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Pareto Efficiency in the Jungle

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Pareto Efficiency in the Jungle

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Abstract

We include initial holdings in the jungle economy of Piccione and Rubinstein (Economic Journal, 2007) in which the unique equilibrium satisfies lexicographic welfare maximization. When we relax assumptions on consumption sets and preferences slightly, equilibria other than lexicographic welfare maximizers can be jungle equilibria. This result is due to myopia. We introduce the concept of farsightedness and show that farsighted jungle equilibria coincide with lexicographic welfare maximization. However, we also find farsighted equilibria that are Pareto inefficient since stronger agents may withhold goods from weaker agents. Here, gift giving by stronger agents is needed to achieve Pareto efficiency. We argue that even trade has a role in the jungle. Our results add to understanding coercion and the subtle role of gift giving and trade in an economy purely based on power relations.

JEL Classification: D51, D61, P52

Keywords: power, coercion, jungle economy, farsightedness, withholding

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1 Introduction

While the competitive market is based on voluntary exchange, the jungle economy is characterized by coercive exchange where stronger agents can take goods from weaker agents. The analysis of the jungle economy provides a complement to the Walrasian equilibrium model with which it shares existence and welfare properties (Piccione and Rubinstein, 2007, P&R hereafter). Embedded within the rich tradition of social contract theory following Hobbes and Locke, it facilitates a better understanding of the allocation of initial endowments, the exogenous primitive of the competitive equilibrium model. P&R propose a stylized model in which coercion governs the bilateral exchange of resources in the jungle. Coercion is driven by the agents' preferences over bounded consumption sets and power relations are described by an exogenous ranking of agents according to their strength. Weaker agents concede to stronger agents without engaging in costly conflict. The jungle economy mirrors the standard model of an exchange economy. The distribution of power in the jungle is the counterpart of the distribution of initial endowments in the market.

In a jungle equilibrium, a stronger agent no longer wants to take goods from any weaker agent nor from a pile of common goods, that no other agent holds. P&R specify certain conditions on consumption sets and preferences under which a unique and Pareto efficient jungle equilibrium exists.[1] This jungle equilibrium coincides with the unique lexicographic welfare maximum in which all of the economy's resources are initially common goods and stronger agents take from the pile of common goods before weaker agents can take.

It is tempting to conclude from P&R's intriguing analysis that exactly the particular strength relation assumed in their paper constitutes the main driving force behind the final distribution of resources in the jungle. However, this conclusion is somewhat premature. The goal of our paper is to provide a more nuanced view on the interaction of strength, preferences and holdings behind the jungle equilibrium concept. Intentionally, we do not

[1] These assumptions are compact and convex consumption sets and smooth, strongly monotone and strictly convex preferences.

deviate from P&R's strength relation throughout the paper.

In our analysis, we assume that initial holdings are distributed over the agents rather than being available as common goods. A stronger agent may take from any weaker agent or from the (remaining) pile of common goods. Under P&R's assumptions on consumption sets and preferences, initial holdings are irrelevant for lexicographic welfare maximization. The intuition is that, when agents take in the lexicographic order induced by power, stronger agents are always able to obtain a lexicographic welfare-maximizing bundle through a sequence of bilateral takings where each taking improves the taker's welfare. The initial distribution of resources among agents in jungle economies is relevant only to determine from whom a stronger agent would take.

However, once we relax the assumptions of strong monotonicity and strict convexity of preferences assumed by P&R, the distribution of initial holdings matters. Imagine an example of a jungle economy with a strong agent and two weaker agents, holding Leontief preferences over pairs of shoes. Suppose one weaker agent holds a left shoe and one holds a right shoe. Since getting only a left shoe or only a right shoe does not increase the strong agent's utility, the strong agent will not take and, therefore, the jungle is in equilibrium.[2] In such a case the jungle equilibrium does not satisfy lexicographic welfare maximization, nor is it Pareto efficient.

