4302 | 0.4270 | 0.4272 | 0.4277 |
| | DIDX ATE | 0.4773 | 0.4555 | 0.4526 | 0.4495 | 0.4497 | 0.4503 |
| | S-DID ADTE | 0.2288 | 0.1508 | 0.1128 | 0.0769 | 0.0585 | 0.0488 |
| | S-DID AITE | 0.2833 | 0.1896 | 0.1370 | 0.0984 | 0.0720 | 0.0578 |
| | S-DID ATE | 0.3714 | 0.2487 | 0.1835 | 0.1230 | 0.0981 | 0.0752 |
| | S-DIDX ADTE | 0.2271 | 0.1491 | 0.1124 | 0.0748 | 0.0554 | 0.0451 |
| | S-DIDX AITE | 0.2127 | 0.1459 | 0.1081 | 0.0754 | 0.0544 | 0.0438 |
| | S-DIDX ATE | 0.3126 | 0.2147 | 0.1615 | 0.1043 | 0.0806 | 0.0624 |

**Table A3:** Bias for different $ATE$ estimators with $0 \leq \rho < 1$ and $\ell = 0.5$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 0.00 | DID ATE | 0.0346 | 0.0257 | 0.0176 | 0.0197 | 0.0238 | 0.0263 |
| | DIDX ATE | 0.0111 | 0.0028 | -0.0055 | -0.0034 | 0.0006 | 0.0032 |
| | S-DID ADTE | 0.0068 | -0.0050 | -0.0152 | -0.0136 | -0.0091 | -0.0075 |
| | S-DID AITE | 0.3659 | 0.3586 | 0.3734 | 0.3725 | 0.3694 | 0.3781 |
| | S-DID ATE | 0.3726 | 0.3536 | 0.3582 | 0.3588 | 0.3603 | 0.3706 |
| | S-DIDX ADTE | 0.0115 | 0.0027 | -0.0053 | -0.0033 | 0.0008 | 0.0027 |
| | S-DIDX AITE | -0.0047 | -0.0033 | 0.0004 | -0.0029 | -0.0027 | 0.0058 |
| | S-DIDX ATE | 0.0068 | -0.0006 | -0.0049 | -0.0062 | -0.0019 | 0.0084 |
| 0.10 | DID ATE | -0.0271 | -0.0209 | -0.0242 | -0.0185 | -0.0227 | -0.0217 |
| | DIDX ATE | -0.0500 | -0.0443 | -0.0473 | -0.0418 | -0.0459 | -0.0449 |
| | S-DID ADTE | -0.0109 | -0.0095 | -0.0117 | -0.0062 | -0.0109 | -0.0089 |
| | S-DID AITE | 0.3722 | 0.3778 | 0.3656 | 0.3693 | 0.3750 | 0.3719 |
| | S-DID ATE | 0.3613 | 0.3683 | 0.3540 | 0.3632 | 0.3642 | 0.3630 |
| | S-DIDX ADTE | -0.0043 | 0.0011 | -0.0019 | 0.0035 | -0.0004 | 0.0009 |
| | S-DIDX AITE | -0.0003 | 0.0002 | -0.0011 | 0.0013 | -0.0002 | -0.0010 |
| | S-DIDX ATE | -0.0046 | 0.0013 | -0.0030 | 0.0049 | -0.0006 | -0.0001 |
| 0.25 | DID ATE | -0.0830 | -0.0958 | -0.0899 | -0.0905 | -0.0967 | -0.0920 |
| | DIDX ATE | -0.1062 | -0.1189 | -0.1130 | -0.1138 | -0.1200 | -0.1152 |
| | S-DID ADTE | 0.0070 | -0.0132 | -0.0082 | -0.0097 | -0.0159 | -0.0104 |
| | S-DID AITE | 0.3580 | 0.3872 | 0.3636 | 0.3725 | 0.3757 | 0.3737 |
| | S-DID ATE | 0.3650 | 0.3740 | 0.3554 | 0.3628 | 0.3597 | 0.3634 |
| | S-DIDX ADTE | 0.0099 | -0.0054 | 0.0020 | 0.0002 | -0.0061 | -0.0008 |
| | S-DIDX AITE | -0.0094 | 0.0146 | -0.0069 | 0.0011 | 0.0025 | -0.0008 |
| | S-DIDX ATE | 0.0005 | 0.0092 | -0.0048 | 0.0012 | -0.0036 | -0.0015 |
| 0.50 | DID ATE | -0.2054 | -0.1980 | -0.1972 | -0.2014 | -0.2077 | -0.2049 |
| | DIDX ATE | -0.2288 | -0.2214 | -0.2206 | -0.2247 | -0.2312 | -0.2283 |
