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Abstract

This paper features an analysis of the effectiveness of a range of portfolio diversification strategies as applied to a set of 17 years of monthly hedge fund index returns on a set of ten market indices representing 13 major hedge fund categories, as compiled by the EDHEC Risk Institute. The 17-year period runs from the beginning of 1997 to the end of August 2014. The sample period, which incorporates both the Global Financial Crisis (GFC) and subsequent European Debt Crisis (EDC), is a challenging one for the application of diversification and portfolio investment strategies. The analysis features an examination of the diversification benefits of hedge fund investments through successive crisis periods. The connectedness of the Hedge Fund Indices is explored via application of the Diebold and Yilmaz (2009, 2014) spillover index. We conduct a series of portfolio optimisation analyses: comparing Markowitz with naive diversification, and evaluate the relative effectiveness of Markowitz portfolio optimisation with various draw-down strategies, using a series of backtests. Our results suggest that Markowitz optimisation matches the characteristics of these hedge fund indices quite well.

Keywords: Hedge Fund Diversification, Spillover Index, Markowitz Analaysis, Downside Risk, CVaR, Draw-Down.
JEL Codes: G11, C61.

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1. Introduction

It is now over 65 years since Alfred Winslow Jones founded the first hedge fund in 1949, and some 60 plus years since Markowitz (1952) developed portfolio theory. This 'alternative investment' vehicle has been the focus of a lot of attention by investors, regulators, and politicians. There have been some incidents that have caught the world's attention, such as the decision by George Soros' Quantum Fund to sell sterling short in the fall of 1992, which is believed to have brought pressure on the currency and hastened its departure from the ERM. In 1997 hedge funds again attracted adverse publicity when the then Prime Minister of Malaysia protested that sales of Asian currencies by hedge funds led to depreciation of the ringitt. More recently there was the collapse of Long Term Capital Management in 2000, and also the multi-billion dollar profits of Paulson & Company during the recent financial crisis.

There has been a spectacular growth in the size of funds under management in this sector. At the time of writing, BarclayHedge reported a total of 6314 reporting funds in their database. Our focus in this paper, however, is not on the performance of individual hedge funds, but on the relative performance of different sectors in the alternative investment universe, as represented by the EDHEC series of indices.

The development by Markowitz (1952) of portfolio theory led to the foundation of classical finance, leading directly to the development of the Capital Asset Pricing Model (CAPM) by Sharpe (1964), Lintner (1965), Mossin (1966) and Treynor (1962). Markowitz (1952, 1959) suggested choosing the portfolio with the lowest risk for a given level of portfolio return and defined such portfolios as being 'efficient'. Merton (1972) demonstrated the parabola that constitutes the efficient frontier in the mean-variance space.

Despite the theoretical elegance and appeal of Markowitz portfolio theory, its practical application has been less successful. Michaud (1989, p. 33) observed that: 'MV optimizers are, in a fundamental sense, "estimation-error maximizers". They have a tendency to over-weight (under-weight) those securities which have large (small) estimated returns, negative (positive) correlations and small (large) variances.'

Various adjustment approaches for tackling estimation risk have been suggested in the literature. Bayesian techniques have featured prominently, and early recommendations were based on the use of diffuse priors; see for example, Barry (1974), and Bawa et al. (1979), or 'shrinkage' estimators, see, for example, Jobson et al. (1979), Jobson and Korkie (1980) and Jorion (1985, 1986) for examples of these approaches. More recently, Pástor (2000) and Pástor and Stambough (2000) have used an asset pricing approach to tackle the same issue.

The early development of portfolio theory was by no means dominated by mean/variance analysis. Markowitz considered a number of downside risk measures as alternatives (1959, 1991) and Roy (1952) developed his 'safety-first' asset selection criteria. Rockafellar et al. (2006a, 2006b, 2007) developed the mean-deviation approach to portfolio selection, providing an extension to the classic mean-variance approach. Rockafellar et al. extended the results to the one fund theorem, (2006a), CAPM (2006b), and provided a derivation of market equilibrium using different deviation measures (2007). Subsequently, Zabarankin et al. (2014) used a draw-down measure to measure betas and alphas based on draw-downs in a CAPM framework.

In this paper our concern is with how well alternative investments as a class performed during the Global Financial Crisis (GFC) and through the subsequent turmoil in Europe, which constituted the European Debt Crisis (EDC). We are concerned with both spillovers and correlations of risk across the sector, as well as the risk diversification properties of alternative investments. Diebold and Yilmaz (2009, 2012) have developed a Spillover Index model which provides precise and separate measures of return spillovers and volatility spillovers. They adopt vector autoregressive (VAR) models in their measurement of return and volatility spillovers, in the broad tradition of Engle et al. (1990). They use variance decompositions to aggregate spillover effects across markets, which permits the concentration of a great deal of information into a single spillover measure.

We use this metric to analyse return spillovers across the various hedge fund sector indices, and then proceed to a portfolio analysis of the diversification properties of the sector using a variety of methods which include: Markowitz mean variance analysis with positive constraints, Conditional Value at Risk (CVar), Conditional Draw-Down (CDaR), Average Draw-Down (AveDD), Maximum-Draw-Down (MaxDD), plus draw-down metrics set at 95% confidence levels (CDaR95) and (CDaRmin95). The effectiveness of these procedures is assessed in a series of out-of-sample hold-out and backtests.

The paper is organised into five sections. The introduction is followed by a discussion of research methods in section 2 which discusses the spillover index model plus the various portfolio optimisation strategies adopted, beginning with Markowitz mean-variance analysis, CVaR, and a variety of optimal draw-down approaches. Section 3 introduces the data set and its characteristics, while section 4 presents the results. A conclusion follows in section 5.

2. Research method

2.1. Spillover Index Model

A VAR framework provides the advantage of capturing a great deal of information about the dynamic structure of the relationships between the variables considered in the analysis without prior specifications or assumptions. Diebold and Yilmaz (2009) use this property in developing their Spillover Index and construct their measure by taking each asset i, and adding the shares of its forecast error variance coming from shocks to asset j, for all $j \neq i$, all in the context of an N variable VAR. They sum these error variances across all $i = 1, ..., N$. If we take the case of a covariance stationary, first-order, two variable VAR, we have:

$$x_t = \Phi x_{t-1} + \varepsilon_t,$$

where $x_t = (x_{1t}, x_{2t})$ and Φ is a 2×2 parameter matrix. The error ε_t is s.t. $E(\varepsilon_t \mid F_{t-1}) = 0$. F_{t-1} is the information set at t_{-1}. In the empirical analysis which follows, x is a vector of hedge fund index returns. The VAR can be written in a moving average representation, given the existence of covariance stationarity, as:

$$x_t = \Theta(L)\varepsilon_t,$$

where $\Theta(L) = (1 - \Phi L)^{-1}$. L is the back-shift operator and the roots of $\mid \Theta(L) \mid \neq 0$ for $\mid L \mid \leq 1$. This can then be conveniently written as a matrix with:

$$x_t = A(L)u_t,$$

where $A(L) = \Theta(L)Q_t^{-1}$, $u_t = Q_t\varepsilon_t$,$E(u_t u_t^{'}) = I$, and Q_t^{-1} is the unique lower-triangular Choleski factor of the covariance matrix of ε_t.

