Testing for Stock Market Contagion:
A Quantile Regression Approach

Sungyong Park[1]
Wendun Wang[2]
Naijing Huang[3]

[1] *Chung-Ang University, Seoul, Korea;*
[2] *Erasmus School of Economics, Erasmus University Rotterdam, and Tinbergen Institute, the Netherlands;*
[3] *Boston College, United States.*

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Sung Y. Park[†]   Wendun Wang[‡]   Naijing Huang[§]

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Abstract

Regarding the asymmetric and leptokurtic behavior of financial data, we propose a new contagion test in the quantile regression framework that is robust to model misspecification. Unlike conventional correlation-based tests, the proposed quantile contagion test allows us to investigate the stock market contagion at various quantiles, not only at the mean. We show that the quantile contagion test can detect a contagion effect that is possibly ignored by correlation-based tests. A wide range of simulation studies show that the proposed test is superior to the correlation-based tests in terms of size and power. We compare our test with correlation-based tests using three real data sets: the 1994 Tequila crisis, the 1997 Asia crisis, and the 2001 Argentina crisis. Empirical results show substantial differences between two types of tests.

Keywords: Financial contagion, Quantile regression, One-sided score test

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[†]School of Economics, Chung-Ang University, 84 Heukseok-Ro, Dongjak-Gu, Seoul, Korea. E-mail: `sungpark@cau.ac.kr`

[‡]Econometric Institute, Erasmus University Rotterdam and Tinbergen Institute. E-mail: `wang@ese.eur.nl`

[§]Department of Economics, Boston College. E-mail: `huangnaijing@hotmail.com`

1 Introduction

Financial crises occurred throughout the world in the second half of the 1990s; these include the 1994 Tequila crisis, the 1997 Asian flu, and the 2001 Argentina crisis. These crises have stimulated many empirical studies on contagion between stock markets, among which testing the occurrence of contagion has been of particular interest in both theoretical and empirical senses.

Various tests for financial contagion have been proposed according to various definitions of contagion; see Forbes and Rigobon (2001)(pp. 44–47) for a comparative review of these definitions. One popular definition of contagion is a significant increase in cross-market linkages after a shock to the original country (Forbes and Rigobon, 2001). Typically, this linkage is measured by the (mean) correlation of the asset returns; early influential studies using correlation include King and Wadhwani (1990) and Lee and Kim. Forbes and Rigobon (2002) showed that the heteroscedastic behavior of asset returns causes a bias in the correlation-based test and suggested an adjusted correlation measure to avoid such bias. Another definition is based on conditional probability: financial contagion is described as the joint occurrence of the extreme values (Bae et al., 2003). Other tests for financial contagion include the vector autoregression (VAR) approach (Faveroa and Giavazzi (2002), Pesaran and Pick (2007)), the copula method with Markov switching parameters (Rodriguez, 2007), and the quantile-based measure of co-movement (Cappiello et al., forthcoming). Dungey et al. (2005a) compared various tests and related them in a pooled regression framework.

In this paper, we propose to test contagion based on "quantile correlation". In contrast to the mean correlation, the quantile correlation captures the linkage of stock returns

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not only at the average (mean) level but also at different quantiles. The linkage at the lower quantiles is of particular interest for risk-averse agents since they are more concerned with whether the slump of one stock market would have an impact on another. The lower quantile linkage is also crucial for policy makers because it may influence government decisions and actions in the crisis period. In addition, we show that tests based on mean correlation may not detect the change of the linkage in some cases, and thus they may suffer from a poor power property. Our quantile correlation test resembles the probability-based test in that both methods attempt to model the behavior of stock returns at lower quantiles. However, the probability-based tests only focus on the exceedences, a small proportion of the support to the whole distribution, while regression quantiles are obtained using the whole sample.

This paper makes three main contributions. First, we contribute to the econometric literature by proposing two-sided and one-sided score tests for the slope coefficient of the quantile regression in a likelihood framework and study their asymptotic properties. The proposed score tests are robust under possible distributional misspecification and are easy to implement since they only require estimators under the null hypothesis, especially for the one-sided test. Second, we contribute to the empirical finance literature by providing new insight into testing financial contagion. The proposed quantile contagion test allows us to investigate the various impacts of one country's asset returns on those of others at various quantile levels. As we have mentioned, standard correlation-based tests can only indicate the linkage between two asset returns on an average level. However, the risk-averse agents and policy makers are typically more concerned with the linkage at the lower quantiles, and it can also be useful to consider the linkage at the upper quantiles. By implementing our quantile contagion test, one could study whether contagion exists

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when a stock market is experiencing a slump or advance. We compare the size and power property of our proposed tests with three forms of Forbes and Rigobon's correlation-based tests (FR), the most popular contagion tests, in various simulation designs. We find that the quantile contagion test beats the FR tests in most cases, especially when the data are skewed and leptokurtic. Finally, we conduct an empirical analysis of the contagion effect in the Tequila crisis, the Asian crisis, and the Argentine crisis and compare our tests with those of the correlation-based tests. The quantile contagion test generally produces different results and new insights. For example, we find that in most transmission channels of the Tequila crisis and the Argentine crisis FR tests show no financial contagion at mean values, while the quantile contagion test reports significant contagion effects at lower quantiles. In contrast, FR tests report contagion in some transmission channels in the Asian crisis, while the quantile contagion test indicates that contagion may only happen on an average level rather than at all quantiles. We conclude that no contagion in the mean correlation does not imply no contagion in the other quantiles. Therefore, when investors and policy makers study the joint behavior of multiple stock returns they should consider the different quantiles in addition to the average level.

This paper is organized as follows. The next section presents our model setup and compares it to other models in the literature. In Section 3, we derive the symmetric test as a preliminary of the contagion test. The contagion test procedure is performed in Section 4. We conduct Monte Carlo experiments in Section 5, and provide the empirical analysis in Section 6. Finally, Section 7 concludes.

2   Model Setup

To simplify the model without loss of generality, we only consider the relationship between two asset markets. Following Dungey et al. (2005a) we denote the asset returns of country 1 over the tranquil period as $x_{1,t}$ and the asset returns of country 2 over the tranquil period as $x_{2,t}$. We also denote the two countries' asset returns in the turbulent period as $y_{1,t}$ and $y_{2,t}$. Then the relationship between the two asset returns in each regime follows

$$x_{2,t} = \alpha_0 + \alpha_1 x_{1,t} + \eta_{x,t}, \tag{2.1}$$

$$y_{2,t} = \beta_0 + \beta_1 y_{1,t} + \eta_{y,t}, \tag{2.2}$$

where α_1 and β_1 capture the degree of the influence of country 1's asset returns on those of country 2 over the tranquil and volatile periods, respectively, and $\eta_{x,t}$ and $\eta_{y,t}$ are corresponding i.i.d. innovations. This is the standard static setup in the literature (see, for example, Dungey et al., 2005), and it assumes that the observations are serial uncorrelated. Later, we shall relax this assumption and incorporate the dynamic feature, at least to some extent, by including lagged variables.

From (2.1) and (2.2), it can be easily checked that contagion is not detected if $\alpha_1 = \beta_1$, that is, if country 1's returns have the same effect on country 2's during non-crisis and crisis periods. In contrast, if $\alpha_1 < \beta_1$, then country 2 is tied to country 1 to a larger extent in the volatile periods, and thus we find the occurrence of contagion. Therefore, the null and alternative hypotheses for testing for contagion are, respectively,

$$H_0 : \alpha_1 = \beta_1 \qquad \text{and} \qquad H_A : \alpha_1 < \beta_1.$$

The test can be performed based on the sample (mean) correlation of the two asset re-

turns in two regimes, which are denoted by $\hat{\rho}_x$ and $\hat{\rho}_y$, respectively (King and Wadhwani, 1990; Forbes and Rigobon, 2002)

$$\hat{\rho}_x = \frac{\hat{\sigma}_{x,12}}{\sqrt{\hat{\sigma}_{x,1}^2 \hat{\sigma}_{x,2}^2}} = \hat{\alpha}_1 \frac{\hat{\sigma}_{x,1}}{\hat{\sigma}_{x,2}}, \qquad \hat{\rho}_y = \frac{\hat{\sigma}_{y,12}}{\sqrt{\hat{\sigma}_{y,1}^2 \hat{\sigma}_{y,2}^2}} = \hat{\beta}_1 \frac{\hat{\sigma}_{y,1}}{\hat{\sigma}_{y,2}},$$

where $\hat{\sigma}_{x,i}$ and $\hat{\sigma}_{y,i}$ denote the standard error of country i's asset returns in the non-crisis and crisis periods, respectively. Interestingly, Forbes and Rigobon (2002) showed that the sample correlation is biased in the sense that $\rho_y > \rho_x$ even when $\alpha_1 = \beta_1$ because of $\sigma_{y,1} > \sigma_{x,1}$. To avoid a spurious test, they proposed an adjusted estimator of ρ_y

$$\hat{\nu}_y = \frac{\hat{\rho}_y}{\sqrt{1 + (\frac{\hat{\sigma}_{y,1}^2 - \hat{\sigma}_{x,1}^2}{\hat{\sigma}_{x,1}^2})(1 - \hat{\rho}_y^2)}}.$$

Thus, the null hypothesis of the test is $H_0 : \hat{\rho}_x = \hat{\nu}_y$, and it can be tested using the t statistic

$$FR = \frac{\hat{\nu}_y - \hat{\rho}_x}{\sqrt{\frac{1}{T_x} + \frac{1}{T_y}}},$$

where T_x and T_y are the numbers of observations in the tranquil and turmoil regimes, respectively. The asymptotic variance of the estimator $\hat{\nu}_y - \hat{\rho}_x$ is approximated by $T_x + T_y$ under the assumption of independence. Forbes and Rigobon (2002) proposed three alternative forms of the FR test to improve the asymptotic approximation using Fisher's Z-transformation

$$FR_1 = \frac{\frac{1}{2}\ln r_y - \frac{1}{2}\ln r_z}{\sqrt{\frac{1}{T_2-3} + \frac{1}{T-3}}}, \quad FR_2 = \frac{\frac{1}{2}\ln r_y - \frac{1}{2}\ln r_x}{\sqrt{\frac{1}{T_2-3} + \frac{1}{T_1-3}}}, \quad FR_3 = \frac{\frac{1}{2}\ln r_y - \frac{1}{2}\ln r_z}{\sqrt{\frac{1}{T_1-3} - \frac{1}{T-3}}}.$$

where $r_y := (1 + \hat{\nu}_y)/(1 - \hat{\nu}_y)$, $r_z := (1 + \hat{\rho}_z)(1 - \hat{\rho}_z)$, and $r_x := (1 + \hat{\rho}_x)(1 - \hat{\rho}_x)$.

Note that α_1 and β_1 coincide with the adjusted correlations ρ_x and ν_y if we scale the asset returns of the individual countries in (2.1) and (2.2) by their corresponding standard

deviations of the tranquil period $\sigma_{x,1}$ and $\sigma_{x,2}$, respectively. Thus, we can test contagion using the regression coefficients α_1 and β_1.

Next, we consider the following pooled regression model proposed by Dungey et al. (2005a)

$$z_{2,t} = \gamma_0 + \gamma_1 d_t + \gamma_2 z_{1,t} + \gamma_3 z_{1,t} d_t + \eta_t, \tag{2.3}$$

where $z_{i,t} = (x_{i,1}, x_{i,2}, \ldots, x_{i,T_x}, y_{i,1}, y_{i,2}, \ldots, y_{i,T_y})'$, $i = 1, 2$, and d_t is the dummy variable such that $d_t = 1$ for the volatile period and $d_t = 0$ otherwise. Note that γ_3 is identical to $\beta_1 - \alpha_1$ in (2.1) and (2.2). Thus, the contagion test is simplified to test whether γ_3 is larger than zero. Moreover, (2.3) can generalize the contagion test by allowing multivariate countries. The hypotheses for testing contagion in the pooled equation are

$$H_0 : \gamma_3 = 0 \quad \text{and} \quad H_A : \gamma_3 > 0.$$

Considering the possibility of serial dependent innovations, the above models (2.1), (2.2), and (2.3) can be readily extended by adding lagged variables:

$$x_{2,t} = \alpha_0 + \sum_{j=0}^{k_x} \alpha_{1,j} x_{1,t-j} + \sum_{l=0}^{k_x} \alpha_{2,l} x_{2,t-l} + \epsilon_{x,t},$$

$$y_{2,t} = \beta_0 + \sum_{j=0}^{k_y} \beta_{1,j} y_{1,t-j} + \sum_{l=0}^{k_y} \beta_{2,l} y_{1,t-l} + \epsilon_{y,t},$$

and

$$z_{2,t} = \gamma_0 + \sum_{i=0}^{k} \gamma_{1,i} d_{t-i} + \sum_{j=0}^{k} \gamma_{2,j} z_{1,t-j} + \sum_{q=0}^{k} \gamma_{3,q} z_{1,t-q} d_{t-q} + \epsilon_t, \tag{2.4}$$

where $\epsilon_{x,t}, \epsilon_{y,t}$ and ϵ_t are i.i.d innovations. The hypotheses also expand to test $\gamma_{3,0} = \gamma_{3,1} = \ldots = \gamma_{3,k} = 0$ jointly. The main purpose of introducing the lagged variables is to emphasize the dynamic structure in which the present asset returns are not only determined by the present quantities of another market, but are also affected by its previous values. One can

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further extend this framework by considering autoregressive conditional heteroscedastic (ARCH) errors. Then the estimation and testing procedure should be adjusted; see, for example, Koenker and Zhao (1996). We leave this generalization for the future research.

3   Quantile Regression Model

Both (2.3) and (2.4) aim at describing the relationship between the stock returns based on the conditional mean process. However, considering the fact that time series data might exhibit different behavior across quantiles, the methods described in Section 2 cannot provide complete information on the data distribution, and they fail to capture heterogeneity across quantiles. More specifically, the relationship between the two stock returns may remain the same during the non-crisis and crisis periods on average but vary at some quantiles. To model such heterogeneity, we consider the quantile version of (2.3)

$$Q_{z_{2,t}}(\tau|x_t) = \gamma_0(\tau) + \gamma_1(\tau)d_t + \gamma_2(\tau)z_{1,t} + \gamma_3(\tau)z_{1,t}d_t. \tag{3.1}$$

where $Q_{z_{2,t}}(\tau|x_t)$ denotes the τ-conditional quantile function of country 2's asset returns, $\tau \in (0,1)$ is a fixed value and $x_t = (1, d_t, z_{1,t}, z_{1,t}d_t)$. By letting τ vary from 0 to 1 continuously, we can trace out the conditional distribution of country 2's asset returns given the information on country 1's asset returns. Model (2.3) allows the interaction between the two countries' asset returns to differ over quantiles, and thus the contagion test based on quantile regression (2.3) can examine the occurrence of contagion at different quantiles. For example, a significant γ_3 at lower quantiles means that the association between the two countries' markets is enhanced when country 2's market is in a decline.

More importantly, the estimated coefficient of the quantile regression (3.1) is related

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to quantile correlation, which can be considered a quantile version of the mean correlation. To demonstrate this, we start with the definition of "quantile uncorrelation" recently proposed by Komarova et al. (2012). Extending Komarova et al. (2012), quantile uncorrelation between two l-dimensional random vectors y and x can be defined as

$$M(cy, x) = 0 \quad \text{for all} \quad c \in \mathcal{R}^l,$$

where

$$M(cy, x) := \arg \min_{\beta} E\rho_\tau[cy - x'\beta - \text{Quant}(cy - x'\beta)]$$

and $\text{Quant}(cy - x'\beta) := \inf\{u : P(cy - x'\beta \geq u) \geq \tau\}$ for a given τ. If we consider the conditional quantile function of y in a bivariate regression model with i.i.d. errors

$$Q_y(\tau|x) = \beta_0 + x'\beta_1 + F_u^{-1}(\tau),$$

the solution to the optimization problem

$$\{\hat{\alpha}(\tau), \hat{\beta}_1(\tau)\} = \arg \min_{\alpha, \beta} E\rho_\tau(y - \alpha - x'\beta)$$

estimates the population parameters $(\beta_0 + F_u^{-1}(\tau), \beta_1)$ for a given τ. Note that $\hat{\beta}_1(\tau)$ is equivalent to $M(cy, x)$ in the sense that $\hat{\beta}_1(\tau) = 0$ implies $M(cy, x) = 0$ because of the invariance property. Thus, we may say that the estimated coefficient of the quantile regression is an *indicator* of quantile correlation (but not a direct measure).