| | S-DID ADTE | -0.0042 | -0.0010 | -0.0031 | -0.0065 | -0.0131 | -0.0097 |
| | S-DID AITE | 0.3451 | 0.3658 | 0.3736 | 0.3767 | 0.3758 | 0.3713 |
| | S-DID ATE | 0.3409 | 0.3648 | 0.3705 | 0.3702 | 0.3626 | 0.3616 |
| | S-DIDX ADTE | 0.0019 | 0.0074 | 0.0065 | 0.0035 | -0.0029 | 0.0000 |
| | S-DIDX AITE | -0.0185 | -0.0048 | 0.0047 | 0.0019 | -0.0002 | -0.0018 |
| | S-DIDX ATE | -0.0166 | 0.0026 | 0.0112 | 0.0054 | -0.0031 | -0.0018 |
| 0.75 | DID ATE | -0.3170 | -0.3272 | -0.3142 | -0.3148 | -0.3152 | -0.3205 |
| | DIDX ATE | -0.3402 | -0.3506 | -0.3378 | -0.3382 | -0.3386 | -0.3439 |
| | S-DID ADTE | -0.0031 | -0.0190 | -0.0044 | -0.0076 | -0.0075 | -0.0112 |
| | S-DID AITE | 0.3722 | 0.3691 | 0.3767 | 0.3725 | 0.3753 | 0.3738 |
| | S-DID ATE | 0.3691 | 0.3500 | 0.3723 | 0.3649 | 0.3678 | 0.3626 |
| | S-DIDX ADTE | 0.0044 | -0.0096 | 0.0049 | 0.0029 | 0.0030 | -0.0013 |
| | S-DIDX AITE | -0.0007 | 0.0018 | 0.0050 | 0.0021 | 0.0016 | -0.0004 |
| | S-DIDX ATE | 0.0037 | -0.0078 | 0.0099 | 0.0050 | 0.0046 | -0.0017 |
| 0.90 | DID ATE | -0.3914 | -0.3882 | -0.3884 | -0.3873 | -0.3861 | -0.3874 |
| | DIDX ATE | -0.4152 | -0.4119 | -0.4119 | -0.4108 | -0.4095 | -0.4109 |
| | S-DID ADTE | -0.0054 | -0.0053 | -0.0134 | -0.0111 | -0.0101 | -0.0104 |
| | S-DID AITE | 0.3639 | 0.3665 | 0.3685 | 0.3783 | 0.3774 | 0.3758 |
| | S-DID ATE | 0.3585 | 0.3611 | 0.3552 | 0.3671 | 0.3673 | 0.3654 |
| | S-DIDX ADTE | -0.0018 | 0.0011 | -0.0021 | -0.0013 | 0.0001 | -0.0004 |
| | S-DIDX AITE | 0.0029 | 0.0021 | -0.0098 | 0.0052 | 0.0047 | 0.0022 |
| | S-DIDX ATE | 0.0011 | 0.0031 | -0.0119 | 0.0039 | 0.0048 | 0.0018 |

**Table A4:** RMSE for different $ATE$ estimators with $0 \leq \rho < 1$ and $\ell = 0.5$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 0.00 | DID ATE | 0.2206 | 0.1509 | 0.1146 | 0.0761 | 0.0578 | 0.0500 |
| | DIDX ATE | 0.2190 | 0.1493 | 0.1137 | 0.0733 | 0.0548 | 0.0452 |
| | S-DID ADTE | 0.2230 | 0.1510 | 0.1153 | 0.0741 | 0.0568 | 0.0458 |
| | S-DID AITE | 0.4201 | 0.3708 | 0.3760 | 0.3725 | 0.3694 | 0.3781 |
| | S-DID ATE | 0.4616 | 0.3848 | 0.3682 | 0.3597 | 0.3603 | 0.3706 |
| | S-DIDX ADTE | 0.2205 | 0.1507 | 0.1144 | 0.0736 | 0.0549 | 0.0453 |
| | S-DIDX AITE | 0.1999 | 0.1321 | 0.1066 | 0.0674 | 0.0512 | 0.0412 |
| | S-DIDX ATE | 0.2914 | 0.1974 | 0.1539 | 0.0957 | 0.0716 | 0.0586 |
| 0.10 | DID ATE | 0.2307 | 0.1456 | 0.1113 | 0.0758 | 0.0586 | 0.0503 |
| | DIDX ATE | 0.2332 | 0.1498 | 0.1155 | 0.0819 | 0.0674 | 0.0601 |
| | S-DID ADTE | 0.2333 | 0.1469 | 0.1115 | 0.0750 | 0.0576 | 0.0483 |
