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Regulation of Road Accident Externalities when Insurance Companies have Market Power

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Regulation of road accident externalities when insurance companies have market power[☆]

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Abstract

Accident externalities are among the most important external costs of road transport. We study the regulation of these when insurance companies have market power. Using analytical models, we compare a public-welfare maximizing monopoly with a private profit-maximizing monopoly, and markets where two or more firms compete. A central mechanism in the analysis is the accident externality that individual drivers impose on one another via their presence on the road. Insurance companies will internalize some of these externalities, depending on their degree of market power. We derive optimal insurance premiums, and "manipulable" taxes that take into account the response of the firm to the tax rule applied by the government. Furthermore, we study the taxation of road users under different assumptions on the market structure. We illustrate our analytical results with numerical examples, in order to better understand the determinants of the relative performance of different market structures.

Keywords: accident externalities, traffic regulation, safety, second-best, market power

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1. Introduction

Road accidents account for a large share of the social costs of road transport. For example, Parry et al. (2007) estimated that the social costs of road accidents for the US are around 4.3% of the GDP, whereas the World Health Organization estimates these costs at around 1%–2% of GDP (WHO, 2004). Such estimates include medical costs, production losses, human losses, property damage, settlement costs and accident-induced congestion costs. Governments have several possibilities to regulate the accident externalities, including efficient control of the insurance markets, upgrading of the road network and influencing road users' attitudes and behavior (Boyer and Dionne, 1987; Cohen and Einav, 2003; Cohen and Dehejia, 2004; Hultkrantz et al., 2012; Kuo, 2011; Rubin and Shepherd, 2007).

Road accidents constitute external costs that road users impose on one-another without trade. This has motivated economists to typically study accident regulation from the perspective regulatory Pigouvian taxation (Edlin and Mandic, 2006; Pigou, 1920; Small and Verhoef, 2007). This paper develops stylized theoretical models that study the efficiency of regulated and unregulated insurance markets, taking into account the interactions between the markets for road trips and the market for traffic safety insurance. Our model acknowledges that insurance companies will internalize a part of the accident externalities when setting their optimal insurance premiums, where the degree of internalization will depend on their market power. The paper is consequently closely related to the growing body of literature on externality regulation in aviation and private roads when firms have market power (Brueckner, 2002; Brueckner and Verhoef, 2010; Daniel, 1995; Engel et al., 2004; Verhoef et al., 1996). Although some of the results on social and monopolistic premiums we derive have close resemblance to this earlier work, there are important differences. In particular, we also study a more complex case of a (symmetric) duopoly, where each of the companies takes into account the behavior of his own clients and that of the clients of the other firm. In such an oligopolistic setting, a firm partly internalizes the accident externalities imposed on clients of the rival firm as well. This contrasts sharply with the case of private congestion pricing by private firms, where the firm only internalizes externalities that its own customers impose on one another.

We analyze a three-level hierarchical market, with government on the top; insurance companies in the middle; and atomistic drivers at the lowest level (see also Delhaye (2007)). It is assumed that the actors are equally aware of the expected monetary and non-monetary accident costs, which are functions of the endogenous aggregate traffic volume, which is measured by total distance traveled. To reflect that in reality not all costs of accidents are covered, notably immaterial costs, we introduce an exogenous insurance percentage coverage of accident costs. That is, the government sets the mandatory liability insurance, and insurance companies may also offer casco insurance. Taking these into account, insurance companies set their premiums, and drivers choose the number of kilometers that they will travel.

For theoretical tractability we consider bilateral accidents (collision of two cars) only. In line with laws in the majority of countries, it is assumed that the liability insurance that covers the third party loss (physical injury and damage costs), is obligatory, but there is an option to choose a more inclusive general casco insurance in which also one's own risks are insured. Because our main focus is to investigate the implications of market power of the insurance firms, cars and drivers are assumed identical in the model. Consistent with this assumption, we assume drivers involved in an accident are in expected terms equally guilty. Expected costs of an accident may include, material damage, medical claims and loss due to disability or death. We will not distinguish between these components explicitly.

We develop stylized equilibrium models that allow assessing the design and impact of second-best policy instruments for traffic regulation on safety externalities, and we study three different governmental taxation rules, assuming different market structures. First, we study taxation of insurance companies with market power, where it is assumed that insurance companies treat these taxes as parametric. Second, we analyze 'manipulable' taxation rules, which we designed to take into account the notion that when the insurance company understands the tax rule that is used, it has an incentive to use that information in optimizing profits, by manipulating the equilibrium level of the tax (Brueckner and Verhoef, 2010). Third, we study the taxation of drivers rather than firms under different market structures.

In line with the earlier literature on airport congestion and private road operation, we find that market power is of key importance when insurance markets are regulated. The discussion on private internalization of externalities is therefore not only relevant for airport congestion or the operation of private roads, but also impacts optimal road transport pricing when regulating the accident externalities. Furthermore, our results may guide empirical research that aims to estimate accident externalities from data on insurance premiums (see for example Edlin and Mandic (2006)). Our models predicts that these premiums will not only reflect marginal costs but also the internalized external costs and a mark-up related to the market power. Controlling for these effects is important in empirical work, and should aid the interpretation of the estimated marginal effects.

The paper is organized as follows. Section 2 studies the first-best benchmark situation where a welfare-maximizing monopolist sets the socially optimal insurance premiums. We then compare this benchmark with a private profit-maximizing monopolist, and continue with competitive markets with two or more firms in Section 3. The manipulable tax rule is introduced in Section 4, and a model for taxation of drivers is developed in Section 5. Section 6 provides numerical sensitivity analysis, to better understand the relative performance of the different market structures, and Section 7 concludes.

2. Public welfare-maximizing monopoly

We start our exposition by presenting the analytical model, and deriving its optimum by considering the situation, where a public welfare-maximizing monopolist sets the insurance premiums.

Let α and β be casco and liability coverage percentage ($1 > \alpha > \beta > 0$), meaning that the insurance company reimburses a fraction β of the third-party accident costs as a result of (obligatory for drivers) liability insurance, and a fraction α of the insuree's own accident costs with a voluntary casco insurance. Both α and β are treated as exogenous parameters. While c_A is average accident cost, βc_A is the reimbursement of the aggrieved party, and αc_A is the coverage of a casco insured driver. K_α stands for casco-covered kilometrage, K_β is kilometrage of the drivers with a liability insurance. We denote with $\mathcal{C}_A = \mathcal{C}_A(K)$ the expected accident costs of each driver, per kilometer driven,[1] and $K = K_\alpha + K_\beta$ is the total kilometrage. We assume that kilometrage is proportional to the number of road users, and we therefore do not consider the two interrelated stages of a consumer's problem to decide on both vehicle ownership and vehicle usage. This is consistent with the

[1] For simplicity we exclude speed/flow parameters from our model, that is why we assume that the expected accident costs $\mathcal{C}_A$ and the driver's accident costs if an accident occurs c_A depend only on aggregate vehicle kilometrage driven, and do not depend on factors such as drivers' speed, speed difference, or vehicle technology.

assumption that an increase of kilometrage implies an increase of the number of drivers, and therewith the risk to be involved into an accident.

We seek insurance premiums π^o_β and π^o_α that the social insurer charges for mandatory liability insurance, and for insurance voluntarily upgraded to casco (which includes liability), respectively, so as to maximize social welfare.

We use social surplus as the measure for social welfare $\mathcal{W}$. It is defined as the net social benefit $\mathcal{B}(K_\alpha, K_\beta)$ of traveling (after correcting for all private costs), minus the expected costs of accidents:

$$\mathcal{W}(K_\alpha, K_\beta) = \mathcal{B}(K_\alpha, K_\beta) - K\mathcal{C}_A(K).$$

The partial derivatives $\frac{\partial \mathcal{B}}{\partial K_\alpha}$ and $\frac{\partial \mathcal{B}}{\partial K_\beta}$ represent the inverse demand functions for insured kilometrage of both types; *i.e.*, the marginal willingness to pay for kilometers driven:

$$\mathcal{D}_\alpha(K_\alpha, K_\beta) = \frac{\partial \mathcal{B}}{\partial K_\alpha}, \quad \mathcal{D}_\beta(K_\alpha, K_\beta) = \frac{\partial \mathcal{B}}{\partial K_\beta}.$$

Equilibrium kilometrage, therefore, is determined by equating this marginal willingness to pay to the "full price" of driving, including the insurance premium and the non-covered part of expected accident costs. The equilibrium conditions for kilometrage driven are the following:

$$\mathcal{D}_\alpha = \pi_\alpha + (1-\alpha)\mathcal{C}_A(K), \quad \mathcal{D}_\beta = \pi_\beta + (1-\frac{\beta}{2})\mathcal{C}_A(K). \tag{1}$$

Maximizing social surplus with respect to (1), we find that socially optimal premiums are:

$$\pi^o_\alpha = \alpha\mathcal{C}_A(K) + K\frac{\partial \mathcal{C}_A}{\partial K_\alpha}, \quad \pi^o_\beta = \frac{\beta}{2}\mathcal{C}_A(K) + K\frac{\partial \mathcal{C}_A}{\partial K_\beta}. \tag{2}$$

for the casco and liability insurance, respectively. The premiums (2) are equal to the corresponding expected payments received from the insurance company, effectively implying that full self-insurance is optimal, plus the marginal external costs imposed on other drivers; which is isomorphic to the conventional Pigouvian congestion toll $\tau = Flow\frac{\partial Cost}{\partial Flow}$, (Small and Verhoef, 2007, pg. 122). Intuitively, the public welfare-maximizing monopolist therefore fully internalizes the externalities caused by the road users, and makes them face the full "own" expected cost per kilometer, by charging a premium equal to the expected accident cost compensation.

