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Second-best urban tolls in a monocentric city with housing market regulations

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Abstract This paper investigates second-best congestion pricing in a monocentric city characterized by distortionary, rigid regulatory mechanisms in the housing market (building height restrictions, zoning and property taxation). The Pigouvian toll is shown to retain its optimality under any setting with quantity restrictions in the housing market. However, the extent of the quantity restriction determines the volume of the welfare gains in a non-monotonic fashion. This finding introduces a warning to cost-benefit analyses: our numerical results suggest that the actual gains of a road tax might be 40% lower than the gains predicted by a model that disregards maximum building height restrictions, and 80% higher than the gains suggested by a model that disregards zoning. In general, this implies that decision making on urban road pricing can ignore quantitative restrictions in the related markets of land, housing and labor insofar as the determination of optimal marginal tax rules is concerned; the tax levels stemming from those rules will be affected by the restrictions. However, this is not the case in the presence of a tax-induced distortion. Introducing an ad-valorem property tax on housing, we show that adjustments of the Pigouvian toll can lead to small, but not negligible welfare gains.

Keywords: road pricing, building height restrictions, zoning, property tax, monocentric city, cost-benefit analysis

JEL classification: R48, R52, R13, H21, H23, D61

1. Introduction

Second-best issues are a prominent theme in the contemporary literature on externality regulation. This is true for the transport economics literature in the context of road pricing (Verhoef et al., 1996), the urban economics literature in the discussion on urban growth boundaries (Brueckner, 2007; Anas and Rhee, 2007), and the environmental economics literature, where the double dividend hypothesis is an important theme (e.g. Goulder et al. 1997). Second-best issues may emerge for a variety of reasons. One is that the market for the commodity under consideration (whose consumption generates the externality) is directly related to the market of another imperfectly priced commodity, because the two commodities are either substitutes or complements. This provides a motive to adjust Pigouvian taxes in the market of primary interest: welfare losses due to deviations of the tax from marginal external costs in one market are motivated by welfare gains in the related market. As an example, in the absence of road congestion pricing, second-best fares in public transport will reflect unpriced congestion on the road: they will be

adjusted downwards (upwards) when public transport functions as a substitute (complement) to private road use. These considerations, and the corresponding second-best adjustments, vanish once optimal road pricing is implemented.

In this paper we examine optimal road pricing from a new perspective, i.e. under the impact of a series of regulations in the housing and land market that are: i) *distortionary*, and thus suboptimal to begin with, and ii) *non-adjustable*, i.e. they are *assumed* to be fixed, in contrast to road pricing schemes which will be optimized. Both assumptions are relevant.

Regarding the first assumption, most urban land and housing markets are heavily distorted due to a range of suboptimal pricing (e.g. housing taxation, mortgage interest deduction) and quantity-restriction policies (e.g., regulated building height, zoning). Although the impact of tax-induced distortions in urban labor markets on optimal road pricing has received some attention in the recent literature (Parry and Bento, 2001; Mayeres and Proost, 2001; Tikoudis et al, 2013), the associated impact of imperfectly functioning land and housing markets on the latter has remained under-researched. Empirical observation suggests that, in a substantial part of the world, quantity restrictions are common. For example, in the Netherlands, most municipalities set maximum building height restrictions for buildings including residences. In most other European countries, similar regulatory restrictions have been implemented.¹ In the US, zoning is a common practice. The bottom line is that, in almost every case, these regulations are decided in the sphere of politics and do not correspond to a welfare maximization plan.

The second assumption is also reasonable for several reasons. First, command and control regulations might be more difficult to adjust than tolls in a road pricing scheme. Current homeowners have strong incentives to lobby for building height restrictions, as well as zoning (Fischel, 2001; Schuetz, 2009). These regulations have been shown to increase housing prices (e.g. Ihlanfeldt, 2007) and are, therefore, very likely to remain intact in the years to come. Similarly, housing property taxation (as well as their counterpart, subsidies through mortgage interest deduction) is frequently observed in most countries and has been established as a standard way to raise public revenue.² Property taxes might in theory be easier to adjust, but in most cases the authority that sets its level will be a different principle than the regulator who controls the road tax.³ Second, specific command and control restrictions (such as minimum and maximum building heights, as well as urban growth boundaries) cannot be implemented in an *already formed urban landscape and network*. For instance, there is no way to introduce an urban growth boundary in the interior of a city, or a maximum building height below the one observed in a specific location. The first would outlaw existing private property outside the boundary; the second would impose a heavy financial cost associated with demolition and reconstruction.

Our paper is, therefore, closely related to a rapidly developing literature in urban economics that explores the potential of land-use practices to substitute for congestion pricing when it comes to curbing traffic-induced externalities. Looking at the same issue from a totally different perspective, this stream of research is (implicitly) motivated by the large implementation costs and the limited political acceptability

¹ Building height restrictions is one specific form of land-use planning. Cheshire and Sheppard (2002) find a negative welfare effect of land-use planning in general.

² Taxation in the housing market include transaction as well as property taxes, but taxation may also be negative in case of mortgage interest deduction. Other regulatory channels include rent control (Gyourko et al. 1989, 1990; Arnott, 1995) and public housing. Via different channels, these regulatory mechanisms may cause welfare losses through a suboptimal allocation of space across economic agents and activities (Glaeser and Luttmer, 2003). See Cheshire and Hilber (2008) for the effect of building height restrictions in the market of office space.

³ But even if the different levels of government could coordinate, the primary function of the property tax would still be the raise of a pre-determined revenue.

associated with road pricing. Brueckner (2007) and Anas and Rhee (2007) investigate the efficiency of an urban growth boundary relative to a congestion toll. Optimal floor-to-area ratio (*FAR*) regulations, have been examined in conjunction with population externalities by Joshi and Kono (2009) and Kono et al. (2010). More recently, Kono et al. (2012) evaluate the efficiency of regulations on building size and city size relative to the gains that can be achieved by a first-best road toll. Pines and Kono (2012) discuss second-best allocations based on space-varying property taxation and *FAR* regulations. The critical differentiator between the above literature and this paper is that we investigate welfare-enhancing policies exclusively through road pricing, i.e. by assuming that housing market regulations are given and rigid.

Our contribution is, however, much more general than the investigation of the specific interaction between transport and the housing market. We show – in a rather general setting – that second-best issues as arising from imperfectly-functioning related markets have different impacts, and therefore different policy implications, depending on the nature of the distortion. We compare two types of distortions arising from policy interventions: *distortionary taxation* (for example motivated by the desire to raise tax revenues) and *quantitative restrictions* generated by policies that are not justified from a welfare perspective. In the context of a city where residents commute to a given workplace, we investigate both types of policy interventions by focusing on housing taxation and building height restrictions respectively (Arnott and MacKinnon, 1977; Bertaud and Brueckner, 2005). Both interventions have in common that they affect the welfare gains that externality pricing in the primary market of interest (road transport, in our example) could bring. But an important difference is that, whereas tax-induced distortions invoke deviations from the Pigouvian principle in the primary market (i.e. the optimal tax deviates from the marginal external cost), such deviation is *not* efficiency enhancing in the case of quantitative restrictions (command and control regulations). The question of whether regulatory taxes must be adjusted given distortionary policies in related markets does, therefore, not so much depend on *whether* there are distortions on these other markets, but much more on *the type of* distortions.

In the specific context of the study, we provide answers to a series of policy-relevant questions. Given the presence of land-use regulations we examine i) if there is any road pricing scheme superior to the Pigouvian toll, and ii) if the gains from the Pigouvian toll vary substantially (and in which direction) across cities with identical household preferences and road technologies but different building height limits or areas subject to zoning. Answers to both questions are valuable to the transport planner, because they provide knowledge on the extent to which road demand management can be detached from urban planning and public finance decisions (which are usually taken at different levels of government). In addition, knowledge over the extent of Pigouvian toll gains/losses across different levels of quantity restrictions is useful to understand to what extent the gains from congestion charges for cities without any housing regulation (as commonly assumed in the literature) can be extrapolated to cities with restrictive land-use regulations.