Diebold and Yilmaz (2009) then proceed to consider the optimal 1 step ahead forecast, given as:

$$x_{t+1,t} = \Phi x_t,$$

with the corresponding one-step ahead error vector:

$$e_{t+1,t} = x_{t+1} - x_{t+1,t} = A_0 u_{t+1} = \begin{bmatrix} a_{0,11} & a_{0,12} \\ a_{0,21} & a_{0,22} \end{bmatrix} \begin{bmatrix} u_{1,t+1} \\ u_{2,t+1} \end{bmatrix},$$

which has the covariance matrix given by:

$$E\left(e_{t+1,t} e_{t+1,t}^{'}\right) = A_0 A_0^{'}.$$

This suggests that the variance of a one-step ahead error in forecasting $x_{1,t}$ is $a_{0,11}^2 + a_{0,12}^2$ and the variance of the one-step ahead error in forecasting $x_{2,t}$ is $a_{0,21}^2 + a_{0,22}^2$. Diebold and Yilmaz (2009) split the forecast error variances of each variable into components attributable to the various system shocks. This means it is possible to distinguish between shocks to the variable itself x_i and shocks to the other variable x_j, for $i,\ j = 1, 2,\ i \neq j$.

Their spillover index in the two variable case is constructed as;

$$S = \frac{a_{0,12}^2 + a_{2,1}^2}{trace(A_0 A_0^{'})} \times 100. \tag{1}$$

They generalise the measure to take into account multiple securities and multiple periods as shown below:

$$S = \frac{\sum_{h=0}^{H-1} \sum_{\substack{i,j=1 \\ i \neq j}}^{N} a_{h,ij}^2}{\sum_{h=0}^{H-1} trace(A_h A_h^{'})}. \tag{2}$$

Diebold and Yilmaz (2012) extend their (2009) Spillover Index into a *generalized* form which eliminates the possible impact of ordering on the results. They develop their model by considering a covariance stationary N-variable $VAR(p)$, $x_t =$

$\sum_{i=1}^{p} \Phi_i x_{t-1} + \varepsilon_t$, where $\varepsilon \sim (0, \sum)$ is a vector of i.i.d. disturbances. They suggest a moving average representation can be written as:

$$x_t = \sum_{i=0}^{\infty} A_i \varepsilon_{t-i},$$

where the $N \times N$ coefficient matrices A_t obey the recursion, $A_t = \Phi_1 A_{t-1} + \Phi_2 A_{t-2} + ... + \Phi_p A_{t-p}$, with A_0 an $N \times N$ identity matrix and $A_t = 0$ for $i < 0$. Diebold and Yilmaz (2009, 2012) use the fact that the moving average coefficients (or transformations in the form of impulse responses or variance decompositions, provide a key to the understanding of the dynamics of the system. They separate the forecast error variances from the variance decompositions into the parts attributable to the various system shocks. These variance decompositions enable them to to assess the fraction of H-step-ahead error variance in forecasting x_t that is due to shocks to x_j, $\forall \neq i$, for each i.

Diebold and Yilmaz (2012) construct their revised Spillover Index in a way that avoids the restriction that the use of the Cholesky factorization produces variance decompositions that are dependent on the ordering of the variables. They adopt the generalised VAR framework of Koop, Pesaran and Potter (1996) and Pesaran and Shin (1998). This provides variance decompositions that are invariant to variable ordering.

They proceed by defining *own variance shares* to be the fraction of the *H-step-ahead* error variances in forecasting x_t due to shocks to x_j, for i, $j = 1, 2, ..., N$, such that $i \neq j$.

They define their *H-step-ahead* forecast error variance decompositions by $\theta^g_{ij}(H)$. For $H = 1, 2, ...$,they have

$$\theta^g_{ij}(H) = \frac{\sigma_{ij}^{-1} \sum_{h=0}^{H-1} (e_i^{'} A_h \sum e_j)^2}{\sum_{h=0}^{H-1} (e_i^{'} A_h \sum A_h^{'} e_i)} \tag{3}$$

where $\sum$ is the variance matrix for the error vector ε, , σ_{ij} is the standard deviation of the error term of the ith equation, while e_i is the selection vector with one as the *ith* element and zero otherwise. They note that the sum of the elements of each row in the variance decomposition table is not equal to 1, $\sum_{j=1}^{N} \theta^g_{ij}(H) \neq 1$.

Diebold and Yilmaz (2012) normalize each entry of the variance decomposition matrix by the row sum as:

$$\tilde{\theta}^s_{ij}(H) = \frac{\theta^s_{ij}(H)}{\sum_{j=1}^{N} \theta^g_{ij}(H)}. \tag{4}$$

They construct their total volatility spillover index as:

$$S^g(H) = \frac{\sum\limits_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}^g_{ij}(H)}{\sum\limits_{i,j=1}^{N} \tilde{\theta}^g_{ij}(H)} \bullet 100 = \frac{\sum\limits_{\substack{i,j=1 \\ i \neq j}}^{N} \tilde{\theta}^g_{ij}(H)}{N} \bullet 100. \quad (5)$$

We use the Diebold and Yilmaz (2012) version of the spillover index to analyse the spillover of return shocks across the 13 EDHEC hedge fund indices for the various categories of hedge funds. Allen et al. (2014) use this in a parallel study of return and volatility spillovers from Australia's major trading partners. Diebold and Yilmaz (2014 have recently further generalised their connectedness measure.

2.2. Naive diversification

In this strategy we consider holding a portfolio where the weights for the asset, $\omega_j = 1/N$, is applied for each of the N risky assets. This strategy ignores the data and does not involve any estimation or optimisation. DeMiguel et al. (2009) suggest that this can be considered as equivalent to imposing the restriction that $\mu_t \propto \sum_t 1_N$ for all t, implying that expected returns are proportional to total risk rather than systematic risk.

