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Car User Taxes, Quality Characteristics and Fuel Efficiency: Household Behavior and Market Adjustment

Bruno de Borger[1]
Jan Rouwendal[2]

[1] *Department of Economics, University of Antwerp, Belgium;*
[2] *Faculty of Economics and Business Administration, VU University Amsterdam, and Tinbergen Institute, The Netherlands.*

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Bruno De Borger and Jan Rouwendal (*)

Abstract

We study the impact of fuel taxes and kilometer taxes on households' choices of vehicle quality, on their demand for kilometers driven, and on fuel consumption. Moreover, embedding this information in a model of the car market, we analyze the implications of these taxes for the opportunity costs of owning cars of different quality. Higher quality raises the fixed cost of car ownership, but it may raise (engine size, acceleration speed, etc.) or reduce (fuel technology, etc.) the variable user cost. Our results show that kilometer charges and fuel taxes have very different implications. For example, a higher fuel tax raises household demand for more fuel efficient cars, provided that the demand for car use is inelastic; it reduces the demand for characteristics that raise variable user costs. Surprisingly, however, a kilometer tax unambiguously reduces the demand for more fuel efficient cars. Incorporating price adjustments at the market level, we find that fuel taxes raise the *marginal* fixed opportunity cost of better fuel efficiency at all quality levels. *Total* annual opportunity costs of owning highly fuel efficient cars increase, while they decline for cars of low fuel efficiency. We further find that both a fuel tax and a kilometer charge reduce the *total* annual fixed ownership cost for car attributes that raise the variable cost of driving (engine power, acceleration speed, etc.). There is thus in general a trade-off between fixed and variable car costs: if the latter increase – due to higher fuel prices or a kilometer charge – total demand for cars decreases and a return to equilibrium is only possible by a decrease in fixed costs. All theoretical results are illustrated using a numerical version of the model. The analysis shows that modeling the effect of tax changes on household behavior alone can produce highly misleading results.

(*) Bruno De Borger, Department of Economics, University of Antwerp, Prinsstraat 13, B-2000 Antwerp, Belgium (bruno.deborger@ua.ac.be); Jan Rouwendal, Department of Spatial Economics, VU University, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands, and Tinbergen Institute, Gustav Mahlerplein117, 1082 MS Amsterdam, The Netherlands (jrouwendal@feweb.vu.nl). Errors are our own responsibility.

1. Introduction

Many countries are revising their fiscal treatment of transport services to cope with the negative side-effects of road transportation. To tackle congestion and pollution, a variety of tax and pricing instruments have been considered in the academic literature. For example, congestion pricing has been extensively studied by Arnott, de Palma and Lindsey (1993), Verhoef, Nijkamp and Rietveld (1996), De Borger and Proost (2001), Parry and Bento (2001), and Van Dender (2003). The introduction of kilometer charges on freight transport was studied in Calthrop, De Borger and Proost (2007). The use of emission taxes, kilometer charges, and subsidies to fuel efficient vehicles was analyzed in, among others, Fullerton and West (2002), Fullerton and Gan (2005), and Fischer, Harrington and Parry (2007). The role of fuel taxes in coping with transport externalities has been investigated by Parry and Small (2005) and Bento, Goulder, Jacobsen and Von Haefen (2009).

In this paper we study, using both theoretical analysis and numerical simulation, the effect of different tax instruments on households' demand for quality characteristics of cars. We further analyze how these demand shifts -- through price adjustments on the car market – affect the annual opportunity cost of owning cars of different quality. Unlike the discrete choice literature, we assume that observable quality attributes can be selected continuously within a certain range of available qualities (for example, engine power, fuel efficiency, 'newness', etc.).[1] This allows us to study the effect of marginal increases in different taxes on the demand for different types of quality characteristics of cars. We distinguish between two prototypes of car quality. The first one refers to better fuel technology that increases fuel efficiency *per se*. As cars of higher fuel efficiency are more expensive, ceteris paribus, owning a more fuel efficient car raises the annual fixed ownership cost, but lower fuel use decreases the variable user cost. The second type includes car characteristics that make driving more comfortable, while decreasing fuel efficiency. They usually increase both fixed and variable cost. Examples are size, acceleration speed, weight, engine power, etc. For the first type of quality the trade-off is between fixed and variable cost, while for the second type

[1] In a discrete choice setting in which cars are also differentiated in unobserved (make-specific) aspects, any switch to a different car type implies a shift to another value of the unobserved quality characteristic. However, in a theoretical setting an exclusive focus on observable quality seems quite justified. Almost all car brands offer the possibility to buy slightly different versions of the same model, and the catalogue of possible combinations of characteristics is often large, approaching a situation in which slightly different values of one or a few characteristics can be chosen while keeping all others constant. Note that, apart from being more realistic in such cases, our approach avoids some peculiar and unattractive properties discrete choice models of the commonly used Generalized Extreme Value (GEV) type can have. See, for instance, the discussions in Berry and Pakes (2007) and Bajari and Benkard (2003).

the impact of better quality on the utility of driving must compensate for the higher fixed as well as variable cost.

Within this framework, the paper studies the impact of fuel taxes, kilometer taxes and taxes on car ownership on households' choices of vehicle quality, on their demand for kilometers driven, and on fuel consumption. Moreover, we investigate the impact of variable user taxes (fuel taxes, kilometer taxes) on the annual opportunity cost of owning various types of cars. The recent empirical literature convincingly argues that higher fuel prices raise the demand for high fuel economy vehicles, pushing up their relative prices (see, e.g., Klier and Linn (2009), Allcott and Wozny (2010)). These price adjustments suggest that the annual fixed opportunity cost of owning a car with given quality characteristics does depend on variable user taxes.[2] To study the effect of fuel and kilometer taxes on fixed ownership costs we embed the model of individual consumer behavior in a simple model for the car market. This allows us to model the adjustments in fixed annual ownership cost of cars of different quality that re-establish market equilibrium after a tax increase. As new car demand and scrappage of old cars constitute a small fraction of the car stock – and in line with recent empirical evidence -- our focus is on price adjustments on the second hand car market, where total supply is close to fixed in the short run.[3]

Of course, a substantial economic literature exists on modeling households' car ownership and car use decisions and on the implications of various types of transport tax reform. A large empirical discrete choice literature studies households' car ownership choices (including the type of car to own) and the associated demand for car kilometres. Early contributions include, among many others, Mannering and Winston (1985), Train (1986) and De Jong (1990). More recently, various authors have exploited advances in econometric methodologies to estimate more detailed empirical models of car ownership and car use, and used the results to shed some light on specific policy issues. For example, Hausman and Newey (1995), West (2004) and West and Williamson (2005) focus on the efficiency and

[2] Klier and Linn (2009) find that higher gas prices drives up prices of high fuel efficient vehicles. Demand responses are significant; the authors argue that nearly half of the decline in market share of U.S. manufacturers from 2002-2007 was due to the increase in the price of gasoline. Increases in the gasoline tax were found to have remarkably modest effects on average fuel economy of new cars. In line with intuition, changes in market share of cars of high fuel efficiency, due to increased production of such vehicles and scrappage of low fuel economy vehicles, attenuate price changes (Li, Timmins, and Von Haefen (2009), and Allcott and Wozny (2010)).

[3] For example, Busse, Knittel and Zettelmayer (2009) find that the adjustment of equilibrium market shares and prices in response to changes in usage cost varies dramatically between new and used markets. In the new car market, the adjustment is primarily in market shares, while in the used car market, the adjustment is primarily in prices. They explain the difference in how gasoline costs affect new and used automobile markets by differences in the supply characteristics of new and used cars.

distributive effects of gasoline taxes. In two related papers, Fullerton and Gan (2005) and Feng, Fullerton and Gan (2005) specify a model in which people choose first among large vehicle categories (small van, small regular car, etc), and then choose kilometres and age of the vehicle simultaneously, where fuel efficiency depends on age. To keep a linear budget constraint they transform age into 'wear' using a constant depreciation rate. The estimated empirical models are used to study the relative efficiency of different emission reduction policies such as an emission tax, a fuel tax, changes in annual registration fees, etc. A recent paper by Bento et al. (2009) incorporates the supply side of the car market into an analysis that distinguishes the market for new, used and scrapped vehicles; they reconsider the distributional effects of fuel taxes, distinguishing 256 types of cars. Finally, a number of papers have developed numerical simulation models to analyze the implications of taxing car use and/or car ownership. For example, Chia, Tsui and Whalley (2001) analyze the relative efficiency of taxes on car ownership and taxes on car use showing, not surprisingly, that the latter are more efficient to control externalities. Parry and Small (2005) construct a detailed simulation model to numerically evaluate the relative optimality of fuel taxes in the US and the UK. Recently, De Borger and Mayeres (2007) study the optimal taxation of the ownership and use of different types of cars, focusing on taxation of cars operating on different fuels (diesel and gasoline) [4].