We use as a starting point the voluminous literature on the welfare costs of land-use and housing market regulations (Brueckner, 1996; Bertaud and Renaud, 1997; Bertaud and Brueckner, 2005; Glaeser et al., 2005; Cheshire and Hilber, 2008).⁴ In the current paper, a monocentric city framework is expanded to incorporate congestion as in Verhoef (2005) and, more recently, Tikoudis et al. (2013). A numerical

⁴ In Bertaud and Brueckner (2005), the welfare cost from a certain level of building height restrictions in a monocentric city is shown to be equal to the difference in commuting costs that the household in the city fringe would face if the building height restrictions were removed. Numerical simulations suggest the welfare cost to be 2% of household income. For an empirical application of this framework for India, see Brueckner and Shridhar (2012).

version of the model is calibrated such that the benchmark equilibrium reproduces a set of stylized facts that characterize a representative monocentric city in the Western world, in absence of congestion charges.

Using the parameters of the benchmark equilibrium, we compute the welfare effect of the Pigouvian toll as a function of the maximum floor-to-area ratio (*FAR*) allowed. We show that this effect is not only very sensitive to the underlying *FAR* regulation, but also non-monotonic: a Pigouvian toll imposed in a city without height restrictions may produce up to 40% larger welfare gains than Pigouvian tolling in a city with a mild, uniform in space, *FAR* restriction. Similar computations for the case of zoning suggest that the welfare gains might be 80% larger in an equilibrium with a large zoned area close to *CBD*. At the same time, we demonstrate that the Pigouvian toll retains its optimality independent of the extent and the type of quantity restriction. And finally, we show that this optimality ceases when other relevant preexisting taxes (e.g. a housing property tax) are added to the model.⁵ Additional sensitivity analysis is used to examine the efficiency of the Pigouvian toll relative to the optimal space-varying tax.

The structure of the paper is as follows. Section 2 introduces the analytical model, Section 3 discusses the performance of the Pigouvian toll under a generic building height restriction, Section 4 presents the calibration and the stylized facts that characterize the benchmark equilibrium (absence of housing market regulations). Section 5.1 discusses the welfare effects of the Pigouvian toll at various levels of a *uniform FAR* restriction. Section 5.2 shows the respective results for the case of zoning. Section 5.3 presents numerical results, i.e. the deviation of the optimal space varying-tax from the corresponding Pigouvian toll and the corresponding relative efficiency, for the case of a distortionary housing property tax. Section 5 summarizes and concludes.

2. Model

The analytical model presented in this section is based to the models used in Verhoef (2005) and Tikoudis et al. (2013).⁶ The extension uses elements from Bertaud and Brueckner (2005) and Muth (1969).

2.1. Households

Households are located anywhere within a linear monocentric city, i.e. a city with a single central business district (hereafter, *CBD*) where all jobs are located. Let z and \bar{z} denote the distances of an arbitrary household and the city fringe, respectively, from the *CBD*. Utility is derived from the consumption of a numéraire good, y , floor space, s , and leisure time, T_F . Assuming CES preferences, the household that locates at distance z from *CBD* maximizes:

$$U_z = [(\alpha y_z)^\rho + (\beta s_z)^\rho + (\gamma T_{Fz})^\rho]^{\frac{1}{\rho}}. \quad (1)$$

The total time endowment, T (e.g. a year), is spent on commuting, T_C , working, T_L , and leisure, T_F :

$$T = T_{Cz} + T_{Lz} + T_{Fz}. \quad (2)$$

Labor supply is inelastic throughout a working day, thus the working day is of fixed duration, t_L . Every

⁵ The efficiency of such anti-sprawl tax instruments has been discussed in Bento et al. (2006, 2011)

⁶ The exposition in Section 2 draws heavily from these two contributions. Literal citations from these sources are not marked as such for legibility; duplicating equations are provided in order to keep this paper self-contained.

trip to work requires t_z units of time. For D_{Wz} working days the time constraint becomes:

$$T = D_{Wz} t_z + D_{Wz} t_L + T_{Fz}. \quad (3)$$

Normalizing the duration of the working day, t_L , to one, the time constraint becomes:

$$T = D_{Wz} (1 + t_z) + T_{Fz} \Leftrightarrow D_{Wz} = (T - T_{Fz}) / (1 + t_z). \quad (4)$$

The net wage per working day is defined as the difference between wage, w , and the total pecuniary commuting cost, which consists of the total road toll charged between z and the CBD, τ_{Rz} , and the operational cost (gasoline, vehicle depreciation etc.), mz , which is a linear function of commuting distance. Full income of the household at distance z , M_z , is the maximum income that can be realized when leisure time is set to zero:

$$M_z = B + \frac{(w - \tau_{Rz} - mz)}{1 + t_z} T, \quad (5)$$

where B denotes a lump-sum transfer (discussed below) from the government to the household, independent of its location. Full income can be used to buy back leisure at its shadow price, $(w - \tau_{Rz} - mz)/(1 + t_z)$, the composite good (with price normalized to one) and residential space (priced at $p_z(1 + \tau_V)$, where τ_V denotes an ad-valorem property tax rate):

$$M_z = \frac{(w - \tau_{Rz} - mz)}{1 + t_z} T_{Fz} + y_z + p_z(1 + \tau_V) S_z. \quad (6)$$

Maximizing (1) subject to (4), (5) and (6), and defining $\chi = \rho/(\rho - 1)$ and $\sigma = 1/(1 - \rho)$ yields the conditional (on z) Marshallian demand functions for the composite good, space and leisure time respectively:

$$y_z^* = M_z \frac{(1/\alpha^\rho)^{-\sigma}}{\left(\frac{1}{\alpha}\right)^\chi + \left[\frac{p_z(1 + \tau_V)}{\beta}\right]^\chi + \left[\frac{w - \tau_{Rz} - mz}{\gamma(1 + t_z)}\right]^\chi}, \quad (7)$$

$$s_z^* = M_z \frac{(p_z(1 + \tau_V)/\beta^\rho)^{-\sigma}}{\left(\frac{1}{\alpha}\right)^\chi + \left[\frac{p_z(1 + \tau_V)}{\beta}\right]^\chi + \left[\frac{w - \tau_{Rz} - mz}{\gamma(1 + t_z)}\right]^\chi}, \quad (8)$$

$$T_{Fz}^* = M_z \frac{\left[\frac{w - \tau_{Rz} - mz}{\gamma^\rho(1 + t_z)}\right]^{-\sigma}}{\left(\frac{1}{\alpha}\right)^\chi + \left[\frac{p_z(1 + \tau_V)}{\beta}\right]^\chi + \left[\frac{w - \tau_{Rz} - mz}{\gamma(1 + t_z)}\right]^\chi}. \quad (9)$$

Finally, substituting (7)-(9) into the objective function yields the *conditional indirect utility*:

$$V_z^* = \left[B + \frac{(w - \tau_{Rz} - mz)}{1 + t_z} T \right] \left\{ \left(\frac{1}{\alpha} \right)^\chi + \left[\frac{p_x(1 + \tau_V)}{\beta} \right]^\chi + \left[\frac{w - \tau_{Rz} - mz}{\gamma(1 + t_z)} \right]^\chi \right\}^{-1/\chi}. \quad (10)$$

The spatial equilibrium is characterized by locational indifference, i.e. constant utility, u , over space. The space-derivative of equation (10) can then be reformulated to express the price of floor space as a function of equilibrium utility:

$$p_z(u) = \frac{\beta}{(1 + \tau_V)} \left\{ u^{-\chi} \left[B + \frac{(w - \tau_{Rz} - mz)}{1 + t_z} T \right]^\chi - \left(\frac{1}{\alpha} \right)^\chi - \left[\frac{w - \tau_{Rz} - mz}{\gamma(1 + t_z)} \right]^\chi \right\}^{1/\chi}. \quad (11)$$

Now, equation (11) can be plugged into (7), (8) and (9) to yield the compensated demands for the composite good, space and leisure at any arbitrary distance z , which we denote by $C_z(u)$, $s_z(u)$, and $T_{Fz}(u)$ respectively.

