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Technological Change During the Energy Transition*

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Abstract

The energy transition from fossil fuels to alternative energy sources has important consequences for technological change and resource extraction. We examine these consequences by incorporating a non-renewable resource and an alternative energy source in a market economy model of endogenous growth through expanding varieties. During the energy transition, technological progress is non-monotonic over time: it declines initially, starts increasing when the economy approaches the regime shift, and jumps down once the resource stock is exhausted. A moment of peak-oil does no longer necessarily occur, and simultaneous use of the resource and the alternative energy source will take place if the return to innovation becomes too low.

JEL codes: O30, Q32, Q42, Q56

Keywords: Alternative energy sources, endogenous growth, energy transition, non-renewable resources, technological change

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1 Introduction

Economic growth and natural resource use have been intrinsically linked throughout history. While in the Malthusian era land improvement and expansions allowed for population increases, in the modern economy era coal and later oil made the steady growth of manufactured output per capita possible. Because fossil resources have seemed so abundant for most of the time since the industrial revolution, our theories of growth could safely ignore the role of resources and focus on capital investment and technological change. However, fossil resources are non-renewable and at some point resource scarcity will be likely to restrict growth. The limited availability of our main current sources of energy gives rise to two possible scenarios: either we need to gradually reduce energy use and prevent sudden declines in energy supply, or substitutes for fossil energy need to be introduced. Both scenarios involve costs and the natural question is to what extent growth will be influenced. In particular, the question is how the engine of growth in our modern economies, namely investment and innovation, will be affected.

To answer this question, we propose a model in which growth is driven by R&D and that integrates the use of energy from potentially two sources: non-renewable (fossil) resources that can be extracted without cost from the earth's crust and a form of energy that is produced by using renewable resources like solar energy or wind. Nordhaus (1973) was the first one to introduce such a substitute technology that is not constrained by exhaustibility, which he called a 'backstop technology'. Examples of already available backstop technologies for natural resources are nuclear energy, solar energy, and wind energy. We contribute to the literature by studying the effects of the availability of a backstop technology on the rate of technological progress and on the resource extraction path in an analytically tractable, general equilibrium model.

Intuition suggests that technological progress as the engine of growth might falter in the long run, because incentives for developing labor- and capital-augmenting technology become smaller as resource stocks dwindle and the increasing resource income share puts downward pressure on the income shares of capital and labor. Taking the existence of a substitute for fossil fuels into account, however, we find the opposite result: technological progress prospers instead of falters when resource stocks dwindle during the energy transition. Underlying the surge in innovation is a consumption smoothing motive: agents convert part of the resource stock into knowledge, thereby transferring some resource wealth to the backstop technology era. Moreover, we show that, if the backstop technology is expensive, a large increase in R&D investment is required for a smooth transition. As a result, the marginal return to innovation falls sharply and may even become equal to the return to conserving some fossil when the backstop technology is already used. In this case, part of the consumption smoothing will take place through a regime of simultaneous use of the resource and the backstop technology. Finally, we find that due to the availability of the backstop technology, the time profile of resource extraction may remain upward-sloping until the stock is depleted.

The first building block of our analysis is the so-called Dasgupta-Heal-Solow-Stiglitz (DHSS) model. The DHSS model integrates non-renewable resources into the neoclassical exogenous growth framework.[1] Although the DHSS model does not focus on the energy transition towards backstop technologies, some of the early studies do take the existence of substitutes for the non-renewable resource into account. Dasgupta and Heal (1974) and Dasgupta and Stiglitz (1981) allow for the invention of a backstop technology, which occurs each period with an exogenously given probability. Kamien and Schwartz (1978) introduce the possibility of undertaking R&D to affect the probability of invention. In partial equilibrium settings, Hoel (1978) and Stiglitz and Dasgupta (1982) assume that a backstop technology already exists. They show that the relative price of the resource compared to the backstop technology increases over time and the backstop is adapted once prices are equalized.

In the neoclassical models discussed so far, gradual technological progress was either absent or exogenous. Barbier (1999) was one of the first to study the role of endogenous technological change in alleviating resource scarcity. Scholz and Ziemes (1999) investigate the effect of monopolistic competition on steady state growth in a model with a necessary non-renewable resource.[2] More recently, Bretschger and Smulders (2012) explore the consequences of poor input substitution possibilities and induced structural change for long-run growth prospects in a multi-sector economy. These three endogenous growth models, however, ignore the existence of a backstop technology for the natural resource. Tsur and Zemel (2003) fill this gap in the literature, by introducing R&D directed at a backstop technology. In their model, accumulation of knowledge gradually decreases the per unit cost of the backstop technology.

[1]The DHSS model consists of Dasgupta and Heal (1974), Solow (1974a; 1974b), and Stiglitz (1974a; 1974b). Recently, Benchekroun and Withagen (2011) have developed a technique to calculate a closed form solution.

[2]Following Dasgupta and Heal (1979), a natural resource is defined to be 'necessary' if production is zero without input of the resource.

Alternatively, Chakravorty, Leach, and Moreaux (2009) assume that per unit costs of the backstop technology decrease over time through learning by doing. Both studies, however, are casted in a partial equilibrium framework.

Accordingly, the existing literature on non-renewable resources in which technological progress is explained endogenously appears to suffer from a dichotomy: either backstop technologies or general equilibrium effects are being ignored. A synthesis of both strands of the literature is, however, desirable and likely to generate new insights (cf. Valente, 2011). After all, contrary to the presumption in the partial equilibrium literature that imposes a fixed resource demand function, output growth and biased technological change both affect the demand for the resource. Moreover, changes in the rate of interest induced by the energy transition should be taken into account, because they affect the level of investment and innovation, and the extraction path through Hotelling's rule.

There are a few notable exceptions that are not subject to the dichotomy criticism. First, Tsur and Zemel (2005) develop a general equilibrium model where the unit costs of the backstop technology decrease as a result of R&D. However, R&D is only possible in the backstop sector, so that effects on aggregate technological progress cannot be addressed. Second, Tahvonen and Salo (2001) study the transition between renewable and non-renewable resource in general equilibrium. In their model, though, technological change results from learning-by-doing and does not come from intentional investments. Moreover, they resort to a Cobb-Douglas specification for final output, thereby ignoring poor substitution between resources and man-made inputs. Finally, Valente (2011) constructs a general equilibrium model in which the social planner optimally chooses whether and when to abandon the traditional resource-based technology in favor of the backstop technology. The differences with our analysis are that Valente abstracts from poor input substitution by imposing Cobb-Douglas production, assumes a costless endowment of the backstop technology, and derives the social optimum instead of the decentralized market equilibrium. Moreover, his focus on the optimal timing of backstop technology adoption and on the optimal jumps in output and consumption at the regime switching instant is different from ours.

The remainder of the paper is structured as follows. Section 2 presents the structure of the model. Section 3 discusses the energy transition and regime shifts. Section 4 provides a numerical illustration. Finally, Section 5 concludes.

2 The Model

We model an economy in which final output is produced with intermediate goods and energy. The production of intermediate goods requires labor. Energy is derived from a non-renewable natural resource that can be extracted at zero costs, or generated by a backstop technology that uses labor. The elasticity of substitution between energy and intermediate goods is assumed to be smaller than unity, in line with the empirical evidence in Koetse, de Groot, and Florax (2008) and van der Werf (2008). Technological progress in the model is driven by labor allocated to R&D directed at the invention of new intermediate goods. The remainder of this section describes the structure of the model in more detail.