Note that, in expected terms, self-insurance appears to be a rather pointless action to take; usual motivations for this via arguments of risk aversion are absent from our model because we prefer to consider expected utility maximizing behaviour, so that our policy conclusions are solely motivated by the distortions from the accident externality and market power of insurance companies.

3. Markets with private profit maximizing firms

3.1. Monopoly

We next consider a private monopoly, where the monopolist seeks to maximize its profit rather than social welfare:

$$\Pi(K_\alpha, K_\beta) = \pi_\alpha K_\alpha + \pi_\beta K_\beta - \alpha K_\alpha \mathcal{C}_A(K) - \frac{\beta}{2} K_\beta \mathcal{C}_A(K).$$

Using the equilibrium conditions (1), we obtain:

$$\pi_\alpha^m = \pi_\alpha^o - K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\alpha} - K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\alpha}, \quad \pi_\beta^m = \pi_\beta^o - K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\beta} - K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\beta}. \tag{3}$$

Thus, the profit-maximizing premiums of private monopolist add to the expression for the first-best premium, demand-related mark-up's $-K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\beta} - K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\alpha}$ and $-K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\alpha} - K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\beta}$. These are conventional monopolistic mark-ups. Note that the terms $-K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\beta}$, $-K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\alpha}$, and $-K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\alpha}$, $-K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\beta}$ are positive with downward sloping demands. While setting a premium, the monopolist has to take into account the fact that different types of insurances are (imperfect) substitutes. With perfectly elastic demands, the private monopolist charges the same premiums as the social monopolist (as expected, monopolistic mark-ups generally vanish if demand approaches perfect elasticity). The monopolist fully internalizes the accident externality for the same reason why a single road owner has the incentive to perfectly internalize congestion externalities: the increase in cost for other users depresses their willingness to pay, and hence the revenue that the monopolist can extract from them, for a given level of demand, on a dollar-by-dollar basis. Therefore, the externality between drivers is entirely internal to the firm, which is why the firm finds it profitable to have prices optimally reflect the marginal externality.

3.2. Private duopoly

In this section we extend our analysis to a private duopoly insurance market. Let us consider two firms, i and j. The firms are considered as imperfect substitutes by consumers, perfect substitutes being a special case, and casco- and liability insurances are in general imperfect substitutes as well. Total benefit is a function $\mathcal{B}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j})$. Then, the inverse demands $\mathcal{D}_{\alpha i} = \frac{\partial \mathcal{B}}{\partial K_{\alpha i}}$ and $\mathcal{D}_{\beta i} = \frac{\partial \mathcal{B}}{\partial K_{\beta i}}$ will be a function of firm i's own choice of kilometrage, as well as the rival j's strategy $K_{\alpha j}, K_{\beta j}$.

The liability payment received by the third party and covered by the insurer of a guilty driver remains βc_A, while the payment received by a casco insured driver is $\alpha_i c_A$ or $\alpha_j c_A$ depending on the firm (s)he has insurance from, with $\alpha_{i(j)} > \beta$. Again, we define kilometrage covered by insurer i's clients as $K_{\alpha i} + K_{\beta i} = K_i$, and similar for firm j: $K_{\alpha j} + K_{\beta j} = K_j$, while $K_i + K_j = K$ is the aggregate vehicle kilometrage driven.

Without loss of generality, we provide analytical results only for company i. It maximizes its own profit:

$$\begin{aligned} &\max \Pi_i(K_{\alpha i}, K_{\beta i}; K_{\alpha j}, K_{\beta j}) \\ &\qquad = K_{\alpha i}\pi_{\alpha i} + K_{\beta i}\pi_{\beta i} - K_{\alpha i}(\alpha_i - \frac{\beta}{2})\mathcal{C}_A(K) - K_i\frac{\beta}{2}\mathcal{C}_A(K), \end{aligned} \tag{4}$$

under the constraints implied by the equilibrium conditions that look similar to (1):

$$\mathcal{D}_{\alpha i} = \pi_{\alpha i} + (1-\alpha_i)\mathcal{C}_A(K), \quad \mathcal{D}_{\beta i} = \pi_{\beta i} + (1-\frac{\beta}{2})\mathcal{C}_A(K). \tag{5}$$

The sum of the first and second terms of (4) represents the firm's revenue, while the last two terms represent the expenditures: firm i has to fully cover its casco-drivers' accident costs, and these costs are partially reimbursed by the other firm in accordance with the liability insurance in case it is its customer is at fault (with the probability of 1/2). The last term is the liability coverage, when firm i's own driver is at fault. Note that for the sake of brevity, we omit the arguments of functions, unless doing so may cause confusion.

We assume that the firms compete in a Cournot fashion, taking the other firm's kilometrage as given. Solving the maximization problem (4)-(5), we find the profit-maximizing premiums:

$$\begin{aligned}
\pi^d_{\alpha i} &= \alpha_i \mathcal{C}_A + K_i \frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K_{\alpha i}\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} - K_{\beta i}\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}}, \\
\pi^d_{\beta i} &= \frac{\beta}{2}\mathcal{C}_A + K_i \frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K_{\alpha i}\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}} - K_{\beta i}\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}}.
\end{aligned} \tag{6}$$

One might have expected the term $\frac{\beta}{2}\mathcal{C}_A$ (with $\mathcal{C}_A$ being external cost per kilometer) in the first expression in (6) as well, as in the second, but it exactly cancels against liability compensation that the insuree receives from the other insurance company ($\frac{\beta}{2}\mathcal{C}_A$ with $\mathcal{C}_A$ being own cost per kilometer), and that compensation implies that $\alpha_i\mathcal{C}_A$ is an "overestimate" for the net cost of compensation by firm i to its casco drivers.

Thus, the duopoly premiums consist of the covered expected accident costs, plus the marginal external accident costs imposed by the firm's additional customer on its own customers, plus a demand-related mark-up. The accident externality imposed by a driver on other customers of the same insurance company, is a full loss for the company, as one part of it is covered by the firm's own compensation to those other customers, while the remaining part suppresses these other customers' willingness to pay for a premium. The firm therefore has the incentive to fully internalize the externalities that its own consumers impose upon one-another.

The own demand effects $-K_{\alpha i}\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}}$ and $-K_{\beta i}\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}}$, and the cross-demand terms $-K_{\beta i}\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}}$ and $-K_{\alpha i}\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}}$, are again positive. As demand sensitivities are now multiplied by a fraction of market demand, the demand-related mark-ups are generally smaller than in the private monopoly.

Note that the effects from the other duopolist's behavior (terms with K_j, $K_{\alpha j}$, and $K_{\beta j}$) are implicitly captured both in $\mathcal{C}_A$, which depends on total kilometrage K, and in the inverse demand functions $\mathcal{D}_{\alpha i}$ and $\mathcal{D}_{\beta i}$, which have $K_{\alpha j}$ and $K_{\beta j}$ as parametric arguments.

3.3. Insurance market with N firms

Using a free-entry insurance market, we may now analyze companies' strategies and market efficiency under increasing competition, with possibly more than two suppliers, so that monopoly/oligopoly behavior may be mitigated. We assume the existence of N firms. Maintaining notational conventions from the previous subsection, an insurer i sells liability so that it covers $K_{\beta i}$ kilometers, and $K_{\alpha i}$ kilometers are casco-insured. Then total kilometrage $K = \sum_{i=1}^{N} K_i = \sum_{i=1}^{N}(K_{\beta i} + K_{\alpha i})$. By repeating our derivations for the duopoly case, we have

$$\pi_{\alpha i}^{n} = \alpha_i \mathcal{C}_A + K_i \frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} - K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}}, \tag{7}$$
$$\pi_{\beta i}^{n} = \frac{\beta}{2} \mathcal{C}_A + K_i \frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}} - K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}}.$$

As N approaches infinity, the last three terms from (7) become zero due to the shrinking market shares, hence $\lim_{N\to\infty} \pi_{\alpha i}^{n} = \alpha_i \mathcal{C}_A$ and $\lim_{N\to\infty} \pi_{\beta i}^{n} = \frac{\beta}{2}\mathcal{C}_A$. Comparison with the first-best premiums (2) shows that the existence of too many firms on the market may in fact lead to bellow-optimal premium levels, and hence a decrease in social welfare. This is a strong contrast to the case with optimal road congestion prices when the number of (parallel) private suppliers goes to infinity. Then, the equilibrium toll converges to the optimal value (Engel et al., 2004; Small and Verhoef, 2007). The root of this difference between the two problems is that for private roads, the congestion externality remains entirely internal for the firm as it is typically assured that the speed on a road does not depend directly on the flow on other roads. For insurance companies, in contrast, the accident externality is to an increasing extent external to the firm when the number of the firms increases. We therefore face a classical externality problem in the limit of an infinite number of firms, but now between atomistic firms, rather than between atomistic drivers.