2.2. Developers

Developers use constant returns to scale technology to convert land and capital into residential space. The capital intensive form of the Cobb-Douglas production function for residential space per unit of land (hereafter, *FAR*) is:

$$\hat{s}_z = g \hat{k}_z^\delta, \quad (12)$$

(where \hat{s} denotes floor space per unit of land) with corresponding profit per unit of land:

$$\pi = p_z g \hat{k}_z^\delta - p_K \hat{k}_z - p_{Lz}, \quad (13)$$

where $g > 0$ is a technology constant, \hat{k} denotes the units of capital over one unit of land (hereafter, *structural density*), p_K the exogenous price of capital and p_{Lz} the per-unit price of land at z . Dividing the supplied space in (12) with the compensated space demand at distance z , s_z , yields the *household density* at point z :

$$n_z = \frac{g \hat{k}_z^\delta}{s_z}. \quad (14)$$

From the first-order condition for profit maximization, we derive the (privately) optimal structural density:

$$\hat{k}^* = \left(\frac{p_K}{\delta p_z g} \right)^{1/(\delta-1)}. \quad (15)$$

Inserting (15) into (13) and solving for zero profits, yields the land rent:

$$p_{Lz} = (p_K)^{\delta/(\delta-1)} (p_z g)^{1/(1-\delta)} (\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)}). \quad (16)$$

2.3. Firms

A representative firm, which is located at the CBD, operates under constant returns to scale, producing the composite good with capital and labor:

$$Y = K^\zeta L^{1-\zeta}. \quad (17)$$

From cost minimization we derive the *conditional factor demands* for labor:

$$L = Y \left(\frac{(1-\zeta)p_K}{\zeta w} \right)^\zeta, \quad (18)$$

and capital:

$$K = Y \left(\frac{\zeta w}{(1-\zeta)p_K} \right)^{1-\zeta}. \quad (19)$$

Using (18) and (19), it can be shown that the marginal cost is:

$$p_K^\zeta w^{1-\zeta} [(1-\zeta)^{\zeta/(1-\zeta)} + \zeta^{(1-\zeta)/\zeta}]. \quad (20)$$

2.4. Commuting

Commuting from any given location z to the CBD is taking place through a single road, used by all households. Letting n_z denote the residential density at location z , we can write the travel time per unit of distance at location z as:

$$t_0 + t_1 \int_z^{\bar{z}} n_\zeta D_{W\zeta} d\zeta, \quad (21)$$

i.e. the sum of the free-flow travel time per unit of distance, t_0 , and a term that represents the congestion delay caused by the aggregate traffic flow at z .⁷ This flow contains all commuting trips by households located between z and the city limit, \bar{z} . Multiplying the aggregate flow with a sensitivity parameter, t_1 , yields the time delay per unit of distance at each point z . Integrating (21) over the interval $(0, z)$ yields the commuting time for the household at z :⁸

$$t_z = z t_0 + t_1 \int_0^z \min\{z, \zeta\} n_\zeta D_{W\zeta} d\zeta. \quad (22)$$

An additional commuting trip generated by a household at distance z increases the travel time of a commuter located at $\zeta \geq z$ by $t_1 z$, and the travel time of a commuter located at $\zeta \leq z$ by $t_1 \zeta$. Multiplying this delay with the *shadow value of time* at ζ , $(w - \tau_L - \tau_{R\zeta})/(1 + t_\zeta)$, and labor supply at

⁷ Therefore, the essence of congestion is captured by assuming that the travel time per unit of distance increases with the number of vehicles passing that point; the linear form is chosen for analytical convenience.

⁸ Note that in (21), z refers to a location along the road and ζ to household locations, while in (22) and (23) z refers to household location.

ζ , $n_\zeta D_{W\zeta}$, provides a measure for the marginal external cost of an additional unit of labor imposed by the commuter at z to the commuter at ζ .⁹ Integrating over the interval $(0, \bar{z})$ yields the *marginal external cost of congestion*, generated by the household located at z :

$$mecc(z) = t_1 \int_0^{\bar{z}} \min\{z, \zeta\} n_\zeta D_{W\zeta} \left(\frac{w - \tau_L - \tau_R \zeta}{1 + t_\zeta} \right) d\zeta. \quad (23)$$

2.5. Government and public budget

The government can tax (or subsidize) road use, and recycle the tax revenue and land rents in a manner that ensures a balanced budget. The total land revenue collected is the sum:

$$R_L = R_a + R = \bar{z} r_A + \int_0^{\bar{z}} (p_{Lz} - r_A) dx, \quad (24)$$

of which the first term, $R_a = \bar{z} r_A$, is transferred to an absentee land owner and the second term, R , i.e. the excess land rent, is returned to the consumers in a lump-sum manner.¹⁰ This assumption ensures that the city cannot expand without cost, and that the excess rents remain within the urban economy.

When imposed, a location-based road tax generates the revenue:

$$G = \int_0^{\bar{z}} n_z D_{Wz} \tau_{Rz} dz, \quad (25)$$

which is endogenous, since it depends on density, n_z , labor supply, D_{Wz} , and city size, \bar{z} . Furthermore, the revenue from the ad-valorem tax on property is:

$$Q = \int_0^{\bar{z}} p_z \tau_V g \hat{k}_z^\delta dz. \quad (26)$$

Toll and tax revenues are returned in the form of a lump-sum transfer. Therefore, the lump-sum transfer per household, B (see equation (5)), can be written as:

$$B = \frac{G + R + Q}{N}. \quad (27)$$

2.6. Equilibrium without distortions in the housing markets

In equilibrium, the (closed) city must accommodate the exogenous population, N , thus:

⁹ The volume delay function is the same used in Verhoef (2005), and in line with those used in other monocentric city models, for instance by Mun et al. (2003) and Brueckner (2007).

¹⁰ The model is a general equilibrium analogue of the *closed-city under public ownership* model (CCP) proposed in Fujita (1989).

$$\int_0^{\bar{z}} n_z dz = N. \quad (28)$$

We assume an absentee landlord concedes the land at an exogenous opportunity cost of land, i.e. the agricultural rent, r_A . Thus, the city fringe, \bar{z} , is endogenously determined via the condition:

$$r_A = (p_K)^{\delta/(\delta-1)} (p_{\bar{z}}g)^{1/(1-\delta)} (\delta^{\delta/(1-\delta)} - \delta^{1/(1-\delta)}), \quad (29)$$

where the right hand side gives the equilibrium value of (16) evaluated at \bar{z} . The rest of the equations comprise the clearing of the labor market:

$$\int_0^{\bar{z}} \left(\frac{T - T_{Fz}}{1 + t_z} \right) n_z dz = Y \left(\frac{(1 - \zeta)p_K}{\zeta w} \right)^\zeta, \quad (30)$$

the zero profit condition for the representative firm:

$$p_K^\zeta w^{1-\zeta} [(1 - \zeta)^{\zeta/(1-\zeta)} + \zeta^{(1-\zeta)/\zeta}] = 1, \quad (31)$$

(which is obtained by setting the normalized price of the numéraire equal to the marginal cost of the composite good) and the closing identity:

$$Y - \int_0^{\bar{z}} y_z n_z dz = \bar{z} r_A + p_K \left[Y \left(\frac{\zeta w}{(1 - \zeta)p_K} \right)^{1-\zeta} + \int_0^{\bar{z}} \left(\frac{p_K}{\delta p_z g} \right)^{1/(\delta-1)} dz \right], \quad (32)$$

which requires the value of the city export (left hand side) to be equal to the opportunity cost of land, $\bar{z} r_A$, and the value of imported capital; the latter is the product of its exogenous price, p_K , and the sum of: i) the demanded quantity by the representative firm, given in equation (19), and ii) the demanded quantity by the construction sector developers, i.e. the integral of (15) across space.

