TI 2013-011/III
Tinbergen Institute Discussion Paper

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December 2012

Abstract

Changing time series properties of US inflation and economic activity are analyzed within a class of extended Phillips Curve (PC) models. First, the misspecification effects of mechanical removal of low frequency movements of these series on posterior inference of a basic PC model are analyzed using a Bayesian simulation based approach. Next, structural time series models that describe changing patterns in low and high frequencies and backward as well as forward inflation expectation mechanisms are incorporated in the class of extended PC models. Empirical results indicate that the proposed models compare favorably with existing Bayesian Vector Autoregressive and Stochastic Volatility models in terms of fit and predictive performance. Weak identification and dynamic persistence appear less important when time varying dynamics of high and low frequencies are carefully modeled. Modeling inflation expectations using survey data and adding level shifts and stochastic volatility improves substantially in sample fit and out of sample predictions. No evidence is found of a long run stable cointegration relation between US inflation and marginal costs. Tails of the complete predictive distributions indicate an increase in the probability of disinflation in recent years.

Keywords: New Keynesian Phillips curve, unobserved components, level shifts, inflation expectations
JEL Classification: C11, C32, E31, E37

*We thank the conference participants at the Tinbergen Institute workshop 'Recent Theory and Applications of DSGE Models' (Rotterdam, 2012), Federal Reserve Bank of Atlanta workshop on 'Methods and Applications for Dynamic Stochastic General Equilibrium Models' (2012), European Seminar on Bayesian Econometrics (ESOBE'12, Vienna) and 5th CSDA international conference on computational and financial econometrics (CFE'12, Oviedo). We thank the seminar participants at the ECARES (Brussels, 2012) and Tinbergen Institute (Amsterdam, 2012). We thank Gianni Amisano, Sylvia Fruehwirth-Schnatter, Domenico Giannone, James Heckman, Alejandro Justitiano, Frank Kleibergen, Michele Lenza, Richard Paap, Georgio Primicieri, Ricardo Reis, and Frank Schorfheide for constructive discussions and helpful comments on the preliminary version of the paper. All remaining errors are the authors responsibility. Nalan Basturk and Herman K. van Dijk are supported by the NWO Grant number 400–09–340.

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1 Introduction

Modelling the relation between inflation and fluctuations in economic activity has been one of the building blocks of macroeconomic policy analysis. Often, the analysis of this relation, denoted as Phillips Curve (PC) models, is conducted using the short-run variations in inflation and economic activity. The conventional method for extracting this short run variation in the observed series is to demean and detrend the data prior to analysis, see Galí and Gertler (1999); Smets and Wouters (2003); Mavroeidis (2004); DeJong and Dave (2011). However, mechanical removal of the low frequency movements in the data may lead to misspecification in the models, as suggested in Canova (2012) for DSGE models. The existence of complex low frequency movements, such as potential structural breaks and level shifts in the observed series, requires more sophisticated models, which can handle this time variation together with the standard PC parameters. Unfortunately, there is no consensus on the appropriate method of detrending these series, see Gorodnichenko and Ng (2010) for a comprehensive list of such methods used in the literature.

The existence of complex low frequency movements, in particular in the inflation series, is well documented in the literature (McConnell and Perez-Quiros, 2000; Stock and Watson, 2008; Zhang et al., 2008; Bianchi, 2010). For instance two distinct periods with different patterns can be observed for the non-filtered inflation series. The period between the beginning of 1970s and beginning of 1980s is often labelled as a high inflationary period compared to the latter periods. The decline in the level and volatility after this period is linked to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see McConnell and Perez-Quiros (2000); Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007); Cecchetti et al. (2007). A similar discussion is also relevant for the economic activity in the sense that the real marginal cost series, often used as a proxy for the economic activity, see Galí and Gertler

(1999); Clarida et al. (2000); Galí et al. (2001), follows a negative trend which is amplified further in the recent decade. The importance of the joint analysis of such high and low frequency movements in macroeconomic data has recently been documented in the literature (Delle Monache and Harvey, 2010; Canova, 2012).

In this paper we model the low and high frequency movements in the inflation and marginal cost series jointly, by extending the Phillips curve models in order to explain the observed time series instead of the a priori filtered series. As a preliminary step we illustrate the possible effect of prior filtering of the data on posterior inference using simulated datasets from a generic PC type mode. The issue is that the observed inflation levels have a complex time series structure, which is not taken into account in standard Phillips curve models. We show that this misspecification deteriorates posterior inference of the structural Phillips curve parameters. Specifically, the estimated persistence in inflation levels tends to be higher than the actual persistence.

Next, we specify extended Phillips Curve models, namely the New Keynesian Phillips Curve (NKPC) and the Hybrid New Keynesian Phillips Curve (HNKPC), with complex time series structures which allow for stochastic trends and/or structural breaks in the inflation and marginal cost series. In addition to modeling the low frequency movements we also include changing patterns in high frequency movements by incorporating a stochastic volatility structure for inflation. This complex model structure enables the identification of the relation between macroeconomic variables inherent in the Phillips Curve models, together with possible long and short run dynamics in each series. For the proposed HNKPC model, richer expectational mechanisms are employed depending on inflation expectations obtained from survey data.

We apply the proposed models to quarterly U.S. data over the period between the first quarter of 1960 and the first quarter of 2012. We compare the forecasting performance of the proposed models with NKPC models with demeaned and/or detrended data, with a standard stochastic volatility model proposed by Stock and Watson (2007) and, further, with an extended Bayesian vector autoregressive model which accounts for changing levels and trends the data. The model comparison is based on predictive likelihood and out-of-sample Mean Squared Forecast Error (MSFE) comparisons.

The proposed class of models capture time variation in the low frequency moments of both inflation and marginal cost data. For the inflation series, the model identifies two distinct periods with different inflation levels. The relatively high inflationary period spans the period between the beginning of 1970s and beginning of 1980s. This period is replaced rapidly by a relatively low inflation period, where annual inflation is anchored at a level around 2%, accompanying the changing monetary policy in the U.S.. This changing behavior of the inflation levels cannot be accurately captured by the conventional NKPC models using a priori filtered data. In terms of the marginal cost series, the trend specification accommodates the smoothly changing trend observed in the series, specifically after 2000.

For all models we consider, posterior and predictive results are obtained using a simulation based Bayesian approach. The Bayesian approach we adopt has several appealing features particularly for the NKPC models considered. In terms of inflation predictions, several measures of interest, such as disinflation probabilities obtained from the lower tail of the complete the predictive densities, are obtained automatically for each model. Furthermore, for the models with general trend and level structures, the non-existence of a stable long-run relationship, i.e. possible cointegration relation, between inflation and marginal cost series, can be easily assessed using the posterior draws of the trends and levels.

The structure of this paper is as follows: Section 2 illustrates the effects of misspecified low frequency moments on inference and prediction using a canonical backward looking Phillips Curve model with filtered data. Section 3 presents the extensions to the standard NKPC model and extended NKPC models. Section 4 summarizes the likelihood, prior and the posterior sampling algorithm. Section 5 provides the application of the proposed models and the standard NKPC model on U.S. inflation and marginal cost data. Section 6 concludes. In the appendices details on parametric structures, state space specification of our models and the sampling algorithm are provided.

2 Effect of misspecified level shifts on posterior estimates of inflation persistence

The linear Backward Looking Phillips Curve (BLPC) captures the relation between real marginal cost $\tilde{z}_t$ and inflation $\tilde{\pi}_t$. We illustrate in this section that model misspecification resulting from ignoring level shifts in inflation data leads to overestimation of persistence in the inflation equation within a linear BLPC.

The linear BLPC model can be written as

$$\begin{aligned} \tilde{\pi}_t &= \lambda \tilde{z}_t + \gamma_b \tilde{\pi}_{t-1} + \epsilon_{1,t}, \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{1}$$

with $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID(0, \Sigma)$. This model is a triangular simultaneous equations model and can also be interpreted as an instrumental variable model with two instruments. We specify an AR(2) model for the marginal cost in order to mimic for the cyclical behavior of the observed series, see Basistha and Nelson (2007); Kleibergen and Mavroeidis (2011) for a similar specification. The AR(2) parameters are restricted to the stationary region $|\phi_1| + \phi_2 < 1$, $|\phi_2| < 1$, and the lagged adjustment parameter in the inflation equation is restricted as $0 \leq \gamma_b < 1$. The structural parameter λ is restricted as $0 \leq \lambda < 1$ using the economic considerations underlying BLPC.

Since BLPC in (1) specifies the relation between the *short-run* stationary fluctuations in real marginal cost and inflation, $\tilde{\pi}_t$ and $\tilde{z}_t$ can be interpreted as the *transitory* components of inflation and marginal cost, in deviation from their long-run components. In fact, the observed non-filtered data can be decomposed into permanent and transitory components in a straightforward way as

$$\begin{aligned} \pi_t &= \tilde{\pi}_t + c_{\pi,t}, \\ z_t &= \tilde{z}_t + c_{z,t}, \end{aligned} \tag{2}$$

where π_t and z_t are the inflation and marginal cost data, respectively, and $c_{\pi,t}$ and $c_{z,t}$ are the permanent components of the series.

In our simulation experiment, we model the steady state inflation as a constant level subject to regime shifts that mimic the high inflationary period during the 1970s. For modelling the permanent component of the real marginal cost series, we use a linear negative trend in order to mimic the declining real marginal cost levels in the U.S. over the sample starting from the 1960s. This specification can be formulated as follows

$$\begin{aligned} c_{\pi,t} &= c_{\pi,t-1} + \kappa_t \eta_{t-1}, \;\; c_{z,t} = c_{z,t-1} + \mu_{z,t-1}, \\ \mu_{z,t} &= \mu_{z,t-1}, \;\; \eta_t \sim NID(0, \omega^2), \end{aligned} \tag{3}$$

where κ_t is a binary variable indicating a level shift in the level series, $c_{\pi,t}$ and $c_{z,t}$ indicate the level value of inflation and real marginal cost, respectively, in period t and $\mu_{z,t}$ is the slope of the trend in the real marginal cost series. By excluding the stochastic component for the slope and the trend of the real marginal cost in (3), we specify a deterministic trend for this series.

We simulate three sets of data from the model in (1)–(3). For the first set, the inflation series show no level shifts, i.e. $\kappa_t = 0$, $\forall t$. For the other two sets of data, we impose different level shifts with moderate ($\omega^2 = 2.5$) and large ($\omega^2 = 5$) changes in the level values, respectively. For each specification we simulate 100 datasets with $T = 200$ observations, where two level shifts occur in periods $t = 50$ and $t = 150$. The observation error variance is set to $\left(\begin{smallmatrix} 1 & 0.01 \\ 0.01 & 0.01 \end{smallmatrix}\right)$, which leads to a correlation of 0.1 between the disturbances, and parameter λ is set to 0.1. Note that parameters $\phi_1 = 0.1$ and $\phi_2 = 0.5$ are chosen such that the transitory component of the series is stationary.

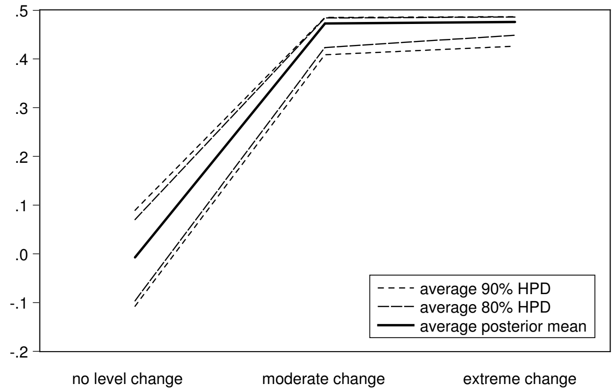
In order to capture the effect of model misspecification on posterior inference, when computing the transitory component, we ignore level shifts in the simulated inflation series and simply demean the series. For the marginal cost series, we remove the linear trend prior to the analysis and only focus on the effect of misspecification in the inflation series. This implies that for the simulated data with no level shifts, the model is correctly specified and the posterior results should be close to the true values. For each simulated data set we estimate the model in (1) using flat priors on restricted parameter regions:

$$p(\phi_1, \phi_2, \gamma_b, \lambda) \propto \begin{cases} 1, & \text{if } |\phi_1| + \phi_2 < 1,\ \ |\phi_2| < 1,\ \ 0 \leq \gamma_b < 1,\ \ 0 \leq \lambda < 1 \\ 0, & \text{otherwise} \end{cases}. \tag{4}$$

Given that model (1) is equivalent to an instrumental variables model with 2 instruments, it can be shown that the likelihood function for such a model combined with the flat prior on a large space yields a posterior distribution that exists but it has no first or higher moments. Due to the bounded region condition on the parameters, where the structural parameter λ in restricted to the unit interval, all moments exist. For details, we refer to Zellner, Ando, Baştürk, Hoogerheide and Van Dijk (2012). We mention this existence result since it explains why it is often difficult to estimate model for macro-economic data (1). Since the posterior surface will be rather flat, in particular, when ϕ_2 is close to zero. Posterior moments are in our case computed by means of standard Metropolis-Hastings method on ϕ_1 and ϕ_2 and λ and γ_b. Other Monte Carlo methods like Gibbs sampling are also feasible in this case.

Figure 1 presents the overestimation results from 100 different simulations for each setting we consider. We report the average overestimation in posterior γ_b estimates and 95% intervals for this overestimation.

Figure 1: Overestimation illustration for the backward looking Phillips curve model



Note: The figure presents overestimation probability of parameter γ_b for simulated data from the BLPC model with different structural breaks structures. We report average quantiles of overestimation based on 100 simulation replications for each parameter setting.

The persistence parameter γ_b is overestimated in all cases except for the correctly specified model. The degree of overestimation becomes larger with a larger shift in the level in of inflation. Note that the average 95% HPDI of overestimation becomes tighter for data with extreme changes in levels. Hence the effect of model misspecification on the persistence estimates is more pronounced if the regime shifts are extreme.

In summary, our simulation experiments using BLPC show that when the shifts in the inflation level are not modelled, inference on model persistence parameters may be severely biased due to the model misspecification. This will also hold for predictive estimates.

We note that we focused on misspecification effects on persistence measures when level shifts in the series are ignored. Similar experiments can be set up for the BLPC with weak identification (or weak instruments) by setting $\phi_2 \approx 0$. The effect of misspecification on posterior and predictive estimates in the case of weak identification is a topic outside the scope of the present paper. We refer to Kleibergen and Mavroeidis (2011) for details on Bayesian estimation in case of weak identification.

3 New Keynesian Philips Curve models

In this section we specify several members of the class of New Keynesian Phillips Curve (NKPC) models. In the pure forward looking form of the NKPC model, the expectations of economics agents are explicitly taken into account by replacing the first lag of inflation in the BLPC by the one period ahead inflation expectation. The NKPC model can be constructed using pricing decision of the firms when prices are sticky (Galí and Gertler, 1999). Using the Calvo formulation, see (Calvo, 1983), sticky prices are modeled as $p_t = \psi p_{t-1} + (1-\psi)p_t^*$, where $\psi \in [0,1]$ is the Calvo parameter indicating the weight firms allocate to previous price level in comparison to the expected optimal reset price p_t^*. The optimal reset price is determined by the current and future stream of the marginal cost, $p_t^* = (1-\gamma_f\psi)\sum_{k=0}^{\infty}(\gamma_f\psi)^k E_t(z_{t+k})$, taking the Calvo price stickiness parameter, ψ, and discount factor, $\gamma_f \in [0,1]$, into account.

We start with an NKPC model based on filtered data. Next, we extend this model with a structural time series model with time varying components in order to deal with low and high frequencies that are present in non-filtered data. Thirdly, we extend the latter NKPC model by introducing a Hybrid NKPC model (HNKPC) with both backward and forward looking inflation expectations where the long-run expectations are anchored around observed values of inflation expectations obtained from survey data.