4. Taxes imposed on companies

In order to correct for the difference between the social monopoly premiums in (2) and the private firms' premiums, and therewith to achieve optimal social welfare, the government may introduce taxes/subsidies. These could be imposed either on insurance companies or on drivers. An aspect of market regulation via taxation of firms with market power gives room to a firm with sufficient market power to affect or manipulate the equilibrium tax level when it knows the tax rule applied, just like it will affect the equilibrium output price if doing so increases achievable profits. In this section, we take this into account, and analyze taxation rules that are targeted to allow both the regulator to maximize the social welfare and the firms to achieve the maximum (possible under taxation) profit level. Because such "manipulable taxation" is quite uncommon to consider (Brueckner and Verhoef, 2010), we will also consider the conventional case of "parametric taxation", where firms are unaware of the underlying tax rule , or for other reasons treat taxes as parametric.

4.1. Monopoly

When a parametric tax/subsidy is used to bridge the difference between social welfare-maximizing (2) and the private monopolist (3) premiums, the taxes should be equal to

$$\mathcal{P}^m_\alpha = K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\alpha} + K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\alpha}, \qquad \mathcal{P}^m_\beta = K_\beta \frac{\partial \mathcal{D}_\beta}{\partial K_\beta} + K_\alpha \frac{\partial \mathcal{D}_\alpha}{\partial K_\beta}. \tag{8}$$

Although parametric taxes are standard tools for dealing with negative externalities, in case of large enough agents the assumption of regulatees treating tax as parametric becomes problematic, and the taxation system may no longer direct the market to the social optimum if regulatees understand that they can affect the level of the tax by their own behavior. In order to avoid it, manipulable tax rules can be designed to achieve the social optimum (Brueckner and Verhoef, 2010).

Let us assume now that private monopolist is aware of the rule used by the state for the calculation of the subsidy[2] $\sigma(K_\alpha, K_\beta)$. In that case, the monopolist maximizes its profit

$$\max \Pi = \pi_\alpha K_\alpha + \pi_\beta K_\beta - \alpha K_\alpha \mathcal{C}_A - \frac{\beta}{2} K_\beta \mathcal{C}_A + \sigma(K_\alpha, K_\beta).$$

Given the equilibrium conditions (1), we redo the analysis of the corresponding Lagrange function and compare the resulting premiums with the rates (2), that maximize social welfare, we find that the taxation function has to meet the following conditions

$$\tfrac{\partial \sigma}{\partial K_\alpha} = -K_\alpha \tfrac{\partial \mathcal{D}_\alpha}{\partial K_\alpha} - K_\beta \tfrac{\partial \mathcal{D}_\beta}{\partial K_\alpha}, \qquad \tfrac{\partial \sigma}{\partial K_\beta} = -K_\beta \tfrac{\partial \mathcal{D}_\beta}{\partial K_\beta} - K_\alpha \tfrac{\partial \mathcal{D}_\alpha}{\partial K_\beta}. \tag{9}$$

Integrating (9), and assuming $\sigma(0,0) = 0$ to fix the constant of integration, we get the taxation rule

$$\sigma(K_\alpha, K_\beta) = \mathcal{B}(K_\alpha, K_\beta) - \big(K_\alpha \mathcal{D}_\alpha(K_\alpha, K_\beta) + K_\beta \mathcal{D}_\beta(K_\alpha, K_\beta)\big). \tag{10}$$

[2]The term σ will represent subsidization rule if it is possible, and taxation rule if it is negative.

Equation (10) is equal to the consumer surplus from driving under insurance.

The subsidies in (10) can only be positive as the minuses compensate the negative partial derivatives, which implies that the monopolist receives a net subsidy under this particular choice of the constant $\sigma(0,0)$. Note that formulas (9) represent the decrease of the insurance premiums, and the reduced premiums lead to an increase of the aggregate vehicle kilometrage driven by the road users.

4.2. Duopoly

In duopoly, a firm internalizes only a part of externalities generated by its customers. In order to find a manipulable taxation rule $\sigma_i(K_{\alpha i}, K_{\beta i}; K_{\alpha j}, K_{\beta j})$ for insurance company i, we equate the marginal manipulable tax to the optimal parametric tax:

$$\begin{aligned} \frac{\partial \sigma_i}{\partial K_{\alpha i}} &= \mathcal{P}^d_{\alpha i} = -K_j \frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} - K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}}, \\ \frac{\partial \sigma_i}{\partial K_{\beta i}} &= \mathcal{P}^d_{\beta i} = -K_j \frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}} - K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}}. \end{aligned} \tag{11}$$

Integrating (11) and setting $\sigma_i(0,0;\cdot) = 0$ provide us with the solution:

$$\begin{aligned} \sigma_i(K_{\alpha i}, K_{\beta i}; \cdot) = & \mathcal{B}(K_{\alpha i}, K_{\beta i}; \cdot) - \mathcal{B}(0,0;\cdot) \\ & - \big(K_{\alpha i}\mathcal{D}_{\alpha i}(K_{\alpha i}, K_{\beta i}; \cdot) + K_{\beta i}\mathcal{D}_{\beta i}(K_{\alpha i}, K_{\beta i}; \cdot)\big) \\ & - K_j\big(\mathcal{C}_A(K_i + K_j) - \mathcal{C}_A(K_j)\big). \end{aligned} \tag{12}$$

Here, the monopoly subsidization rule of (10) is decreased, as the accident externalities imposed by firm i's customers on other drivers are now part of the pricing rule. The liabilities (12) may now be negative or positive, and subsidies are in place if consumer surplus is large relative to the accident externality. Specifically, firm i receives a subsidy that is equal to the drivers' consumer surplus it generates, over the supply generated by the competitor, minus the total external cost its customers impose on the other firm's customers. Quite intuitively, this subsidy transforms the firm's optimization problem into the maximization of social surplus.

4.3. Taxation in N-firm market

The model can be extended for N firms. The duopoly results are easily generalized to the case of a market with an arbitrary number N of firms.

Equations (11) then become:

$$\frac{\partial \sigma_i}{\partial K_{\alpha i}} = \mathcal{P}^e_{\alpha i} = -K_{-i}\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K_{\alpha i}\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} - K_{\beta i}\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}}, \tag{13}$$
$$\frac{\partial \sigma_i}{\partial K_{\beta i}} = \mathcal{P}^e_{\beta i} = -K_{-i}\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K_{\beta i}\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}} - K_{\alpha i}\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}},$$

where K_{-i} substitutes K_j in (11), and stands for total kilometrage of other firms. For the optimal manipulable tax, we then obtain the following expression:

$$\begin{aligned}\sigma_i(K_{\alpha i}, K_{\beta i}; \cdot) = &\mathcal{B}(K_{\alpha i}, K_{\beta i}; \cdot) - \mathcal{B}(0, 0; \cdot) \\ &- \big(K_{\alpha i}\mathcal{D}_{\alpha i}(K_{\alpha i}, K_{\beta i}; \cdot) + K_{\beta i}\mathcal{D}_{\beta i}(K_{\alpha i}, K_{\beta i}; \cdot)\big) \\ &- K_{-i}\big(\mathcal{C}_A(K_i + K_{-i}) - \mathcal{C}_A(K_{-i})\big).\end{aligned} \tag{14}$$

In a limiting case, with N approaching infinity, the optimal tax on atomistic companies (who will treat it atomistically due to lack of market power) is the standard Pigouvian marginal (accident) externality tax. In the other extreme, with one single firm, (14) reduces into the monopolistic rule in (10). Clearly, for $N = 2$, the duopoly results above are reproduced.

5. Taxation of drivers

Given that the taxes and subsidies derived above aim to correct for non-optimal pricing by insurance companies, it seemed natural to assume — as we did — that it is these firms who are primarily taxed or subsidized. Still, externality pricing in road transport is usually associated with the pricing of individual travelers, or vehicles, and it is equally natural to consider optimal taxation of travelers when they are insured by firms with market power. this section will consider that option. In doing so, we will make the conventional assumption that firms take taxes to be paid by road users as parametric.

5.1. Monopoly

On a monopolistic market, the regulator maximizes the social welfare

$$\max \mathcal{W} = \mathcal{B} - K\mathcal{C}_A \tag{15}$$

subject to the equilibrium conditions, which now also contain the user taxes,[3] denoted as τ_α^m, τ_β^m

$$\begin{aligned} \mathcal{D}_\alpha - (1-\alpha)\mathcal{C}_A - \pi_\alpha^m - \tau_\alpha^m &= 0, \\ \mathcal{D}_\beta - (1-\beta/2)\mathcal{C}_A - \pi_\beta^m - \tau_\beta^m &= 0. \end{aligned}$$

The Lagrangian to this constrained optimization problem looks as follows:

$$\begin{aligned} \mathcal{L}(\cdot, \lambda_\alpha, \lambda_\beta) =& \mathcal{W} + \lambda_\alpha\big(\mathcal{D}_\alpha - (1-\alpha)\mathcal{C}_A - \pi_\alpha^m - \tau_\alpha^m\big) \\ &+ \lambda_\beta\big(\mathcal{D}_\beta - (1-\beta/2)\mathcal{C}_A - \pi_\beta^m - \tau_\beta^m\big). \end{aligned}$$

The Lagrangian multipliers are zero:

$$\frac{\partial \mathcal{L}}{\tau_\alpha^m} = \frac{\partial \mathcal{L}}{\tau_\beta^m} = -\lambda_\alpha = -\lambda_\beta = 0.$$

And the F.O.C. with respect to K_α is

$$\frac{\partial \mathcal{L}}{\partial K_\alpha} = \mathcal{D}_\alpha - \mathcal{C}_A - K\frac{\partial \mathcal{C}_A}{\partial K_\alpha} = -\alpha\mathcal{C}_A + \pi_\alpha^m + \tau_\alpha^m - K\frac{\partial \mathcal{C}_A}{K_\alpha} = 0.$$

Therefore,

$$\tau_\alpha^m = \alpha\mathcal{C}_A + K\frac{\partial \mathcal{C}_A}{K_\alpha} - \pi_\alpha^m.$$

Thus, for the premiums π_α^m, π_β^m defined in (3), the form of the taxes in monopoly market is

$$\tau_\alpha^m = K_\alpha\frac{\partial \mathcal{D}_\alpha}{\partial K_\alpha} + K_\beta\frac{\partial \mathcal{D}_\beta}{\partial K_\alpha}, \quad \tau_\beta^m = K_\alpha\frac{\partial \mathcal{D}_\alpha}{\partial K_\beta} + K_\beta\frac{\partial \mathcal{D}_\beta}{\partial K_\beta}.$$

These are net subsidies (as τ_α^m, τ_β^m are negative), and they provide exactly the discounts in the subsidization of the firm found in (3). The interpretation stays the same: the firm already internalizes the accident externality, and furthermore overcharges by using the conventional monopolistic mark-up. The tax, therefore, needs not reflect the former, and corrects for the latter.