Our findings in this paper show that kilometer charges and fuel taxes have very different implications, both for the demand for quality and for price adjustments on the car market. First, consider the demand for car attributes that improve fuel efficiency (say, better fuel technology). As expected, an increase in the fuel tax induces households to choose cars of better fuel efficiency, at least provided that the demand for car use is inelastic. Surprisingly, a higher kilometer tax reduces the demand for fuel efficiency. A general increase in fixed taxes on car ownership reduces the demand for better fuel technology although, obviously, lower marginal tax rates on more fuel efficient cars raise demand for better fuel technology. Second, looking at quality characteristics that raise variable user costs (engine size, acceleration speed,…), we show that higher fuel taxes plausibly induce households to demand cars of lower quality, whereas the effect of higher kilometer taxes is ambiguous. Higher non-

[4] The model studied in this paper is also related to two other strands of recent literature. One is the literature on aggregate but separate effects of policy parameters on the vehicle stock, fuel efficiency and energy use (Johansson and Schipper (1997), Small and Van Dender (2007)). Another link is to the literature on multiple discreteness, that has recently received substantial attention. See, in particular, Bhat (2005, 2008). These models were originally developed to deal with consumers who use varying amounts of different types of nondurable consumption goods, or participate in various activities for time spells of varying length; however, there have also been some applications to automobile demand (see, e.g., Bhat and Sen (2006) and Bhat (2008)).

differentiated fixed taxes raise the demand for such quality attributes. Third, different demand impulses of fuel taxes and kilometer charges also imply different adjustments on the car market. We find that fuel taxes raise the annual opportunity costs of owning very fuel efficient cars, while reducing ownership costs of cars of low fuel efficiency. The marginal fixed cost of better fuel efficiency on the market rises at all quality levels, but mostly so for the most fuel efficient cars. Both a fuel tax and a kilometer charge reduce the annual fixed ownership cost for car attributes that raise the variable cost of driving (engine power, acceleration speed, etc.). This holds at all quality levels.

Of course, this paper has a number of limitations. The model we develop considers the effects of taxes on the demand side of the market only. Like most other models (Bento et al. (2009) being an obvious exception) we ignore the supply side of the car market, assuming supply is fixed in the short-run. Moreover, unlike a discrete choice approach, our model is deterministic; it does not introduce randomness in preferences for unobserved characteristics to generate car ownership decisions. In the numerical simulations below, we use differences in income to generate differences in demand among households. Finally, note that we focus on illustrating the effects of taxes on household decisions; we neither attempt to design an optimal tax system, nor do we concentrate on the overall welfare effects of marginal tax changes.

The paper is structured as follows. In section 2 we develop a model of household choice of car quality and demand for kilometres, and we analyse the impact of different variable user taxes and fixed ownership taxes on household behaviour. Since tax changes affect the car ownership decision as well as the choice of car quality, they affect the equilibrium on the car market. In section 3 we therefore embed the choice model into a simple model of the car market. We use this model to investigate the changes in annual fixed ownership costs necessary to re-establish market equilibrium after a change in the fuel tax or the kilometre charge. Section 4 illustrates the theoretical findings using a numerical version of the model. In Section 5 we discuss extensions of the model. Section 6 concludes.

2. Taxes and the demand for car quality attributes

In this section, we present a simple theoretical model of individual household behavior. Although the car ownership decision is considered towards the end of this section, the main purpose of the model is to study the impact of various transport taxes on households’ choice of car quality attributes and on their demand for car use, conditional on car ownership[5]. We are specifically interested in the differential effects of fuel taxes and kilometer taxes on fuel efficiency and on the demand for kilometers (and, hence, on overall fuel consumption). Car attributes are summarized into two general quality characteristics that have very different implications for cars' fuel efficiency and for overall energy use[6]. The first one is a technological attribute k that improves fuel efficiency (call it 'fuel technology'), but it does not directly affect the utility of driving. The second one is a quality attribute m that does affect the utility of driving (engine size, engine power, car size, acceleration speed, etc.), but it raises fuel consumption and, hence, the variable cost per kilometer. Both characteristics also raise the fixed annual cost of car ownership, see below.

2.1. Structure of the model

We assume the consumer's choice of car quality characteristics and the number of kilometers to drive can be seen as the result of the following problem:

$$\max_{x,q,k,m} u(x,q;m) \quad s.t. \quad x + [f(k,m) + t(k,m)] + [p(\pi, k.m) + \tau]q = y$$

In this formulation, x is a general consumption good, q is kilometers driven by car, and k and m are the quality attributes described above; finally, y is exogenous income. The fixed annual cost of the car consists of the net-of-tax cost $f(k,m)$ plus the annual ownership tax payments $t(k,m)$. It is realistically assumed that both quality attributes raise the annual fixed cost: better quality cars are more expensive, and it is assumed that spreading this increase over the lifetime of the car is not fully compensated by, for example, lower annual maintenance costs. We specifically assume the fixed cost is increasing in quality at an increasing rate:

$$\frac{\partial f(k,m)}{\partial k} > 0; \frac{\partial f(k,m)}{\partial m} > 0; \frac{\partial f^2(k,m)}{\partial k^2} > 0; \frac{\partial f^2(k,m)}{\partial m^2} > 0$$

Note that, although the relative prices of different cars with different quality characteristics will respond to changes in taxes due to aggregate demand effects, in the short-run consumers treat the fixed ownership cost as exogenously given. The relation of the fixed annual tax payments *t(.)* and the quality indicators is left unspecified for the time being, see below. The

[5] We limit the discussion to households that have decided to own a single car. Extending the model to allow for multiple car ownership is conceptually straightforward, but it does substantially complicate the technical analysis. Apart from the potential effect of transport taxes on the relative intensity of using different cars (substitution between cars within the household), assuming multiple household car does not yield major additional insights in the topic we study in this paper. For information on empirical analyses of substitution between cars in the household, see De Borger, Mulalic and Rouwendal (2012)).

[6] Again, it is not difficult to generalize the model to more quality characteristics, but for the topic of this paper nothing is gained by doing so.

variable price per kilometer $p(\pi,k,m)$ gives the cost per kilometer as a function of the fuel price π (inclusive of fuel tax) and the quality indicators. As our focus will be on fuel efficiency and fuel use, we specify the variable cost as

$$p(\pi,k,m)=\pi e(k,m); \quad \frac{\partial e(k,m)}{\partial k}<0; \frac{\partial e(k,m)}{\partial m}>0$$

In other words, the variable cost is the fuel price π (inclusive of tax) times fuel use *e(k,m)* per kilometer. The function *e(k,m)* reflects the inverse of fuel efficiency; it declines in the fuel technology characteristic *k*, and rises in the quality characteristic *m*. Finally, as argued above, the government has the possibility to introduce a kilometer tax, denoted τ.

2.2. Taxes and the demand for car quality attributes

In this section, we study the impact of taxes on the demand for the two quality characteristics defined above. For pedagogic reasons, it will be useful to deal with the two quality attributes separately. The effect of the various taxes on both car characteristics jointly is considered in Appendix 1. It does not affect the main insights derived from the results described in this section. The sequential treatment offered here is more transparent, however.

2.2.1. Taxes and the demand for fuel-saving technology

Let us first focus on the effect of various taxes on the consumer's optimal demand for better fuel technology, as captured by *k*. In other words, we hold *m* constant and, for notational convenience, drop it from the analysis that follows.

Consider first the individual's choice of kilometers driven for a car with given fuel technology *k*:

$$\underset{x,q}{Max} \quad u(x,q) \; s.t. \quad x+\left[\pi e(k)+\tau\right]q=y-f(k)-t(k)$$

This leads to the 'short-run' demand function for car use

$$q^{SR}=q\left\{y-f(k)-t(k),\pi e(k)+\tau\right\}$$

Indirect utility, for a given quality *k*, is written as:

$$v^{SR}=v\left\{y-f(k)-t(k),\pi e(k)+\tau\right\}$$

Next, optimal quality choice is found by maximizing indirect utility with respect to *k*. We have the first-order condition (the second-order condition requires $\frac{\partial^2 v}{\partial k^2}<0$):

$$-\frac{\partial v}{\partial y}\left(\frac{\partial f}{\partial k}+\frac{\partial t}{\partial k}\right)+\frac{\partial v}{\partial p}\pi\frac{\partial e}{\partial k}=0 \tag{1}$$

This condition holds at the optimal quality *k**. Using Roy's theorem and rearranging, we find that (1) relates equilibrium demand at optimal quality to the effects of quality on fixed and variable costs as follows:

$$q\left\{y-f(k^*)-t(k^*),\pi e(k^*)+\tau\right\}=\frac{-\frac{\partial f}{\partial k}-\frac{\partial t}{\partial k}}{\pi\frac{\partial e}{\partial k}} \tag{2}$$

Interestingly, expression (2) implies a strong relationship between optimal demand for kilometers (on the left-hand side) and the technical relations that determine how quality affects the fixed and variable costs.

To analyze the effect of various taxes on optimal quality choice and optimal car use, we rewrite (2) and define Z_k as follows

$$Z_k=q\left\{y-f(k^*)-t(k^*),\pi e(k^*)+\tau\right\}\left(\pi\frac{\partial e}{\partial k}\right)+\frac{\partial f}{\partial k}+\frac{\partial t}{\partial k}=0 \tag{3}$$

The implicit function theorem then yields the effects of fuel and kilometer taxes on optimal quality choices:

$$\begin{aligned}\frac{dk^*}{d\pi}&=-\frac{Z_{k\pi}}{Z_{kk}}=-\frac{1}{Z_{kk}}\left[\frac{\partial e}{\partial k}\left(q+\pi e(k)\frac{\partial q}{\partial p}\right)\right]\\ \frac{dk^*}{d\tau}&=-\frac{Z_{k\tau}}{Z_{kk}}=-\frac{1}{Z_{kk}}\left[\pi\frac{\partial e}{\partial k}\frac{\partial q}{\partial p}\right]\end{aligned} \tag{4}$$

Here Z_{kr} denotes the derivative of Z_k with respect to *r*. It is easy to verify that the second-order condition of the consumer's optimization problem implies $Z_{kk}>0$.