Next, we examine how the quantile regression coefficient is related to the quantile correlation. Komarova et al. (2012) used the $\mathcal{L}_1$ analogue of R^2 as a measure of median correlation. It takes a value of zero when y is median uncorrelated with x. Extending the median correlation, we follow Koneker and Machado (1999) and measure the quantile correlation using

$$quanrsq(y, x) := 1 - \frac{\min_\beta E\rho_\tau(y - x'\beta - F_{\hat{u}}^{-1})}{\min_\beta E\rho_\tau(y - F_{\tilde{u}}^{-1})},$$

where $\hat{u} = y - x'\beta$ and $\tilde{u} = y$. According to Theorem 3.3 in Komarova et al. (2012) if $\beta = 0$ then $quanrsq(y, x) = 0$. Hence, the regression quantile reflects the degree of quantile correlation even though it is a direct measure. In our case, if we find that $\gamma_3(\tau) = 0$ for a given τ, namely $\alpha_1(\tau) = \beta_1(\tau)$, then we can infer that the quantile correlations of the two countries' stock returns are similar, and there is thus no quantile contagion.

The conditional quantile function $Q_{z_{2,t}}(\tau|x_t)$ can be estimated by $\hat{\gamma}(\tau) = [\hat{\gamma}_0, \hat{\gamma}_1(\tau), \hat{\gamma}_2(\tau), \hat{\gamma}_3(\tau)]$ which solves the following minimization problem

$$\min_{\gamma \in \mathbb{R}^4} \sum_{t=1}^{T} \rho_\tau \left[z_{2,t} - \gamma_0(\tau) - \gamma_1(\tau)d_t - \gamma_2(\tau)z_{1,t} - \gamma_3(\tau)z_{1,t}d_t \right], \tag{3.2}$$

where $\rho_\tau(\eta_t) = \eta_t[\tau - I(\eta_t < 0)]$ and $I(\eta_t < 0)$ is an indicator function (Koenker and Bassett, 1978). This is equivalent to maximizing the log-likelihood function (ML) of (2.3)

$$\ln L(\gamma, \tau) = T \ln \tau(1 - \tau) - \sum_{t=1}^{T} \rho_\tau(z_{2,t} - \gamma x_t), \tag{3.3}$$

assuming that η_t follows the asymmetric Laplace density (ALD)

$$f(\eta_t, \sigma, \gamma) = \frac{1}{\sigma}\tau(1 - \tau) \exp\left[-\frac{1}{\sigma}\rho_\tau(z_{2,t} - \gamma x_t) \right],$$

with $\sigma = 1$ and fixed $\tau \in (0, 1)$.

In this likelihood framework, we can conveniently derive the score tests for quantile regression. Moreover, the likelihood approach enables us to consider the quasi maximum likelihood estimation (QMLE), which leads to robust estimates under distributional misspecification. Bera et al. (2008) showed that the quasi-maximum likelihood estimator $\hat{\gamma}$ converges to the normal distribution as

$$\sqrt{T}(\hat{\gamma} - \gamma_0) \xrightarrow{d} N(0, V_2^{-1}V_1V_2^{-1})$$

where

$$V_1 := E\left[\frac{1}{T}\left(\frac{\partial \ln L(\gamma, \tau)}{\partial \gamma} \right) \cdot \left(\frac{\partial \ln L(\gamma, \tau)}{\partial \gamma} \right)' \right]_{\gamma=\gamma_0}, \quad V_2 := \frac{\partial^2 E[1/T \ln L(\gamma, \tau)]}{\partial \gamma \partial \gamma'}\bigg|_{\gamma=\gamma_0}.$$

The expectations in V_1 and V_2 are taken with the true density function.

4 Quantile Symmetric Test

We first test the coefficient of the quantile regression with the two-sided alternative hypothesis, that is, $H_0 : \gamma_3 = 0$ against $H_A : \gamma_3 \neq 0$, and we call this the quantile symmetric test (QST). We use Rao's score test based on the likelihood function (3.3) because it is not affected even if parameters might lie on the boundary of the maintained hypothesis. In addition, it is computationally easier than the Wald and likelihood ratio tests in many cases since it only requires the restricted estimator under the null hypothesis. We start with four necessary assumptions:

A1: $f(\eta_t, \gamma)$ is measurable in η_t for every γ in a compact set Γ and continuous in γ for every η_t.

A2: $|\log f(\eta_t, \gamma)| \leq m(\eta_t)$ for every γ where $m(\eta_t)$ is integrable with respect to the true density distribution $G(\eta_t)$.

A3: $\tilde{J} := \frac{1}{T} \sum [\partial ln f(\eta_t, \gamma)/\partial \gamma] \cdot [\partial ln f(\eta_t, \gamma)/\partial \gamma]'$ and $\tilde{K} := \frac{1}{T} \sum -\partial^2 ln f(\eta_t, \gamma)/\partial \gamma \partial \gamma'$ exist and are continuous in γ and $E(\sup_\gamma |\tilde{J}|) < \infty$, $E(\sup_\gamma |\tilde{K}|) < \infty$.

A4: The Kullback-Leibler information criterion (KLIC),$I(g, f, \gamma) := E(\log[g(\eta_t)/f(\eta_t, \gamma)])$, reaches its unique minimum γ^* interior to the parameter space Γ.

A1 ensures the existence of QMLE and a well-defined KLIC is guaranteed by A2. Both $f(\eta_t, \gamma)$ and $\partial \log f(\eta_t, \gamma)/\partial \gamma$ are continuous and differentiable except at the points where $\eta_t = 0$; however, they have directional derivatives in all directions at these points (Koenker

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(2005) pp. 32–33). Therefore, A3 is satisfied to allow the application of the uniform law of large numbers to J and K.

It is well known that when the true data generating process (DGP) deviates from the assumed distribution that we assume, the standard Rao's score statistic $RS = s(\tilde{\gamma})'K(\tilde{\gamma})^{-1}s(\tilde{\gamma})$ is not valid because a distribution misspecification would lead to breaks in information matrix equality, that is, $J(\gamma_g) \neq K(\gamma_g)$, where

$$J(\gamma_g) = E_g\left[\frac{\partial \ln f(\gamma)}{\partial \gamma} \cdot \frac{\partial \ln f(\gamma)}{\partial \gamma'}\right] \quad \text{and} \quad K(\gamma_g) = E_g\left[-\frac{\partial^2 \ln f(\gamma)}{\partial \gamma \partial \gamma'}\right].$$

White (1982) suggested modifying the standard Rao's score test with asymptotic robust variance to ensure proper size. Then, under A1–A4, the score test under misspecification is

$$RS^* = \frac{1}{T}s'(\tilde{\gamma})'K(\tilde{\gamma})^{-1}H(\tilde{\gamma})[H(\tilde{\gamma})'V(\tilde{\gamma})H(\tilde{\gamma})]^{-1}H(\tilde{\gamma})'K(\tilde{\gamma})^{-1}s'(\tilde{\gamma}), \tag{4.1}$$

where $\tilde{\gamma}$ is the restricted estimator under the null hypothesis, $H(\tilde{\gamma}) = \partial h(\gamma)/\gamma'$ where $h(\gamma)$ denotes the restrictions, and $V(\tilde{\gamma}) = K(\tilde{\gamma})^{-1}J(\tilde{\gamma})K(\tilde{\gamma})^{-1}$. The modified score test RS^* is asymptotically distributed as χ_q^2 under H_0 even in presence of misspecification, where q is the number of restrictions.

To test $H_0 : \gamma_3 = 0$ against $H_A : \gamma_3 \neq 0$, we partition the parameter vector γ' as $(\gamma_A, \gamma_B)'$, where $\gamma_A = (\gamma_0, \gamma_1, \gamma_2)$ is the nuisance parameters and $\gamma_B = \gamma_3$ is the parameter of interest. Under the null hypothesis, the QMLE for γ is $\tilde{\gamma} = (\tilde{\gamma}_A, 0)'$.

Using the likelihood function (3.3), the score vector, $J(\gamma)$, and $K(\gamma)$ under the null hypothesis can be obtained consistent with the partition by

$$s(\tilde{\gamma}) = \left(\mathbf{0}, s_B(\tilde{\gamma})'\right) = \left(\mathbf{0}, \sum_{t=1}^{T}(\frac{1}{2}\text{sgn}(\xi_t) + \tau - \frac{1}{2})z_{1,t}d_t\right),$$

$$J(\tilde{\gamma}_g) = \tau(1 - \tau)E^*[x'x], \qquad K(\tilde{\gamma}_g) = E^*[g(\xi)x'x],$$

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where $\xi_t = z_{2,t} - \tilde{\gamma}_0 - \tilde{\gamma}_1 d_t - \tilde{\gamma}_2 z_{1,t}$ denotes the residual in the restricted model, $E^*[s] = \lim_{t \to \infty} \frac{1}{T} \sum_{t=1}^{T} s_t$ and $x_t = (1, d_t, z_t, z_t d_t)$. The restriction $h(\gamma) = \gamma_3$ leads to $H(\tilde{\gamma}) = \partial h(\gamma)/\gamma' = (0, 0, 0, 1)$.

For notational simplicity, we drop the arguments and let '~' denote the quantity evaluated at $\gamma = \tilde{\gamma}$; then

$$s(\tilde{\gamma})' K^{-1}(\tilde{\gamma}) H(\tilde{\gamma}) = \begin{pmatrix} 0' & s_B(\tilde{\gamma}) \end{pmatrix} \begin{pmatrix} \tilde{K}^{AA} & \tilde{K}^{AB} \\ \tilde{K}^{BA} & \tilde{K}^{BB} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \tilde{s}'_B \tilde{K}^{BB},$$

and the sandwich variance $\tilde{V}$ in (4.1) can be computed by

$$\tilde{V} = \begin{pmatrix} \tilde{K}^{AA} & \tilde{K}^{AB} \\ \tilde{K}^{BA} & \tilde{K}^{BB} \end{pmatrix} \begin{pmatrix} \tilde{J}_{AA} & \tilde{J}_{AB} \\ \tilde{J}_{BA} & \tilde{J}_{BB} \end{pmatrix} \begin{pmatrix} \tilde{K}^{AA} & \tilde{K}^{AB} \\ \tilde{K}^{BA} & \tilde{K}^{BB} \end{pmatrix},$$

where the subscript represents the partition of the matrix and the superscript denotes the corresponding element in its inverse matrix; for example

$$\tilde{K} = \begin{pmatrix} \tilde{K}_{AA} & \tilde{K}_{AB} \\ \tilde{K}_{BA} & \tilde{K}_{BB} \end{pmatrix} \quad \text{and} \quad \tilde{K}^{-1} = \begin{pmatrix} \tilde{K}^{AA} & \tilde{K}^{AB} \\ \tilde{K}^{BA} & \tilde{K}^{BB} \end{pmatrix}.$$

In addition, $\tilde{H}' \tilde{V} \tilde{H}$ in (4.1) can be obtained in a similar fashion

$$\tilde{H}' \tilde{V} \tilde{H} = \begin{pmatrix} 0, & 1 \end{pmatrix} \begin{pmatrix} \tilde{V}_{AA} & \tilde{V}_{AB} \\ \tilde{V}_{BA} & \tilde{V}_{BB} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \tilde{V}_{BB}.$$

With every parameter estimated, the symmetry quantile score test is summarized by the following proposition.

Proposition 1 *Under A1–A4, the robust Rao's score test in the quantile regression is*

$$RS_s = \frac{1}{T} \tilde{s}'_B \tilde{K}^{BB} \left(\tilde{K}^{AB} \tilde{K}^{BA} \tilde{J}_{AA} + \tilde{K}^{AB} \tilde{K}^{BB} \tilde{J}_{BA} + \tilde{K}^{BB} \tilde{K}^{BA} \tilde{J}_{AB} + \tilde{K}^{BB} \tilde{K}^{BB} \tilde{J}_{BB} \right)_{BB}^{-1} \tilde{K}^{BB} \tilde{s}_B. \qquad (4.2)$$

It asymptotically follows χ_q^2 under the null hypothesis where q is the number of restrictions, and it is invariant to the scaling transformation.

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The proof of invariance to the scaling transformation follows directly from the equivariance in quantile regression (see the Appendix). The invariance property suggests a direct test procedure for contagion simply with the usage of unscaled parameters instead of scaled ones.

5 Quantile Contagion Test

Since contagion is defined as $\gamma_3 > 0$ in (2.3), the one-sided nature of the alternative hypothesis should be taken into account to obtain a more appropriate test. In this section, we derive the quantile one-sided test (QOT) for testing the financial contagion.

One major difference between the one-sided and two-sided tests is their asymptotic distributions under the null hypothesis. Like the Wald and likelihood ratio tests with a one-sided alternative, the one-sided score test no longer asymptotically follows the χ^2 distribution but the mixture of χ^2 distribution. One advantage of the score test over the Wald and likelihood ratio tests is that its one-sided nature under the alternative hypothesis can be ignored since only the restricted model needs to be estimated under the null hypothesis. To ensure the limiting distribution of the efficient score test under the null hypothesis, an additional regularity condition should be satisfied (Gourieroux et al., 1982)

A5: There exists a non-singular matrix $J(\gamma)$ such that $\frac{1}{\sqrt{T}}s(\gamma)$ follows a normal distribution with variance $J(\gamma)$.

Given the asymptotic normal distribution of score vector $\frac{1}{\sqrt{T}}J^{-1}s(\gamma) \sim N\big(\iota, J^{-1}(\gamma)\big)$ under the sequence of hypotheses $H_T : \gamma_3 = T^{-1/2}\iota$, the one-sided score test for $H_0 : \gamma_3 = 0$ against $H_A : \gamma_3 > 0$ can be implemented equivalently by performing a likelihood ratio test for

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$H_0 : \iota = T^{1/2}\gamma_3 = 0$ against $H_A : \iota = T^{1/2}\gamma_3 > 0$ since $\gamma_3 > 0$ is equivalent to $\iota > 0$ (Silvapulle and Silvapulle, 1995). The likelihood ratio test statistic or $\bar{\chi}^2$-statistic for $\iota = 0$ against $\iota > 0$ was proposed by (Kudo, 1963)

$$\bar{\chi}^2 = T\left\{\bar{X}'\Lambda^{-1}\bar{X} - \min_{\mu \geqslant 0,}(\bar{X} - \mu)'\Lambda^{-1}(\bar{X} - \mu)\right\}, \tag{5.1}$$

where $\bar{X} = T^{-1/2}\Lambda^{-1}d$. Here d is the realization of ι with the asymptotic distribution $T^{-1/2}\Lambda^{-1}d \sim N(\iota, \Lambda^{-1})$ and Λ is the known non-singular variance matrix. With the equivalence of the likelihood ratio test and the score test, we can perform the one-sided score test with a similar procedure to that of the likelihood ratio test according to (5.1). Letting $\gamma = (\gamma_A, \gamma_B)$, the matrices $M = \frac{1}{\sqrt{T}}(s_B - K_{BA}K_{AA}^{-1}s_A)$ and $U = K^{BB}M$ evaluated at restricted MLE $\tilde{\gamma}$ can be estimated by $\tilde{M} = \frac{1}{\sqrt{T}}(\tilde{s}_B - \tilde{K}_{BA}\tilde{K}_{AA}^{-1}\tilde{s}_A)$ and $\tilde{U} = \tilde{K}^{BB}\tilde{M}$. Since $\frac{1}{\sqrt{T}}J(\gamma)^{-1}s(\gamma) \sim N(\iota, J(\gamma)^{-1})$, it follows that M is also normally distributed with variance $C = J_{BB} - K_{BA}K_{AA}^{-1}J_{AB} - (J_{BA} - K_{BA}K_{AA}^{-1}J_{AA})(K_{AA}^{-1})'K_{BA}'$ under the null hypothesis. Then, for a given $\iota > 0$ the asymptotic normality of U under $H_T : \gamma_3 = T^{-1/2}\iota$ can be obtained by

$$U \xrightarrow{d} N(\iota, V_{BB}).$$

With the parameters readily estimated, the one-sided score test statistic in our model is given in the following proposition

Proposition 2 *Under A1–A5, the one-sided robust score test in the quantile regression can be obtained as*
$$RS_c = \tilde{U}'\tilde{V}_{BB}^{-1}\tilde{U} - \inf_{b>0}\left\{(\tilde{U} - b)'\tilde{V}_{BB}^{-1}(\tilde{U} - b)\right\} \tag{5.2}$$
which asymptotically follows the mixture chi-square under the null hypothesis.