[3] Positive τ_α^m, τ_β^m are taxes, which increase the total price incurred by a driver, so that negative τ_α^m, τ_β^m are in fact subsidies, decreasing these prices.

5.2. Duopoly

For the analysis of drivers' taxation under duopolistic supply of insurance, it matters whether or not we assume that taxes can be differentiated over road users based on the identity of their insurance company. We start the unconstrained case, where the social welfare (15) is maximized subject to the equilibrium conditions, accounting for firm-specific taxes $\tau^d_{\alpha i(j)}$, $\tau^d_{\beta i(j)}$

$$\begin{aligned}
\mathcal{D}_{\alpha i} - (1-\alpha_i)\mathcal{C}_A - \pi^d_{\alpha i} - \tau^d_{\alpha i} &= 0, \\
\mathcal{D}_{\beta i} - (1-\beta/2)\mathcal{C}_A - \pi^d_{\beta i} - \tau^d_{\beta i} &= 0, \\
\mathcal{D}_{\alpha j} - (1-\alpha_j)\mathcal{C}_A - \pi^d_{\alpha j} - \tau^d_{\alpha j} &= 0, \\
\mathcal{D}_{\beta j} - (1-\beta/2)\mathcal{C}_A - \pi^d_{\beta j} - \tau^d_{\beta j} &= 0.
\end{aligned}$$

The Lagrangian is

$$\begin{aligned}
\mathcal{L}_d(\cdot, \lambda_{\alpha i}, \lambda_{\beta i}, \lambda_{\alpha j}, \lambda_{\beta j}) =& \mathcal{W} + \lambda_{\alpha i}\big(\mathcal{D}_{\alpha i} - (1-\alpha_i)\mathcal{C}_A - \pi^d_{\alpha i} - \tau^d_{\alpha i}\big) \\
&+ \lambda_{\beta i}\big(\mathcal{D}_{\beta i} - (1-\beta/2)\mathcal{C}_A - \pi^d_{\beta i} - \tau^d_{\beta i}\big) \\
&+ \lambda_{\alpha j}\big(\mathcal{D}_{\alpha j} - (1-\alpha_j)\mathcal{C}_A - \pi^d_{\alpha j} - \tau^d_{\alpha j}\big) \\
&+ \lambda_{\beta j}\big(\mathcal{D}_{\beta j} - (1-\beta/2)\mathcal{C}_A - \pi^d_{\beta j} - \tau^d_{\beta j}\big).
\end{aligned} \tag{16}$$

When taxation of a driver can be differentiated on basis of the identity of the firm-insurer, the Lagrangian multipliers are again all individually equal to zero. Given our assumption on the premiums (6), the optimal differentiated taxes are:

$$\begin{aligned}
\tau^d_{\alpha i} &= K_j \frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} + K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} + K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}}, \\
\tau^d_{\beta i} &= K_j \frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} + K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}} + K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}}, \\
\tau^d_{\alpha j} &= K_i \frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} + K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha j}} + K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha j}}, \\
\tau^d_{\beta j} &= K_i \frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} + K_{\alpha i} \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta j}} + K_{\beta i} \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta j}}.
\end{aligned} \tag{17}$$

These taxes internalize the part of the externality (imposed on the other firm's clients) that the firm ignores, and, again, correct for the mark-ups. The positive part of the taxes, represented by the first terms in (17), grows along with the other firm's market power, while the absolute value of the negative part grows with the size of the own firm.

The regulation by introducing the road users' taxes (17) (provided treated as parametric by firms) is equivalent to the use of taxes imposed on the insurance firms; hence, it delivers the first-best equilibrium.

It is, however, not inconceivable that the taxation liability depends only on the type of insurance, and not on the firm providing the insurance. We then have to look for a generic τ_α^d as a tax for all drivers with an extended insurance, and a τ_β^d as the generic tax for all drivers with a mandatory insurance only. Then, the first-order conditions state that $\lambda_{\alpha i} + \lambda_{\alpha j} = 0$ and $\lambda_{\beta i} + \lambda_{\beta j} = 0$, and the relevant Lagrangian is

$$\begin{aligned}
&\bar{\mathcal{L}}_d(\cdot, \lambda_{\alpha i}, \lambda_{\beta i}, \lambda_{\alpha j}, \lambda_{\beta j}) \qquad (18)\\
=&\mathcal{W} + \lambda_{\alpha i}\big(\mathcal{D}_{\alpha i} - (1-\alpha_i)\mathcal{C}_A - \pi_{\alpha i}^d - \tau_\alpha^d\big) + \lambda_{\beta i}\big(\mathcal{D}_{\beta i} - (1-\beta/2)\mathcal{C}_A - \pi_{\beta i}^d - \tau_\beta^d\big)\\
&+ \lambda_{\alpha j}\big(\mathcal{D}_{\alpha j} - (1-\alpha_j)\mathcal{C}_A - \pi_{\alpha j}^d - \tau_\alpha^d\big) + \lambda_{\beta j}\big(\mathcal{D}_{\beta j} - (1-\beta/2)\mathcal{C}_A - \pi_{\beta j}^d - \tau_\beta^d\big)\\
=&\mathcal{W} + \lambda_{\alpha}\big(\mathcal{D}_{\alpha i} - (1-\alpha_i)\mathcal{C}_A - \pi_{\alpha i}^d - \tau_\alpha^d\big) + \lambda_{\beta}\big(\mathcal{D}_{\beta i} - (1-\beta/2)\mathcal{C}_A - \pi_{\beta i}^d - \tau_\beta^d\big)\\
&- \lambda_{\alpha}\big(\mathcal{D}_{\alpha j} - (1-\alpha_j)\mathcal{C}_A - \pi_{\alpha j}^d - \tau_\alpha^d\big) - \lambda_{\beta}\big(\mathcal{D}_{\beta j} - (1-\beta/2)\mathcal{C}_A - \pi_{\beta j}^d - \tau_\beta^d\big)\\
=&\mathcal{W} + \lambda_{\alpha}\big(\mathcal{D}_{\alpha i} - (1-\alpha_i)\mathcal{C}_A - \pi_{\alpha i}^d - \mathcal{D}_{\alpha j} + (1-\alpha_j)\mathcal{C}_A + \pi_{\alpha j}^d\big)\\
&+ \lambda_{\beta}\big(\mathcal{D}_{\beta i} - (1-\beta/2)\mathcal{C}_A - \pi_{\beta i}^d - \mathcal{D}_{\beta j} + (1-\beta/2)\mathcal{C}_A + \pi_{\beta j}^d\big)\\
=&\mathcal{W} + \lambda_{\alpha}\big(\mathcal{D}_{\alpha i} + (\alpha_i-\alpha_j)\mathcal{C}_A - \pi_{\alpha i}^d - \mathcal{D}_{\alpha j} + \pi_{\alpha j}^d\big) + \lambda_{\beta}\big(\mathcal{D}_{\beta i} - \pi_{\beta i}^d - \mathcal{D}_{\beta j} + \pi_{\beta j}^d\big),
\end{aligned}$$

where $\lambda_\alpha = \lambda_{\alpha i} = -\lambda_{\alpha j}$ and $\lambda_\beta = \lambda_{\beta i} = -\lambda_{\beta j}$.