2.3. Markowitz Mean-Variance Analysis

The Markowitz (1952) approach can be presented as the following non-linear programming problem:

$$\begin{gathered} \underset{\omega}{min} \; \frac{1}{n} \sum_{i=1}^{n} \left(\sum_{j=1}^{m} \omega_j (r_{i,j} - \mu_j \right)^2 \\ s.t. \\ \sum_{j=1}^{m} \omega_j \mu_j = C \\ \sum_{j=1}^{m} \omega_j = 1 \\ \omega_j \geqslant 0, \forall j \in \{1, ..., m\}. \end{gathered} \quad (6)$$

In the above formulation, ω are the portfolio weights for the universe of the $j = 1, ...m$ assets available, $i = 1, ..., n$ are the number of periods considered for the returns r and for μ_j, which is the forecast return. The optimisation involves minimizing the portfolio variance subject to the portfolio forecast return being set to a level C. A full investment constraint and positive constraints on the weights are included, effectively ruling out short sales.

Jagganathan and Ma (2003) demonstrate that the placement of a short-sale constraint on the minimum variance portfolio is equivalent to shrinking the elements of the covariance matrix. For this reason, we do not make any other adjustments for estimation risk, (see for example, the discussions in Best and Grauer (1992), Chan, Karceski and Lakonishok (1999), and Ledoit and Wolf (2004)).

2.4. Optimising Conditional Value at Risk (CVaR)

In a series of papers, Uryasev and Rockafellar (1999) have advocated CVaR as a useful risk metric. Pflug (2000) proved that CVaR is a coherent risk measure with a number of attractive properties, such as convexity and monotonicity, among other desirable characteristics. A number of papers apply CVaR to portfolio optimization problems, (see, for example, Rockafeller and Uryasev (2002, 2000), Andersson et al. (2000), Alexander, Coleman and Li (2003), Alexander and Baptista (2003) and Rockafellar et al. (2006)).

The conditional value at risk of X at level $\alpha \in (0,1)$ is defined by:

$$CVaR_\alpha(X) = expectation\, of\, X\, in\, its\, \alpha - tail, \tag{7}$$

which can also be expressed as:

$$CVaR_\alpha(X) = \frac{1}{1-\alpha} \int {}_\alpha^1 VaR_\tau(X)dt. \tag{8}$$

In terms of portfolio selection, CVaR can be represented as a non-linear programming minimisation problem, with an objective function given as:

$$\begin{matrix} min \\ \omega, \upsilon \end{matrix} \; \frac{1}{na} \sum_{i=1}^{n} \left[max(0, \upsilon - \sum_{j=1}^{m} \omega_j r_{i,j} \right] - \upsilon \tag{9}$$

where υ is the $\alpha-$quantile of the distribution. In the discrete case, this was shown by Rockfellar and Uryasev (2000) to be capable of being represented by using auxiliary variables in the linear programming formulation below:

$$\begin{matrix} min \\ \omega, d, \upsilon \end{matrix} \; \frac{1}{na} \sum_{i=1}^{n} d_i + \upsilon$$

$$s.t.$$

$$\sum_{j=1}^{m} \omega_j r_{i,j} + \upsilon \geq -d_i, \forall \in \{1, ..., n\}$$

$$\sum_{j=1}^{m} \omega_j \mu_{j=C} \tag{10}$$

$$\sum_{j=1}^{m} \omega_j = 1$$

$$\omega_j \geq 0, \forall j \in \{1, ..., n\}$$

$$d_i \geq 0, \forall i \in \{1, ..., n\}$$

where υ represents the VaR at the α coverage rate and d_i the deviations below the VaR.

2.5. Optimal draw-down portfolios

Chekhlov et al. (2000, 2004, 2005) considered the optimization of portfolios with respect to the portfolio's drawdown. The Conditional Drawdown (CDD) measure includes the Maximum Drawdown (MaxDD) and Average Drawdown (AvDD) as limiting cases. The CDD family of risk functional measures is similar to Conditional Value-at-Risk (CVaR). Chekhlov et al. (2005) suggest that portfolio managers would like to avoid large drawdowns and/or extended drawdowns as it may lead to a loss of mandate or withdrawal of business.

The analysis can be developed as follows. Let a portfolio be optimised over some time interval $[0, T]$, and let $W(t)$ be the portfolio value at some moment in time $t \in [0, T]$. The portfolio drawdown is defined as:

$$max_{\tau \in [0,t]} W(\tau) - W(t)/W(t). \tag{11}$$

If we think in terms of the portfolio's constituent assets and write $W(\omega, t) = y_t^{'} \omega$ as the uncompounded portfolio value at time t, with ω the portfolio weights for the N constituent assets, and write y_t for the cumulated returns, the Draw-down can be written as:

$$D(\omega, t) = \underset{0 \leq \tau \leq t}{max} \{W(\omega, \tau)\} - W(\omega, t). \tag{12}$$

This definition can be converted into the three previously mentioned functional risk measures; MaxDD, AvDD and Conditional Draw-down at Risk (CDaR). CDaR is dependent on the chosen confidence level α in the same way as CVaR. CDaR can be defined as:

$$CDaR(\omega)_\alpha = \underset{\varsigma}{min}\{\varsigma + \frac{1}{(1-\alpha)T} \int_0^T [D(\omega, t) - \varsigma]^+ dt, \tag{13}$$

where ς is the threshold value for drawdowns, so that only $(1-\alpha)T$ observations exceed this value. The limiting cases of this family of risk functions are MaxDD and the AvDD. In the case that $\alpha \to 1$, CDaR approaches the maximum draw-down, $CDaR(\omega)_{\alpha\to 1} = MaxDD(\omega) = max_{0\le t\le T}\{D(\omega,t)dt$. The AvDD results from the case in which $\alpha = 0$. That is $CDaR(\omega)_{\alpha\to 0} = AvDD(\omega) = (1/T\int_0^T D(\omega,t)dt$.

These risk functionals can be used in terms of the optimization of a portfolio's drawdown and implemented as inequality constraints for a fixed share of the wealth at risk.

The goal of maximizing the average annualised portfolio return with respect to limiting the maximum draw-down can be written as:

$$P_{MaxxDD} = \underset{\omega,u}{argmax}\, R(\omega) = \frac{1}{dC} y_T^{'}\omega, \tag{14}$$

$$u_k - y_k^{'}\omega \le v_1 C,$$

$$u_k \ge y_k^{'}\omega,$$

$$u_k \ge u_{k-1},$$

$$u_0 = 0,$$

where u denotes a $(T+1\times 1)$ vector of slack variables in the program formulation, in effect, the maximum portfolio values up to time period k with $1 \le k \le T$.

We include these three approaches to portfolio optimisation, CDaR, MaxDD, and AvDD, in our portfolio analyses. We use programs from the R library to conduct our analysis, in particular the packages fPortfolio, FRAPO and PerformanceAnalytics. We also modify the R code from Pfaff (2013) to undertake the various draw-down optimisations.