NKPC models with filtered time series

The structural form (SF) representation for the basis NKPC model derived from the firm's price setting for filtered data is given as

$$\begin{aligned} \tilde{\pi}_t &= \lambda \tilde{z}_t + \gamma_f E_t(\tilde{\pi}_{t+1}) + \epsilon_{1,t}, \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{5}$$

where $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID\,(0, \Sigma)$ and standard stationary restrictions hold for ϕ_1, ϕ_2.

The model can be solved for the inflation expectation by iterating the model forward. This implies that the entire stream of future inflation expectations are taken into account. The NKPC model together with AR(2) dynamics for the forcing variable takes the form of triangular simultaneous equations model with nonlinear parameters in the inflation equation[1]

$$\begin{aligned} \tilde{\pi}_t &= \tfrac{\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f} \tilde{z}_t + \tfrac{\phi_2\gamma_f\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f} \tilde{z}_{t-1} + \epsilon_{1,t} \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{6}$$

One way to estimate the structural parameters is to start from the unrestricted reduced form representation of the above system. As there exists a one-to-one mapping between the unrestricted reduced form and the structural parameters one can generate random draws from the reduced form posterior and solve for the structural posterior draws. However, this transformation involves a complex Jacobian structure that includes ratios of structural model parameters. This may seriously obscure the inference on the structural parameters, even though posterior inference of the reduced form parameters is straightforward. Hence, we opt for estimating the structural parameters directly, without relying on the reduced form estimation,

[1] The model in (6) can be written as a triangular simultaneous equations model:
$\begin{pmatrix} 1 & -\alpha_1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \pi_t \\ z_t \end{pmatrix} = \begin{pmatrix} 1 & -\alpha_1 & -\alpha_2 & 0 \\ 0 & 1 & -\phi_1 & -\phi_2 \end{pmatrix} \left(c_{\pi,t},\, c_{z,t},\, c_{z,t-1},\, c_{z,t-2} \right)' + \begin{pmatrix} 0 & \alpha_2 \\ 0 & \phi_1 \end{pmatrix} \begin{pmatrix} \pi_{t-1} \\ z_{t-1} \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & \phi_2 \end{pmatrix} \begin{pmatrix} \pi_{t-2} \\ z_{t-2} \end{pmatrix} + \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix}$,
where unobserved states $c_{\pi,t}$ and $c_{z,t}$ follow from the last three equations in (10), and the following parameter restrictions hold: $\alpha_1 = \lambda / \left(1 - (\phi_1 + \phi_2\gamma_f)\gamma_f\right)$ and $\alpha_2 = \phi_2\gamma_f\lambda / \left(1 - (\phi_1 + \phi_2\gamma_b)\gamma_b\right)$.

see Kleibergen and Mavroeidis (2011) for a discussion. In Appendix A, we provide further details of the transformation from reduced form parameters to structural parameters for the NKPC model.

Extended NKPC models with non-filtered time series

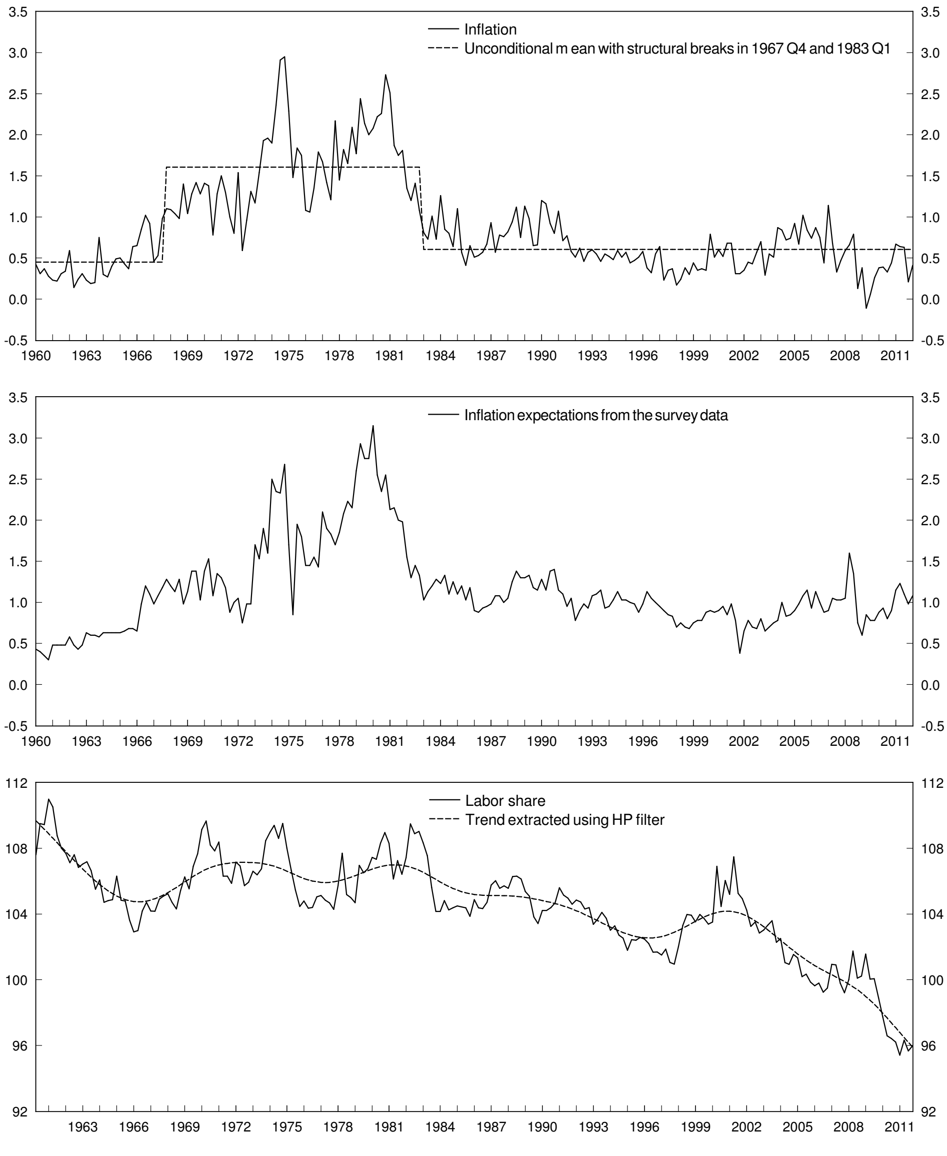
We first consider the data features for the empirical application. For the empirical analysis, we consider U.S. inflation and real marginal cost series over the period from the first quarter of 1960 until the first quarter of 2012. Inflation is computed as the growth rate of the implicit GDP deflator and for the real marginal cost series we use labor share in non-farm business sector[2], see Galí and Gertler (1999) for details. The non-filtered series of US inflation is displayed in the top panel of in Figure 2 and real marginal cost is displayed in the bottom panel of in Figure 2.

From the top panel in Figure 2, we observe two stylized facts. First, there exist distinct periods with differing patterns for the inflation series. The period between the beginning of the 1970s and the beginning of the 1980s can be labelled as a high inflationary period compared to the remaining periods. Existing evidence shows that the decline in level and volatility is due to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties, see McConnell and Perez-Quiros (2000); Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007); Cecchetti et al. (2007). We include the level (unconditional mean) of the inflation series in the upper panel of Figure 2 with level shifts in the fourth quarter of 1967 and the first quarter of 1983 in line with the existing findings.[3] Indeed, the figure demonstrates a temporary increase in the level of inflation during 1970s, while this increase in inflation switches back to the earlier levels after the second break in the first quarter of 1983. One way

[2]http://research.stlouisfed.org/fred2/

[3]This pattern does not change with marginal changes in terms of the timing of the breaks, which correspond to the period where the Federal Reserve Board reserve-targeting policies had been replaced with the interest rate-targeting policy rule. Moreover, Cecchetti et al. (2007), among other papers, point out another shift in the level of inflation around the late 1960s as the start of the high inflationary period.

Figure 2: Inflation, inflation expectations and real marginal cost series over first quarter of 1960 and the first quarter 2012



to model this changing behavior of the series to allow for regime changes in parameters to capture the change in the structure of the series, see Cogley and Sargent (2005); Canova and Gambetti (2006); Kim and Nelson (2006); Sims and Zha (2006); Cogley and Sbordone (2008), among others. We consider two cases. In the first case, we assume that the level shifts occur in each time period continuously. Then we can model the changing inflation level using a random walk process for the level of inflation as follows

$$c_{\pi,t+1} = c_{\pi,t} + \eta_{1,t+1}, \tag{7}$$

where $\eta_{1,t} \sim NID(0, \sigma^2_{\eta 1})$.

Alternatively, we assume that inflation is subject to occasional and discrete shifts. For this case we model the level of the inflation allowing for permanent level shifts. This can be incorporated to the previous case using a regime indicator function as follows

$$c_{\pi,t+1} = c_{\pi,t} + \kappa_t \eta_{1,t+1} \tag{8}$$

where κ_t is a binary variable taking the value of 1 with probability p_κ if there is level shift and it takes the value 0 with probability $1 - p_\kappa$ if the level does not change and $\eta_{1,t} \sim N(0, \sigma^2_{\eta 1})$. This model structure allows for occasional level shifts depending on the probability p_κ of the binomial process preserving a parsimonious model structure with only a single additional parameter. Occasional and large level shifts corresponds to low values of $1 - p_\kappa$ together with relatively high values of $\sigma^2_{\eta 1}$ and the opposite case corresponds to the local level model, see Giordani et al. (2007) for a similar approach. We will use both specifications (7) and (8) in the empirical analysis.

The real marginal cost series is analyzed in the bottom panel of Figure 2. For a visual inspection, we also include a time varying trend extracted using the Hodrick-Prescott (HP) filter (Hodrick and Prescott, 1997). Unlike the inflation series we do not observe discrete changes during the course of time for the the real marginal cost series. Instead, it exhibits a continuously changing pattern around a negative trend, which can be attributed to technology shocks. Since in the figure this trend is more prominent in the second half of the sample period, we allow for a changing trend using a local linear trend specification as follows

$$\begin{aligned} c_{z,t+1} &= \mu_{z,t} + c_{z,t} + \eta_{2,t+1}, \\ \mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1}, \end{aligned} \tag{9}$$

where $\eta_{2,t} \sim NID(0, \sigma^2_{\eta 2})$ and $\eta_{3,t} \sim NID(0, \sigma^2_{\eta 3})$, see Durbin and Koopman (2001) for details. This specification is flexible enough to encompass many types of filters used for detrending, see Delle Monache and Harvey (2011), see also Canova (2012) for a similar specification in the more general context of DSGE models. When $\sigma^2_{\eta_3} = 0$, for example, the level of the real marginal cost follows a random walk with a drift, μ_z. Additionally, when $\sigma^2_{\eta_2} = 0$, a deterministic trend is obtained. Note that, setting only $\sigma^2_{\eta_2} = 0$ but allowing $\sigma^2_{\eta_3}$ to be positive results in an integrated random walk process which can approximate many types of nonlinear trends including the Hodrick-Prescott (HP) filter.

Together with the level specifications of the inflation and real marginal cost series the NKPC model in (6) using (2) and (3) takes the following form

$$\begin{aligned} \pi_t - c_{\pi,t} &= \tfrac{\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\left(z_t - c_{z,t}\right) + \tfrac{\phi_2\gamma_f\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\left(z_{t-1} - c_{z,t-1}\right) + \epsilon_{1,t}, \\ z_t - c_{z,t} &= \phi_1\left(z_{t-1} - c_{z,t-1}\right) + \phi_2\left(z_{t-2} - c_{z,t-2}\right) + \epsilon_{2,t}, \\ c_{\pi,t+1} &= c_{\pi,t} + \kappa_t\eta_{1,t+1}, \\ c_{z,t+1} &= \mu_{z,t} + c_{z,t} + \eta_{2,t+1}, \\ \mu_{z,t+1} &= \mu_{z,t} + \eta_{3,t+1}, \end{aligned} \tag{10}$$

where $(\epsilon_{1,t},\ \epsilon_{2,t})' \sim NID\left(0, \begin{pmatrix} \sigma^2_{\epsilon_1} & \rho\sigma_{\epsilon_1}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon_1}\sigma_{\epsilon_2} & \sigma^2_{\epsilon_2} \end{pmatrix}\right)$, $(\eta_{1,t},\ \eta_{2,t},\ \eta_{3,t})' \sim NID\left(0, \begin{pmatrix} \sigma^2_{\eta_1} & 0 & 0 \\ 0 & \sigma^2_{\eta_2} & 0 \\ 0 & 0 & \sigma^2_{\eta_3} \end{pmatrix}\right)$

and the residuals $(\epsilon_{1,t},\ \epsilon_{2,t})'$ and $(\eta_{1,t},\ \eta_{2,t},\ \eta_{3,t})'$ are independent for all t.

Adding Stochastic Volatility

A further refinement in the NKPC model can be achieved allowing for time dependency in residual variances. This extension is particularly appealing for the inflation series, as the variance of this series changes over time substantially, see e.g. Stock and Watson (2007) for a reduced form model with a stochastic volatility component. To extend the NKPC model with a stochastic volatility process in the inflation shocks, we add the following state equation to the system

$$h_{t+1} = h_t + \eta_{4,t+1},\ \eta_{4,t+1} \sim NID(0, \sigma^2_{\eta_4}), \tag{11}$$

where the error term of the first equation in (10) has a time-varying variance $\sigma^2_{\epsilon_{1,t}} = \exp(h_t/2)$. We follow the practice in Stock and Watson (2007) by fixing the value of $\sigma^2_{\eta_4}$ prior to analysis to facilitate inference. We set $\sigma^2_{\eta_4} = 0.5$, which seems to work well for the U.S. inflation series.

Hybrid NKPC

While the BLPC model only considers the backward looking dynamics in inflation, the NKPC model replaces this backward looking dynamics with forward looking inflation expectations. The 'Hybrid' NKPC (HNKPC) model combines both backward and forward looking dynamics by including the first lag of inflation deviation in the model along with forward looking dynamics, see Galí and Gertler (1999); Galí et al. (2001) for details. Hence, the hybrid NKPC model takes the form of

$$\begin{aligned} \tilde{\pi}_t &= \lambda \tilde{z}_t + \gamma_f E_t(\tilde{\pi}_{t+1}) + \gamma_b \tilde{\pi}_{t-1} + \epsilon_{1,t}, \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{12}$$

together with the AR(2) process for the forcing variable $\tilde{z}_t$. Iterating the first equation forward the HNKPC implies the following triangular simultaneous equations system nonlinear in the parameters and still containing an expectations operator

$$\begin{aligned}\tilde{\pi}_t &= \tfrac{\lambda}{(1-\gamma_b\gamma_f)(1-(\phi_1+\phi_2\gamma_f)\gamma_f)}\tilde{z}_t + \tfrac{\phi_2\gamma_f\lambda}{(1-\gamma_b\gamma_f)(1-(\phi_1+\phi_2\gamma_f)\gamma_f)}\tilde{z}_{t-1} \\ &+ \tfrac{\gamma_b\gamma_f}{(1-\gamma_b\gamma_f)}\textstyle\sum_{k=1}^{\infty}\gamma_f^k E_t(\tilde{\pi}_{t+k}) + \tfrac{\gamma_b}{(1-\gamma_b\gamma_f)}\tilde{\pi}_{t-1} + \tfrac{1}{(1-\gamma_b\gamma_f)}\epsilon_{1,t} \\ \tilde{z}_t &= \phi_1\tilde{z}_{t-1} + \phi_2\tilde{z}_{t-2} + \epsilon_{2,t}.\end{aligned} \tag{13}$$

Since this system involves the infinite sum of expectations, a closed form solution only exists under certain assumptions such as rational expectations. Here, we do not follow this practice but model the inflation expectations using an unobserved component to be estimated along with other parameters. Specifically, let S_t be the next period inflation expectation, $S_t = E_t(\pi_{t+1})$. We assume that inflation expectations are anchored around long-term expectations, μ, and deviations from this long-term expectations follow an AR(1) process as follows

$$S_{t+1} = \mu + \beta(S_t - \mu) + \eta_{5,t+1}, \tag{14}$$

where $|\beta| < 1$ such that inflation expectations converge to the long-run expectations, μ. When $\beta = 1$, we arrive at the random walk process for the inflation expectations and μ is dropped out from the specification indicating that inflation expectations are not anchored around μ. Notice that this formulation specifies a Bayesian learning rule for the inflation expectations in the sense that each period when the new information about the inflation arrives, the states including the inflation expectations S_t are updated using this new piece of information. We do not estimate μ explicitly nor we assume a constant long-run inflation expectations, instead we use the inflation expectations data from University of Michigan Research Center, which provide quarterly one year ahead inflation expectations, shown in the middle panel of Figure 2.[4][5]

[4]This approach is also followed in other applications, such as trend-cycle decomposition techniques. For example, the Beveridge-Nelson decomposition (Beveridge and Nelson, 1981) defines the trend as the long-horizon expectation of an integrated time series.