The F.O.C. with respect to kilometrage $K_{\alpha i}$ is

$$\begin{aligned}
\frac{\partial \bar{\mathcal{L}}_d}{\partial K_{\alpha i}} =&\mathcal{D}_{\alpha i} - \mathcal{C}_A - K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} + \lambda_\beta\left[\frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}} - \frac{\partial \mathcal{D}_{\beta j}}{\partial K_{\alpha i}} - \frac{\partial \pi_{\beta i}^d}{\partial K_{\alpha i}} + \frac{\partial \pi_{\beta j}^d}{\partial K_{\alpha i}}\right]\\
&+ \lambda_\alpha\left[\frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} - \frac{\partial \mathcal{D}_{\alpha j}}{\partial K_{\alpha i}} + (\alpha_i-\alpha_j)\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - \frac{\partial \pi_{\alpha i}^d}{\partial K_{\alpha i}} + \frac{\partial \pi_{\alpha j}^d}{\partial K_{\alpha i}}\right]\\
=&-\alpha_i\mathcal{C}_A + \pi_{\alpha i}^d + \tau_\alpha^d - K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} + \lambda_\alpha Aai + \lambda_\beta Bai = 0, \qquad (19)
\end{aligned}$$

where

$$\begin{aligned}
Aai &= \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha i}} - \frac{\partial \mathcal{D}_{\alpha j}}{\partial K_{\alpha i}} + (\alpha_i-\alpha_j)\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - \frac{\partial \pi_{\alpha i}^d}{\partial K_{\alpha i}} + \frac{\partial \pi_{\alpha j}^d}{\partial K_{\alpha i}},\\
Bai &= \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha i}} - \frac{\partial \mathcal{D}_{\beta j}}{\partial K_{\alpha i}} - \frac{\partial \pi_{\beta i}^d}{\partial K_{\alpha i}} + \frac{\partial \pi_{\beta j}^d}{\partial K_{\alpha i}}.
\end{aligned}$$

The other three equations look similar:

$$
\begin{aligned}
&-\alpha_j \mathcal{C}_A + \pi^d_{\alpha j} + \tau^d_\alpha - K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} + \lambda_\alpha Aaj + \lambda_\beta Baj = 0, && (20)\\
&-\frac{\beta}{2}\mathcal{C}_A + \pi^d_{\beta i} + \tau^d_\beta - K\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} + \lambda_\alpha Abi + \lambda_\beta Bbi = 0, && (21)\\
&-\frac{\beta}{2}\mathcal{C}_A + \pi^d_{\beta j} + \tau^d_\beta - K\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} + \lambda_\alpha Abj + \lambda_\beta Bbj = 0. && (22)
\end{aligned}
$$

The auxiliary notations[4] in (19)-(22) are:

$$
\begin{aligned}
Aaj &= \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\alpha j}} - \frac{\partial \mathcal{D}_{\alpha j}}{\partial K_{\alpha j}} + (\alpha_i - \alpha_j)\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} - \frac{\partial \pi^d_{\alpha i}}{\partial K_{\alpha j}} + \frac{\partial \pi^d_{\alpha j}}{\partial K_{\alpha j}},\\
Baj &= \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\alpha j}} - \frac{\partial \mathcal{D}_{\beta j}}{\partial K_{\alpha j}} - \frac{\partial \pi^d_{\beta i}}{\partial K_{\alpha j}} + \frac{\partial \pi^d_{\beta j}}{\partial K_{\alpha j}},\\
Abi &= \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta i}} - \frac{\partial \mathcal{D}_{\alpha j}}{\partial K_{\beta i}} + (\alpha_i - \alpha_j)\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - \frac{\partial \pi^d_{\alpha i}}{\partial K_{\beta i}} + \frac{\partial \pi^d_{\alpha j}}{\partial K_{\beta i}},\\
Bbi &= \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta i}} - \frac{\partial \mathcal{D}_{\beta j}}{\partial K_{\beta i}} - \frac{\partial \pi^d_{\beta i}}{\partial K_{\beta i}} + \frac{\partial \pi^d_{\beta j}}{\partial K_{\beta i}},\\
Abj &= \frac{\partial \mathcal{D}_{\alpha i}}{\partial K_{\beta j}} - \frac{\partial \mathcal{D}_{\alpha j}}{\partial K_{\beta j}} + (\alpha_i - \alpha_j)\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} - \frac{\partial \pi^d_{\alpha i}}{\partial K_{\beta j}} + \frac{\partial \pi^d_{\alpha j}}{\partial K_{\beta j}},\\
Bbj &= \frac{\partial \mathcal{D}_{\beta i}}{\partial K_{\beta j}} - \frac{\partial \mathcal{D}_{\beta j}}{\partial K_{\beta j}} - \frac{\partial \pi^d_{\beta i}}{\partial K_{\beta j}} + \frac{\partial \pi^d_{\beta j}}{\partial K_{\beta j}}.
\end{aligned}
$$

Therefore, we have a system of four equations (20)-(22) with respect to four unknowns τ^d_α, τ^d_β, λ_α, and λ_β, to determine the second-best taxes. The system can be rewritten in matrix form as follows

$$
\begin{pmatrix} 1 & 0 & Aai & Bai \\ 1 & 0 & Aaj & Baj \\ 0 & 1 & Abi & Bbi \\ 0 & 1 & Abj & Bbj \end{pmatrix}
\begin{pmatrix} \tau^d_\alpha \\ \tau^d_\beta \\ \lambda_\alpha \\ \lambda_\beta \end{pmatrix}
=
\begin{pmatrix} \alpha_i \mathcal{C}_A - \pi^d_{\alpha i} + K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} \\ \alpha_j \mathcal{C}_A - \pi^d_{\alpha j} + K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} \\ \frac{\beta}{2}\mathcal{C}_A - \pi^d_{\beta i} + K\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} \\ \frac{\beta}{2}\mathcal{C}_A - \pi^d_{\beta j} + K\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} \end{pmatrix}. \qquad (23)
$$

[4]The first letter A (B) corresponds to α (β) in the numerators of the fractions, the other two are from the denominator, e.g. ai mean that the partial derivatives are taken over $K_{\alpha i}$.

If the determinant Δ of the matrix at the left side of (23) is not equal to zero, then the system has a unique solution. The transformations of the system's solution can be found in Appendix A.

The firm-indifferent taxation liability rules can be expressed as weighted sums of the differentiated taxes $\tau_{\alpha i(j)}$, $\tau_{\beta i(j)}$ in (17), as follows

$$\begin{aligned}\tau^d_\alpha = &- \big[(\tau^d_{\beta i} - \tau^d_{\beta j})(AaiAbj - AajBai)\\ &- \tau^d_{\alpha i}(AbjBai - AbjBaj - AajBbi + AajBbj)\\ &+ \tau^d_{\alpha j}(Bai^2 - BaiBaj - AaiBbi + AaiBbj)\big]/\Delta,\\ \tau^d_\beta = &- \big[(\tau^d_{\alpha i} - \tau^d_{\alpha j} - (\alpha_i - \alpha_j)\mathcal{C}_A)(BaiBbj - BajBbi)\\ &+ \tau^d_{\beta j}(AbjBai - AajBbi + AaiBbi - Baj^2)\\ &- \tau^d_{\beta i}(AbjBaj - BaiBaj + AaiBbj - AajBbj)\big]/\Delta.\end{aligned} \tag{24}$$

Therefore, the indifferent α-tax includes not only $\alpha i(j)$- but also the difference between the βi- and βj-taxes; and similarly for the β-tax. In a symmetric duopoly with the firms being perfect substitutes $\tau_{\alpha i} = \tau_{\alpha j}$ and $\tau_{\beta i} = \tau_{\beta j}$, but these are not a special case of the solution (24) as the Lagrangian (18) shrinks to the social welfare $\mathcal{W}$. Instead, one can calculate the differentiated taxes, which provide the first-best equilibrium.

6. Numerical sensitivity analysis

The marginal conditions derived in the foregoing provide quite some insight into the economic properties of the problem at hand. However, to understand the relative performance of the different market forms, and the factors determining these, we have to perform a comparative static analysis using an equilibrium model. In order to perform such a sensitivity analysis, we first do some calibration[5]. We assume the social benefit $\mathcal{B}$ to be quadratic, thus the inverse demand functions are linear. The insurances are assumed to be imperfect substitutes, while the firms are perfect substitutes. The form of the expected accident costs is linear: $\mathcal{C}_A = \gamma K$, where the positive constant γ represents the per-kilometer risk of being involved into an accident.

[5] We give detailed description of the calibration process in Appendix B, and only mention the forms of the inverse demand and accident cost functions, and other essential market characteristics here.

Following Arnott et al. (1991), we use, as an indicator of efficiency, relative efficiency of a market setup

$$\omega = \frac{\mathcal{W}^{\#} - \mathcal{W}_{ref}}{\mathcal{W}_{fb} - \mathcal{W}_{ref}},$$

where $\mathcal{W}^{\#}$ is the social welfare that the market under investigation generates, $\mathcal{W}_{ref}$ is the "reference" market social welfare, and $\mathcal{W}_{fb}$ is the maximum social welfare. We choose a perfectly competitive insurance market, characterized by zero-profit and the insurance premiums equal to average expected costs, to be our reference point for determining $\mathcal{W}_{ref}$.

In our analysis we will focus on three market characteristics that can be expected to be rather decisive for the market performance: the number of firms N, the cross-effect $d = \frac{\partial \mathcal{D}_\alpha}{\partial K_\beta} = \frac{\partial \mathcal{D}_\beta}{\partial K_\alpha}$, and the price elasticity of demand ε.

The presence of $N > 1$ firms on the (unregulated) insurance market prevents them from full internalization of accident externalities. With an increasing number of firms, the effect upon social welfare therefore depends on the relative importance of two conflicting forces: lower losses for social welfare due to the decrease of market power on the one hand, versus higher losses due to increased externalities. Therefore, one may expect a certain "optimal" number of firms on the insurance market, for which the corresponding social welfare is closest to the social maximum level. Fig. 1 shows the change of relative efficiency of a market as a function of the number of firms. In our calibration, ω reaches its maximum value $\omega_{max} = 0.964893$ at $N = 13$.