As this example illustrates, the jungle equilibrium concept is myopic. It fails to recognize that the stronger agent can gain by coercing both the medium and the weak agent even if each individual taking does not improve the strong agent's welfare. We show that if we include farsightedness into the equilibrium concept, jungle equilibria coincide with lexicographic welfare maximization under rather weak assumptions.[3]

Furthermore, with the use of an example, we derive a continuum of farsighted equilibria in which the strongest agent holds goods in excess of her satiation point. As the strongest

[2] For the sake of the argument, assume the weaker agents cannot take from each other.

[3] These assumptions are non-empty, compact and strictly comprehensive consumption sets and complete, transitive and continuous preferences.

agent might have no incentive to dispose of these excess goods, she may withhold them from weaker agents, who cannot take them. This withholding of goods is Pareto inefficient and only gift giving by stronger agents can remove this inefficiency. In another example, we show that even gift giving may sometimes be insufficient to achieve Pareto efficiency. Then, trade is needed.

These examples demonstrate why we believe that our analysis adds to a better understanding of the crucial assumptions underlying jungle economies. Pareto efficiency in the jungle is not a result of coercion alone. On the contrary, depending on the kind of preferences present in the jungle, gift giving and trade, behavior that is in sharp contrast to coercion, is needed to keep the jungle efficient. Thus, our conclusions diverge strikingly from P&R, who see no role for trade in the jungle.

We proceed as follows. Section 2 presents a formal account of a jungle economy with initial holdings of which P&R's jungle economy is a special case. Section 3 investigates lexicographic welfare maximization and provides two examples to motivate our analysis. The farsighted jungle equilibrium is investigated in Section 4. The subtle role of withholding, giving and trade is discussed in Section 5. Section 6 concludes.

2 The Jungle Economy

We consider a finite and ordered set of agents $N = \{1, \dots, n\}$ of size $n \geq 2$ and a finite number $m \geq 1$ of goods that are present in positive quantities. We refer to z^i as the holdings of agent $i \in N$. Her consumption set is denoted $C^i \subseteq \mathbb{R}^m_+$. This set is non-empty, compact and strictly comprehensive, i.e., for all $z^i \in C^i$ and $\hat{z}^i \in \mathbb{R}^m_+$ such that $\hat{z}^i \leq z^i$ it holds that $\hat{z}^i$ lies in the interior of C^i.[4] The preference relation of agent i on C^i, denoted $\succeq^i$, is complete, transitive and continuous. In line with P&R we assume that an agent's holdings cannot exceed her consumption set.

[4] Vector inequalities: $a \leqq b$, $a \leq b$ and $a < b$. Furthermore, $\subseteq$ denotes a subset and $\subset$ a strict subset.

An allocation $z = (z^1, \ldots, z^n, z^{n+1})$ assigns holdings to each agent in N, while $z^{n+1} \in \mathbb{R}^m_+$ indicates the bundle of common goods that is held by none of the agents. The economy's total endowment equals $\widetilde{\omega} \in \mathbb{R}^m_+$. An allocation z is feasible when $\sum_{i=1}^{n+1} z^i = \widetilde{\omega}$ and $z^i \in C^i$ for all $i \in N$.[5] Unlike P&R we consider that initial endowments are allocated to the agents. Initial holdings are defined as the feasible allocation $\omega = (\omega^1, \ldots, \omega^n, \omega^{n+1})$.

Coercion governs the bilateral exchange of goods in the jungle economy and it is driven by the agents' preferences and strengths. The order of the agents reflects their strength. As in the jungle economy of P&R, the strength structure is extreme. The strongest agent of any pair of agents has the power to take everything that the weaker agent possesses, while the weaker agent cannot take anything from the stronger agent.[6] The agents in N are ordered such that agent 1 is stronger than agent 2, who is stronger than agent 3, and so on. Thus $i < j$ implies that i is stronger than j.

Thus far, we have defined an economy driven by coercion as a tuple $\langle N, \{C^i, \succeq^i\}_{i \in N}, \omega \rangle$. This tuple extends the jungle economy of P&R by introducing the initial holdings ω. Furthermore, in the jungle economy of P&R each C^i is a convex set and preferences in the jungle economy are strongly monotone and strictly convex.[7] We relax this assumption later on.