[5]The data is taken from http://www.src.isr.umich.edu/.

This implies that the inflation expectations are anchored around the survey values, see Roberts (1995, 1997); Del Negro and Schorfheide (2012) for a similar approach.

Specifying inflation expectations as in (14), the HNKPC model becomes

$$\begin{aligned}
\pi_t - c_{\pi,t} &= \tfrac{\lambda}{(1-\gamma_b\gamma_f)(1-(\phi_1+\phi_2\gamma_f)\gamma_f)}\left(z_t - c_{z,t}\right) + \tfrac{\phi_2\gamma_f\lambda}{(1-\gamma_b\gamma_f)(1-(\phi_1+\phi_2\gamma_f)\gamma_f)}\left(z_{t-1} - c_{z,t-1}\right), \\
&+ \tfrac{\gamma_b\gamma_f}{(1-\gamma_b\gamma_f)}\tfrac{\gamma_f}{1-\gamma_f\beta}(S_t - \mu_t) + \tfrac{\gamma_b}{(1-\gamma_b\gamma_f)}\left(\pi_{t-1} - c_{\pi,t-1}\right) + \tfrac{1}{(1-\gamma_b\gamma_f)}\epsilon_{1,t}, \\
z_t - c_{z,t} &= \phi_1\left(z_{t-1} - c_{z,t-1}\right) + \phi_2\left(z_{t-2} - c_{z,t-2}\right) + \epsilon_{2,t}.
\end{aligned} \tag{15}$$

Similar to the NKPC model, we consider three case of the HNKPC model: (i) continuous changes for the inflation level; (ii) discrete but occasional changes for the inflation level; and (iii) discrete but occasional changes for the inflation level and stochastic volatility for inflation.

4 Bayesian inference

In this section we summarize the Bayesian inference steps for the proposed models, which are obtained by the product of the likelihood function and the prior density for the model parameters. The likelihood functions of the proposed models are multivariate normal densities, as we assume normal error distributions. We elaborate on the prior specifications and the posterior sampler in this section. More details are presented in Appendices B and C.

The prior specification in the NKPC models is of primary importance since the likelihood of the NKPC models is often flat (see Kleibergen and Mavroeidis (2011)). One way to overcome this difficulty is to impose informative priors on the model parameters. However, this may obscure posterior inference of the structural parameters. Therefore we use flat priors for the structural parameters but we specify informative priors for the observation variances.

For the structural parameters of the NKPC and HNKPC models, we define independent flat priors on restricted regions. The choice of these regions are based on the underlying economic theory. We restrict parameters γ_b, γ_f and λ to be in the unit interval. For the β parameter in the HNKPC models, we use a flat prior on the unit interval. We restrict the autoregressive parameters, ϕ_1 and ϕ_2, to be in the stationary region.

The prior specifications of the observation and state covariance matrices are important in this class of models and for the case of macroeconomic data. Since the sample size is typically small, differentiating the short-run variation in series (the observation variances) from the variation in the long-run behavior (the state variation) can be cumbersome (Canova, 2012). For this reason, we impose a data based prior structure on the observation covariance matrices. We first estimate the implied unrestricted reduced form VAR model using demeaned inflation series and (linear) detrended real marginal cost series, and base the observation variance priors on the covariance matrix estimates from this model. This specification imposes smoothness for the estimated levels and trends and ensures that the state errors do not capture all variation in the observed variables. For the states, we assume a diagonal covariance matrix with an uninformative prior implying that the shocks to the long-run inflation and real marginal cost are independent.

When estimating the models with stochastic volatility together with the level shifts in inflation, the prior specifications on the covariance matrices play also an important role for the identification of the level shifts and observation volatility. This leads us to consider also informative priors on the state covariance matrix when estimating models with a stochastic volatility component, where we specify informative priors that limit the variation in the states enabling identification of the stochastic volatility in the inflation series.

For the models with level shifts, we fix the level shift probability as 0.04 (0.01) for the NKPC (HNPC) models, implying an expected number of shifts of 8 (2) for

200 observations in the sample. Alternatively, we could also estimate this parameter together with other model parameters. However, often the limited level shift observations plague the inference of this parameter. Hence, we opt for setting the value prior to analysis where the values are selected trough an extensive search over intuitive values of this parameter.

We note that we also anchor the inflation expectations by using survey data in the NKPC models. Therefore we use slightly informative priors on the variance of the error term in (14). This ensures that the implied inflation expectations of the models do not diverge from the survey expectations.

Posterior distributions are obtained as the product of the prior distributions and the likelihood function. As the number and the location of the structural breaks are unknown the likelihood function is intractable. Therefore, we set up an MCMC algorithm to sample from the full conditional posterior distributions. Specifically, we use Gibbs sampling together with data augmentation (see Geman and Geman, 1984; Tanner and Wong, 1987) to obtain posterior results. Gibbs sampling steps are based on Kim and Nelson (1999); Gerlach et al. (2000); Çakmaklı et al. (2011). Details of the MCMC algorithm are given in Appendix B.

5 Posterior and Predictive Evidence

In this section we present posterior and predictive evidence on several features of New Keynesian Phillips Curve (NKPC) models using U.S. data on inflation and marginal costs. We compare these results with those obtained from alternative reduced form models like Bayesian Vector Auto Regressive models and the stochastic volatility model from Stock and Watson (2007). Specifically, we estimate eight NKPC models, where the first two models use a Linear Trend filter, labelled as NKPC-LT, and the HP filter, labelled as NKPC-HP. In the other six NKPC models we make use of structural time series models to specify low and high frequencies and level shifts in the inflation series and a time-varying trend in the marginal cost series. Three models use the NKPC framework, where in the first model we only allow for continuous changes in the level of inflation, denoted as NKPC-TVP, in the second we allow for discrete occasional level shifts, denoted as NKPC-TVP-LS and in the third we allow for stochastic volatility for the inflation in addition to the level shifts, denoted as NKPC-TVP-LS-SV. The final three models use the Hybrid form of NKPC framework and corresponding extensions are denoted as HNKPC-TVP, HNKPC-TVP-LS and HNKPC-TVP-LS-SV similar to the NKPC based models.

The evidence reported refers to such posterior features as the slope of the Phillips curve, the value of the Calvo parameter on price stickiness, the strength of endogeneity in the inflation equation, persistence in the dynamics of the model and the relative importance of forward and backward looking expectations. Next, the models are compared in terms of fit of estimated inflation and cost levels, their volatilities and break probabilities. Predictive performances of the models are reported using mean squared forecast errors, predictive likelihoods and full predictive densities which enable us to report on the tail probability of disinflation. Finally, we present evidence on the absence of a long term stable relation between inflation and marginal costs.

Posterior evidence

We display the estimation results in Table 1. The first two rows of the table show the estimation results of the NKPC-LT and NKPC-HP models where the data is demeaned and detrended prior to analysis. We focus on three features. First, the slope of the Phillips curve is estimated around 0.07 and 0.09 which is slightly higher than the conventional estimates of the Philips curve slope, that indicate an almost flat curve (see e.g. Galí and Gertler (1999); Gali et al. (2005)).

Table 1: Posterior results of alternative Phillips curve models

Model	λ	γ_f	γ_b	Calvo	β	ρ	ϕ_1	ϕ_2
NKPC-LT	0.067 (0.028)	0.349 (0.254)		0.911 (0.017)		-0.011 (0.024)	0.837 (0.045)	0.074 (0.045)
NKPC-HP	0.090 (0.046)	0.432 (0.279)		0.871 (0.048)		-0.066 (0.051)	0.657 (0.045)	-0.003 (0.045)
NKPC-TVP	0.052 (0.026)	0.380 (0.256)		0.926 (0.023)		-0.045 (0.037)	0.815 (0.052)	0.065 (0.052)
NKPC-TVP-LS	0.054 (0.029)	0.375 (0.258)		0.924 (0.028)		-0.044 (0.043)	0.817 (0.053)	0.066 (0.052)
NKPC-TVP-LS-SV	0.063 (0.001)	0.322 (0.056)		0.919 (0.000)		-0.016 (0.005)	0.871 (0.003)	0.093 (0.003)
HNKPC-TVP	0.041 (0.020)	0.011 (0.022)	0.463 (0.138)	0.927 (0.032)	0.461 (0.242)	0.006 (0.043)	0.812 (0.056)	0.066 (0.055)
HNKPC-TVP-LS	0.032 (0.021)	0.009 (0.009)	0.557 (0.140)	0.926 (0.055)	0.407 (0.242)	0.009 (0.034)	0.821 (0.064)	0.069 (0.063)
HNKPC-TVP-LS-SV	0.043 (0.018)	0.008 (0.013)	0.428 (0.104)	0.930 (0.027)	0.439 (0.249)	0.001 (0.008)	0.824 (0.064)	0.072 (0.062)

Note: The table presents posterior means and standard deviations (in parentheses) of parameters for the competing New Keynesian Phillips Curve (NKPC) type models estimated for quarterly inflation and real marginal cost over over the period from the first quarter of 1960 and the first quarter of 2012. NKPC-LT (NKPC-HP) refers to the NKPC model where the real marginal cost series is detrended using linear trend (Hodrick-Prescott) filter. NKPC-TVP refers to the NKPC model with time varying levels and trends defined in (7) and (9). NKPC-TVP-LS refers to the NKPC model with time varying levels and trends defined in (8) and (9). NKPC-TVP-LS-SV refers to the NKPC model with time varying levels, trends and volatility defined in (8), (9) and (11). HNKPC-TVP refers to the Hybrid NKPC model with time varying levels, trends and inflation expectations defined in (7), (9) and (14). HNKPC-TVP-LS refers to the HNKPC model with time varying levels, trends and inflation expectations defined in (8), (9) and (14). HNKPC-TVP-LS-SV refers to the HNKPC model with time varying levels, trends, inflation expectations and volatility defined in (8), (9), (14) and (11). λ is the slope of the Phillips Curve in (6). γ_f is the coefficient of inflation expectations in NKPC in (6). γ_b is the coefficient of the backward looking component in the HNKPC model in (15). 'Calvo' is the parameter representing the degree of price stickiness. β is the autoregressive parameter for the deviation inflation expectations from the long-run trend, as defined in (14). ρ is the correlation coefficient of the residuals ϵ_1 and ϵ_2. ϕ_1 and ϕ_2 are the autoregressive parameters for the real marginal cost specification in (6). Posterior results are based on 40000 simulations of which the first 20000 are discarded for burn-in.

Second, the coefficient of the short-run inflation expectation, γ_f is much lower than the conventional estimates, which is above 0.9 in most of the cases. A potential reason for this finding is the methodology used for inference. Conventional analysis replaces inflation expectation of the next period by the real leading value of the inflation relying on the rational expectations hypothesis, see e.g. McCallum (1976); Roberts (1995); Galí and Gertler (1999); Sims (2002). However, we opt for explicitly solving for expectations resulting in a highly nonlinear system of simultaneous equations. We also notice a relatively higher posterior standard deviation for this parameter, hence another potential cause of this parameter is the relatively low information content in the data about this parameter. Still more conventional values of this parameter is inside the 95% HPDI.

Third, the posterior mean of the Calvo parameter estimates are around 0.9, indicating a high degree of price stickiness in the new Keynesian model. Low values of posterior standard deviation indicate that the data are highly informative about this parameter. These values are in line with the previous findings suggesting that on average, prices remain fixed for between roughly 6-8 quarters (see Galí and Gertler (1999) for a comparison).

In the second panel of Table 1 results are given for cases: NKPC-TVP, NKPC-TVP-LS and NKPC-TVP-LS-SV. Posterior means and standard deviations of the structural form parameters are similar across all three models with the NKPC structure. The posterior means for the Phillips curve slope, λ, are around 0.055, slightly lower than those obtained from the NKPC model with demeaned and detrended data. As for the posterior means, posterior standard deviations are also lower compared to the NKPC-LT and NKPC-HP models. Hence, 0 is outside the 95% HPDI for most cases.

Posterior distribution of the γ_f parameter closely resembles those obtained from the NKPC models with a prior detrended data. This implies that the data information about this parameter is limited regardless of the level and trend specification. As in the first two models, the estimated Calvo parameters in the NKPC models indicate a high degree of price stickiness. Furthermore, the estimates from the NKPC models are slightly higher than those obtained from the NKPC models estimated using demeaned and detrended data.

Posterior results for the HNKPC models are displayed in the third panel of Table 1. Posterior slopes of the Phillips curve for the HNKPC models are close to the NKPC counterparts, albeit slightly lower. These lower values, however, are accompanied by lower standard deviations. Consequently, 0 is outside the 95% HPDI, indicating an almost flat but a significant positive slope for the Phillips curve. A striking result from Table 1 is related to the relative importance of the forward and backward looking components of the Phillips curve, measured by parameters γ_f and γ_b. While the evidence in Clarida et al. (2000); Galí et al. (2001); Gali et al. (2005) suggests a dominant forward looking effect, in contrast, many studies including Fuhrer and Moore (1995); Rudd and Whelan (2005) document a dominant backward looking effect in NKPC. Our results favor the latter view since the effect of the backward looking components of inflation estimated by the HNKPC models in the bottom panel of Table 1 are substantially higher than those of the forward looking components. More specifically, Table 1 shows that the HNKPC and NKPC model results differ in terms of the forward looking components' coefficient γ_f.

Posterior means of the β parameter, which shows the persistence in deviations of inflation expectations from the long-run trend, are given in the fifth column of Table 1. All HNKPC models indicate a mediocre persistence for these deviations, as the posterior means are around 0.4. This implies that inflation expectations in subsequent periods are dependent on the current inflation expectations, albeit to a limited extend.

The estimated degree of price stickiness, i.e. the Calvo parameter, from the

HNKPC models are in line with those obtained from the NKPC models where levels and trends are estimated along with other model parameters. Hence, Calvo parameter estimates are not sensitive to the inclusion of the backward looking component for the models where levels and trends are modelled explicitly.

A further consideration in these models is the contemporaneous correlation between observation errors determining the degree of endogeneity of real marginal cost in the Phillips curve specification. The estimates of this correlation parameter ρ are displayed in the sixth column of Table 1. Posterior means of ρ from all NKPC models are negative and close to 0, with a high standard deviation. Consequently, 0 is inside the 95% HPDI. For the HNKPC models, posterior means of ρ are positive with an even smaller magnitude. Therefore, the endogeneity problem does not seem to be severe for these models.