3. Data set

We use a sample of the monthly values of thirteen EDHEC Alternative Indexes: Convertible Arbitrage, CTA Global, Distressed Securities, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Funds of Funds, Global Macro, Long / Short Equity, Merger Arbitrage, Relative Value, and Short Selling (see: http:www.edhec-risk.com), from the end of January 1997 until the end of August 2014, providing a total of 212 monthly observations on each sector series.

EDHEC construct the indices using factor analysis to provide one-dimensional summaries of information conveyed by the various competing indexes for a given style, and claim the method captures the the largest fraction of the variance explained. EDHEC suggest that their Alternative Indexes, which are generated as the first component in a factor analysis, have a built-in element of optimality, given there is no other linear combination of competing indexes that implies a lower information loss.

Table 1: Descriptive Statistics of the Edhec alternative indices, monthly arithmetically compounded returns

Alternative Index	Mean	Median	Min	Max	Std. Dev	Skewness	Ex. Kurtosis
Convertible Arbirtage	0.00637123	0.00800000	-0.123700	0.0611000	0.01793080	-2.69038	18.5372
CTAGlobal	0.00501274	0.0029500	-0.0543000	0.0691000	0.0238803	0.178014	-0.117779
Distressed Securities	0.00826604	0.0100000	-0.0836000	0.0504000	0.0176540	-1.53124	5.70168
Emerging Markets	0.00737594	0.0114500	-0.192200	0.123000	0.0349562	-1.23175	5.71631
Equity marketNeutral	0.00522642	0.00580000	-0.0587000	0.0253000	0.00844519	-2.36582	15.5672
Event Driven	0.00742406	0.00940000	-0.0886000	0.0442000	0.0173793	-1.58917	5.65087
Fixed Income Arbitrage	0.00489528	0.00605000	-0.0867000	0.0365000	0.0125808	-3.88509	23.8725
Global Macro	0.00638868	0.00535000	-0.0313000	0.0738000	0.0155871	0.903867	2.28122
Long-Short Equity	0.00719906	0.00980000	-0.0675000	0.0745000	0.0212323	-0.422744	1.22353
Merger-Arbitrage	0.00597217	0.00620000	-0.0544000	0.0272000	0.0101187	-1.47871	6.03397
Relative Value	0.00661509	0.00840000	-0.0692000	0.0392000	0.0122530	-1.98352	9.02728
Short-Selling	-0.000189151	-0.00520000	-0.134000	0.246300	0.0503101	0.726257	2.79397
Funds of Funds	0.00504953	0.00675000	-0.0618000	0.0666000	0.0165642	-0.415499	3.73789

The 17 year sample period we use, which incorporates both the Global Financial Crisis (GFC) and subsequent European Debt Crisis (EDC), is challenging for the application of portfolio investment strategies. The end of month values of these indices are differenced to form arithmetically compounded return series. Graphs of the returns on these indices, for the whole sample period, are shown in Figure 1, together with QQ Plots.

It is clear from the QQ plots, also in Figure 1, that all the index return distributions are non-normal and fat-tailed. Descriptive statistics for the series are provided in Table 1.

The descriptive statistics in Table 1 suggest that the series have the typical characteristics of financial return series in that these hedge fund index return series are skewed, mainly negatively, but the CTAGlobal, Global Macro, and Short-Selling series demonstrate positive skewness. Some of the series demonstrate extreme kurtosis, with values above 5, in the cases of Convertible Arbitrage, Distressed Securities, Emerging Markets, Equity Market Neutral, Event Driven, Fixed Income Arbitrage, Merger-Arbitrage, and Relative Value. This suggests that portfolio analysis based on mean-variance analysis is not likely to match the characteristics of the data sets.

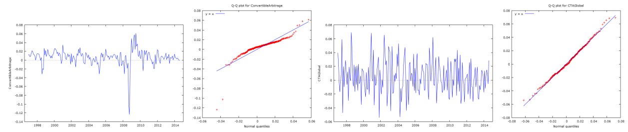
4. Results

4.1. Spillover Index Analysis

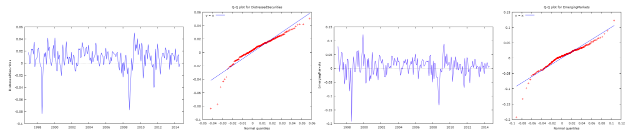
Table 2 presents the results of the Spillover Index analysis using the Diebold and Yilmaz (2012) generalised version of their index, which is invariant to the ordering of variables. The first entry in the first row and column of Table 2 shows the proportion of the forecast error variance of the Convertible Arbitrage index provided by its own shocks, which has a value of 18.8%. The next entry in row 1 of the table shows that the Convertible Arbitrage sector has virtually no impact on the CTA Global index, measured at 0.2%. It has a larger influence on all the other hedge fund sector indices, with the greatest impact of 12.4% on the Relative Value Sector, which makes intuitive sense, given the nature of these two hedge fund sectors. Its total contribution to explanation of the variances of the

Figure 1: Plots of Indices arithmetically compounded monthly returns and QQ Plots

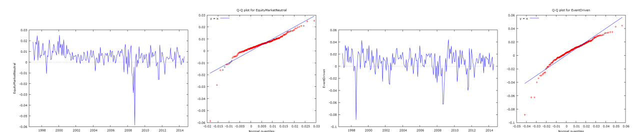
(a) Convertible Arbitrage and CTA Global



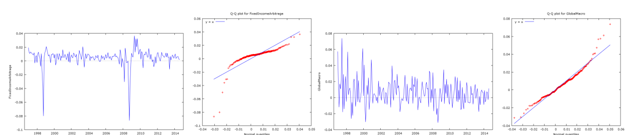
(b) Distressed Securities and Emerging Markets



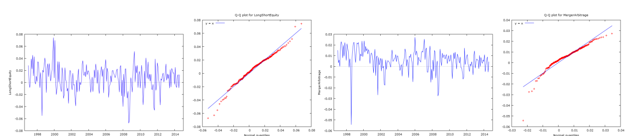
(c) Equity Market Neutral and Event Driven



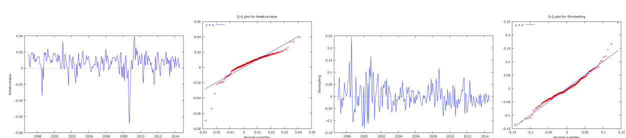
(d) Fixed Income Arbitrage and Global Macro