Since V is nonsingular, symmetric, and positive definite, we can find a non-singular matrix T to transform V into a unit matrix—say $TVT' = I$. By multiplying the same matrix T,

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U converts to the uncorrelated matrix $Q = TU$, and therefore $U'\tilde{V}_{BB}^{-1}U$ reduces to a square form $Q'Q$. In addition, the minimum $\inf_{b>0}\left\{(\tilde{U} - b)'\tilde{V}_{BB}^{-1}(\tilde{U} - b)\right\}$ can be thought of as the distance between the point of vector Q and a cone, and thus the geometric intuition behind the statistics in (5.2) is the difference between the square length of a vector $Q'Q$ and the distance from a point in this vector to a closed convex cone (Kudo, 1963).

The asymptotic distribution of the test statistic can be studied via the asymptotic p-value, which can be obtained by $\sup_{\gamma_A} \zeta(RS_c, V(\gamma_A), \mathscr{C})$, where $\mathscr{C}$ denotes the set of positive real numbers and

$$\zeta(RS_c, V(\gamma_A), \mathscr{C}) = \text{Prob}\left([U'V_{BB}^{-1}U - \inf_{b\geqslant 0}\left\{(U - b)'V_{BB}^{-1}(U - b)\right\}] \geqslant RS_c\right).$$

As an extension of the work by Perlman (1969), the asymptotic p-value $\sup_{\gamma_A} \zeta(RS_c, V(\gamma_A), \mathscr{C})$ lies in the interval between the lower bound $0.5\text{Prob}(\chi_1^2 \geqslant RS_c)$ and the upper bound $0.5[\text{Prob}(\chi_{q-1}^2 \geqslant RS_c) + \text{Prob}(\chi_q^2 \geqslant RS_c)]$, where q is the dimension of the restricted vector (see also Silvapulle and Silvapulle (1995)). The statistic in (5.2) follows a mixture of chi-squared distribution as

$$\text{Prob}\left(\bar{\chi}^2 \geqslant c\right) = \sum_{i=0}^{q} w_{i,q}\text{Prob}\left(\chi_i^2 \geqslant c\right),$$

where $w_{i,q}$, $(i = 0, 1, \ldots, q)$ are the nonnegative weights determined by the nuisance parameter γ_A. The weights can be calculated by

$$w_{i,q}(q, V) = \sum_{|\alpha|=i} p\{(V_{\alpha'})^{-1}\}p\{V_{\alpha,\alpha'}\},$$

where the summation extends over the nonempty subsets α of $\{1, 2, \cdots, q\}$, V_α is the corresponding variance matrix of normal vector U_i, $i \in \alpha$, $V_{\alpha,\alpha'}$ is the same under the condition $U_j = 0$ for $j \notin \alpha$, and $p\{V\}$ denotes the probability of the multivariate normal vector U larger than 0 with the distribution $U \sim N(0, V)$ (Kudo, 1963).

In the pooled regression model (2.3), U is a scalar for the single constraint $\gamma_3 = 0$; therefore, $q = 1$ in this model. The weights can be obtained by

$$w_{0,1}(V) = w_{1,1}(V) = 0.5.$$

Note that $RS_c(\tau)$ is a function of τ and the weights are not dependent on γ_A. Hence, the "$\sup_{\gamma_A}$" in calculating the p-value can be ignored and the p-value varies for different τ as

$$
\begin{aligned}
\sup_{\gamma_A} \zeta(RS_c(\tau), A(\gamma_A), \mathscr{C}) &= \sup_{\gamma_A} \mathrm{Prob}\left([U'A_{BB}^{-1}U - \inf_{b \geq 0}\{(U-b)'A_{BB}^{-1}(U-b)\}] \geq RS_c(\tau)\right).\\
&= 0.5\mathrm{Prob}\left(\chi_1^2 \geq RS_c(\tau)\right).
\end{aligned}
$$

Further computation is required in the case of the dynamic model in (2.4). If we consider the model with one lagged variable

$$z_{2,t} = \gamma_0 + \gamma_{1,0}d_t + \gamma_{2,0}z_{1,t} + \gamma_{3,0}z_{1,t}d_t + \gamma_{1,1}d_{t-1} + \gamma_{2,1}z_{1,t-1} + \gamma_{3,1}z_{1,t-1}d_{t-1} + \epsilon_t, \qquad (5.3)$$

we need to test the hypothesis $H_0 : \gamma_{3,0} = 0, \gamma_{3,1} = 0$ with the alternative hypothesis $H_A : \gamma_{3,0} > 0, \gamma_{3,1} > 0$. Therefore, $q = 2$ in the dynamic model (5.3). In this case, the weights can be calculated by

$$
\begin{aligned}
w_{0,2}(V) &= 0.5\pi^{-1}(\pi - \cos^{-1}\rho_{1,2}),\\
w_{1,2}(V) &= 0.5,\\
w_{2,2}(V) &= 0.5\pi^{-1}\cos^{-1}\rho_{1,2}.
\end{aligned}
$$

where $\rho_{i,q}$ is given by the (i, q)th element of the correlation matrix of U:

$$\left[diag(U)^{-1}\right]^{-\frac{1}{2}}(U)^{-1}\left[diag(U)^{-1}\right]^{-\frac{1}{2}}.$$

Different from the weights in the static model, the weights here are functions of γ_A, namely $w_{i,q}(\gamma_A)$, with the intermediate parameter V. Hence the p-value can be obtained

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by optimizing the probability with γ_A carried by the weights

$$\sup_{\gamma_A} \text{Prob}\left(\left[(U)'(V)^{-1}_{BB}(U) - \inf_{b \geqslant 0}\{(U-b)'(V)^{-1}_{BB}(U-b)\}\right] \geqslant RS_c(\tau)\right)$$

$$= \sup_{\gamma_A} \sum_{i=0}^{2} w_{i,2}(\gamma_A)\text{Prob}\left(\chi^2_i \geqslant RS_c(\tau)\right)$$

where χ^2_i denotes the chi-square distribution with i degrees of freedom.

6   Monte Carlo Simulation

In the simulation we mainly focus on the comparison of the proposed quantile tests and the correlation-based FR tests. FR tests are mostly popular for testing financial contagion because they are based on the most popular definition of contagion, are easy to implement, and have better performance in most cases. Our quantile contagion can be thought as a quantile version of correlation-based tests, and therefore this comparison is of particular interest. More comparison between contagion tests can be found in Dungey et al. (2005a,b).

6.1   Simulation Design

The experiment design follows the setup of Dungey et al. (2005b) in which asset returns in non-crisis and crisis structures are determined by common factors w_t together with idiosyncratic factors u_t (see also Pericoli and Sbracia (2003)). The common factors describe the fundamental elements that determine the average level of asset returns worldwide, while the idiosyncratic component captures the unique factor that differs from individual markets. In our experiments, we study the interrelationship of two asset markets using

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observations during tranquil and volatile periods. Stock market contagion is defined as transmission from country 1 to country 2. We model the non-crisis linkage as

$$x_{1,t} = 4w_t + 2u_{1,t}, \tag{6.1}$$

$$x_{2,t} = 3w_t + 4u_{2,t}, \tag{6.2}$$

where $w_t = \rho w_{t-1} + u_{w,t}$, $u_{w,t} \sim N(0, h_t)$, $h_t = \omega_x^2(1 - \alpha - \beta + \alpha u_{w,t-1}^2 + \beta h_{t-1})$, and $u_{i,t} \sim G_x(u_{i,t})$ for some distribution $G_x(\cdot)$. This setup allows for autocorrelation as well as a GARCH structure in the common factor. The parameter ρ reflects the degree of autocorrelation while α and β jointly control the GARCH process with time varying variance. In addition, ω_x determines the variance of the common factor, and it can shift during the tranquil and volatile periods to generate the common factor structural break. The crisis model is an extension of the non-crisis framework of (6.1) and (6.2) that adds country 1's idiosyncratic factor to the country 2's asset return $y_{2,t}$.

$$y_{1,t} = 4w_t + 2u_{1,t}, \tag{6.3}$$

$$y_{2,t} = 3w_t + 3u_{2,t} + 2\delta u_{1,t}, \tag{6.4}$$

where $w_t = \rho w_{t-1} + u_{w,t}$, $u_{w,t} \sim N(0, h_t)$, $h_t = \omega_y^2(1 - \alpha - \beta + \alpha u_{w,t-1}^2 + \beta h_{t-1})$, $u_{i,t} \sim G_y(u_{i,t})$ and δ denotes the contagion strength. For the distribution of idiosyncratic factor $G_i, i = x, y$, three distributions are considered: (i) the normal distribution, (ii) Student's t distribution with degree of freedom 3[1], and (iii) Hansen (1994)'s skewed t distribution.

In our experiment design, the two markets are relatively uncorrelated with independent idiosyncratic factors in the tranquil period. However, during the crisis period, country 1's idiosyncratic factor affects country 2's asset returns, increasing the correlation be-

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[1]We also perform the experiment under Student's t distribution with degrees of freedom five and seven. The results are substantially unchanged.

tween the two countries. We consider various characteristics of the data, such as the GARCH process, no autocorrelation data, weak autocorrelation data, strong autocorrelation data and data with a structural break. Two structural breaks are studied: (1) a structural break in the world factor w_t, which is generated by changing ω, and (2) a structural break in the idiosyncratic factor, which is generated by changing the variance of the idiosyncratic factor in different periods. When G is normally distributed, $u_{i,x} \sim N(0, \kappa_x^2)$ and $u_{i,y} \sim N(0, \kappa_y^2)$, the idiosyncratic structural break comes from the deviation between κ_x and κ_y. When G follows Student's t distribution, the idiosyncratic break is generated by changing the degree of freedom. For the skewed t distribution, we use Hansen (1994)'s generalized t distribution (see also Jondeau and Rockinger (2003))

$$
u(m|v, \phi) = \begin{cases} c\left(1 + \frac{1}{v-2}\left(\frac{m}{1-\phi}\right)^2\right)^{(v+1)/2} & \text{if } m < -a/b, \\[2ex] c\left(1 + \frac{1}{v-2}\left(\frac{m}{1+\phi}\right)^2\right)^{(v+1)/2} & \text{if } m \geqslant -a/b, \end{cases} \tag{6.5}
$$

where $c \equiv \Gamma[(v+1)/2]/[\sqrt{\pi(v-2)}\Gamma(v/2)]$ normalizes function $u(\cdot)$ as a p.d.f. with the mean $4\phi c(v-2)/(v-1)$ and variance $1 + 3\phi^2 - a^2$ (Jondeau and Rockinger, 2003). The scaling parameter v, interpreted as the order of moment, characterizes the height and tail of the distribution, while ϕ controls the skewness of the distribution. Based on (6.5), we can generate the break by changing either ϕ or v.

Although the model (6.1)–(6.4) is a typical way of generating contagion (Dungey et al., 2005a,b), we can show that under some specific parameterization the mean correlation decreases as contagion strength δ increases. This can be seen in Figure 1, and a mathematical derivation is given in the Appendix. In this case, correlation-based tests would incorrectly report no contagion. Therefore, we compare our quantile contagion with *FR* tests in two scenarios when correlation increases and when correlation decreases as contagion strengthens. In the following, we call these "increasing correlation" and "decreasing

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correlation" for short. The decreasing correlation scenario can be generated by choosing a smaller variance of the idiosyncratic factor than that of the common factor.[2]

[**Figure1**, **Table1**]

The whole sample period T is taken by 200, 500 and 1000 with sample sizes of 100, 250, and 500 of returns in the crisis period respectively. We let δ vary in a range from 0 to 12, and each experiment is duplicated 3000 times. The parameterization in both the increasing and decreasing correlation cases is given in Table 1.

6.2 Size Property

Tables 2 and 3 report the size of the QST, QOT, and FR tests in the increasing and decreasing correlation cases. Each of the experiments are based on critical values of 5%. We first consider the increasing correlation case. In this case, FR_1 is always undersized in all experiments, which implies that the null hypothesis of no contagion is over accepted. The poor size property of FR_1 may be due to the inappropriate assumption of independence between the non-crisis and crisis samples in its asymptotic variance. Thus, we focus on comparing QST and QOT with FR_2 and FR_3 in the following. FR_2 and FR_3 are competitive with QST and QOT in the normal distribution case, but not in the Student's t and skewed t cases. The four test statistics exhibit good size properties in most experiments of the normal distribution except for Experiments I and IV. We note that oversize of all the tests in Experiment IV of common the structural break may be because an increase in the volatility of the common factor leads to a higher degree of similarity between the two

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[2]The GARCH experiments with decreasing correlation are implemented by generating an idiosyncratic factor with a smaller amplitude than the common factor.

markets even though the contagion does not occur. Under the Student's t distribution, FR_2 yields relatively good size performance in most experiments except for Experiments IV and V (structure break), and the size of FR_3 is inflated in most cases. Compared to the FR tests, QST and QOT maintain better size conduction in most of the experiments. The results under the skewed t distribution are very similar to the Student's t case in that QST and QOT generally beat the FR tests. We thus conclude from the above results that QST and QOT perform better than all forms of FR in terms of size when the data distribution is generated with fatter tail and skewness.

Although the size of FR_2 and FR_3 is comparable to QST and QOT in the increasing correlation case, it is heavily dominated by our quantile tests in the decreasing correlation case, as reported in Table 3. FR_1 remains severely undersized for all experiments. FR_2 and FR_3 have an undesirable size property in most of the experiments when we have decreasing correlation. For example, under the normal distribution the size of FR_2 is less than 0.020 in all experiments, and FR_3 has an oversized value in Experiments I, II and III. Under the skewed t distribution, FR_2 and FR_3 tend to be oversized in most experiments. In contrast, QST and QOT report much better size properties for all experiments. For most experiments, the best size performance of QOT occurs for $\tau \in (0.3, 0.7)$ except Experiment IV and V.

[**Table2**, **Table3**]

6.3   Power Property

The finite sample power property, which reflects the efficiency of contagion detection, is reported in Figures 2–4. Each of the experiments are based on critical values of 5%.

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We consider the increasing correlation case. The highest power property of QOT appears in the median (0.5 quantile), and the power decreases when the quantile goes to two extremes. FR_2 and FR_3 show a good power property, which is consistent with the results in Dungey et al. (2005b). Among the three forms of FR tests, FR_2 exhibits the best power property in all experiments, and FR_2 even beats the quantile tests the under the normal distribution case (Figure 2). This shows the importance of correctly specifying that the non-crisis and crisis periods should be separated to satisfy the independence. Comparing the power of the FR tests to that of QOT at the 0.5 quantile, we see that QOT is superior to FR_1 and FR_3 but not to FR_2 in the normal case. However, when we consider the Student's t and skewed t cases where the data are characterized by a fatter tail and higher peak, quantile tests are shown to have better power than FR tests. Since financial data are mostly non-normally distributed, these cases are of more practical interest. For example, under Student's t distribution QOT beats all FR tests in all experiments.

We next compare the tests in the decreasing correlation case. In this case the quantile tests show particular superiority. All forms of the FR tests fail to capture the contagion when the contagion strength δ rises (Figures 2– 4), and none of their powers are monotonically increasing with δ. For example, FR_1 and FR_2 diminish to zero when δ exceeds 8 in Experiments I, II, III, and IV under the normal distribution (Figure 2). In contrast, the power of QOT remains monotonically increasing, although it is less powerful than in the increasing correlation case. Under the Student's t distribution, the FR tests behave differently from the normal cases (Figure 3). When δ increases from 0 to 2, the FR tests have increasing power, but the power drops when δ exceeds 2. Similar behavior of the FR tests is found under the skewed t distribution. However, in all cases QOT produces monotonically increasing power as δ increases.

Based on the above comparison, we conclude that the proposed quantile tests (QST and QOT) generally produce better size and power properties than FR tests, especially when the data are skewed and leptokurtic. Moreover, correlation-based tests may fail to capture contagion when it occurs not in the mean, and our quantile contagion test QOT works well and is therefore particularly useful in this case. In this sense, QOT is more robust than the FR tests.