Estimated Levels, Volatilities, Breaks and Inflation Expectations

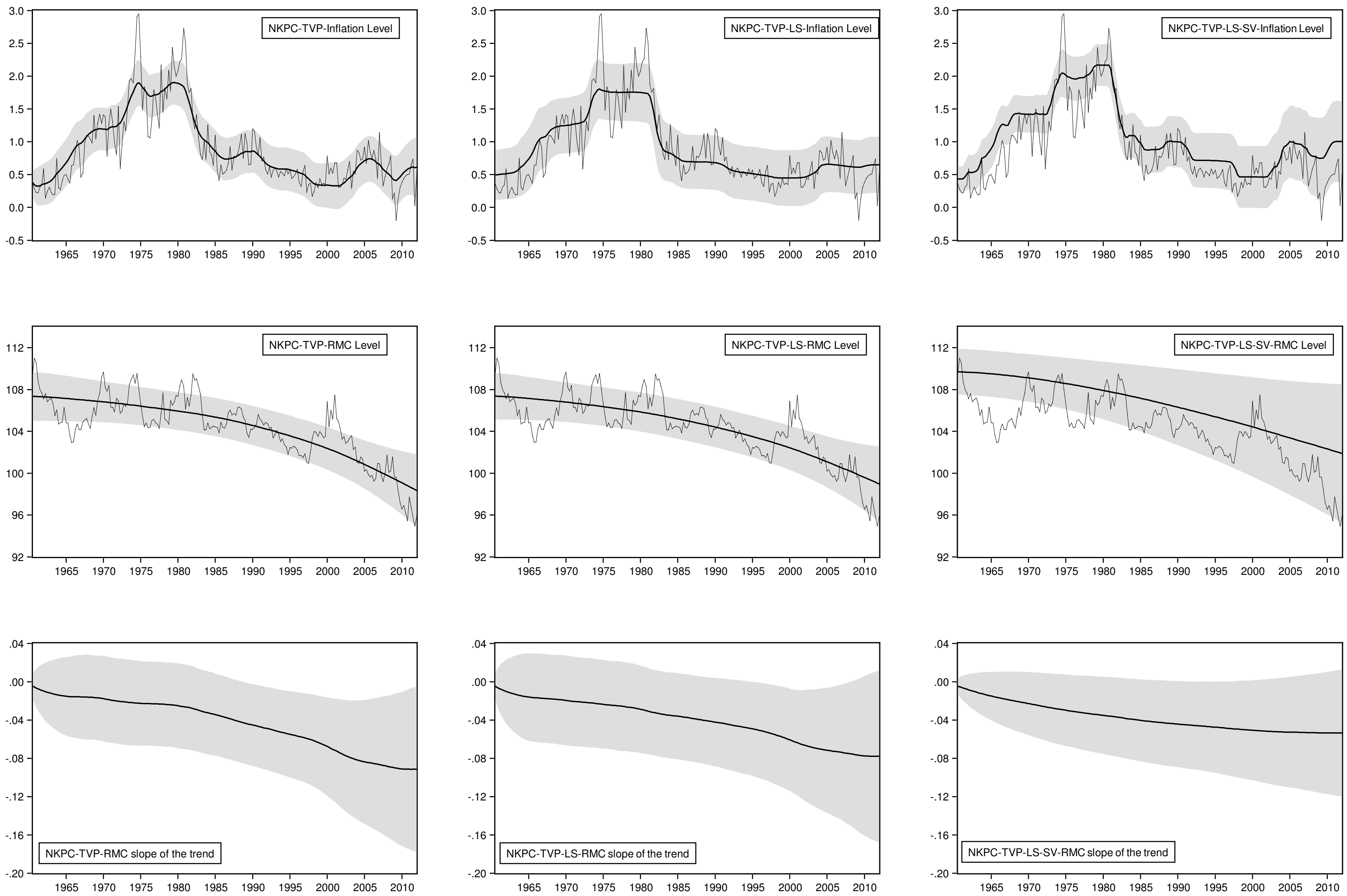
Figure 3 shows the estimated levels from the three NKPC models. Estimated inflation levels, computed as the posterior mean of the smoothed states, are given in the first row of Figure 3. Shaded areas around the posterior means represent the 95% HPDI for the estimated levels. For all three models, estimated inflation levels nicely track the observed inflation. Effects of the level specification are reflected in the estimates in various ways. First, when we model inflation level changes as discrete level shifts rather than continuous changes, we observe a relatively smoother pattern in estimated inflation levels. This effect can be seen by comparing the second and first graphs in the first row of Figure 3. While estimated inflation level in the first graph follows the observed inflation patterns closely, estimated inflation level in the second (and third to a less extent) graph mostly indicates three distinct periods. These periods are the high inflation periods capturing 1970s with a constant inflation level around 1.7% (quarterly inflation) following a low inflation period in 1960s, and the period after the beginning of 1980s with a stable inflation level around 0.5%, see

Cecchetti et al. (2007) for similar findings. Second, when we include the stochastic volatility component in inflation series, the uncertainty of the estimated inflation levels decreases as part of the uncertainty is reflected in the stochastic volatility process. Put differently, a more flexible volatility structure captures part of the inflation uncertainty. This is also visible when we compare the second and the third graphs in the first row of the figure. Adding the stochastic volatility together with level shifts results in discrete level shifts in inflation which are more frequent than the model with only level shifts.

The second panel in Figure 3 presents the estimated levels for the real marginal cost series for all models. A common feature of all these estimates is the smoothness of the estimated levels. In all models, marginal cost series follows a slightly nonlinear trend during the sample period. The estimated slopes of these trends for all models are given in the bottom panel of Figure 3, together with the 95% HPDIs. Nonlinearity of the negative trend is reflected in the negative values for the slope of the trend, with an increasing magnitude at the end of the sample. This change in the slope of the trend is accompanied by the increasing uncertainty about the slope. The difference between the models in terms of the estimated marginal cost structures is negligible.

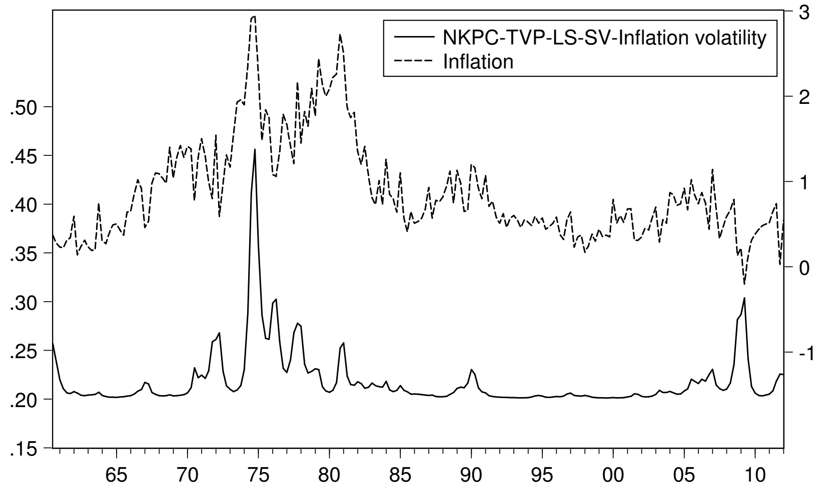
Figure 4 presents the estimated inflation volatilities for the NKPC model with level shifts and stochastic volatility. The stochastic volatility pattern in the figure coincides nicely with the findings on Great Moderation, which refer to the decline of the volatility of many U.S. macroeconomic series, see McConnell and Perez-Quiros (2000) among others. The period before the beginning of 1980s are characterized by high inflation levels accompanied by a high volatility, whereas inflation becomes more stable in the second half of the sample period. The decline in inflation volatility after 1980s is linked to credible monetary policy that stabilized inflationary expectations at a low level via commitment to a nominal anchor since the early eighties,

Figure 3: Level, trend and slope estimates from the NKPC models



Note: The top panel exhibits estimated inflation levels. The middle and the bottom panels show estimated real marginal cost levels and the slopes of the levels, respectively. Grey shaded areas correspond to the 95% HPDI. Model abbreviations are as in Table 1 Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

Figure 4: Estimated inflation volatility from the NKPC-TVP-LS-SV model

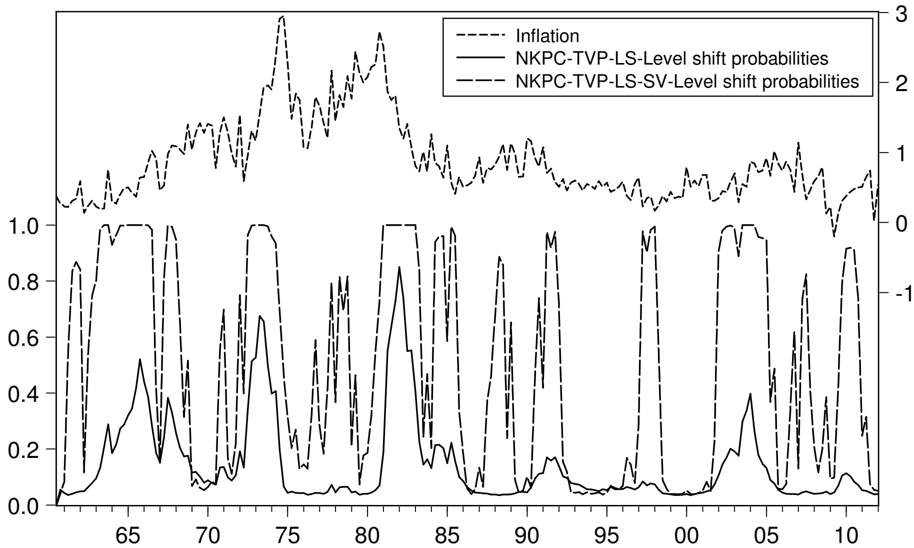


Note: The solid line is the posterior mean of the time varying inflation volatility. The dashed line is the observed inflation level. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

see Stock and Watson (2002); Ahmed et al. (2004); Stock and Watson (2007). This period of low volatility is replaced by a highly volatile period after 2005 and during the recent financial crisis.

We next report the break probabilities for the NKPC models with level shifts in Figure 5 for the NKPC-TVP-LS and NKPC-TVP-LS-SV models. On the one hand, estimated level shift probabilities from the NKPC-TVP-LS model identify two major shifts in the inflation level around 1973 and 1982, which comprise the beginning and the end of the high inflationary periods. The models indicate two further level shifts around 1966 and 2005 although the estimated level shift probabilities in these years are much lower than those for the high inflationary periods. On the other hand, estimated shift probabilities in the NKPC-TVP-LS-SV model demonstrate the complementarity of level shifts with the changing volatility. The probabilities follow a similar pattern with the NKPC-TVP-LS model, however, the periods subject to level shifts are much longer. During the highly volatile periods of 1970s, the model produces quite clear signals of changing inflation levels, as high volatility levels cause

Figure 5: Estimated level shift probabilities for the NKPC models



Note: The solid and long-dashed lines are the posterior means of the estimated level shift probabilities from the NKPC-TVP-LS model and the NKPC-TVP-LS models, respectively. The dashed line is the observed inflation level. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

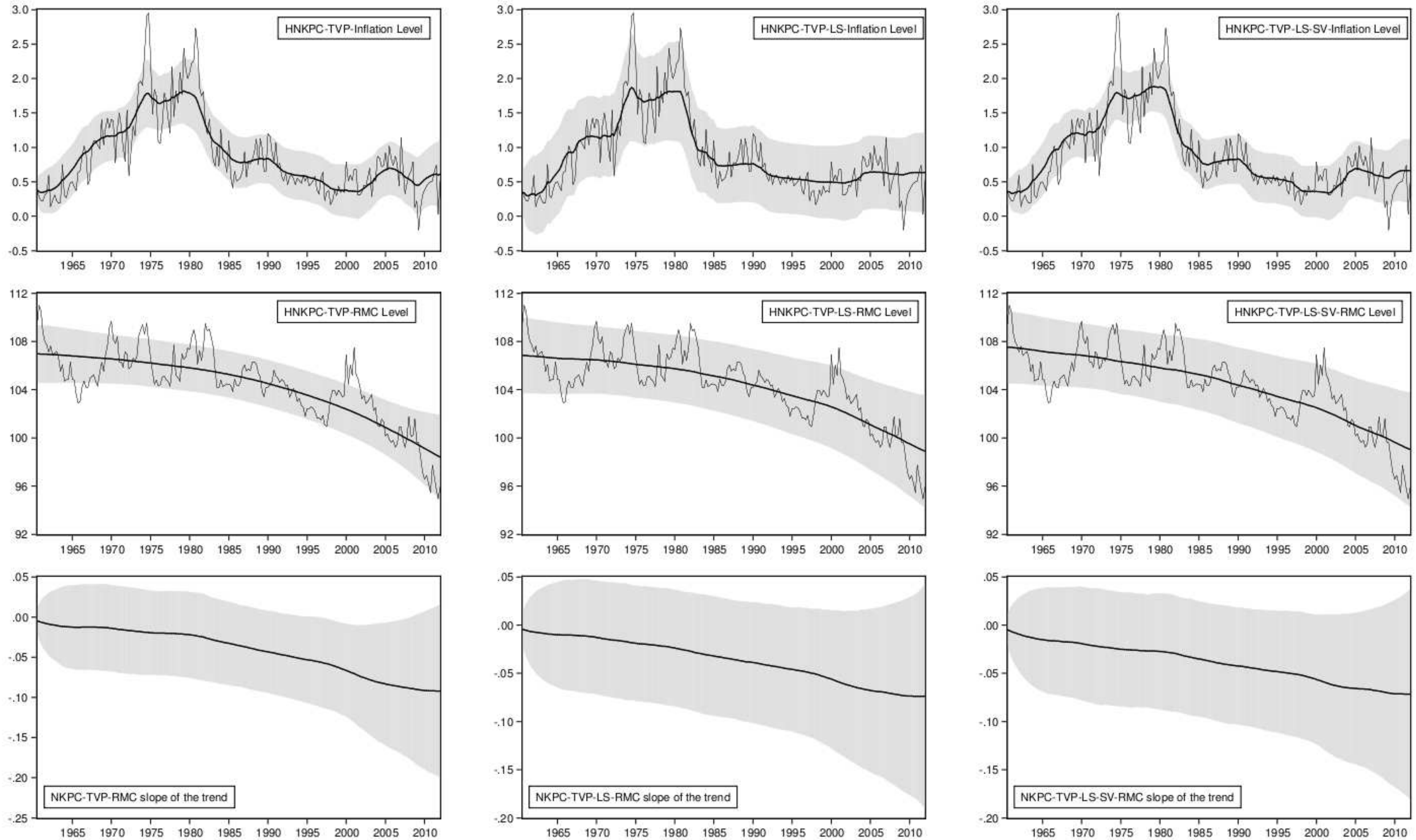
rapid changes in inflation. Accordingly, low volatility periods are characterized by mild changes in inflation, leading to a stable inflation level. Still, for the low volatility periods, mild but significant changes in the inflation level are attributed to level shifts leading to higher level shift probabilities and more clear signals of level shifts.

The top panel of Figure 6 shows estimated levels from all three HNKPC models. In line with the NKPC models' findings, models that only allow for discrete and occasional level shifts lead to smoother inflation level estimates compared to the model that allows for continuous level changes, especially in the second half of the sample period. Furthermore, the model with a stochastic volatility component, presented in the third graph, provides more precise inflation level estimates. As in the NKPC counterpart, the model with stochastic volatility indicates frequent level shifts with a more stable inflationary pattern between these level shifts. Estimated levels for the real marginal cost series for all models are given in the middle panel of Figure 6. Similar to the NKPC model results, marginal cost series follows a slightly nonlinear trend during the sample period.

Figure 7 presents estimated volatility levels from the HNKPC model with level shifts and the stochastic volatility component. Comparable to the findings of the NKPC models, highly volatile periods of 1970s and the beginning of 1980s, together with the recent recession are nicely captured by the volatility process. A slight difference between the two models is related to the volatility peaks during 1972 and 1978, which are higher than the volatility estimates of the NKPC model. Accordingly, the volatility peak around 1975 is lower in the HNKPC model. It seems that the high volatility is distributed more evenly in the HNKPC model with stochastic volatility, whereas for the NKPC counterpart, high volatility is concentrated around 1975. Finally, the peak points of estimated volatility coincide with rapid and substantial changes in inflation.