(e) Long-Short Equity and Merger Arbitrage



(f) Relative Value and Short Selling



(g) Funds of Funds

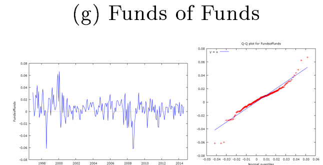


Table 2: Spillover Index variance decomposition of the monthly hedge fund index returns

	Convertible Arbitrage	CTA Global	Distressed Securities	Emerging Markets	Equity Market Neutral	Event Driven	Fixed Income Arbitrage	Global Macro	Long Short Equity	Merger Arbitrage	Relative Value	Short Selling	Fund of Funds	Contribution from others
Convertible Arbitrage	18.8	0.2	9.4	7.4	3.4	9.9	9..7	3.8	7	7.3	12.4	3.1	7.6	81
CTA Global	0.8	58.2	1	14	4.2	1.8	0.5	20.3	2.2	1.5	1.1	1.8	5.3	42
Distressed Securities	8.1	0	12.9	9.2	4.5	12.1	4.8	4.3	9.7	7.7	10.5	6.7	9.5	87
Emerging Markets	5.6	0.1	9.2	14.3	4.4	10.3	4.2	6.8	10.5	6.8	9.2	7.6	11	86
Equity Market Neutral	5.7	1	7.7	7.5	17.5	9.6	2.3	6.8	10.5	7.8	8.7	4.4	10.5	82
Event Driven	7	0.1	10.8	8.9	5.1	12.6	3.7	4.6	10.4	9.7	10.7	6.9	9.7	87
Fixed Income Arbitrage	14.8	0.3	9.3	8.6	2.8	9.5	16.9	3.9	6	6.3	11.6	2.8	7	83
Global Macro	3.3	6.3	6.5	9.8	6.8	7.8	3.7	18.4	9.8	5.1	6.5	3.5	12.5	82
Long Short Equity	4.6	0.2	8.9	9.6	6.3	11	2.6	6.5	13.6	7.3	9.6	8.4	11.3	86
Merger Arbitrage	5.7	0.2	8.8	7.6	5.4	13	2.7	4	9.5	18.1	10.1	6	9	82
Relative Value	9.9	0.1	9.5	8.7	4.6	11.2	6	4.4	9.9	8.2	13.2	5.5	8.7	87
Short Selling	3	0.2	9.3	9.9	3.8	10.7	1.2	3.7	12.7	7	8	20.2	10.2	80
Fund of Funds	5.2	0.7	9	10.2	6.1	10.4	3.4	8.1	11.4	7.3	8.6	6.8	12.8	87
Contribution to others	74	10	99	99	57	117	45	77	110	82	107	63	112	1053
Contribution including own	93	68	112	113	75	130	62	96	123	100	120	84	125	81.00%

errors of the other sectors is recorded at the end of the row, in the last column entry, which shows that it contributed 81% to the forecast error variances of the other sectors.

The diagonal entries in Table 2, showing the influence of each sector index on itself, reveal that by far the most independent of the hedge fund sector indices is the CTA Global sector, which explains 58.2% of its own variance. The next largest entry on the diagonal is Short Selling, which explains 20.2% of its own variance. The smallest entry on the diagonal is 12.8% for Fund of Funds, revealing that this sector explains the least amount of its own variance. This also makes intuitive sense, in that Fund of Funds is a conduit for investment in all the other hedge fund sectors. The penultimate entries at the foot of each column in Table 2 show the contribution of that sector to the other sectors. The Event Driven sector appears to make the biggest contribution, recorded at 110%. It is closely followed by Long Short Equity at 110%, and by Relative Value at 107%.

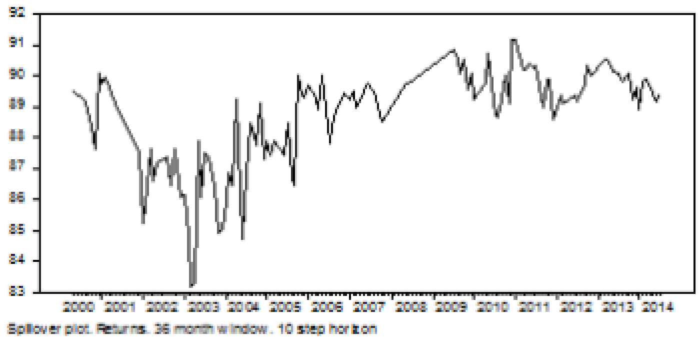
Figure 2 provides a rolling window analysis of the spillovers, using a forecast period of 10 months, and a window of 36 months. It can be seen clearly in Figure 2, that the total size of the spillovers varies over time, and becomes more pronounced in times of financial distress, as suggested by the peak in 2008/2009, and then again late in 2010.

The results in Table 3 are produced by applying Markowitz portfolio optimisation, with a positive weights constraint, as applied to a five year estimation period window, and then the weights are maintained for the next year in a one-year out-of-sample test. The procedure is then rolled forward through the data one year at a time. Given that we only had 8 observations for 2013 we did not run the out-of-sample test in 2014. The results suggest that Markowitz optimisation with positive weights produces a higher Sharpe ratio in 10 years of the 12 years of hold-out periods. Paradoxically, the return produced by a strategy of equal weighting is higher in 8 of the 12 years. However, the increase in risk associated with the higher level of return produces a lower Sharpe ratio. This performance is much better than the previous findings of DeMiguel

