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The Methodology of Modern Macroeconomics and the Descriptive Approach to Discounting: a Thought Experiment

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Abstract

Critics of modern macroeconomics often raise concerns about unwarranted welfare conclusions and data mining. This paper illustrates these concerns with a thought experiment, based on the debate in environmental economics about the appropriate discount rate in climate change analyses: I set up an economy where a social evaluator wants to determine the optimal time path of emission levels, and seeks advice for this from an old-style neo-classical macroeconomist and a new neo-classical (modern) macroeconomist; I then describe how both economists analyze the economy, their policy advice, and their mistakes. I then use the insights from this thought experiment to point out some pitfalls of the modern macroeconomic methodology.

Keywords: modern macroeconomics, methodology, prescriptive, desriptive, discount rate

JEL Codes: B22, B41, E13, O44, Q52, Q54

1 Introduction

This paper presents a thought experiment to illustrate some concerns about the methodology of modern macroeconomics. The thought experiment is inspired by the recent debate among climate change economists about the appropriate way to discount the costs and benefits of environmental policy.

It is good to be clear at the outset about the objective of this paper: its sole objective is to illustrate some methodological pitfalls of modern macroeconomics; its objective is *not* to argue that the modern macroeconomic methodology should be given up. Every methodology has its drawbacks, and an alternative methodology may not be unambiguously superior. Nevertheless, it is important to identify the risks and weaknesses of a methodology, and to understand their implications. This paper aims to contribute to this.

In this introduction, I first review the main methodological principles of modern macroeconomics, and why they are criticized. I then summarize the debate among climate change economists about the descriptive and prescriptive approach to discounting, and explain how it is related to the debate about the methodology of modern macroeconomics. Finally, I sketch this paper's thought experiment and give a preview of its main insights.

1.1 The methodology of modern macroeconomics

Modern macroeconomics (or *new* neo-classical macroeconomics) arose in the wake of the Lucas critique on the dominant macroeconomic paradigm in the 1950s and the 1960s, which I will refer to (for lack of a better name) as *old* or *old-style* neo-classical macroeconomics. Old-style neo-classical macroeconomists simply posited some stylized relations between aggregate variables in their models. Lucas (1976), however, pointed out that relations between aggregate variables may not be stable: they may change if policy changes, especially if economic agents behave strategically in anticipation of or in response to policy changes. So to make sure that their models are immune against the Lucas critique, modern macroeconomists build their models from microfoundations: they specify the objectives and the constraints of a number of forward-looking agents, they then derive their optimal behavior, and then impose an equilibrium device to derive the aggregate behavior of the economy.[1]

[1]This is what is usually understood with the term *microfoundations*. Note, however, that microfoundations were also implicit in much of the work by old-style neo-classical economists (such as James Tobin and Franco Modigliani). Hoover (2009) traces back the role of microfoundations in macroeconomics to the very origin of the terms *microeconomics* and *macroeconomics*, when they were coined by (presumably) Ragnar Frisch in the 1930s.

Unfortunately, these microfoundations often lack a strong empirical foundation: the amount of behavioral complexity, heterogeneity and institutional detail in most modern macroeconomic models is too small to establish clear links with microeconomic evidence; and the available microeconomic evidence is a cheese with many holes (Hansen and Heckman, 1996; and Browning et al., 1999). Modern macroeconomic models therefore combine a set of maintained assumptions about the microfoundations for which there is little or no evidence (for instance, the assumption that aggregate consumption can be derived by maximizing the utility of a representative household), with values of deep structural parameters (such as those that describe the representative household's preferences) that are often[2] calibrated or estimated such that the aggregate behavior of the model mimics macroeconomic data.

The microfoundations in modern macroeconomic models are therefore not descriptively realistic; at best, they mimic some aspects of the microeconomic reality.[3] But, according to modern macroeconomists, this does not necessarily invalidate these models as descriptive models of the macro-economy: modern macroeconomic models are tested not by testing their microfoundations with microeconomic data, but "by subjecting them to shocks for which we are fairly certain how actual economies, or part of economies would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions" (Lucas, 1980).

This methodology has two implications, however, which are the source of much of the criticism that has recently been raised against modern macroeconomics.

The first implication is that, notwithstanding their explicit microfoundations, modern macroeconomic models may not reveal much about the preferences of the population: as their microfoundations are not meant to be descriptively realistic, the calibrated or estimated preference parameters may have substantially different values than their real world counterparts.[4] Never-

[2]...often, but not always: there are several examples where key parameters of the microfoundations do have clear links with microeconomic evidence, such as Storesletten et al. (2004), and Bartelsman et al. (2009); Caballero (2010) gives several other examples from what he calls "the periphery of modern macroeconomics".

[3]Prescott (2006) illustrates this as follows: "In the case of production technology, the nature of the aggregate production function in the empirically interesting cases is very different from that of the individual production units being aggregated. The same is true for the aggregate or a stand-in household's utility function in the empirically interesting cases." And Sims (2012) argues that the way how price stickiness is modeled in DSGE models is "clearly at odds with empirical micro evidence or common sense".

[4]Prescott (2006) illustrates this by pointing out that the aggregate labor supply elasticity

theless, modern macroeconomists sometimes seem to claim that the calibrated or estimated preference parameters in their microfoundations do reveal the preferences of the population, and use them accordingly to draw welfare conclusions - which is criticized by, among others, Hoover (2006), Wren-Lewis (2007) and Atkinson (2009).[5]

A second implication of the modern macroeconomic methodology follows from its instrumentalist nature. Since Friedman (1953), instrumentalism in economics usually refers to the position that assumptions do not need to be true in order to be useful for explaining and predicting economic phenomena. In this spirit, modern macroeconomics entertains the hypothesis that a set of microfoundations that are descriptively unrealistic may nevertheless be useful for explaining and predicting the macro-economy. But it is not clear how this set of microfoundations is defined: the microfoundations and their parameterization differ from model to model, and there does not seem to be a core that is truly invariant across the modern macroeconomic literature. This causes concerns about data mining,[6] which have been raised by both critics and practitioners of modern macroeconomics alike - see, for instance, Solow (2008), Caballero (2010), Chari et al. (2009), Kocherlakota (2010) and Sims (2012).[7]

The objective of this paper's thought experiment is to clarify the concerns about unwarranted welfare conclusions and data mining, and to show how they may interact with each other. The thought experiment is inspired by the debate among environmental economists about the appropriate discount rate for climate change analyses, which I briefly summarize in the next subsection.

(which is used to calibrate the preferences of the representative household in many modern macroeconomic models) is much greater than the individual labor supply elasticity estimated in microeconomic studies.

[5]Atkinson (2009) goes even further: even if these preference parameters revealed the population's preferences, it still would be highly questionable whether it is appropriate to use them in welfare analysis.

[6]In this paper, I use the word "data mining" to refer to the practice of making assumptions (about the microfoundations and their parameterization) that are not supported by evidence from outside the model (not even by instrumentalist evidence that these assumptions seem to work in other models), but that are chosen simply to make sure that the model mimics the data which it is supposed to explain.

[7]...even though opinions differ about how serious and widespread this problem is. For instance, Solow (2008) and Caballero (2010) worry about data mining in DSGE's in general; Chari et al. (2009), Kocherlakota (2010) and Sims (2012) focus only on the New-Keynesian variant of DSGE's.

1.2 The prescriptive and the descriptive approach to discounting

Much of the literature on climate change economics revolves around the question how the costs and benefits of climate change policy should be discounted. According to most environmental economists, an appropriate consumption discount rate (which translates future consumption into equivalent values of current consumption) can be found with the Ramsey rule. The Ramsey rule expresses the consumption discount rate as the sum of two components: the subjective discount rate (also called the *utility* discount rate, to capture the idea that most people seem to care less about future felicity than about current felicity); and (the absolute value of) the elasticity of the marginal social value of consumption *times* the growth rate of consumption (to capture the idea that the marginal social value of consumption decreases as societies grow richer). But economists disagree about how we should pin down appropriate values for these two components.

To fix ideas, let us assume that there is full agreement about appropriate values for the elasticity of the marginal social value and the future growth rate of consumption,[8] and let us focus on the subjective discount rate.

According to economists such as Ramsey (1928), Pigou (1932), Harrod (1948), Sen (1982) and Cline (1992), the choice of an appropriate subjective discount rate is not an economic question but an ethical issue, and should be set as low as possible - possibly even zero. This *prescriptive approach* is followed by Sir Nicholas Stern in his *Stern Review* (Stern, 2007), where he uses a subjective discount rate of a mere 0.1%.

Many other economists follow the *descriptive approach*, however, and argue that the consumption discount rate should be set equal to the rate of return on capital and that the Ramsey rule then reveals the subjective discount rate of the population[9] - see, for instance, Tol and Yohe (2006), Nordhaus (2007), Weitzman (2007) and Mendelsohn (2007) in their critique on the Stern Review. According to most adherents of the descriptive approach, the revealed

[8]...which, of course, is not the case: Dasgupta (2008) gives an account of the disagreements.

[9]Note that there is a difference between the descriptive approach and the Kaldor-Hicks compensation principle. According to the Kaldor-Hicks compensation principle, the rate of return on capital is the appropriate consumption discount rate to search for potential Pareto-improvements (in some settings, at least). According to the descriptive approach, at the other hand, the rate of return on capital is the socially optimal consumption discount rate. So unlike the Kaldor-Hicks compensation principle, the descriptive approach implicitly assumes that consumption is allocated across time in a socially optimal way. Goulder and Williams (2012) explain the difference in more detail.

subjective discount rate is somewhere between 1 and 3%.[10]

But to claim that the subjective discount rate of the population is revealed by the rate of return in the financial markets requires a very specific set of microfoundations, for which there is no empirical evidence. So the descriptive approach to discounting seems to be an example of drawing unwarranted welfare conclusions from descriptively unrealistic microfoundations - the first concern about the modern macroeconomic methodology which I raised in the previous subsection.[11]

This paper's thought experiment clarifies this concern and shows how it results from data mining - the second concern about modern macroeconomics which I raised above.

1.3 An overview of the thought experiment

I now give a brief overview of the thought experiment, and a preview of its main insight for the modern macroeconomic methodology.

In the next section, I set up a fictitious economy, where production pollutes the environment, and where environmental degradation leads to abatement costs for the government. I assume overlapping generations, which introduces some heterogeneity among households. This heterogeneity has two effects. First, it obscures the link between the interest rate and the households' subjective discount rate. Second, it establishes a link between the interest rate and the environmental quality: as the environmental quality deteriorates and abatement costs for the government increase, the national saving rate goes down, and the interest rate increases.

I then consider a *social evaluator* who wants to determine the optimal path of emission allowances. This social evaluator faces a trade-off between high emission levels which allow firms to produce a lot today, or low emission levels to avoid the impact of environmental degradation in the future - a trade-off which he wants to settle based upon a subjective discount rate. As the social evaluator does not understand well how the economy works, he seeks advice

[10]Note that the difference between a subjective discount rate of almost 0% and one in the range of 1-3% turns out to be very important: Weitzman (2007), for instance, argues that this is the main reason why the Stern Review calls for sharp and immediate reductions in greenhouse gas emissions, while cost benefit analyses based on market-based subjective discount rates such as Tol (2002a, 2002b) and Nordhaus (2008) yield much more moderate policy implications.

[11]The descriptive approach to discounting is actually one of the examples which Atkinson (2009) used to illustrate how welfare statements in modern macroeconomics often rely on undiscussed assumptions. Nelson (2008) and Dietz et al. (2009), among several others, raise similar objections.

from an old-style neo-classical and a new neo-classical (modern) macroeconomist.

In section 3, I describe the advice of the old-style neo-classical economist. I will assume that he does not know much about the household sector and therefore simply assumes that aggregate saving is a constant fraction of aggregate disposable income. Nevertheless, it turns out that his model captures some essential features of the economy; and recognizing that he does not know the social evaluator's subjective discount rate, he computes the optimal emissions policy for a range of possible discount rates.

In section 4, I consider the analysis of a new neo-classical economist, a modern macroeconomist who attempts to model the economy with microfoundations.[12] The modern macroeconomist, not having much empirical information about the household sector either, introduces an immortal representative household in his model, endows her with a utility function, and derives her optimal consumption and saving decisions. He then finds that the relation between the representative household's subjective discount rate, the interest rate and the growth rate of aggregate consumption is described by the Ramsey rule, which he exploits to calibrate the representative household's subjective discount rate. Claiming that his model has revealed the population's subjective discount rate,[13] he then advises the social evaluator to adopt this subjective discount rate, and traces out the corresponding optimal emissions policy.

But his microfoundations are descriptively not realistic, which causes a fallacy of composition: even though applying the Ramsey rule to household data would reveal the subjective discount rate of an individual household,[14] this is not the case with aggregate data. As a result, the modern macroeconomist derives a subjective discount rate that bears no clear relation with the true preferences of the population.

The source of this problem is data mining: the modern macroeconomist selects a value for the subjective discount rate simply to make sure that his model mimics aggregate data, without having any evidence for his maintained assumptions and without being able to test whether the calibrated value for the subjective discount rate is consistent with other data.[15]

[12]It is a dogmatic and short-sighted modern macroeconomist. Real world modern macroeconomists may well be more enlightened.

[13]Even though this is essentially what the descriptive approach to discounting entails, it is not clear whether Prescott would approve of this, given the quote from his Nobel Prize lecture in footnote 3.

[14]...at least in this thought experiment: the real world appears to be far more complicated (see, for instance, Frederick et al, 2002).

[15]In the real world, deriving a reasonable value for the subjective discount rate from microeconomic evidence also appears to be extremely problematic (see Browning et al., 1999).

Furthermore, because of data mining, he does not notice a key misspecification in his model: when the interest rate is higher than what he had predicted, he does not reject the maintained assumptions in his microfoundations and continues to assume that the world is populated by a representative household; and instead of realizing that the higher interest rate is the result of a decline in national saving because of environmental degradation and higher abatement costs, he ascribes the higher interest rate to an increase in the subjective discount rate of the representative household.