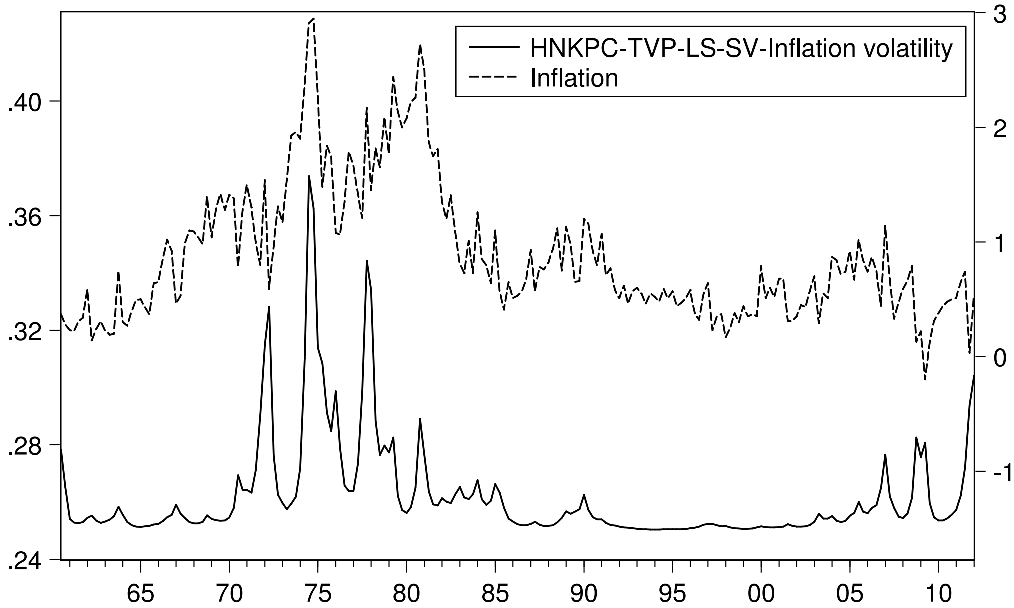
Estimated break probabilities for the HNKPC models with and without the stochastic volatility component are given in Figure 8. Both models indicate subsequent level shifts from the beginning of the sample period until 1975, which corresponds to the period during which inflation increased from around 0.20% to around 3%. Unlike the NKPC model, HNKPC based models indicate continuous inflation changes during this period. This picture is reversed for the remaining sample period, as the level shift probabilities for both HNKPC models are considerably smaller. The model with only level shift signals a clear level change in the inflation at the beginning of 1980s, where inflation is subject to a rapid decrease. However, for the period of Great Moderation, the model implies a stable inflationary pattern with moderate signals of level shifts around 1990 and around 2005. As for the NKPC model with level shifts and stochastic volatility, the periods of level changes indicated by high break probabilities are longer and more clear compared to the counterpart with-

Figure 6: Level, trend and slope estimates from the HNKPC models



Note: The top panel exhibits estimated inflation levels. The middle and the bottom panels show estimated real marginal cost levels and the slopes of the levels, respectively. Grey shaded areas correspond to the 95% HPDI. Model abbreviations are as in Table 1 Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

Figure 7: Estimated inflation volatility from the HNKPC-TVP-LS-SV model

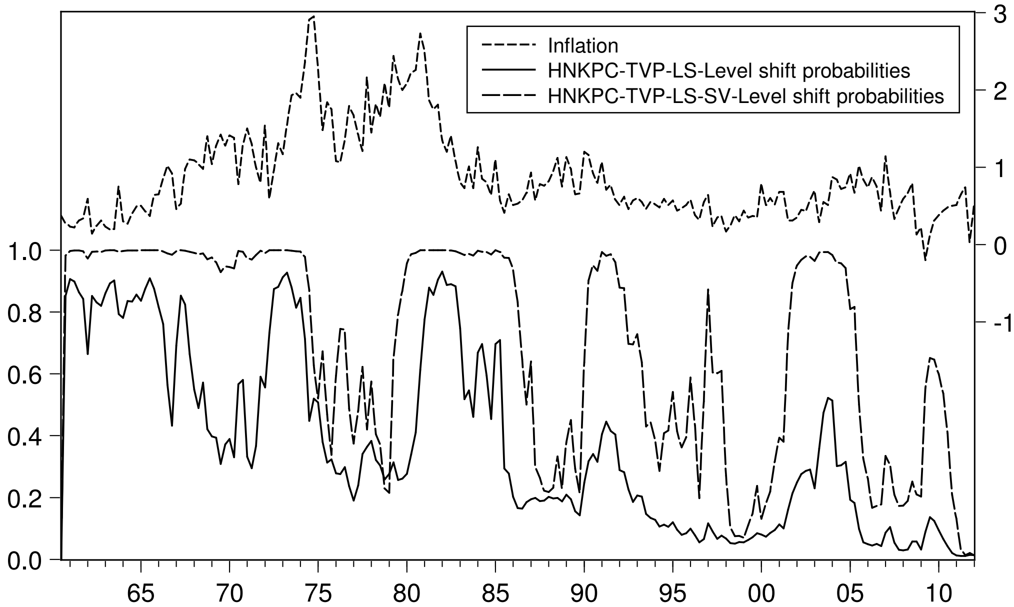


Note: The solid line is the posterior mean of the time varying inflation volatility. The dashed line is the observed inflation level. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

out stochastic volatility. Again, this shows the complementarity of the stochastic volatility component to the level shifts.

Finally, we report implied inflation expectations, computed as the posterior mean of the unobserved component S_t, for all HNKPC models in Figure 9. The shaded areas around the posterior means represent the 95% HPDI for the estimated long-term inflation expectations. All models estimate similar inflation expectations that track nicely the observed long-term inflation expectations. A noticeable difference between unobserved inflation expectations and the survey data is that the former are smoother than the latter. Particularly around 1970s, the implied expectations by the HNKPC models are lower than those based on the survey data. This results in repeated negative deviations from the survey expectations for this period. This is also apparent in the estimates of the β parameter, which is around 0.45 in Table 1, indicating a positive correlation between these deviations and the inflation level. In

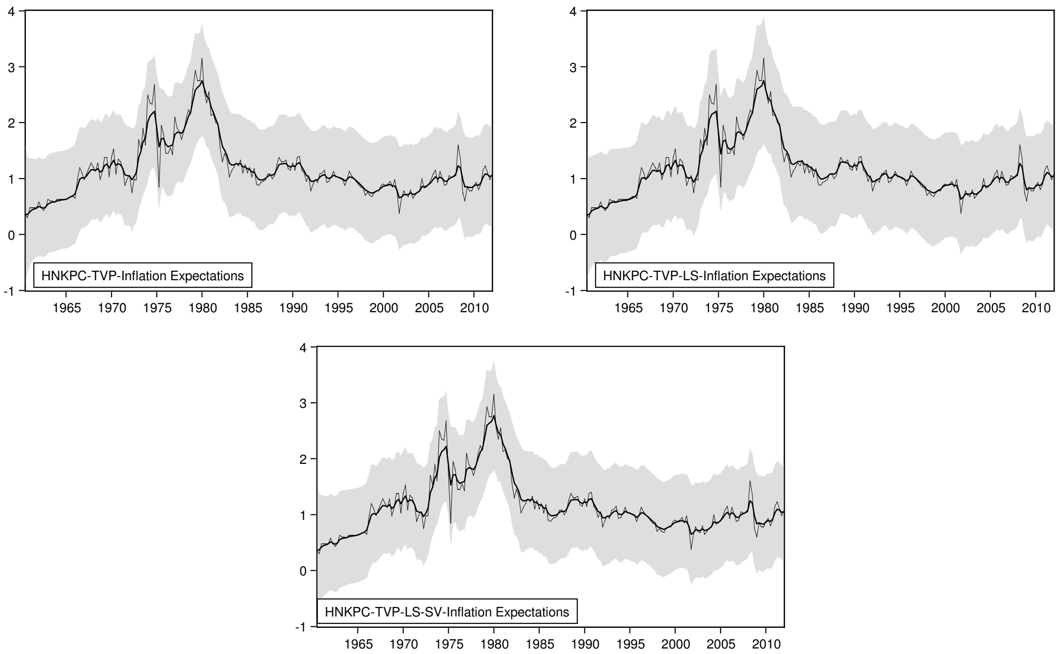
Figure 8: Estimated level shift probabilities for the HNKPC models



Note: The solid and long-dashed lines are the posterior means of the estimated level shift probabilities from the HNKPC-TVP-LS model and the HNKPC-TVP-LS models, respectively. The dashed line is the observed inflation level. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

line with the volatility findings, these deviations become considerably smaller during the second half of the sample period. This indicates that model based expectations are quite close to survey expectations for the latter half of the sample period.

Figure 9: Implied inflation expectations by HNKPC models



Note: The thick solid lines are the posterior means of inflation expectations from the HNKPC models. The thin solid lines are the observations of inflation expectations from survey data. Grey shaded areas are the 95% HPDI for estimated inflation expectations. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

Predictive Performance

Our next consideration is to evaluate the performance of the eight NKPC models in terms of their ability to predict inflation. The first metric we consider is the predictive likelihoods of all models in order to compare the density forecasts ability of the models. The one-step ahead predictive likelihood of the observation at $t_0 + 1$, y_{t_0+1}, conditional on the previous observations $y_{1:t_0}$, is given by

$$f(y_{t_0+1}|y_{1:t_0}) = \int p(y_{t_0+1}|X_{t_0+1}, \theta)p(X_{t_0+1}, \theta|y_{1:t_0})dX_{t_0+1}d\theta. \tag{16}$$

This can be computed as by first generating $\{X_{t_0+1}\}_{m=1}^{M}$ for M posterior draws, using the corresponding state equations of the models. Next, the predictive likelihood of the observation at t_0+1 can be computed as

$$f(y_{t_0+1}|y_{1:t_0}) \approx \frac{1}{M}\sum_{m=1}^{M} p(y_{t_0+1}|X_{t_0+1}^{m}, \theta_{1:t_0}^{m}), \tag{17}$$

where $p(y_{t_0+1}|X_{t_0+1}^{m}, \theta_{1:t_0}^{m})$ is a multivariate normal density and M is a sufficiently large number. A feature of the predictive likelihoods is that these can be used to compute the marginal likelihood as

$$p(y_{t_0+1:T}) = \prod_{t=t_0}^{T} f(y_{t+1}|y_{1:t}). \tag{18}$$

This provides a tool to analyze the contribution of each observation at time period t to the (log) marginal likelihoods as in (18), see Geweke and Amisano (2010).

Accurate point predictions of inflation is of key importance for economic agents such as investors and central banks. Therefore, we also consider the MSFE which is computed as the mean of the sum of squares of the prediction errors. For inflation forecasts we use mean of the posterior predictive distribution of inflation consistent with a quadratic loss function. We consider the MSFE for one and four period ahead forecasts in order to examine the forecasting ability of the models also for longer horizons.

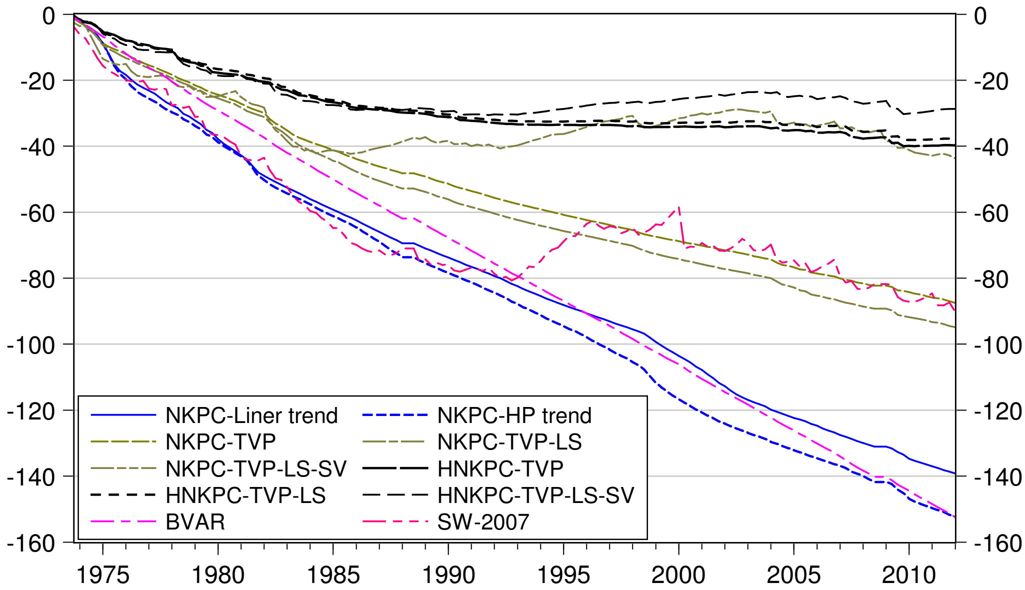
As a third performance criteria, we report the disinflation risk indicated by each model. Typically, increased uncertainty about future inflation is penalized by the predictive likelihood comparisons. This uncertainty, however, may simply indicate the increasing inflationary risk. We include this criterion in order to gain insights on the inflationary risk implied by each model. Disinflation probabilities are computed as the tail probability of the predictive distributions such that the one step ahead predicted inflation values are below zero.

Apart from the models we considered so far, we also consider alternative reduced form models that are proven to have superior predictive abilities. The first model we include is the unobserved component model proposed by Stock and Watson (2007), henceforth denoted as SW2007. This model captures the unobserved trend in inflation where both the inflation and the trend volatility follow a stochastic process. We refer to Stock and Watson (2007) for the details of this model. The second model we consider is an unrestricted Bayesian VAR (BVAR) model with four lags of quarterly inflation and real marginal cost series. BVAR models are one of the workhorse models used for forecasting macroeconomic series. For the sake of brevity, we do not provide the details of this model, and refer to standard textbooks such as Canova (2011). As for the structural models, we use the identical structural time series methods for modeling the level and the trends of the inflation and marginal cost series in the BVAR. Hence, both SW2007 and the BVAR models are strong competitors models for the extended NKPC and HNKPC models we propose.

Marginal likelihoods and the MSFE of the alternative models are presented in Table 2. The likelihood contribution of each observation and the corresponding cumulative predictive likelihoods are displayed in Figure 10.

We present the (log) marginal likelihood of the competing models in the first column of Table 2. These values together with Figure 10 indicate three groups of models in terms of their predictive performances. The first group of models include the BVAR and the conventional NKPC models with demeaned and detrended data (NKPC-LT and NKPC-HP). The second group consists of the NKPC models with discrete and continuous changes in inflation levels (NKPC-TVP, NKPC-TVP-LS) and the SW2007 model. The models in the second group have much superior performance in terms of the marginal likelihood values. A second increase in the marginal likelihood values can be observed when we consider the models in the third group, namely the HNKPC models (HNKPC-TVP, HNKPC-TVP-LS, HNKPC-TVP-LS-

Figure 10: Predictive likelihoods from competing models



Note: The figure displays the evolution of the (log) predictive likelihoods for the computing models between the third quarter of 1973 and the first quarter of 2012. Model abbreviations are based on Table 1. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

Table 2: Predictive performance of alternative Phillips curve models

Model	(Log) Marg. Likelihood	MSFE 1 period ahead	MSFE 4 period ahead
SW2007	-90.10	0.182	0.295
BVAR	-152.53	0.133	0.252
NKPC-LT	-139.23	0.352	0.398
NKPC-HP	-152.34	0.449	0.438
NKPC-TVP	-87.48	0.175	0.297
NKPC-TVP-LS	-94.90	0.203	0.292
NKPC-TVP-LS-SV	-43.76	0.146	0.231
HNKPC-TVP	-40.83	0.117	0.216
HNKPC-TVP-LS	-38.98	0.090	0.203
HNKPC-TVP-LS-SV	-30.11	0.084	0.205

Note: The table reports the predictive performances of all competing models for the prediction sample over the second quarter of 1973 and the first quarter of 2012. '(Log) Marg. Likelihood' stands for the natural logarithm of the marginal likelihoods. 'MSFE' stands for the Mean Squared Forecast Error. Marginal likelihood values in the first column are calculated as the sum of the predictive likelihood values in the prediction sample. Posterior results are based on 2000 burn-in and 4000 posterior draws. 'SW2007' stands for the model proposed by Stock and Watson (2007), and 'BVAR' stands for the Bayesian VAR model with time varying levels and trends. Remaining abbreviations are as in Table 1.