Table 3: Markowitz with positive constraints, whole sample analysis

Markowitz Pos. Constraints					Naive Diversification (1/N)		
year	Indices included (weights)	Return (%)	St.Dev	Sharpe Ratio	Return (%)	St.Dev	Sharpe Ratio
1997-2014	CTAGlobal (0.02)+EquityMarketNeutral(0.42)+FixedIncomeArb(0.39)+ShortSelling(0.09)	0.0000007061	0.00729705	0.00000959	3.379556	0.0105301	320.94
2002	CTAGlobal(0.01)+EquityMarketNeutral(0.74)+RelativeValue(0.19)+Shortselling(0.06)	1.057591	0.00371252	15.512	1.056935	0.00591664	9.623
2003	ConvertibleArbitrage(0.92)+CTAGlobal(0.025)+ShortSelling().055)	1.086280854	0.011066	98.16	1.117157	0.00406031	275.1
2004	DistressedSecurities(0.05)+EventDriven(0.51)+GlobalMacro(0.31)+RelativeValue(0.09)+Shortselling(0.04)	1.116112537	0.0086662	128.78	1.067563684	0.00744878	144.40
2005	DistressedSecurities(0.03)+EmergingMarkets(0.024)+EquityMarketsNeutral(0.44)+FixedIncomeArbitrage(0.40)+MergerArbitrage(0.064)+ShortSelling(0.04)	1.030537	0.00328509	313.7	1.067473693	0.00806884	132.30
2006	EquityMarketNeutral(0.67)+FixedIncomeArbitrage(0.28)+MergerArbitrage(0.03)+ShortSelling(0.02)	1.073117	0.00353134	303.88	1.099187	0.00816933	134.55
2007	EquityMarketsNeutral(0.56)+FixedIncomeArbitrage(0.30)+MergerArbitrage(0.10)+ShortSelling(0.04)	1.077053	0.00581800	185.12	1.096651	0.00966964	113.41
2008	EquityMarketsNeutral(0.44)+FixedIncomeArbitrage(0.36)+MergerArbitrage(0.12)+ShortSelling((0.08)	0.918493	0.0158075	58.10	0.883542	0.0196917	44.86
2009	CTAGlobal(0.03)+EquityMarketsNeutral((0.08)+FixedIncomeArbitrage(0.07)+MergerArbitrage(0.61)+ShortSelling(0.21)	1.031226	0.00680753	151.48	1.158968	0.0100555	115.26
2010	CTAGlobal(0.03)+EquityMarketsNeutral((0.07)+FixedIncomeArbitrage(0.09)+MergerArbitrage(0.61)+ShortSelling(0.20)	1.019417	0.00578795	176.12	1.073962	0.00991864	108.28
2011	CTAGlobal(0.03)+EquityMarketsNeutral((0.15)+FixedIncomeArbitrage(0.04)+MergerArbitrage(0.61)+ShortSelling(0.17)	1.0287	0.00217551	472.85	0.984303	0.00916986	107.34
2012	CTAGlobal(0.01)+EquityMarketsNeutral((0.16)+FixedIncomeArbitrage(0.01)+MergerArbitrage(0.66)+ShortSelling(0.16)	1.000117	0.00377598	264.87	1.045324	0.00646125	161.78
2013	EquityMarketsNeutral((0.16)+MergerArbitrage(0.69)+ShortSelling(0.15)	1.010506	0.00242167	417.28	1.053932	0.00653502	161.27

Figure 2: Total Spillover Rolling Window Analysis, 36 month window, 10 month forecasts



et al. (2009), who suggested, that in their sample and simulation analysis, it took around 3000 months, for a portfolio of 25 assets, to outperform the naive diversification strategy. Similarly, Allen et al. (2014), in their analysis of European markets, found no evidence that a Markowitz optimisation strategy outperformed naive diversification.

By contrast, the results in Table 3 are much more favourable to a Markowitz optimisation strategy. This is probaby because the 'securities' analysed are hedge fund sector indices, and their return behaviour does not seem to be subject to as much estimation risk as are normal equity securities. For example, the EquityMarketNeutral sector appears in 9 of the 12 hold-out portfolios, ShortSelling also appears in 12 of 12, and FixedIncomeArbitrage also in 12 from 12.

A non-parametric sign test on the Sharpe ratios, in which the Sharpe ratio for the Markoitz strategy is superior in 10 of 12 cases, produced a probability value of 0.019 in a one-tail test. Similarly, a Wilcoxon signed rank test of the differences also had a probability value of 0.019 in a one-tail test. Even a two-tailed t-test of the difference in the means of the Sharpe ratios, obtained for Markowitz optimisation in the hold-out samples, and those obtained by naive diversification, gave a probability of 0.059.

The other notable feature of Table 3 is that FundofFunds never appears as a component of an optimal portfolio. It appears that certain hedge fund sectors have such different investment strategies that they are powerful components of an effective diversification strategy. However, these benefits are not so readily available to the average retail investor.

The Spillover analysis, reported in Table 2, suggested that CTAGlobal is the most self-contained of the sectors, making the smallest contribution to the variances of the returns of the other sectors. This sector appeared in 6 of the 12 hold-out sample portfolios. FixedIncomeArbitrage, EquityMarketNeutral and ShortSelling also made low contributions to the variances of other sectors and appeared as very regular components of the Markowitz hold-out portfolios, in 8, 10, and 12 cases of 12, respectively.

They also appeared in the Markowitz portfolio retrospectively fitted to the entire sample, shown in the top line of Table 3. By contrast, this portfolio fitted across the whole period, had a lower Sharpe ratio than a naive diversification strategy.

4.2. Draw-down portfolio analyses

In the next part of the analysis, we explore the characteristics of portfolio draw-downs.

Figure 3 shows the draw-downs of the global minimum variance portfolio. The trajectory of draw-downs of the global minimum variance portfolio, shown in Figure 3, reveals that the biggest impact on the hedge fund sectors was in 2008-2009.

A comparison of the draw-downs for the various strategies is shown in Figure 4. The imposition of an average draw-down constraint to optimise the portfolio can still result in large draw-downs, as shown in the first graph labelled "(a) AveDD", in the top left-hand panel of Figure 4. The draw-down of -80% is much greater than the other draw-down optimiser outcomes, with the minimum CDaR, in panel (d) of Figure 4, producing the smallest draw-down.

In Table 4, we analyse these portfolios fitted to historic data, in terms of their weights, risk contributions and diversification ratios.

Table 4 demonstrates how the portfolio weights vary if we apply the various strategies across the entire 17-year sample period. The GMV strategy with positive weights, places 1.73% of the portfolio in the CTAGlobal sector, 41.22% in the Equity Market Neutral Sector, 9.7% in Fixed Income Arbitrage, 38.85% in Merger Arbitrage, and the remainder of around 8.5% in Short Selling. The other strategies, which concentrate on minimising the maximum draw-down, average draw-down, or conditional average draw-downs, or minimum draw-downs, at a 95% confidence level, produce much less diversified portfolios, with MaxDD placing 100% in the Global Macro Sector, AveDD placing 49.3% in Equity Market Neutral, 46.06% in Fixed Income Arbitrage and 4.64% in Global Macro. The CDaR95 strategy places 35.19% in Emerging Markets, 1.72% in Equity Market Neutral, 0.58% in Fixed Income Arbitrage, 62.51% in Global Macro. The CDaRMIN95 strategy places 35.19% in Emerging Markets, 1.72% in Equity Market Neutral, 0.58% in Fixed Income Arbitrage, and 62.51% in Global Macro.

The impact on reducing diversification is shown in the bottom line of Table 6, which reports the Diversification Ratio, which is lowest for the MaxDD strategy, for which it has a value of 1, reflecting that this strategy places 100% in Global Macro. The Diversification Ratio was developed by Choueifaty and Cognard (2008) and Choueifaty et al. (2011), and provides a measure of the degree of diversification of long only portfolios. It has a lower bound of one, which is achieved in single asset portfolios. The most diversified portfolio of hedge fund sectors is the GMV strategy, which has a diversification ratio of 2.11.