An agent's feasible consumption is the set of bundles that this agent is able to reach from her own and the weaker agents' current holdings. It is convenient to define $y^{i,j} \leq z^j$ as agent i's bilateral takings that results from a single coercive exchange between agents i and j with $i < j$.

As a benchmark, we adopt the jungle equilibrium of P&R in which stability against bilateral takings by stronger agents is the key idea.

[5] We include $z^i \in C^i$ into the definition of feasible allocations because technically speaking agent i's preferences on $\mathbb{R}^m_+ \backslash C^i$ are undefined.

[6] The interpretation is that the weaker agent concedes to the stronger agent and does not initiate a costly conflict knowing it will be lost for sure.

[7] In fact, monotone and strictly convex preferences imply strongly monotone preferences. However, for the sake of consistency with P&R, we keep referring to the preferences in the jungle as strongly monotone and strictly convex.

Definition 1 *A jungle equilibrium is a feasible allocation* z *for which there does not exist an ordered pair of agents* i, j*, bilateral takings* $y^{i,j}$ *and a feasible bundle* $\hat{z}^i = z^i + y^{i,j} \in C^i$ *such that* $\hat{z}^i \succ^i z^i$*.*

Note that the definition of a jungle equilibrium is independent of the initial allocation which may assign goods to the agents or to the pile of common goods as in P&R.

3 Lexicographic welfare maximization

In the jungle economy of P&R a jungle equilibrium is obtained by lexicographic welfare maximization. The lexicographic welfare maximum is obtained when agents take sequentially in the order of strength from a pile of common goods. Assuming strongly monotone and strictly convex preferences, the jungle economy has a unique jungle equilibrium (P&R, Proposition 3). This result still holds, if we allow for an arbitrary division of initial holdings. Consider a sequence of takings starting with the strongest agent, agent 1, who collects her preferred bundle from the holdings of all weaker agents $i > 1$. Then agent 2 collects her preferred bundle from agents $3, ..., n$, and so on. This procedure implements the unique lexicographic welfare maximizing allocation. Obviously, this allocation is a jungle equilibrium.

However, when the assumption of strongly monotone and strictly convex preferences of P&R is relaxed to monotone and convex preferences, to allow for Leontief preferences, the lexicographic welfare maximizing allocation need no longer be unique. Example 1 illustrates this argument.

Example 1 *Consider an economy with two agents and two goods. The economy's total resources are* $\widetilde{\omega} = (2, 1)$*. The agents' consumption sets are identical and given by* $C^1 = C^2 = \left\{x \in \mathbb{R}^2_+ | x \leq \widetilde{\omega}\right\}$ *for simplicity. For* $i = 1, 2$*, agent* i*'s best element of* $\succeq^i$ *on* C^i *maximizes the Leontief preferences* $\min\left\{z^i_1, z^i_2\right\}$*. It is easy to see that allocations*

$$\omega^1 = (1, 1)\,,\ \omega^2 = (1, 0)\ \text{ and common goods } \omega^3 = (0, 0)$$

$$\omega'^1 = (1, 1)\,,\ \ \omega'^2 = (0, 0)\ \text{ and common goods } \omega'^3 = (1, 0)$$

are both equilibria according to Definition 1 because neither agent 1 *nor agent* 2 *can gain from bilateral takings.*

An agent with Leontief preferences may not have an incentive to take a good, because a larger bundle, even though it belongs to her consumption set, is not necessarily better.

Moreover, there may exist jungle equilibria that are not lexicographic welfare maximizers. Example 2 illustrates this argument.