2.1 Production

Final output Y is produced with energy E and an intermediate input M, according to

$$Y = \left[\bar{\theta} E^{\frac{\sigma-1}{\sigma}} + (1-\bar{\theta}) M^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}},$$

where $0 < \bar{\theta} < 1$ and $\sigma \in (0,1)$ denotes the elasticity of substitution between energy and the intermediate input.[3]

The intermediate input is a CES aggregate of intermediate goods k with an elasticity of substitution between varieties of $1/(1-\beta) > 1$. At time t, there exists a mass of $N(t)$ different intermediate goods. When intermediate goods producers are identical, the equilibrium quantity of variety j is the same for all varieties, so that $k_j = k, \forall j$. By defining aggregate intermediate goods as $K \equiv Nk$, the intermediate input can be written as

$$M = \left(\int_0^N k_j^{\beta} dj\right)^{\frac{1}{\beta}} = N^{\phi} K,$$

where $\phi \equiv (1-\beta)/\beta$ measures the gains from specialization: while keeping aggregate intermediate goods K constant, the intermediate input M rises with the number of varieties N through increased specialization possibilities in the use of intermediate goods (cf. Ethier, 1982; Romer, 1987, 1990).

[3] Time arguments are omitted if there is no possibility of confusion.

Energy is generated by the non-renewable resource R and a backstop technology H:

$$E = R + A_H H,$$

where A_H is a productivity index.

Final goods producers maximize profits in a perfectly competitive market. They take their output price p_Y, the prices of intermediate goods p_{K_j}, the resource price p_R and the price of the backstop technology p_H as given. Because R and H are perfect substitutes, final good producers will only use the energy source with the lowest relative price per unit of energy and they are indifferent between the two if their prices are equal. Relative demand for intermediate goods and energy is therefore given by:[4]

$$\begin{array}{llll} K/R = & \left(\frac{p_R}{p_K}\right)^{\sigma} \left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^{\sigma} N^{-\phi(1-\sigma)}, & H = 0 \quad \text{if} & p_H/A_H > p_R \\ K/H = & \left(\frac{p_H}{p_K}\right)^{\sigma} \left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^{\sigma} (A_H N^{-\phi})^{1-\sigma}, & R = 0 \quad \text{if} & p_H/A_H < p_R \\ K/E = & \left(\frac{p_E}{p_K}\right)^{\sigma} \left(\frac{1-\bar{\theta}}{\bar{\theta}}\right)^{\sigma} N^{-\phi(1-\sigma)}, & \quad\quad\quad\;\, \text{if} & p_H/A_H = p_R \end{array} , \tag{1}$$

where p_E denotes the price of energy.

Firms in the intermediate goods sector need a patent to produce one specific variety according to the production function $k_j = l_{K_j} \Rightarrow K = L_K$, where l_{K_j} denotes labor demand by firm j and L_K is aggregate labor demand by the intermediate goods sector. Imperfect substitutability between varieties implies that the intermediate goods market is characterized by monopolistic competition. Each producer maximizes profits and faces a price elasticity of demand demand equal to $(1 + \phi)/\phi$. As a result, all firms charge the same price

$$p_K = (1 + \phi)w, \tag{2}$$

where w denotes the wage rate. Profits of intermediate goods producers are used to cover the costs of obtaining a patent. Combining (2) with the intermediate goods production function, we obtain an expression for individual profits:

$$\pi = p_K k - wk = \frac{\phi w K}{N}. \tag{3}$$

[4]Appendix A.1 derives the relative factor demand by solving the profit maximization problem of final good producers.

Firms in the perfectly competitive backstop technology sector use labor to produce energy according to the production function $H = \eta L_H$, where L_H denotes aggregate labor demand by the backstop technology sector. The price of one unit of the backstop equals its marginal cost:

$$p_H = \frac{w}{\eta}. \tag{4}$$

2.2 Research and Development

Research and development (R&D) undertaken by firms in the research sector leads to the invention of new intermediate goods varieties. Following Romer (1990), we assume that the stock of public knowledge evolves in accordance with the number of invented intermediate goods. New varieties are created according to

$$\dot{N} = \frac{1}{a} L_R N, \tag{5}$$

where L_R denotes labor allocated to research and a is a productivity parameter. The right hand side of (5) features the stock of public knowledge, to capture the 'standing on shoulders effect': researchers are more productive if the available stock of public knowledge is larger (cf. Romer, 1990). Moreover, we assume spillovers from the stock of public knowledge to the backstop technology sector by imposing $A_H = N^{\phi}$.[5] We define the innovation rate as $g \equiv \dot{N}/N$.

Free entry of firms in the research sector implies that whenever the cost of inventing a new variety, aw/N, is lower than the market price of a patent, p_N, entry of firms in the research sector will take place until the difference is competed away. Therefore, free entry gives rise to the following condition:

$$aw/N \geq p_N \qquad \text{with equality (inequality) if } g > 0 \ (g = 0). \tag{6}$$

Throughout, we restrict our attention to the case of a positive innovation rate. In equilibrium,

[5]The assumption of $A_H = N^{\phi}$ implies Hicks-neutral technological change between intermediate goods and the backstop technology. Technically, the assumption ensures that the energy income share is constant in the backstop regime, as discussed in Section 3.3. Making this assumption is equivalent to assuming that the backstop is produced by using final output instead of labor.

investors earn the market interest rate r on their investment in patents:

$$\pi + \dot{p_N} = r p_N, \tag{7}$$

By combining (2), (3), (6), and (7), we obtain an expression for the return to innovation:

$$r = \frac{\phi}{a} K + \hat{w} - g \text{ if } g > 0, \tag{8}$$

where hats denote growth rates. The return to innovation depends positively on K, because of a market size effect. The term $\hat{w} - g$, takes account of the change in the patent price over time. The parameter a has a negative effect on the return to innovation, because it is related negatively to the productivity of researchers. The parameter ϕ has a positive effect, because of its positive relationship with the mark-up on the price of intermediate goods.

2.3 Factor Markets

Equilibrium on the labor market requires that aggregate labor demand from the intermediate goods sector, the backstop technology sector, and the research sector equals the fixed labor supply $L_K + L_H + L_R = K + \frac{H}{\eta} + ag = L$. We define the income shares of energy and intermediate goods, and the expenditure shares of the backstop technology and the resource in total energy costs as follows:

$$\theta \equiv \frac{p_E E}{p_Y Y}, \qquad 1 - \theta = \frac{p_K K}{p_Y Y}, \qquad \omega \equiv \frac{p_H H}{p_E E}, \qquad 1 - \omega = \frac{p_R R}{p_E E}. \tag{9}$$

Using these definitions together with (2) and the backstop production function, labor market equilibrium implies:

$$K = \frac{1 - \theta}{(1 + \phi)\omega\theta + 1 - \theta} \left(L - ag \right). \tag{10}$$

Resource extraction depletes the resource stock S according to:

$$\dot{S}(t) = -R(t), \quad S(0) = S_0, \quad R(t) \geq 0, \quad S(t) \geq 0, \tag{11}$$

which implies that total extraction cannot exceed the initial resource stock.

2.4 Households

The representative household lives forever, derives utility from consumption of the final good, and inelastically supplies L units of labor at each moment. It owns the resource stock with value $p_R S$ and all equity in intermediate goods firms with value $p_N N$. The household maximizes lifetime utility $U(t) = \int_t^\infty \ln Y(z) e^{-\rho(z-t)} dz$, subject to its flow budget constraint $\dot{V} = r(V - p_R S) + \dot{p}_R S + wL - p_Y Y$, and the transversality condition $\lim_{z\to\infty} \lambda(z) V(z) e^{-\rho z} = 0$, where ρ denotes the pure rate of time preference, V total wealth, and λ the shadow price of wealth.[6]

Straightforward manipulations of the standard first-order conditions for the optimization problem of the representative household yield two familiar rules:[7]

$$\hat{p}_Y + \hat{Y} = r - \rho, \tag{12a}$$

$$\hat{p}_R = r. \tag{12b}$$

The first one, (12a), is the Ramsey rule, which relates the growth rate of consumer expenditures to the difference between the nominal interest rate and the pure rate of time preference. Equation (12b) is the Hotelling rule, which ensures that owners of the resource stock are indifferent between (i) selling an additional unit of the resource and investing the revenue at the interest rate r, and (ii) conserving it and earn a capital gain at rate $\hat{p}_R$.