| | S-DID AITE | 0.4259 | 0.3894 | 0.3674 | 0.3695 | 0.3750 | 0.3719 |
| | S-DID ATE | 0.4634 | 0.3952 | 0.3645 | 0.3637 | 0.3642 | 0.3630 |
| | S-DIDX ADTE | 0.2304 | 0.1451 | 0.1099 | 0.0746 | 0.0565 | 0.0472 |
| | S-DIDX AITE | 0.2122 | 0.1390 | 0.1032 | 0.0673 | 0.0510 | 0.0415 |
| | S-DIDX ATE | 0.3123 | 0.1935 | 0.1490 | 0.0964 | 0.0719 | 0.0608 |
| 0.25 | DID ATE | 0.2417 | 0.1677 | 0.1373 | 0.1068 | 0.1034 | 0.0951 |
| | DIDX ATE | 0.2476 | 0.1777 | 0.1489 | 0.1234 | 0.1234 | 0.1166 |
| | S-DID ADTE | 0.2339 | 0.1529 | 0.1176 | 0.0768 | 0.0574 | 0.0463 |
| | S-DID AITE | 0.4152 | 0.3997 | 0.3660 | 0.3725 | 0.3757 | 0.3737 |
| | S-DID ATE | 0.4593 | 0.4016 | 0.3626 | 0.3628 | 0.3597 | 0.3634 |
| | S-DIDX ADTE | 0.2319 | 0.1494 | 0.1155 | 0.0755 | 0.0558 | 0.0443 |
| | S-DIDX AITE | 0.2078 | 0.1346 | 0.1052 | 0.0710 | 0.0514 | 0.0413 |
| | S-DIDX ATE | 0.2931 | 0.1920 | 0.1463 | 0.0998 | 0.0735 | 0.0585 |
| 0.50 | DID ATE | 0.2921 | 0.2237 | 0.2066 | 0.2023 | 0.2077 | 0.2049 |
| | DIDX ATE | 0.3044 | 0.2412 | 0.2272 | 0.2250 | 0.2312 | 0.2283 |
| | S-DID ADTE | 0.2402 | 0.1528 | 0.1140 | 0.0779 | 0.0586 | 0.0480 |
| | S-DID AITE | 0.4034 | 0.3767 | 0.3751 | 0.3767 | 0.3758 | 0.3713 |
| | S-DID ATE | 0.4530 | 0.3979 | 0.3784 | 0.3703 | 0.3627 | 0.3616 |
| | S-DIDX ADTE | 0.2370 | 0.1510 | 0.1122 | 0.0761 | 0.0573 | 0.0468 |
| | S-DIDX AITE | 0.1987 | 0.1396 | 0.1024 | 0.0694 | 0.0529 | 0.0416 |
| | S-DIDX ATE | 0.3030 | 0.2023 | 0.1501 | 0.0981 | 0.0771 | 0.0590 |
| 0.75 | DID ATE | 0.3555 | 0.3358 | 0.3157 | 0.3148 | 0.3152 | 0.3205 |
| | DIDX ATE | 0.3728 | 0.3571 | 0.3388 | 0.3382 | 0.3386 | 0.3439 |
| | S-DID ADTE | 0.2320 | 0.1488 | 0.1141 | 0.0767 | 0.0573 | 0.0466 |
| | S-DID AITE | 0.4306 | 0.3803 | 0.3787 | 0.3726 | 0.3753 | 0.3738 |
| | S-DID ATE | 0.4629 | 0.3755 | 0.3782 | 0.3657 | 0.3678 | 0.3626 |
| | S-DIDX ADTE | 0.2239 | 0.1465 | 0.1132 | 0.0751 | 0.0556 | 0.0454 |
| | S-DIDX AITE | 0.2083 | 0.1343 | 0.1038 | 0.0702 | 0.0526 | 0.0418 |
| | S-DIDX ATE | 0.2856 | 0.1903 | 0.1437 | 0.0959 | 0.0746 | 0.0592 |
| 0.90 | DID ATE | 0.4166 | 0.3908 | 0.3887 | 0.3873 | 0.3861 | 0.3874 |
| | DIDX ATE | 0.4361 | 0.4137 | 0.4121 | 0.4108 | 0.4095 | 0.4109 |
| | S-DID ADTE | 0.2303 | 0.1526 | 0.1165 | 0.0760 | 0.0560 | 0.0469 |
| | S-DID AITE | 0.4076 | 0.3798 | 0.3714 | 0.3783 | 0.3774 | 0.3758 |
| | S-DID ATE | 0.4540 | 0.3924 | 0.3647 | 0.3671 | 0.3673 | 0.3654 |
| | S-DIDX ADTE | 0.2275 | 0.1496 | 0.1143 | 0.0738 | 0.0548 | 0.0454 |
| | S-DIDX AITE | 0.1977 | 0.1350 | 0.1009 | 0.0709 | 0.0517 | 0.0421 |
| | S-DIDX ATE | 0.3033 | 0.1947 | 0.1517 | 0.0961 | 0.0748 | 0.0597 |