Example 2 *Consider an economy with two agents and three goods. The economy's total resources are $\widetilde{\omega} = (1, 1, 1)$. The agents' consumption sets are identical and given by $C^1 = C^2 = \left\{x \in \mathbb{R}^3_+ | x \leq \widetilde{\omega}\right\}$ for simplicity. For $i = 1, 2$, agent i's best element of $\succeq^i$ on C^i maximizes the Leontief preferences $\min\left\{z^i_1, z^i_2, z^i_3\right\}$. For all allocations ω, the unique lexicographic welfare maximum $\bar{z}$ is given by $\bar{z}^1 = \widetilde{\omega}$ and $\bar{z}^2 = \bar{z}^3 = 0$. However, for $\alpha \in [0, 1]$,*

$$\omega^1 = (1, \alpha, \alpha), \; \omega^2 = (0, 1 - \alpha, 0) \; \textit{and common goods} \; \omega^3 = (0, 0, 1 - \alpha)$$

form an equilibrium according to Definition 1 because neither agent 1 *nor agent* 2 *can gain from bilateral takings. For $\alpha < 1$, it differs from the lexicographic welfare maximum and it is Pareto inefficient because $z^1 = (\alpha, \alpha, \alpha)$, $z^2 = (1 - \alpha, 1 - \alpha, 1 - \alpha)$ and $z^3 = (0, 0, 0)$ is welfare improving. Besides permutations, no other equilibria exist.*

There are two intriguing issues about this example. First, the jungle equilibrium concept fails to recognize that stronger agents might evaluate the aggregate of sequences of bilateral takings instead of single bilateral takings. This can be interpreted as myopia. In the next section, we introduce farsightedness in the jungle. Second, for initial holdings $\omega^1 = (1, \alpha, \alpha)$, the amount $1 - \alpha$ of good 1 does not contribute additional welfare to agent 1 and this agent is indifferent between keeping it or disposing it. This observation gives a glimpse of the issue of withholding, which we discuss in Section 5.

4 The farsighted jungle economy

In this section, we include farsightedness into the jungle equilibrium concept and show that all farsighted jungle equilibria maximize lexicographic welfare.

Formally, a farsighted agent i should not only consider bilateral takings from a single weaker agent j, as in a jungle equilibrium, but rather consider sequences of bilateral takings from some or all weaker agents. We model this dynamic sequence in a static manner. We denote agent i's bilateral net takings from all weaker agents by the tuple $y^i = (y^{i,i+1}, \ldots, y^{i,n+1})$. Given allocation z, agent i's bilateral takings y^i are feasible if

$$z^i + \sum\nolimits_{j=i+1}^{n+1} y^{i,j} \in C^i \quad \text{and } y^{i,j} \leqq z^j \text{ for all } j \in \{i+1, \ldots, n+1\}. \tag{1}$$

We define the farsighted version of the jungle equilibrium as follows:

Definition 2 *A farsighted jungle equilibrium is a feasible allocation z such that there does not exist an agent $i \in N$ and feasible bilateral takings y^i for which $z^i + \sum_{j=i+1}^{n+1} y^{i,j} \succ^i z^i$.*

We denote the set of lexicographic welfare maximizers by $\overline{Z}$. Then, the following equivalence holds:

Theorem 1 *Each lexicographic welfare maximizing $\bar{z} \in \bar{Z}$ is a farsighted jungle equilibrium and vice versa.*

Proof. The result follows from the following equivalences: $\bar{z} \in \bar{Z} \Leftrightarrow$ for each $i \in N$ and feasible bilateral takings y^i it holds that $\bar{z}^i \succeq^i \bar{z}^i + \sum_{j=i+1}^{n+1} y^{i,j} \Leftrightarrow$ for each $i \in N$ there does not exist feasible bilateral takings y^i such that $\bar{z}^i + \sum_{j=i+1}^{n+1} y^{i,j} \succ^i \bar{z}^i \Leftrightarrow \bar{z}$ is a farsighted jungle equilibrium. ■

Note that $\bar{Z}$ is non-empty and therefore a farsighted jungle equilibrium always exists. Recall that consumption sets are non-empty, compact and strictly comprehensive and preferences are complete, transitive and continuous. Therefore, the assumptions for theorem 1 are rather weak.