Equations (28)-(31) define a non-linear system in four unknowns: Y , \bar{z} , w , u . The remaining endogenous variables $(M, y, s, T_F, p, p_L, \hat{s}, n, G, K, L, R, B, t, \hat{k})$ are completely determined, given the values of the above four variables, through the equations in sections 2.1-2.5.

2.7. Equilibrium with a uniform building height restriction

In this paper, we consider the case of a uniform floor-to-area ratio restriction as a possible *quantity-induced* distortion in the housing market. This introduces a new endogenous variable, \hat{z} , which is the distance from CBD at which the maximum height constraint ceases binding (hereafter, the *building height restriction boundary*, or *BHR boundary*). Equation (12) becomes:

$$\hat{s}_z = \begin{cases} \bar{h} & \text{if } z < \hat{z} \\ g \hat{k}_z^\delta & \text{if } z > \hat{z} \end{cases} \quad (33)$$

where \bar{h} is the maximum number of floors (floor-to-area ratio) permitted at any point in the city. The equilibrium population density in (14) becomes:

$$n_z = \begin{cases} \frac{\bar{h}}{s_z} & \text{if } z < \hat{z} \\ \frac{g\hat{k}_z^\delta}{s_z} & \text{if } z > \hat{z}. \end{cases} \quad (34)$$

And the equilibrium condition in (28) is now written as:

$$\left[\int_0^{\hat{z}} \frac{\bar{h}}{s_z} dz + \int_{\hat{z}}^{\bar{z}} \frac{g\hat{k}_z^\delta}{s_z} dz \right] = N. \quad (35)$$

The additional endogenous variable, \hat{z} , is accompanied by a new equation in order for the model to close. This states that the building height at point \hat{z} is equal to the restricted height limit, \bar{h} . Inserting (15) into (12), this condition can be written as:

$$\hat{s}_z = g \left(\frac{p_K}{\delta p_z g} \right)^{\delta/(\delta-1)} = \bar{h}. \quad (36)$$

2.8. Equilibrium under zoning

The other quantity restriction examined in this paper is zoning. Under zoning, the structural density, \hat{k} , and floor-to-area ratio, \hat{s} , are restricted to be zero in an interval (z_L, z_U) specified by government. That is:

$$\hat{s}_x = \begin{cases} g\hat{k}_z^\delta & \text{if } z < z_L \text{ or } z > z_U \\ 0 & \text{if } z_L \leq z \leq z_U. \end{cases} \quad (37)$$

Furthermore, travel times in (22) and marginal external costs in (23) are adjusted for the fact that the flow at any point in the interval (z_L, z_U) remains constant. As in the unintervented case, the rent at city fringe is given by (29); no specific restrictions apply to points z_L and z_U . Despite the fact that zoned land is required to be vacant, it is still acquired from the absentee landlord at the opportunity cost of land.

3. Pigouvian taxation under non-price regulation in related markets

It is well known that non-optimal pricing in a market interacting with the market in which an externality is to be regulated typically calls for an adjustment of the standard Pigouvian prescription, i.e. setting taxes equal to marginal external costs. Such non-optimal pricing in the related market may result from pre-existing taxation, in which case the tax on the externality usually has to be adjusted to reduce the distortionary impact of the pre-existing tax in the other market. For the optimal design of policies addressing externalities, it is therefore important to take into account distortions elsewhere in the economy, including those arising from other taxes. An important question is then whether a similar reasoning also holds when the distortion in the related market is not due to a tax policy, but rather to other types of regulation. In this section we focus on quantity policies in a rather general setting. Applied to the topic of the paper, the more specific question would be whether application of the conventional Pigouvian congestion toll becomes suboptimal when the city has building height restrictions.

We will argue that the answer to this question is negative: the marginal external cost toll remains optimal, and this reflects an important distinction between a *tax-induced distortion* and *distortions from quantitative policy measures* in related markets. That is, a road tax can strongly interact with taxes imposed in the urban labor or housing market (see section 5.3), but does not do so with quantitative restrictions in related markets. The reason is that, *at the margin* in the Pigouvian equilibrium, an adjustment of the road toll will leave the supply of floor space intact at all locations where the constraint is binding, that is the interval $(0, \hat{z})$. Therefore, there is no efficiency gain in the land or housing market to be realized from a marginal adjustment of the Pigouvian toll in response to this distortion.

In this section we seek to put this argument in a rather general setting, where an externality tax is to be levied in one market and there is an undetermined number of related markets in which other policies may or may not be pursued; these policies can be taxes or quantitative non-tax measures. To that end, consider an economy with M markets. Denote the marginal social benefit on market m as MB_m , the marginal social cost as MC_m , and the quantity as q_m . Consider the case where social surplus maximization is the relevant social objective. The marginal impact on social welfare from a perturbation of a given tax, τ_m , can then be compactly written as:

$$\Delta W(\tau_m) = \sum_{\mu=1}^M \left\{ (MB_{\mu} - MC_{\mu}) \frac{dq_{\mu}}{d\tau_m} \right\}. \quad (38)$$

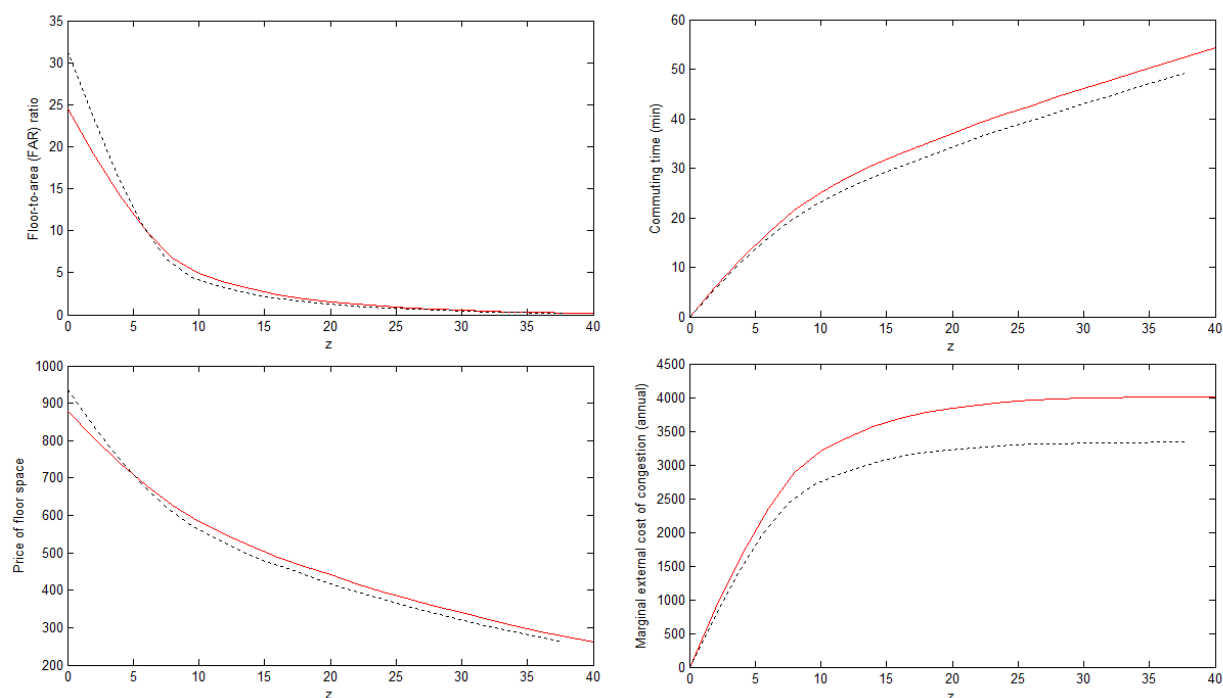
where $dq_{\mu}/d\tau_m$ may also include policy-induced effects via adjustments in other taxes (e.g. when considering tax recycling). For a single market, m , the optimum $\Delta W(\tau_m) = 0$ requires that $MB_m - MC_m = 0$ if $dq_m/d\tau_m \neq 0$. This would call for a Pigouvian tax equal to the marginal external cost when present. With multiple markets, welfare effects in other markets μ affect the second-best level of τ_m when the corresponding term in (38) for that market is non-zero; i.e., if $MB_{\mu} \neq MC_{\mu}$ (the market is distorted) and $dq_{\mu}/d\tau_m \neq 0$ (the equilibrium quantity in market μ has a non-zero response to a marginal change in τ_m). Stated differently, indirect effects in other markets vanish if these markets operate efficiently or if the quantity is insensitive, possibly because of a quantitative policy measure such as the building height restriction considered in this paper. Therefore, a tax in market μ that drives a wedge between MB_{μ} and MC_{μ} directly affects the optimality condition for the tax, τ_m , whereas markets for which q_{μ} is fixed would not affect the first-order condition – even though they may of course affect the equilibrium level of the tax.¹¹