**Table A5:** Bias for different $ATE$ estimators with $0 \leq \rho < 1$ and $\ell = 0.9$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 0.00 | DID ATE | 0.0289 | 0.0303 | 0.0198 | 0.0205 | 0.0255 | 0.0283 |
| | DIDX ATE | 0.0043 | 0.0062 | -0.0046 | -0.0040 | 0.0012 | 0.0040 |
| | S-DID ADTE | -0.0722 | -0.0732 | -0.0878 | -0.0872 | -0.0803 | -0.0782 |
| | S-DID AITE | 0.6311 | 0.6150 | 0.6337 | 0.6363 | 0.6330 | 0.6389 |
| | S-DID ATE | 0.5589 | 0.5418 | 0.5459 | 0.5491 | 0.5527 | 0.5607 |
| | S-DIDX ADTE | 0.0054 | 0.0068 | -0.0037 | -0.0035 | 0.0015 | 0.0034 |
| | S-DIDX AITE | -0.0053 | -0.0044 | -0.0065 | -0.0024 | -0.0024 | 0.0037 |
| | S-DIDX ATE | 0.0000 | 0.0024 | -0.0102 | -0.0059 | -0.0009 | 0.0071 |
| 0.10 | DID ATE | -0.0224 | -0.0164 | -0.0150 | -0.0159 | -0.0178 | -0.0167 |
| | DIDX ATE | -0.0468 | -0.0411 | -0.0394 | -0.0404 | -0.0422 | -0.0411 |
| | S-DID ADTE | -0.0832 | -0.0836 | -0.0796 | -0.0802 | -0.0831 | -0.0797 |
| | S-DID AITE | 0.6318 | 0.6393 | 0.6216 | 0.6321 | 0.6381 | 0.6385 |
| | S-DID ATE | 0.5485 | 0.5557 | 0.5420 | 0.5519 | 0.5550 | 0.5588 |
| | S-DIDX ADTE | -0.0035 | 0.0001 | 0.0029 | 0.0009 | -0.0007 | 0.0007 |
| | S-DIDX AITE | -0.0094 | 0.0003 | -0.0046 | 0.0017 | 0.0012 | 0.0005 |
| | S-DIDX ATE | -0.0129 | 0.0004 | -0.0017 | 0.0026 | 0.0004 | 0.0011 |
| 0.25 | DID ATE | -0.0761 | -0.0822 | -0.0796 | -0.0782 | -0.0848 | -0.0828 |
| | DIDX ATE | -0.1007 | -0.1066 | -0.1041 | -0.1028 | -0.1093 | -0.1073 |
| | S-DID ADTE | -0.0732 | -0.0849 | -0.0809 | -0.0795 | -0.0869 | -0.0835 |
| | S-DID AITE | 0.6155 | 0.6426 | 0.6260 | 0.6340 | 0.6384 | 0.6389 |
| | S-DID ATE | 0.5423 | 0.5577 | 0.5451 | 0.5545 | 0.5515 | 0.5555 |
| | S-DIDX ADTE | 0.0056 | -0.0042 | 0.0017 | 0.0023 | -0.0053 | -0.0025 |
| | S-DIDX AITE | -0.0087 | 0.0129 | -0.0073 | -0.0025 | 0.0020 | -0.0001 |
| | S-DIDX ATE | -0.0032 | 0.0086 | -0.0056 | -0.0001 | -0.0033 | -0.0027 |
| 0.50 | DID ATE | -0.1807 | -0.1741 | -0.1751 | -0.1832 | -0.1861 | -0.1840 |
| | DIDX ATE | -0.2055 | -0.1989 | -0.1998 | -0.2080 | -0.2110 | -0.2087 |
| | S-DID ADTE | -0.0695 | -0.0712 | -0.0746 | -0.0804 | -0.0850 | -0.0799 |
| | S-DID AITE | 0.6209 | 0.6250 | 0.6333 | 0.6400 | 0.6397 | 0.6359 |
| | S-DID ATE | 0.5513 | 0.5537 | 0.5586 | 0.5596 | 0.5547 | 0.5560 |
| | S-DIDX ADTE | 0.0076 | 0.0099 | 0.0072 | 0.0013 | -0.0023 | 0.0007 |
| | S-DIDX AITE | -0.0148 | -0.0042 | 0.0041 | 0.0018 | 0.0006 | -0.0016 |
| | S-DIDX ATE | -0.0072 | 0.0057 | 0.0113 | 0.0031 | -0.0018 | -0.0009 |
| 0.75 | DID ATE | -0.2772 | -0.2889 | -0.2827 | -0.2845 | -0.2846 | -0.2895 |