5 Withholding, giving and trade

In principle, an agent is not forced to consume all her holdings and may freely dispose or waste some of the resources available to her. In the jungle economy of P&R agents may freely dispose goods. This is captured by the assumption that the consumption set is comprehensive. But given strongly monotone preferences in the jungle economy of P&R, all agents consume their holdings in equilibrium. Example 2, however, illustrates that the distinction between holdings and consumption is more subtle and can matter. Holdings that are not consumed are withheld from other agents in the economy. We now investigate withholding of goods in farsighted equilibria.

First, we motivate our analysis with a third example. It is similar in spirit to Example 2 but differs in that a continuum of farsighted jungle equilibria are Pareto inefficient due to withholding, a phenomenon that is not present in Example 2.

Example 3 *Consider an economy with two agents and two goods. The economy's total resources are $\widetilde{\omega} = (2,1)$. The agents' consumption sets are identical and given by $C^1 = C^2 = \left\{x \in \mathbb{R}^2_+ | x \leq \widetilde{\omega}\right\}$ for simplicity. Agent 1's best element of $\succeq^1$ on C^1 maximizes the Leontief preferences $\min\{z^1_1, z^1_2\}$ and agent 2's best element of $\succeq^2$ on C^2 maximizes $\sqrt{z^2_1} + \sqrt{z^2_2}$. For all allocations ω, the set of lexicographic welfare maximizers $\bar{Z}$ is given by*

$$\left\{\left(\bar{z}^1, \bar{z}^2, \bar{z}^3\right) \in C^1 \times C^2 \times \widetilde{\omega} | \bar{z}^1 = (1+\varepsilon, 1), \bar{z}^2 = (1-\varepsilon, 0), \bar{z}^3 = (0,0), \varepsilon \in [0,1]\right\}.$$

This set coincides with the set of jungle equilibria as well as the set of farsighted equilibria because neither agent 1 or agent 2 can gain from a sequence of bilateral takings. For $\varepsilon > 0$, the lexicographic welfare maximum is Pareto inefficient because $z^1 = (1,1)$, $z^2 = (1,0)$ and $z^3 = (0,0)$ is welfare improving. So, only the lexicographic welfare maximum corresponding to $\varepsilon = 0$ is Pareto efficient. Withholding can be said to occur whenever $\varepsilon > 0$.

The economic issue, left open in this paper, is whether agent 1 has an incentive to keep (or give away) her excess holdings $\varepsilon > 0$ of good 1. By keeping ε of good 1, this agent withholds it from agent 2, for whom it would increase welfare.

In Example 3 preferences are defined for holdings. One can raise the question whether an agent with Leontief preferences consumes all holdings or only consumes the corner point while disposing the rest.[8] How to conceptualize this issue? From here on, we distinguish between an agent's holdings and her consumption. This distinction requires that we redefine the agents' preference relations. Whenever an agent compares two bundles of holdings, z^i and $\hat{z}^i$, she actually compares best elements attainable from z^i with best elements attainable from $\hat{z}^i$ and prefers the holdings that allow the most favorable of such best elements.

We introduce some extra notation. For all i in N, we denote agent i's consumption under free disposal as $x^i \in \mathbb{R}^m_+$ and define feasible consumption as $x^i \leqq z^i$ and $x^i \in C^i$. By comprehensiveness of the consumption set, $x^i \in C^i$ whenever $x^i \leqq z^i$. Free disposal implies that agent i's preference relation over holdings, $\succeq^i$, must be distinguished from agent i's preference relation over consumption bundles, denoted $\succeq^i_x$. Both preference relations have to be logically consistent. We take preference relations $\succeq^i_x$ as the primitive and assume it is complete, transitive and continuous. These assumptions on $\succeq^i_x$ and feasibility on the set $\{x^i \in C^i | x^i \leqq z^i\}$ guarantee a non-empty and compact set of best elements, denoted as $\beta^i(z^i)$, on the set of feasible consumptions. By definition, $x^i, \hat{x}^i \in \beta^i(z^i)$ implies $x^i \sim^i \hat{x}^i$. The set $\beta^i(z^i)$ defines $\succeq^i$ as follows: Agent i prefers holdings z^i to $\hat{z}^i$, if she prefers the best elements attainable from z^i to the best elements attainable to $\hat{z}^i$. That is, $z^i \succeq^i \hat{z}^i$ if and only if for every $x^i \in \beta^i(z^i)$ and $\hat{x}^i \in \beta^i(\hat{z}^i)$ it holds that $x^i \succeq^i_x \hat{x}^i$. The redefined preference relation $\succeq^i$ on C^i is also complete, transitive and continuous.