The effect of a variation of the cross-effect d is shown on Fig. 2. The decrease of the (absolute value of) coefficient d makes the peak of a market performance sharper, and moves it closer to monopoly, i.e. fewer firms provide maximum possible social welfare on a unregulated market. Larger d makes the graph smoother, while bringing the peak farther away from oligopoly. The reason why a smaller d makes a smaller number of firms optimal, is that it makes the firms less inclined to raise premiums as consumers would respond to that by going to the firm's other product. Premiums are therefore lower, meaning that the optimal premium is achieved for a lower number of firms.

The decreased market power and lower degree of internalization both cause premiums to decline with the number of firms, and hence the total kilometrage to increase (Fig. 3). Under manipulable taxes (14), firms adjust

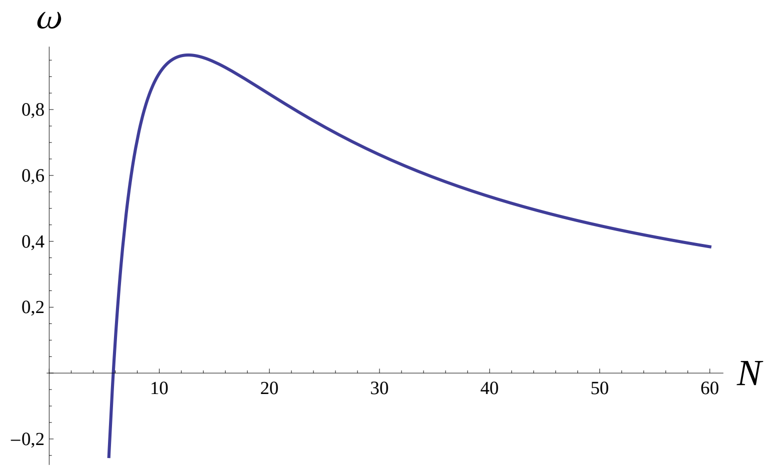

Figure 1: Relative efficiency ω of a market performance as a function of number of firms N

the insurance premiums according to the subsidies or taxes they face, and the equilibrium kilometrage and social welfare will be equal to the first-best level (lower straight line). However, on a free non-regulated market, equilibrium approaches the reference market level, which is normalized to 1 (Fig. 3).

The total manipulable tax per firm is shown in Fig. 4, and Fig. 5 helps understanding the nature of the taxation pattern presented on Fig. 4. The declining line shows the consumer surplus from buying insurance from firm i

$$\sigma_i^+(K_{\alpha i}, K_{\beta i}; \cdot) = \mathcal{B}(K_{\alpha i}, K_{\beta i}; \cdot) - \mathcal{B}(0, 0; \cdot) \\ - \big(K_{\alpha i}\mathcal{D}_{\alpha i}(K_{\alpha i}, K_{\beta i}; \cdot) + K_{\beta i}\mathcal{D}_{\beta i}(K_{\alpha i}, K_{\beta i}; \cdot)\big).$$

It approaches zero as number of firms grows. Meanwhile, the other market failure rises, as lack of market power leads to insufficient internalization of accident externalities by the firm

$$\sigma_i^-(K_{\alpha i}, K_{\beta i}; \cdot) = K_{-i}\big(\mathcal{C}_A(K_i + K_{-i}) - \mathcal{C}_A(K_{-i})\big).$$

The result of these combined mechanisms is a switch from subsidization (for $N = 1 \ldots 6$) to taxation (as of $N = 7$ onwards).

Finally, Fig. 6 show the relative efficiency ω as a function of the price elasticity of demand ε for a profit maximizing monopolist and for duopoly with two symmetric firms being perfect substitutes. The relatively large losses in both figures are partly due to a small denominator in the computation of

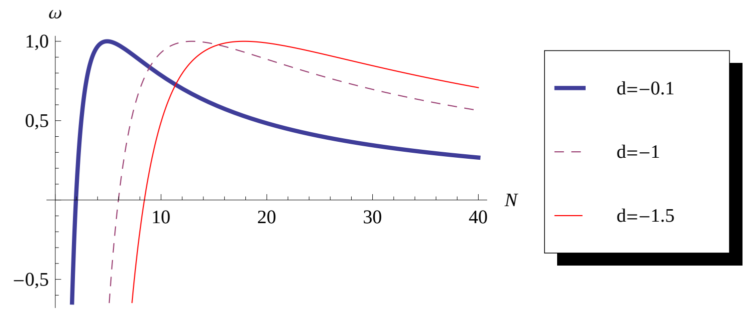

Figure 2: Relative efficiency ω of a market performance as a function of N and the cross effect d

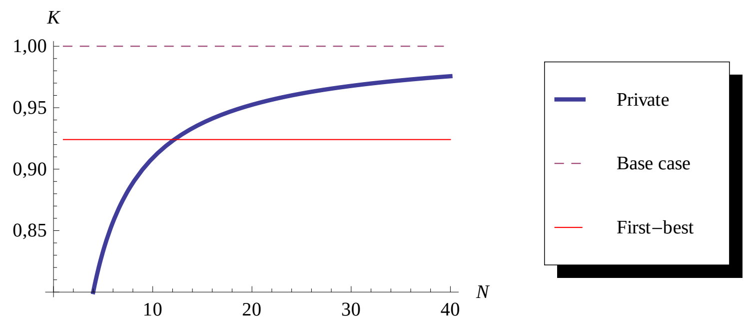

Figure 3: Aggregate kilometrage as a function of N

ω. Both figures show that non-regulated markets produce higher efficiency losses as demand becomes less elastic. The reason is that a higher elasticity (in absolute value) makes market premiums closer to socially optimal for the monopoly and duopoly markets, which tend to overprice as implied by Fig. 4 so that a higher elasticity and the implied smaller mark-up lead to a higher relative efficiency. Decrease of the market power, therefore, leads to a lessening of the efficiency loss.

7. Conclusion

In this paper we designed a simple model of a road accident insurance market. The analysis of the model, while having clear parallels with other literature on externalities in transport (notably the literature on congested airports and parallel road pricing, Small and Verhoef (2007)), brings new interesting results. In particular, although — as is true for competing private

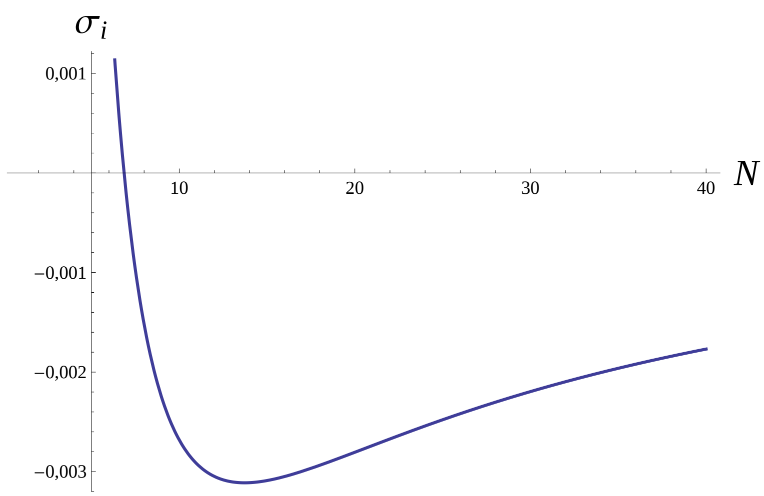

Figure 4: Manipulable tax

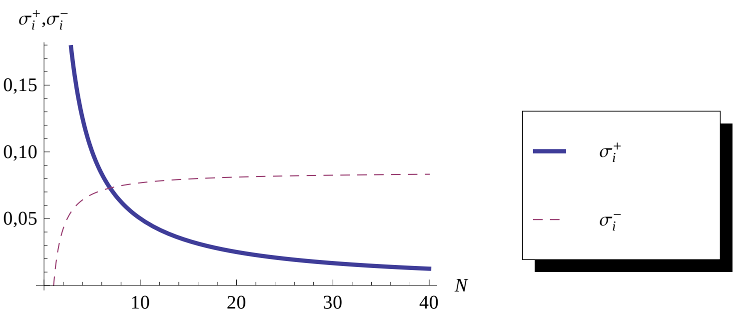

Figure 5: Positive and negative terms of a tax

road operators — insurance firms face an incentive to internalize the externalities that its customers impose upon one-another, an important difference arises because drivers from different insurance companies also directly impose mutual externalities.

The similarity between private road pricing and insurance pricing therefore weakens once we step away from a monopoly case. The presence of $N > 1$ firms on the insurance market prevents them from full internalization of accident externalities. Mathematically, it leads to additional terms in the profit-maximizing premiums' expressions, and with N going to infinity, these premiums do not converge to the socially optimal, as one might have expected from basic micro economics theory.