This may be a good illustration of a potential pitfall of the modern macroeconomic methodology: the danger is that a modern macroeconomist who is confronted with data that differ from what he had predicted, ascribes this to an unexpected change in some unobservable structural parameter, rather than rejecting the maintained assumptions in his microfoundations for which he does not have strong empirical evidence - which reduces the scope for learning from mistakes, and which may lead to misguided policy advice. It is fair to point out that the set-up of this thought experiment clearly stacks the deck against the modern macroeconomist (as the key parameter is unobservable, leading the modern macroeconomist astray); another set-up of the economy may shuffle the cards in a different way. But it does illustrate Caballero's remark (Caballero, 2010) that a good (descriptive) macroeconomic model is one where "the main object of study is anchored by sensible assumptions" - which is not the case in the analysis of the fictitious modern macroeconomist in this thought experiment, and which may not always be the case in real world modern macroeconomic models either.

The rest of the paper is organized as follows. In section 2, I set up the economy and derive the optimal environmental policy of a social planner, given his subjective discount rate. In sections 3 and 4, I describe how an old-style neo-classical macroeconomist and a modern macroeconomist would analyze this economy, how they would derive their policy advice, and which mistakes they would make. Section 5 concludes. Please keep in mind, however: a thought experiment is a caricature.

2 An economy with pollution

The thought experiment takes place in a simple economy where production causes environmental damage, and where environmental damage leads to production losses and abatement costs for the government; the extent to which

In addition, people seem to have different discount rates for different types of intertemporal trade-offs (see Frederick et al., 2002).

production pollutes the environment depends on the level of emissions, which is determined by the government's environmental policy.

I first present the set-up of the model. I then show how the economy's steady state depends on the steady state emission level. I conclude this section by assuming a social welfare function and by deriving how the optimal environmental policy depends on the social evaluator's subjective discount rate; this will then serve as a benchmark to assess the policy advice of the old-style neo-classical and the new neo-classical (modern) macroeconomists in sections 3 and 4.

2.1 The set-up

The model is set up in three steps. First, I explain the relation between production, pollution and environmental quality, and its consequences for the taxes that are needed to cover the government's abatement costs. I then describe the economy's population and their consumption and saving decisions. I complete the economy's set-up by characterizing its equilibrium.

2.1.1 Production, pollution, the environmental quality, abatement costs and taxes

The supply side of the economy is described by the production decisions of a representative firm, operating under perfect competition. The representative firm produces output Y according to a Cobb-Douglas production function with capital K, technology A and labor input L; in addition, the production function depends on the emissions E that are allowed by the government and on the environmental quality M:

$$Y_t = E_t^{\varepsilon} M_t^{\mu_Y} K_t^{\alpha} (A_t L_t)^{1-\alpha} \tag{1}$$

...where the subscript t denotes the time period, and $0 < \alpha < 1$. E and M are measured on a scale from 0 to 1 and ε and μ_Y are both positive: $\varepsilon > 0$ and $\mu_Y > 0$. So the higher the permitted emission level E and the better the environmental quality M, the more the representative firm can produce.

A period lasts very long (several decades). I therefore assume that the capital stock fully depreciates within a period, such that next period's capital stock is always equal to current period's investment. In addition, I assume that the state of technology grows at an exogenous rate g, and that labor input remains constant over time and is normalized to 1:

$$K_{t+1} = I_t \tag{2}$$

$$
\begin{aligned}
A_{t+1} &= A_t(1+g) && (3)\\
L_t &= 1 && (4)
\end{aligned}
$$

The firm hires labor and invests in new capital taking as given the real wage w, the real interest rate r, the permitted emission level E and the environmental quality M. Profit maximization yields then the following first-order conditions:

$$
\begin{aligned}
(1-\alpha)Y_t &= w_t && (5)\\
\alpha\frac{Y_t}{K_t} &= 1+r_t && (6)
\end{aligned}
$$

The environmental quality M not only affects aggregate production, it also determines the abatement costs G which the government has to incur. I assume that these abatement costs are proportional with aggregate output; and as the government balances its budget in every period, taxes T are proportional with aggregate output as well:

$$
G_t = T_t = \tau_t Y_t \qquad \text{where} \quad \tau_t = z - \zeta M_t^{\mu_Z} \tag{7}
$$

...with $0 < \zeta < z < 1$ and $\mu_Z > 0$. So the lower the environmental quality, the higher are the abatement costs for a given production level and the higher is the share of taxes in aggregate income. Note that as M is measured on a scale from 0 to 1, τ_t is always between z (for $M_t = 0$) and $z - \zeta$ (for $M_t = 1$).

I assume that initially, in period 0, the economy is in a steady state where the environmental quality is optimal, and where emissions do not pollute the environment; consequently, the government sets E to its maximum level in period 0. From period 1 onwards, however, emission levels affect the dynamics of M according to the following law of motion:

$$
M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t \quad \text{for } t \geq 1 \tag{8}
$$

...where $0 < \phi < 1$ and $0 < \psi < 1 - \phi$. So from period 1 onwards, the government faces a trade-off between setting a high emission level and allowing firms to produce a lot today, or setting a low emission level to protect the environment and avoid the impact of environmental degradation on production and government spending in the future.

2.1.2 Consumption and saving

Households live for two periods. In the beginning of every period, a new generation is born, and at the end of every period, the oldest generation dies. In the first period of life, households supply labor, earn labor income, pay a lump sum tax, and consume part of their disposable income; the rest of their disposable income is saved to finance their consumption in their second period of life, when they are retired.

I assume that all households have the same preferences. The consumption and saving decisions of the generation born in period t can then be derived by maximizing the utility function of a representative household,

$$U_t = \ln c_{1,t} + \frac{1}{1+\theta} \ln c_{2,t+1} \quad \text{with } \theta > 0 \tag{9}$$

subject to her lifetime budget constraint,

$$c_{1,t} + \frac{1}{1+r_{t+1}} c_{2,t+1} = w_t - T_t \tag{10}$$

...where $c_{1,t}$ and $c_{2,t+1}$ are her consumption in her first and second period of life, and T_t are the lump sum taxes which she has to pay when she is young; θ is her subjective discount rate. As the representative household supplies one unit of labor when she is young, her labor income is equal to w_t, and her disposable income in her first period of life is $w_t - T_t$. Note that I assume that households do not leave bequests.

Utility maximization leads then to the following expressions for $c_{1,t}$ and $c_{2,t+1}$:

$$c_{1,t} = (1 - s_Y)(w_t - T_t) \tag{11}$$

$$c_{2,t+1} = (1 + r_{t+1}) s_Y (w_t - T_t) \tag{12}$$

...where s_Y is the saving rate of the young generation:

$$s_Y = \frac{1}{2+\theta} \tag{13}$$

2.1.3 Equilibrium

In every period the goods market clears, such that aggregate saving is always equal to aggregate investment. Saving by the young generation in period t is $s_Y(w_t - T_t)$; saving by the old generation is zero, as they only have capital income, which they completely consume. Equilibrium in the goods market therefore implies that

$$s_Y(w_t - T_t) = I_t \tag{14}$$

Substituting the firm's first-order condition (5) in equation (14), and using (7) to eliminate T_t, shows then how aggregate investment depends on aggregate output:

$$I_t = s_Y (1 - \alpha - \tau_t) Y_t \tag{15}$$

Aggregate consumption C_t, which is the sum of the consumption of the young generation and the elderly, can be found as follows: substitute equation (14) in the expression for the consumption of the elderly, equation (12), and recall that $I_t = K_{t+1}$ according to the law of motion (2); we then find that $c_{2,t+1} = (1+r_{t+1})K_{t+1}$; now rewrite this equation and the consumption equation for the young generation, equation (11), by exploiting the firm's first-order conditions (5) and (6), and eliminate T_t with equation (7). We then find how aggregate consumption depends on aggregate output:

$$\begin{aligned} C_t &= c_{1,t} + c_{2,t} \\ &= (1-s_Y)(1-\alpha-\tau_t)Y_t + \alpha Y_t \\ &= [(1-s_Y)(1-\alpha-\tau_t)+\alpha]\,Y_t \end{aligned} \tag{16}$$

The gross interest rate $1+r_{t+1}$ follows from the firm's first-order condition (6): use the fact that $I_t = K_{t+1}$, and use (15) to express I_t as a function of Y_t. We then find that

$$1 + r_{t+1} = \alpha \frac{1+g_{t+1}}{s_Y(1-\alpha-\tau_t)} \tag{17}$$

where g_{t+1} is the growth rate of aggregate output from period t to period $t+1$: $1+g_{t+1} = Y_{t+1}/Y_t$. For the further discussion, it is useful (but not necessary) to assume that $\alpha/(s_Y(1-\alpha)) > 1$, such that the interest rate is always higher than the growth rate of aggregate output.

I therefore conclude that aggregate investment and aggregate consumption are negatively affected by τ, the share of taxes in aggregate income. And as the tax share increases as the environmental quality goes down, we find that the share of aggregate income that is invested or consumed is lower the more the environment has been destroyed.

Similarly, given the growth rate of aggregate output, the interest rate is positively affected by τ - which means that environmental degradation increases the extent to which the interest rate is above output growth.

2.2 The steady state as a function of the emission level

I assume that the economy starts off in period 0 in a steady state where pollution is not a concern for policy makers. So in period 0, the environmental quality is optimal, emissions do not pollute the environment, and the government consequently sets the permitted emission level to its maximum value:

$$M_0 = E_0 = 1 \tag{18}$$

As the environmental quality is optimal, the share of aggregate income that goes to taxes is at its lower bound:

$$\tau_0 \quad = \quad z - \zeta \tag{19}$$

Substituting in equations (15) and (17) shows that the share of investment in aggregate income is at its highest possible level,

$$\frac{I_0}{Y_0} \quad = \quad s_Y(1 - \alpha - \tau_0) \tag{20}$$

while the interest rate is at its lowest possible level

$$1 + r_0 \quad = \quad \alpha \frac{1 + g}{s_Y(1 - \alpha - \tau_0)} \tag{21}$$

(where I use the fact that the steady state growth rate of aggregate output is equal to the technological growth rate g).

For future reference, I define $\bar{Y}_t$, the output level which the economy would attain in period t if it could grow along this initial steady state without suffering any environmental degradation or cuts in emission levels. The value for $\bar{Y}_t$ follows from substituting (21) in the first-order condition (6), combined with the production function (1) where E_t and M_t are assumed to be equal to 1:

$$\bar{Y}_t \quad = \quad \left(\frac{s_Y(1 - \alpha - \tau_0)}{1 + g} \right)^{\frac{\alpha}{1-\alpha}} A_t \tag{22}$$

The economy will not attain $\bar{Y}_t$, however: from period 1 onwards, the dynamics of M are given by the law of motion (8), and the government has to weigh the costs and benefits of the emission levels which it allows. Suppose that the economy converges to a new steady state, which depends on the government's environmental policy, and let E^* be the emission level which the government allows in this new steady state. From (8) follows then the new steady state value of the environmental quality:

$$M^* \quad = \quad 1 - \frac{\psi}{1 - \phi} E^* \tag{23}$$

Substituting in equation (7) yields the new steady state share of taxes in aggregate income:

$$\tau^* \quad = \quad z - \zeta M^{*\mu z} \tag{24}$$

Substituting in equations (15) and (17), taking into account that aggregate output grows at rate g in the new steady state, gives the new steady state values of the investment share and the interest rate:

$$\frac{I_t^*}{Y_t^*} = s_Y(1-\alpha-\tau^*) \tag{25}$$

$$1+r^* = \alpha\frac{1+g}{s_Y(1-\alpha-\tau^*)} \tag{26}$$

From (26), the firm's first-order condition (6) and the production function (1) follows then the new steady state level of aggregate output:

$$Y_t^* = \left(\frac{s_Y(1-\alpha-\tau^*)}{1+g}\right)^{\frac{\alpha}{1-\alpha}} \left(E^{*\varepsilon}M^{*\mu_Y}\right)^{\frac{1}{1-\alpha}} A_t \tag{27}$$

The percentage output loss compared with the case where the economy could move along the initial steady state, without suffering any environmental degradation or cuts in emission levels, follows from equations (22) and (27):

$$\begin{aligned}\triangle Y^* &= \frac{\bar{Y}_t - Y_t^*}{\bar{Y}_t} \\ &= 1-\left(\frac{1-\alpha-\tau^*}{1-\alpha-\tau_0}\right)^{\frac{\alpha}{1-\alpha}} \left(E^{*\varepsilon}M^{*\mu_Y}\right)^{\frac{1}{1-\alpha}} \end{aligned}\tag{28}$$

This expression identifies three reasons why the economy moves to a lower output level if the environment is affected by emissions: first, the higher tax share τ^* lowers the investment share, and therefore also the steady state capital stock; second, the steady state emission level E^* is lower; and third, the steady state level of the environmental quality M^* is lower.

2.3 Optimal environmental policy

Let us now consider a *social evaluator* (a private citizen, a government official, perhaps even an economist), who wants to figure out the optimal time path of emission allowances E once the environment starts getting polluted as of period 1.[16]

I assume that all social evaluators agree that the optimal path of emission allowances can be found by maximizing a social welfare function, given by the

[16] I deliberately do not assume a social planner. The title of "social planner" suggests wide powers, including the power to determine government saving. The social evaluator, in contrast, takes the workings of the economy as given, including the assumption that the government budget is balanced in every period.

present discounted value of a stream of logarithmic felicity specifications of aggregate consumption:

$$W_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\rho}\right)^{s-t} \ln C_s \quad \text{with } \rho > 0 \tag{29}$$

...where ρ is called *the social evaluator's subjective discount rate.*

Note that ρ may well be different from the households' subjective discount rate θ. The households' subjective discount rate shows how households trade off consumption based felicity when they are young with consumption based felicity when they are retired. So it determines their personal consumption and saving decisions over their own lifetime. It does not say anything about how they would trade off aggregate consumption of the generations that are currently alive with aggregate consumption of the generations that are alive at some point in the future.

Therefore, the best thing we can do at this point is to derive the optimal path for emission allowances for a range of possible values of the social evaluator's subjective discount rate, given the firms' and the households' production and consumption behavior, and given the relations between the emission levels, the environmental quality, production, abatement costs and taxes. Note that I will thus assume full knowledge about the set-up of the economy as described in section 2.1. I will then use this analysis as a benchmark in sections 3 and 4, where I will assess the policy advice of old-style neo-classical and new neo-classical economists who do not have full knowledge about the economy's set-up and who are therefore forced to make some simplifying assumptions in their models.