There is another way to rationalize the optimality of the Pigouvian toll when policy-induced distortions in related markets involve fixed quantities rather than taxes. Returning to the example of building height restrictions, assume the existence of a non-smooth production function, alternative to (12), where developers use a constant return to scale technology to convert land and capital into residential space only up to a certain building height, \bar{h} , but where the capital cost of buildings exceeding \bar{h} is infinite. It is straightforward that a maximum *FAR* restriction and this building technology will result in identical equilibria. But a Pigouvian toll would be optimal under the alternative building technology,

¹¹ This effect is not to be confused with the mirror case where a quantity restriction on an externality-generating commodity (i.e. a polluting good) generates a non-negligible welfare effect in a primary factor market distorted by a pre-existing tax (e.g. labor tax). Parry (1996) shows that, for plausible parameters, an environmental quota can cause losses in labor market (i.e. through a decrease in labor supply) that are large enough to outweigh the beneficial partial equilibrium effect, i.e. the direct welfare benefit of an environmental quota.

since all conditions of the first welfare theorem, apart from the externality, then hold. Thus, a Pigouvian toll must also be optimal in the twin equilibrium under *FAR* where for a different reason building heights are restricted, but in essentially the same manner (Appendix C offers a numerical illustration, using computations of the optimal road tax for different starting values, with all of them converging to the value of the Pigouvian toll).

Figure 1. Floor-to-area ratio (upper left panel), commuting time (upper right), residential space prices (lower left), and marginal external congestion cost (lower right) in the two equilibria without building height regulations: unregulated (solid) and Pigouvian (dashed).



The above optimality has a clear policy implication: possibly distortive policies in other markets can be ignored for the determination of optimal Pigouvian taxes as long as these taxes are based on correctly computed marginal external costs. In the context of this paper: decisions on road pricing can in general ignore building height restrictions (or other quantitative restrictions such as zoning), in that sense allowing a detachment of optimal road pricing from urban planning. Of course, to the extent that the restrictions will affect the behavioral responses to pricing and hence the marginal external costs in the optimum, there remains an impact of the *FAR* on the optimal *levels* of road prices; it is however not the tax *rule* that is affected. As shown in section 5.3, the detachment of optimal road pricing from urban planning ceases in the presence of tax interactions.

4. Calibration

4.1. Unregulated equilibrium

We now turn to the numerical version of the model presented in section 2. The calibration of the unregulated equilibrium (hereafter, the *free market* equilibrium) seeks to create a city that, despite the model's abstract nature, resembles reality as closely as possible, in line with Tikoudis et al. (2013). We set the parameters such that equilibrium characterizes a quite sprawled (40 kilometers), congested metropolitan area in absence of government intervention.¹² The distance between the CBD and the city fringe is then covered in 47 minutes. The average speed ranges between 15 and 50 kilometers per hour, depending on the location from which the commuter departs. The average speed of the median household is approximately 37 km per hour, which is roughly consistent with the average commuting speed reported for large US cities in the national household travel survey. The speed near the CBD is one-fifth of the speed in the city fringe. The floor-to-area ratio ranges between 25.0 (CBD) and 0.2 floors (fringe). Housing prices in the CBD are roughly 3.5 times those in the fringe. This price variation is accompanied by a respective variation in the household size (2.5 times larger in the fringe) and a steeper variation in land prices and structural density.

Households work between 288 (CBD) and 233 (fringe) days per year.¹³ The (endogenous) annual income of the representative household is €43000, almost all of which (92%) is earned by labor. This income is spent on consumption (57%), housing (39%) and transport (4%). The maximum marginal external congestion cost, i.e. as generated by the most remote household, is roughly €4000, approximately €17 per working day and slightly above 9% of the mean income. The solid lines in Figure 1 show the spatial patterns of some of the key endogenous variables of interest.

4.2. Pigouvian toll

The dashed lines in Figure 1 indicate how the key variables of interest are affected (in the long-run) when a road charge equal to the marginal external cost of congestion, i.e. the scheme in (23), is imposed throughout the metropolitan area in the otherwise optimal situation where no constraints apply.

Clearly, floor-to-area ratio adjusts upwards in locations fairly close to the CBD and downwards in locations further away. Accompanied by a similar adjustment in prices, this reflects changes in relative accessibility occurring after the introduction of the policy. The small elasticity of labor supply with respect to pecuniary commuting costs underlies negligible adjustments in labor supply. In contrast, with adjustments through the behavioral margin of relocation, significant changes take place in commuting times and speeds. The reduction in the total external costs in the model is associated with a welfare gain; the compensating variation from the Pigouvian toll is approximately €66 per household, annually. The next section investigates the performance of the Pigouvian toll and other policies in a city where the floor-to-area ratio is regulated.

¹² The free market equilibrium parameter vector values are: $\alpha = 7.0$, $\beta = 0.11$, $\gamma = 0.9$, $\rho = 0.2$, $t_0 = 0.02$, $t_1 = 0.12$, $g = 0.01$, $N = 1$, $m = 417.5$, $r_A = 10.0$, $p_K = 1.0$, $\delta = 0.8$, $\zeta = 0.2$.

¹³ This is 0.79 of a total time endowment, which is normalized to one.

5. Policy analysis

5.1. Pigouvian toll in cities with different maximum FAR

A natural question arising in the context of this paper is to what extent welfare gains of the Pigouvian toll vary across different levels of preexisting building height regulations. The left panel of Figure 2 illustrates the population density in a series of cities emerging from the same parameters used in the benchmark equilibrium of section 4.1, but under various maximum floor-to-area ratios imposed by the planning authorities.¹⁴ The associated values of the *BHR boundary* (\hat{z}) and sprawl (\bar{z}), before (\hat{z}_0 and \bar{z}_0) and after (\hat{z}_1 and \bar{z}_1) the introduction of Pigouvian toll are given in Table 1. The values of column \bar{z}_0 confirm that the total sprawl increases as the *FAR* restriction becomes more severe.