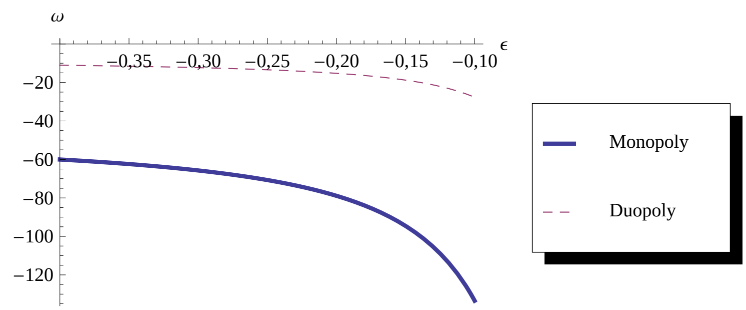

Figure 6: Relative efficiency $\omega = \omega(\varepsilon)$

We also acknowledged that a firm with a large enough market share would most likely use the opportunity to affect the level of regulatory tax while optimizing its decisions. However, in such a case, regulatory taxation based on the erroneous assumption that firms treat taxes parametrically does not reach its goal of maximum social welfare. Introducing of manipulable taxation functions avoids that problem, and should produce an equilibrium corresponding to the social optimum.

Further research will make an emphasis on endogenizing of drivers' decisions on car ownership and use, type of insurance, and speed choice, as it was done in Verhoef and Rouwendal (2004). The number of firms on the market and its interrelation with market-entry costs, and fractions of mandatory and casco coverage, will be investigated in future research.

References

Arnott, R., de Palma, A., Lindsey, R., 1991. Does providing information to drivers reduce traffic congestion? Transportation research Part A 25(5), 309–318.

Boyer, M., Dionne, G., 1987. The economics of road safety, Transportation Research Part B: Methodological, 21(5), 413–431.

Brueckner, J.K., 2002. Airport congestion when carriers have market power, American Economic Review 92(5), 1357–1375.

Brueckner, J.K., Verhoef, E., 2010. Manipulable congestion tolls, Journal of Urban Economics 67, 315–321.

Cohen, A., Einav, L., 2003, The effects of mandatory seat belt laws on driving behavior and traffic fatalities, Review of Economics and Statistics 85, 828-843.

Cohen, A., Dehejia, R., 2004. The effect of automobile insurance and accident liability laws on traffic fatalities, Journal of Law and Economics 47(2), 357–393.

Daniel, J.I., 1995. Congestion pricing and capacity of large hub airports: A bottleneck model with stochastic queues. Econometrica 63(2), 327–370.

Delhaye, E., 2007. Economic analysis of traffic safety. Ph.D. Thesis. Katholieke Universiteit Leuven, Nummer 256. 208 pp.

Edlin, A.S., Mandic, P.K., 2006. The accident externality from driving, Journal of Political Economy 114(5), 931–955.

Engel, E.M., Fischer, R.D., Galetovic, A., 2004. Toll competition among congested roads, The B.E. Journal of Economic Analysis & Policy, Berkeley Electronic Press, 1(4). Available at: http://bit.ly/POoSJ8

Hultkrantz, L., Nilsson, J.-E., Arvidsson, S., 2012. Voluntary internalization of speeding externalities with vehicle insurance, Transportation Research Part A 46(6), 926—937.

Knight, F.H., 1924. Some fallacies in the interpretation of social cost, Quarterly Journal of Economics 38(4), 582–606.

Kuo, T.C., 2011. Evaluating Californian under-age drunk driving laws: endogenous policy lags, Journal of Applied Econometrics. Available at: http://dx.doi.org/10.1002/jae.1243

Parry, I., 2004. Comparing alternative policies to reduce traffic accidents, Journal of Urban Economics 56, 364–387.

Parry, I., Walls, M., Harrington, W., 2007. Automobile externalities and policies, Journal of Economic Literature 45(2), 373–399.

Pigou, A.C., 1920. The Economics of Welfare. Macmillan, London.

Rubin, P.H., Shepherd, J., 2007. Tort reform and accidental deaths, Journal of Law and Economics 50(2), 221–238.

Small, K., Verhoef, E., 2007. The Economics of Urban Transportation. Routledge, London.

SWOV Institute for Road Safety Research, 2009. SWOV Fact Sheet: Risk in traffic. Leidschendam, the Netherlands, September 2009.

SWOV Institute for Road Safety Research, 2010. SWOV Fact Sheet: Mobility on Dutch roads. Leidschendam, the Netherlands, July 2010.

Verhoef, E., Nijkamp, P., Rietveld, P., 1996. Second-best congestion pricing: the case of an untolled alternative, Journal of Urban Economics 40(3), 279–302.

Verhoef, E., Rouwendal, J. 2004. A behavioural model of traffic congestion: Endogenizing speed choice, traffic safety and time losses, Journal of Urban Economics 56, 408–434.

World Health Organization, 2004. World Report on Road Traffic Injury Prevention, Geneva. Available at: http://whqlibdoc.who.int/publications/2004/9241562609.pdf

Appendix A. Taxes imposed on drivers

The system (19)-(22) has a closed form solution[6]:

$$\begin{aligned}
\tau_\alpha^d =& -\Bigg[(K\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} - \pi_{\beta i}^d + \pi_{\beta j}^d)(AaiAbj - AajBai) \\
&- (K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} + \alpha_i \mathcal{C}_A - \pi_{\alpha i}^d)(AbjBai - AbjBaj - AajBbi + AajBbj) \\
&+ (K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} + \alpha_j \mathcal{C}_A - \pi_{\alpha j}^d)(Bai^2 - BaiBaj - AaiBbi + AaiBbj)\Bigg] /\Delta \\
\overset{*}{=}& - \big[(\tau_{\beta i}^d - \tau_{\beta j}^d)(AaiAbj - AajBai) \\
&- \tau_{\alpha i}^d(AbjBai - AbjBaj - AajBbi + AajBbj) \\
&+ \tau_{\alpha j}^d(Bai^2 - BaiBaj - AaiBbi + AaiBbj)\big] /\Delta,
\end{aligned}$$

[6]Here the transformations marked with asterisk are based on the assumption that the duopolists charge the premiums (6), and on the formulas of differentiated taxes (17).

$$
\begin{aligned}
\tau_\beta^d = &- \Bigg[[K\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} + \frac{\beta}{2}\mathcal{C}_A - \pi_{\beta j}^d](AbjBai - AajBbi + AaiBbi - Baj^2) \\
&- [K\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} + \frac{\beta}{2}\mathcal{C}_A - \pi_{\beta i}^d](AbjBaj - BaiBaj + AaiBbj - AajBbj) \\
&+ [K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} - \pi_{\alpha i}^d + \pi_{\alpha j}^d](BaiBbj - BajBbi) \Bigg] / \Delta \\
\overset{*}{=} &- \left[\tau_{\beta j}^d (AbjBai - AajBbi + AaiBbi - Baj^2) \right. \\
&- \tau_{\beta i}^d (AbjBaj - BaiBaj + AaiBbj - AajBbj) \\
&\left. + (\tau_{\alpha i}^d - \tau_{\alpha j}^d - \alpha_i \mathcal{C}_A + \alpha_j \mathcal{C}_A)(BaiBbj - BajBbi) \right] / \Delta,
\end{aligned}
$$

$$
\begin{aligned}
\lambda_\alpha = &\Bigg[(Bai - Abj)(K\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} - \pi_{\beta i}^d + \pi_{\beta j}^d) \\
&- (Bbi - Bbj)(K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} - \pi_{\alpha i}^d + \pi_{\alpha j}^d) \Bigg] / \Delta \\
\overset{*}{=} &\left[(Bai - Abj)(\tau_{\beta i}^d - \tau_{\beta j}^d) - (Bbi - Bbj)(\tau_{\alpha i}^d - \tau_{\alpha j}^d - \alpha_i \mathcal{C}_A + \alpha_j \mathcal{C}_A) \right] / \Delta,
\end{aligned}
$$

$$
\begin{aligned}
\lambda_\beta = &\Bigg[(Aai - Aaj)(K\frac{\partial \mathcal{C}_A}{\partial K_{\beta i}} - K\frac{\partial \mathcal{C}_A}{\partial K_{\beta j}} - \pi_{\beta i}^d + \pi_{\beta j}^d) \\
&- (Bai - Baj)\,(K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha i}} - K\frac{\partial \mathcal{C}_A}{\partial K_{\alpha j}} - \pi_{\alpha i}^d + \pi_{\alpha j}^d) \Bigg] / \Delta \\
\overset{*}{=} &\left[(Aai - Aaj)(\tau_{\beta i}^d - \tau_{\beta j}^d) - (Bai - Baj)(\tau_{\alpha i}^d - \tau_{\alpha j}^d - \alpha_i \mathcal{C}_A + \alpha_j \mathcal{C}_A) \right] / \Delta.
\end{aligned}
$$

Appendix B. Model calibration

Appendix B.1. Monopoly

For calibration purposes we assume quadratic social benefit function, which instantly implies linear inverse demand functions. We choose a monopolistic market with zero profit[7] as the base case, and use it to estimate/evaluate the coefficients in the social benefit functions.

[7]The performance of this market coincides with the perfectly competitive market. Representation it as a monopoly lets us simplify the notations, without changing the essential characteristics.