3 Dynamics of the Model

In this section, we discuss the dynamics of the model. Because the resource and the backstop technology are perfect substitutes, only the cheapest of the two will be used at a particular moment in time. If the two energy sources have equal prices, simultaneous use may occur. Therefore, three different regimes of energy use exist: a fossil regime, a simultaneous use regime, and a backstop regime. We proceed by first describing the dynamic behavior of the economy during each regime. Subsequently, we describe the energy transition by linking the regimes together.

[6] Note that final output cannot be stored, so that consumption equals output.

[7] Appendix A.1 derives the solution to the utility maximization problem of the representative household.

3.1 The Fossil Regime

In the fossil regime, energy generation relies exclusively on the natural resource. The model described in Section 2 with $H = 0$ imposed can be condensed to a three-dimensional block-recursive system of differential equations in the energy income share θ, the innovation rate g, and the reserve-to-extraction rate $y \equiv S/R$. The system is block-recursive in the sense that the system of θ and g can be solved independently from y. Beyond simplifying the mathematical analysis, the re-expression of the model in terms of θ, y, and g also helps to clarify the economics behind our results. These variables, namely, have a clear interpretation as they are indicators of energy scarcity and the rate of technological progress. In this section, we analyze the (θ, g)-subsystem described in Lemma 1, and we postpone the solution of the differential equation for y until Section 3.5.

Lemma 1 *Provided that $g(t) > 0$, the dynamics in the fossil regime are described by the following two-dimensional system of first-order differential equations in $\theta(t)$ and $g(t)$:*

$$\dot{\theta}(t) = \theta(t)[1 - \theta(t)](1 - \sigma)\left[r(t) - \hat{w}(t) + \phi g(t)\right], \tag{13a}$$

$$\dot{g}(t) = \left[\frac{L}{a} - g(t)\right]\left\{\rho + \theta(t)(1 - \sigma)\phi g(t) - [1 - \theta(t)(1 - \sigma)]\left[r(t) - \hat{w}(t)\right]\right\}, \tag{13b}$$

where the term $r(t) - \hat{w}(t)$ is a function of $g(t)$:

$$r(t) - \hat{w}(t) = \phi\frac{L}{a} - (1 + \phi)g(t). \tag{13c}$$

Proof. See Appendix A.2. □

Equation (13a) shows that the energy income share increases if the price per unit of energy increases relative to the price of intermediate goods, i.e. if $r - \hat{w} + \phi g > 0$. Expression (13b) is derived from the labor market equilibrium (10), which requires that the innovation rate declines if the input of labor in the intermediate sector $L_K = K$ increases.

3.2 Simultaneous Use Regime

The simultaneous use regime is characterized by equal effective prices of the resource and the backstop technology. As a result, the energy income share will be constant and the innovation rate will be declining over time, as shown in Lemma 2.

Lemma 2 *In the simultaneous use regime, the income share of energy θ remains constant and is equal to*

$$\theta_S = \left[[\eta(1+\phi)]^{1-\sigma} \left(\frac{1-\bar{\theta}}{\bar{\theta}} \right)^{\sigma} + 1 \right]^{-1}. \tag{14a}$$

The innovation rate is decreasing over time, according to the following differential equation

$$\dot{g} = -g(\phi g + \rho). \tag{14b}$$

Proof. See Appendix A.2. □

Intuitively, as long as $\theta < \theta_S$, the resource is relatively cheaper than the backstop technology so that only the resource will be used for energy generation. If $\theta = \theta_S$, effective prices of the resource and the backstop technology are equal, which enables a regime of simultaneous use as long as θ remains constant. The declining innovation rate follows from the constant energy income share during the simultaneous use regime. A constant income share requires that the relative price of intermediate goods and energy remains unchanged: $r - \hat{w} + \phi g = 0$. As a result, K goes down over time, because the constant income share implies $\hat{K} = r - \hat{w} - \rho$. According to (8), g consequently needs to decline in order to ensure that $r - \hat{w} = -\phi g < 0$ remains satisfied: a decrease in g is needed to keep the return to innovation from dropping below the rate of interest as a result of the declining market size.

3.3 The Backstop Regime

The backstop regime is characterized by a constant energy income share and a constant innovation rate, as described in Lemma 3.

Lemma 3 *In the backstop regime, the energy income share θ and the innovation rate g remain constant and are equal to*

$$\theta_B = \theta_S, \tag{15a}$$

$$g_B = \left(\frac{L}{a} + \rho \right) \frac{\phi}{1+\phi}(1-\theta_B) - \rho. \tag{15b}$$

Proof. See Appendix A.2. □

Intuitively, Hicks-neutral technological change between the backstop and intermediate goods implies a constant energy income share. Given that the resource stock is depleted, innovation is the only remaining investment possibility. The constant income share of intermediate goods implies an unchanging return to innovation, resulting in a constant innovation rate over time.

3.4 The Energy Transition

Assuming that the initial stock is large enough to get $p_H(0)/A_H(0) > p_R(0)$, implying that $\theta(0) < \theta_S = \theta_B$, the economy will start in the fossil regime. Due to increasing scarcity and resource using technological change, the energy income share increases over time until this inequality is no longer satisfied. At this moment, the fossil regime will end. Depending on the productivity of the backstop technology and on characteristics of the innovation process, the switch to the backstop technology can either take place abruptly or gradually through an intermediate regime of simultaneous use. Both cases will be discussed in turn.

3.4.1 Abrupt Shift

By imposing $\dot{\theta} = \dot{g} = 0$ in (13a)-(13b), we obtain the following steady state loci in the fossil regime:

$$g|_{\dot{\theta}=0} = \phi\frac{L}{a}, \tag{16a}$$

$$g|_{\dot{g}=0} = \frac{\phi(L/a)\left[1 - (\theta(1-\sigma)\right] - \rho}{(1+\phi) - \theta(1-\sigma)} < g|_{\dot{\theta}=0}. \tag{16b}$$

Moreover, by combining (9) with (12a) and (12b) we get $\hat{R} = \hat{\theta} - \rho$, so that the resource extraction isocline can be written as

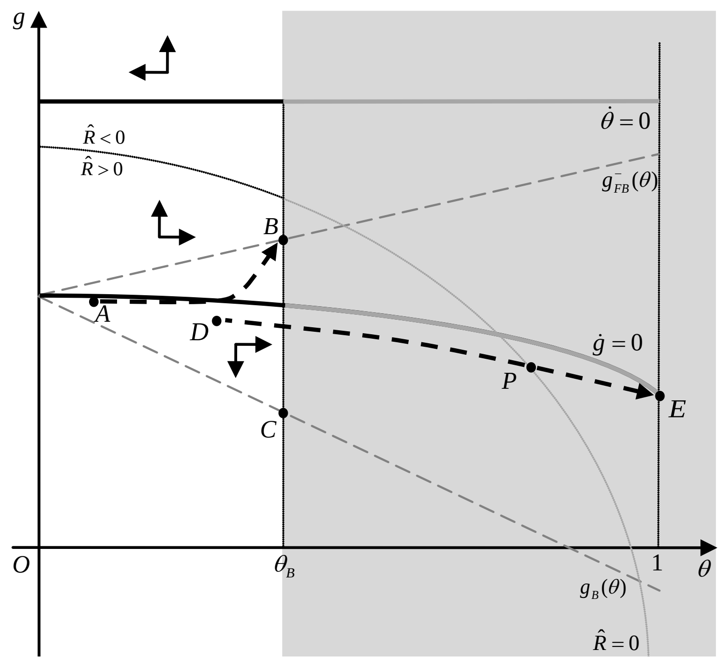
$$g|_{\hat{R}=0} = g|_{\dot{\theta}=0} - \frac{\rho}{(1-\sigma)(1-\theta)}, \tag{16c}$$