The total welfare gain of the Pigouvian toll in this series of cities can be conceptually decomposed into two separate effects. The first regards the *direct benefits* realized on the transport market. When labor supply responds mildly to commuting distance, one may expect the total number of vehicle kilometers to increase with urban sprawl. Cities with a lower maximum FAR produce more vehicle kilometers, longer commuting times and larger total external costs, as suggested by the upper right and lower right panels of Figure 2. Thus, the Pigouvian toll in these cities will produce a larger direct benefit (on the road).

Table 1. Relative efficiency and city size adjustments from a Pigouvian toll in cities with different FAR regulations.

Road toll scheme	Relative efficiency	\bar{z}_0	\bar{z}_1	\hat{z}_0	\hat{z}_1
<i>Maximum FAR allowed</i>					
<i>Free*</i>	1.000	40.0	37.8	-	-
20.0	0.741	40.1	38.3	0.95	1.7
15.0	0.587	40.8	39.3	2.8	3.3
10.0	0.640	42.9	41.7	6.6	6.8
6.00	0.884	47.6	46.4	14.0	13.5
4.00	1.060	53.0	51.8	22.3	21.0

* The resulting floor-to-area ratio in the CBD of the free market equilibrium is approximately 24.5.

Notes: \bar{z} and \hat{z} denote total sprawl and BHR boundary respectively, before (\bar{z}_0, \hat{z}_0) and after (\bar{z}_1, \hat{z}_1) the imposition of a Pigouvian toll. The welfare gains (compensating variations) of a Pigouvian toll in a city with an arbitrary maximum building height are expressed relative to the respective gains in the city without building height restrictions.

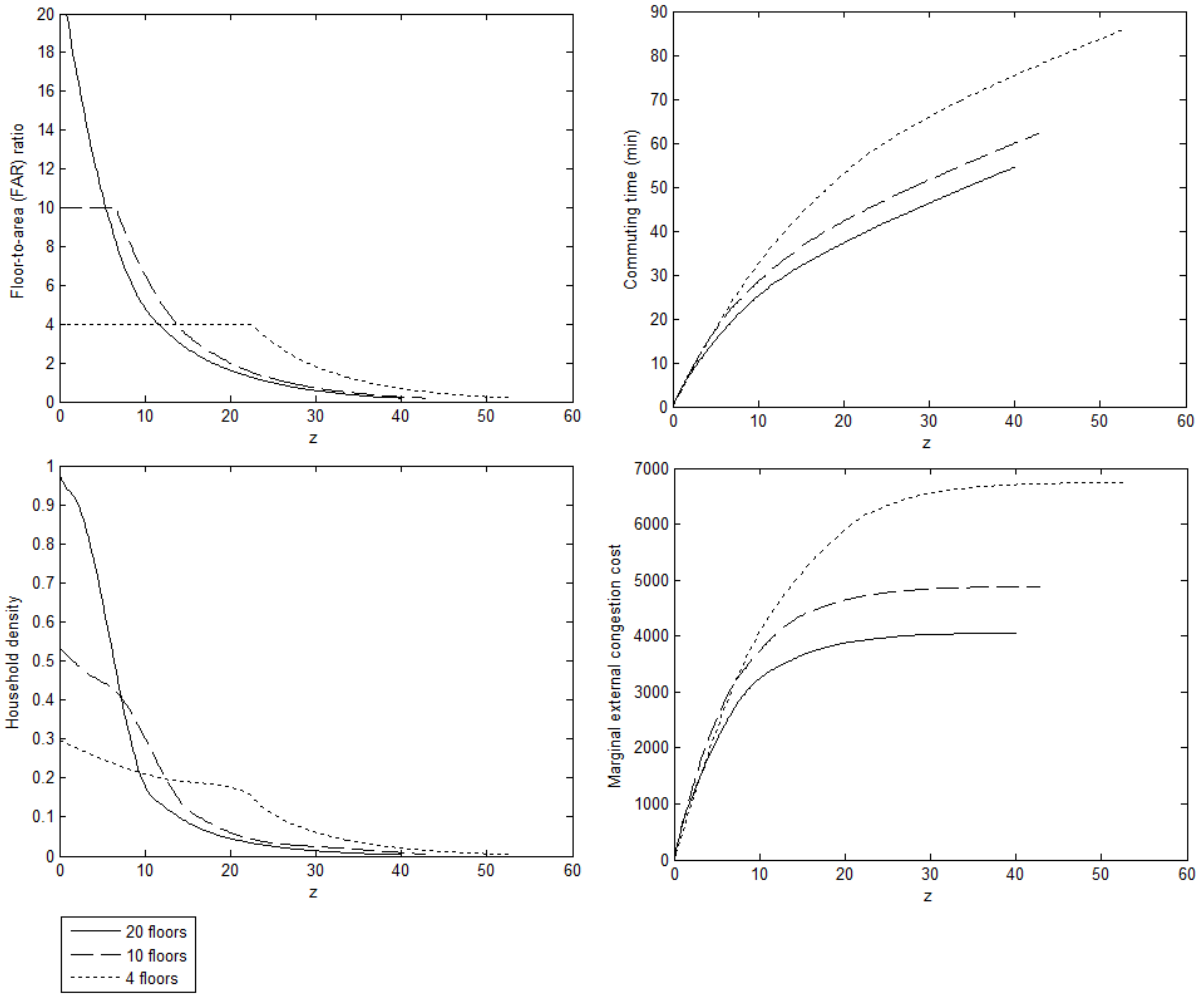
The second effect regards *indirect benefits or losses* realized in the housing market. In particular, the road toll scheme may increase the effective demand for floor space in locations where the maximum *FAR* restriction is already binding, and in locations where *FAR* is close to its maximum level imposed by the regulator.¹⁵ Thus, Pigouvian toll may expand the *BHR boundary* of the city and cause welfare losses in the housing market.

¹⁴ The maximum FAR values are chosen arbitrarily.

¹⁵ Note that, in contrast to section 3, here we are not referring to a marginal adjustment of an existing toll scheme, but to the introduction of a toll scheme equal to the marginal external cost of congestion.

Table 1 suggests that this may be the case for cities with a mild *FAR* restriction, lying not far below the respective *FAR* of the free-market equilibrium. The rationale behind this counter-intuitive result is that the population residing in the close neighborhood of *BHR boundary*, i.e. the population density contained on an interval $(\hat{z}_0, \hat{z}_0 + \Delta\hat{z}_0)$, is larger in these cities (see lower left panel of Figure 2). Subsequently, a larger such population (prior to the Pigouvian toll) will generate a larger expansion of the *BHR boundary* (after the introduction of the Pigouvian toll). This expansion is given by the difference $\hat{z}_1 - \hat{z}_0$ in Table 1, and becomes smaller as the *BHR* regulation becomes more stringent and the pre-toll *BHR boundary*, \hat{z}_0 , moves further away from the CBD.

Figure 2. Floor-to-area ratio (upper left panel), commuting times (upper right panel), household densities (lower left panel) and marginal external congestion costs (lower right panel) in the *BHR* equilibria without road pricing.



The results in the same table suggest that, in cities where FAR is pressed far below its free market equilibrium level (in this numerical example this is, roughly, when maximum FAR falls below 8.00), a Pigouvian toll can even contract the *BHR* boundary. Again, this result can be explained by the (relatively smaller) population density on the associated interval $(z_0, z_0 + \Delta z_0)$ and the location of z_0 itself (larger values are associated with larger commuting costs and lower rents for households residing outside the area where FAR limit is binding).

Combining the two effects yields the total welfare gain induced by a Pigouvian toll in a city with a maximum FAR . Table 1 (second column) reports this gain relative to the welfare gain in a city without building height restrictions.¹⁶ This relative gain turns out to be non-monotonic across cities with different maximum FAR . It is the outcome of two functions: a road market benefit function (increasing in regulation stringency) and a housing market loss function (decreasing in regulation stringency). It is also volatile enough to fall below 60% (that is, 59% in the case of fifteen floors).