SV) and the NKPC model together with discrete level shifts and stochastic volatility for inflation (NKPC-TVP-LS-SV).

A similar clustering of models is observed when we compare the models' performances using the one period ahead MSFE, with the exception of the BVAR model. Unlike the model fit performance, measured by the marginal likelihood values, BVAR model performs considerably better in terms of point prediction.

Three main conclusions can be drawn from these findings. First, the conventional NKPC models with demeaned and detrended data (NKPC-LT and NKPC-HP) perform worse than the competing models both in terms of MSFE and in terms of the marginal likelihood metric. However, the difference between HNKPC and NKPC models in terms of point forecasts is less pronounced compared to the increase in precision when switching from models using demeaned and detrended data to the models that use the raw data. This indicates the importance of estimating levels and trends together with the structural model parameters.

Second, still the difference between the NKPC model with level shifts and stochastic volatility with the remaining NKPC models is considerably large. The performance of this model is comparable to the HNKPC models which perform superior both in terms of point forecasts and the model fit. On the one hand, models with level shifts and stochastic volatility deliver the most accurate point predictions considering the MSFE and marginal likelihood values. These results pinpoint the importance of incorporating the high and low frequency movements in the structural models. On the other hand, this model performance can be increased further by incorporating the survey data and the backward looking component in the HNKPC models.

Third, structural models perform at least as well as the strong reduced form candidates, the SW2007 and BVAR modes. These findings are crucial in the sense that the structural models deliver both structural macroeconomic information and predictive performance, whereas the reduced form models are solely designed for improving the predictive performance. Incorporating high and low frequency movements in structural models increase the predictive power of the structural models substantially while still exploiting the macroeconomic information indicated by the economic theory. These findings also hold for four period ahead forecasts, as shown in the last column of Table 2.

We next consider the evolution of the model performance over the forecast sample in detail, shown in Figure 10. An important finding from the figure is the increasing performance of the HNKPC models and the models with stochastic volatility components after mid 1980s. Note that this period is characterized by a decrease in inflation volatility also denoted as the Great Moderation period. It seems that the stochastic volatility component captures this decrease in volatility accurately. Moreover, the effect of the level shifts can be observed when we compare the NKPC-TVP-LS-SV model with the SW2007 model. Much of the difference in the performance of these models can be attributed to the changes in inflation levels. This shows that the inflation process exhibits rare regime changes and within each regime inflation follows a stable path.

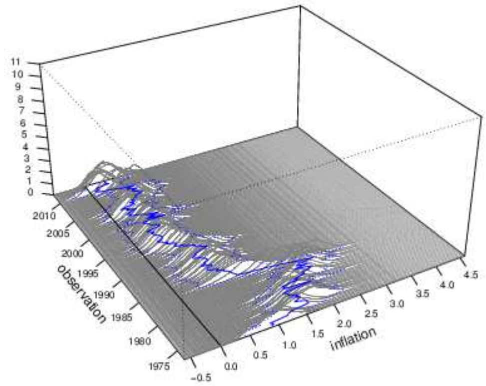
The last metric we use for model comparison considers the implied inflationary risk. Figure 11 shows the entire distribution of the inflation predictions for the NKPC and HNKPC models where the levels and trends are estimated together with the structural parameters. Posterior means of predicted inflation for all models are represented by the solid lines, and the widths of the predictive distributions are indicated by the white areas under the inflation densities. As expected, inflation predictions are concentrated around high (low) values during the high (low) inflationary periods. The uncertainty around the inflation predictions are also high for these periods, together with the periods when inflation is subject to a transition to low values around 1980s. When the observed inflation values are close to the zero bound, the predictive densities indicate disinflationary risk, computed as the fraction of the predictive distribution below zero.

Figure 12 displays this disinflationary risk in detail. The ability to predict the disinflationary risk is of key importance especially for policy making purposes. From the figure it is seen that NKPC models with a priori demeaned and detrended data do not signal any pronounced disinflation risk except for the low disinflation probabilities during mid 1970s and mid 1980s. However, NKPC and HNKPC models incorporating raw data information by exploiting the high and low frequency movements produce clear signals of disinflation risk during the recent recession. This reflects the disinflationary pressure of the recent recession, denoted as the 'Great Recession', and the enlarged NKPC models can predict these effects successfully.

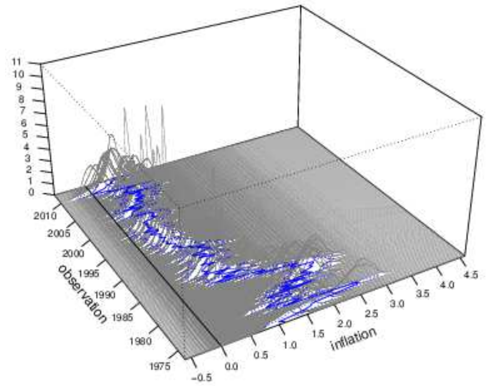
Analysis of cointegration in inflation and marginal cost levels

The models we considered so far rely on the implicit assumption of the absence of a long-run cointegrating relationship between the inflation and marginal cost series.

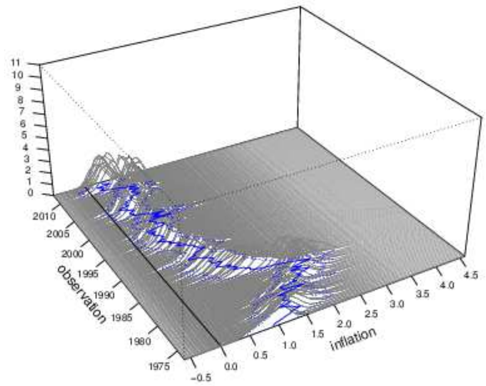
Figure 11: Predicted inflation densities from NKPC and HNKPC models



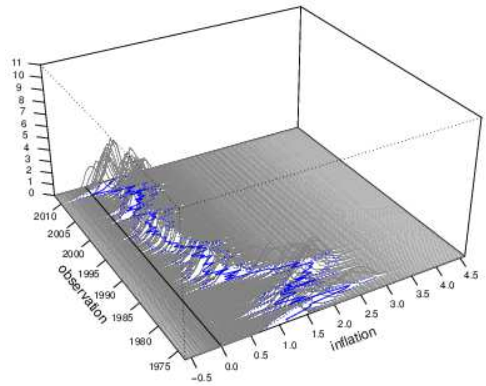
NKPC-TVP



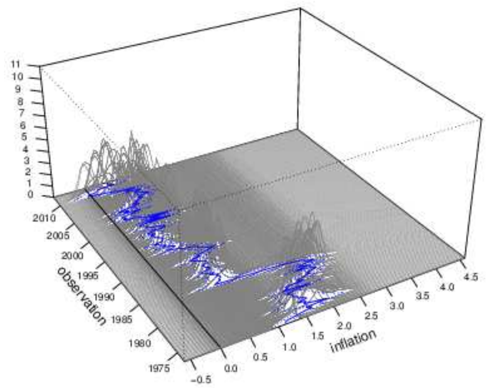
HNKPC-TVP



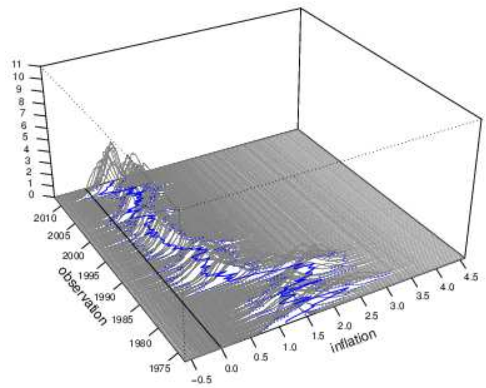
NKPC-TVP-LS



HNKPC-TVP-LS



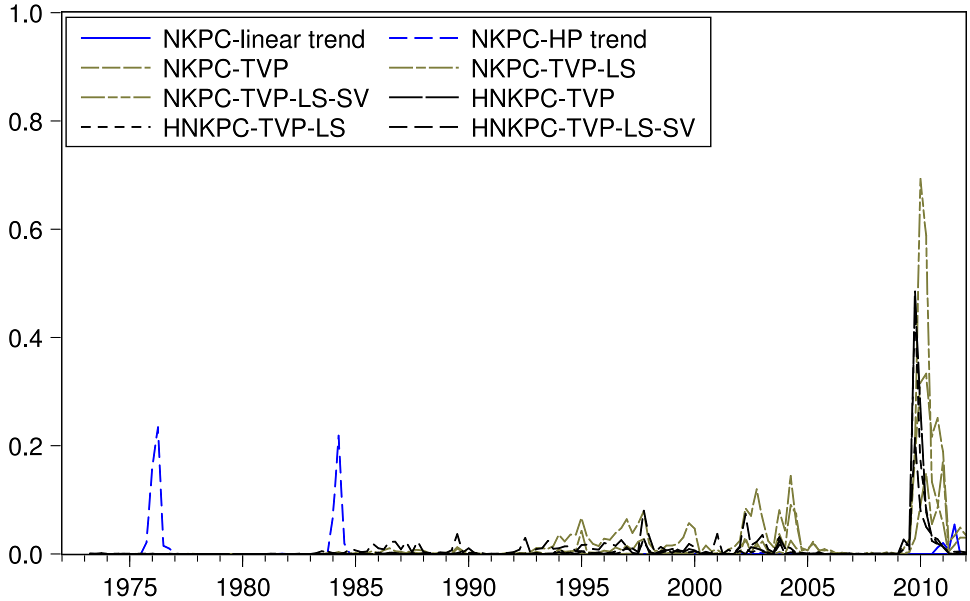
NKPC-TVP-LS-SV



HNKPC-TVP-LS-SV

Note: The figure presents one period ahead predictive distributions of inflation from the NKPC and HNKPC models, for the period between the third quarter of 1973 and the first quarter of 2012. Model abbreviations are based on Table 1. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

Figure 12: Disinflation probabilities implied by different Phillips curve models



Note: The figure presents disinflation probabilities computed using the one period ahead predictive distributions of inflation for the period between the third quarter of 1973 and the first quarter of 2012. Model abbreviations are based on Table 1. Results are based on 4000 simulations of which the first 2000 are discarded for burn-in.

We assess whether this assumption is plausible for the U.S. data. For this reason, we consider the NKPC-TVP model that provides the unobserved levels of both series at each posterior draw. For each of these obtained posterior draws, we perform a simple two-step analysis to check the existence of the cointegrating relationship, which can be seen as a Bayesian extension of the method of Engle and Granger (1987).

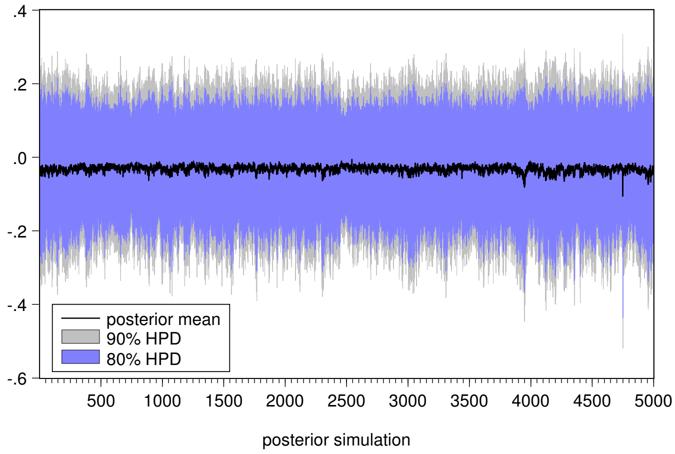
We perform a two step analysis, where in the first step we obtain the residuals from the regression of the estimated level of inflation on a constant and the estimated level of marginal cost, for each posterior draw. This implies that we take the estimation uncertainty in the analysis into account. Next, we obtain the posterior distribution of the autoregressive parameter, ρ, for each set of residuals from the following regression using flat priors on the identified region $\rho \in [-1, 1]$

$$\Delta\hat{\epsilon}_t = \rho\hat{\epsilon}_{t-1} + \eta_t, \;\; \eta_t \sim NID(0, \sigma^2), \tag{19}$$

where $\hat{\epsilon}_t$ denotes the residuals from the first stage, and $\rho = 0$ implies that there is no cointegrating relationship between the series. An HPDI including the value of 0 indicates that a cointegrating relation between inflation and marginal cost is unlikely.

We compute the mean and the quantiles of these individual densities using 5000 posterior draws, and report the average values of the mean and the quantiles of ρ based on 3000 simulations. These results are presented in Figure 13. Posterior means of parameter ρ are around 0 for all posterior draws of inflation and marginal cost levels, and the 80% an 90% percent quantiles of the distribution are around 0 as well. Hence this simulation experiment does not indicate a cointegrating relationship between the inflation and marginal cost levels. This pattern is also found for other TVP-NKPC models we considered for the U.S. data, but these results are not reported for the sake of brevity. We conclude that the underlying assumption of 'no cointegrating relationship' is found to be feasible for the Phillips curve models we consider.

Figure 13: Cointegration analysis for the marginal cost and inflation series, using the NKPC-TVP model



Note: The figure presents the posterior means and quantiles of the ρ parameter from 5×10^3 posterior draws from the NKPC-TVP models, where for each draw, the the reported values are calculated using 3000 simulations. $\rho = 0$ implies that there is no cointegrating relationship between the series.

6 Conclusion

The NKPC model constitutes an integral part of macroeconomic models used for policy analysis. These models are estimated mostly after demeaning and/or detrending the series. In this paper it is shown that mechanical removal of the low frequency movements in the data may lead to misspecification plaguing inference. Potential structural breaks and level shifts as well as changing volatility in the observed series require more complex models, which can handle these time variation together with the standard NKPC parameters. We propose a set of models where low and high frequency movements in the inflation and marginal cost series are taken into account explicitly. This is achieved by modeling the levels and trends of the series together with the volatility process explicitly in the NKPC model and estimating these along with other model parameters simultaneously. Furthermore, we consider richer expectational mechanisms for the inflation series in enlarged Hybrid-NKPC models, where the inflation expectations are anchored around the inflation expectations obtained from survey data.

The proposed models capture time variation in the low frequency moments of both inflation and marginal cost data. For the inflation series we identify three distinct periods with high and low inflation. The high inflationary period corresponds to 1970s, following a low inflationary period of 1960s. The last period starting with 1980s is characterized by low inflation levels corresponding to an annual inflation level around 2%. When this model is blended with the stochastic volatility component, the level shifts can be identified even more precisely.

The use of macroeconomic information in the structural models together with the remaining high and low frequency movements in the data improves the predictive ability also compared to celebrated reduced form models, including the Bayesian VAR and the stochastic volatility model (Stock and Watson, 2007). Furthermore, modelling inflation expectations using survey data and adding stochastic volatility to the NKPC model structure improves in sample fit and out of sample predictive performance substantially. We also analyze the disinflation probabilities indicated by each competing model. The complete predictive densities, most notably from the enlarged models, indicate an increase in the probability of disinflation in the U.S. in recent years.

Modelling forward and backward looking components of inflation has important effects on empirical results. Weak endogeneity and persistence do not appear to be important issues in NKPC model structures. Finally, we also analyze the existence of a long-run relation between the low frequency movements of both series. No evidence is found on a long run stable cointegrating relation between U.S. inflation and marginal costs.

We show that incorporating low and high frequency movements explicitly in macroeconomic models provides additional insights for both policy analysis and more accurate predictions. Hence we plan to enlarge the proposed model to a more general DSGE framework in future work. Another interesting possibility of future research is to combine different NKPC models using their predictive performances, which seems to be time varying.