So let us maximize the social welfare function (29) as of period 1, subject to the aggregate investment and consumption functions (15) and (16), the aggregate production function (1), the tax function (7), the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8), and taking as given the values for the state variables K, A and M in period 1:

$$\begin{aligned} W(K_t, A_t, M_t) &= \max_{E\in(0,1]} \left\{ \ln C_t + \frac{1}{1+\rho} W(K_{t+1}, A_{t+1}, M_{t+1}) \right\} \\ \text{subject to} \quad & C_t = [(1-s_Y)(1-\alpha-\tau_t)+\alpha]\, Y_t \\ & I_t = s_Y (1-\alpha-\tau_t)\, Y_t \\ & Y_t = E_t^{\varepsilon} M_t^{\mu_Y} K_t^{\alpha} (A_t L_t)^{1-\alpha} \\ & \tau_t = z - \zeta M_t^{\mu_Z} \\ & K_{t+1} = I_t \end{aligned} \tag{30}$$

$$
\begin{aligned}
A_{t+1} &= A_t(1+g) \\
M_{t+1} &= 1+\phi(M_t-1)-\psi E_t \\
K_1 &= I_0, \quad A_1 = A_0(1+g) \quad \text{and} \quad M_1 = 1
\end{aligned}
$$

In appendix A, I solve this dynamic programming problem, I sketch a numerical procedure to derive the transitional dynamics if the model is parameterized, and I derive the new steady state to which the economy will converge. Note for future reference that the optimal steady state emission level, denoted by E^*, depends on the social evaluator's subjective discount rate ρ in such a way that

$$
\frac{\varepsilon}{E^*} \;\geq\; \frac{\psi}{1+\rho-\phi}\frac{1}{M^*}(\mu_Y+\mu_Z\lambda^*) \quad \text{and} \quad E^* \;\leq\; 1 \quad \text{with c.s.} \tag{31}
$$

where "c.s." stands for "complementary slackness", M^* is the optimal steady state level of the environmental quality, and

$$
\begin{aligned}
\lambda^* \;&=\; \frac{1+\rho-\alpha}{1+\rho}(1-s_Y)\lambda_c^* + \frac{\alpha}{1+\rho}s_Y\lambda_I^* \\
&\text{with} \quad \lambda_c^* = \frac{\zeta M^{*\mu_Z}}{(1-s_Y)(1-\alpha-z+\zeta M^{*\mu_Z})+\alpha} \\
&\text{and} \quad \lambda_I^* = \frac{\zeta M^{*\mu_Z}}{s_Y(1-\alpha-z+\zeta M^{*\mu_Z})}
\end{aligned} \tag{32}
$$

Eliminating M^* with equation (23) yields an equation in E^*, which can be solved numerically if the model is parameterized. Once we have E^*, we can use equations (23), (24), (25), (26) and (28) to compute the optimal steady state values of the environmental quality, the tax share, the investment share, the interest rate, as well as the aggregate output loss compared with the case where emissions do not pollute the environment.

Let us illustrate this with a numerical example. Let us assume that one period lasts for 30 years. I set the capital share of aggregate income, α, to 1/3. Assuming 1.5% technological growth annually, I set $1+g = 1.015^{30}$. I choose $\phi = 0.95$, such that the half-life of a shock in environmental quality is $30*\ln 0.5/\ln 0.95 \approx 400$ years. μ_Y and μ_Z are both set to 0.5. I assume that government spending in period 0 is 20% of aggregate output; and I assume that if the government forever keeps the emission allowance at its maximum level after period 0, the maximum output loss due to environmental degradation amounts to 10%, while the extra abatement costs for the government are another 10% of aggregate production. To satisfy these assumptions, I set $\psi = 0.0095$, $z = 1.2$ and $\zeta = 1$. I then choose s_Y such that aggregate investment is initially 20% of aggregate output. Finally, I assume that $\varepsilon = 0.01$: in this way, the optimal environmental policy if the social evaluator's subjective discount rate is 0, would lead the economy to a steady state where aggregate output is about 5% below what it would be if the economy could simply continue growing along the initial steady state without any environmental degradation or cuts in emission levels.

The graphs in figure 1 show then how the social evaluator's (annualized) subjective discount rate affects the optimal policy's transitional dynamics and steady state values of the emission level E, the environmental quality M, the tax share τ, the investment share I/Y, the (annualized) interest rate r, and the percentage output loss compared with the case where the economy could move along the initial steady state without suffering any environmental degradation or cuts in emission levels, which is given by $(Y - \bar{Y})/\bar{Y}$. The transitional dynamics are given for 50 periods; the new steady state values are projected on the back plane of the graphs. The transitional dynamics for some selected values of the subjective discount rate ($\rho \in \{0.005, 0.01, 0.015, 0.02, 0.025\}$) are traced out in bold.

The first graph shows that a higher subjective discount rate leads to higher emission levels, which, according to the second graph, causes faster deterioration of the environmental quality. The next three graphs show how this leads to a higher tax share, a lower investment share, and a higher interest rate. The last graph illustrates the trade-off which the social evaluator faces: the higher the social evaluator's subjective discount rate, the lower the impact is on aggregate output in the short run (because of higher emission levels), but the larger the output losses will be in the long run (because of more environmental degradation).

It is important to note that I do not claim that the parameter values which I choose and the graphs in figure 1 are realistic. As a matter of fact, the main motivation of this paper is precisely to point out that the micro-foundations in macroeconomic models are almost always so much simplified that a clear relation with the available real-world micro-economic evidence is hard if not impossible to establish. In this respect, this paper is no exception: it is meant as a thought experiment, not as a positive model.

3 An old-style neo-classical analysis

In the previous section, I derived the social evaluator's optimal environmental policy. But this required that he has a full understanding of how the economy works, which is typically not the case. So let us assume that the social evaluator turns to two experts for advice: an old-style neo-classical economist (economist A) and a new neo-classical (modern) macroeconomist (economist B).

I assume that both economists know the interaction between production, pollution, environmental quality, abatement costs and taxes, as described in subsection 2.1.1; they are also aware that all markets always clear. But unfortunately, they don't know much about the household sector: they lack sufficient micro-economic evidence to model the household sector in detail; and even if they had sufficient micro-economic evidence, they suspect that there is so much heterogeneity and behavioral complexity in the household sector that they would succumb to the curse of dimensionality if they tried to aggregate the consumption and saving decisions of all the individual households to model the macro-economy. So both economist A and economist B are forced to make some drastic simplifications. In this section, I describe how economist A does this. I will then describe in the next section the approach of economist B.

Being an old-style neo-classical economist, economist A tries to find some stylized relations between aggregate variables. For instance, looking at data of aggregate consumption, aggregate output and government spending and taxes, he may find it reasonable to assume that in the long run aggregate saving is more or less a constant fraction of aggregate disposable income, just as in the Solow model (Solow, 1956). Let us assume that his estimate of this constant aggregate saving rate is such that it always perfectly matches the most recent data that are available.[17] As he knows that aggregate saving always equals aggregate investment, his period T estimate of the aggregate saving rate, $\hat{s}_T$, is then given by

$$\hat{s}_T = \frac{I_T}{Y_T - T_T} \tag{33}$$

Armed with this estimate of the aggregate saving rate and recalling that taxes are proportional with aggregate income according to equation (7), economist A then designs a model of the economy for periods $t \geq T$ which features the following aggregate investment and consumption functions:

$$I_t = \hat{s}_T(1-\tau_t)Y_t \tag{34}$$
$$C_t = (1-\hat{s}_T)(1-\tau_t)Y_t \tag{35}$$

Combining equation (34) with the capital stock's law of motion (2) and the

[17] I make this assumption for convenience, but it may have a touch of realism: every period lasts very long (several decades), so if there are no good reasons to believe that there are long cycles, any change in the observed saving rate may well be perceived to be permanent.

firm's first-order condition (6) yields the interest rate equation in his model:

$$1 + r_{t+1} \;=\; \alpha \frac{1 + g_{t+1}}{\hat{s}_T(1 - \tau_t)} \tag{36}$$

Suppose now that economist A wants to use his model to advise the social evaluator on the optimal path for emission allowances as of period 1. He would then take the social welfare function (29) as of period 1, and maximize it subject to the aggregate production function (1), the tax function (7), the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8) (which is all common knowledge), and subject to the investment and consumption functions (34) and (35) which he has estimated in period 0, starting from the values for the state variables K, A and M in period 1 (which are known at the end of period 0):

$$\begin{aligned} W(K_t, A_t, M_t) \;&=\; \max_{E_t \in [0,1]} \left\{ \ln C_t + \frac{1}{1+\rho} W(K_{t+1}, A_{t+1}, M_{t+1}) \right\} \\ \text{subject to} \quad & C_t = (1 - \hat{s}_0)(1 - \tau_t) Y_t \\ & I_t \;= \hat{s}_0 (1 - \tau_t) Y_t \\ & Y_t = E_t^{\varepsilon} M_t^{\mu_Y} K_t^{\alpha} (A_t L_t)^{1-\alpha} \\ & \tau_t = z - \zeta M_t^{\mu_Z} \\ & K_{t+1} \;= I_t \\ & A_{t+1} \;= A_t(1+g) \\ & M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t \\ & K_1 = I_0, \quad A_1 = A_0(1+g) \quad \text{and} \quad M_1 = 1 \end{aligned} \tag{37}$$

Economist A then does his computations, and derives the transitional dynamics and the new steady state as documented in Appendix B. Note that according to his policy advice, the economy will eventually settle down in a steady state which depends on the social evaluator's subjective discount rate in such a way that

$$\frac{\varepsilon}{E^*} \;\geq\; \frac{\psi}{1 + \rho - \phi} \frac{1}{M^*} (\mu_Y + \mu_Z \tilde{\lambda}^*) \quad \text{and} \quad E^* \;\leq\; 1 \quad \text{with c.s.} \tag{38}$$

where

$$\tilde{\lambda}^* \;=\; \frac{\zeta M^{*\mu_Z}}{1 - z + \zeta M^{*\mu_Z}} \tag{39}$$

He then eliminates M^* with equation (23), solves for E^*, and uses equations (23), (24), (25), (26) and (28) to compute the new steady state values of the environmental quality, the tax share, the investment share, the interest rate, as well as the aggregate output loss compared with the case where the economy could move along the initial steady state without loss in environmental quality and emission cuts.

Assuming the same parameter values as in the previous section, economist A then goes back to the social evaluator in the beginning of period 1 with the graphs in figure 2, to show the implications of different values of the social evaluator's subjective discount rate ρ for the transitional dynamics and steady state values; the transitional dynamics for some selected values of the subjective discount rate are traced out in bold. Note, however, that economist A does not take a stand about the subjective discount rate: he cannot, because he doesn't observe it in his macroecomomic data set.

The transitional dynamics in figure 2 look very similar as the transitional dynamics of the optimal policy in figure 1: a higher discount rate leads to higher emission levels and faster environmental degradation, which drives up the tax share and the interest rate, and gradually depresses the investment share; and economist A recognizes the trade-off which the government faces between setting low emission levels to avoid the impact of environmental degradation on aggregate output in the long run, or setting high emission levels which allow firms to produce more in the short run.

How well does economist A do?

Figure 3 compares the steady state predictions of his advice with the steady states if the optimal policy is followed, for different values of the social evaluator's subjective discount rate ρ: the thin red curves are the steady states if the optimal policy is followed, which were also projected at the back plane of the graphs in figure 1; the broken green curves are economist A's predictions as of period 1, which are taken from the back plane of the graphs in figure 2 - note that in the first three graphs in figure 3, the broken green curves coincide with full thick green curves (which will be introduced in a moment).

Naturally, economist A makes some mistakes. His computation of the optimal steady state emission level is not completely correct, for instance: the green curve in the upper left graph in figure 3 deviates a little bit from the thin red curve, which is a consequence of the fact that $\tilde{\lambda}^*$ in equation (39) is not exactly the same as λ^* in equation (32). The reason for this mistake is that economist A assumes that *all* income is taxed. But that is not true: in fact, only the income of the young generation is taxed - which has implications for the effect on aggregate consumption of a change in the tax rate due to environmental degradation. Because of equations (23), (24), (25), (26) and (28), this mistake spills over in all the other graphs of figure 3.

Furthermore, recall that environmental degradation causes higher abatement costs, such that the tax rate steadily increases until the new steady state is reached. But as only the income of the young generation is taxed, and as this is the only generation that saves, the *aggregate* saving rate will in fact go down as the economy moves to the new steady state - and not remain constant as economist A assumes. As a result, economist A, armed with his period 0 estimate of the aggregate saving rate $\hat{s}_0$, overestimates the steady state value of the investment share, and underestimates the steady state interest rate and the output loss - which follows immediately from equations (25), (26) and (28), and which is apparent from the last three graphs in figure 3.

But economist A learns from his mistakes: in the next period, as environmental degradation will have pushed up the tax rate, he will observe a lower aggregate saving rate and revise his estimate of the aggregate saving rate accordingly - and he will continue to do so in all subsequent periods, until the economy reaches a new steady state.

I illustrate this with the thick green curves in figure 3. Let us assume that the social evaluator's subjective discount rate ρ is 1.5% annually, and that the time path of emission levels is set according to economist A's policy recommendations for this particular value of ρ.[18] As economist A will steadily revise his estimate of the aggregate saving rate downwards, the curve that shows his predictions of the new steady state values for the investment share will steadily move downwards as well, while the curves that represent his predictions for the new steady state values of the interest rate and aggregate output loss will steadily move upwards. This goes on until the economy reaches a new steady state, where economist A's estimate of the aggregate saving rate turns out to be correct. In this new steady state (reached after the emission levels have consistently been set following economist A's policy advice for a subjective discount rate of 1.5% annually), the steady state predictions of economist A for a *range* of values of ρ are given by the thick green curves in figure 3. So eventually, his steady state predictions turn out to be almost perfect, as long as the social evaluator does not suddenly prefer a totally different subjective discount rate.[19]

[18] The only reason why I choose a value for ρ of 1.5% is that this is right in the middle of the horizontal axis in figure 3, which helps to make the graphs more transparent.

[19] Note, however, that the optimal emission level will still be computed inaccurately, because of the discrepancy between $\tilde{\lambda}^*$ and λ^*.