The implications are clear. Since $\frac{\partial e(k)}{\partial k}<0$, a sufficient condition for a higher fuel tax to induce people to choose a car of better fuel technology is that the demand for car kilometers is inelastic. In choosing optimal fuel efficiency, consumers trade off the marginal benefits associated with lower variable costs versus the higher fixed cost of buying and owning a more fuel efficient car. At higher fuel prices, the marginal benefit of better fuel efficiency is affected while the marginal cost remains unchanged. The effect of fuel prices on the marginal benefit of better fuel efficiency has two dimensions. For a given number of kilometers, the marginal benefit of a more fuel efficient car rises when the fuel tax rises. However, the higher price per kilometer reduces demand, and this reduces the marginal benefit of a more fuel efficient car. Inelastic demand implies the first effect dominates and the consumer buys a more fuel efficient car. If demand is elastic, the decline in demand makes a more fuel efficient car less beneficial, and the fuel tax actually reduces fuel efficiency.

Next, consider the effect of a kilometer tax on fuel technology choices. Expression (4) implies that this effect is unambiguously negative: introducing kilometer taxes leads people to opt for less fuel efficient cars. The reason is that a kilometer tax reduces the demand for kilometers, making the marginal benefit of better fuel efficiency smaller while not affecting the fixed ownership cost increase to obtain better fuel technology.

Of course, we can also consider the effect of changes in fixed annual car taxes on quality choices and on demand for kilometers. To do so, it seems instructive to specify the fixed tax function more explicitly. Suppose we have a simple linear fixed tax structure that allows for the possibility to impose lower fixed taxes on more fuel efficient cars:

$$t(k) = t_0 - t_k k$$

We find the following effects of the tax function parameters on quality:

$$\frac{dk^*}{dt_0} = -\frac{Z_{kt_0}}{Z_{kk}} = -\frac{1}{Z_{kk}}\left[-\pi \frac{\partial e}{\partial k}\frac{\partial q}{\partial y}\right] < 0$$

$$\frac{dk^*}{dt_k} = -\frac{Z_{kt_k}}{Z_{kk}} = -\frac{1}{Z_{kk}}\left[\pi \frac{\partial e}{\partial k}\frac{\partial q}{\partial y} k\right] > 0$$

A higher general fixed tax t_0 reduces the quality of fuel technology and hence fuel efficiency. The reason is that, provided income effects are nonzero, the tax increase reduces the demand for kilometers; this makes the benefit of better fuel efficiency less important. A specific tax reduction for more fuel efficient vehicles leads to better fuel efficiency choices.

We summarize our findings so far in the following proposition.

Proposition 1: ***Consider a quality characteristic of cars that increases fuel efficiency and hence reduces variable user costs per kilometer (say, fuel technology).***

a. Introducing kilometer taxes unambiguously reduces the demand for fuel efficiency.

b. If demand for car use is inelastic, higher fuel taxes imply that households demand more fuel efficient cars.

c. A general increase in fixed taxes on car ownership reduces the demand for better fuel technology. Lower marginal tax rates on more fuel efficient cars raise demand for better fuel technology.

Two remarks conclude this section. First, qualitatively similar results are expected to hold for other quality indicators, such as 'newness' (i.e., the inverse of age), that also reduce the variable cost via lower fuel consumption and lower maintenance costs. The only difference is that, unlike a technological characteristic that raises fuel efficiency, newness does yield intrinsic utility for the typical consumer. However, as long as taxes not too strongly affect the marginal utility of driving in a newer car, we find qualitatively the same effects as in the case of pure fuel efficiency. So higher fuel taxes plausibly raise demand for newer cars, kilometer charges lead to more demand for older cars. Second, as repeatedly mentioned before, shifts in the demand for quality are likely to lead to price adjustments at the market level (these are studied in Section 3 below). However, if the supply of quality were perfectly elastic so that higher demand for quality could be obtained without affecting the fixed opportunity cost of owning a better car, Proposition 1 can be used to show the existence of rebound effects on demand. When the fuel tax increases, the direct negative effect on demand is partially counteracted by the impact the tax increase has on better fuel efficiency; the latter raises demand (see, for example, Small and Van Dender (2007) for an aggregate model of the rebound effect). A higher kilometer charge in fact generates an 'inverse rebound' effect: the direct demand effect of a kilometer charge is strengthened by the lower demand for fuel efficiency; this raises variable user cost and further reduces demand.[7]

2.2.2 Taxes and the demand for quality attributes that raise variable unit costs

The analysis for the quality characteristic m (for instance, weight, size, engine power, or acceleration speed) is to a large extent analogous to what we did in section 2.2.1, but there are a few important differences that we will emphasize. The characteristic m does yield intrinsic utility; moreover, it raises the variable cost per kilometer as well as the fixed annual ownership cost. We have $p = \pi e(m); \quad \frac{\partial e}{\partial m} > 0$.

Following the same steps as before, we now obtain the following first-order condition with respect to quality

[7] These results easily follow from defining demand at optimal quality $q = q\left\{y - f(k^*) - t(k^*), \tau + \pi e(k^*)\right\}$, differentiating with respect to the fuel tax, and using the results of Proposition 1.

$$-\frac{\partial v}{\partial y}\left(\frac{\partial f}{\partial m}+\frac{\partial t}{\partial m}\right)+\frac{\partial v}{\partial p}\pi\frac{\partial e}{\partial m}+\frac{\partial v}{\partial m}=0 \qquad (5)$$

As both the first and second terms are negative, the pleasure of owning and using a better car must yield sufficient extra intrinsic utility: $\frac{\partial v}{\partial m}$ >0. Using Roy's theorem we find, following the same steps as above (notation is analogous to what we used before):

$$Z_m = q\left\{y - f(m^*) - t(m^*), \pi e(m^*)+\tau, m^*\right\}\left(\pi\frac{\partial e}{\partial m}\right)+\frac{\partial f}{\partial m}+\frac{\partial t}{\partial m}-B=0; \quad B=\frac{\frac{\partial v}{\partial m}}{\frac{\partial v}{\partial y}} \qquad (6)$$

In (6), *B* is the intrinsic (i.e., holding kilometers constant) willingness to pay for extra quality.

We find the following effects of fuel and kilometer taxes on optimal quality choices:

$$\begin{aligned}\frac{dm^*}{d\pi} &= -\frac{Z_{m\pi}}{Z_{mm}} = -\frac{1}{Z_{mm}}\left[\frac{\partial e}{\partial m}\left(q+\pi e(m)\frac{\partial q}{\partial p}\right)-\frac{\partial B}{\partial \pi}\right]\\ \frac{dm^*}{d\tau} &= -\frac{Z_{m\tau}}{Z_{mm}} = -\frac{1}{Z_{mm}}\left[\pi\frac{\partial e}{\partial m}\frac{\partial q}{\partial p}-\frac{\partial B}{\partial \tau}\right]\end{aligned} \qquad (7)$$

In these expressions

$$\frac{\partial B}{\partial \pi}=\frac{\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial m\partial \pi}-\frac{\partial v}{\partial m}\frac{\partial^2 v}{\partial y\partial \pi}}{\left(\frac{\partial v}{\partial y}\right)^2} \qquad \frac{\partial B}{\partial \tau}=\frac{\frac{\partial v}{\partial y}\frac{\partial^2 v}{\partial m\partial \tau}-\frac{\partial v}{\partial m}\frac{\partial^2 v}{\partial y\partial \tau}}{\left(\frac{\partial v}{\partial y}\right)^2}$$

Consider the effect of the fuel tax. As more quality raises fuel use per kilometer (and we again have $Z_{mm} > 0$ by the second-order conditions), a sufficient condition for the first term between the square brackets on the right hand side of (7) to be positive is that demand for kilometers is less than unitary elastic. Moreover, as it probably feels better to have a large or powerful car when fuel prices are low, we may speculate that $\frac{\partial B}{\partial \pi}$ <0. If this is the case, it follows from (7) that a fuel tax increase reduces a quality indicator *m* that raises the variable cost.

Next, consider the effect of a kilometer tax on quality as defined here. The first term between brackets in the relevant equation of (7) is now negative. Unless the effect of the kilometer tax on the intrinsic value of a larger or more powerful car is largely negative ($\frac{\partial B}{\partial \tau}$ <0 and large in absolute value), a higher kilometer tax raises quality. In that case, people drive less because of the tax increase, but they do so in a better car. If the impact on the intrinsic value of better quality is largely negative, however, the opposite may hold.

Consider again a linear fixed tax structure:

$$t(m) = t_0 + t_m m$$

We can think of the tax slope parameter t_m as reflecting a higher fixed ownership tax on larger size or engine power. We find the following effects of the tax function parameters on quality:

$$\frac{dm^*}{dt_0} = -\frac{Z_{mt_0}}{Z_{mm}} = -\frac{1}{Z_{mm}}\left[-\pi\frac{\partial e}{\partial m}\frac{\partial q}{\partial y} - \frac{\partial B}{\partial t_0}\right]$$

$$\frac{dm^*}{dt_m} = -\frac{Z_{mt_m}}{Z_{mm}} = -\frac{1}{Z_{mm}}\left[-\pi\frac{\partial e}{\partial m}\frac{\partial q}{\partial y}m - \frac{\partial B}{\partial t_m}\right]$$

The fixed tax effects operate through two channels here. First, higher fixed taxes reduce net disposable income which reduces the demand for kilometers. This leads consumers to have higher demand for better quality cars; the better quality raises the variable price which reduces kilometer demand further. People opt for better cars driving fewer kilometers. Second, taxes affect the intrinsic willingness to pay for better quality. If the former effects dominate then higher fixed taxes (both t_0, t_m) raise quality indicators such as engine size that raise the variable price. Note that these effects hold, conditional on ownership. Fixed taxes do reduce the demand for car ownership, see below.