7   Empirical Analysis

We evaluate our quantile contagion tests and correlation-based FR tests using three real data sets. We reexamine whether contagion existed in the 1994 Tequila crisis, the 1997 Asian crisis, and the 2001 Argentine crisis.

Before Forbes and Rigobon (2002) the literature using correlation-based tests had generally concluded that cross-market correlation increased significantly during crises, and that there was contagion in these crises. Forbes and Rigobon (2002) pointed out that previous correlation-based tests were upward biased and inaccurate because of heteroscedasticity, and they proposed adjusted tests (FR tests) to overcome spurious contagion problems. They found that only a small proportion of transmission channels were significant in the 1987 U.S. market crash, the 1994 Tequila crisis, and the 1997 Asian crisis. Thus they found no strong evidence of contagion in these crises, only interdependence. However, we argue that the results of the FR tests should be interpreted with caution since they only indicate the relationship at an average level. We will compare the quantile contagion with

the *FR* tests.

The simulation analysis has shown that among the three forms of *FR* tests FR_2 generally possesses the best finite sample properties.[3] Therefore our empirical analysis will focus on comparing FR_2 and quantile contagion test QOT. The stock market returns of each country are calculated as rolling-average two-day returns based on the aggregate stock market index.[4] All the test statistics are considered under a bivariate framework; that is, we only consider the transmission of volatility between two countries. Before we evaluate all the tests, we take the preliminary step of extracting the residual from the bivariate VAR process for the two countries with one lag over the full sample periods to exclude any common factor in the data. We perform our proposed tests, QOT for $\tau \in [0.05, 0.95]$, and the significance level is set at 5% for all cases.

7.1   1997 Asian Crisis

In June 1997 a financial crisis broke out in Asia, causing great devastation in Asia's financial markets. The Thai baht has experienced substantial depreciation following the Thai government's proposal of a floating exchange rate policy on June 2, 1997. At first, only certain southeastern Asian countries, such as Malaysia and Indonesia, appeared to be affected. However, after Hong Kong's market crashed in mid-October, other Asian countries and even some Western countries appeared to be involved in this crisis. Thus, we include the stock returns of nine countries in this analysis: Japan, South Korea, Malaysia, Thailand, Hong Kong, the Unite States, Germany, the Unite Kingdom, and Australia. The indices cover a span of one year and eight months from January 1, 1997 to August 31,

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[3]FR_2 and FR_3 are largely similar, and FR_2 beats FR_3 in most cases.

[4]The daily returns are also adjusted for weekends and holidays

1998 with a sample size of 436.[5] Figure 5 provides time series plots of the stock return rates of some countries in the Asian crisis, from which we note an obvious increase in the volatility of the return rates after October 20, 1997. Accordingly, the entire sample is divided into a tranquil period from January 1, 1997 to October 19, 1997 and a turmoil period from October 20, 1997 to August 31, 1998.

[Figure 5]

Table 4 presents the descriptive statistics and some preliminary tests. All the series are asymmetrically distributed: the returns of Hong Kong, South Korea, Malaysia, Japan, and Thailand are positively skewed, while the returns of the Unite States, the Unite Kingdom, Australia, and Germany are negatively skewed. Some series show excess kurtosis, including Hong Kong, Malaysia, the Unite States, and Australia, indicating a higher probability of the existence of extreme-valued returns. The non-normal characteristics are confirmed by the Jarque–Bera test. The results of the Ljung–Box test indicate that the stock returns of Hong Kong, South Korea, Japan, the Unite Kingdom, Germany, and Thailand exhibit significant autocorrelations with $lag = 5$. The ARCH LM test shows that all the countries except Malaysia have significant ARCH effects with $lag = 5$. These observations suggest that the real data are highly non-normal, and therefore, a simulation design with excess skewness and kurtosis, autocorrelation, and ARCH effects is of more practical interest.

[Table 4]

Tables 5 and 6 report the intercorrelation among the nine stock markets, and we can see that the intercorrelation increases significantly in the crisis period except for between

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[5]The data exclude the holidays and missing values

Hong Kong and South Korea. Therefore, if we only consider intercorrelation, we would tend to conclude the existence of contagion in the Asian crisis. However, Forbes and Rigobon (2002) pointed out that the intercorrelation is conditional, and upward biased. They used adjusted tests based on unconditional correlation and reached the opposite conclusion of no contagion.

[Table 5, Table 6]

In the following, we compare our tests with FR_2 and examine the contagion effect within and outside of Asian countries.

7.1.1   Within Asia

Figure 6 shows representative transmission channels within Asia between Thailand and Malaysia, Thailand and Hong Kong, Japan and South Korea, and Japan and Malaysia. Channels related with Thailand are of interest because it is considered to be the origin of the Asian crisis, and channels related with Japan are considered since it plays a crucial role in the Asian economy. More results are available upon request. The graphs in the upper row give the values of test statistics for $\tau \in [0.05, 0.95]$. Since FR_2 does not change over τ, its curve is a straight line in the graph. The graphs in the bottom row give the p-values of QOT. The quantile contagion test rejects the null hypothesis of no contagion if the p-values of QOT are under the 5% line, while the correlation-based test FR_2 rejects the null hypothesis when its value is greater than 1.644, the critical value of the one-sided t-statistic at size 5%. Since we perform all the tests in a bivariate framework, we test the channels from country 1 to country 2 as well as those from 2 to 1.

We first investigate the channels related with Thailand. QOT reports significant im-

pacts from Thailand to Malaysia and to Hong Kong at almost all quantiles. This is in line with the results of the FR_2 test. However, the results of the two tests differ in the reverse transmission. FR_2 shows that the contagion from Malaysia to Thailand and from Hong Kong to Thailand is weakly significant ($FR_2 \approx 1.69$ and $FR_2 \approx 1.80$, respectively), while QOT suggests that the reverse transmission is significant at some specific quantiles. More precisely, QOT shows that the transmission from Malaysia to Thailand is significant at quantiles of 0.15, 0.19, 0.2, 0.25, [0.47, 0.86], and [0.91, 0.95]; the transmission from Hong Kong to Thailand is significant at quantiles of [0.14, 0.67], 0.69, [0.73, 0.76], and [0.84, 0.91]. These results suggest that Thailand's stock returns are affected by the Malaysian and Hong Kong markets when they are in a decline (at lower quantiles). We note that QOT does not conflict with FR_2 in the sense that QOT also finds contagion around the median and mean levels in these transmission channels. However, QOT provides information on the whole distribution, not only the mean as reported by FR_2. Similar results are found in the transmission channels between Thailand and Korea, and between Thailand and Japan.

Next, we examine the channels related with Japan. FR_2 indicates no contagion from Japan to South Korea ($FR_2 \approx 0.63$). QOT confirms that this channel is indeed not significant at most quantiles, but it is significant at quantiles from 0.50 to 0.79. This implies that Japan's stock returns are relatively unrelated with those of South Korea, but when the South Korean stock market is in an advance it is significantly correlated with the Japanese market. For the reverse channel from South Korea to Japan, QOT and FR_2 both report no significant contagion. FR_2 and QOT also differ in the transmission channels between Japan and Malaysia. FR_2 reports no contagion from Japan to Malaysia ($FR_2 \approx 1.36$) or in the reverse channel ($FR_2 \approx 0.67$). QOT suggests that contagion from Japan to Malaysia

exists at quantiles from 0.19 to 0.94 and in the reverse channel at some lower quantiles (< 0.22) and upper quantiles (> 0.77). The influence from Japan to Malaysia may be due to Japan's large investment in Malaysia, but the Malaysian stock market hardly affects Japan except in some extreme cases.

[Figure 6]

7.1.2 From Asia to the West

The 1997 Asian crisis also affected the Unite States, Australia, and some European countries. Since Hong Kong and Japan may be the most important financial centers in Asia, and since they have strong connections with these Western countries, we focus on the channels related with Hong Kong, Japan, and some Western countries. We present the results for Japan and the Unite States, Japan and Australia, Hong Kong and the Unite States, and Hong Kong and the Unite Kingdom in Figure 7.

We first consider the channels related with Japan. FR_2 reports no contagion from Japan to the Unite States ($FR_2 \approx 1.58$), while QOT suggests significant contagion at some lower quantiles, 0.07 and [0.1,0.44], and at quantiles slightly above the median, 0.65, 0.67, and [0.77,0.81]. The reverse impact is also not significant according to FR_2 ($FR_2 \approx 1.55$), but QOT finds it to be prominent at some lower quantiles, 0.06, [0.08,0.22], and [0.27,0.52]. This implies that the connection between Japan and the Unite States generally increases when either of the two markets suffers from a sharp drop in the crisis period. For channels from Japan and Australia, both FR_2 and QOT find significant contagion. However, FR_2 detects no contagion in the reverse channel, and QOT finds contagion at quantiles [0.28,0.6] and [0.72,0.79].

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Next, we study the channels between Hong Kong and the Western countries. FR_2 shows no contagion from Hong Kong to the Unite States ($FR_2 = 0.86$), while QOT finds it to be significant at some upper quantiles from 0.69 to 0.94. Both tests suggest a significant influence in the reverse channel. This suggests that the U.S. market is more closely related with Hong Kong's market when it is in an advance and that the Hong Kong's market is significantly affected by the U.S. market most of the times. This is evidence that Hong Kong is an international financial center and that its market is easily influenced by some international markets, especially large ones. For the relation between Hong Kong and the Unite Kingdom, we find similar results. From Hong Kong to the Unite Kingdom, FR_2 again reports no contagion ($FR_2 = 1.111$), but QOT detects significant contagion at some lower (< 0.24) and upper quantiles (> 0.56). From the Unite Kingdom to Hong Kong, both tests find significant results. The associations between Hong Kong and Australia, and between Hong Kong and Germany are largely similar. Contagion from Hong Kong to these countries is detected by QOT at some quantiles but not by FR_2, while the reverse transmission is shown to be significant by both tests.

We note from the empirics that if FR_2 finds significant contagion then QOT reports significant results at most quantiles. However, when FR_2 fails to detect any contagion QOT may report significance at a few quantiles. In general, QOT can detect contagion that FR_2 ignores, especially at lower and upper quantiles, and it does not miss significant results if FR_2 also detects them.

[Figure 7]

7.2 1994 Tequila Crisis

In December 1994, an economic crisis began in Mexico following by the sharp decline of the Mexican peso. Subsequently, several South American countries fell into the crisis period, and their stock indices experienced a sudden decline. We examine the contagion effects between the Unite States, Chile, Mexico, Argentina, and Venezuela. These four South American countries particularly suffered from this crisis, and the Unite States is included because of its close connections with these four countries. The indices span a period of nine months from June 1, 1994 to March 2, 1995, with 198 sample points. The change point from the non-crisis period to the crisis period is December 12, 1994.

We also make a preliminary analysis of the characteristics of the data, and we find that all series are skewed and leptokurtic, and therefore non-normally distributed. The Ljung–Box and ARCH tests show that the stock returns of all South American countries are autocorrelated with ARCH effects. The correlations between Chile and Argentina, Mexico and Chile, Venezuela and Chile, and Argentina and Mexico increase significantly in the crisis period. However, the correlations between Chile and the Unite States, Argentina and the Unite States, Argentina and Venezuela, Mexico and the Unite States, Mexico and Venezuela, the Unite States and Venezuela decrease in the crisis period. The results of this preliminary analysis are available upon request.

Figure 8 presents the results of the two tests for the channels between the Unite States and Mexico, Mexico and Argentina, Mexico and Chile, and Chile and Argentina. We first examine the association between the Unite States and Mexico. FR_2 reports no significant impact in either direction of this channel ($FR_2 \approx -0.80$ from Mexico to the Unite States, and $FR_2 \approx 0.12$ for the reverse direction). QOT is generally in line with FR_2 except that it

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detects significant effects from the Unite States to Mexico only at quantiles 0.89 and 0.9. This implies that the association between the Unite States and Mexico did not experience significant change during the crisis. Next we examine the relation between Mexico and Argentina. FR_2 reports no contagion from Mexico to Argentina ($FR_2 \approx 0.10$), while QOT detects contagion at quantiles [0.37,0.74] and [0.77,0.89]. Both FR_2 and QOT find the reverse effect to be not significant ($FR_2 \approx -0.14$ and QOT is not significant except at quantile 0.92). Thus, the remarkable increase in the correlation between Mexico and Argentina is mainly due to the contagion effect from Mexico to Argentina. We next consider the relation between Mexico and Chile. Both FR_2 and QOT find no significant contagion from Mexico to Chile ($FR_2 \approx 0.33$ and QOT is significant only at quantile 0.23). However, QOT indicates strong impacts from Chile to Mexico at lower quantiles [0.09,0.15] and upper quantiles, [0.78,0.79], 0.87, 0.89, and 0.92. This effect is ignored by FR_2. This implies that Chile has a strong impact on the Mexican market when the latter experiences a sharp slump or an advance. Finally we investigate the channel between Chile and Argentina. FR_2 finds no significant results in either direction. QOT indicates a significant effect from Chile to Argentina at quantiles [0.11, 0.86] and [0.88,0.95], but not a reverse effect. Most of the other channels are shown to be not significant by both FR_2 and QOT.

[Figure 8]

Based on the above analysis, we conclude that Mexico and Chile were the two main sources of contagion in the Tequila crisis, and they had impacts on other South American countries to various extents. However, these effects are generally not captured by the correlation-based test FR_2. We also note that this crisis did not spread to the Unite States to a significant degree. In contrast to the Asian crisis, we observe that the correlations be-

tween some South America countries decreased in the Tequila crisis, leading to negative values of the FR_2 test. In this case, QOT can still capture contagion at some extreme quantiles. This again shows the difference between the mean correlation and the correlation at quantiles.

7.3   2001 Argentine Crisis

Finally we test the contagion effect in the Argentine crisis, which began in 2001. In that crisis, some South American countries, including Brazil, Chile, Mexico, suffered from an unfavorable balance of trade, heavy international debts, and high inflation rates. We focus on four South America countries, Argentina, Mexico, Brazil, and Chile, that were involved in this crisis to varying extents. The entire sample period spans from July 9, 2000 to December 3, 2001, with a sample size of 367. We choose July 9, 2001 as the change point from the non-crisis period to crisis period because it is the date Argentina became involved in this crisis.

The descriptive statistics show that all series are non-normal. The stock returns of all countries have less kurtosis than the normal distribution. The Ljung–Box test indicates that all series exhibit significant autocorrelation effects with *lag* = 5 except Argentina and Brazil, and the ARCH test reports significant ARCH effects for Chile and Brazil. The correlation studies suggest that the correlations of most channels increase significantly in the crisis period.

Figure 9 presents the associations between Argentina and Chile, Argentina and Brazil, Chile and Mexico, and Chile and Brazil. We first examine the channels related with Argentina. FR_2 shows that these channels are not significant in either direction. QOT finds

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similar results of no contagion in most of the channels. The only exception is the association between Argentina and Chile. QOT reports a significant effect from Argentina to Chile around the median and a significant reverse effect at lower quantiles, [0.06,0.2] and [0.35,0.41]. This shows that the Argentine market had a strong impact on the Chilean market when the latter was under a normal situation (around the median), but Chile affected Argentina only when the latter was in a bear market (lower quantile).

We next consider the channels related with Chile. QOT finds the impact from Mexico to Chile to be significant at lower quantile [0.05,0.32], while FR_2 ignores this effect ($FR_2 \approx 0.17$). Both tests report insignificant feedback from Chile to Mexico. Both tests find strong contagion between Brazil and Chile in both directions. More particularly, according to QOT the Brazilian market affects the Chilean market significantly at nearly all quantiles, and this is confirmed by FR_2 ($FR_2 \approx 3.05$). QOT shows that the reverse impact is significant at quantiles [0.17,0.21], [0.42,0.72], and [0.9,0.95], while $FR_2 \approx 2.14$ also detects contagion.

[Figure 9]

In summary, there is significant contagion between Brazil and Chile in the Argentine crisis, which is captured by both tests. However, the contagion between Argentina and Chile and between Mexico and Chile is captured by QOT at some quantiles, but not by FR_2. Except for these channels, there is no strong evidence for the spread of the Argentine crisis, at least within our samples.