References

Ahmed S, Levin A, Wilson BA. 2004. Recent U.S. macroeconomic stability: Good policies, good practices, or good luck? *The Review of Economics and Statistics* **86**: 824–832.

Basistha A, Nelson CR. 2007. New measures of the output gap based on the forward-looking new Keynesian Phillips curve. *Journal of Monetary Economics* **54**: 498–511.

Beveridge S, Nelson CR. 1981. A new approach to decomposition of economic time series into permanent and transitory components with particular attention to measurement of the business cycle. *Journal of Monetary Economics* **7**: 151–174.

Bianchi F. 2010. Regime switches, agents' beliefs, and Post-World War II U.S. macroeconomic dynamics. Working Papers 10–39, Duke University, Department of Economics.

Calvo G. 1983. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics* **12**: 383–398.

Canova F. 2011. *Methods for applied macroeconomic research*, volume 13. Princeton University Press.

Canova F. 2012. Bridging DSGE models and the raw data. Working Papers 635, Barcelona Graduate School of Economics.

Canova F, Gambetti L. 2006. Structural changes in the US economy: Bad luck or bad policy? CEPR Discussion Papers 5457, C.E.P.R. Discussion Papers.

Carter CK, Kohn R. 1994. On Gibbs sampling for state space models. *Biometrika* **81**: 541–553.

Çakmaklı C, Paap R, Van Dijk D. 2011. Modeling and estimation of synchronization in multistate Markov-switching models. Working Papers 11-002/4, Tinbergen Institute.

Cecchetti SG, Hooper P, Kasman BC, Schoenholtz KL, Watson MW. 2007. Understanding the evolving inflation process. Paper written for the U.S. Monetary Policy Forum 2007, Washington, DC.

Clarida R, Galí J, Gertler M. 2000. Monetary policy rules and macroeconomic stability: Evidence and some theory. *The Quarterly Journal of Economics* **115**: 147–180.

Cogley T, Sargent TJ. 2005. Drift and volatilities: Monetary policies and outcomes in the post WWII U.S. *Review of Economic Dynamics* **8**: 262–302.

Cogley T, Sbordone AM. 2008. Trend inflation, indexation, and inflation persistence in the New Keynesian Phillips curve. *The American Economic Review* **98**: 2101–2126.

DeJong DN, Dave C. 2011. *Structural macroeconometrics.* Princeton University Press.

Del Negro M, Schorfheide F. 2012. DSGE model-based forecasting. Staff Reports 554, Federal Reserve Bank of New York.

Delle Monache D, Harvey AC. 2010. The effect of misspecification in models for extracting trends and cycles. Manuscript.

Delle Monache D, Harvey AC. 2011. The effect of misspecification in models for extracting trends and cycles. Working Papers EWP 2011/013, Euroindicators.

Durbin J, Koopman SJ. 2001. *Time series analysis by state space methods.* OUP Catalogue. Oxford University Press.

Engle RF, Granger CWS. 1987. Co-integration and error correction: representation, estimation, and testing. *Econometrica* : 251–276.

Frühwirth-Schnatter S. 1994. Data augmentation and dynamic linear models. *Journal of Time Series Analysis* **15**: 183–202.

Fuhrer J, Moore G. 1995. Inflation persistence. *The Quarterly Journal of Economics* **110**: 127–59.

Galí J, Gertler M. 1999. Inflation dynamics: A structural econometric analysis. *Journal of Monetary Economics* **44**: 195–222.

Gali J, Gertler M, David Lopez-Salido J. 2005. Robustness of the estimates of the hybrid new keynesian phillips curve. *Journal of Monetary Economics* **52**: 1107–1118.

Galí J, Gertler M, Lopez-Salido JD. 2001. European inflation dynamics. *European Economic Review* **45**: 1237–1270.

Geman S, Geman D. 1984. Stochastic relaxations, Gibbs distributions, and the Bayesian restoration of images. *IEEE Transaction on Pattern Analysis and Machine Intelligence* **6**: 721–741.

Gerlach R, Carter C, Kohn R. 2000. Efficient Bayesian inference for dynamic mixture models. *Journal of the American Statistical Association* : 819–828.

Geweke J, Amisano G. 2010. Comparing and evaluating Bayesian predictive distributions of asset returns. *International Journal of Forecasting* **26**: 216–230.

Giordani P, Kohn R. 2008. Efficient Bayesian inference for multiple change-point and mixture innovation models. *Journal of Business & Economic Statistics* **26**: 66–77.

Giordani P, Kohn R, van Dijk D. 2007. A unified approach to nonlinearity, structural change, and outliers. *Journal of Econometrics* **137**: 112–133.

Gorodnichenko Y, Ng S. 2010. Estimation of DSGE models when the data are persistent. *Journal of Monetary Economics* **57**: 325–340.

Hastings WK. 1970. Monte Carlo sampling using Markov chains and their applications. *Biometrika* **57**: 97–10.

Hodrick RJ, Prescott EC. 1997. Postwar U.S. business cycles: An empirical investigation. *Journal of Money, Credit and Banking* **29**: 1–16.

Kim CJ, Nelson CR. 1999. *State-space models with regime switching: Classical and Gibbs-Sampling approaches with applications*, volume 1. The MIT Press, 1 edition.

Kim CJ, Nelson CR. 2006. Estimation of a forward-looking monetary policy rule: A time-varying parameter model using ex post data. *Journal of Monetary Economics* **53**: 1949–1966.

Kim S, Shephard N, Chib S. 1998. Stochastic volatility: Likelihood inference and comparison with ARCH models. *Review of Economic Studies* **65**: 361–93.

Kleibergen F, Mavroeidis S. 2011. Identification robust priors for Bayesian analysis in DSGE models. Manuscript.

Mavroeidis S. 2004. Weak identification of forward-looking models in monetary economics. *Oxford Bulletin of Economics and Statistics* **66**: 609–635.

McCallum BT. 1976. Rational expectations and the natural rate hypothesis: Some consistent estimates. *Econometrica* **44**: 43–52.

McConnell MM, Perez-Quiros G. 2000. Output fluctuations in the United States: What has changed since the early 1980's? *American Economic Review* **90**: 1464–1476.

Metropolis N, Rosenbluth AW, Rosenbluth MN, Teller AH, Teller E. 1953. Equations of state calculations by fast computing machines. *Journal of Chemical Physics* **21**: 1087–1092.

Omori Y, Chib S, Shephard N, Nakajima J. 2007. Stochastic volatility with leverage: Fast and efficient likelihood inference. *Journal of Econometrics* **140**: 425–449.

Ritter C, Tanner MA. 1992. Facilitating the Gibbs sampler: The Gibbs stopper and the griddy-Gibbs sampler. *Journal of the American Statistical Association* **87**: 861–868.

Roberts JM. 1995. New keynesian economics and the phillips curve. *Journal of Money, Credit and Banking* **27**: 975–84.

Roberts JM. 1997. Is inflation sticky? *Journal of Monetary Economics* **39**: 173–196.

Rudd J, Whelan K. 2005. New tests of the new-Keynesian Phillips curve. *Journal of Monetary Economics* **52**: 1167–1181.

Sims CA. 2002. Solving linear rational expectations models. *Computational Economics* **20**: 1–20.

Sims CA, Zha T. 2006. Were there regime switches in U.S. monetary policy? *American Economic Review* **96**: 54–81.

Smets F, Wouters R. 2003. An estimated Dynamic Stochastic General Equilibrium model of the Euro Area. *Journal of the European Economic Association* **1**: 1123–1175.

Stock JH, Watson MW. 2002. Has the business cycle changed and why? NBER Working Papers 9127, National Bureau of Economic Research, Inc.

Stock JH, Watson MW. 2007. Why has U.S. inflation become harder to forecast? *Journal of Money, Credit and Banking* **39**: 3–33.

Stock JH, Watson MW. 2008. Phillips Curve inflation forecasts. Working Paper 14322, National Bureau of Economic Research.

Tanner MA, Wong WH. 1987. The calculation of posterior distributions by data augmentation. *Journal of the American Statistical Association* **82**: 528–550.

Zellner A, Ando T, Baştürk N, Hoogerheide L, Van Dijk HK. 2012. Bayesian analysis of instrumental variable models: Acceptance-rejection within Direct Monte Carlo. *Econometric Reviews*, accepted.

Zhang C, Osborn DR, Kim DH. 2008. The new Keynesian Phillips curve: From sticky inflation to sticky prices. *Journal of Money, Credit and Banking* **40**: 667–699.

Appendix

A Structural and reduced form inference of the NKPC model

This section presents the unrestricted reduced form inference (URF) of the NKPC model, and the inference of the corresponding structural form (SF) model parameters. We show that the posterior draws from the structural form parameters can be obtained using the reduced form representation of (5):

$$\begin{aligned} \tilde{\pi}_t &= \alpha_1 \tilde{z}_{t-1} + \alpha_2 \tilde{z}_{t-2} + \epsilon_{1,t}, \\ \tilde{z}_t &= \phi_1 \tilde{z}_{t-1} + \phi_2 \tilde{z}_{t-2} + \epsilon_{2,t}, \end{aligned} \tag{20}$$

where $(\epsilon_{1,t}, \epsilon_{2,t})' \sim NID(0, \Sigma)$, and the restricted reduced form (RRF) representation is obtained by introducing the following restrictions on parameters in (5):

$$\alpha_1 = \tfrac{\lambda(\phi_1+\gamma\phi_2)}{1-\gamma(\phi_1+\gamma\phi_2)}, \ \ \alpha_2 = \tfrac{\lambda\phi_2}{1-\gamma(\phi_1+\gamma\phi_2)}. \tag{21}$$

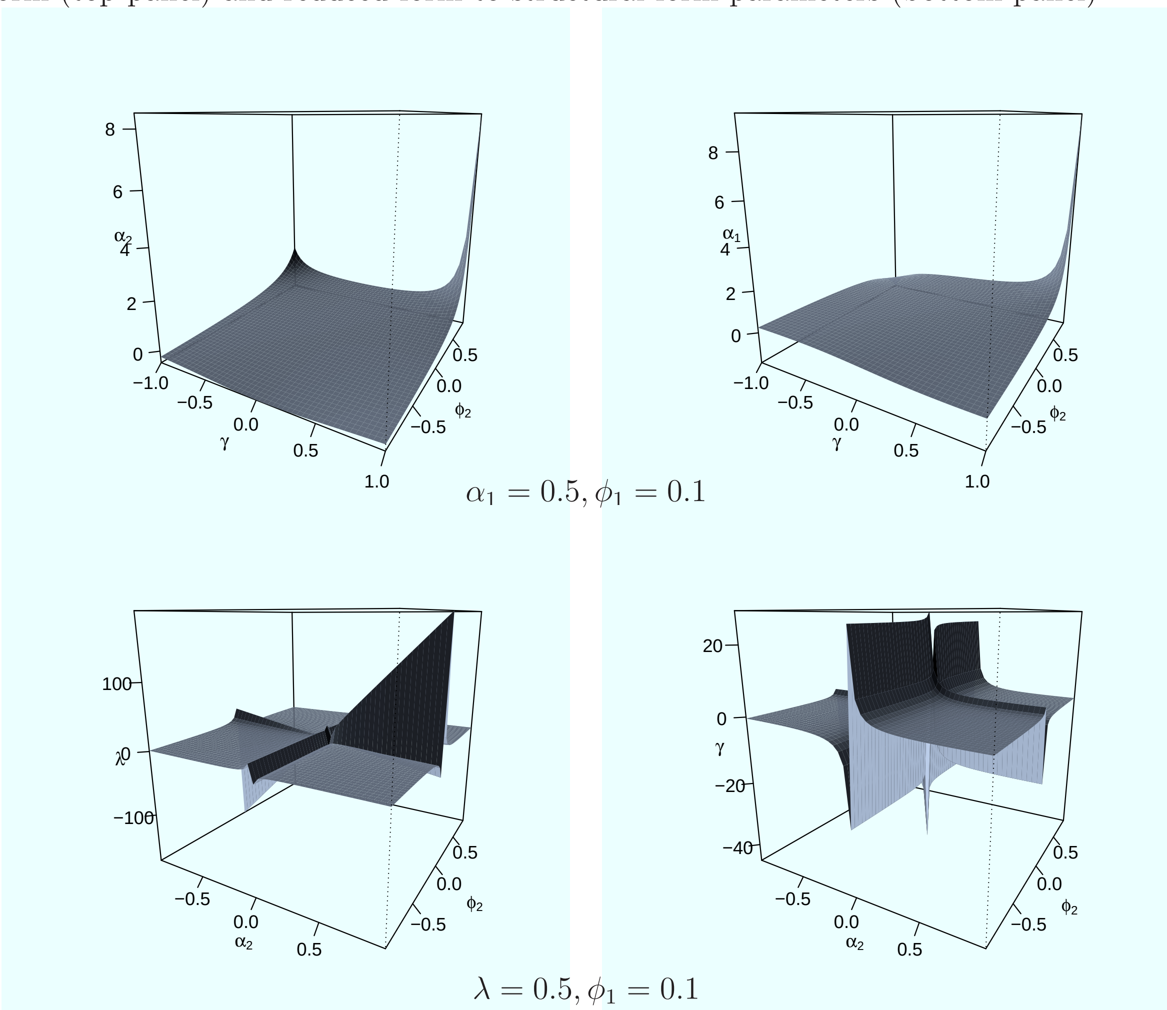
Finally, the model in (5) is related to an Instrumental Variables (IV) model with exact identification. Bayesian estimation of the unrestricted reduced form model in (20) is straightforward under flat or conjugate priors. Given the posterior draws of reduced form parameters, posterior draws of structural form parameters in (5) can be obtained using the transformation in (21). This nonlinear transformation, however, causes difficulties in setting the priors in an adequate way. The determinant of the Jacobian of this nonlinear transformation is $\mid J \mid = \frac{\lambda\phi_2^2}{(1-\gamma(\phi_1+\gamma\phi_2))^2}$, where the Jacobian is non-zero and finite if $\gamma(\phi_1 + \gamma\phi_2) \neq 1$, $\phi_2 \neq 0$ and $\lambda \neq 0$.[6]

Figure 14 illustrates the nonlinear transformation for the SF and RRF repre-

[6] We only consider the transformation from $\{\lambda, \gamma, \phi_1, \phi_2\}$ to $\{\alpha_1, \alpha_2, \phi_1, \phi_2\}$, i.e. variance parameters in the transformed model are left as free parameters.

sentations, for a grid of parameter values from SF representations, and plot the corresponding RRF parameter values, and vice versa. The top panel in Figure 14 shows the transformations from SF to RRF. Reduced form parameters α_1 and α_2 tend to infinity when persistence in inflation and marginal cost series are high, i.e. when the structural form parameters λ and $\phi_1 + \phi_2$ tend to 1. The bottom panel in Figure 14 shows the RRF to SF transformations. The corresponding SF parameters lead to an irregular shape, for example, when the instrument z_{t-2} has no explanatory power with $\phi_2 = 0$ or when $\alpha_2 = 0$.

Figure 14: Nonlinear parameter transformation from structural form to reduced form (top panel) and reduced form to structural form parameters (bottom panel)



Note: The top panel presents the implied unrestricted reduced form parameters in (20) given structural form parameters in (5). The top panel presents implied structural form parameters in (5) given unrestricted reduced form parameters in (20). Parameter transformations are obtained using the RRF restrictions in (21).

B Bayesian inference of the extended NKPC model

This section presents the MCMC scheme for the posterior inference of the NKPC model. Specifically, we use a Gibbs sampler together with data augmentation (see Geman and Geman, 1984; Tanner and Wong, 1987).