| | DIDX ATE | -0.3019 | -0.3138 | -0.3079 | -0.3095 | -0.3095 | -0.3144 |
| | S-DID ADTE | -0.0675 | -0.0872 | -0.0771 | -0.0807 | -0.0785 | -0.0813 |
| | S-DID AITE | 0.6352 | 0.6293 | 0.6389 | 0.6394 | 0.6397 | 0.6385 |
| | S-DID ATE | 0.5677 | 0.5420 | 0.5618 | 0.5587 | 0.5613 | 0.5572 |
| | S-DIDX ADTE | 0.0130 | -0.0031 | 0.0048 | 0.0026 | 0.0031 | -0.0004 |
| | S-DIDX AITE | 0.0002 | -0.0030 | 0.0046 | 0.0013 | 0.0037 | 0.0001 |
| | S-DIDX ATE | 0.0132 | -0.0062 | 0.0094 | 0.0038 | 0.0068 | -0.0003 |
| 0.90 | DID ATE | -0.3614 | -0.3468 | -0.3506 | -0.3500 | -0.3487 | -0.3510 |
| | DIDX ATE | -0.3868 | -0.3720 | -0.3757 | -0.3750 | -0.3737 | -0.3760 |
| | S-DID ADTE | -0.0830 | -0.0766 | -0.0883 | -0.0826 | -0.0815 | -0.0811 |
| | S-DID AITE | 0.6392 | 0.6257 | 0.6339 | 0.6430 | 0.6402 | 0.6383 |
| | S-DID ATE | 0.5562 | 0.5491 | 0.5456 | 0.5605 | 0.5587 | 0.5573 |
| | S-DIDX ADTE | -0.0102 | 0.0020 | -0.0024 | -0.0008 | 0.0007 | 0.0000 |
| | S-DIDX AITE | 0.0118 | 0.0032 | -0.0041 | 0.0058 | 0.0053 | 0.0014 |
| | S-DIDX ATE | 0.0016 | 0.0053 | -0.0065 | 0.0050 | 0.0060 | 0.0014 |

**Table A6:** RMSE for different $ATE$ estimators with $0 \leq \rho < 1$ and $\ell = 0.9$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 0.00 | DID ATE | 0.2214 | 0.1541 | 0.1154 | 0.0766 | 0.0599 | 0.0505 |
| | DIDX ATE | 0.2204 | 0.1522 | 0.1141 | 0.0744 | 0.0563 | 0.0449 |
| | S-DID ADTE | 0.2451 | 0.1729 | 0.1391 | 0.1073 | 0.0904 | 0.0832 |
| | S-DID AITE | 0.6437 | 0.6157 | 0.6337 | 0.6363 | 0.6330 | 0.6389 |
| | S-DID ATE | 0.5876 | 0.5453 | 0.5468 | 0.5491 | 0.5527 | 0.5607 |
| | S-DIDX ADTE | 0.2240 | 0.1556 | 0.1164 | 0.0750 | 0.0565 | 0.0452 |
| | S-DIDX AITE | 0.1965 | 0.1323 | 0.1041 | 0.0656 | 0.0504 | 0.0401 |
| | S-DIDX ATE | 0.2818 | 0.1890 | 0.1432 | 0.0932 | 0.0700 | 0.0559 |
| 0.10 | DID ATE | 0.2327 | 0.1503 | 0.1133 | 0.0762 | 0.0561 | 0.0475 |
| | DIDX ATE | 0.2349 | 0.1537 | 0.1171 | 0.0825 | 0.0644 | 0.0570 |
| | S-DID ADTE | 0.2508 | 0.1738 | 0.1339 | 0.1043 | 0.0924 | 0.0852 |
| | S-DID AITE | 0.6439 | 0.6396 | 0.6216 | 0.6321 | 0.6381 | 0.6385 |
| | S-DID ATE | 0.5914 | 0.5603 | 0.5425 | 0.5519 | 0.5550 | 0.5588 |
| | S-DIDX ADTE | 0.2334 | 0.1515 | 0.1141 | 0.0759 | 0.0555 | 0.0457 |
| | S-DIDX AITE | 0.2063 | 0.1368 | 0.0988 | 0.0657 | 0.0496 | 0.0406 |
| | S-DIDX ATE | 0.3042 | 0.1915 | 0.1447 | 0.0926 | 0.0675 | 0.0564 |
| 0.25 | DID ATE | 0.2380 | 0.1635 | 0.1305 | 0.0994 | 0.0936 | 0.0872 |
| | DIDX ATE | 0.2438 | 0.1726 | 0.1425 | 0.1157 | 0.1139 | 0.1093 |
| | S-DID ADTE | 0.2462 | 0.1752 | 0.1373 | 0.1038 | 0.0960 | 0.0889 |