The distinction between consumption and holdings requires to modify the procedure of lexicographic welfare maximization. As before, agents take from strongest to weakest. The sequence of bilateral takings by a stronger agent from weaker agents is expressed as bilateral takings from the weaker agents' remaining holdings after stronger agents took before. But now, maximization happens with respect to the preference relation over consumption bundles under an additional constraint. For $j \in N$ and after bilateral takings $\bar{y}^1, \ldots, \bar{y}^{j-1}$

[8] Such ambiguity between consumptions and holdings cannot occur for satiated preferences.

by agents that are stronger than agent j, the j-th level of the modified lexicographic welfare maximization is defined as the consumption bundle x^j, holdings z^j and the tuple y^j of bilateral takings by agent j that are a best element according to $\succeq_x^j$ on C^j subject to

$$\begin{aligned} x^j &\leqq z^j, \\ z^j &= \omega^j - \sum\nolimits_{i=1}^{j-1} \bar{y}^{i,j} + \sum\nolimits_{k=j+1}^{n+1} y^{j,k} \in C^j \text{ and} \\ y^{j,k} &\leqq \omega^k - \sum\nolimits_{i=1}^{j-1} \bar{y}^{i,k} \text{ for all } k \in \{j+1, \ldots, n+1\}. \end{aligned} \tag{2}$$

We denote a modified lexicographic welfare maximum by an upper bar. As before, the best element $\bar{y}^j$ will not be uniquely determined and we suppress it from our notation in what follows. Also, for each best element $\bar{x}^j$, all feasible $z^j \geqq \bar{x}^j$ can be used to construct another best element. However, in order to study excess holdings we maintain $\bar{z}^j$ in our notation. So, we write $\bar{x} = (\bar{x}^1, \ldots, \bar{x}^n)$ for the allocation of consumption bundles, $\bar{z} = (\bar{z}^1, \ldots, \bar{z}^{n+1})$ for the allocation of holdings and the set of such maximizers as $\bar{X} \times \bar{Z}$. The associated excess holdings are denoted as the tuple $\bar{e} = (\bar{z}^1 - \bar{x}^1, \ldots, \bar{z}^n - \bar{x}^n)$, where each vector $\bar{e}^j$ is nonnegative. The set of all excess holdings is $\bar{E}$.

We modify the farsighted jungle equilibrium of Definition 2 to distinguish between consumption and holdings.

Definition 3 *A modified farsighted jungle equilibrium is a feasible pair (x, z) such that there does not exist an agent $i \in N$, a tuple of feasible bilateral takings y^i such that $z^i + \sum_{j=i+1}^{n+1} y^{i,j} \in C^i$ and a consumption bundle $\hat{x}^i \leqq z^i + \sum_{j=i+1}^{n+1} y^{i,j}$ for which $\hat{x}^i \succ_x^i x^i$.*

The following equivalence holds:

Theorem 2 *Each modified lexicographic welfare maximizing pair $(\bar{x}, \bar{z}) \in \bar{X} \times \bar{Z}$ is a modified farsighted jungle equilibrium $(\bar{x}, \bar{z})$ and vice versa. Moreover, $\bar{X} \times \bar{Z}$ is non-empty.*

Note that each modified farsighted jungle equilibrium induces nonnegative excess holdings and that there exists a modified farsighted jungle equilibrium with no excess holdings.

Furthermore, strongly monotone preferences exclude excess holdings. This condition holds in the jungle economy of P&R. Example 3 shows that it is impossible to relax the conditions of this result to monotone preferences.