Summarizing, we have Proposition 2.

Proposition 2: ***Consider quality characteristics of cars that raise variable user costs (engine size, acceleration speed,…).***

- ***a.*** ***Higher fuel taxes then plausibly reduce households' demand for quality***
- ***b.*** ***The effect of higher kilometer taxes is ambiguous; it depends on how the tax affects intrinsic utility of having a better car.***
- ***c.*** ***Higher fixed taxes raise the demand for quality.***

2.3. The ownership decision

We model car ownership decisions in the simplest possible way. First, we assume that people not owning a car are public transport users. Denoting the public transport fare by p^{PT} and its quality by m^{PT} (this is intrinsic quality valued by the user), the demand for public transport – conditional on not owning a car -- follows from standard utility maximizing behavior. It results in indirect utility $v^{PT} = v(y, p^{pt}, m^{pt})$.

In the long-run, the consumer will use public transport if this yields higher utility than owning a car of optimal quality[8]. This condition can be written as

$$v^{PT}(y, p^{PT}, m^{PT}) > v^{LR}(y) = \underset{k,m}{Max}\, v^{SR}\left(y - f(k,m) - t(k,m), p(\pi,k,m) + \tau, m\right)$$

The left-hand side is the utility of using public transport, given the fare and service quality provided by the operator. The right-hand side can be interpreted as the long run indirect utility of owning a car; we will denote this in shorthand notation as $v^{LR}(y)$.

Provided that the utilities of public transport and car use satisfy a single crossing condition

$$\frac{\partial v^{PT}(y, p^{PT}, m^{PT})}{\partial y} < \frac{\partial v^{LR}(y)}{\partial y}$$

the solution of the above equation gives the critical income y^c such that all individuals with higher incomes than y^c are car owners, whereas all people with lower incomes use public transport. Note that this setup assumes, consistent with empirical evidence, that car ownership rises with income. Importantly, since taxes increase the cost of driving, this critical income is an increasing function of all tax parameters.

3. Taxes and equilibrium on the car market

In the previous section, we presented a simple model to analyze the implications of changes in taxes on car use and car ownership on households' choices of car quality characteristics and demand for kilometers. The model realistically assumed that from the viewpoint of the individual household, the annual fixed ownership cost function was fixed. Of course, changes in individual demands of households for cars of different qualities may lead to price adjustments at the market level. In the short-run, the total car stock cannot substantially adapt to the pressures of higher user taxes (as new car sales and scrappages are a

[8] Technically, the individual has selected the optimal values of both quality characteristics *k* and *m*.

very small part of the total car market) and there is a distortion to the original equilibrium. Consequently, fuel prices and kilometer charges will lead to changes in the prices of cars on the second-hand market and, hence, to changes in the annual opportunity cost of owning different types of cars. As noted in the introduction, there is strong empirical evidence that rising fuel prices affect prices of cars of different fuel efficiency on the second-hand car market so as to re-establish market equilibrium (see Busse et al. (2009)). Some car types will become more expensive, others cheaper.

To study these processes, in this section we embed the model of individual consumer behavior developed in Section 2 in a simple model of the car market. We assume that the car stock is currently allocated to a population of heterogeneous consumers that differ only in income y. Moreover, the model assumes the car stock is unaffected by the tax changes. As observed before, although the tax changes possibly affect the demand for new cars and the scrappage of old cars, these changes have such a small impact on the total car stock that we treat it as negligible. Within this simple framework our interest is to find the fixed opportunity cost function *f(.)* that equilibrates demand for cars of all kinds of qualities with the existing stock, and to analyze how this function adjusts after a change in taxes so as to re-establish market equilibrium. For what follows, we need the assumption that car kilometers are a normal good. We further assume that fuel efficiency and the ownership tax are convex in k and m. As in the previous section, for pedagogic reasons we assume initially that cars differ only in one-dimensional quality (k or m).[9]

3.1 Fuel saving technology *k*

To get started, the income distribution is denoted $G(y)$, and the car stock is described by a distribution function $H(k)$. Incomes and car qualities belong to closed intervals $Y = [y^{min}, y^{max}]$ and $K = [k^{min}, k^{max}]$. The total number of households is $G(y^{max})$, and the total car stock is $H(k^{max})$. We take the stock of cars as given and assume that there are more households than cars.

Rewrite the first order condition (1), using Roy's identity, as:

$$\frac{\partial f}{\partial k} = -\frac{\partial t}{\partial k} - q^{SR}(y - f(k), \pi e(k) + \tau)\pi \frac{\partial e}{\partial k} \qquad (8)$$

This condition equates the marginal cost of better quality (higher fixed cost) to the marginal benefit (tax savings plus fuel expenditure savings). The left-hand side of this equation can

[9] A generalized version in which several of the above assumptions are relaxed is discussed below (see Section 4.3).

therefore be interpreted as the maximum the consumer is willing to pay to realize a marginal quality increase. Since the demand for car kilometers is increasing in income, this maximum is itself increasing in income. It then easily follows that, given our assumptions on the functions *t(.)* and *e(.)* mentioned above and given a convex fixed cost function f , consumers with a higher income will demand a car of higher quality.[10] Hence, there must be an increasing one-to-one relationship between household y and car quality k. We denote this 'matching function' as $y(k)$.

This result has two consequences. First, cars will be owned by the households with the highest incomes, so that we can find the critical income y^c previously defined (see Section 2.3) from the equation $G(y^c) = G(y^{max}) - H(k^{max})$. Second, for every value of $k \in K$ we can determine $y(k)$ -- i.e., the income of the household that will own the car of this particular quality -- from the equation $G\big(y(k)\big) = G(y^c) + H(k)$, or $y(k) = G^{-1}\big(G(y^c) + H(k)\big)$. The matching function is determined only by the income distribution and the car stock. Put differently: as long as the income distribution and the car stock do not change, the equilibrium choice of car quality of a household will not change. Any change in the demand for car quality that follows from a change in one of the tax parameters must be balanced by a change in the fixed cost function.

After substitution of $y = y(k)$ in the first order condition (8), we find:

$$\frac{\partial f}{\partial k} = -\frac{\partial t}{\partial k} - q^{SR}\Big(G^{-1}\big(G(y^c) + H(k)\big) - f(k) - t(k), \pi e(k) + \tau\Big)\pi\frac{\partial e}{\partial k}. \qquad (9)$$

This is a differential equation in *f(.)*. Its solution gives us the fixed opportunity cost (of owning cars of different qualities) that equilibrates the car market as a function of the tax parameters.[11] We therefore write the solution as $f(k;\pi,\tau)$; it describes how the fixed opportunity cost function adapts in order to restore market equilibrium after a tax change. A closed form solution of (9) is in general not available, but provided we have an initial value, $f(k;\pi,\tau)$ can under general conditions be found by numerical methods (see, e.g., Judd (1998)). In Appendix 2 we show that an initial condition is readily available.

The method just described will be used to recover $f(k;\pi,\tau)$ for a numerical application of the model. That is, we will solve (9) and compare an initial market equilibrium with

[10] Note that fuel efficiency is decreasing in *k*.

[11] It is obvious from (9) that this solution guarantees that for all consumers the first-order condition for utility maximization (1) is satisfied at the car quality that is allocated to them. It may also be noted that our setup implies that the critical income must be the same before and after the tax change. Since we have implicitly assumed that the utility of using public transport does not change, the utility of a household with the critical income remains unchanged. Of course, one could easily use the model to study the effect of changes in public transport prices on fixed car costs. Alternatively, simultaneous changes in public transport fares and car taxes could be studied.

alternative equilibria that result from a higher fuel tax or a kilometer charge. We again expect different cost adjustments in response to the two different tax instruments. To see why, note that (9) can be rewritten in simplified notation as $\frac{\partial f}{\partial k} = -\frac{\partial t}{\partial k} - EXP\frac{\partial e}{\partial k}$, where *EXP* denotes expenditure on fuel at the equilibrium assignment of cars to households. Both a higher fuel tax and a higher kilometer charge lead to a decrease in the demand for car kilometers. However, as higher kilometer charges reduce demand, they lead to an unambiguous *decrease* in fuel expenditures *EXP*, whereas the higher fuel tax will lead to *increased EXP* if the demand for car kilometers is price inelastic (as suggested by all empirical evidence). This is a clear difference between the impact of the two tax instruments on the left-hand-side of (9). Of course, the previous argument ignored the fact that *EXP* depends itself on fixed car costs, which will be affected by tax changes. However, it does suggest that the two taxes have a qualitatively different impact on the fixed cost function. In view of the above discussion, noting that $\frac{\partial e}{\partial k} < 0$, a kilometer charge is more likely to reduce the marginal fixed cost ($d\frac{\partial f}{\partial k}/d\pi < 0$); a fuel tax is more likely to raise it ($d\frac{\partial f}{\partial k}/d\tau < 0$).