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Appendix

Appendix 1: Proof of the Invariance Property of the Scaling Transformation

In this appendix, we show the invariance property of the scaling transformation in quantile regression. Let

$$\psi(\tau, z_T, X; \gamma) = \sum_{t: z_{2,t} > \gamma x_t} \tau |z_{2,t} - \gamma x_t| + \sum_{t: z_{2,t} < \gamma x_t} (1 - \tau) |z_{2,t} - \gamma x_t|.$$

Note that the following properties are possessed by $\psi(\tau, z_T, X; \gamma)$ (Koenker and Bassett 1978): (1) $\kappa \psi(\tau, z_T, X; \gamma) = \psi(\tau, \kappa z_T, X; \kappa \gamma)$, $\kappa \in [0, \infty)$; (2) $\psi(\tau, z_T, X; \gamma) = \psi(\tau, z_T, XA; A^{-1}\gamma)$, $|A| \neq 0$; and (3) $\psi(\tau, z_T, X; \gamma) = \psi(\tau, z_T + X\beta, X; \gamma + \beta)$. Therefore, we have

$$\psi[\tau, \frac{z_T}{\sigma_{2,t}}, XA; \gamma_{st}] = \frac{1}{\sigma_{2,t}} \psi(\tau, z_T, X; \gamma),$$

and take the first order derivatives

$$\frac{\partial}{\partial \gamma} \psi(\tau, \frac{z_T}{\sigma_{2,t}}, XA; \gamma_{st}) = \frac{1}{\sigma_{2,t}} \frac{\partial}{\partial \gamma} \psi(\tau, z_T, X; \gamma), \tag{7.1}$$

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where

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma_{2,t}} & 0 \\ 0 & 0 & 0 & \frac{1}{\sigma_{2,t}} \end{pmatrix}$$

and $\gamma_{st} = A^{-1}\gamma$ denotes the coefficient after the scaling transformation.

Note that the first order derivative of ψ is equivalent to the score of the log likelihood function of the asymmetric Laplace density if we fix τ and σ. Thus (7.1) implies $s_{st}(\gamma) = \frac{1}{\sigma_{2,t}}s(\gamma)$, where $s_{st}(\gamma)$ is the score function in the scaling transformation model. Similarly, we have the invariant property of the information matrix and other matrices,

$$J_{st} = (\frac{1}{\sigma_{2,t}})^2 J \quad \text{and} \quad K_{st} = \frac{1}{\sigma_{2,t}}K;$$

therefore $V_{st} = V$ where the subscript "st" denotes the corresponding variables after the scaling transformation.

Hence, we have

$$
\begin{aligned}
RS_{st} &= \frac{1}{T}\tilde{s}'_{2,st}\tilde{K}^{BB}_{st}\tilde{V}_{BB,st}\tilde{K}^{BB}_{st}\tilde{s}_{2,st} \\
&= \frac{1}{T}\left(\frac{1}{\sigma_{2,t}}\tilde{s}'_2\right)\left(\sigma_{2,t}\tilde{K}^{BB}\right)\tilde{V}_{BB}\left(\sigma_{2,t}\tilde{K}^{BB}\right)\left(\frac{1}{\sigma_{2,t}}\tilde{s}_2\right) \\
&= RS,
\end{aligned}
\tag{7.2}
$$

which shows that Rao's score statistic in the regression quantile is invariant to the scaling transformation. The invariance property of the scaling transformation suggests a direct test procedure for contagion simply by usage of the unscaled parameters instead of the scaled ones.

Appendix 2: Derivation of Parameterization in the Monte Carlo Simulation

In this appendix, the derivation of the parameterization in the Monte Carlo simulation is provided.

Suppose the non-crisis model is the combination of the common factor w_t and the idiosyncratic factor $u_{i,t}$. Then

$$x_{i,t} = \lambda_i w_t + \phi_i u_{i,t},$$

where λ_i and ϕ_i denote the loadings of the two factors. The crisis period model is

$$y_{1,t} = \lambda_1 w_t + \phi_1 u_{1,t}, \quad \text{and} \quad y_{2,t} = \lambda_2 w_t + \phi_2 u_{2,t} + \delta \phi_1 u_{1,t}, \tag{7.3}$$

where δ captures the strength of the contagion effect. For convenience, we define $v_{i,t} = \lambda_i w_t + \phi_i u_{i,t}$ and $c_{1,t} = \phi_1 u_{1,t}$ then the crisis model for country i can be rewritten as

$$y_{1,t} = v_{1,t} \quad \text{and} \quad y_{2,t} = v_{2,t} + \delta c_{1,t},$$

In this setup, we show that the correlation between the two asset returns in the crisis period y_1 and y_2 is not necessarily monotonic with respect to δ as follows.

Consider the denominator and numerator in the correlation

$$corr(y_1, y_2) = \frac{Cov(y_1, y_2)}{\sqrt{Var(y_1)Var(y_2)}}.$$

Expanding the numerator and denominator we can explicitly express the correlation as a function of δ

$$
\begin{aligned}
corr(y_1, y_2) &= \frac{E(v_1 v_2) - E(v_1)E(v_2) + \delta[E(v_1 c_1) - E(v_1)E(c_1)]}{\sqrt{Var(v_1)Var(v_2) + \delta^2 Var(v_1)Var(c_1) + 2\delta Var(v_1)Cov(v_2, c_1)}} \\
&= \frac{DE(v_1, v_2) + \delta DE(v_1, c_1)}{\sqrt{PV}},
\end{aligned} \tag{7.4}
$$

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where $DE(v_1, c_1) = E(v_1 c_1) - E(v_1)E(c_1)$, $DE(v_1, v_2) = E(v_1 v_2) - E(v_1)E(v_2)$ and $PV = Var(v_1)Var(v_2) + \delta^2 Var(v_1)Var(c_1) + 2\delta Var(v_1)Cov(v_2, c_1)$.

To study the behavior of the correlation with the increasing contagion effect δ, we take the first order derivatives of Eq.(7.4) with respect to δ

$$\frac{\partial corr(y_1, y_2)}{\partial \delta} = \frac{\frac{\partial Cov(y_1, y_2)}{\partial \delta} \sqrt{PV} - Cov(y_1, y_2)\frac{\partial \sqrt{PV}}{\partial \delta}}{\sqrt{PV}}$$

$$= \frac{DE(v_1, c_1) - \frac{1}{2}\left[DE(v_1, v_2) + \delta DE(v_1, c_1)\right]\left[2\delta Var(v_1)Var(c_1) + 2Var(v_1)Cov(v_2, c_1)\right]/PV}{PV^{3/2}}$$

In order to determine the sign of the first order derivatives, we focus on the sign of the numerator since the denominator is guaranteed to be positive. The numerator can be rewritten as

$$\frac{DE(v_1, c_1)PV - \frac{1}{2}\left[DE(v_1, v_2) + \delta DE(v_1, c_1)\right]\left[2\delta Var(v_1)Var(c_1) + 2Var(v_1)Cov(v_2, c_1)\right]}{PV} \quad (7.5)$$

and its sign is determined by its numerator

$$\begin{aligned} Num \quad = \quad & DE(v_1, c_1)Var(v_1)Var(v_2) - DE(v_1, v_2)Var(v_1)Cov(v_2, c_1) \\ & + \delta\left[DE(v_1, c_1)Var(v_1)Cov(v_2, c_1) - DE(v_1, v_2)Var(v_1)Var(c_1)\right] \end{aligned}$$

Note that the sign of Num depends on the parameterization. In this sense, we study its sign by assuming the distribution of random variables.

We consider that the data generation process is based on the normal distribution[6]. Suppose that w_t, $u_{1,t}$ and $u_{2,t}$ are independently distributed as

$$w_t \sim N(\mu_w, \sigma_w^2), \quad u_{1,t} \sim N(\mu_{u_1}, \sigma_{u_1}^2), \quad u_{2,t} \sim N(\mu_{u_2}, \sigma_{u_2}^2),$$

then we can obtain the distribution of $v_{1,t}$, $v_{2,t}$ and $c_{1,t}$ from the linearity of the normal distribution as

$$v_{1,t} \sim N(\lambda_1 \mu_w + \phi_1 \mu_{u_1}, \lambda_1^2 \sigma_w^2 + \phi_1^2 \sigma_{u_1}^2), \quad v_{2,t} \sim N(\lambda_2 \mu_w + \phi_2 \mu_{u_2}, \lambda_2^2 \sigma_w^2 + \phi_2^2 \sigma_{u_2}^2),$$

[6]The cases of the t distribution and skewed t distribution follow similar patterns to the normal case.

$$c_{1,t} \sim N(\phi_1 \mu_{u_1}, \phi_1^2 \sigma_{u_1}^2).$$

Furthermore, we express each term in *Num* with the parameter definition above as follows

$$
\begin{aligned}
DE(v_1, c_1) &= E(v_1 c_1) - E(v_1)E(c_1) = \phi_1^2 \sigma_{u_1}^2, \\
DE(v_1, v_2) &= E(v_1 v_2) - E(v_1)E(v_2) = \lambda_1 \lambda_2 \sigma_w^2, \\
Cov(v_2, c_1) &= 0.
\end{aligned}
$$

The last equation holds because v_2 and c_1 are assumed to be independent in this model. Therefore, *Num* can be written as

$$Num = \phi_1^2 \sigma_{u_1}^2 (\lambda_1^2 \sigma_w^2 + \phi_1^2 \sigma_{u_1}^2)(\lambda_2^2 \sigma_w^2 + \phi_2^2 \sigma_{u_2}^2 - \delta \lambda_1 \lambda_2 \sigma_w^2). \tag{7.6}$$

It can be verified that when

$$\delta > \frac{\lambda_2^2 \sigma_w^2 + \phi_2^2 \sigma_{u_2}^2}{\lambda_1 \lambda_2 \sigma_w^2}$$