where we have used (13a) and (13c) to substitute for $\hat{\theta}$. Figure 1 shows the phase diagram for the fossil regime in (θ, g)-space. The growth rates of the effective prices of intermediate goods and energy are equal along the flat $\dot{\theta} = 0$ line, leading to constant income shares. At points below the income share locus, the effective price of energy relative to the intermediate goods increases, i.e. $r - \hat{w} + \phi g > 0$, so that the income share of energy rises over time and *vice versa*. The dynamic behavior of θ is illustrated by the horizontal arrows in the phase diagram. The real interest rate in the fossil regime can be found by combining $\hat{p}_Y = \theta r + (1-\theta)(\hat{w} - \phi g)$ and (13c), which gives

$$r - \hat{p}_Y = (1-\theta)\left(\phi \frac{L}{a} - g\right). \tag{17}$$

At points above the downward-sloping innovation locus, the real interest rate and output growth are lower than in steady state equilibrium, so that $L_K = K$ declines and the innovation rate increases over time and *vice versa*. The dynamic behavior of g is illustrated by the vertical arrows in the phase diagram. The figure also contains the dotted extraction isocline $\hat{R} = 0$, which slopes downward and has a vertical asymptote at $\theta = 1$. At points above the $\hat{R} = 0$ isocline, the real interest rate and therefore output growth are lower than required for constant extraction, so that extraction growth becomes negative and *vice versa*.

Figure 1: Phase Diagram: Fossil Regime



Notes: The dashed arrow from point A to point B represents the equilibrium path when a backstop technology is available. The shaded area of the phase diagram is not relevant when a backstop technology is available. The dashed arrow from point D to point E represents the equilibrium path when no backstop technology is available.

Without the existence of a backstop technology, the fossil regime lasts forever and the economy converges along the stable manifold from point D to point E in Figure 1.[8] This equilibrium path is characterized by an ever decreasing innovation rate and an energy income share that converges to unity. Peak-oil occurs at point P, where the equilibrium path crosses the extraction isocline. Because of the vertical asymptote of the extraction isocline at $\theta = 1$, resource use is necessarily declining in the long run. The occurrence of peak-oil, however, depends crucially on the elasticity of substitution between intermediate goods and energy. If this elasticity is high enough, the extraction isocline is located entirely below the equilibrium path. In that case, point P does not exist and extraction is declining throughout.[9]

When a backstop technology exists, however, the resource will not be used anymore if $\theta > \theta_B$, which is the case in the shaded area of the figure. In equilibrium, the resource will then be exhausted at the moment when θ hits θ_B and the economy will shift abruptly to the backstop technology. The negatively-sloped dashed line in the figure represents (15b) and gives g_B for each possible θ_B. Hence, point C shows the steady state equilibrium in the backstop regime, where the economy ends up immediately after the switch. The end point of the fossil regime can be found by using the Ramsey rule (12a), which implies that consumption should be continuous at each point in time as long as the interest rate is finite. Output in either regime can be written as

$$Y = N^{\phi} \left(\frac{1-\bar{\theta}}{1-\theta} \right)^{\frac{\sigma}{\sigma-1}} K.$$

Hence, given that prices and therefore income shares are continuous, due to the required continuity of output, K needs to be continuous as well. Accordingly, labor market equilibrium (10) with $\omega = 0$ before the switch and $\omega = 1$ after the switch gives

$$L - ag_{FB}^{-} = \left(\frac{1-\theta_B}{\theta_B(1+\phi)+1-\theta_B} \right) (L - ag_B),$$

where g_{FB}^{-} denotes the innovation rate just before the switch at time T_{FB} from the fossil to the backstop regime.[10] Substitution of (15b) into this expression yields the innovation rate

[8] Appendix A.3 shows that point E in Figure 1 is the only attainable steady state of the model without a backstop technology that satisfies the transversality condition.

[9] Note from (16c) that $\frac{\partial g|_{\hat{R}=0}}{\partial \sigma} < 0$ and $\lim_{\sigma \to 1} g|_{\hat{R}=0} = -\infty$, so that extraction would be declining throughout with Cobb-Douglas production.

[10] We use the conventional shortcut notation $x_j^{+} \equiv \lim_{t \downarrow T_{ij}} x_j(t)$ and $x_{ij}^{-} \equiv \lim_{t \uparrow T_{ij}} x_i(t)$.

at the end of the fossil regime:

$$g_{FB}^{-} = \frac{L}{a} - \frac{1-\theta_B}{1+\phi}\left(\frac{L}{a} + \rho\right). \tag{18}$$

The positively-sloped dashed line in Figure 1 gives g_{FB}^{-} for each possible value of θ_B. Point B denotes the end point (θ_B, g_{FB}^{-}) of the fossil regime. The equilibrium path that leads to the end point B is indicated by the dashed arrow starting at A. Along this path, the income share of energy is increasing over time and the innovation rate is higher than it would have been in an economy without the backstop technology available. The innovation rate is initially decreasing, but as soon as the economy crosses the innovation locus, the growth rate starts to increase until the moment of the regime switch. Intuitively, in order to prevent consumption from falling discontinuously when the resource stock is exhausted, the representative household now starts to increase savings when the regime switch comes near. In so doing, the household effectively smooths consumption by converting part of the resource wealth into knowledge, thereby transferring consumption possibilities to the future regime in which the resource stock is depleted. In the figure, the extraction path is upward-sloping throughout the fossil regime, as the equilibrium path is entirely located below the extraction isocline. At time T_{FB}, the economy shifts from the fossil to the backstop regime and the innovation rate jumps down to free enough labor for the production of energy with the backstop technology while keeping output unaffected. Note that the end point (θ_B, g_{FB}^{-}) is located below the $\dot{\theta} = 0$ line, i.e. $\frac{\theta_B - \phi^2}{1-\theta_B}\frac{L}{a} < \rho \Leftrightarrow g_{FB}^{-} < \phi\frac{L}{a}$, which is a necessary condition for the abrupt shift from the fossil to the backstop regime to occur. Proposition 1 summarizes the findings of this section:

Proposition 1 *If $\frac{\theta_B - \phi^2}{1-\theta_B}\frac{L}{a} < \rho$, the economy shifts abruptly from the fossil regime to the backstop regime and the innovation rate jumps down at T_{FB}.*

Proof. The case in which the economy relies upon the resource forever without switching to the backstop technology can be excluded, because eventually $\theta > \theta_B$ would hold, implying that the backstop technology is cheaper than the resource. Hence, there exists a time T_{FB} at which the fossil regime ends. Moreover, due to the inequality the end point of the fossil regime is located below the income share locus so that the price of the resource relative to the backstop keeps on rising until the stock is depleted, which implies that simultaneous use cannot take place, so that the economy shifts from the fossil to the backstop regime at T_{FB}. The downward jump in the innovation rate follows immediately when subtracting (15b) from (18), yielding:

$$g_{FB}^{-} - g_B = \theta_B \left(\frac{L}{a} + \rho \right) > 0. \quad \square$$

3.4.2 Gradual Transition

If the inequality in Proposition 1 is violated, the economy will not experience an abrupt shift from the fossil to the backstop regime. In this case, the shift in energy usage occurs more gradually, through a regime in which both energy sources are used simultaneously.

Proposition 2 *If $\frac{\theta_B - \phi^2}{1-\theta_B} \frac{L}{a} > \rho$, the economy shifts from the fossil regime to an intermediate regime of simultaneous use at T_{FS}.*[11] *Subsequently, the economy shifts from the simultaneous use to the backstop regime at $T_{SB} > T_{FS}$. The innovation rate is continuous and equal to*

$$g_{FS}^{-} = g_{FS}^{+} = \phi \frac{L}{a},$$

at T_{FS}. The innovation rate decreases during the simultaneous use regime and jumps down from

$$g_{SB}^{-} = \frac{\phi(1-\theta_S)\left(\frac{L}{a} + \rho\right)}{(1+\phi)(1-\phi)} > 0, \tag{19}$$

to g_B at T_{SB}. During the simultaneous use regime, the real interest rate equals zero, the backstop expenditure share ω increases, while resource extraction declines gradually over time.