Finally, consider the change in the fixed cost function that results from a change in the ownership tax $t(k)$. Given the structure of our model, equation (9) tells us that in equilibrium any change in the car ownership tax $t(k)$ must be completely compensated by a change in the fixed cost.

3.2 Quality attributes that increase variable car user cost

The analysis for the case where quality increases the variable user cost is conceptually similar, except that we have to take into account that characteristic *m* is an argument of the utility function. The relevant first-order condition is (5), which we rewrite as:

$$\frac{\partial f}{\partial m} + \frac{\partial t}{\partial m} = q^{SR}(y - f(m) - t(m), \pi e(m) + \tau)\left(-\frac{\partial v/\partial m}{\partial v/\partial p} - \pi\frac{\partial e}{\partial m}\right) \quad (10)$$

The left-hand side is the marginal gross fixed cost of better quality; it consists of a larger annual fixed cost and, plausibly, larger fixed tax payments. The right-hand side is the net marginal benefit. For this to be positive, the consumer must value the increase in quality enough to be willing to bear the additional variable cost. Technically, it must be the case that $-\frac{\partial v/\partial m}{\partial v/\partial p}$ -- which gives the marginal increase in variable cost that keeps the consumer on the same indifference curve after a change in car quality -- exceeds $\pi\frac{\partial e}{\partial m}$.

Assuming the existence of an increasing matching function $y(m)$, substitution in (10) yields a differential equation in car quality m that can be solved numerically in the same way as (9).[12]

With respect to the ownership tax it is again the case that for the market equilibrium only the gross fixed cost matters. Hence a higher ownership tax will imply that in market equilibrium depreciation, capital cost and other fixed cost must decrease by the same amount to keep gross fixed cost constant.

4. Numerical illustration

We illustrate the theoretical results derived in the previous sections using a simple numerical representation of the model. The model is a generic numerical exercise not intended to be a fully realistic representation of any particular given situation, but sufficiently rich to capture all major ingredients of the effects of the tax changes considered. We explain the model in some detail for the case of a fuel-saving technology; the case of attributes that raise the variable user cost is more briefly dealt with towards the end of this section. Extensions are discussed in Section 5 below.

4.1 Fuel saving technology

We proceed in three steps. We present the specification of the model of individual household demand used in the numerical exercise. Next we illustrate the implications of increases in fuel taxes and of imposing kilometer charges on the demand for car quality characteristics. Finally, we consider the changes in the fixed annual opportunity costs of car ownership that establish a new market equilibrium.

4.1.1 Specification of the model: functional forms and reference parameters

[12] This an be shown when $-\frac{\partial v/\partial m}{\partial v/\partial p}$ is non-decreasing in *y*, both *f* and *t* are convex in *m*, and $\left(-\frac{\partial v/\partial m}{\partial v/\partial p} - \pi\frac{\partial e}{\partial m}\right)$ is decreasing in *m*. The latter condition is satisfied when $-\frac{\partial v/\partial m}{\partial v/\partial p}$ is non-increasing in *m* and *e* is convex in *m*. These appear to be reasonable conditions, which we assume to be valid.

Our specification was based on the desire to use a simple functional form for the demand equation and parameter values that imply elasticities that are close to empirical estimates. Our basic assumption is that the demand function is linear:

$$q^{SR} = \alpha + \beta(\pi e(k) + \tau) + \gamma(y - f(k) - t(k)).$$

Hausman (1981) has shown that the indirect utility function implied by a linear demand curve is:[13]

$$v = \left(\frac{\beta+\alpha\gamma+\beta\gamma(\pi e(k)+\tau)+\gamma^2\left(y-f(k)-t(k)\right)}{\gamma^2}\right) e^{-\gamma(\pi e(k)+\tau)}$$

We also assume that inverse fuel efficiency is linear in quality:

$$e(k) = \theta_0 + \theta_1 k \text{ with } \theta_1 < 0.$$

It will be shown later on in this paper that these assumptions, in combination with uniform distributions of car quality and income implies the following annual fixed cost function:

$$f(k) = A + Bk + Ce^{D\left(k-k^{min}\right)}$$

where *A*, *B*, *C* and *D* are constants, and $k^{\min}$ is the minimum car quality on the market.

Some comments are usefulat this point. It is not difficult to verify that the annual fixed ownership cost is increasing in k when $B + CDe^{D\left(k-k^{min}\right)} > 0$ and convex when $C > 0$. Moreover, note that when $k = k^{min}$we have $f(k^{\min}) = A + Bk^{\min} + C$. At this fixed cost, the parameters must be such that the household is indifferent between having a car and using public transport, while the marginal cost *B* is determined by (9). Therefore, the value of *A+C* must be such that these two requirements are met. In the numerical exercise below, the parameters will be chosen such that all the above conditions are satisfied.

Consumers without a car use public transport. The demand for public transport q^{PT} of these consumers is determined by the same tastes as that for cars. That is, we treat public transport as if it is a car without fixed annual cost, but with a lower quality and a different value of the variable cost, p^{pt}. The demand for kilometers driven by public transport thus contains a quality indicator m^{pt}which takes on a negative value:

$$q^{PT} = \alpha + \beta p^{pt} + \gamma(y) + \delta m^{pt}$$

where p^{pt} denotes the variable cost of public transport. The associated indirect utility of public transport users is, of course, similar to that of car users:

$$v^{PT} = (\alpha\gamma + \beta + \beta\gamma p^{pt}) + \gamma^2 y + \delta\gamma m^{pt})\frac{1}{\gamma^2} e^{-\gamma\left(p^{pt}+(\delta/\beta)m^{pt}\right)}.$$

The parameter δ is positive, so demand for public transport is increasing in quality.[14]

[13] It is easy to verify, through Roy's identity, that the demand for kilometers implied by this indirect utility function is indeed linear.

The parameter values that will be used in the simulations are listed in Table 1. A number of assumptions underlie the choice of the parameter set. Consider the (inverse) fuel efficiency function. We let the quality indicator k vary continuously between 0 and 10. Fuel use per kilometer driven is assumed to equal 0.20 (or 20 liters per 100 kilometer) for cars with the lowest quality (k^{min}=0); hence, we set $\theta_0 = .2$. We further choose $\theta_1 = -0.015$, which implies that fuel use decreases to 5 liter per 100 kilometer for cars with the highest quality (k^{max}=10). This figure is quite realistic as an estimate of average fuel consumption (averaging over urban and non-urban road sections) for the most fuel efficient cars on the market.

The demand parameters were determined such that the price elasticity of demand for car kilometers is equal to -0,10 and the income elasticity for a specific reference case equal to 0.5 (viz., when income equals €50.000, fixed costs equals €5,624, 15,000 kilometers are driven annually, variable cost is €0.17 and quality equals 5.84). We let income vary between €10,000 and €75,000. The chosen values of the parameters imply that the critical income y^c equals €15,000. At this critical income a car of the lowest quality (k=0) is optimal, while at the highest income the highest quality (k=10) is chosen.

The fixed cost function was specified increasing and convex in k, see above. Moreover, to keep the interpretation as transparent as possible, parameters were calibrated so as to make the relation between demand for quality and income linear[15]. The parameter values that achieve this are also given in Table 1.

4.1.2. The effects of tax changes on demand for car quality

We use the model specified above to study the impact of various tax changes on the demand for car quality. For simplicity, we assume that in the reference situation, the ownership tax t and the kilometer charge τ are equal to zero, and we let the initial fuel price be equal to 1.5. The marginal fixed cost of better quality increases from slightly more than €150 for k=0 to well over €300 for k=10. The annual number of kilometers driven is slightly higher than 10,000 for consumers with the critical income level (€ 15,000), and it increases to more than 19,000 for consumers with the maximum income (€ 75,000). The effect of the higher income is mitigated by the increase in fixed car cost.

Higher income raises the demand for kilometers, but also leads to higher quality demand and, hence, a higher fixed cost. Since the fixed cost is nonlinear in quality (which is

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[14] It is not difficult to verify that the single crossing condition is satisfied when $p^{pt} + (\delta/\beta)m^{pt} > \pi\, e(k)$; parameters are such that this is the case for any admissible value of car quality k.

itself linearly related to income) this effect is stronger for the higher incomes. Car quality itself has a limited positive effect on the annual number of kilometers driven: one additional unit of quality implies that some 120 extra kilometers will be driven per year.

Table 1 Numerical values for the simulations with one-dimensional quality

Parameter	Interpretation	Numerical value
α	Intercept demand function	9,547
β	Price sensitivity of demand	-9,375
γ	Income sensitivity of demand	0.15
$\boldsymbol{\delta}$	Quality sensitivity of demand	240
θ_0	Intercept of fuel use function	0.20
θ_1	Quality sensitivity of fuel use	-0.015
A	Parameters of the cost function	-2.2827×10^6
B		7906.25
C		2.2867×10^6
D		-0.003375
y^{max}	Maximum income	75,000
y^{min}	Minimum income	10,000
k^{max}	Maximum quality	10
k^{min}	Minimum quality	0
p^{pt}	Price per km of public transport	0.05
m^{pt}	Quality indicator public transport	-50

We first analyze the implications of an increase in fuel taxes. Starting from the reference situation, we investigate what happens if a tax increase raises the fuel price by 20% (from €1.5 to €1.8). The theory showed that this raises the demand for better fuel efficiency, and the simulation results reported in Figure 1 suggest that these effects may be substantial. The curve denoted ‘original price’ gives the demand for quality as a function of income before the price increase. The curve denoted ‘high price’ is the simulated relation between quality and income after the increase in fuel tax.[16] In line with expectations, we see that there is now excess demand for cars of the highest quality, whereas demand for cars of the lowest qualities disappears completely. More specifically, we find that cars of the highest available quality (*k*=10) are now demanded by all consumers with an income of €54,807 or more. Since car driving becomes more expensive, the critical income shifts upwards to €18,488. As a consequence, cars of the lowest qualities (below *k=2.0*) are no longer demanded. Note that demand increases are most pronounced at the high income levels.