| | S-DID AITE | 0.6257 | 0.6431 | 0.6260 | 0.6340 | 0.6384 | 0.6389 |
| | S-DID ATE | 0.5768 | 0.5637 | 0.5457 | 0.5545 | 0.5515 | 0.5555 |
| | S-DIDX ADTE | 0.2323 | 0.1517 | 0.1150 | 0.0757 | 0.0554 | 0.0459 |
| | S-DIDX AITE | 0.2008 | 0.1332 | 0.1013 | 0.0684 | 0.0493 | 0.0412 |
| | S-DIDX ATE | 0.2779 | 0.1872 | 0.1409 | 0.0953 | 0.0701 | 0.0571 |
| 0.50 | DID ATE | 0.2819 | 0.2074 | 0.1896 | 0.1847 | 0.1863 | 0.1840 |
| | DIDX ATE | 0.2936 | 0.2243 | 0.2096 | 0.2086 | 0.2110 | 0.2087 |
| | S-DID ADTE | 0.2555 | 0.1653 | 0.1349 | 0.1045 | 0.0940 | 0.0847 |
| | S-DID AITE | 0.6318 | 0.6256 | 0.6333 | 0.6400 | 0.6397 | 0.6359 |
| | S-DID ATE | 0.5923 | 0.5583 | 0.5589 | 0.5596 | 0.5547 | 0.5560 |
| | S-DIDX ADTE | 0.2392 | 0.1500 | 0.1143 | 0.0759 | 0.0586 | 0.0455 |
| | S-DIDX AITE | 0.1967 | 0.1346 | 0.0995 | 0.0667 | 0.0525 | 0.0407 |
| | S-DIDX ATE | 0.2901 | 0.1876 | 0.1428 | 0.0920 | 0.0740 | 0.0559 |
| 0.75 | DID ATE | 0.3246 | 0.3025 | 0.2857 | 0.2847 | 0.2846 | 0.2895 |
| | DIDX ATE | 0.3414 | 0.3242 | 0.3098 | 0.3096 | 0.3095 | 0.3144 |
| | S-DID ADTE | 0.2407 | 0.1731 | 0.1381 | 0.1042 | 0.0915 | 0.0858 |
| | S-DID AITE | 0.6496 | 0.6297 | 0.6389 | 0.6394 | 0.6397 | 0.6385 |
| | S-DID ATE | 0.6043 | 0.5441 | 0.5626 | 0.5587 | 0.5613 | 0.5572 |
| | S-DIDX ADTE | 0.2222 | 0.1481 | 0.1176 | 0.0745 | 0.0553 | 0.0463 |
| | S-DIDX AITE | 0.2067 | 0.1293 | 0.1011 | 0.0668 | 0.0512 | 0.0397 |
| | S-DIDX ATE | 0.2785 | 0.1843 | 0.1362 | 0.0908 | 0.0711 | 0.0562 |
| 0.90 | DID ATE | 0.3931 | 0.3523 | 0.3510 | 0.3500 | 0.3487 | 0.3510 |
| | DIDX ATE | 0.4125 | 0.3761 | 0.3760 | 0.3750 | 0.3737 | 0.3760 |
| | S-DID ADTE | 0.2404 | 0.1639 | 0.1419 | 0.1029 | 0.0924 | 0.0860 |
| | S-DID AITE | 0.6454 | 0.6263 | 0.6339 | 0.6430 | 0.6402 | 0.6383 |
| | S-DID ATE | 0.5856 | 0.5530 | 0.5458 | 0.5605 | 0.5587 | 0.5573 |
| | S-DIDX ADTE | 0.2239 | 0.1462 | 0.1163 | 0.0724 | 0.0564 | 0.0467 |
| | S-DIDX AITE | 0.1920 | 0.1309 | 0.0972 | 0.0679 | 0.0499 | 0.0409 |
| | S-DIDX ATE | 0.2835 | 0.1811 | 0.1466 | 0.0900 | 0.0694 | 0.0573 |

**Table A7:** Bias for different $ATE$ estimators with $1 \leq \rho \leq 5$ and $\ell = 0$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 1 | DID ATE | -0.4767 | -0.4736 | -0.4777 | -0.4824 | -0.4757 | -0.4797 |
| | DIDX ATE | -0.4996 | -0.4961 | -0.5003 | -0.5049 | -0.4982 | -0.5022 |
| | S-DID ADTE | 0.0369 | 0.0288 | 0.0239 | 0.0194 | 0.0243 | 0.0209 |
| | S-DID AITE | -0.0026 | -0.0034 | -0.0058 | -0.0042 | -0.0024 | 0.0022 |
| | S-DID ATE | 0.0342 | 0.0254 | 0.0181 | 0.0152 | 0.0219 | 0.0231 |