increasing δ can lead to a negative and decreasing *Num* and, therefore, a decreasing correlation.
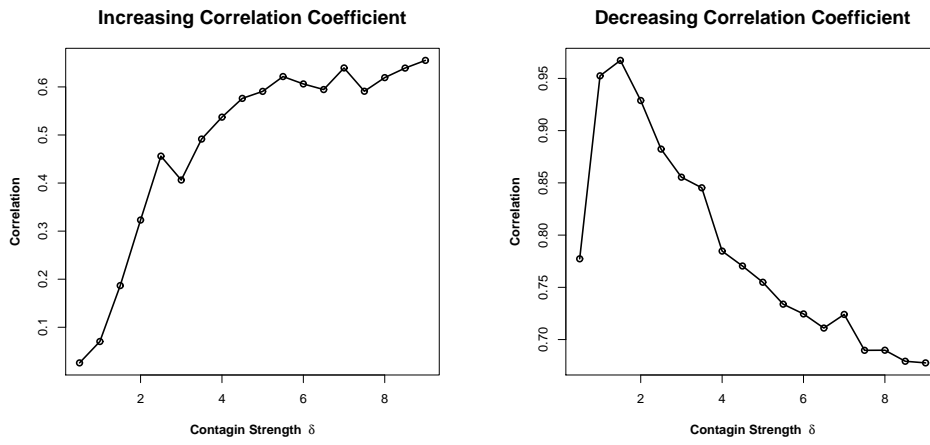
Figure 1: Correlation under Different Parameterizations
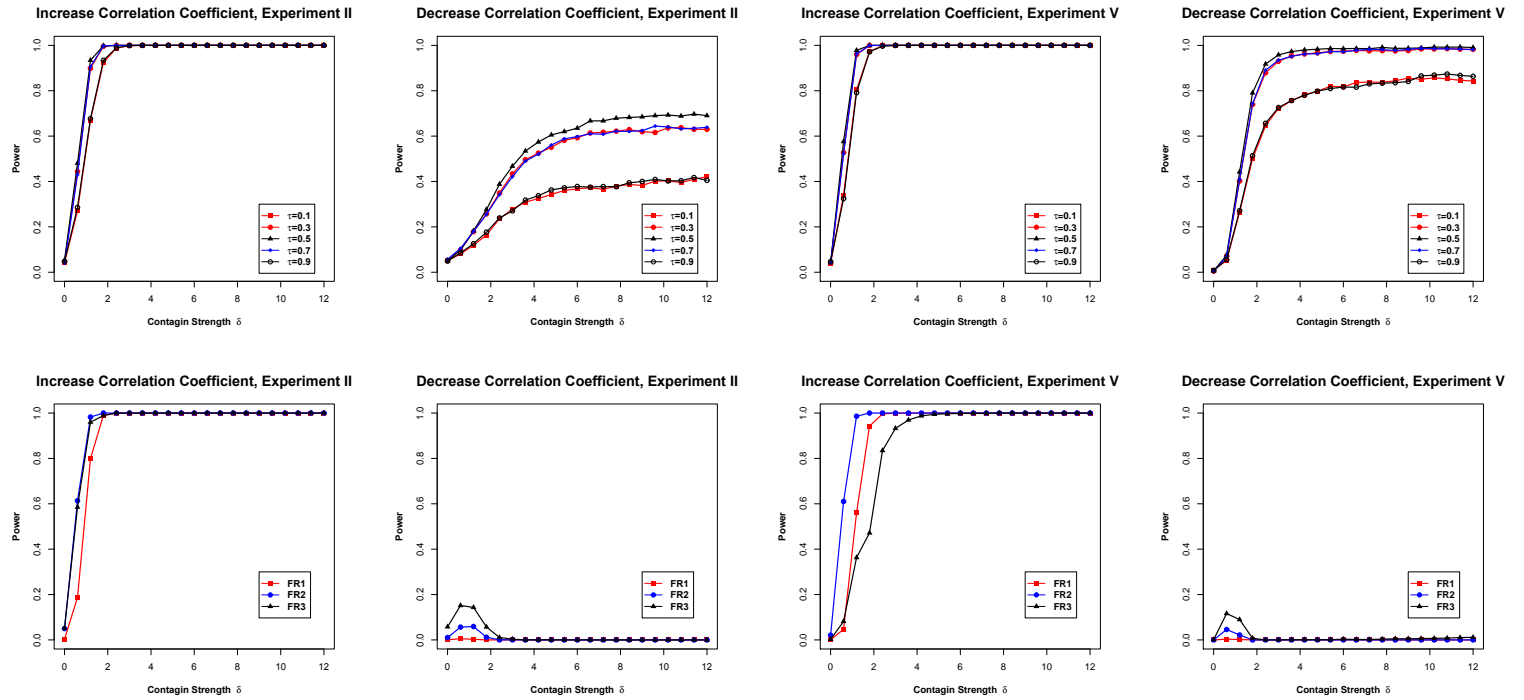
Figure 2: Power Property under the Normal Distribution with Sample Size 200
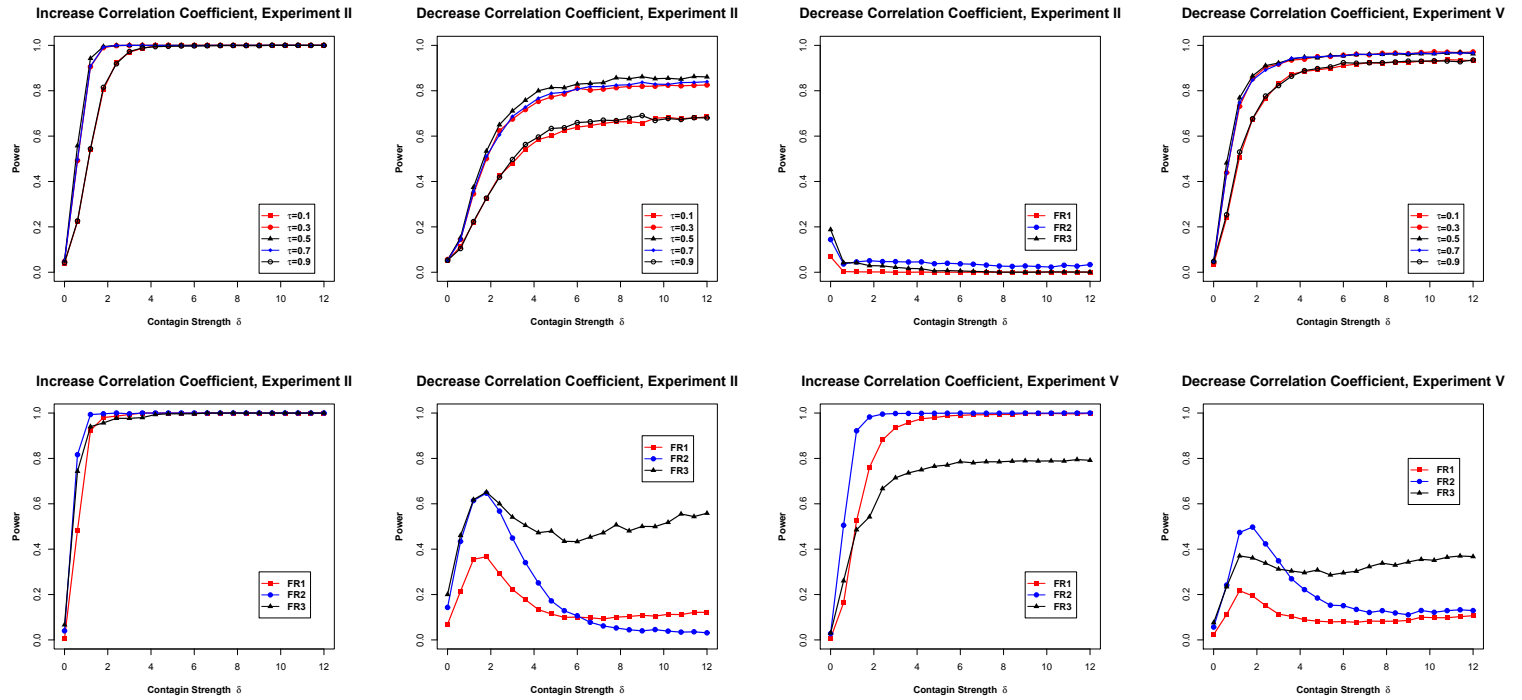
Figure 3: Power Property under the Student's *t* Distribution with Sample Size 200
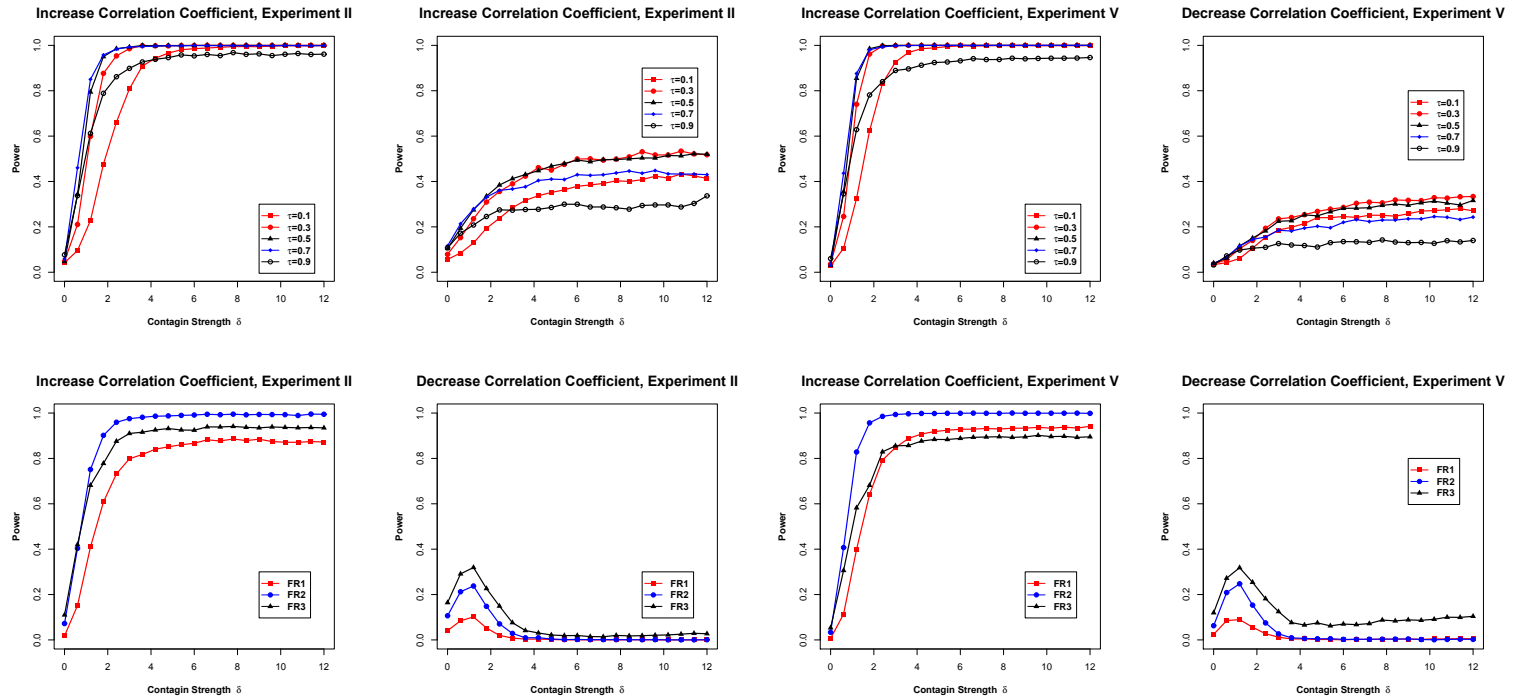
Figure 4: Power Property under the Skewed *t* Distribution with Sample Size 200
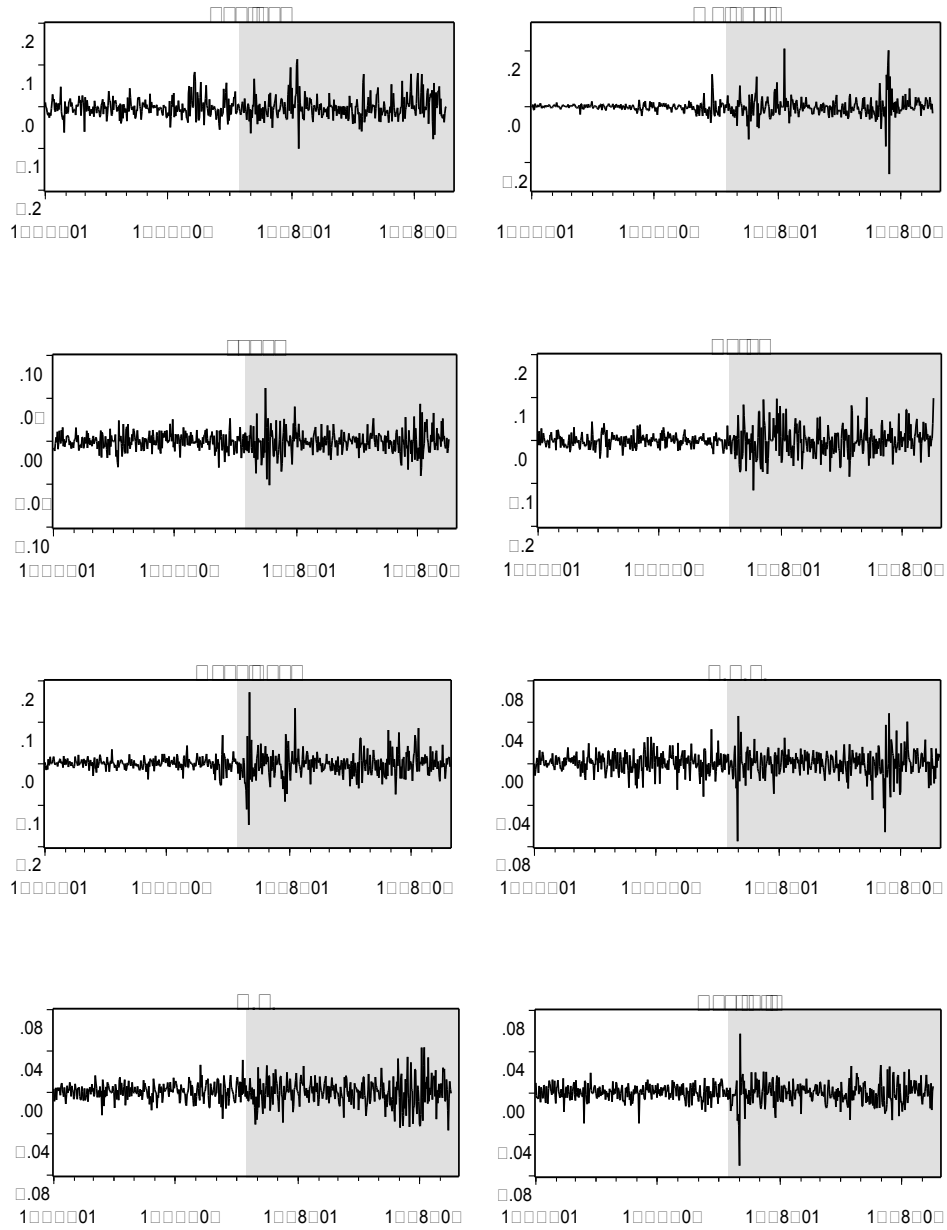
Figure 5: Daily percentage equity returns for some countries around the Asian Crisis. The start of the sample period is Jan 1, 1997 (vertical axis). The end of the sample period is Aug 31, 1998. The crisis period is from Oct 20, 1997 to Aug 31, 1998 (shaded area).
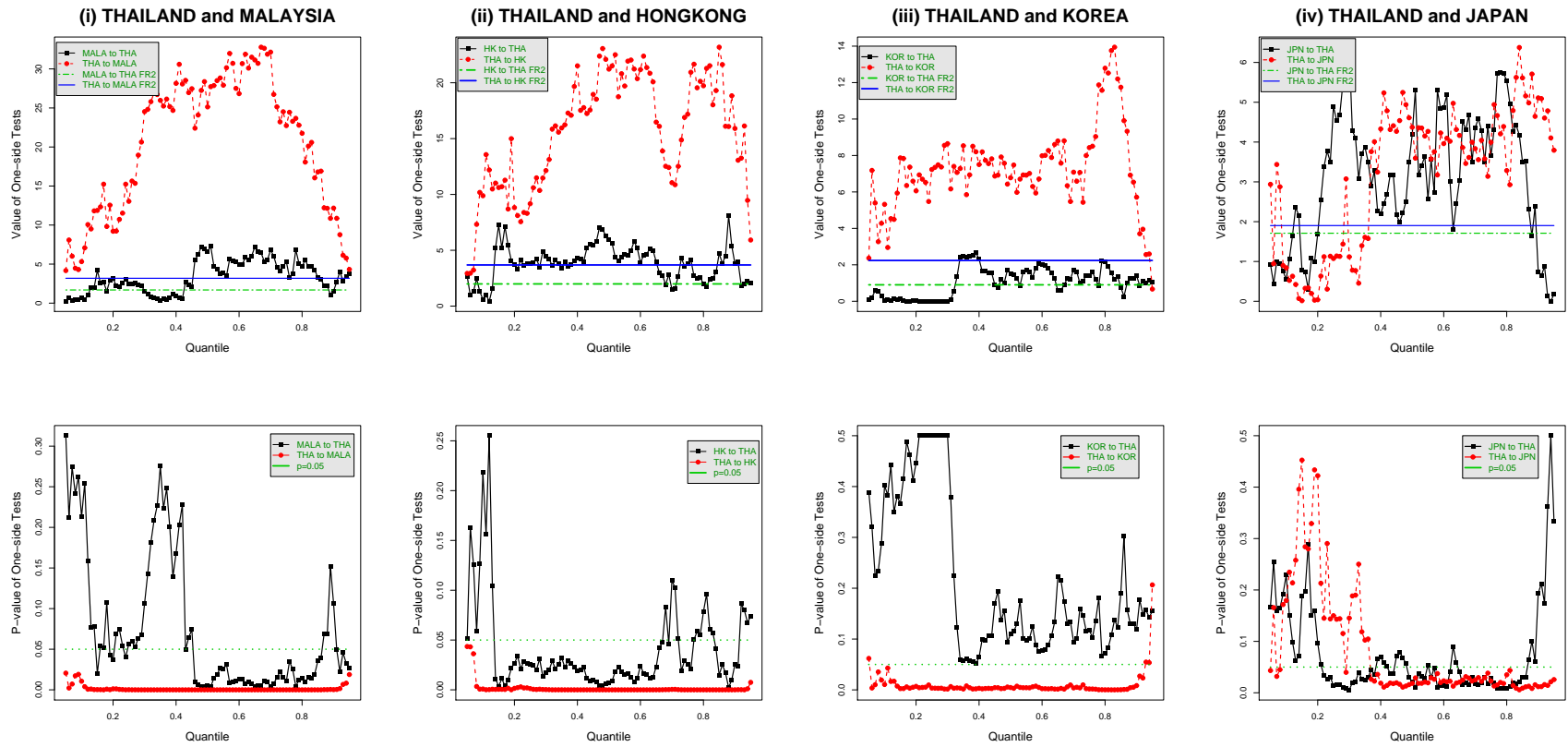
Figure 6: Statistics and *p*-value in the Asian Crisis: Within Asia
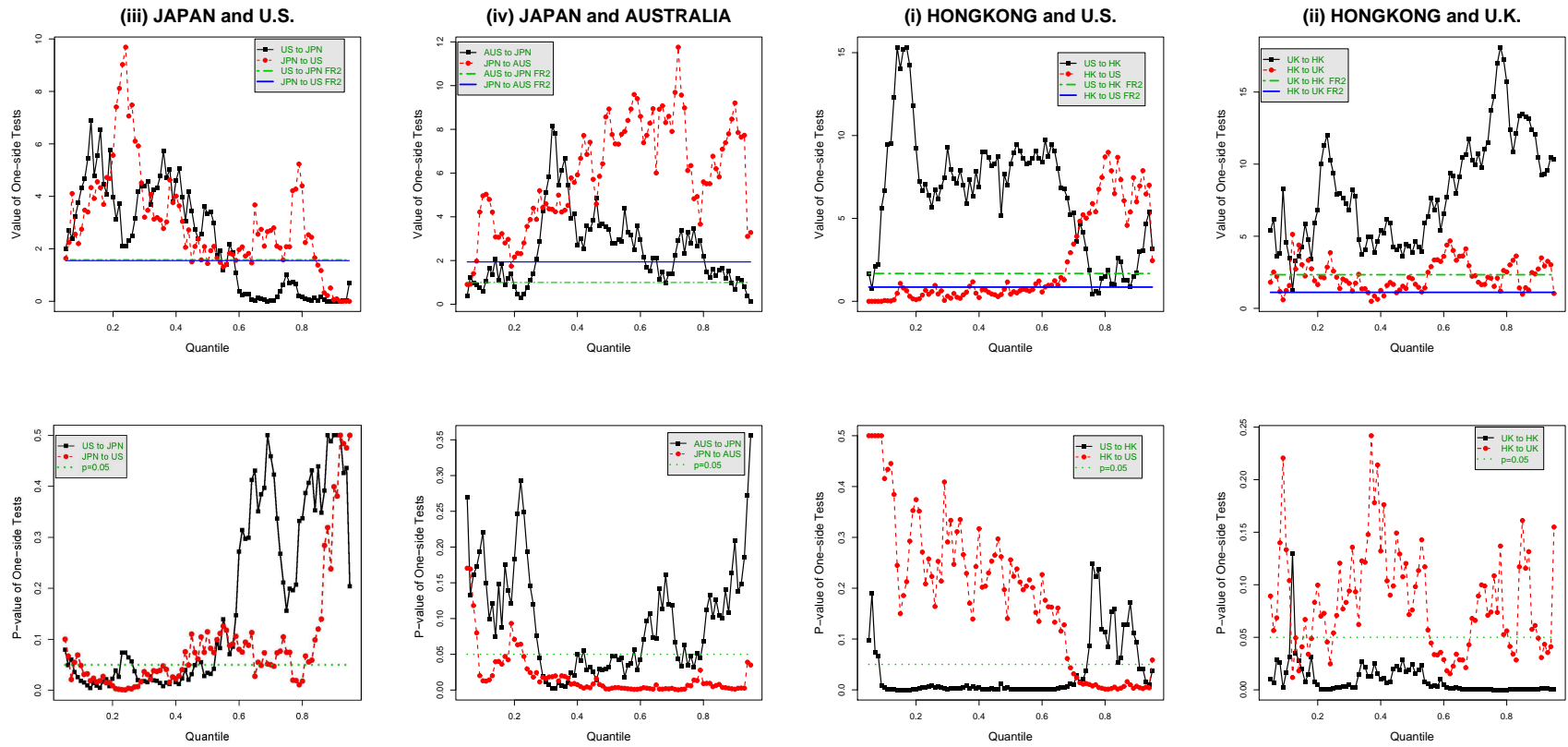
Figure 7: Statistics and *p*-value in the Asian Crisis: From Asia to the West
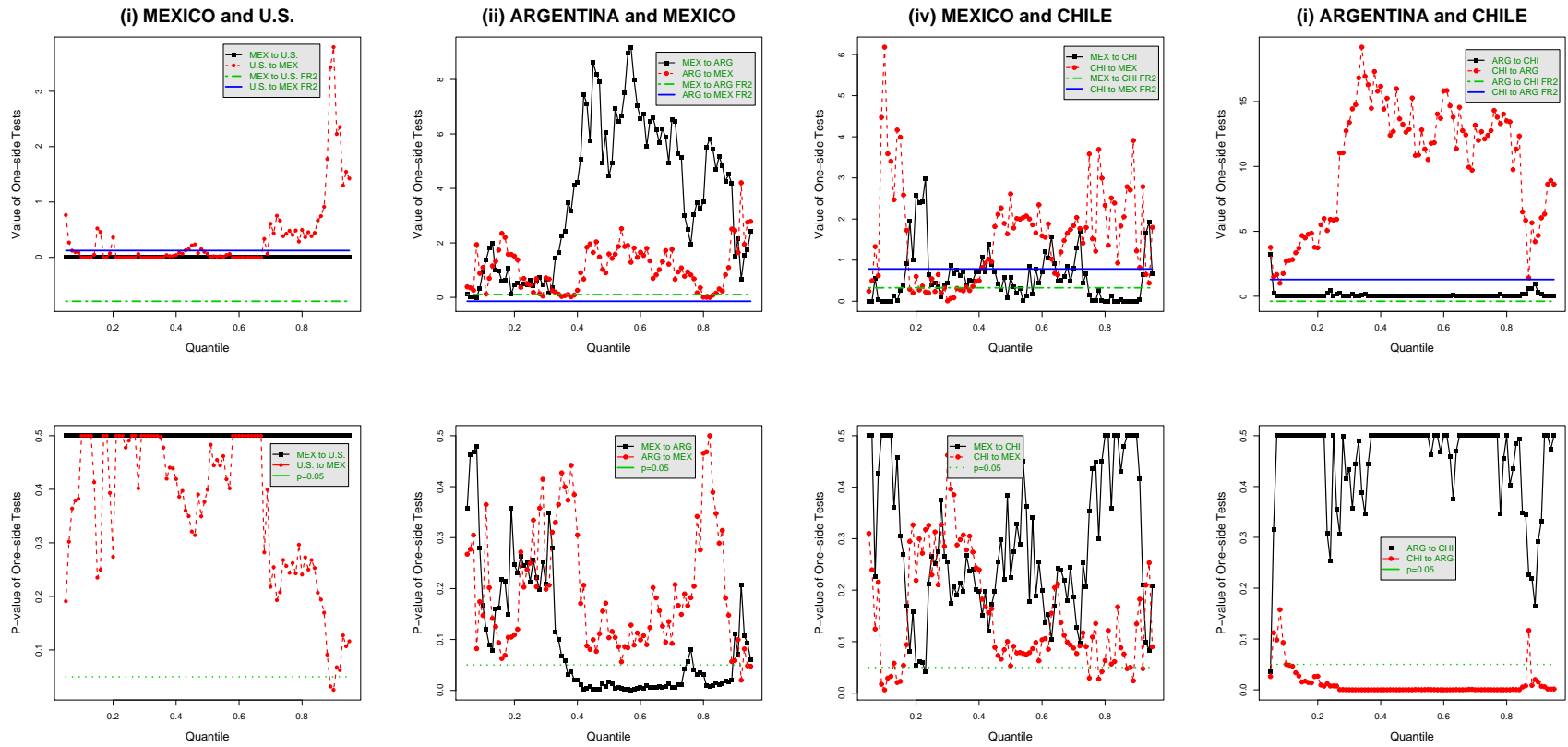
Figure 8: Statistics and p-value in the Tequila Crisis
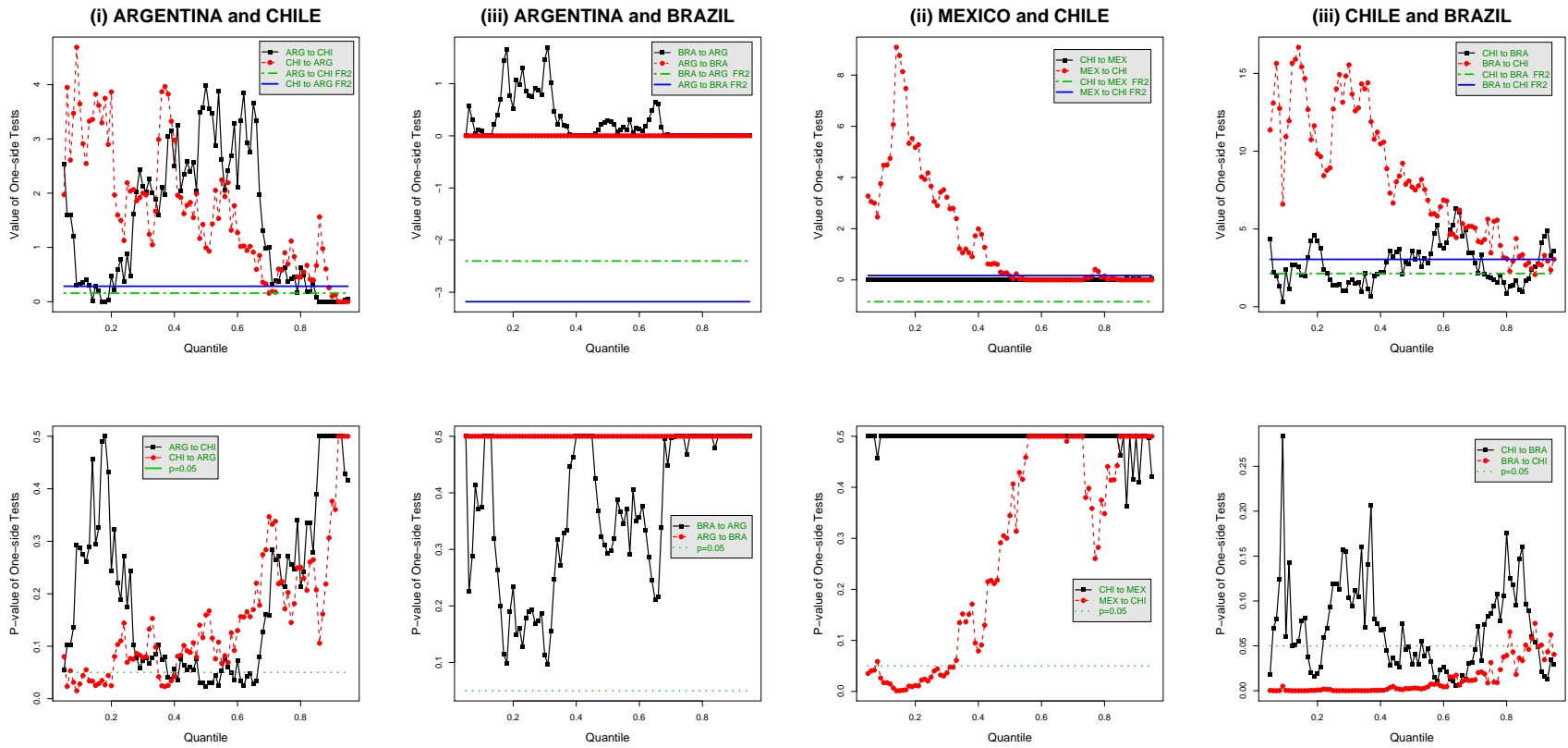
Figure 9: Statistics and *p*-value in the Argentine Crisis