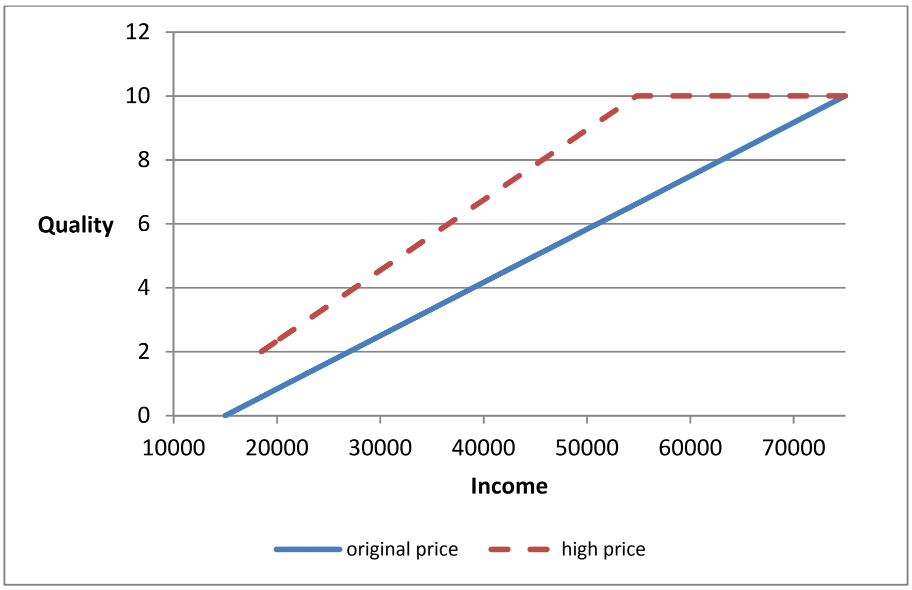


Figure 1. The shift in demand for fuel-saving technology: an increase in the fuel price

Next, let us see what happens if we keep the fuel price at its reference level (€1.50) and introduce a kilometer charge. An increase in the fuel price of €0.30 implied an increase in the variable cost of approximately €0.0375 for the average car of quality *k*=5; to make the results comparable to those found for the fuel tax increase, we set the kilometer charge at €0.0375.

As convincingly argued in the theoretical sections of the paper, the resulting shift in demand is indeed quite different from that implied by the fuel tax change. A kilometer tax leads to a decline in the demand for fuel efficiency enhancing technology. This is clearly illustrated in Figure 2. Although the changes in quality demand are modest, demand declines for car owners at all income levels. The implication is that demand for the top quality cars disappears. We find that the new critical income level is € 17,344 and at this income the optimal quality level is 0. This implies a decrease in optimal quality of 0.39. At the highest income of € 75,000, optimal quality is 9.6, which means that demand for the highest quality levels (k>9.6) disappears. Together these findings imply that more households would decide to no longer own a car at the prevailing fixed annual cost function.

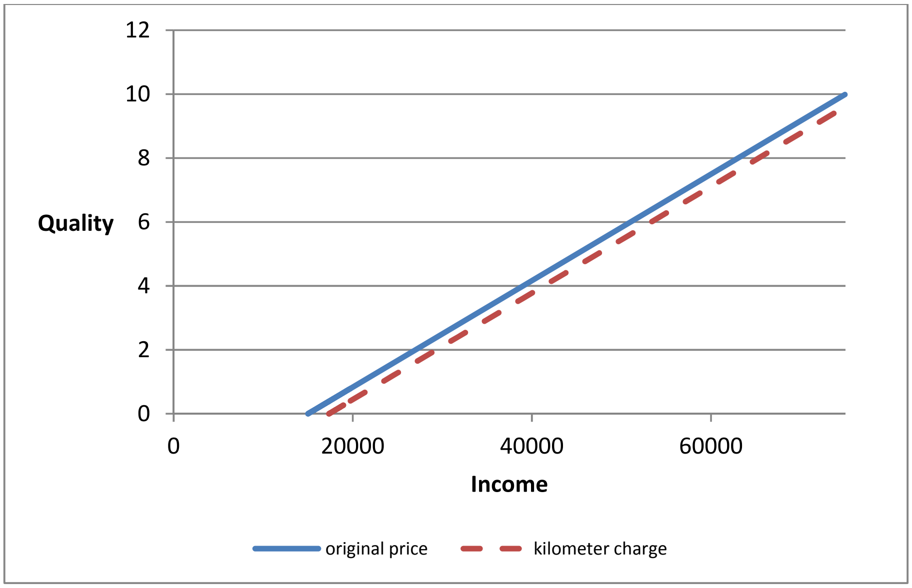


Figure 2. The shift in demand for quality *k* (fuel-saving technology) caused by a kilometer charge

4.1.3. Taxes and price adjustments at the market level

The simulations we just presented show that increases in taxes have substantial effects on the demand for car characteristics by individual households. We suggested above that, since the total car stock is given in the short run (as new car sales are a small part of the total car market), these demand adjustments may imply a substantial distortion in the original equilibrium. In this section, we illustrate the adaptations in the annual fixed opportunity cost of owning different quality cars that restore market equilibrium.

As the numerical model only serves illustrative purposes, we use very simple income and quality distributions. Specifically, we assume G and H are both uniform:

$$G(y) = \frac{y - y^{min}}{y^{max} - y^{min}}, \quad H(k) = \frac{k - k^{min}}{k^{max} - k^{min}}.$$

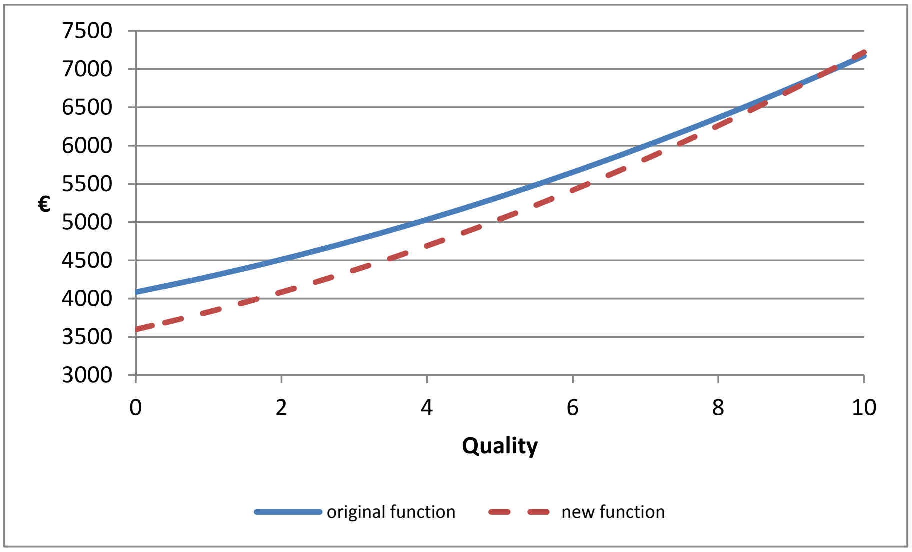
It then follows that $G^{-1}\big(G(y^c)+H(k)\big)=y^c+\big(k-k^{min}\big)\big(y^{max}-y^{min}\big)/\big(k^{max}-k^{min}\big)$. In combination with the linear demand equation specified before, differential equation (9) becomes:

$$\frac{\partial f}{\partial k}=-\frac{\partial t}{\partial k}-(\alpha+\beta p(k)+\gamma\left(y^c+\big(k-k^{min}\big)\frac{(y^{max}-y^{min})}{(k^{max}-k^{min})}-f(k)-t(k)\right)\pi\theta_1$$

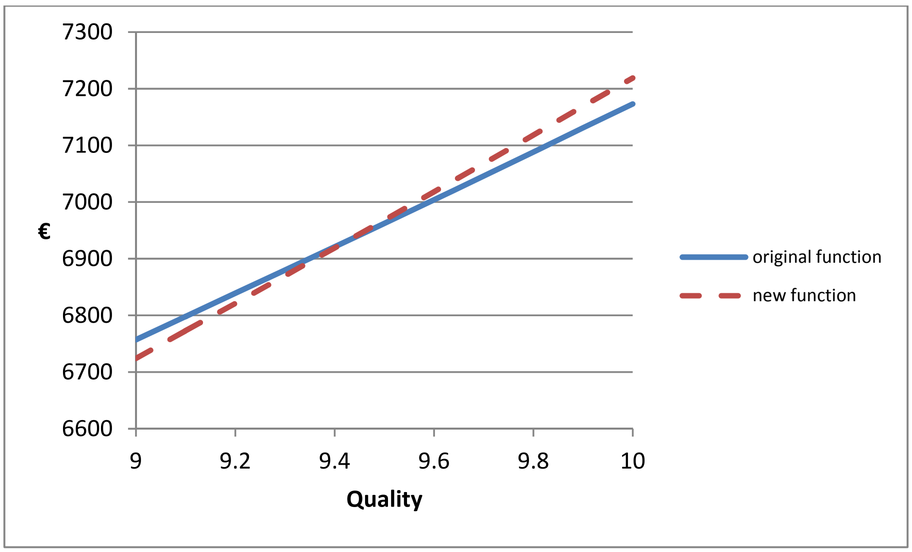
In Appendix 3 we discuss the solution of this differential equation for *f(k;* π,τ *)*. Here we just report the results of using *f(k;* π,τ *)* to study the impact of tax changes on the equilibrium opportunity cost of owning cars of different qualities.