4 A new neo-classical (modern) analysis

New neo-classical or modern macroeconomists, aware of the Lucas critique, find the old-style neo-classical approach of the previous section *ad hoc*. Simply looking for some stylized relations between aggregate variables makes the analysis vulnerable for the Lucas critique. Furthermore, it is sometimes argues, as old-style neo-classical economists remain silent about the households' objectives, they cannot carry out a proper welfare analysis: old-style neo-classical economists have to impose a social welfare function, without being sure that it is somehow based on the preferences of the economic agents.

Modern macroeconomists therefore propose to build macroeconomic models from microfoundations. But just as old-style neo-classical economists, they have to make simplifying assumptions. A typical modern macroeconomic approach, especially for questions where the focus is on the supply side, is then to assume that the economy is populated by households who live infinitely long and all have the same preferences and budget constraints - which implies that their consumption and saving decisions can be derived by maximizing the utility of an immortal *representative* household.

So let us consider a modern macroeconomist (economist B) who assumes that the economy is populated by a representative household. The representative household maximizes the utility function

$$U_t = \sum_{s=t}^{\infty} \left(\frac{1}{1+\theta^R}\right)^{s-t} \ln C_s \quad \text{with } \theta^R > 0 \tag{40}$$

subject to the budget constraint and the transversality condition

$$K_{t+1} = K_t(1+r_t) + w_t - T_t - C_t \tag{41}$$

$$\lim_{s\to\infty} \Pi_{s'=t+1}^{s} \frac{1}{1+r_{s'}} K_s = 0 \tag{42}$$

and taking K_t, the factor prices w and r and the lump sum taxes T as given. θ^R is the representative household's subjective discount rate. As in the previous section, the set-up of the supply side (the aggregate production function (1), the tax function (7), and the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8)) is assumed to be common knowledge. The representative household uses this information to forecast factor prices and lump sum taxes, given the government's environmental policy.

The representative household's maximization problem leads then to the Euler equation

$$\frac{1}{C_t} = \frac{1+r_{t+1}}{1+\theta^R}\frac{1}{C_{t+1}} \tag{43}$$

Moving the dynamic budget constraint (41) forward and combining with the transversality condition (42) yields the household's lifetime budget constraint. By combining with the Euler equation (43), current consumption C_t can then be written as a function of K_t and the current and future factor prices and lump sum taxes.

The current and future factor prices and lump sum taxes depend on the time path of the emission levels, which are set by the government. How the government does this, was not specified in the set-up in section 2. But let us assume that the representative household thinks that the government's emission policy only depends on the state of the economy as described by the state variables K, A and M:

$$E_{t'} \quad = \quad E(K_{t'}, A_{t'}, M_{t'}), \quad \forall\, t' \geq t \tag{44}$$

Given the laws of motion of K, A and M, it then follows that current consumption C_t can be written as a function of the current state variables:

$$C_t \quad = \quad C(K_t, A_t, M_t) \tag{45}$$

If the model is parameterized, the consumption function $C(\cdot)$ can be derived for any environmental policy $E(\cdot)$ by, for instance, a time iteration procedure.

Economist B faces one unknown parameter, however: the representative households' subjective discount rate θ^R. But an estimate of θ^R follows immediately from the Euler equation (43). Let us assume that economist B always estimates θ^R in such a way that it perfectly matches the most recent data that are available.[20] His period T estimate of the representative household's subjective discount rate, $\hat{\theta}^R_T$, is then given by

$$\hat{\theta}^R_T \quad = \quad \frac{1 + r_T}{1 + g^c_T} - 1 \tag{46}$$

where $g^c_T = C_T/C_{T-1}$ is the growth rate of aggregate consumption between periods $T-1$ and T.

Note that equation (46) is approximately equivalent with $r_T = \hat{\theta}^R_T + g^c_T$, which is the Ramsey rule (taking into account that the elasticity of the marginal social value of consumption is -1 because of the logarithmic felicity specification). So we find that the economist B uses the market interest rate to calibrate the subjective discount rate with the Ramsey rule, where the Ramsey rule is derived from his macroeconomic model with microfoundations.

[20] A similar remark as in footnote 17 holds also here: I make this assumption for convenience, but given that every period lasts very long, it may have a touch of realism.

Suppose now that in the beginning of period 1, the social evaluator asks economist B for advice on the optimal path of emission levels E. Armed with his period 0 estimate of the representative household's subjective discount rate, $\hat{\theta}_0^R$, economist B then replies that he has a model that does not only describe how the economy works, but that has even revealed the population's preferences about the subjective discount rate which the social evaluator should use. So he tells the social evaluator not to worry about the subjective discount rate, and that he will use his model to figure out what environmental policy maximizes the utility of some representative household in the economy.[21]

The challenge which economist B then faces is to find an emissions function $E(\cdot)$ that maximizes the utility function (40), where the subjective discount rate is set to $\hat{\theta}_0^R$, and taking the representative household's consumption behavior as described in the consumption function (45) as given (and where this consumption function is derived by using the optimal emissions function $E(\cdot)$). He solves this problem as follows. He first computes the emissions function that maximizes the utility function (40) as of period 1 *for a given consumption function* (45), subject to the aggregate production function (1), the tax function (7), the laws of motion for the capital stock, the state of technology, and the environmental quality (2), (3) and (8), and starting from the values for the state variables K, A and M in period 1:

$$\begin{aligned} U(K_t, A_t, M_t) &= \max_{E_t \in (0,1]} \left\{ \ln C_t + \frac{1}{1+\hat{\theta}_0^R} U(K_{t+1}, A_{t+1}, M_{t+1}) \right\} \\ \text{subject to} \quad & C_t = C(K_t, A_t, M_t) \\ & I_t = (1-\tau_t) Y_t - C_t \\ & Y_t = E_t^{\varepsilon} M_t^{\mu_Y} K_t^{\alpha} (A_t L_t)^{1-\alpha} \\ & \tau_t = z - \zeta M_t^{\mu_Z} \\ & K_{t+1} = I_t \\ & A_{t+1} = A_t(1+g) \end{aligned} \tag{47}$$

[21] This is actually *not* what is usually done in climate change economics, even not by environmental economists who favor the descriptive approach to cost-benefit analysis. Some integrated assessment models (IAMs) that follow the descriptive approach, such as FUND (Tol, 2002a and 2002b), are not based on a utility maximizing immortal representative agent, but simply assume a constant saving rate - very much in the same spirit as the old-style neo-classical analysis in the previous section; however, to justify the descriptive approach, it is necessary that the dynamics of aggregate consumption and saving can be derived by maximizing the utility function of a representative agent, which is inconsistent with a constant saving rate. Other IAMs, such as DICE (Nordhaus, 2008), do provide a consistent framework for the descriptive approach, but only by assuming a social planner (who determines both the time path of optimal emission allowances and the time path of consumption and investment).

$$
\begin{aligned}
&M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t \\
&K_1 = I_0, \quad A_1 = A_0(1+g) \quad \text{and} \quad M_1 = 1
\end{aligned}
$$

Once he knows the optimal emissions function *for a given consumption function*, he uses a simple iteration scheme to find the emissions and consumption functions that are *jointly* consistent with the maximization problem of the representative household and the maximization problem of the social evaluator, i.e. the emissions function which solves the social evaluator's problem (47), while the consumption function maximizes the utility function (40) subject to the budget constraint (41) and the transversality condition (42), where the factor prices and lump sum taxes (which the representative household takes as given) are consistent with the optimal environmental policy. Mathematical and numerical details are provided in Appendix C.

In Appendix C, I also show that according to this policy advice, the economy will eventually converge to a steady state which depends on the estimate of the representative household's subjective discount rate, $\hat{\theta}_0^R$, in such a way that

$$
\frac{\varepsilon}{E^*} \geq \frac{\psi}{1+\hat{\theta}_0^R-\phi}\frac{1}{M^*}(\mu_Y+\mu_Z\tilde{\lambda}^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.} \tag{48}
$$

where $\tilde{\lambda}^*$ is a variable that also appeared in the economist A's computations, and is defined in equation (39). The steady state values of the optimal emission level, the environmental quality, the tax share, the investment share, the interest rate and output loss can then be computed in a similar way as in the previous sections.

The graphs in figure 4 show the results of the computations of economist B (assuming the same parameter values as in the previous sections). Note that his policy advice is much more clear-cut than the advice of economist A. Economist A did not take a stand about the subjective discount rate; so the best thing he could do is to show the implications of different values of the subjective discount rate, leaving it to the social evaluator to pick the value that he finds most appropriate. Economist B, at the other hand, traces out the implications of just one value for the subjective discount rate, namely the subjective discount rate which - according to his analysis - reflects the preferences of the population.

How well does economist B do?

Figure 5 compares the steady state predictions of his advice with the steady states if the optimal policy is followed. As in figure 3, the thin red curves are taken from the back plane of the graphs in figure 1, and represent the steady states if the optimal policy is followed for a range of values of the subjective discount rate ρ. The broken vertical line indicates economist B's estimate in period 0 of the representative agent's subjective discount rate, $\hat{\theta}_0^R$. The small green dots are taken from the right vertical axes in the graphs in figure 4, and give the steady state values which the economy will arrive at if the emission levels are set according to economist B's recommendations in period 0.

In computing the optimal emission levels, economist B makes exactly the same mistake as economist A, for essentially the same reason: by lumping all households together and not taking into account that in fact only the income of the young generation is taxed, the effect on aggregate consumption of a change in the tax rate due to environmental degradation is distorted - a mistake which affects all the other graphs in figure 5.

In addition (and unlike economist A), economist B misses the point that environmental degradation leads to a higher interest rate: according to his advice in the beginning of period 1 (based on $\hat{\theta}_0^R$), the steady state interest rate will not change compared with period 0. As a result, his steady state projections for the investment share and output loss are off-track.

But his most serious mistake is that he claims to perceive the population's preferences for the subjective discount rate in the social welfare function. Of course that is not possible: how households trade off aggregate consumption today with aggregate consumption in the next period or in a period when they are not alive anymore, is even not specified in the set-up of section 2. It is an open question how households would want to do this. And suppose that the households want to make this trade-off in the same way as how they trade off their own consumption: even then economist B makes a mistake. His estimate of the subjective discount rate is based on the aggregate Euler equation (43). But the aggregate Euler equation suffers from a fallacy of composition: even though the Euler equation holds at the micro level for individual households, there is no guarantee that it also holds at the macro level for aggregate consumption. The Euler equation only holds for aggregate consumption if the growth rate of aggregate consumption is equal to the growth rate of the consumption of individual households. But this is not the case with the parameterization which was used in the previous sections.[22] And it is not the case in the real world either, as is documented by Attanasio and Weber (1993) and Blundell and Stoker (2005).

To make matters even worse, economist B does not learn from his mistakes. At the end of period 1, he observes that the interest rate has gone up (which he

[22]With the parameterization presented in section 2, the growth rate of the consumption of individual households in the initial steady state is about 2.3% annually, while the growth rate of aggregate consumption is 1.5% annually; the households' subjective discount rate θ is about 1% annually, while economist B's period 0 estimate of θ is about 1.7% annually.

did not expect in the beginning of period 1). But instead of revising his view on how macroeconomic variables are related with each other, as economist A did, economist B uses equation (46) to revise upwards his estimate of the subjective discount rate of the representative household. But as he increases his estimate of the subjective discount rate, his policy recommendations change as well, and he will now advise higher emission levels - which causes even more environmental damage and drives up the interest rate even further. Suppose now that in every period, the emission levels are set according to the latest update of economist B's policy advice. The economy will then eventually converge to a steady state where his prediction of the interest rate turns out to be correct. His recommended subjective discount rate in this new steady state is indicated by the full vertical lines in figure 5, leading to higher emission levels, lower environmental quality, a higher tax share, a lower investment share, a higher interest rate and more output loss than what he had predicted in the beginning of period 1 - as is shown by the big green dots.

The reason why economist B does not learn from his mistakes, is that his assumption that the world is populated by an immortal representative agent belongs to the maintained assumptions of his model. Consequently, this assumption cannot be rejected. So when economist B is confronted with observations that deviate from what he had predicted, he simply adjusts an unobservable preference parameter of his representative household. So there is no scope for learning - which results in misguided and time-inconsistent policy advice.

5 Conclusion

Of course, this thought experiment is a caricature. But nevertheless, it identifies a possible pitfall of the modern macroeconomic methodology. So it is instructive to summarize why the old-style neo-classical ecomomist fares so much better in this thought experiment than his modern colleague, and what we can learn from this.

Both economists make simplifying assumptions, for which they don't have evidence: the old-style neo-classical economist assumes that aggregate saving is a constant fraction of aggregate disposable income; the new neo-classical economist assumes (in a Friedmanian instrumentalist spirit) that the economy behaves as if it is populated by an immortal representative household.

But the assumption of the old-style neo-classical economist seems fairly innocuous for his analysis. The most important conclusion from his analysis, which drives his policy advice, is that the higher abatement costs that come with environmental degradation decrease disposable income, and therefore also aggregate saving and investment. This conclusion seems fairly robust for alternative specifications of the relation between aggregate saving and disposable income: it survives as long as aggregate saving goes down when disposable income goes down - which seems plausible enough.

The assumption of the modern macroeconomist, however, is much more problematic. The key variable in his analysis, and the core of his policy advice, is the representative household's subjective discount rate. But whether this is a good measure of the preferences of the population, crucially depends on the maintained assumption that the economy indeed behaves as if it is populated by an immortal representative household, for which there is no evidence; and even if this assumption turns out to be correct, it is not necessarily the case that the preferences of the representative household are the same as those of the population.

So the difference between both analyses is that the conclusions of the old-style neo-classical economist do not crucially hinge on the specific assumptions which he has made but for which he does not have any evidence, while the policy advice of the modern macroecomist crucially depends on a variable which is not directly observable and assumptions for which he does not have any evidence - which leads to data mining and unwarranted welfare conclusions.

This identifies a possible pitfall of the methodology of modern macroeconomics. Modern macroeconomic models are almost always based on at least some microfoundations that lack a strong empirical foundation. But if the predictions of the model crucially depend on these empirically unfounded microfoundations, we may go astray in the same way as the modern macroeconomist in the thought experiment: prediction errors may be ascribed to unexpected changes in some unobservable structural parameters, rather than to empirically unfounded microfoundations that belong to the model's maintained assumptions; and empirically unfounded microfoundations may lead to unwarranted welfare conclusions.