Proof. See Appendix A.2. $\square$

The negatively-sloped dashed-dotted line in panel (a) of Figure 2 represents (19) and gives g_{SB}^{-} for each possible θ_B. The end point of the fossil regime is indicated by B. The economy moves along the equilibrium path from point A to point B during the fossil regime. The income share of energy increases over time, while the innovation rate again exhibits a non-monotonic time profile: it decreases initially but starts to increase once the economy has

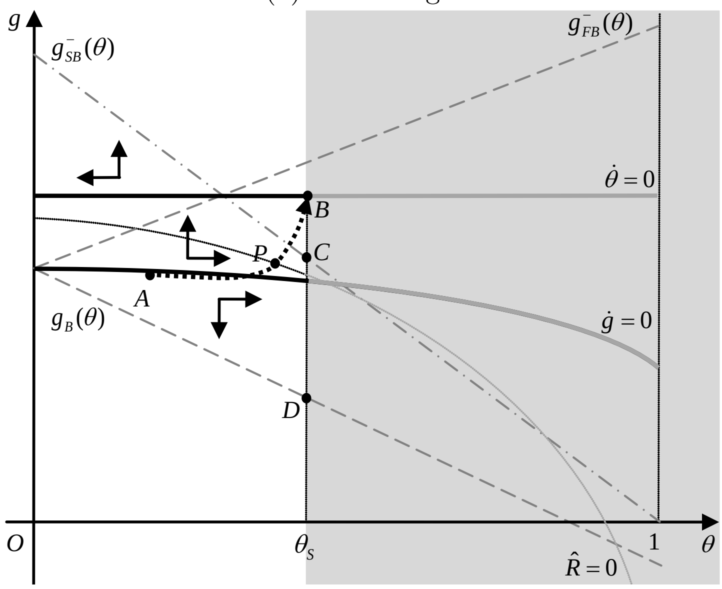
[11] If $\frac{\theta_B - \phi^2}{1-\theta_B} \frac{L}{a} = \rho$, the simultaneous use regime is degenerate with $T_{FS} = T_{SB}$.

passed the innovation locus. If the equilibrium path starts out below the extraction isocline, resource extraction peaks at point P and decreases afterwards. Once point B is reached, both the levels and the growth rates of the effective resource and backstop price are equal, and the economy shifts to the simultaneous use regime. The evolution of the innovation rate from g_{FS}^{+} to g_{SB}^{-} during this regime of simultaneous use of the resource and the backstop is indicated by the dashed arrow from point B to point C in panel (b). Once point C is reached, the economy shifts to the backstop technology and jumps to point D in panel (b) of the figure to free labor for the backstop production while leaving output unchanged.

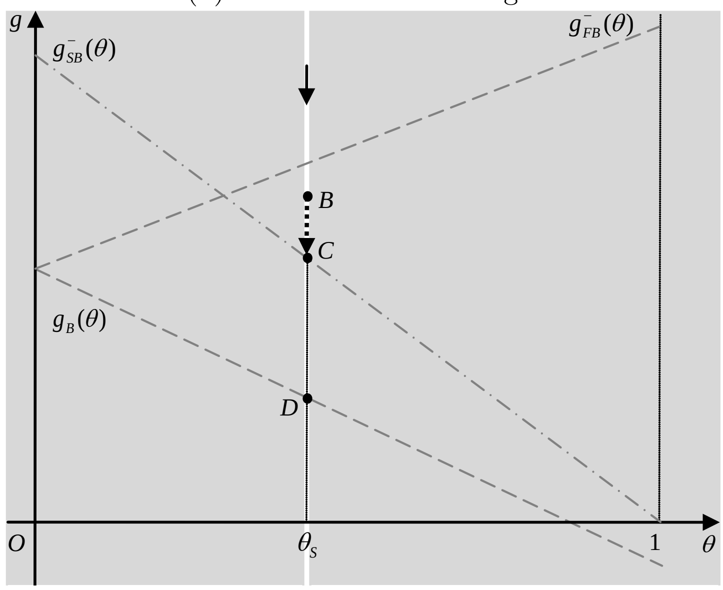
The existence of simultaneous use can be explained by the desire to smooth consumption between the different regimes of energy generation. Because resource conservation and investment in innovation each provide a channel for households to smooth consumption, the existence of a simultaneous use regime depends on the profitability of innovation (i.e., on ϕ) and on the costs of the backstop technology, (i.e., on θ_B through η). If innovation revenues would be zero (i.e, if $\phi = 0$), there would be no investment in R&D at all. Without investment in R&D, there necessarily exists a regime of simultaneous use. The reason is that if $g = 0$, $L = L_K + L_H$, so that any jump in L_H will imply a jump in L_K and hence in consumption and marginal utility. Therefore, L_H must gradually increase from zero to its long-run value. In a market equilibrium with positive R&D (when $\phi > 0$), labor market equilibrium reads $L = L_K + L_H + ag$ so that households have an additional way to smooth consumption: by reducing innovation at the time of the regime shift, labor becomes available for energy generation with the backstop technology without a need to reduce consumption. In scenarios with profitable innovation possibilities and a relatively cheap backstop technology, consumption smoothing may completely take place through this new channel: simultaneous use will not occur. If, however, innovation is less profitable, or the backstop technology is relatively expensive so that it will absorb a substantial amount of the labor supply after the regime switch, part of the consumption smoothing still takes place through a temporary regime of simultaneous use with a zero real interest rate, during which the production of the backstop technology starts from zero at the beginning of this regime and gradually increases, until it jumps up to its constant long-run level at time T_{SB}.

Figure 2: Phase Diagrams

(a) Fossil Regime



(b) Simultaneous Use Regime



Notes: In panel (a), the dotted arrow represents the equilibrium path of the fossil regime. In panel (b), the dotted arrow shows the equilibrium path of the simultaneous use regime. In both panels, the irrelevant parts of the phase diagrams are shaded in gray.

3.5 Initial Condition

To determine the initial value for the energy income share θ, i.e. to find the location of point A in Figures 1 and 2, we exploit the fact that total resource extraction over time should be equal to the initial resource stock. From the demand function (1) we derive a relationship between the income share and the reserve-to-extraction rate $y \equiv S/R$ at the beginning of the fossil regime:

$$\frac{\theta(0)}{1-\theta(0)} = \frac{\bar{\theta}}{1-\bar{\theta}} \left(\frac{y(0)\,[L - af(\theta(0))]\, N_0^{\phi}}{S_0} \right)^{\frac{1-\sigma}{\sigma}}, \tag{20}$$

where the function $g = f(\theta)$ is defined by the equilibrium path in (θ, g)-space. A second relationship between $\theta(0)$ and $y(0)$ is obtained by deriving from (1) a differential equation for y:

$$\dot{y} = -1 + \left[\rho + \frac{\omega}{1-\omega}\hat{\omega} - (1-\theta)(1-\sigma)\,(r - \hat{w} + \phi g)\right] y.$$

By using (8)-(10), $r - \hat{w} = -\phi g$, and $\theta = \theta_S$, this differential equation can be expressed in terms of y and ω in the simultaneous use regime. In the fossil regime, substitution of (13c) and $\omega = \hat{\omega} = 0$ yields an expression in terms of y, θ, and g. The end condition for y is given by $y(T_{FB}^{-}) = 0$ in the scenario without simultaneous use and by $y(T_{SB}^{-}) = 0$ in the scenario with simultaneous use. Using the already determined paths for θ, g, and ω, the differential equation for y gives a unique equilibrium path in (θ, y)-space. The initial point $(\theta(0), y(0))$ is given by the intersection of this equilibrium path with (20) in the (θ, y)-plane.