The expected shortfalls at the 95% level, are shown in the penultimate row of Table 4. The lowest expected shortfall at a 95% confidence level, not sur-

Figure 3: Comparison of draw-downs

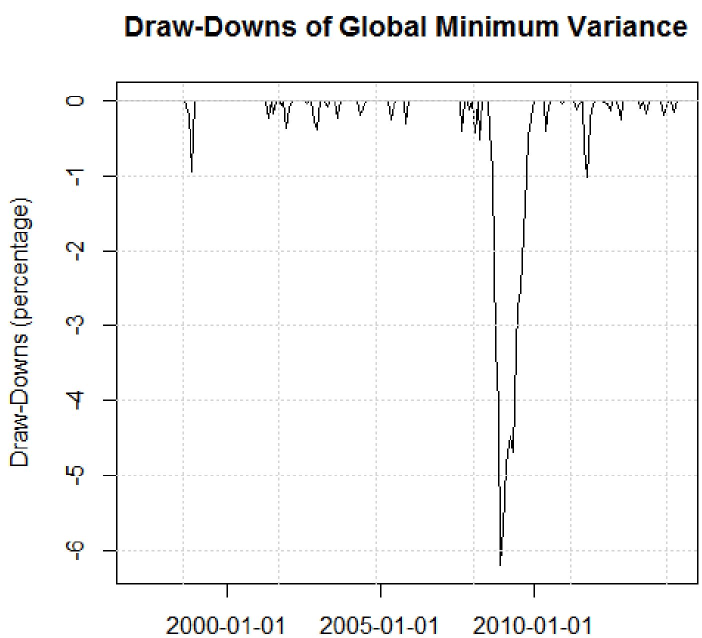


Table 4: Comparison of portfolio allocations and characteristics

	GMV	MaxDD	AveDD	CDaR95	CDaRMin95
ConvertibleArbitrage					
Weight	0.00	0.00	0.00	0.00	0.00
CTAGlobal					
Weight	1.73	0.00	0.00	0.00	0.00
DistressedSecurities					
Weight	0.00	0.00	0.00	0.00	0.00
EmergingMarkets					
Weight	0.00	0.00	0.00	35.19	35.19
EquityMarketNeutral					
Weight	41.22	0.00	49.30	1.72	1.72
EventDriven					
Weight	0.00	0.00	0.00	0.00	0.00
FixedIncomeArbitrage					
Weight	9.70	0.00	46.06	0.58	0.58
GlobalMacro					
Weight	0.00	100.00	4.64	62.51	62.51
LongShortEquity					
Weight	0.00	0.00	0.00	0.00	0.00
MergerArbitrage					
Weight	38.85	0.00	0.00	0.00	0.00
RelativeValue					
Weight	0.00	0.00	0.00	0.00	0.00
ShortSelling					
Weight	8.50	0.00	0.00	0.00	0.00
FundofFunds					
Weight	0.00	0.00	0.00	0.00	0.00
Overall					
ES 95%	0.788	0.088	0.095	0.113	0.091
Div Ratio	2.110	1.00	1.207	1.093	1.093

Table 5: Draw-Downs Comparisons

	Drawdowns(MVRet)						
	From	Trough	To	Depth	Length	To Trough	Recovery
1	31/05/2011	30/09/2011	31/12/20131	-0.0114	8	5	3
2	31/05/2010	31/05/2010	31/07/2010	-0.0045	3	1	2
3	30/09/2012	31/10/2012	30/11/2012	-0.0030	3	2	1
4	31/03/2014	31/03/2014	31/05/2014	-0.002	3	1	2
5	31/08/2013	31/08/2013	30/09/2013	-0.0081	2	1	1
	Drawdowns(CDRet)						
	From	Trough	To	Depth	Length	To Trough	Recovery
1	31/05/2013	30/09/2011	31/12/2012	-0.0346	20	5	15
2	31/05/2010	31/05/2010	31/08/2010	-0.0138	4	1	3
3	31/12/2009	31/01/2010	31/03/2010	-0.0098	4	2	1
4	30/11/2010	30/11/20130	31/12/2010	-0.0061	2	1	1
5	310/06/2013	30/06/2013	31/07/2013	-0.0052	2	1	1

prisingly, is obtained via the CDaRMin95 strategy, which yields an expected shortfall of 0.091.

These results are obtained by fitting the optimisations to the entire data set and are of limited use. The crucial tests are the out of sample ones, and these are considered next, using rolling one year windows for analysis purposes. In the next section, we compute the draw-down portfolio solutions, and use the maximum draw-down of the minimum variance portfolio as a benchmark value. The CDaR portfolios are calculated for a confidence level of 95%.

4.3. Portfolio comparisons using back-tests

We conducted further analyses to compare the results of the minimum variance strategy with the various conditional draw-down as risk strategies. The back-tests are carried out using a recursive window of 60 months, or five years of monthly data. The CDaR portfolio is optimised for a conditional draw-down of 10% at a 95% confidence level. The GMV portfolio is again constrained to be long only.

Figure 5 provides a graph of the wealth trajectories of the CDaR strategy, contrasted with the GMV one. An initial wealth of 100 units is assumed. The surprising feature of Figure 5 is that, for most of the period considered, the wealth trajectory of the CDaR portfolio is above that of GMV. This demonstrates that a portfolio of hedge fund sectors, combined with risk-minimising strategies, is a very effective way of preserving wealth.

Table 5 provides an analysis of the five greatest draw-downs, that resulted from the implementation of each strategy. The first draw-down for the CDaR strategy is surprisingly deeper (-0.0346) than for MVRet (-0.0114), and with a longer total period of 15 months as compared to 3 months for MVRet.