The Friedmanian instrumentalist nature of the microfoundations in modern macroeconomics can therefore easily degenerate into data mining, and may turn out to be a slippery route to scientific progress. Recognizing this does not imply that the modern macroeconomic methodology should be given up, however: as I wrote in the opening paragraphs of this paper, every methodology has its drawbacks, and an alternative methodology may not be unambiguously superior. But it is important to be aware of the pitfalls of a methodology, and to try to avoid them.

Appendix A: The optimal environmental policy

In this appendix, I derive the optimal environmental policy.

I first turn problem (30) in a stationary problem. Define $k_t = K_t/A_t$ and $c_t = C_t/A_t$. Problem (30) is then equivalent with the following stationary problem:

$$\begin{aligned}
w(k_t, M_t) &= \max_{E\in(0,1]} \left\{ \ln c_t + \frac{1}{1+\rho} w(k_{t+1}, M_{t+1}) \right\} \\
\text{subject to} \quad & c_t = \left[(1-s_Y)\left(1-\alpha-z+\zeta M_t^{\mu_Z}\right)+\alpha\right] E_t^{\varepsilon} M_t^{\mu_Y} k_t^{\alpha} \\
& k_{t+1} = \frac{1}{1+g} s_Y \left(1-\alpha-z+\zeta M_t^{\mu_Z}\right) E_t^{\varepsilon} M_t^{\mu_Y} k_t^{\alpha} \\
& M_{t+1} = 1+\phi(M_t-1)-\psi E_t \\
& k_1 = k_0 \quad \text{and} \quad M_1 = 1
\end{aligned}$$

The first-order condition for the control variable E is:

$$\frac{\varepsilon}{E_t}\left(1+\frac{1}{1+\rho}\frac{\partial w}{\partial k_{t+1}}k_{t+1}\right) \geq \frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{A.1}$$

The envelope conditions for the state variables k and M are:

$$\frac{\partial w}{\partial k_t} = \frac{\alpha}{k_t}\left(1+\frac{1}{1+\rho}\frac{\partial w}{\partial k_{t+1}}k_{t+1}\right) \tag{A.2}$$

$$\begin{aligned}
\frac{\partial w}{\partial M_t} = {} & \frac{1}{M_t}\left[\mu_Y+(1-s_Y)\mu_Z\lambda_{c,t}\right] + \frac{1}{1+\rho}\frac{\partial w}{\partial k_{t+1}}\frac{k_{t+1}}{M_t}\left(\mu_Y+s_Y\mu_Z\lambda_{I,t}\right) \\
& + \frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\phi
\end{aligned} \tag{A.3}$$

with

$$\lambda_{c,t} = \frac{\zeta M_t^{\mu_Z}}{(1-s_Y)(1-\alpha-z+\zeta M_t^{\mu_Z})+\alpha} \quad \text{and} \quad \lambda_{I,t} = \frac{\zeta M_t^{\mu_Z}}{s_Y(1-\alpha-z+\zeta M_t^{\mu_Z})}$$

As $\alpha/(1+\rho) < 1$, equation (A.2) implies that $(\partial w/\partial k_t)k_t$ must remain constant over time in order to rule out exploding paths:

$$\frac{\partial w}{\partial k_t}k_t = \frac{\alpha(1+\rho)}{1+\rho-\alpha}, \quad \forall t$$

Substituting in the first-order condition (A.1) and the law of motion (A.3) yields:

$$\frac{\epsilon}{E_t} \geq \frac{1+\rho-\alpha}{1+\rho}\frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{A.4}$$

$$\frac{\partial w}{\partial M_t} = \frac{1+\rho}{1+\rho-\alpha}\frac{1}{M_t}(\mu_Y+\mu_Z\lambda_t) + \frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\phi \tag{A.5}$$

with

$$\lambda_t = \frac{1+\rho-\alpha}{1+\rho}(1-s_Y)\lambda_{c,t} + \frac{\alpha}{1+\rho}s_Y\lambda_{I,t}$$

The optimal emission policy is therefore independent of the capital stock. Note also that if $E_t < 1$, combining equations (A.4) and (A.5) leads to an expression for $\partial w/\partial M_t$:

$$\frac{\partial w}{\partial M_t} = \left[\frac{\phi}{\psi}\frac{\epsilon}{E_t} + \frac{1}{M_t}(\mu_Y + \mu_Z\lambda_t)\right]\frac{1+\rho}{1+\rho-\alpha} \tag{A.6}$$

Expressions (A.4), (A.5) and (A.6) suggest then the following time iteration procedure to find the policy function $E(M_t)$:

1. Construct a grid $[M_1, M_2, ..., M_n]$ and choose associated starting values for the emissions $[E_1^{\{0\}}, E_2^{\{0\}}, ..., E_n^{\{0\}}]$.

2. Compute in each iteration $j = 1, 2, ...$ new values $[E_1^{\{j\}}, E_2^{\{j\}}, ..., E_n^{\{j\}}]$ as follows:

   (a) If $E_i^{\{j-1\}} < 1$, compute the implied value of E in the previous period by substituting equation (A.6) (moved forward with one period) in expression (A.4):

   $$\frac{\epsilon}{\tilde{E}_i^{\{j\}}} \geq \frac{\phi}{1+\rho}\frac{\epsilon}{E_i^{\{j-1\}}} + \frac{\psi}{1+\rho}\frac{1}{M_i}(\mu_Y + \mu_Z\lambda_i)$$
   $$\text{and} \quad \tilde{E}_i^{\{j\}} \leq 1 \quad \text{with c.s.}$$

   (b) If $E_i^{\{j-1\}} = 1$, derive the implied value of M in the next period from the law of motion (8) and compute the associated value of E by interpolating on $[E_1^{\{j-1\}}, E_2^{\{j-1\}}, ..., E_n^{\{j-1\}}]$. Continue until a value for E is found that is less than one - suppose this happens in period t'. Then use equation (A.6) to compute $\partial w/\partial M_{t'}$, and move backwards with the law of motion (A.5) until period $t+1$. Then use expression (A.4) to find $\tilde{E}_i^{\{j\}}$. If no value for E is found that is less than one, set $\tilde{E}_i^{\{j\}}$ equal to one.

   (c) Find for each grid point i the value $\tilde{M}_i^{\{j\}}$ such that the law of motion (8) holds, i.e. such that

   $$M_i = 1 + \phi(\tilde{M}_i^{\{j\}} - 1) - \psi\tilde{E}_i^{\{j\}}$$

   (d) We now have a grid $[\tilde{M}_1^{\{j\}}, \tilde{M}_2^{\{j\}}, ..., \tilde{M}_n^{\{j\}}]$ and associated emission values $[\tilde{E}_1^{\{j\}}, \tilde{E}_2^{\{j\}}, ..., \tilde{E}_n^{\{j\}}]$, which define the policy rule in iteration j. Use then inter- and extrapolation to find the emission values $[E_1^{\{j\}}, E_2^{\{j\}}, ..., E_n^{\{j\}}]$ that are associated with $[M_1, M_2, ..., M_n]$ according to this policy rule.

3. Continue until $\max_i\left(|E_i^{\{j\}} - E_i^{\{j-1\}}|\right)$ is less than the tolerance level.

The steady state is found as follows. From equation (A.5) follows the steady state value of $\partial w/\partial M$:

$$\left(\frac{\partial w}{\partial M}\right)^* = \frac{1+\rho}{1+\rho-\phi}\frac{1+\rho}{1+\rho-\alpha}\frac{1}{M^*}(\mu_Y + \mu_Z\lambda_t^*)$$

Substituting in expression (A.4) yields then equation (31):

$$\frac{\epsilon}{E^*} \geq \frac{\psi}{1+\rho-\phi}\frac{1}{M^*}(\mu_Y + \mu_Z\lambda^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.}$$

Appendix B: The old-style neo-classical policy advice

The derivation of the old-style neo-classical policy advice (the advice of economist A) is very similar as for the optimal policy. First turn problem (37) in a stationary problem:

$$\begin{aligned} w(k_t, M_t) &= \max_{E\in(0,1]}\left\{\ln c_t + \frac{1}{1+\rho}w(k_{t+1}, M_{t+1})\right\} \\ \text{subject to} \quad & c_t = (1-\hat{s}_0)\left(1 - z + \zeta M_t^{\mu_Z}\right)E_t^{\varepsilon}M_t^{\mu_Y}k_t^{\alpha} \\ & k_{t+1} = \frac{1}{1+g}\hat{s}_0\left(1 - z + \zeta M_t^{\mu_Z}\right)E_t^{\varepsilon}M_t^{\mu_Y}k_t^{\alpha} \\ & M_{t+1} = 1 + \phi(M_t - 1) - \psi E_t \\ & k_1 = k_0 \quad \text{and} \quad M_1 = 1 \end{aligned}$$

The first-order condition for the control variable E and the envelope condition for the state variable k are the same as for the optimal policy, and are given by expressions (A.1) and (A.2). The envelope condition for the state variable M is slightly different, however:

$$\begin{aligned} \frac{\partial w}{\partial M_t} &= \frac{1}{M_t}\left[\mu_Y + \mu_Z\tilde{\lambda}_t\right] + \frac{1}{1+\rho}\frac{\partial w}{\partial k_{t+1}}\frac{k_{t+1}}{M_t}\left(\mu_Y + \mu_Z\tilde{\lambda}_t\right) \\ &\quad + \frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\phi \end{aligned} \tag{B.1}$$

with

$$\tilde{\lambda}_t = \frac{\zeta M_t^{\mu_Z}}{1 - z + \zeta M_t^{\mu_Z}}$$

Substituting the solution for $(\partial w/\partial k_t)k_t$, (A.4), in the first-order condition (A.1) and the envelope condition (B.1) for M yields:

$$\frac{\epsilon}{E_t} \geq \frac{1+\rho-\alpha}{1+\rho}\frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{B.2}$$

$$\frac{\partial w}{\partial M_t} = \frac{1+\rho}{1+\rho-\alpha}\frac{1}{M_t}(\mu_Y+\mu_Z\tilde{\lambda}_t)+\frac{1}{1+\rho}\frac{\partial w}{\partial M_{t+1}}\phi \tag{B.3}$$

As the optimal policy, economist A's emission policy is independent of the capital stock. If $E_t < 1$, combining equations (B.2) and (B.3) leads to an expression for $\partial w/\partial M_t$, similar to equation (A.6):

$$\frac{\partial w}{\partial M_t} = \left[\frac{\phi}{\psi}\frac{\epsilon}{E_t}+\frac{1}{M_t}(\mu_Y+\mu_Z\tilde{\lambda}_t)\right]\frac{1+\rho}{1+\rho-\alpha} \tag{B.4}$$

Expressions (B.2), (B.3) and (B.4) suggest then a similar time iteration procedure to find economist A's policy advice $E(M_t)$ as the procedure that was used to derive the optimal policy.

The steady state is also found in the same way as for the optimal policy: from equation (B.3) follows the steady state value of $\partial w/\partial M$; substituting in expression (B.2) yields then equation (38).

Appendix C: The new neo-classical policy advice

I now derive the new neo-classical policy advice (the advice of economist B). I first solve the representative household's and the social evaluator's maximization problem. I then sketch a numerical procedure to find the emission and consumption functions that jointly solve the representative household's problem and the social evaluator's problem. I will conclude with a derivation of the new steady state values if emission levels are set according to economist B's advice.

I first turn the representative household's problem in a stationary problem, using the same notation as in Appendix A:

$$u(k_t, M_t) = \max_c \left\{\ln c_t + \frac{1}{1+\hat{\theta}_0^R}u(k_{t+1}, M_{t+1})\right\}$$

$$\text{subject to} \quad k_{t+1} = \frac{1}{1+g}\left(k_t(1+r_t)+w_t/A_t-T_t/A_t-c_t\right)$$

$$\lim_{s\to\infty}\Pi_{s'=t+1}^{s}\frac{1+g}{1+r_{s'}}k_s = 0$$

The first-order condition for the control variable c is:

$$\frac{1}{c_t} = \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial k_{t+1}}\frac{1}{1+g} \tag{C.1}$$

The envelope condition for the state variable k is:

$$\frac{\partial u}{\partial k_t} = \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial k_{t+1}}\frac{1+r_t}{1+g} \tag{C.2}$$

Now substitute (C.1) in (C.2), move the resulting equation one period forward, and substitute it again in (C.1). This yields the consumption Euler equation:

$$\frac{1}{c_t} = \frac{1+r_{t+1}}{(1+\hat{\theta}_0^R)(1+g)}\frac{1}{c_{t+1}} \tag{C.3}$$

I now turn the social evaluator's problem (47) in a stationary problem:

$$\begin{aligned}
u(k_t, M_t) &= \max_{E\in(0,1]}\left\{\ln c_t + \frac{1}{1+\hat{\theta}_0^R}u(k_{t+1}, M_{t+1})\right\} \\
\text{subject to} \quad & c_t = c(k_t, M_t) \\
& k_{t+1} = \frac{1}{1+g}\left[(1-z+\zeta M_t^{\mu_Z})\, E_t^{\varepsilon} M_t^{\mu_Y} k_t^{\alpha} - c_t\right] \\
& M_{t+1} = 1+\phi(M_t-1)-\psi E_t \\
& k_1 = k_0 \quad \text{and} \quad M_1 = 1
\end{aligned}$$

The first-order condition for the control variable E is:

$$\begin{aligned}
\frac{1}{1+g}\frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial k_{t+1}}(1-z+\zeta M_t^{\mu_Z})\,\varepsilon\frac{y_t}{E_t} &\geq \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial M_{t+1}}\psi \\
\text{and} \quad E_t &\leq 1 \quad \text{with c.s.}
\end{aligned} \tag{C.4}$$

where $y_t = Y_t/A_t = E_t^{\varepsilon}M_t^{\mu_Y}k_t^{\alpha}$.