Table 1: Parameterization in the Experiments

Normal Distribution	α	β	ρ	ω_x	ω_y	κ_x	κ_y
Increasing Correlation							
(I) Strong Autocorrelation	0	0	0.95	0.50	0.50	4	4
(II) Weak Autocorrelation	0	0	0.20	0.50	0.50	4	4
(III) No Autocorrelation	0	0	0	0.50	0.50	4	4
(IV) Common Factor Structural Break	0	0	0	0.50	1	4	4
(V) Idiosyncratic Factor Structural	0	0	0	0.50	0.50	4	6
(VI) GARCH	0.05	0.95	0	0.50	0.50	4	4
Decreasing Correlation							
(I) Strong Autocorrelation	0	0	0.95	2	2	0.50	0.50
(II) Weak Autocorrelation	0	0	0.20	2	2	0.50	0.50
(III) No Autocorrelation	0	0	0	2	2	0.50	0.50
(IV) Common Factor Structural Break	0	0	0	2	4	0.50	0.50
(V) Idiosyncratic Factor Structural	0	0	0	2	2	0.50	1
(VI) GARCH	0.10	0.90	0	4	4	0.01	0.01

Student's t Distribution	α	β	ρ	ω_x	ω_y	df_x	df_y
Increasing Correlation							
(I) Strong Autocorrelation	0	0	0.95	0.50	0.50	3	3
(II) Weak Autocorrelation	0	0	0.20	0.50	0.50	3	3
(III) No Autocorrelation	0	0	0	0.50	0.50	3	3
(IV) Common Factor Structural Break	0	0	0	0.50	1	3	3
(V) Idiosyncratic Factor Structural	0	0	0	0.50	0.50	3	2
(VI) GARCH	0.05	0.95	0	0.50	0.50	3	3
Decreasing Correlation							
(I) Strong Autocorrelation	0	0	0.95	2	2	3	3
(II) Weak Autocorrelation	0	0	0.20	2	2	3	3
(III) No Autocorrelation	0	0	0	2	2	3	3
(IV) Common Factor Structural Break	0	0	0	2	4	3	3
(V) Idiosyncratic Factor Structural	0	0	0	2	2	3	2
(VI) GARCH	0.10	0.90	0	4	4	3	3

Skew t Distribution	α	β	ρ	ω_x	ω_y	ϕ_x	ϕ_y
Increasing Correlation							
(I) Strong Autocorrelation	0	0	0.95	0.50	0.50	-0.3	-0.3
(II) Weak Autocorrelation	0	0	0.20	0.50	0.50	-0.3	-0.3
(III) No Autocorrelation	0	0	0	0.50	0.50	-0.3	-0.3
(IV) Common Factor Structural Break	0	0	0	0.15	0.30	-0.3	-0.3
(V) Idiosyncratic Factor Structural	0	0	0	0.50	0.50	-0.3	-0.7
(VI) GARCH	0.05	0.95	0	0.50	0.50	-0.3	-0.3
Decreasing Correlation							
(I) Strong Autocorrelation	0	0	0.95	2	2	-0.3	-0.3
(II) Weak Autocorrelation	0	0	0.20	2	2	-0.3	-0.3
(III) No Autocorrelation	0	0	0	2	2	-0.3	-0.3
(IV) Common Factor Structural Break	0	0	0	2	4	-0.3	-0.3
(V) Idiosyncratic Factor Structural	0	0	0	2	2	-0.3	-0.7
(VI) GARCH	0.10	0.90	0	4	4	-0.3	-0.3

Notes: The parameter υ in (6.5) equals 4 for all experiment designs in the skewed t distribution.

Table 2: Size Property under Increasing Correlation Cases

Normal Distribution		sample size 200						sample size 500						sample size 1000					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
$\tau = 0.10$	QST	0.0507	0.0487	0.0500	0.0530	0.0650	0.0563	0.0453	0.0460	0.0487	0.0577	0.0533	0.0513	0.0560	0.0497	0.0537	0.0660	0.0617	0.0483
	QOT	0.0393	0.0497	0.0467	0.0613	0.0383	0.0407	0.0327	0.0440	0.0467	0.0987	0.0350	0.0443	0.0283	0.0477	0.0447	0.1270	0.0380	0.0450
$\tau = 0.30$	QST	0.0610	0.0510	0.0550	0.0653	0.0647	0.0573	0.0547	0.0513	0.0470	0.0717	0.0600	0.0547	0.0530	0.0513	0.0523	0.0807	0.0543	0.0447
	QOT	0.0387	0.0510	0.0510	0.0970	0.0510	0.0520	0.0273	0.0487	0.0410	0.1253	0.0377	0.0490	0.0267	0.0513	0.0507	0.1763	0.0320	0.0467
$\tau = 0.50$	QST	0.0547	0.0493	0.0513	0.0597	0.0567	0.0520	0.0657	0.0460	0.0390	0.0700	0.0520	0.0530	0.0640	0.0593	0.0557	0.0837	0.0527	0.0570
	QOT	0.0347	0.0427	0.0493	0.0930	0.0460	0.0517	0.0277	0.0487	0.0460	0.1237	0.0313	0.0513	0.0213	0.0530	0.0500	0.1677	0.0377	0.0543
$\tau = 0.70$	QST	0.0517	0.0557	0.0413	0.0640	0.0587	0.0523	0.0573	0.0457	0.0470	0.0623	0.0520	0.0467	0.0640	0.0470	0.0557	0.0857	0.0477	0.0480
	QOT	0.0343	0.0523	0.0517	0.0933	0.0483	0.0547	0.0297	0.0453	0.0490	0.1120	0.0357	0.0477	0.0217	0.0450	0.0530	0.1743	0.0410	0.0507
$\tau = 0.90$	QST	0.0487	0.0593	0.0563	0.0633	0.0617	0.0553	0.0493	0.0480	0.0477	0.0617	0.0560	0.0560	0.0617	0.0493	0.0507	0.0733	0.0513	0.0503
	QOT	0.0367	0.0587	0.0527	0.0750	0.0423	0.0547	0.0307	0.0443	0.0543	0.0913	0.0410	0.0527	0.0287	0.0483	0.0480	0.1373	0.0337	0.0510
	FR_1	0.0027	0.0040	0.0030	0.0043	0.0000	0.0030	0.0013	0.0047	0.0033	0.0087	0.0000	0.0010	0.0000	0.0020	0.0017	0.0140	0.0000	0.0023
	FR_2	0.0343	0.0470	0.0470	0.0890	0.0197	0.0520	0.0310	0.0517	0.0487	0.1333	0.0223	0.0553	0.0243	0.0433	0.0493	0.2007	0.0173	0.0480
	FR_3	0.0470	0.0500	0.0490	0.0673	0.0017	0.0547	0.0423	0.0520	0.0477	0.0980	0.0017	0.0567	0.0433	0.0460	0.0507	0.1443	0.0020	0.0513

Student's t Distribution		sample size 200						sample size 500						sample size 1000					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
$\tau = 0.10$	QST	0.0677	0.0563	0.0677	0.1117	0.1140	0.0707	0.0720	0.0567	0.0640	0.1383	0.1217	0.0600	0.0830	0.0700	0.0577	0.2400	0.1383	0.0487
	QOT	0.0340	0.0447	0.0393	0.1423	0.0277	0.0360	0.0293	0.0427	0.0510	0.2650	0.0180	0.0453	0.0203	0.0653	0.0537	0.4147	0.0143	0.0360
$\tau = 0.30$	QST	0.0667	0.0593	0.0670	0.1317	0.1090	0.0660	0.0690	0.0527	0.0587	0.2830	0.1163	0.0667	0.0880	0.0517	0.0603	0.5447	0.1350	0.0470
	QOT	0.0323	0.0463	0.0597	0.2647	0.0223	0.0520	0.0153	0.0533	0.0540	0.4957	0.0120	0.0570	0.0073	0.0553	0.0587	0.7737	0.0060	0.0437
$\tau = 0.50$	QST	0.0587	0.0510	0.0570	0.1523	0.0983	0.0633	0.0653	0.0460	0.0490	0.3347	0.1083	0.0580	0.0993	0.0573	0.0523	0.6397	0.1400	0.0537
	QOT	0.0270	0.0483	0.0483	0.2987	0.0207	0.0603	0.0123	0.0490	0.0567	0.5590	0.0097	0.0523	0.0037	0.0600	0.0537	0.8390	0.0033	0.0457
$\tau = 0.70$	QST	0.0603	0.0540	0.0657	0.1563	0.1047	0.0673	0.0723	0.0530	0.0623	0.2967	0.1117	0.0590	0.0890	0.0560	0.0497	0.5627	0.1403	0.0503
	QOT	0.0243	0.0477	0.0497	0.2727	0.0247	0.0570	0.0140	0.0477	0.0570	0.4987	0.0127	0.0503	0.0087	0.0590	0.0497	0.7710	0.0037	0.0520
$\tau = 0.90$	QST	0.0737	0.0727	0.0817	0.1107	0.1540	0.0880	0.0773	0.0703	0.0637	0.1523	0.1517	0.0640	0.0800	0.0627	0.0690	0.2547	0.1710	0.0603
	QOT	0.0253	0.0450	0.0497	0.1433	0.0330	0.0420	0.0290	0.0517	0.0527	0.2533	0.0247	0.0447	0.0210	0.0600	0.0597	0.4143	0.0130	0.0483
	FR_1	0.0193	0.0160	0.0163	0.0497	0.0060	0.0123	0.0287	0.0183	0.0187	0.0997	0.0053	0.0037	0.0280	0.0240	0.0223	0.3980	0.0007	0.0060
	FR_2	0.0697	0.0620	0.0737	0.2300	0.0293	0.0647	0.0907	0.0773	0.0767	0.4540	0.0150	0.0523	0.0900	0.0820	0.0820	0.8587	0.0070	0.0503
	FR_3	0.1040	0.0970	0.1060	0.1140	0.0347	0.0737	0.1330	0.1160	0.1143	0.1523	0.0150	0.0627	0.1290	0.1170	0.1170	0.4407	0.0063	0.0590