The above findings have significant policy implications. They indicate that the same policy rule in a series of seemingly identical cities (in terms of road technology, preferences, structure, size, and population) can generate rather diverse welfare effects. Improvised extrapolations of benefits across such cities may lose the point, and thus should not constitute the basis for decision making if not accompanied by elaborate cost-benefit analyses. The latter, as already shown in section 3, do not require background information on building height restrictions, as long as traffic forecasts and in particular marginal external cost estimates are consistent with these restrictions.

4.3. Pigouvian toll in a city with zoning

Similar to a maximum FAR , zoning constitutes a quantity distortion. In fact, it can be seen as a special case of a building height restriction, in which FAR is forced to be zero in a specified area. For this reason, the Pigouvian toll is optimal, as it is the case with a uniform maximum FAR . In contrast to a uniform maximum FAR , however, the extent of the distortion in the housing market remains intact after the introduction of a Pigouvian toll. As a result, road externalities determine completely the extent of welfare gains in any arbitrary choice of zoning area (z_L, z_U).

Table 2. *Pigouvian toll welfare gains in cities with zoning.*

<i>Extent of zoned area (km)</i>	<i>1.00</i>	<i>2.00</i>	<i>3.00</i>	<i>4.00</i>
<i>Zoning lower bound</i>				
<i>0.40</i>	1.134	1.338	1.597	1.870
<i>4.00</i>	1.021	1.023	0.998	0.954
<i>30.00</i>	0.995	0.992	0.989	0.986

Notes: All welfare gains are expressed in terms of compensating variations relative to the case without zoning.

When the lower bound, z_L , is placed close to the CBD, expanding the zoned area forces households to relocate further away. With mild adjustments in labor supply, total external costs of congestion rise. Then, Pigouvian toll produces gradually larger welfare gains as the extent of the vacant area increases (first row of Table 2). The opposite is true when the bound is placed relatively close to city fringe. In this case, extension of zoning causes relocation closer to CBD, subsequently decreasing the welfare gains from a Pigouvian toll (third row). Apart from having opposite signs, the above effects also differ in magnitude due to the initial distribution of population over space, which is far from uniform.

¹⁶ For instance, letting CV_s denote the compensating variation of the Pigouvian toll in a city where maximum FAR is restricted to be equal to s (both before and after the introduction of the policy), and CV_{free} the associated compensating variation in a city without a maximum FAR (either before or after the introduction of the policy), the relative efficiency is $RE_s = CV_s/CV_{free}$.

Therefore, the welfare gains of a Pigouvian toll produced under policy extrapolations similar to those discussed in section 5.1. are much more likely to be underestimated rather than the opposite.¹⁷ Between the two polar cases, gradual expansion of the upper zoning bound, z_U , produces non-monotonic results; that is, the direction of household relocation becomes ambiguous.

4.4. Optimal toll in the presence of ad valorem property taxation

We now investigate the role a pre-existing tax induced distortion, by considering an ad-valorem tax (value tax) on housing property, paid by the consumer. We consider a wide range (0-8%) in order to capture the various levels of property taxation observed across Europe and North America. Because unpriced congestion is present in the benchmark city, this tax (at low levels) may be welfare improving: it generally reduces dwelling size, resulting in a more compact city where fewer vehicle kilometers are produced, thus replicating one of the behavioral reactions that also an optimal road price would bring. At the same time, the equilibrium tax per square meter is higher closer to CBD; this generates a *push-out effect* which increases urban sprawl. Therefore, the total impact of the ad-valorem tax on the size of the city is ambiguous.

Table 3. Relative efficiency gains from the Pigouvian and optimal toll in cities with a different ad-valorem tax on housing property.

Road toll scheme	Pigouvian toll	Optimal tax interpolant	% change in welfare gains	% change in city size
<i>Value tax rate</i>				
0%	1.0	1.0	0%	0%
4%	1.021	1.028	0.7%	-0.87%
6%	1.030	1.059	2.8%	-1.05%
8%	1.038	1.095	5.2%	-1.30%

Notes: The welfare gains (compensating variations) of the Pigouvian and optimal toll in a city with an arbitrary ad-valorem tax rate (second and third column) are expressed relative to the respective gains in the city without property taxation. Therefore, a value above 1 does not signify a welfare level that exceeds that in the first-best, but merely a welfare gain from road pricing that exceeds the gain in absence of the housing tax. The percentage changes in relative welfare gains and city size refer to the transition from the Pigouvian to the *optimal tax interpolant* equilibrium.

The imposition of this policy generates a clear *tax interaction*: a location-specific tax for the use of the road coexists with a location-specific tax on property. However, this interaction is relatively weak compared to other interactions investigated in relevant literature: for instance, a labor tax and a road toll might be perfect substitutes (from the viewpoint of policymaker) in settings where labor supply is inelastic in the intensive margin (fixed duration of working day, such as in Parry and Bento, 2001 or Verhoef, 2005) and there is no substitute transport mode for car.

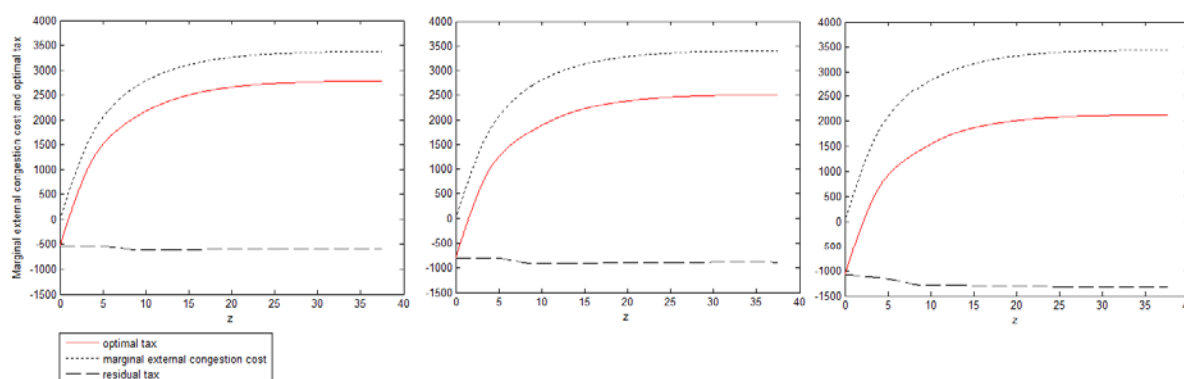
As shown in the second column of Table 3, the welfare gains from a Pigouvian toll are generally stable across different levels of the ad-valorem tax. The third (fourth) column displays the gains

¹⁷ Note that the welfare effects of zoning per se are negative. That is, equilibria with a larger zoning area, $x_U - x_L$, are associated with lower utility, despite Pigouvian toll might produce larger welfare gains.

(percentage difference in welfare gains) from an alternative road pricing scheme based on the optimal tax interpolant, discussed in Appendix B. This is the numerically established tax schedule that maximizes social welfare. Despite the (numerical) difficulty to differentiate between the Pigouvian and the optimal tax at low levels of property taxation, the fourth column shows that the former ceases to be optimal even at moderate levels of an ad-valorem tax. Intuitively, at higher levels of the tax, the difference between the optimal and Pigouvian tax increases, reflecting the increased marginal excess burden.

As Figure 3 shows, the optimal road tax lies below the Pigouvian level. This implies that the interaction between the road toll and the property tax is negative. That is, the Pigouvian toll erodes the base of the property value-tax by reducing apartment sizes, city size and thus total floor space stock of the city. Note that, despite the fact that interpolation yields a strictly monotonous, concave price scheme, the total toll is negative in a considerable space interval, in all three cases. In fact, forcing the tax to be non-negative in the entire interval $(0, \bar{z})$ exhausts almost all of the additional gains.