The envelope condition for the state variable M is:

$$\begin{aligned}
\frac{\partial u}{\partial M_t} &= \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial k_{t+1}}\frac{1}{1+g}\left[(1-z+\zeta M_t^{\mu_Z})\mu_Y\frac{y_t}{M_t} + \zeta\mu_Z M_t^{\mu_Z}\frac{y_t}{M_t} - \frac{\partial c}{\partial M_t}\right] \\
&\quad + \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial M_{t+1}}\phi + \frac{1}{c_t}\frac{\partial c}{\partial M_t}
\end{aligned} \tag{C.5}$$

Substituting the representative household's first-order condition (C.1) in expressions (C.4) and (C.5) yields:

$$\frac{\varepsilon}{E_t} \geq \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial M_{t+1}}\frac{c_t}{x_t}\psi \quad \text{and} \quad E_t \leq 1 \quad \text{with c.s.} \tag{C.6}$$

$$\frac{\partial u}{\partial M_t} = \frac{x_t}{c_t}\frac{1}{M_t}(\mu_Y + \mu_Z\tilde{\lambda}_t) + \frac{1}{1+\hat{\theta}_0^R}\frac{\partial u}{\partial M_{t+1}}\phi \tag{C.7}$$

with

$$x_t = (1-z+\zeta M_t^{\mu_Z})y_t \quad \text{and} \quad \tilde{\lambda}_t = \frac{\zeta M_t^{\mu_Z}}{1-z+\zeta M_t^{\mu_Z}}$$

Note that if $E_t < 1$, combining equations (C.6) and (C.7) leads to an expression for $\partial u/\partial M_t$:

$$\frac{\partial u}{\partial M_t} = \left[\frac{\phi}{\psi}\frac{\varepsilon}{E_t} + \frac{1}{M_t}(\mu_Y + \mu_Z\tilde{\lambda}_t)\right]\frac{x_t}{c_t} \tag{C.8}$$

Expressions (C.3), (C.6), (C.7) and (C.8) suggest then the following time iteration procedure to find the policy functions $c(k_t, M_t)$ and $E(k_t, M_t)$:

1. Construct grids $[k_1, k_2, ..., k_n]$ and $[M_1, M_2, ..., M_m]$ and construct two (n,m)-matrices with associated starting values for consumption c and emission levels E.

2. Perform a time iteration procedure with expressions (C.6), (C.7) and (C.8) in a similar way as explained in Appendix A; this yields a new (n,m)-matrix with the optimal emission levels associated with the k- and M-grids, assuming that the consumption function $c(k_t, M_t)$ is described by the (n,m)-matrix with consumption levels.

3. Perform a time iteration procedure based on (C.3) to determine the optimal consumption levels associated with the k- and M-grids, assuming that the emissions function $E(k_t, M_t)$ is described by the (n, m)-matrix with emission levels.

4. Repeat steps 2 and 3 until convergence, which is reached when both time iteration procedures are finished after one step.

The steady state is found as follows. From equation (C.7) follows the steady state value of $\partial u/\partial M_t$:

$$\left(\frac{\partial u}{\partial M}\right)^* = \frac{1+\hat{\theta}_0^R}{1+\hat{\theta}_0^R-\phi}\frac{1}{M^*}(\mu_Y + \mu_Z\tilde{\lambda}_t^*)\frac{x_t}{c_t}$$

Substituting in expression (C.6) yields then equation (48):

$$\frac{\epsilon}{E^*} \geq \frac{\psi}{1+\hat{\theta}_0^R-\phi}\frac{1}{M^*}(\mu_Y + \mu_Z\tilde{\lambda}^*) \quad \text{and} \quad E^* \leq 1 \quad \text{with c.s.}$$

References

Atkinson, Anthony B. (2009), “Economics as a Moral Science”, Economica, 76, 791-804.

Attanasio, Orazio P., and Guglielmo Weber (1993), ”Consumption Growth, the Interest Rate and Aggregation”, Review of Economic Studies, 60, 631-649.

Bartelsman, Eric J., John C. Haltiwanger, and Stefano Scarpetta (2009), ”Cross-Country Differences in Productivity: the Role of Allocation and Selection”, NBER Working Paper Series, Working Paper 15490.

Browning, Martin J., Lars Peter Hansen, and James J. Heckman (1999), ”Micro Data and General Equilibrium Models”, in John B. Taylor and Michael Woodford (eds.), *Handbook of Macroeconomics*, Vol. 1, Elsevier, Amsterdam: North-Holland, Chapter 8, 543-633.

Blundell, Richard, and Thomas M. Stoker (2005), ”Heterogeneity and Aggregation”, Journal of Economic Literature, Vol. XLIII (June), 347-391.

Caballero, Ricardo J. (2010), “Macroeconomics after the Crisis: Time to Deal with the Pretense-of-Knowledge Syndrome”, Journal of Economic Perspectives, 24(4), 85-102.

Chari, V.V., Patrick J. Kehoe, and Ellen R. McGrattan (2009), “New Keynesian Model: Not Yet Useful for Policy Analysis”, American Economic Journal: Macroeconomics, 1(1), 242-266.

Cline, William R. (1992), *The Economics of Global Warming*, Washington D.C.: Institute for International Economics.

Dasgupta, Partha S. (2008), ”Discounting climate change”, Journal of Risk and Uncertainty, 37, 141-169.

Dietz, Simon, Cameron Hepburn, and Nicholas Stern (2009), ”Economics, ethics and climate change”, in Kaushik Basu and Ravi Kanbur (eds.), *Arguments for a Better World: Essays in Honour of Amartya Sen (Volume 2: Society, Institutions, and Development)*, Oxford University Press, Oxford.

Frederick, Shane, George Loewenstein, and Ted O’Donoghue (2002), ”Time Discounting and Time Preference: A Critical Review”, Journal of Economic Literature, Vol. XL (June), 351-401.

Friedman, Milton (1953), “The Methodology of Positive Economics”, in Milton Friedman, *Essays in positive economics*, University of Chicago Press, Chicago, 3-43.

Goulder, Lawrence H., and Roberton C. Williams, III (2012), “The Choice of Discount Rate for Climate Change Policy Evaluation”, Climate Change Economics, 1-18.

Hansen, Lars Peter, and James J. Heckman (1996), ”The Empirical Foundations of Calibration”, Journal of Economic Perspectives, 10(1), 87-104.

Harrod, Roy F. (1948), *Towards a dynamic economics*, London: MacMillan.

Hoover, Kevin D. (2006), ”A Neowicksellian in a New Classical World: the Methodology of Michael Woodford’s *Interest and Prices*”, Journal of the History of Economic Thought, 28(2), 143-149.

Hoover, Kevin D. (2009), ”Microfoundational Programs”, mimeo, Department of Economics and Department of Philosophy, Duke University.

Kocherlakota, Narayana (2010), ”Modern Macroeconomic Models as Tools for Economic Policy”, The Region, May, Federal Reserve Bank of Minneapolis.

Lucas, Robert E., Jr. (1976), ”Econometric Policy Evaluation: A Critique”, Carnegie-Rochester Conference Series on Public Policy 1: 19-46.

Lucas, Robert E., Jr. (1980), ”Methods and problems in business cycle theory”, Journal of Money, Credit, and Banking, 12, 696-715.

Mendelsohn, Robert O. (2007), ”A Critique of the Stern Report”, Regulation, 29(4):40-47.

Nelson, Julie A. (2008), ”Economists, value judgments, and climate change: A view from feminist economics”, Ecological Economics, 65, 441-447.

Nordhaus, William D. (2007), ”A Review of the *Stern Review on the Economics of Climate Change*”, Journal of Economic Literature, Vol. XLV (Sept.), 686-702.

Nordhaus, William D. (2008), *A Question of Balance: Economic Modeling of Global Warming*, New Haven, Yale University Press.

Pigou, Arthur C. (1932), *The economics of welfare*, 4th edition, London: MacMillan.

Prescott, Edward C. (2006), “Nobel Lecture: The Transformation of Macroeconomic Policy and Research”, Journal of Political Economy, 114(2), 203-235.

Ramsey, Frank P. (1928), ”A mathematical theory of saving”, Economic Journal, 38(152), 543-59.

Sen, Amartya K. (1982), ”Approaches to the choice of discount rate for social benefit-cost analysis”, in R.C. Lind (ed.), *Discounting for Time and Risk in Energy Policy*, Washington D.C.: Resources for the Future, 325-353.

Sims, Christopher A. (2012), “Statistical Modeling of Monetary Policy and its Effects”, American Economic Review, 102(4), 1187-1205.

Solow, Robert M. (1956), ”A Contribution to the Theory of Economic Growth”, Quarterly Journal of Economics, 70(Feb.), 65-94.

Solow, Robert M. (2008), ”The State of Macroeconomics”, Journal of Economic Perspectives, 22(1), 243-249.

Stern, Nicholas (2007), *The Economics of Climate Change: The Stern Review*, Cambridge: Cambridge University Press.

Storesletten, Kjetil, Chris I. Telmer, and Amir Yaron (2004), ”Consumption and Risk Sharing over the Life Cycle”, Journal of Monetary Economics, (51), 609-633.

Tol, Richard S.J. (2002a), ”Estimates of the damage costs of climate change. Part 1: Benchmark estimates”, Environmental & Resource Economics, 21(1),47-73.

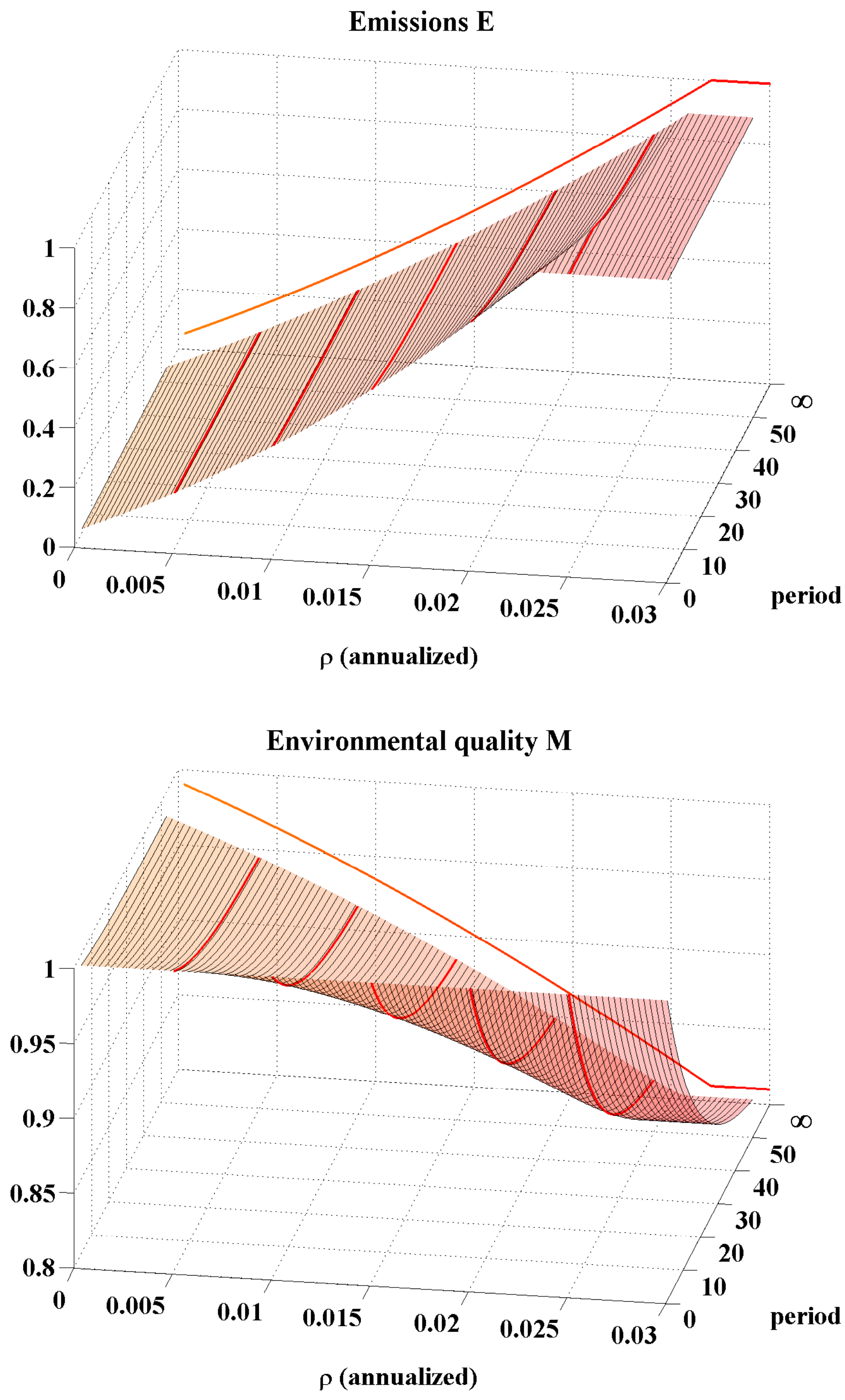
Tol, Richard S.J. (2002b), ”Estimates of the damage costs of climate change. Part 2: Dynamic estimates”, Environmental & Resource Economics, 21(2), 135-160.

Tol, Richard S.J., and Gary W. Yohe (2006), ”A Review of the *Stern Review*”, World Economics, 7(4):233-50.

Weitzman, Martin L. (2007), ”A Review of *The Stern Review on the Economics of Climate Change*”, Journal of Economic Literature, Vol. XLV (Sept.), 703-724.