| | S-DIDX ADTE | 0.0122 | 0.0065 | 0.0012 | -0.0031 | 0.0018 | -0.0016 |
| | S-DIDX AITE | -0.0053 | -0.0061 | -0.0055 | -0.0020 | -0.0039 | 0.0001 |
| | S-DIDX ATE | 0.0069 | 0.0003 | -0.0043 | -0.0052 | -0.0021 | -0.0015 |
| 2 | DID ATE | -0.9929 | -0.9694 | -0.9788 | -0.9814 | -0.9841 | -0.9763 |
| | DIDX ATE | -1.0154 | -0.9919 | -1.0014 | -1.0039 | -1.0066 | -0.9989 |
| | S-DID ADTE | 0.0238 | 0.0326 | 0.0272 | 0.0194 | 0.0201 | 0.0241 |
| | S-DID AITE | 0.0042 | 0.0038 | 0.0079 | -0.0024 | -0.0012 | 0.0031 |
| | S-DID ATE | 0.0280 | 0.0363 | 0.0350 | 0.0169 | 0.0188 | 0.0272 |
| | S-DIDX ADTE | 0.0013 | 0.0095 | 0.0046 | -0.0030 | -0.0025 | 0.0016 |
| | S-DIDX AITE | 0.0033 | 0.0025 | 0.0037 | -0.0029 | -0.0023 | 0.0023 |
| | S-DIDX ATE | 0.0046 | 0.0120 | 0.0084 | -0.0059 | -0.0048 | 0.0039 |
| 3 | DID ATE | -1.5018 | -1.4979 | -1.4802 | -1.4836 | -1.4788 | -1.4730 |
| | DIDX ATE | -1.5244 | -1.5204 | -1.5027 | -1.5062 | -1.5014 | -1.4956 |
| | S-DID ADTE | 0.0244 | 0.0203 | 0.0275 | 0.0211 | 0.0217 | 0.0286 |
| | S-DID AITE | -0.0233 | 0.0094 | 0.0019 | 0.0006 | -0.0001 | 0.0003 |
| | S-DID ATE | 0.0011 | 0.0296 | 0.0294 | 0.0217 | 0.0216 | 0.0289 |
| | S-DIDX ADTE | 0.0031 | -0.0022 | 0.0051 | -0.0014 | -0.0009 | 0.0059 |
| | S-DIDX AITE | -0.0101 | 0.0095 | 0.0040 | 0.0008 | -0.0013 | 0.0020 |
| | S-DIDX ATE | -0.0070 | 0.0073 | 0.0091 | -0.0005 | -0.0022 | 0.0080 |
| 4 | DID ATE | -1.9994 | -1.9829 | -1.9790 | -1.9789 | -1.9854 | -1.9822 |
| | DIDX ATE | -2.0222 | -2.0054 | -2.0017 | -2.0015 | -2.0080 | -2.0048 |
| | S-DID ADTE | 0.0288 | 0.0241 | 0.0246 | 0.0252 | 0.0211 | 0.0211 |
| | S-DID AITE | -0.0210 | 0.0036 | 0.0100 | 0.0032 | -0.0032 | 0.0019 |
| | S-DID ATE | 0.0078 | 0.0277 | 0.0346 | 0.0284 | 0.0179 | 0.0231 |
| | S-DIDX ADTE | 0.0075 | 0.0010 | 0.0021 | 0.0026 | -0.0014 | -0.0015 |
| | S-DIDX AITE | -0.0117 | 0.0053 | 0.0048 | 0.0034 | -0.0027 | 0.0013 |
| | S-DIDX ATE | -0.0042 | 0.0063 | 0.0070 | 0.0060 | -0.0041 | -0.0002 |
| 5 | DID ATE | -2.5108 | -2.5095 | -2.5023 | -2.4785 | -2.4820 | -2.4800 |
| | DIDX ATE | -2.5332 | -2.5322 | -2.5249 | -2.5011 | -2.5045 | -2.5026 |
| | S-DID ADTE | 0.0277 | 0.0108 | 0.0196 | 0.0255 | 0.0222 | 0.0221 |
| | S-DID AITE | 0.0005 | 0.0077 | 0.0134 | -0.0018 | -0.0007 | -0.0004 |
| | S-DID ATE | 0.0282 | 0.0185 | 0.0330 | 0.0237 | 0.0215 | 0.0217 |
| | S-DIDX ADTE | 0.0083 | -0.0120 | -0.0031 | 0.0028 | -0.0003 | -0.0005 |
| | S-DIDX AITE | 0.0017 | 0.0043 | 0.0043 | 0.0021 | -0.0002 | -0.0006 |
| | S-DIDX ATE | 0.0100 | -0.0077 | 0.0012 | 0.0049 | -0.0005 | -0.0012 |