Figure 3. Optimal tax (solid) versus marginal external cost of congestion (short dash) and their difference (dash) in three different levels of an ad-valorem tax: 4% (left panel), 6% (middle) and 8% (right).



This implies that, to a large extent, these gains stem from household relocation closer to CBD, revealing that the non-congestion benefits from a downward adjustment of the toll close to CBD (and therefore attracting households close to CBD) outweigh the losses from non-optimal pricing of the congestion externality. The optimal tax interpolant provides a general downward adjustment and local corrections which account for the inefficiency of the property tax. These adjustments, which may not be feasible without the use of an additional policy instrument, can provide considerable improvements in terms of welfare gains.

6. Concluding remarks

This paper has investigated road pricing policies in the presence of various types of housing market regulations. The topic is highly relevant because the existing regulations in the land and housing markets, especially command and control regulations (such as maximum building heights and urban growth boundaries), might be incomparably more difficult to adjust than tolls in a road pricing scheme.

A general result, with relevance beyond the case of road pricing with distorted land and housing markets studied here, is that given a quantity restriction in a related market the Pigouvian toll rule remains optimal. An intuitive explanation is that the effect of a toll adjustment at the margin of the Pigouvian equilibrium will not affect the quantity produced in the distorted market, because that quantity is fixed.

The optimality of the Pigouvian toll in such cases suggests that decision making regarding urban road pricing can ignore quantitative restrictions in the parallel markets of land, housing and labor. That is, such restrictions will not affect the optimal road toll *rule*, although the equilibrium *levels* will generally be affected. This is a useful result for policymakers, because the socially-optimal road tax scheme can then be computed exclusively with data from the road market. However, the Pigouvian toll ceases to be optimal when a tax-induced housing market distortion is present.

Our numerical application examined the presence of maximum floor-to-area ratio and zoning. We demonstrated that the welfare gain from a Pigouvian toll is a non-monotonic function of the building height regulation stringency. This non-monotonicity is the consequence of two major, but opposing, effects that a Pigouvian toll generates. The first regards the welfare gains on the road, which increase as the maximum building height is reduced, because travel distances and hence congestion costs increase within the city. The second regards the losses in housing market caused by a road toll, which are more severe at milder levels of the *FAR* restriction. With zoning, the second result is absent: the efficiency of land use (in the areas where zoning does not apply) does not depend on road pricing. The welfare effects of road pricing are then entirely determined on the road.

Our findings have significant policy implications. Our computations suggest that the Pigouvian toll imposed in a city with a mild, uniform over space, floor-to-area ratio restriction may produce up to 40% smaller welfare gains relative to the gains of a Pigouvian toll in a city without height restrictions. On the other hand, welfare gains might be 80% larger in a city with an extensive zoned area close to the CBD. Therefore, welfare gains of road pricing across cities with different housing market regulations may differ wildly from each other.

Appendices

A. Notation

Table A. Basic model variables and parameters

z	distance from CBD	p	price of floor space
\bar{z}	city size (sprawl)	p_L	price of land
\hat{z}	BHR boundary	p_K	exogenous price of capital
\hat{k}	structural density	r_A	opportunity cost of land
\hat{s}	floor-to-area (FAR) ratio	R	aggregate differential rents
M	disposable income	R_a	total opportunity cost of land
m	pecuniary commuting cost (per km)	R_L	aggregate rents paid
τ_z	total road tax from z to CBD	G	toll revenue
B	lump-sum income	n	population density at distance z
T_F	leisure time	K	total capital costs
t	commuting time	N	exogenous population
s	apartment size	\bar{h}	policy imposed FAR ceiling
y	composite good (numéraire)	Q	tax revenue

B. Computational details

Tax interpolation and direction set optimization

We have divided the road intervals $(0, \hat{z})$ and (\hat{z}, \bar{z}) in two and (respectively) three segments of equal length. The points that define these segments are given by the vector:

$$\boldsymbol{\delta}' = [0, 0.33 \hat{z}, 0.66 \hat{z}, \hat{z}, \hat{z} + 0.33 (\bar{z} - \hat{z}), \hat{z} + 0.66 (\bar{z} - \hat{z}), \bar{z}], \quad (\text{B1})$$

when FAR regulations are present and by the vector:

$$\boldsymbol{\delta}' = [0, 0.25 \bar{z}, 0.50 \bar{z}, 0.75 \bar{z}, \bar{z}], \quad (\text{B2})$$

when not. Define any pricing scheme in the paper as a vector \mathbf{c}' of equal size as $\boldsymbol{\delta}'$, and let c'_i denote the total price charged to the household located at distance δ'_i . Under piecewise linear interpolation, for any $\delta'_i < z < \delta'_{i+1}$, the charge is given by:

$$c_z = c'_i + \frac{z - \delta'_i}{\delta'_{i+1} - \delta'_i} (c'_{i+1} - c'_i). \quad (\text{B3})$$

Under (non-linear) Lagrange interpolation, we define the Vandermonde matrix:

$$\mathbf{V} = \begin{bmatrix} 1 & \delta'_0 & \delta_0'^2 & \delta_0'^3 & \delta_0'^4 \\ 1 & \delta'_1 & \delta_1'^2 & \delta_1'^3 & \delta_1'^4 \\ 1 & \delta'_2 & \delta_2'^2 & \delta_2'^3 & \delta_2'^4 \\ 1 & \delta'_3 & \delta_3'^2 & \delta_3'^3 & \delta_3'^4 \\ 1 & \delta'_4 & \delta_4'^2 & \delta_4'^3 & \delta_4'^4 \end{bmatrix} \quad (\text{B4})$$

and solve $\mathbf{V}\mathbf{a} = \mathbf{c}$ for the coefficient vector \mathbf{a} which is used to construct the interpolating polynomial:

$$c_z = \mathbf{a}'_0 z^0 + \mathbf{a}'_1 z^1 + \mathbf{a}'_2 z^2 + \mathbf{a}'_3 z^3 + \mathbf{a}'_4 z^4. \quad (\text{B5})$$

Throughout the paper, the expressions in (B3) and (B5) are called *tax interpolants*. Any optimization technique employed attempts to approximate the vector \mathbf{c}' that belongs to policy space and maximizes (10) subject to the equilibrium conditions.

C. Numerical illustration of Pigouvian optimality

To illustrate the optimality of the Pigouvian toll, we perform a series of numerical computations based on the idea that the toll function in (23) can be approximated with the use of a piecewise linear interpolant (see Appendix B). Then, the optimal tax at the selected points used in the interpolation (distances from CBD) can be computed with standard numerical optimization techniques. We use starting values that account for 70% of the Pigouvian toll in the interpolation points. Various different levels of initial perturbations have been tried and resulted in slightly different relative efficiencies; however, the general result remains intact: in none of the cases has the relative efficiency been observed to fall below 1.0. Table C displays the efficiency of the Pigouvian toll relative to the optimized tax interpolant.

To establish that the result does not depend on the assumed elasticity of substitution, the computations are repeated for two different cities in which σ is lower (higher). In each case, the model

parameters are recalibrated in order for the base equilibrium to loosely resemble the unregulated equilibrium of section 4.1. The results are displayed in the second (fourth) column of Table C.

Table C. Relative efficiency of the Pigouvian toll (benchmark: optimal tax interpolant).

<i>Elasticity of substitution (σ)</i>	<i>0.40</i>	<i>1.25</i>	<i>1.54</i>
<i>Maximum FAR allowed</i>			
<i>15.0</i>	1.014	1.035	1.025
<i>10.0</i>	1.009	1.011	1.010
<i>6.00</i>	1.006	1.008	1.002

Notes: all results in the table produced with linear piecewise interpolation and Newton-Raphson with line search.

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