Consider the increase in the fuel price from 1.5 to 1.8 euro we looked at before. There are three aspects to the adjustment in the fixed ownership opportunity cost function. First, the higher demand for fuel efficiency at all income levels we found before (see Figure 1) raises the relative demand for more fuel efficient cars. For cars of the highest quality, there was excess demand at the original cost function. Second, the higher marginal value of fuel saving makes the cost function steeper. Third, the car stock is fixed, so that the lower part of the quality distribution must necessarily become cheaper.

Figure 3a shows the final result of the adjustment; it gives *f(.)* in the initial and in the new equilibrium. It shows indeed a downward shift in the fixed opportunity costs over quite an extended range of quality levels. Fixed costs of owning the best quality cars increase. This is also clear from Figure 3a; Figure 3b shows it more in detail. Finally, note from Figure 4 that the marginal fixed cost of a change in quality increases at all levels of quality, but more so for the highest qualities. The marginal fixed cost also increases at relatively low quality to prevent the realization of the upward shift in quality that results from the higher returns to better fuel efficiency.



a) Shift in equilibrium fixed cost *f(k)* due to fuel tax increase (whole quality range)



b) Shift in equilibrium fixed cost *f(k)* due to fuel tax increase (highest qualities)

Figure 3. Equilibrium fixed cost functions (fuel saving quality) before and after a fuel tax increase

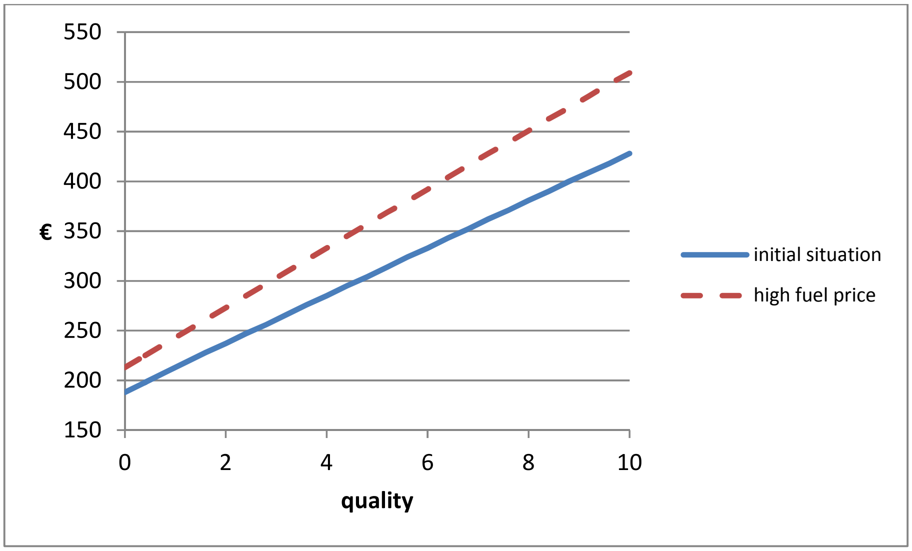


Figure 4. Marginal fixed cost before and after the fuel tax increase

Figures 5 and 6 illustrate the adjustment to a new market equilibrium after imposing the kilometer tax discussed in the previous section. At given fixed costs, this was shown before to reduce the demand for fuel efficiency so that one expects a downward adjustment in market prices towards a new equilibrium. Figure 5 indeed illustrates this downward shift. Marginal fixed costs decrease slightly, consistent with the discussion in Section 3.1. The decrease is virtually independent of the quality level *k*.

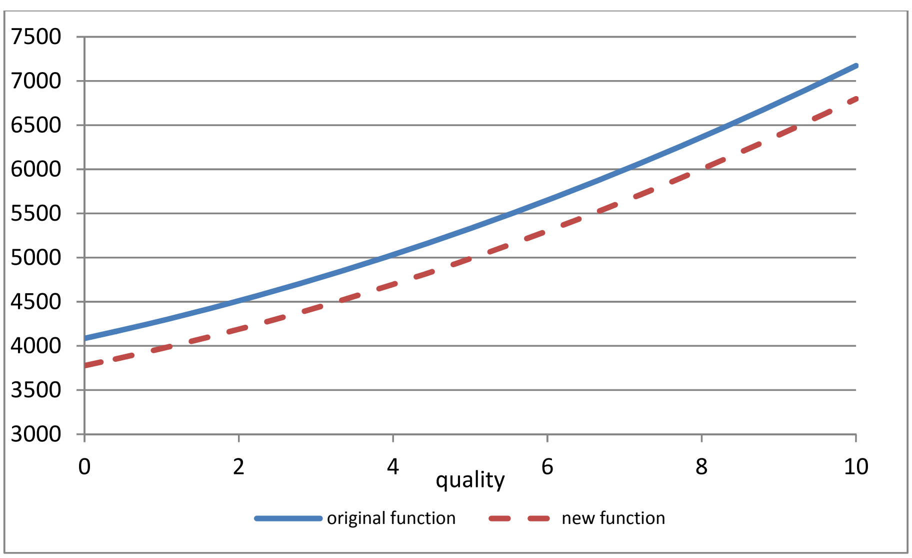


Figure 5. Equilibrium fixed cost functions (fuel saving quality) before and after introduction of a kilometer charge

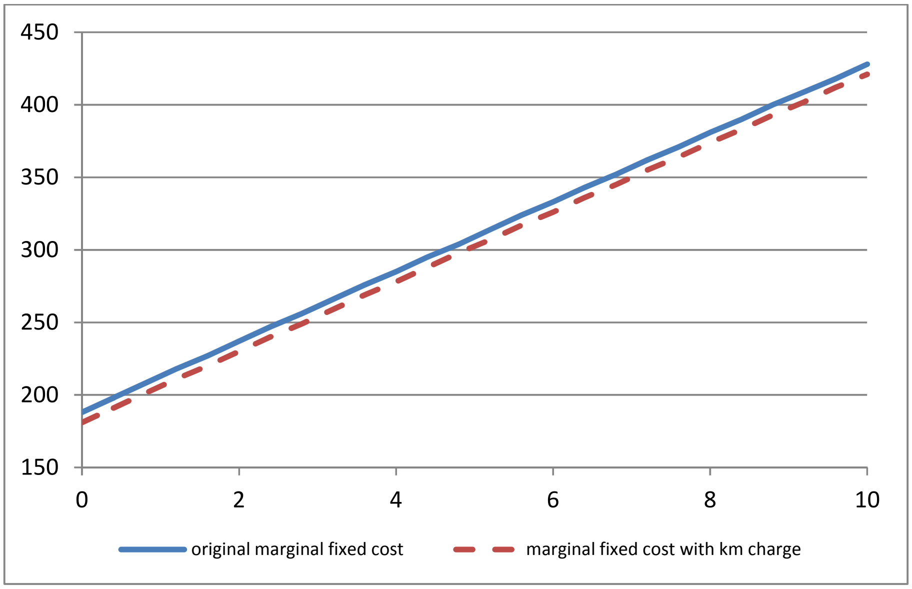


Figure 6. Marginal fixed cost function (fuel saving quality)
in equilibrium before and after introduction of a kilometer charge

Finally, we provide some numerical results of the simulations in Table 2. It should be kept in mind that there is large heterogeneity in the effects of the tax changes. For households with the critical income, a fuel tax increase is compensated exactly by the reduction in fixed cost, whereas households with the highest incomes experience an increase in fixed annual costs on top of the increase in variable costs. With a kilometer charge, all households experience some compensation for the higher variable cost through the induced change in fixed opportunity cost. Table 2 therefore illustrates the implications of the fuel tax and the kilometer charge at three income levels: the critical income level (the income for the marginal car owner), average income and the highest income considered.