Wren-Lewis, Simon (2007), “Are there Dangers in the Microfoundations Consensus?”, in Philip Arestis (ed.), *Is there a New Consensus in Macroeconomics?*, Palgrave Macmillan

Figure 1: *Transitional dynamics according to the optimal policy*



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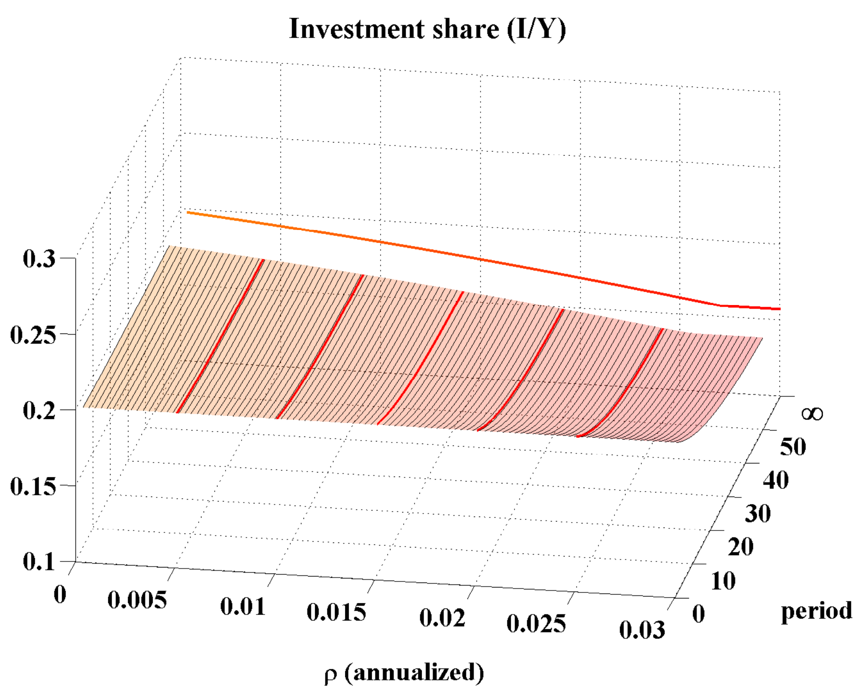

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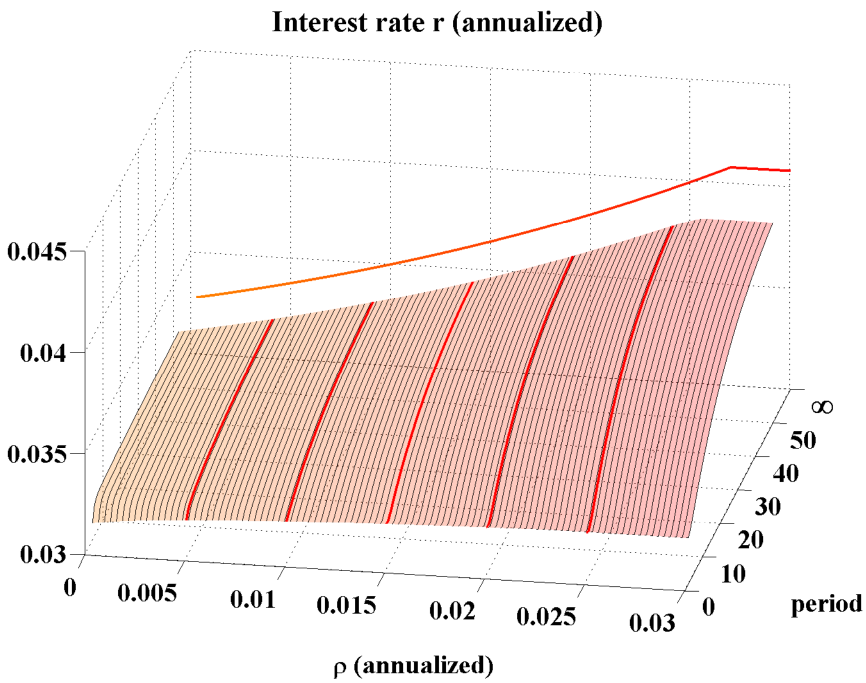


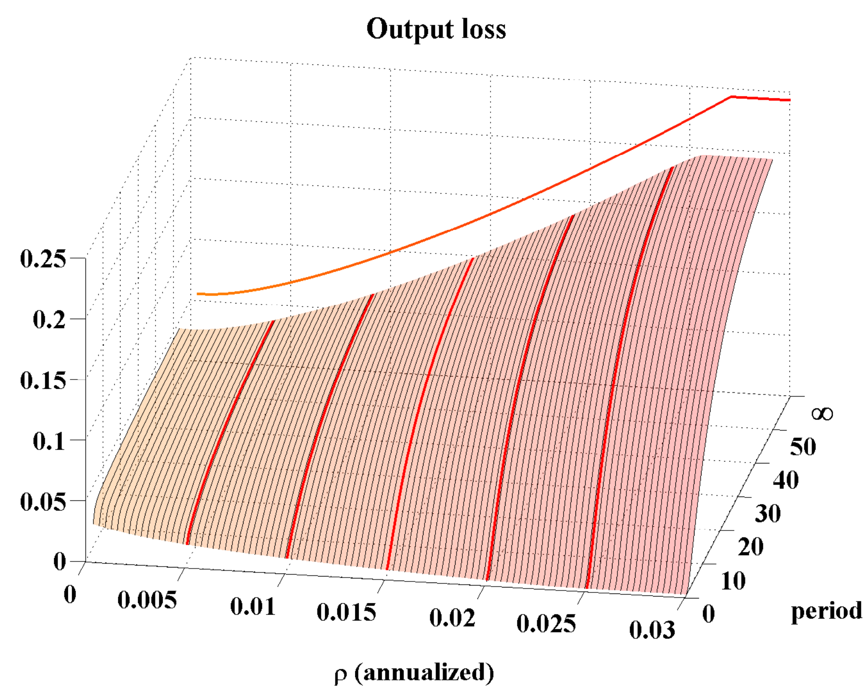
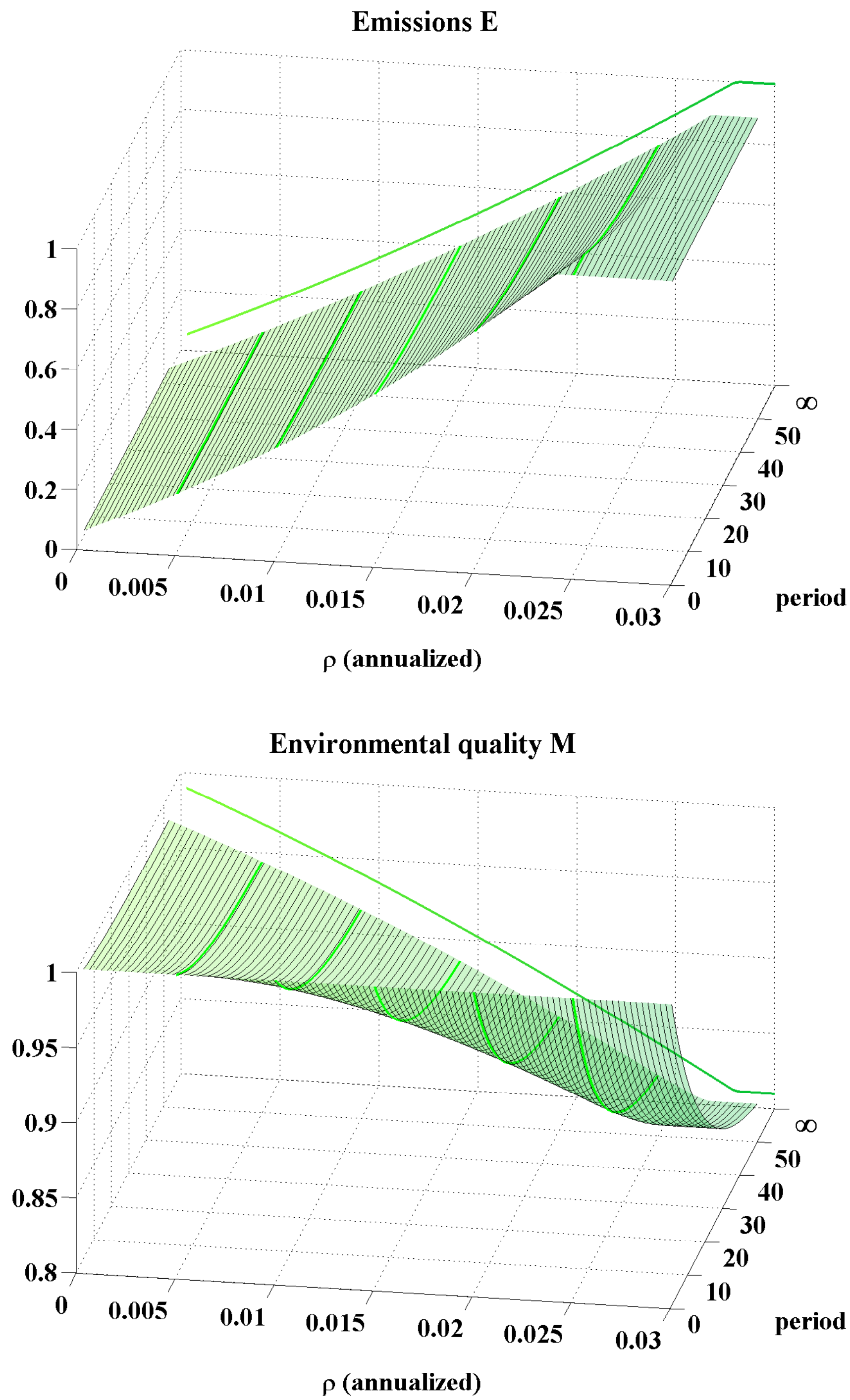


Figure 2: *Transitional dynamics according to the neo-classical policy advice*



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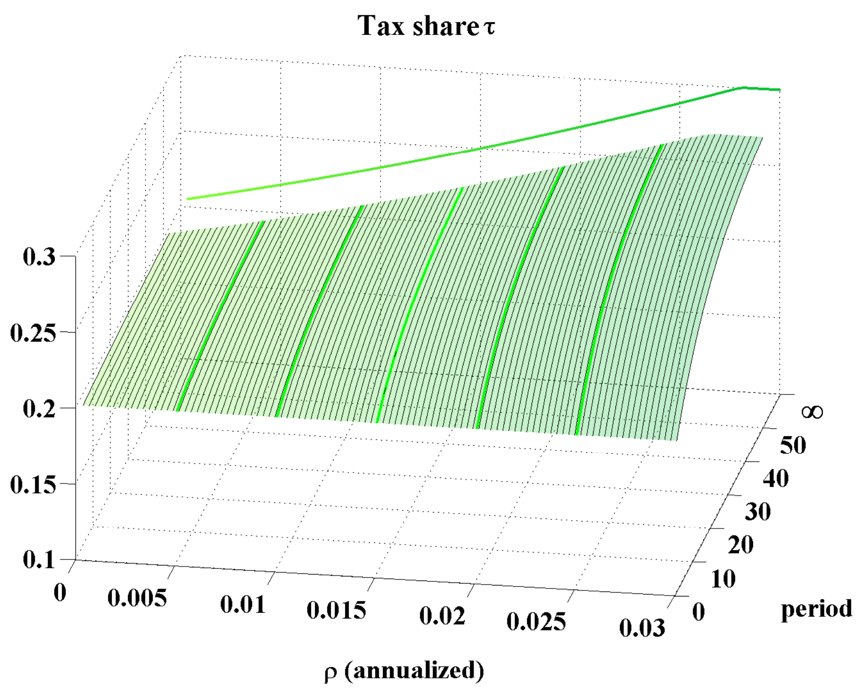


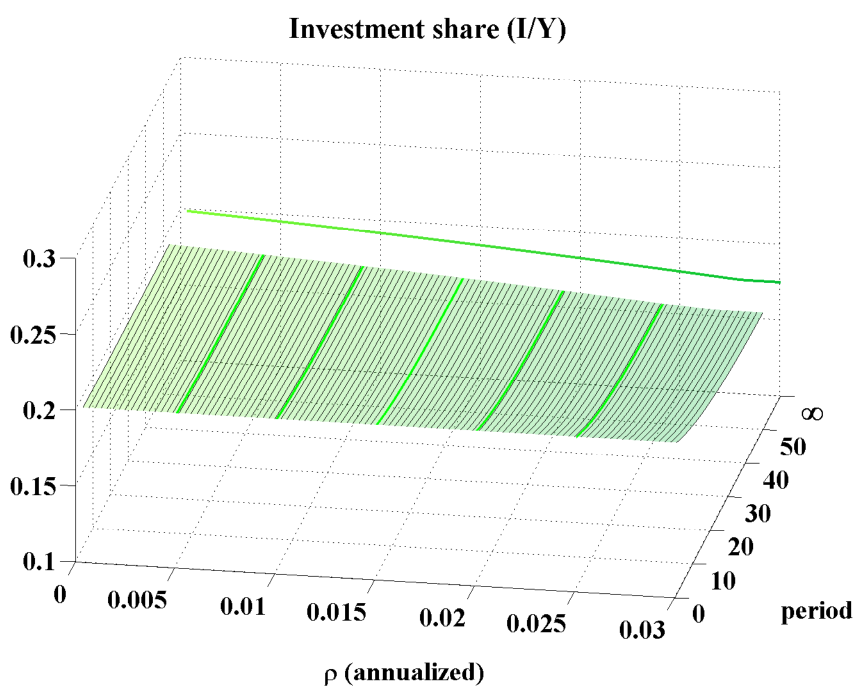
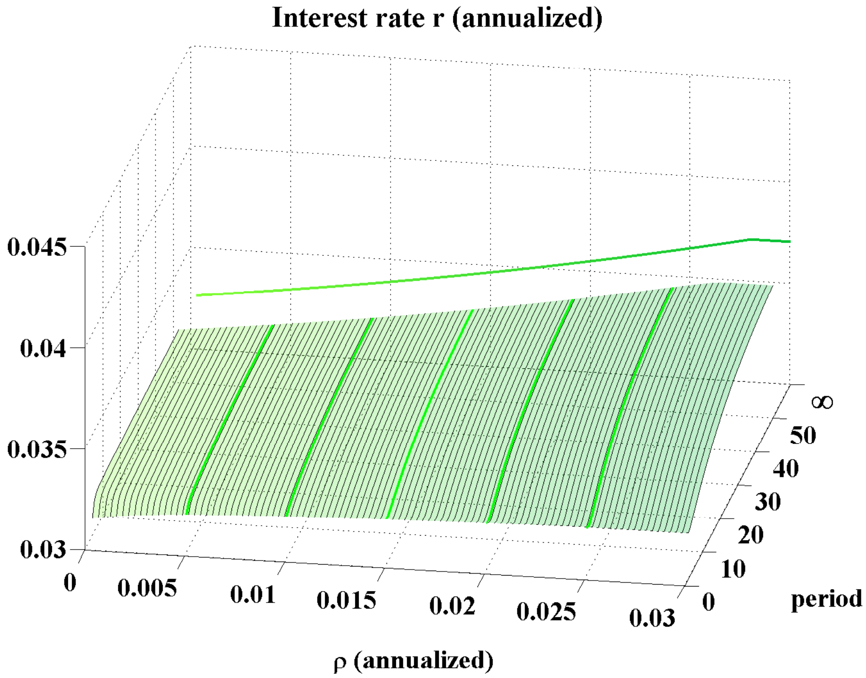