**Table A8:** RMSE for different $ATE$ estimators with $1 \leq \rho \leq 5$ and $\ell = 0$.

| $\rho$ | Estimator | $n = 100$ | $n = 225$ | $n = 400$ | $n = 900$ | $n = 1600$ | $n = 2500$ |
|---|---|---|---|---|---|---|---|
| 1 | DID ATE | 0.4900 | 0.4740 | 0.4777 | 0.4824 | 0.4757 | 0.4797 |
| | DIDX ATE | 0.5108 | 0.4962 | 0.5003 | 0.5049 | 0.4982 | 0.5022 |
| | S-DID ADTE | 0.2216 | 0.1522 | 0.1124 | 0.0765 | 0.0588 | 0.0481 |
| | S-DID AITE | 0.2688 | 0.1843 | 0.1440 | 0.0957 | 0.0718 | 0.0598 |
| | S-DID ATE | 0.3455 | 0.2457 | 0.1846 | 0.1250 | 0.0929 | 0.0761 |
| | S-DIDX ADTE | 0.2205 | 0.1501 | 0.1109 | 0.0751 | 0.0556 | 0.0457 |
| | S-DIDX AITE | 0.2097 | 0.1409 | 0.1069 | 0.0756 | 0.0537 | 0.0444 |
| | S-DIDX ATE | 0.3033 | 0.2111 | 0.1551 | 0.1074 | 0.0775 | 0.0645 |
| 2 | DID ATE | 0.9930 | 0.9694 | 0.9788 | 0.9814 | 0.9841 | 0.9763 |
| | DIDX ATE | 1.0155 | 0.9919 | 1.0014 | 1.0039 | 1.0066 | 0.9989 |
| | S-DID ADTE | 0.2239 | 0.1548 | 0.1179 | 0.0777 | 0.0590 | 0.0474 |
| | S-DID AITE | 0.2816 | 0.1852 | 0.1438 | 0.0946 | 0.0719 | 0.0586 |
| | S-DID ATE | 0.3725 | 0.2412 | 0.1867 | 0.1206 | 0.0940 | 0.0776 |
| | S-DIDX ADTE | 0.2235 | 0.1517 | 0.1159 | 0.0766 | 0.0568 | 0.0433 |
| | S-DIDX AITE | 0.2164 | 0.1394 | 0.1081 | 0.0722 | 0.0544 | 0.0438 |
| | S-DIDX ATE | 0.3259 | 0.2049 | 0.1576 | 0.1040 | 0.0807 | 0.0633 |
| 3 | DID ATE | 1.5018 | 1.4979 | 1.4802 | 1.4836 | 1.4788 | 1.4730 |
| | DIDX ATE | 1.5244 | 1.5204 | 1.5027 | 1.5062 | 1.5014 | 1.4956 |
| | S-DID ADTE | 0.2314 | 0.1583 | 0.1170 | 0.0767 | 0.0585 | 0.0511 |
| | S-DID AITE | 0.2836 | 0.1822 | 0.1402 | 0.0941 | 0.0716 | 0.0583 |
| | S-DID ATE | 0.3679 | 0.2494 | 0.1820 | 0.1232 | 0.0925 | 0.0786 |
| | S-DIDX ADTE | 0.2289 | 0.1573 | 0.1148 | 0.0748 | 0.0562 | 0.0456 |
| | S-DIDX AITE | 0.2152 | 0.1437 | 0.1080 | 0.0729 | 0.0538 | 0.0455 |
| | S-DIDX ATE | 0.3158 | 0.2196 | 0.1597 | 0.1028 | 0.0765 | 0.0654 |
| 4 | DID ATE | 1.9994 | 1.9829 | 1.9790 | 1.9789 | 1.9854 | 1.9822 |
| | DIDX ATE | 2.0222 | 2.0054 | 2.0017 | 2.0015 | 2.0080 | 2.0048 |
| | S-DID ADTE | 0.2347 | 0.1552 | 0.1122 | 0.0750 | 0.0572 | 0.0478 |
| | S-DID AITE | 0.2699 | 0.1794 | 0.1380 | 0.0974 | 0.0738 | 0.0591 |
| | S-DID ATE | 0.3694 | 0.2429 | 0.1760 | 0.1236 | 0.0910 | 0.0756 |
| | S-DIDX ADTE | 0.2322 | 0.1536 | 0.1110 | 0.0718 | 0.0544 | 0.0453 |
| | S-DIDX AITE | 0.2081 | 0.1378 | 0.1077 | 0.0757 | 0.0533 | 0.0457 |
| | S-DIDX ATE | 0.3162 | 0.2090 | 0.1538 | 0.1069 | 0.0769 | 0.0640 |
| 5 | DID ADTE | 2.5108 | 2.5095 | 2.5023 | 2.4785 | 2.4820 | 2.4800 |
| | DIDX ADTE | 2.5332 | 2.5322 | 2.5249 | 2.5011 | 2.5045 | 2.5026 |
| | S-DID ADTE | 0.2277 | 0.1490 | 0.1148 | 0.0748 | 0.0595 | 0.0479 |
| | S-DID AITE | 0.2806 | 0.1861 | 0.1426 | 0.0943 | 0.0719 | 0.0559 |
| | S-DID ATTE | 0.3618 | 0.2439 | 0.1872 | 0.1223 | 0.0902 | 0.0728 |
| | S-DIDX ADTE | 0.2269 | 0.1498 | 0.1141 | 0.0720 | 0.0559 | 0.0450 |
| | S-DIDX AITE | 0.2106 | 0.1442 | 0.1086 | 0.0722 | 0.0560 | 0.0423 |
| | S-DIDX ATTE | 0.3034 | 0.2112 | 0.1581 | 0.1041 | 0.0776 | 0.0601 |