Skew t Distribution		sample size 200						sample size 500						sample size 1000					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
$\tau = 0.10$	QST	0.0457	0.0507	0.0400	0.0480	0.0467	0.0507	0.0407	0.0460	0.0457	0.0527	0.0443	0.0457	0.0450	0.0473	0.0410	0.0493	0.0493	0.0527
	QOT	0.0237	0.0387	0.0330	0.0433	0.0287	0.0393	0.0243	0.0403	0.0427	0.0697	0.0297	0.0427	0.0197	0.0360	0.0420	0.0983	0.0263	0.0497
$\tau = 0.30$	QST	0.0510	0.0477	0.0477	0.0583	0.0423	0.0537	0.0590	0.0510	0.0520	0.0803	0.0467	0.0520	0.0700	0.0467	0.0490	0.1023	0.0493	0.0517
	QOT	0.0223	0.0497	0.0417	0.1017	0.0293	0.0473	0.0227	0.0527	0.0553	0.1717	0.0287	0.0553	0.0067	0.0410	0.0427	0.2390	0.0210	0.0537
$\tau = 0.50$	QST	0.0577	0.0510	0.0533	0.0853	0.0597	0.0613	0.0690	0.0553	0.0607	0.1407	0.0660	0.0607	0.0953	0.0487	0.0473	0.2270	0.0667	0.0447
	QOT	0.0237	0.0480	0.0563	0.1550	0.0300	0.0507	0.0173	0.0543	0.0570	0.2573	0.0257	0.0570	0.0090	0.0537	0.0480	0.4087	0.0163	0.0517
$\tau = 0.70$	QST	0.0720	0.0663	0.0683	0.1123	0.0727	0.0597	0.0870	0.0650	0.0607	0.1893	0.0827	0.0607	0.1377	0.0590	0.0600	0.3387	0.1027	0.0517
	QOT	0.0240	0.0677	0.0660	0.1937	0.0343	0.0493	0.0197	0.0630	0.0677	0.3433	0.0253	0.0677	0.0053	0.0603	0.0587	0.5397	0.0190	0.0437
$\tau = 0.90$	QST	0.0973	0.0943	0.0883	0.1037	0.1130	0.0717	0.1180	0.0920	0.0957	0.1193	0.1237	0.0957	0.1697	0.0927	0.0807	0.1597	0.1383	0.0503
	QOT	0.0383	0.0817	0.0747	0.1300	0.0487	0.0460	0.0367	0.1027	0.1017	0.2053	0.0420	0.1017	0.0147	0.0780	0.0833	0.2757	0.0290	0.0463

Table 3: Size Property under Decreasing Correlation Cases

		sample size 200						sample size 500						sample size 1000					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
τ = 0.10	QST	0.0557	0.0510	0.0447	0.0700	0.0613	0.0543	0.0557	0.0520	0.0527	0.0700	0.1370	0.0460	0.0500	0.0453	0.0527	0.0610	0.0933	0.0483
	QOT	0.0413	0.0437	0.0407	0.0533	0.0153	0.0413	0.0530	0.0493	0.0500	0.0533	0.0027	0.0450	0.0450	0.0457	0.0533	0.0903	0.0117	0.0460
τ = 0.30	QST	0.0577	0.0520	0.0487	0.0677	0.0717	0.0490	0.0590	0.0547	0.0463	0.0677	0.2123	0.0553	0.0497	0.0497	0.0470	0.0653	0.1183	0.0437
	QOT	0.0520	0.0487	0.0473	0.0723	0.0130	0.0467	0.0560	0.0560	0.0467	0.0723	0.0027	0.0593	0.0533	0.0513	0.0517	0.1180	0.0047	0.0477
τ = 0.50	QST	0.0493	0.0537	0.0467	0.0503	0.0830	0.0573	0.0527	0.0497	0.0560	0.0503	0.2140	0.0460	0.0463	0.0457	0.0470	0.0663	0.1147	0.0470
	QOT	0.0543	0.0457	0.0507	0.0720	0.0117	0.0547	0.0603	0.0483	0.0443	0.0720	0.0007	0.0570	0.0513	0.0433	0.0480	0.1147	0.0053	0.0493
τ = 0.70	QST	0.0557	0.0457	0.0553	0.0580	0.0743	0.0527	0.0507	0.0477	0.0560	0.0580	0.2080	0.0607	0.0493	0.0450	0.0487	0.0637	0.1037	0.0440
	QOT	0.0507	0.0450	0.0470	0.0753	0.0123	0.0533	0.0560	0.0500	0.0470	0.0753	0.0023	0.0530	0.0450	0.0410	0.0463	0.1123	0.0043	0.0423
τ = 0.90	QST	0.0520	0.0490	0.0583	0.0793	0.0640	0.0570	0.0587	0.0540	0.0543	0.0793	0.1313	0.0487	0.0493	0.0457	0.0547	0.0657	0.0910	0.0480
	QOT	0.0463	0.0453	0.0440	0.0550	0.0133	0.0493	0.0503	0.0520	0.0490	0.0550	0.0040	0.0457	0.0550	0.0437	0.0513	0.0897	0.0087	0.0450
	FR$_1$	0.0003	0.0013	0.0007	0.0003	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0013	0.0003	0.0000	0.0010	0.0000	0.0000	0.0023
	FR$_2$	0.0173	0.0187	0.0140	0.0143	0.0003	0.0480	0.0123	0.0120	0.0117	0.0130	0.0000	0.0516	0.0143	0.0180	0.0167	0.0210	0.0000	0.0470
	FR$_3$	0.1230	0.0777	0.0707	0.0097	0.0023	0.0530	0.1090	0.0683	0.0667	0.0003	0.0000	0.0523	0.0897	0.0680	0.0620	0.0000	0.0000	0.0490

Student's t Distribution		sample size 200						sample size 500						sample size 1000					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
τ = 0.10	QST	0.0970	0.0623	0.0567	0.0737	0.1113	0.0610	0.0607	0.0580	0.0580	0.0883	0.1593	0.0523	0.0640	0.0580	0.0580	0.1413	0.2080	0.0627
	QOT	0.0360	0.0470	0.0490	0.0940	0.0243	0.0360	0.0513	0.0623	0.0647	0.1750	0.0107	0.0400	0.0430	0.0623	0.0647	0.2750	0.0060	0.0460
τ = 0.30	QST	0.0973	0.0537	0.0650	0.0700	0.0680	0.0587	0.0557	0.0597	0.0567	0.1330	0.1020	0.0587	0.0523	0.0597	0.0567	0.2210	0.1443	0.0673
	QOT	0.0360	0.0477	0.0460	0.1203	0.0167	0.0443	0.0400	0.0587	0.0577	0.2570	0.0083	0.0450	0.0303	0.0587	0.0577	0.4320	0.0050	0.0547
τ = 0.50	QST	0.0933	0.0443	0.0500	0.0823	0.0663	0.0637	0.0543	0.0520	0.0590	0.1387	0.0803	0.0573	0.0480	0.0520	0.0590	0.2530	0.1207	0.0523
	QOT	0.0333	0.0503	0.0503	0.1357	0.0220	0.0463	0.0393	0.0563	0.0540	0.2813	0.0080	0.0433	0.0260	0.0563	0.0540	0.4587	0.0047	0.0523
τ = 0.70	QST	0.0923	0.0480	0.0510	0.0867	0.0697	0.0717	0.0527	0.0547	0.0523	0.1320	0.0923	0.0733	0.0573	0.0547	0.0523	0.2443	0.1333	0.0573
	QOT	0.0444	0.0483	0.0480	0.1237	0.0230	0.0473	0.0400	0.0570	0.0570	0.2480	0.0113	0.0487	0.0317	0.0570	0.0570	0.4327	0.0053	0.0543
τ = 0.90	QST	0.102	0.0650	0.0587	0.0927	0.0950	0.0827	0.0610	0.0670	0.0600	0.1030	0.1367	0.0753	0.0537	0.0670	0.0600	0.1640	0.2013	0.0733
	QOT	0.0446	0.0513	0.0590	0.0867	0.0217	0.0380	0.0507	0.0600	0.0643	0.1663	0.0167	0.0450	0.0433	0.0600	0.0643	0.2877	0.0080	0.0473
	FR$_1$	0.0513	0.0513	0.0563	0.0673	0.0260	0.0087	0.0660	0.0690	0.0657	0.1047	0.0253	0.0090	0.0650	0.0737	0.0890	0.1393	0.0200	0.0067
	FR$_2$	0.1110	0.1163	0.1213	0.1670	0.0570	0.0517	0.1410	0.1613	0.1523	0.2553	0.0473	0.0567	0.1387	0.1563	0.1707	0.3497	0.0330	0.0527
	FR$_3$	0.2137	0.1767	0.1770	0.0390	0.0803	0.0640	0.2447	0.2140	0.2053	0.0143	0.0560	0.0607	0.2580	0.1973	0.2123	0.0057	0.0357	0.0590

Skew t Distribution		sample size 200						sample size 500						sample size 1000					
		I	II	III	IV	V	VI	I	II	III	IV	V	VI	I	II	III	IV	V	VI
τ = 0.10	QST	0.0487	0.0470	0.0550	0.0703	0.0467	0.0483	0.0517	0.0407	0.0533	0.0600	0.0407	0.0540	0.0540	0.0507	0.0480	0.0647	0.0460	0.0490
	QOT	0.0357	0.0343	0.0420	0.0443	0.0357	0.0313	0.0413	0.0400	0.0413	0.0650	0.0390	0.0397	0.0453	0.0427	0.0453	0.0890	0.0423	0.0413
τ = 0.30	QST	0.0473	0.0540	0.0487	0.0633	0.0483	0.0617	0.0527	0.0450	0.0530	0.0693	0.0433	0.0527	0.0513	0.0527	0.0493	0.0847	0.0490	0.0440
	QOT	0.0493	0.0373	0.0420	0.0830	0.0443	0.050	0.0483	0.0497	0.0437	0.1127	0.0380	0.0513	0.0493	0.0487	0.0500	0.1683	0.0380	0.0510
τ = 0.50	QST	0.0490	0.0557	0.0467	0.0637	0.0567	0.0613	0.0513	0.0477	0.0440	0.0990	0.0510	0.0470	0.0533	0.0477	0.0527	0.1187	0.0577	0.0553
	QOT	0.0413	0.0477	0.0507	0.0920	0.0463	0.0543	0.0427	0.0433	0.0457	0.1640	0.0403	0.0480	0.0463	0.0483	0.0523	0.2383	0.0377	0.0527
τ = 0.70	QST	0.0563	0.0627	0.0530	0.0653	0.0507	0.0680	0.0530	0.0553	0.0540	0.1010	0.0560	0.0567	0.0603	0.0507	0.0540	0.1593	0.0557	0.0523
	QOT	0.0477	0.0523	0.0553	0.0993	0.0440	0.0573	0.0440	0.0560	0.0500	0.1953	0.0303	0.0527	0.0497	0.0500	0.0543	0.2987	0.0340	0.0493
τ = 0.90	QST	0.0777	0.0617	0.0597	0.0963	0.0577	0.070	0.0557	0.0597	0.0607	0.0987	0.0540	0.0587	0.0577	0.0523	0.0610	0.1587	0.0663	0.0517
	QOT	0.0480	0.0500	0.0540	0.0867	0.0350	0.0543	0.0447	0.0527	0.0533	0.1710	0.0290	0.0447	0.0520	0.0597	0.0660	0.2810	0.0187	0.0467

Table 4: Descriptive Statistics of Countries in the Asia Crisis

	Mean	Variance	Median	Skewness	Excess Kurtosis	$J-B$ [1] Test	$Ljung-Box$ [1] Box Test	$ARCH$ [1] Test
Hong Kong	-0.00142	0.000673	0	0.379216	6.056046	1511.839991*[2]	26.73796242*	102.3380846*
South Korea	-0.00171	0.000784	0	0.137872	-0.97908	76.94547603*	17.35117772*	56.36415268*
Malaysia	-0.00324	0.000672	-0.00269	1.541682	9.571008	3063.658555*	9.091541876	3.9842208
Japan	-0.00036	0.000138	0	0.021421	-0.08357	156.7419826*	11.8900159*	46.32324159*
U.K.	0.000559	0.0000997	0.000517	-0.2638	-2.14174	18.94853472*	20.90735135*	33.27060309*
U.S.	0.00036	0.00013	0.000546	-1.13983	3.963259	983.0770432*	1.818650886	29.16018572*
Australia	0.0000512	0.000546	0.00092	-0.57487	2.63192	605.8060695*	3.054880627	117.825644*
Germany	0.001126	0.000215	0.00225	-0.86824	0.614144	295.1270798*	10.40811878*	67.6223594*
Thailand	-0.00312	0.000588	-0.00388	0.94095	0.220109	255.3603359*	19.25763484*	65.01672425*

Notes:

[1] In the above table, the values of the Jarque–Bera test, the Ljung–Box test and the ARCH LM test are calculated with $lags = 5$.

[2] * indicates the rejection of the null hypothesis.

Table 5: Correlations Among Several Countries in the Asian Crisis (Before Crisis[1])

	Hong Kong	South Korea	Malaysia	Japan	U.K.	U.S.j	Australia	Germany	Thailand
Hong Kong	1								
South Korea	0.151735	1							
Malaysia	0.273213	0.047137	1						
Japan	0.308531	0.031992	0.088048	1					
U.K.	0.151702	0.065907	-0.01891	0.130064	1				
U.S.	0.081478	0.008263	0.001455	-0.04569	0.331209	1			
Australia	0.273855	0.05636	0.137215	0.235572	0.117007	-0.01811	1		
Germany	0.266234	0.024454	0.101507	0.125796	0.457624	0.242969	0.397739	1	
Thailand	0.082663	0.036424	0.132881	0.016986	0.023094	-0.03897	0.103829	0.04785	1

Table 6: Correlations Among Several Countries in the Asian Crisis (During Crisis[2])

	Hong Kong	South Korea	Malaysia	Japan	U.K.	U.S.	Australia	Germany	Thailand
Hong Kong	1								
South Korea	0.130589	1							
Malaysia	0.444207	0.285872	1						
Japan	0.399132	0.13909	0.296769	1					
U.K.	0.489817	0.204314	0.290062	0.396905	1				
U.S.	0.261643	0.169513	0.10811	0.136181	0.518994	1			
Australia	0.589521	0.261065	0.36159	0.490537	0.449319	0.190915	1		
Germany	0.622381	0.172951	0.291964	0.394909	0.684973	0.396108	0.568798	1	
Thailand	0.442736	0.328666	0.479853	0.241142	0.278739	0.137684	0.380339	0.268605	1

[1] Before crisis: from January 1, 1997 to October 19, 1997. [2] After crisis: from October 20, 1997 to August 31, 1998