The NKPC model in (10) can be cast into the state-space form as follows

$$\begin{aligned} Y_t &= HX_t + BU_t + \epsilon_{\mathbf{t}}, \quad \epsilon_{\mathbf{t}} \sim N(0, Q_t) \\ X_t &= FX_{t-1} + R_t\eta_{\mathbf{t}}, \qquad \eta_{\mathbf{t}} \sim N(0, I) \end{aligned} \tag{22}$$

where

$$Y_t = \begin{pmatrix} \pi_t \\ z_t \end{pmatrix}, \quad X_t = \begin{pmatrix} c_{\pi,t}, & c_{z,t}, & \mu_{z,t}, & c_{z,t-1}, & c_{z,t-2} \end{pmatrix}', \quad U_t = \begin{pmatrix} z_t \\ z_{t-1} \\ z_{t-2} \end{pmatrix}, \quad \epsilon_{\mathbf{t}} = \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix},$$

$$H = \begin{pmatrix} 1 & -\alpha_1 & 0 & -\alpha_2 & 0 \\ 0 & 1 & 0 & -\phi_1 & -\phi_2 \end{pmatrix}, \quad B = \begin{pmatrix} \alpha_1 & \alpha_2 & 0 \\ 0 & \phi_1 & \phi_2 \end{pmatrix}, \quad Q_t = \begin{pmatrix} \sigma^2_{\epsilon_1,t} & \rho\sigma_{\epsilon_1,t}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon_1,t}\sigma_{\epsilon_2} & \sigma^2_{\epsilon_2} \end{pmatrix},$$

$$F = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{pmatrix}, \quad R_t = \begin{pmatrix} \kappa_t\sigma_{\eta_1} & 0 & 0 \\ 0 & \sigma_{\eta_2} & 0 \\ 0 & 0 & \sigma_{\eta_3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \eta_{\mathbf{t}} = \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \end{pmatrix},$$

where $\alpha_1 = \frac{\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}$ and $\alpha_2 = \frac{\lambda\gamma\phi_2}{1-(\phi_1+\phi_2\gamma)\gamma}$.

Once the state-space form of the model is set as in (22) standard inference techniques in state-space models can be carried out. Let $Y_{1:T} = (Y_1, Y_2, \ldots, Y_T)'$, $X_{1:T} = (X_1, X_2, \ldots, X_T)'$, $U_{1:T} = (U_1, U_2, \ldots, U_T)'$, $\sigma^2_{\epsilon_1,1:T} = (\sigma^2_{\epsilon_1,1}, \sigma^2_{\epsilon_1,2}, \ldots, \sigma^2_{\epsilon_1,T})'$ and $\theta = (\phi_1, \phi_2, \gamma_f, \lambda)'$. For the most general NKPC model with level shifts and stochastic volatility, the simulation scheme is as follows

1. Initialize the parameters by drawing κ_t using the prior for κ and unobserved states X_t, h_t for $t = 1, 2, \ldots, T$ from standard normal distribution and conditional on κ_t for $t = 0, 1, \ldots, T$. Initialize $m = 1$.

2. Sample $\theta^{(m)}$ from $p(\theta|Y_{1:T}, X_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$.

3. Sample $X_t^{(m)}$ from $p(X_t|\theta^{(m)}, Y_{1:T}, h_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$ for $t = 1, 2, \ldots, T$.

4. Sample $h_t^{(m)}$ from $p(h_t|X_{1:T}^{(m)}, \theta^{(m)}, Y_{1:T}, X_{1:T}, U_{1:T}, R_{1:T}, \rho^{m-1}, \sigma_{\epsilon_2}^{2,(m-1)}, \sigma_{\eta_4}^{2,(m-1)})$ for $t = 1, 2, \ldots, T$.

5. Sample $\kappa_t^{(m)}$ from $p(\kappa^{(m)}|\theta^{(m)}, Y_{1:T}, h_{1:T}, U_{1:T}, R_{1:T}, Q_{1:T})$ for $t = 1, 2, \ldots, T$.

6. Sample $\sigma_{\eta_i}^{2,(m)}$ from $p(\sigma_{\eta_i}^{2,(m)}|X_{1:T}^{(m)}, h_{1:T}^{(m)}, \kappa_{1:T}^{(m)})$ for $i = 1, 2, 3, 4$.

7. Sample $\rho^{(m)}$ from from $p(\rho^{(m)}|X_{1:T}^{(m)}, h_{1:T}^{(m)}, Y_{1:T}, X_{1:T}, U_{1:T}, \theta^{(m)}, \sigma_{\epsilon_2}^{2,(m-1)})$.

8. Sample $\sigma_{\epsilon_2}^{2,(m)}$ from from $p(\sigma_{\epsilon_2}^{2,(m)}|\rho^{(m)}, X_{1:T}^{(m)}, h_{1:T}^{(m)}, Y_{1:T}, X_{1:T}, U_{1:T}, \theta^{(m)})$.

9. Set $m = m + 1$, repeat (2)-(9) until $m = M$.

Steps (3)-(5) are common to many models in the Bayesian state-space framework, see for example Kim and Nelson (1999); Gerlach et al. (2000). Note that parameter p_κ is set a priori using heuristics.

Sampling of θ

Conditional on the states $c_{\pi,t}, c_{z,t}$ and h_t for $t = 1, 2, \ldots, T$, redefining the variables such that $\tilde{\pi}_t = \pi_t - c_{\pi,t}$, $\tilde{z}_t = z_t - c_{z,t}$ and $\varepsilon_t = \epsilon_t / \exp(h_t/2)$, the measurement equation in (22) can be rewritten as

$$\begin{aligned} \tilde{\pi}_t &= \tfrac{\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\tilde{z}_t + \tfrac{\phi_2\gamma_f\lambda}{1-(\phi_1+\phi_2\gamma_f)\gamma_f}\tilde{z}_{t-1} + \varepsilon_t \\ \tilde{z}_t &= \phi_1\tilde{z}_{t-1} + \phi_2\tilde{z}_{t-2} + \epsilon_{2,t}. \end{aligned} \tag{23}$$

Posterior distributions of the structural parameters under flat priors are non-standard since z_t term also is on the right hand side of (23) and the model is highly non-linear in parameters. We therefore use two Metropolis Hastings steps to sample these structural parameters (Metropolis et al., 1953; Hastings, 1970). For sampling ϕ_1, ϕ_2 conditional on λ, γ_f and other model parameters, the candidate density is a multivariate student-t density on the stationary region with a mode and scale with the posterior mode and scale using only the second equation in (23) and 1 degrees of freedom. For sampling λ, γ_f conditional on ϕ_1, ϕ_2 and other model parameters, the candidate is a uniform density.

Sampling of states, X_t

Conditional on the remaining model parameters, drawing $X_{0:T}$ can be implemented using standard Bayesian inference. This constitutes running the Kalman filter first and running a simulation smoother using the filtered values for drawing smoothed states as in Carter and Kohn (1994) and Frühwirth-Schnatter (1994). We start the recursion for $t = 1, \ldots, T$

$$\begin{aligned}
X_{t|t-1} &= FX_{t-1|t-1} \\
P_{t|t-1} &= FP_{t-1|t-1}F' + R_t'R_t \\
\eta_{t|t-1} &= y_t - HX_{t|t-1} - BU_t \\
\zeta_{t|t-1} &= HP_{t|t-1}H' + Q_t \\
K_t &= P_{t|t-1}H'\zeta_{t|t-1}' \\
X_{t|t} &= X_{t|t-1} + K_t\eta_{t|t-1} \\
P_{t|t} &= P_{t|t-1} - K_tH'\zeta_{t|t-1}',
\end{aligned} \tag{24}$$

and store $X_{t|t}$ and $P_{t|t}$. The last filtered state $X_{T|T}$ and its covariance matrix $P_{T|T}$ correspond to the smoothed estimates of the mean and the covariance matrix of the states for period T. Having stored all the filtered values, simulation smoother involves the following backward recursions for $t = T-1, \ldots, 1$

$$\begin{aligned}
\eta^*_{t+1|t} &= X_{t+1} - FX_{t|t} \\
\zeta^*_{t+1|t} &= FP_{t|t}F' + R'_{t+1}R_{t+1} \\
X_{t|t,X_{t+1}} &= X_{t|t} + P_{t|t}F'\zeta^{*-1}_{t+1|t}\eta^*_{t+1|t} \\
P_{t|t,P_{t+1}} &= P_{t|t} - P_{t|t}F'\zeta^{*-1}_{t+1|t}FP_{t|t}.
\end{aligned} \tag{25}$$

Intuitively, the simulation smoother updates the states using the same principle as in the Kalman filter, where at each step filtered values are updated using the smoothed values obtained from backward recursion. For updating the initial states, using the state equation $X_{0|t,X_1} = F^{-1}(X_1)$ and $P_{0|t,P_1} = F^{-1}(P_1 + R'_1R_1)F'^{-1}$ can be written for the first observation. Given the mean $X_{t|t,X_{t+1}}$ and the covariance matrix $P_{t|t,P_{t+1}}$, the states can be sampled from $X_t \sim N(X_{t|t,X_{t+1}}, P_{t|t,P_{t+1}})$ for $t = 0, ..., T$.

Sampling of inflation volatilities, h_t

Conditional on the remaining model parameters, we can draw $h_{0:T}$ using standard Bayesian inference as in the case of X_t. One important difference, however, stems from the logarithmic transformation of the variance in (11). As the transformation concerns the error structure, the square of which follows a χ^2 distribution, the system is not Gaussian but follows a log-χ^2 distribution. Noticing the properties of log-χ^2 distribution, Kim et al. (1998) and Omori et al. (2007) approximate this distribution using mixture of Gaussian distributions. Hence, conditional on these mixture components the system remains Gaussian allowing for standard inference outlined above. For details, see Omori et al. (2007).

Sampling of structural break parameters, κ_t

Sampling of structural break parameters, κ_t relies on the conditional posterior of the binary outcomes, i.e. the posterior value in case of a structural break in period t and the posterior value of the case of no structural breaks. However, evaluating this posterior requires one sweep of filtering, which is of order $O(T)$. As this evaluation should be implemented for each period t the resulting procedure would be of order $O(T^2)$. When the number of sample size is large this would result in an infeasible scheme. Gerlach et al. (2000) propose an efficient algorithm for sampling structural break parameters, κ_t, conditional on the observed data, which is still of order $O(T)$. We implement this algorithm for estimation of the structural breaks and refer to Gerlach et al. (2000); Giordani and Kohn (2008) for details.

Sampling of state error variances, σ^2_η

Using standard results from a linear regression model with a conjugate prior for the variances in (22), it follows that the conditional posterior distribution of $\sigma^2_{\eta_i}$, with $i = 1, 2, 3, 4$ is an inverted χ^2 distribution with scale parameter $\Phi_{\eta_i} + \sum_{t=1}^{T} \eta^2_{i,t}$ and with $T + \nu_{\eta_i}$ degrees of freedom for $i = 2, 3, 4$ where Φ_{η_i} and ν_{η_i} are the scale and degrees of freedom parameters of the prior density. For $i = 1$ the parameters of the inverted χ^2 distribution becomes $\Phi_{\eta_1} + \sum_{t=1}^{T} \kappa_t \eta^2_{1,t}$ and $\sum_{t=1}^{T} \kappa_t + \nu_{\eta_1}$.

Sampling of marginal cost variance and correlation coefficient

To sample the variance of marginal cost and correlation coefficient, we decompose the multivariate normal distribution of $\epsilon_{\mathbf{t}}$ into the conditional distribution of $\epsilon_{2,t}$ given $\epsilon_{1,t}$ and the marginal distribution of $\epsilon_{1,t}$, as in Çakmaklı et al. (2011). This results in

$$\prod_{t=1}^{T} f(\epsilon_{\mathbf{t}}) = \prod_{t=1}^{T} \frac{1}{\sigma_{\epsilon_{1,t}}} \, \phi\left(\frac{\epsilon_{1,t}}{\sigma_{\epsilon_{1,t}}}\right) \frac{1}{\sigma_{\epsilon_{2,t}} \sqrt{(1-\rho^2)}} \, \phi\left(\frac{\epsilon_{2,t} - \rho\epsilon_{1,t}}{\sigma_{\epsilon_{2,t}}(1-\rho^2)}\right), \tag{26}$$

Hence, together with prior for the variance in (22), variance of the marginal cost series can be sampled using (26) by setting up a Metropolis-Hasting step using an inverted χ^2 candidate density with scale parameter $\sum_{t=1}^{T} \epsilon^2_{2,t}$ and with T degrees of freedom. To sample ρ from its conditional posterior distribution we can again use

(26). Conditional on the remaining parameters the posterior becomes

$$(1-\rho^2)^{-\frac{3}{2}} \prod_{t=1}^{T} \left(\frac{1}{\sqrt{(1-\rho^2)}} \, \phi\left(\frac{\epsilon_{2,t} - \rho\epsilon_{1,t}}{\sigma_{\epsilon_{2,t}}(1-\rho^2)} \right) \right). \tag{27}$$

We can easily implement the griddy Gibbs sampler approach of Ritter and Tanner (1992). Given that $\rho \in (-1,1)$ we can setup a grid in this interval based on the precision we desire about the value of ρ.

C Bayesian inference of the extended HNKPC model

Posterior inference of the HNKPC models with time varying parameters follow similar to Appendix B, using the Gibbs sampler with data augmentation. The HNKPC models with time varying parameters (HNKPC-TVP), with level shifts in inflation (HNKPC-TVP-LS), and with level shifts and stochastic volatility in inflation (HNKPC-TVP-LS-SV) in (9) and (12), and the inflation expectation specification in (14) can be cast into the state-space form in (22) using the following definitions

$$Y_t = \begin{pmatrix} \pi_t \\ z_t \end{pmatrix}, \; X_t = \begin{pmatrix} c_{\pi,t} & c_{z,t} & \mu_{z,t}, & c_{z,t-1} & c_{z,t-2} & S_t & c_{\pi,t-1} \end{pmatrix}', \; \epsilon_{\mathbf{t}} = \begin{pmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{pmatrix},$$

$$U_t = \begin{pmatrix} z_t & z_{t-1} & z_{t-2} & \mu_t & \pi_{t-1} \end{pmatrix}', \; B_t = \begin{pmatrix} \alpha_1 & \alpha_2 & 0 & -\alpha_3 & \alpha_4 \\ 0 & \phi_1 & \phi_2 & 0 & 0 \end{pmatrix},$$

$$H_t = \begin{pmatrix} 1 & -\alpha_1 & 0 & -\alpha_2 & 0 & \alpha_3 & -\alpha_4 \\ 0 & 1 & 0 & -\phi_1 & -\phi_2 & 0 & 0 \end{pmatrix}, \; Q_t = \begin{pmatrix} \sigma^2_{\epsilon_1,t} & \rho\sigma_{\epsilon_1,t}\sigma_{\epsilon_2} \\ \rho\sigma_{\epsilon_1,t}\sigma_{\epsilon_2} & \sigma^2_{\epsilon_2} \end{pmatrix},$$

$$F_t = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \beta & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \; R_t = \begin{pmatrix} \kappa_t\sigma_{\eta_1} & 0 & 0 & 0 \\ 0 & \sigma_{\eta_2} & 0 & 0 \\ 0 & 0 & \sigma_{\eta_3} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{\eta_5} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \; \eta_{\mathbf{t}} = \begin{pmatrix} \eta_{1,t} \\ \eta_{2,t} \\ \eta_{3,t} \\ \eta_{5,t} \end{pmatrix},$$

where parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ are defined as functions of the structural form parameters

$$\alpha_1 = \frac{\lambda}{(1 - (\phi_1 + \phi_2\gamma_f)\gamma_f)(1 - \gamma_b\gamma_f)}, \quad \alpha_2 = \frac{\lambda\gamma_f\phi_2}{(1 - (\phi_1 + \phi_2\gamma_f)\gamma_f)(1 - \gamma_b\gamma_f)},$$
$$\alpha_3 = \frac{\gamma_b\gamma_f}{(1 - \gamma_b\gamma_f)}\frac{\gamma_f}{(1 - \gamma_f\beta)}, \quad \alpha_4 = \frac{\gamma_b}{(1 - \gamma_b\gamma_f)}.$$

Given this setup, posterior inference can be carried out using the steps outlined in Appendix B.