From Example 3 it seems that only gift giving by stronger agents can remedy Pareto inefficiency. However, in our final example we contend that this is not necessarily true. We argue that even trade can have a role in the jungle.

Example 4 *Consider an economy with two agents and two goods. The economy's total resources are* $\widetilde{\omega} = (1, 1)$*. Agent* 1*'s consumption set is given by* $C^1 = \left\{x \in \mathbb{R}^2_+ | x_1 + x_2 \leq 1\right\}$ *and, for simplicity, agent* 2*'s consumption set is given by* $C^2 = \left\{x \in \mathbb{R}^2_+ | x \leq \widetilde{\omega}\right\}$*. Agent* 1*'s best element of* $\succeq^1_x$ *on* C^1 *maximizes the preferences* $x^1_1 + x^1_2$ *and agent* 2*'s best element of* $\succeq^2_x$ *on* C^2 *maximizes* $\sqrt{x^2_1} + \sqrt{x^2_2}$*. Due to strongly monotone preferences, we obtain* $\succeq^i$ *on* C^i *by substituting* z *for* x *in* $\succeq^i_x$*. For all allocations* ω*, the set of lexicographic welfare maximizers* $\bar{Z}$ *is given by*

$$\left\{\left(\bar{z}^1, \bar{z}^2, \bar{z}^3\right) \in C^1 \times C^2 \times \widetilde{\omega} | \bar{z}^1 = (\varepsilon, 1 - \varepsilon), \bar{z}^2 = (1 - \varepsilon, \varepsilon), \bar{z}^3 = (0, 0), \varepsilon \in [0, 1]\right\}.$$

This set coincides with the set of jungle equilibria as well as the set of farsighted jungle equilibria because neither agent 1 *nor agent* 2 *can gain from a sequence of bilateral takings. Obviously, no withholding occurs. For* $\varepsilon \neq \frac{1}{2}$*, however, the lexicographic welfare maximum is Pareto inefficient because* $z^1 = (\frac{1}{2}, \frac{1}{2})$*,* $z^2 = (\frac{1}{2}, \frac{1}{2})$ *and* $z^3 = (0, 0)$ *is welfare improving. So, only the lexicographic welfare maximum corresponding to* $\varepsilon = \frac{1}{2}$ *is Pareto efficient. It can only be reached from allocations with* $\varepsilon \neq \frac{1}{2}$ *by voluntary trade in which goods* 1 *and* 2 *are exchanged one-for-one.*

As before, the economic issue remains whether agent 1 has an incentive to trade or not.

6 Conclusion

This paper provides an extensive analysis of lexicographic welfare maximization. Our work can be interpreted as a sensitivity analysis on the crucial assumptions underlying jungle economies, as first described by P&R. Throughout the paper, we maintain the assumption that stronger agents have coercive power over weaker agents. We discuss two extensions of P&R. First, we allow initial holdings of the agents, while P&R assume that all goods are taken from a common pool. Second, we allow agents in the jungle to be endowed with Leontief preferences. In this case, Example 2 shows that stronger agents may not be able to gain by any bilateral takings. Hence, jungle equilibria are no longer lexicographic welfare maximizers. This result is due to myopia in the equilibrium concept, where only bilateral takings are considered. We introduce the concept of a farsighted jungle equilibrium that does not have this drawback. It requires that no player can gain through a sequence of takings. Farsighted jungle equilibrium implements lexicographic welfare maximization under weak assumptions. However, Example 3 shows that a farsighted jungle equilibrium need not be Pareto efficient. Stronger agents can withhold from weaker agents goods they do not wish to consume. Only voluntary giving can restore Pareto efficiency in this case. Example 4 shows that, in some non-generic cases, even trade is necessary to achieve Pareto efficiency. The microeconomic idea of an efficient jungle has its philosophical underpinning in John Locke's (1690, section 31) no-spoilage proviso. In his famous "Second Treatise of Government" Locke argues that legitimate property rights are incompatible with wasting resources. Locke's second proviso that one can only privately acquire goods from the common pool as long as "there is enough, and as good left in common for others" (section 27) is, however, violated in the jungle.

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