Figure 2 (continued)



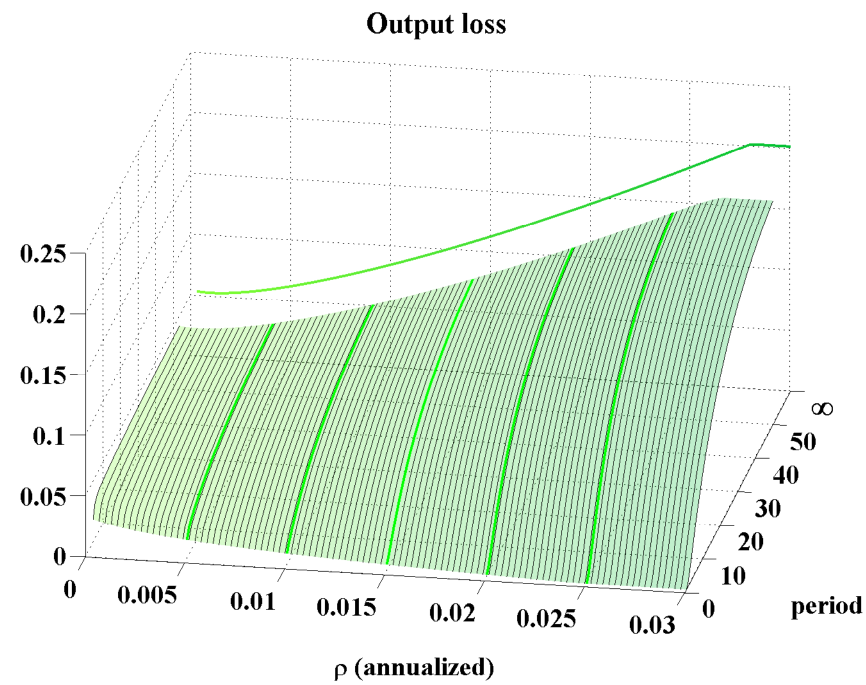
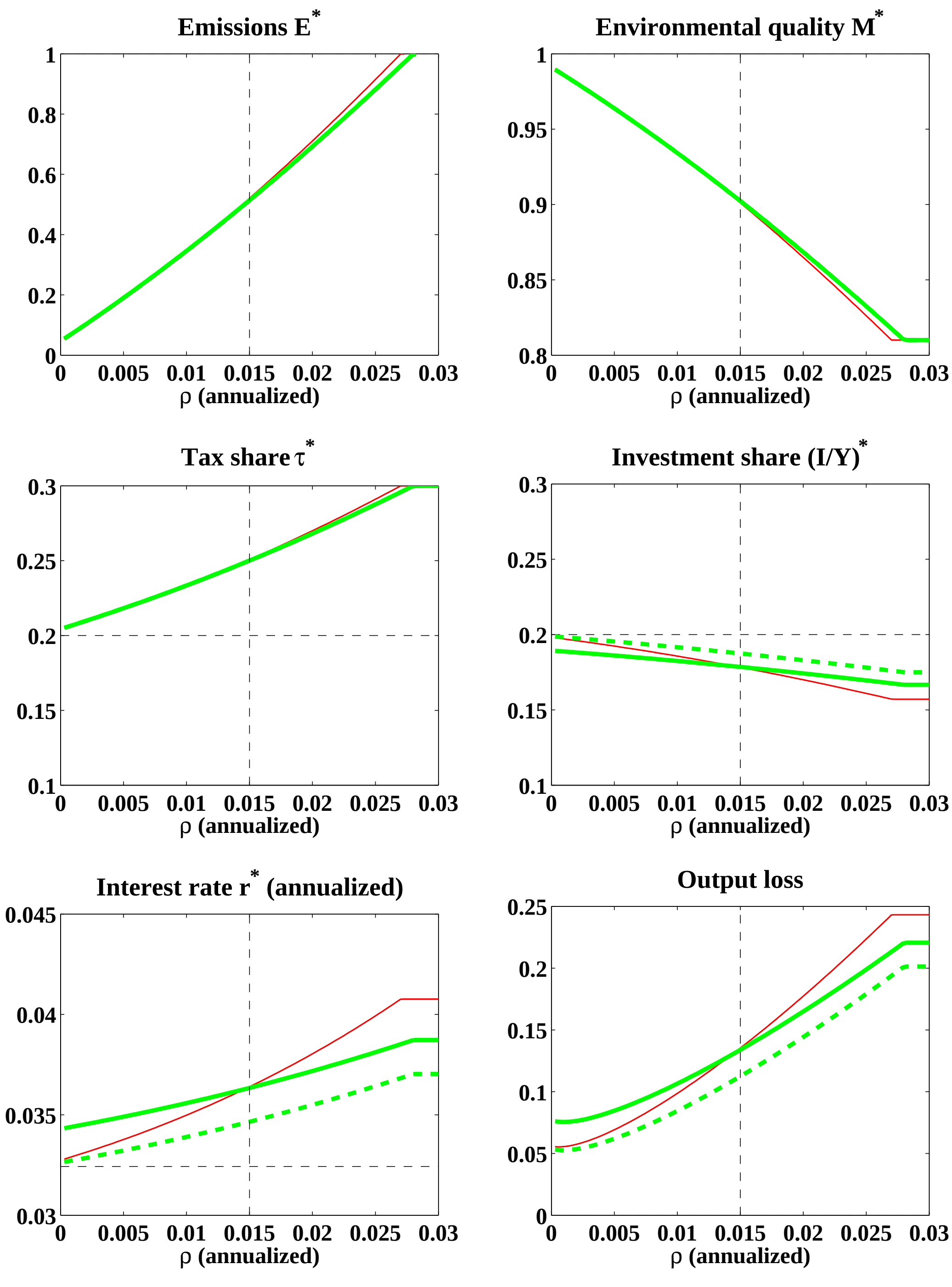
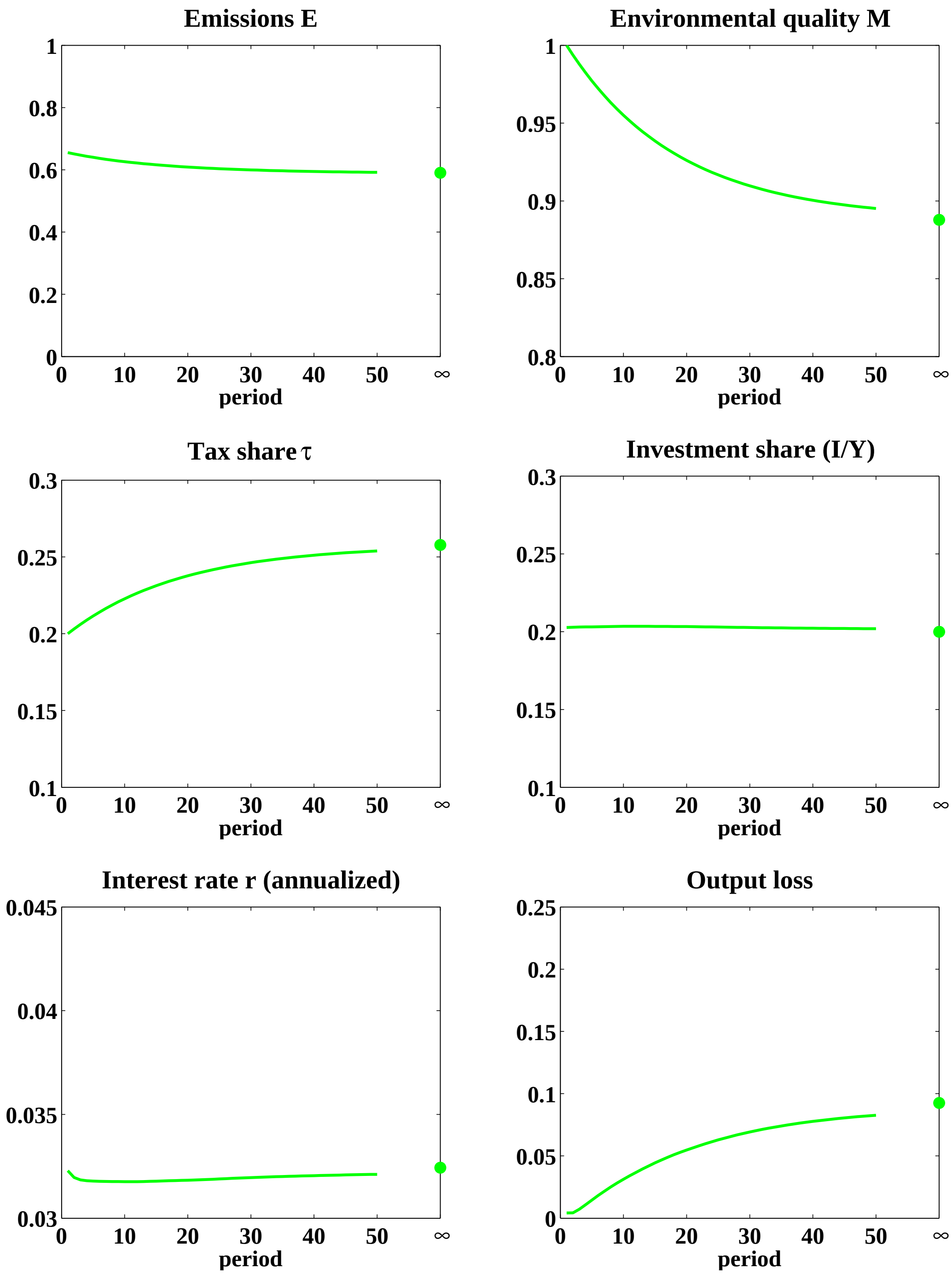


Figure 3: *Steady state according to the neo-classical policy advice*



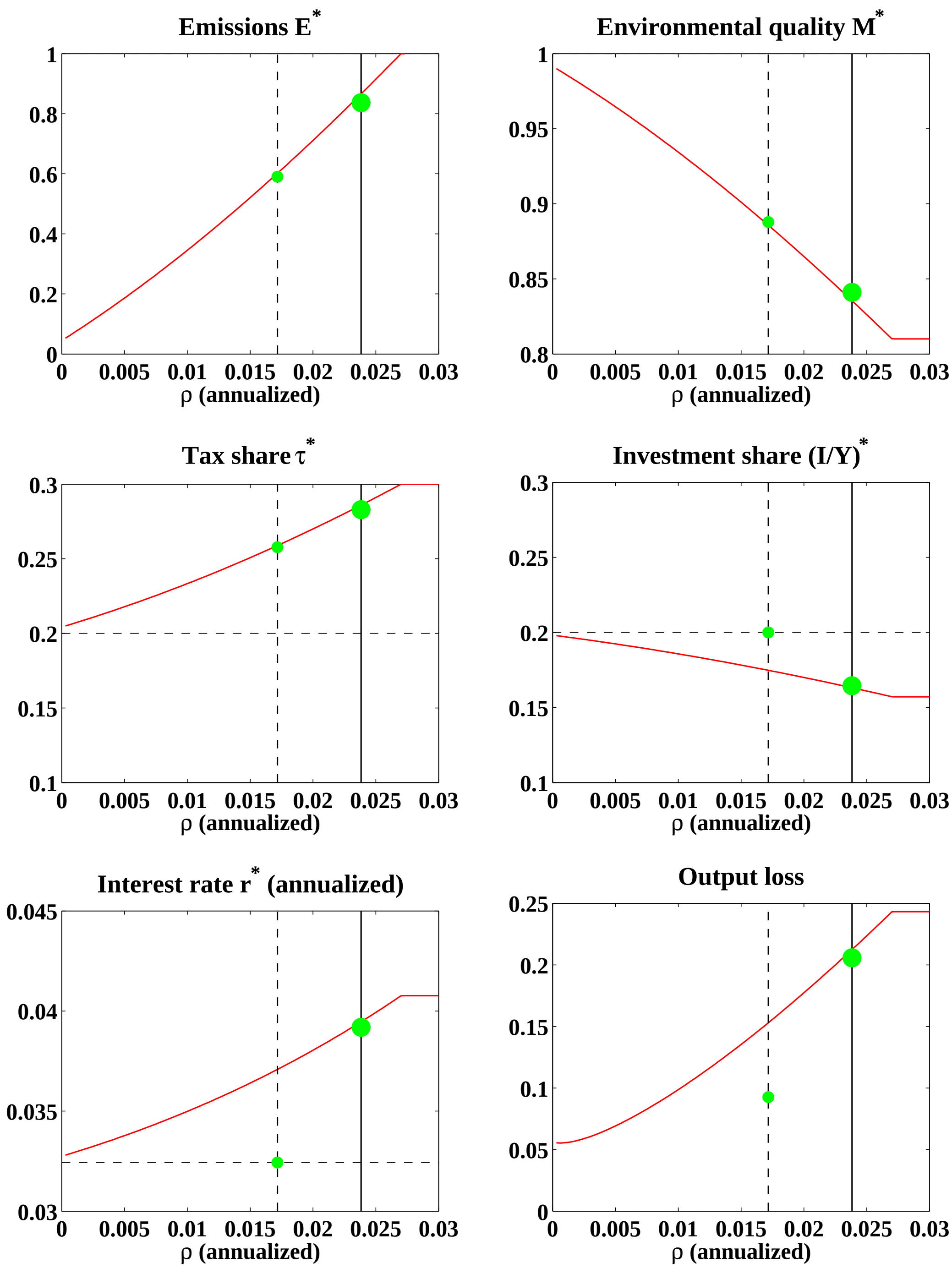
Thin red line: steady state according to the optimal policy. *Broken black line, vertical*: social evaluator's subjective discount rate. *Broken green line and thick green line*: steady state as predicted by a neo-classical economist at time 0 and in steady state. *Broken black line, horizontal*: values in period 0.

Figure 4: *Transitional dynamics according to the new-classical policy advice*



Note: the green dots on the vertical axes are the steady state levels as predicted at time 0.

Figure 5: *Steady state according to the new-classical policy advice*



Thin red line: steady state according to the optimal policy. *Broken black line, vertical; and small green dot; full black line, vertical; and big green dot*: optimal subjective discount rate and steady state according to a new-classical economist at time 0 and in steady state. *Broken black line, horizontal*: values in period 0.