4 Numerical Illustration

In this section, we perform a simulation analysis to quantify the transitional dynamics of the model. We focus on a scenario in which an intermediate regime of simultaneous use exists. As a robustness check, we also provide simulation results for a formulation of our model in which the resource and the backstop technology are good instead of perfect substitutes, according to a CES function.[12] We first calibrate the model and then present the simulation results.

[12]The specification of the CES function is $E = \left(\bar{\omega}(A_H H)^{\frac{\gamma-1}{\gamma}} + (1-\bar{\omega})R^{\frac{\gamma-1}{\gamma}}\right)^{\frac{\gamma}{\gamma-1}}$ with $\bar{\omega} = 0.9$ and $\gamma = 50$.

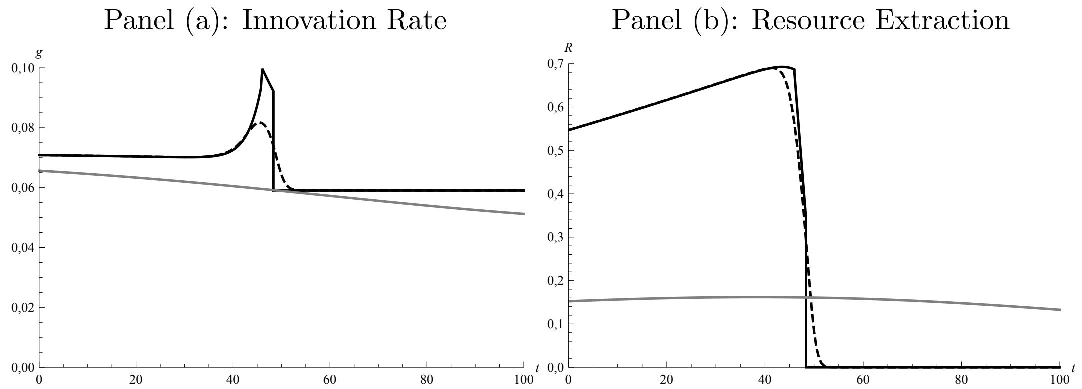
Van der Werf (2008) reports elasticities of substitution between a capital-labor aggregate and energy ranging from 0.17 to 0.61. We take the average of these values to obtain $\sigma = 0.4$. By choosing $\phi = 0.25$, we get a markup of prices over marginal costs within the range of estimation results that Roeger (1995) reports for the US manufacturing sector. We set the production function parameter $\bar{\theta}$ and the rate of pure time preference ρ to 0.1 and 0.01, respectively. Labor supply L and the initial knowledge stock N_0 are normalized to 1 and 0.1, respectively. The initial resource stock is chosen such that the initial share of resource expenditures in GDP $\theta(0)$ equals 8 percent, to match the average US energy expenditure share in GDP over the period 1970-2009 (U.S. Energy Information Administration, 2012). We use the research productivity parameter a to obtain an initial consumption growth rate $\hat{C}(0)$ of 1.7 percent, which is equal to the average yearly growth rate of GDP per capita in the US over the period 1970-2010 (The Conference Board, 2012). Our benchmark calibration implies an initial reserve-to-extraction rate of $y(0) = 55$, which lies within the range of the reserve-to-production ratios for oil, natural gas, and coal in 2008 of 44, 58, and 127, respectively (U.S. Energy Information Administration, 2012). Initially, the ratio between the per unit of energy price of the backstop technology and the resource amounts to 3.6.[13] The current era in which energy generation relies on the non-renewable resource ends in roughly 4 decades.[14]

The solid line in panel (a) of Figure 3 depicts the time profile of the innovation rate. The innovation rate first decreases slightly over time, but starts to increase after the stable manifold has crossed the innovation locus. During the simultaneous use regime, the innovation rate is declining. Once the shift to the backstop technology takes place, the innovation rate jumps down to its constant long-run level. The gray line shows the innovation rate in a world similar to the benchmark economy, but without the availability of a backstop technology. In contrast to the benchmark case, innovation in such a world decreases monotonically over time and starts out lower. The dashed line represents the time path in a model in which the resource and the backstop technology are good, but imperfect substitutes. The imperfect substitutes model yields time paths that, though smoother, are quite similar to those generated by our simpler model in which the resource and the backstop technology are perfect substitutes. Panel (b) shows that resource extraction is increasing initially, peaks just

[13] Using (1), this ratio is given by $\beta/\eta[\bar{\theta}/(1-\bar{\theta})]^{\frac{\sigma}{1-\sigma}}[(1-\theta(0))/\theta(0)]^{\frac{1}{1-\sigma}}$.

[14] The initial expenditures on innovation as a share of GDP is equal to 0.25. Given that expenditure on innovation is the only investment possibility in the model, this number should be interpreted as the aggregate investment share in the economy.

Figure 3: Time Profiles



Notes: The solid line represents the benchmark scenario, in which a backstop technology that provides a perfect substitute for the resource is available. The gray line represents the scenario in which there is no backstop technology available. The dashed line represents the scenario in which a backstop technology provides a good, but imperfect substitute for the resource. In the latter scenario, η is adjusted to obtain $\theta_\infty = \theta_B$.

before the economy switches to the simultaneous use regime and then decreases rapidly until the stock is exhausted. Due to the finite exhaustion time, extraction starts out considerably higher than in the model without a backstop technology.

5 Conclusion

We have investigated the effects of the availability of a backstop technology on the time paths of resource extraction and the rate of technological progress, taking into account that natural resources and man-made inputs are poor substitutes and that generation of energy with the backstop technology is costly. To this end, we introduce a non-renewable resource and a backstop technology in a simple endogenous growth model. The backstop technology can be used to produce a perfect substitute for the natural resource. Technological progress is driven by workers in R&D, who build upon previously generated knowledge. We solve the model analytically and develop a graphical apparatus to visualize its transitional dynamics and regime shifts. Moreover, we quantify the results by calibrating the model and performing a simulation analysis. The results are robust to relaxing the assumption of perfect substitutability between the resource and the backstop technology.

Our main findings can be divided into three categories: energy regimes, technological change, and resource extraction. Regarding the first category, we find that the economy experiences different regimes of energy generation. Initially, the economy relies exclusively on the natural resource. In the long run, the natural resource will be abandoned in favor of the backstop technology. In between these two regimes there may exist an intermediate era during which the resource and the backstop technology are used simultaneously. This feature is noteworthy, because the model does not impose the convexities in resource extraction or backstop production costs that are normally required to obtain this result. The reason for the existence of a regime of simultaneous use is the desire to smooth consumption: by introducing the backstop technology gradually during the simultaneous use regime, households effectively shift part of the resource wealth to the future.

Second, the introduction of a backstop technology in the model crucially affects the shape of the time path of technological progress, measured by the rate of innovation. Instead of monotonically decreasing as it would be without the backstop technology, the rate of innovation exhibits a non-monotonic development over time: it first decreases gradually, but during the run-up to the backstop technology it starts to increase. The reason for the surge in innovation is again consumption smoothing: by investing in innovation, households effectively convert resource wealth into knowledge, thereby shifting consumption possibilities to the future in which energy generation is costly. If the return to investment in innovation remains high enough, consumption smoothing entirely takes place through investment in innovation so that the simultaneous use regime disappears. Once the economy enters the backstop regime, the rate of innovation jumps down to its long-run value to release resources for production in the backstop sector. At any moment during the resource regime, the rate of innovation is strictly higher than it would have been without the availability of a backstop technology.

Third, the introduction of the backstop technology has notable implications for the development of resource extraction over time. The resource extraction path does no longer have to become downward-sloping eventually. Depending crucially on the elasticity of substitution between energy and man-made inputs, the extraction path can be monotonically upward-sloping or downward-sloping until exhaustion, or exhibit an internal maximum, known as 'peak-oil'.