Figure 4: Comparison of wealth trajectories

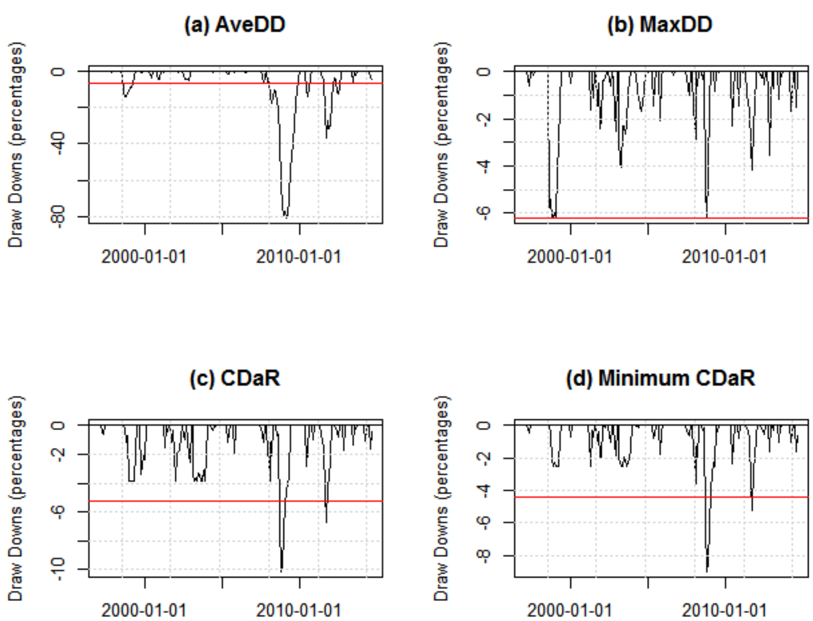


Figure 5: Comparison of Wealth Trajectories

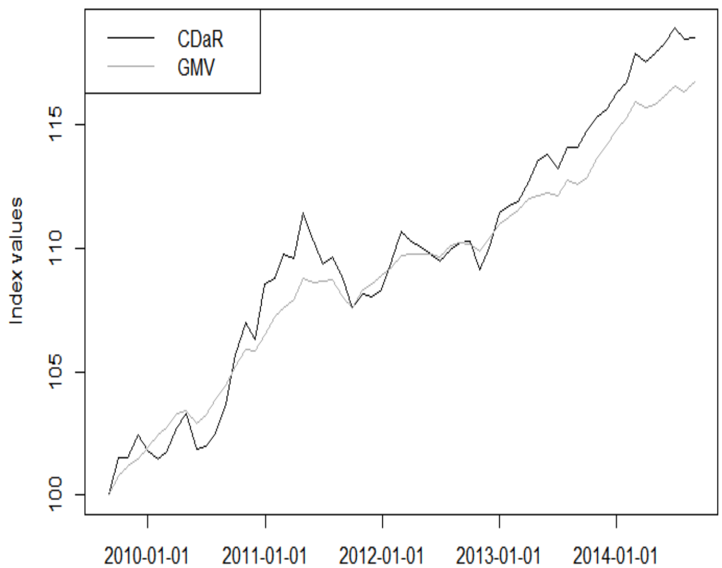


Figure 8 provides a comparison of the draw-down trajectories. It is readily apparent that the CDaR strategy successfully minimises draw-downs, but it does not necessarily provide compensating returns.

It can be seen in Table 6 that the GMV optimiser works better than the CDaR optimiser in terms of this set of hedge fund sector returns. GMV has a lower VaR and ES, than the CDaR optimiser at 95% levels, and it has a higher Sharpe ratio than CDaR. The number of draw-downs is the same, but is always relatively smaller for GMV than for CDaR.

Figure 6: Comparison of Draw-Down Trajectories

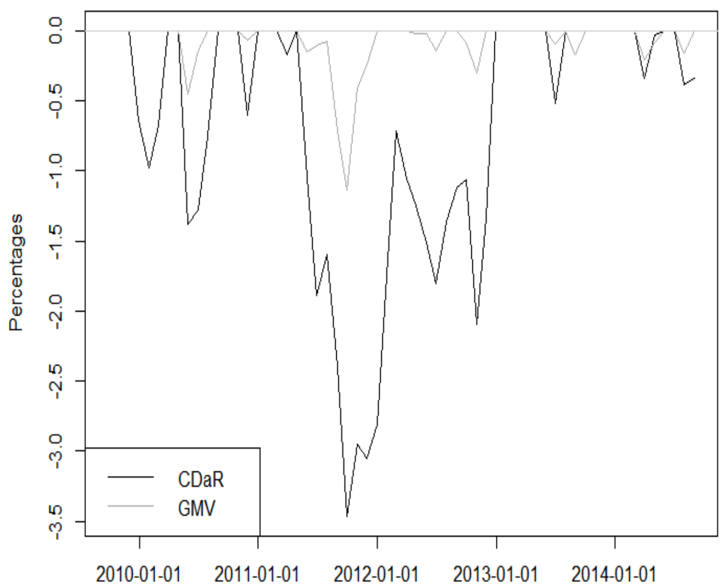


Table 6: Relative performance statistics GMV versus CDaR

Statistics	GMV	CDaR
Risk/return		
VaR 95%	0.271	0.929
ES 95%	0.406	1.24
Sharpe ratio	0.4341370	0.05088481
Return annualised %	0.143	0.159
Draw-down		
Count	9	9
Minimum	0.0679	0.001469
1st Quartile	0.1452	0.336000
Median	0.1691	0.516500
Mean	0.3034	0.869400
3rd Quartile	0.3002	0.975700
Maximum	1.1400	3.464000

The maximum draw-down in Table 6 is practically 3 times larger for CDaR than for GMV.

5. Conclusion

In this paper we examined the diversification and portfolio optimisation capabilities of a set of 13 major hedge fund sector indices, as represented by the EDHEC series, for a 17-year period of monthly returns. We commenced our application by using the Diebold and Yilmaz (2012) Spillover Index to analyse the inter-connectedness of these series. This analysis revealed that the least connected hedge fund sectors, on this measure, were the CTAGlobal and Short-Selling sectors.

We then contrasted a naive diversification strategy with a Markowitz diversification strategy, using a 5-year estimation period and one year hold-out samples. The Markowitz strategy appeared to work well, on this investment universe of hedge fund indices, out-performing a naive diversification strategy across the hold out samples, as demonstrated by superior Sharpe ratios, which were confirmed as being significant in non-parametric tests.

Then we examined the effectiveness of a variety of portfolio optimisation strategies using CVaR optimisers, plus a further set using four different applications of draw-down optimisers: MaxDD, AveDD, CDaR95, CDaRMin95. These were evaluated using a series of rolling five-year window back tests.

The most successful of the optimisation strategies was Markowitz with positive constraints. The CVaR strategy did not seem to dominate Markowitz. Even more surprising was the fact that the draw-down optimisation techniques neither dominated Markowitz, nor successfully diminished extreme adverse outcomes, as compared with Markowitz optimisation.

These results contrast with the results in Allen et al. (2014), in their previous work on portfolio diversification in European equity markets, which suggested the primacy of naive diversification, consistent with the results of DeMiguel et al. (2009). These results on hedge fund sector indices favour Markowitz optimisation techniques, and possible reflect the distinctive characteristics of the Alternative Investment universe. The Markowitz portfolios did not appear to be plagued by the customary estimation risk, and the securities and weights chosen were reasonably consistent from year to year, in the annual hold-out portfolios.

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