Social benefit function is as follows

$$\mathcal{B}_{mon}(K_\alpha, K_\beta) = \frac{a}{2}K_\alpha^2 + \frac{b}{2}K_\beta^2 + dK_\alpha K_\beta + h_\alpha K_\alpha + h_\beta K_\beta,$$

where K_α is casco-kilometrage, and K_β is liability insured kilometrage. Then the inverse demand functions are:

$$\mathcal{D}_\alpha(K_\alpha, K_\beta) = \frac{\partial \mathcal{B}_{mon}}{\partial K_\alpha} = aK_\alpha + dK_\beta + h_\alpha, \tag{B.1}$$

$$\mathcal{D}_\beta(K_\alpha, K_\beta) = \frac{\partial \mathcal{B}_{mon}}{\partial K_\beta} = bK_\beta + dK_\alpha + h_\beta.$$

The coefficients a, b, and d are negative and represent direct and cross demand effects:

$$\frac{\partial \mathcal{D}_\alpha}{\partial K_\alpha} = a < 0, \quad \frac{\partial \mathcal{D}_\beta}{\partial K_\beta} = b < 0, \quad \frac{\partial \mathcal{D}_\alpha}{\partial K_\beta} = \frac{\partial \mathcal{D}_\beta}{\partial K_\alpha} = d < 0.$$

The insurance premiums are equal to the average costs

$$\pi_\alpha = \alpha \mathcal{C}_A(K), \quad \pi_\beta = \frac{\beta}{2}\mathcal{C}_A(K)$$

and provide zero profit to the insurance company

$$\Pi = \pi_\alpha K_\alpha + \pi_\beta K_\beta - \alpha K_\alpha \mathcal{C}_A(K) - \frac{\beta}{2}K_\beta \mathcal{C}_A(K) = 0.$$

We assume linear expected costs per vehicle kilometer driven

$$\mathcal{C}_A(K) = \gamma K = \gamma(K_\alpha + K_\beta), \tag{B.2}$$

where γ represents accident risk per vehicle kilometer. We can define accident risk γ as casuality rate: the number of road casualties per billion kilometers traveled. For instance, in the Netherlands in the year 2003 (SWOV, 2009, 2010), γ was approximately €12.3 per 118 km, that is €0.104 per km. We assume $\gamma = 0.1$.

For K normalized to 1 (and $K_\alpha := \theta$, $K_\beta := 1 - \theta$, $\theta \in [0, 1]$) and linear inverse demand (B.1) and accident cost functions (B.2), the premiums are

$$\pi_\alpha = \alpha\gamma, \quad \pi_\beta = \frac{\beta}{2}\gamma,$$

and market equilibrium conditions[8] are

$$a\theta + d(1-\theta) + h_\alpha = \gamma, \quad b(1-\theta) + d\theta + h_\beta = \gamma. \tag{B.3}$$

In order to deliver expressions for price elasticities, we first transform the marginal willingness to pay for the insurances into the demand functions as follows:

$$p_\alpha = aK_\alpha + dK_\beta + h_\alpha, \quad p_\beta = dK_\alpha + bK_\beta + h_\beta.$$

For $ab \neq d^2$ these are equivalent to

$$K_\alpha = -\frac{bh_\alpha - dh_\beta - bp_\alpha + dp_\beta}{ab - d^2}, \quad K_\beta = \frac{dh_\alpha - ah_\beta - dp_\alpha + ap_\beta}{ab - d^2}.$$

Then for the price elasticities of demands[9], we have

$$\varepsilon_\alpha = \frac{p_\alpha}{K_\alpha}\frac{b}{ab - d^2}, \quad \varepsilon_\beta = \frac{p_\beta}{K_\beta}\frac{a}{ab - d^2},$$

which for the base kilometrage $K_\alpha = \theta$, $K_\beta = 1-\theta$, are equivalent to

$$\frac{\theta d + (1-\theta)b + h_\beta}{1-\theta} \cdot \frac{a}{ab - d^2} = \varepsilon_\beta, \quad \frac{a\theta + (1-\theta)d + h_\alpha}{\theta} \cdot \frac{b}{ab - d^2} = \varepsilon_\alpha,$$

that is, using (B.3), we get

$$\frac{\gamma}{\theta} \cdot \frac{b}{ab - d^2} = \varepsilon_\alpha, \quad \frac{\gamma}{(1-\theta)} \cdot \frac{a}{ab - d^2} = \varepsilon_\beta. \tag{B.4}$$

The final system of equations (B.3) and (B.4) for calibration of the coefficients a, b, h_α, and h_β, with $\gamma = 0.1$, $\varepsilon_\alpha = \varepsilon_\beta = -0.3$, and $d = -1$, is

$$\begin{cases} a\theta - (1-\theta) + h_\alpha = 0.1, & b(1-\theta) - \theta + h_\beta = 0.1, \\ \frac{0.1}{1-\theta} \cdot \frac{a}{ab-1} = -0.3, & \frac{0.1}{\theta} \cdot \frac{b}{ab-1} = -0.3. \end{cases} \tag{B.5}$$

[8]See (1).

[9]Value of elasticities is taken $\varepsilon_\alpha = \varepsilon_\beta = -0.3$. (It is consistent with the road use and health insurance literature.) Although we take equal elasticities for our numerical examples, we keep the subscripts for better understanding of the model.

Solving it, we have the following results

θ	ε	d
0.4	-0.3	-1

=>

a	b	h_α	h_β
-1.71035	-1.14023	1.38414	1.18414

The socially optimal premiums are

$$\pi^o_\alpha = \alpha\gamma(K_\alpha + K_\beta) + (K_\alpha + K_\beta)\gamma = (\alpha + 1)\gamma(K_\alpha + K_\beta),$$
$$\pi^o_\beta = \frac{\beta}{2}\gamma(K_\alpha + K_\beta) + (K_\alpha + K_\beta)\gamma = (\frac{\beta}{2} + 1)\gamma(K_\alpha + K_\beta).$$

In order to find the first-best K_α and K_β, we solve the following equations

$$aK_\alpha + dK_\beta + h_\alpha = 2\gamma(K_\alpha + K_\beta),$$
$$dK_\alpha + bK_\beta + h_\beta = 2\gamma(K_\alpha + K_\beta),$$

using the coefficients a, b, d, and the free terms (maximal prices) h_α and h_β found from solving the system (B.5). The private monopoly case is solved in the same manner (see Table B.1).

Appendix B.2. Symmetric duopoly

Using similar approach, we consider two identical firms in the market, both presenting two types of goods, namely casco (α-) and liability (β-) insurances. The coverage percentage is the same for each of the firms. Social benefit function is

$$\begin{aligned}
&\mathcal{B}_{sduop}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j}) \\
&\quad = \frac{a}{2}(K^2_{\alpha i} + K^2_{\alpha j}) + \frac{b}{2}(K^2_{\beta i} + K^2_{\beta j}) + d(K_{\alpha i}K_{\beta i} + K_{\alpha j}K_{\beta j}) \\
&\qquad + c_\alpha K_{\alpha i}K_{\alpha j} + c_\beta K_{\beta i}K_{\beta j} + c^\times K_{\alpha i}K_{\beta j} + c^{\times\times} K_{\alpha j}K_{\beta i} \\
&\qquad + h_\alpha(K_{\alpha i} + K_{\alpha j}) + h_\beta(K_{\beta i} + K_{\beta j}).
\end{aligned}$$

We assume that the firms are perfect substitutes, this implies $c_\alpha = a$, $c_\beta = b$, $c^\times = c^{\times\times} = d$, and then

$$\begin{aligned}
&\mathcal{B}_{sduop}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j}) \\
&\quad = \frac{a}{2}(K_{\alpha i} + K_{\alpha j})^2 + \frac{b}{2}(K_{\beta i} + K_{\beta j})^2 \\
&\qquad + d(K_{\alpha i}K_{\beta i} + K_{\alpha j}K_{\beta j} + K_{\alpha i}K_{\beta j} + K_{\alpha j}K_{\beta i}) \\
&\qquad + h_\alpha(K_{\alpha i} + K_{\alpha j}) + h_\beta(K_{\beta i} + K_{\beta j}).
\end{aligned}$$

The inverse demand functions are

$$
\begin{aligned}
\mathcal{D}_{\alpha i}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j}) &= \mathcal{D}_{\alpha j}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j}) \\
&= a(K_{\alpha i} + K_{\alpha j}) + d(K_{\beta i} + K_{\beta j}) + h_\alpha, \\
\mathcal{D}_{\beta i}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j}) &= \mathcal{D}_{\beta j}(K_{\alpha i}, K_{\beta i}, K_{\alpha j}, K_{\beta j}) \\
&= d(K_{\alpha i} + K_{\alpha j}) + b(K_{\beta i} + K_{\beta j}) + h_\beta.
\end{aligned}
$$

The optimal duopolists' premiums are

$$
\begin{aligned}
\pi_\alpha =& \alpha\gamma(K_{\alpha i} + K_{\beta i} + K_{\alpha j} + K_{\beta j})(K_{\alpha i} + K_{\beta i})\gamma - aK_{\alpha i} - dK_{\beta i}, \\
\pi_\beta =& \frac{\beta}{2}\gamma(K_{\alpha i} + K_{\beta i} + K_{\alpha j} + K_{\beta j})(K_{\alpha i} + K_{\beta i})\gamma - dK_{\alpha i} - bK_{\beta i}.
\end{aligned}
$$

The results of final calculations are presented in Table B.1

Table B.1: Key characteristics of the various market structures

	Base case	First-best	Private monopoly	Duopoly
α-kilometrage K_α	0.4	0.387483	0.2	0.1(3)
β-kilometrage K_β	0.6	0.536594	0.3	0.2
Aggregate kilometrage K	1	0.924076	0.5	0.(6)
Social benefit $\mathcal{B}$	0.68207	0.671257	0.372771	0.597206
Social welfare $\mathcal{W}$	0.58207	0.585866	0.361064	0.552761