The most important direction for further research is the introduction of stock-dependent extraction costs and pollution from combustion of the natural resource. In combination with the backstop technology these features make it interesting to compare the decentralized outcome to the social optimum, in order to shed light on optimal environmental policy. Another useful extension of the current analysis would be the introduction R&D activities in the resource and backstop sector, so that the direction of technological progress becomes endogenous.

Appendix

This Appendix contains the derivations of the mathematical results in the main text.

A.1 Household and Firm Behavior

The Lagrangian associated with the profit maximization problem of firms in the final output sector reads:

$$\mathcal{L} = p_Y \left[\bar{\theta} E^{\frac{\sigma-1}{\sigma}} + (1-\bar{\theta}) \left(\int_0^N k_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - \int_0^N p_{Kj} k_j dj - p_R R - p_H H$$
$$+ p_E \left(R + N^{\phi} H - E \right). \tag{A.1}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}}{\partial k_j} = p_Y \left[\bar{\theta} E^{\frac{\sigma-1}{\sigma}} + (1-\bar{\theta}) \left(\int_0^N k_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}} \right]^{\frac{\sigma}{\sigma-1}-1} (1-\bar{\theta}) K^{-\frac{1}{\sigma}} N^{-\phi\frac{1-\sigma}{\sigma}} - p_K = 0, \tag{A.2a}$$

$$\frac{\partial \mathcal{L}}{\partial E} = p_Y \left[\bar{\theta} E^{\frac{\sigma-1}{\sigma}} + (1-\bar{\theta}) \left(\int_0^N k_j^{\beta} dj \right)^{\frac{\sigma-1}{\beta\sigma}} \right]^{\frac{\sigma}{\sigma-1}-1} \bar{\theta} E^{-\frac{1}{\sigma}} - p_E = 0, \tag{A.2b}$$

$$\frac{\partial \mathcal{L}}{\partial R} = p_E - p_R \leq 0, \quad (p_E - p_R) R = 0, \tag{A.2c}$$

$$\frac{\partial \mathcal{L}}{\partial H} = p_E N^{\phi} - p_H \leq 0, \quad (p_E N^{\phi} - p_H) H = 0, \tag{A.2d}$$

where we have used $p_{Ki} = p_{Kj} \equiv p_K,\ \forall\ i, j$. Combining (A.2a)-(A.2d) with $H = 0$ ($R = 0$) gives the first (second) row in (1). The third row of (1) follows from combining (A.2a)-(A.2d) with $p_H N^{-\phi} = p_R$ imposed.

Before moving to the utility maximization problem of the representative household, we derive the flow budget constraint of the households. Total wealth is equal to $V = p_N N + p_R S$, so that the change in wealth is given by

$$\dot{V} = \dot{p}_N N + p_N \dot{N} + \dot{p}_R S + p_R \dot{S} = \dot{p}_N N + p_N \dot{N} + \dot{p}_R S - p_R R, \tag{A.3}$$

where the second equality uses (11). Nominal GDP can be written as

$$\begin{aligned} p_Y Y &= p_K K + p_R R + p_H H = \pi N + w L_K + p_R R + p_H H \\ &= r p_N N - \dot{p}_N N + w L_K + p_R R + p_H H, \end{aligned} \tag{A.4}$$

where the second and third equality use (3) and (7), respectively. Using (A.4) to substitute for $p_R R$ in (A.3), we obtain:

$$\dot{V} = p_N \dot{N} + \dot{p}_R S - p_Y Y + r p_N N + w L_K + p_H H = r p_N N + \dot{p}_R S + w L - p_Y Y, \tag{A.5}$$

where we have used (4), (5), (6), the labor market equilibrium, and the backstop production function for the second equality. Using the definition of wealth again, we get the flow budget constraint in the main text.

The Hamiltonian associated with the utility maximization problem of the representative household reads:

$$\mathcal{H} = \ln(C) + \lambda_V \left[r(V - p_R S) + \dot{p}_R S + w L - p_C C \right], \tag{A.6}$$

where λ_V denotes the shadow price of wealth. The necessary first-order conditions for an optimum are given by:

$$\frac{\partial \mathcal{H}}{\partial C} = 0 \Rightarrow \frac{1}{C} - \lambda_V p_C = 0 \Rightarrow \hat{C} + \hat{p}_C = -\hat{\lambda}_V, \tag{A.7}$$

$$\frac{\partial \mathcal{H}}{\partial S} = 0 \Rightarrow -\lambda_V r p_R + \lambda_V \dot{p}_R = 0 \Rightarrow \hat{p}_R = r, \tag{A.8}$$

$$\frac{\partial \mathcal{H}}{\partial V} = -\dot{\lambda}_V + \rho \lambda_V \Rightarrow \lambda_V r = -\dot{\lambda}_V + \rho \lambda_V. \tag{A.9}$$

Combining (A.7) and (A.9) gives the Ramsey rule (12a). The first order condition (A.8) is the Hotelling rule (12b).

A.2 Proofs of Lemmata and Propositions

Proof of Lemma 1. By substituting the labor market equilibrium (10) with $\omega = 0$ imposed into (8), we find expression (13c) for the return to innovation in the fossil regime. We use the expenditure share definitions in (9) to rewrite the first line of the relative demand function (1):

$$\frac{\theta}{1-\theta} = \left(\frac{p_R}{p_K}\right)^{1-\sigma} \left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma} N^{\phi(1-\sigma)} \Rightarrow \hat{\theta} = (1-\theta)(1-\sigma)\left[r - \hat{w} + \phi g\right]. \tag{A.10}$$

This completes the derivation of expression (13a) in Proposition 1. To obtain the second expression in the proposition, we first differentiate the labor market equilibrium condition (10) to get

$$\hat{K} = -\frac{\dot{g}}{\frac{L}{a} - g}. \tag{A.11}$$

By converting the energy income share definition (9) into growth rates while using the intermediate goods price (2) and the Ramsey rule (12a), we obtain:

$$\hat{\theta} = -\frac{1-\theta}{\theta}\left[\hat{w} + \hat{K} - (r-\rho)\right]. \tag{A.12}$$

Combining (A.10), (A.11), and (A.12), we find (13b) in Lemma 1. □

Proof of Lemma 2. In the simultaneous use regime, the effective prices of the resource and the backstop technology must be equal, as in the third line of (1):

$$p_H N^{-\phi} = p_R. \tag{A.13}$$

Substitution of $p_E = p_H N^{-\phi}$ and (A.13) into the third line of the relative demand function (1) and by using $p_K/p_H = \eta(1+\phi)$ from (2) and (4) gives

$$\frac{\theta}{1-\theta} = [(1+\phi)\eta]^{\sigma-1}\left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma}, \tag{A.14}$$

which implies $\theta = \theta_S$ and therefore proofs the first part of the lemma. To proof the second part, we convert (A.13) into growth rates:

$$\hat{p}_H - \phi g = \hat{p}_R \Rightarrow r - \hat{w} + \phi g = 0, \tag{A.15}$$

where the latter expression uses (4) and (12b). Substituting (A.15) into (8), we find

$$-\phi g = \phi \frac{K}{a} - g. \tag{A.16}$$

Using (2), (9), (12a), (12b) and $\hat{\theta} = 0$ together with (A.16), we obtain:

$$\hat{g} = \hat{K} = -\phi g - \rho, \tag{A.17}$$

which gives rise to the differential equation in Lemma 2. □

Proof of Lemma 3. Using $p_K/p_H = \eta(1+\phi)$ from (2) and (4), the relative factor demand function (1) gives

$$\left(\frac{\theta}{1-\theta}\right) = [\eta(1+\phi)]^{\sigma-1} \left(\frac{\bar{\theta}}{1-\bar{\theta}}\right)^{\sigma}, \tag{A.18}$$

which can be solved for θ to obtain θ_B. Combining the innovation return (8), the income share definition (9), labor market equilibrium (10), the Ramsey rule (12a), and the relative demand function (A.18), we find a differential equation for the innovation rate:

$$\dot{g} = -\left(\frac{L}{a} - g\right)\left[\phi\left(\frac{1-\theta_B}{\theta_B(1+\phi)+1-\theta_B}\right)\left(\frac{L}{a} - g\right) - g - \rho\right]. \tag{A.19}$$

Because this differential equation is unstable in g, the innovation rate immediately settles down at its steady-state value given by the second expression in Lemma 3. □

Proof of Proposition 2. The case in which the economy relies upon the resource forever without switching to the backstop technology can be excluded, because eventually $\theta > \theta_B$ would hold, implying that the backstop technology is cheaper than the resource. Hence, there exists a time at which the fossil regime ends. If there would not exist a regime of simultaneous use, continuity of consumption requires that the end point of the fossil regime would be given by (θ_B, g^-_{FB}). However, the inequality in the proposition implies $g^-_{FB} > \phi\frac{L}{a}$. Therefore, the dynamic path in the fossil regime that leads to (θ_B, g^-_{FB}) would necessarily intersect the vertical θ_B line before the fossil regime has ended. This would imply that only the resource is being used while the backstop technology is relatively cheaper, which violates optimality of the behavior of final good producers. As a result, there exists a time T_{FS} at which the economy shifts from the fossil to the simultaneous use regime. The simultaneous use regime cannot last forever, because the innovation rate is decreasing throughout a regime of simultaneous use, according to (14b), while (A.21) implies a strictly positive lower bound on g due to $\omega \leq 1$. Therefore, there exists a time $T_{SB} \geq T_{FS}$ at which the economy shifts from the simultaneous use to the backstop regime.

We continue by showing that the innovation rate is continuous at T_{SB}. Together with the labor market equilibrium (10) with $\omega^-_{FS} = 0$, the continuity of output requires:

$$L - ag^-_{FS} = \frac{1-\theta_B}{\omega^+_{FS}\theta_B(1+\phi) + 1 - \theta_B}\left(L - ag^+_{FS}\right). \tag{A.20}$$

Substituting the labor market equilibrium (10) into the innovation return equation (8) and noting that $r - \hat{w} = -\phi g$, we get:

$$\omega = \frac{\phi L - ag}{ag(1+\phi)(1-\phi)}\frac{1-\theta_S}{\theta_S}. \tag{A.21}$$

Using this relationship to substitute for ω^+_{FS} in (A.20), the matching condition reduces to

$$L - ag^-_{FS} = \frac{a}{\phi}(1-\phi)g_B. \tag{A.22}$$

We have already argued that the innovation rate g^-_{FS} cannot exceed $\phi(L/a)$. Moreover, given that $\omega \geq 0$, it follows from (A.21) that the innovation rate g^+_{FS} cannot exceed $\phi(L/a)$ either. Consequently, the only solution to (A.22) reads $g^-_{FS} = g^+_{FS} = \phi(L/a)$.

To proof the downward jump of the innovation rate at T_{SB}, first note that, as a result of the required continuity of output, the labor market equilibrium (10) with $\omega_B = 1$ implies:

$$\frac{1-\theta_S}{\omega^-_{SB}\theta_S(1+\phi) + 1 - \theta_S}\left(L - ag^-_{SB}\right) = \frac{1-\theta_B}{\theta_B(1+\phi) + 1 - \theta_B}\left(L - ag_B\right).$$

Substitution of (A.21) for ω_{SB}^{-} on the left hand side and (15b) for g_B on the right hand side, gives equation (19), where $g_{SB}^{-} > 0$ follows from $\phi < 1$, which is required for the simultaneous use regime to exist.[15] Subtracting (15b) from (19), we find:

$$g_{SB}^{-} - g_B = \frac{L\phi^2(1-\theta_B) + a\left[(1-\phi)(1+\phi) + \phi^2(1-\theta_B)\right]\rho}{a(1-\phi)} > 0,$$

implying that the innovation rate jumps down at T_{SB}.

Finally, we show that the real interest rate equals zero, that the backstop expenditure share increases, and that resource extraction decreases over time during the simultaneous use regime. Using $\hat{p}_Y = \theta r + (1-\theta)(\hat{w} - \phi g)$, the real interest rate can be written as

$$r - \hat{p}_Y = -\theta_S\left(r - \hat{w} + \phi g\right) = 0, \tag{A.23}$$

where the second equality follows from (A.15). Taking the time derivative of (A.21), we find:

$$\hat{\omega} = \frac{L/a}{L/a - g}\frac{\beta(1-\theta_S) + \omega\theta_S}{\omega\theta_S}(\phi g + \rho) > 0. \tag{A.24}$$

By using the expenditure share definition (9), the Hotelling rule (12b), the backstop price (4), and $\hat{E} = \hat{K} = -\phi g - \rho$, we obtain:

$$\hat{R} = -\frac{\omega}{1-\omega}\hat{\omega} - \rho = -\frac{\phi^2(1-\theta_S)L + a\left[(1-\phi)(1+\phi) + \phi^2(1-\theta_S)\right]\rho}{a\theta_S(1-\phi)(1-\omega)(1+\phi)} < 0, \tag{A.25}$$

where the last equality uses (A.21) and the labor market equilibrium (10). □

A.3 Steady States

Here we show that point E in Figure 1 is the only attainable steady state of the model without a backstop technology that satisfies the transversality condition.

Proposition 3 *The only attainable internal steady state of the model without a backstop technology that satisfies the transversality conditions is given by point E in Figure 1.*

[15] The simultaneous regime can only exist if the g_{FB}^{-} line intersects the income share locus of the fossil regime, which requires $\phi < 1$.

Proof. Using asterisks (*) to denote steady state values of this model, the other three steady states of the model satisfy:

$$g^* = \frac{L}{a}, \quad \theta^* = 1, \tag{A.26a}$$
$$g^* = \frac{L}{a}, \quad \theta^* = 0, \tag{A.26b}$$
$$g^* = \frac{\phi}{1+\phi}\frac{L}{a} - \beta\rho, \quad \theta^* = 0. \tag{A.26c}$$

The first two steady states (A.26a) and (A.26b) do not satisfy the transversality condition, because substitution of $K^* = L - ag^* = 0$ into (13c) implies $(r - \hat{w})^* = -g^* < 0$ and the transversality condition in growth rates requires:

$$\lim_{t\to\infty} = \hat{p}_N(t) + \hat{N}(t) - r(t) \leq 0 \Rightarrow \lim_{t\to\infty} r(t) - \hat{w}(t) \geq 0, \tag{A.27}$$

where the second inequality uses (6) and (12b). Hence, the two steady states with $(r - \hat{w})^* = -g^* < 0$ do not satisfy the transversality condition. Steady state (A.26c) is located at the intersection of the innovation locus with the $\theta = 0$ line, and below the income share locus in (θ, g)-space. It is immediately clear from the dynamics around this point in Figure 1 ($\dot{\theta} > 0$) that this steady state cannot be attained. The economy can only be situated here if there is an infinite amount of oil available from the beginning (so that $\theta^* = 0$), which is impossible. Point E in Figure 1 satisfies the transversality condition, as $(r - \hat{w})^* = \rho > 0$ in this equilibrium. □

A.4 Initial Condition

By using (8)-(10), $r - \hat{w} = -\phi g$, and $\theta = \theta_S$ in the simultaneous use regime, and (13c) and $\omega = \hat{\omega} = 0$ in the fossil regime, the differential equation for y can be expressed as:

$$\begin{aligned}
\dot{y} &= -y(1-\theta)(1-\sigma)\left[\phi\tfrac{L}{a} - g\right] + y\rho - 1, && \text{if } t < T_{FS} \\
\dot{y} &= \tfrac{L\phi^2(1-\theta_S) + a\left[(1-\phi)(1+\phi)+\phi^2(1-\theta_S)\right]\rho}{a\theta_S(1-\phi)(1-\omega)(1+\phi)} y - 1, && \text{if } T_{FS} \leq t \leq T_{SB}.
\end{aligned}$$

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