This leads to several observations. First, it confirms that a fuel price increase raises variable costs but reduces fixed annual opportunity costs, except for the high income users of the most fuel efficient cars; the demand increases for these cars push annual fixed opportunity costs of ownership upward. The marginal fixed cost of a car of rises at all quality levels, but mostly so for the most fuel efficient car types. Note that total annual costs (variable plus fixed) for the least fuel efficient cars actually go down: these cars become cheaper in terms of ownership cost, and they are used for substantially fewer kilometers. Second, the table illustrates that kilometer taxes have a smaller effect on demand for kilometers and on fuel consumption, especially for the least fuel efficient cars. Third, the results illustrate that fuel

Table 2. Implications for drivers with low, average and high income (fuel saving quality)

Income y	15000	45000	75000
Initial situation			
Quality k	0	5	10
Kilometer demand q	8,371	13,739	19,018
fuel efficiency	0.200	0.125	0.050
total fuel use	1,674	1,717	951
total variable cost	2,512	2,576	1,426
fixed cost	4,085	5,330	7,173
marginal fixed cost	188	309	428
total cost	6,597	7,906	8,590
Increase in fuel Price			
Quality k	0	5	10
Kilometer demand q	7,882	13,432	18,870
fuel efficiency	0.200	0.125	0.050
total fuel use	1,576	1679	944
total variable cost	2,838	3022	1698
fixed cost	3,598	5038	7219
marginal fixed cost	213	363	509
total cost	6,435	8,060	8,918
compensating variation ex ante	-488	-510	-285
compensating variation ex post	0	-218	-330
Kilometer charge			
Quality k	0	5	10
Demand for kilometers q	8,066	13,440	18,723
fuel efficiency	0.200	0.125	0.050
total fuel use	1,613	1,680	936
total variable cost	2,722	3,024	2,106
fixed cost	3,777	4.988	6,798
marginal fixed cost	181	302	421
total cost	6,500	8,012	8,904
compensating variation ex ante	-308	-510	-709
compensating variation ex post	0	-168	-333

taxes (except for most fuel efficient cars) and kilometer charges both reduce fixed opportunity costs. However, in line with theory, a fuel tax raises the marginal fixed cost of better fuel efficiency, a kilometer tax reduces it. Moreover, the fixed annual costs of the most fuel efficient cars decline much less in the case of a kilometer charge. Fourth, note that the cost adaptations at the market level imply that tax changes have fairly limited effects on demand and fuel consumption.

Finally, we also calculated the compensating variation as an indicator of the welfare effects of the fuel price increase and the kilometer charge. As suggested by Table 2, they are quite large when computed *ex ante* (before the reaction in the fixed opportunity cost is taken into account), ranging between 300 and 700 euro. They are much larger for a kilometer tax that for a fuel tax for those car drivers owning highly fuel efficient cars; the opposite holds for low income individuals driving cars that are not very fuel efficient. When calculated *ex post* (after the fixed cost adjustment) the compensation variations are substantially smaller, except in the case of high income people when the fuel tax is increased. They range between 0 at the critical income to about 350 euro.

4.2 Variable cost increasing in quality

To simulate the effect of tax changes on the demand for quality characteristics that yield direct utility to the driver but raise the variable cost per kilometer (acceleration speed, engine power, etc.), we have to adapt the model on two accounts. First, we now introduce car quality *m* into the (indirect) utility function as follows:[17]

$$v^{SR} = \left(\alpha\gamma + \beta + \beta\gamma(\pi e(m) + \tau) + \gamma^2\big(y - f(m)\big) + \delta\gamma m\right)\frac{1}{\gamma^2}e^{-\gamma\left((\pi e(m)+\tau)+(\delta/\beta)m\right)}$$

The demand function is then

$$q^{SR} = \alpha + \beta(\pi e(m) + \tau) + \gamma(y - f(m) - t(m)) + \delta m.$$

The value of δ already reported in Table 1 implies a modest effect of quality on the demand for kilometers: an increase of one point implies that annually 240 km per year more will be driven.

Second, we have to adapt the parameters of the variable and fixed cost functions. The parameters of the variable cost function $e(m) = \theta_0 + \theta_1 m$ are now both positive, because better quality raises fuel consumption. We specify the parameters so that, for the car with average quality, fuel use remains unchanged to what it was in the previous subsection. The parameters of the fixed opportunity cost function have been chosen in such a way that the allocation of quality to incomes is initially identical to that of the previous subsection. That is, for

[17] This means that we introduce quality of cars in the same way as we did already for public transport (see 4.1.1).

consumers with a particular income *y* the optimal quality *m* has the same value as the optimal quality *k* associated with that income in the previous subsection. This implies, among other

Table 3. Numerical values for the variable cost function (fuel-using quality)

θ_0	Intercept of fuel use function	0.075
θ_1	Quality sensitivity of fuel use	0.010
A	Parameters of the cost function	-4.4331×10^6
B		7162.5
C		4.4389×10^6
D		-0.00159

things, that the critical income is also identical to that in the previous subsection. We summarize parameter values in Table 3.

We simulate the same tax changes as in the previous subsection. Consistent with the theory, a fuel tax increase shifts demand for better quality downwards, because higher quality raises fuel consumption. The effects are illustrated on Figure 7. A 20% increase in fuel prices clearly has a large impact on demand. A large number of households (all households with incomes up to more than € 42400) prefer to have cars of the lowest quality. Given the prevailing fixed annual ownership cost, nobody demands a car with quality higher than *m*=3.6 anymore.

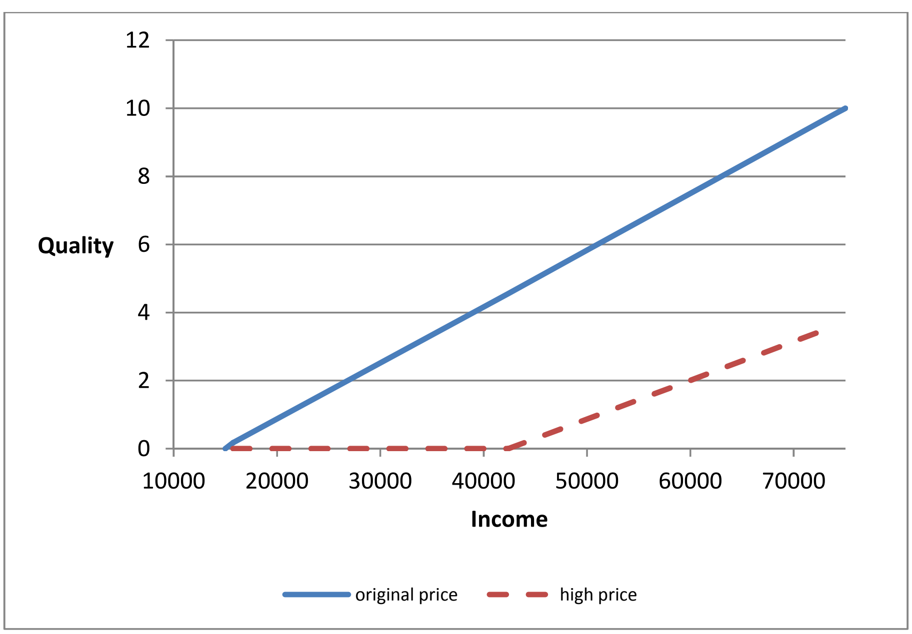


Figure 7. The shift in demand for quality (fuel-using quality) caused by an increase in the fuel tax

For the effects of a kilometer charge no graphical illustration is provided. Again there is a downward shift in optimal car quality that varies between -0.42 at the new critical income (€ 17,500) to -1.15 at the highest income (€ 75,000).

Finally, using the specifications introduced before, the shift in the fixed annual cost function that reestablishes car market equilibrium can be analyzed by solving the following differential equation

$$\frac{\partial f}{\partial m}+\frac{\partial t}{\partial m}=\left(\alpha+\beta p(m)+\gamma\left(y^{c}+\left(m-m^{min}\right)\frac{(y^{max}-y^{min})}{(m^{max}-m^{min})}-f(k)-t(k)\right)+\delta m\right)\left(-\frac{\delta}{\beta}-\pi\theta_{1}\right)$$

Let, as before, a fuel tax increase raise the unit user cost by 20%. A graphical illustration of the adjustment in the annual fixed cost is given on Figure 8: it shows the fixed cost functions that correspond to the initial and the new equilibrium. Fixed costs go down for all quality levels, but mostly so for the highest quality cars: they become less attractive when fuel prices increase.

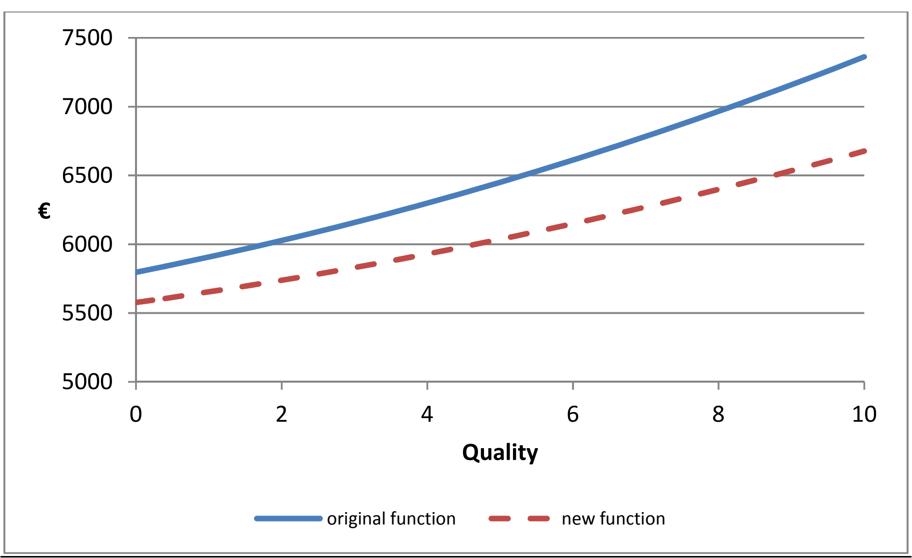


Figure 8. Equilibrium fixed cost functions (fuel-using quality) before and after a change in the fuel tax

Figure 9 shows the change in the marginal fixed cost induced by an increase in the fuel price. Given the large distortion in the original market equilibrium that was caused by the fuel price increase, we find that the changes in the fixed costs needed to restore a market equilibrium are substantial. The marginal fixed cost declines, and it declines most at high quality. Note that it is somewhat lower than in the case of a fuel saving quality indicator; quality now raises variable costs, and it provides additional utility.

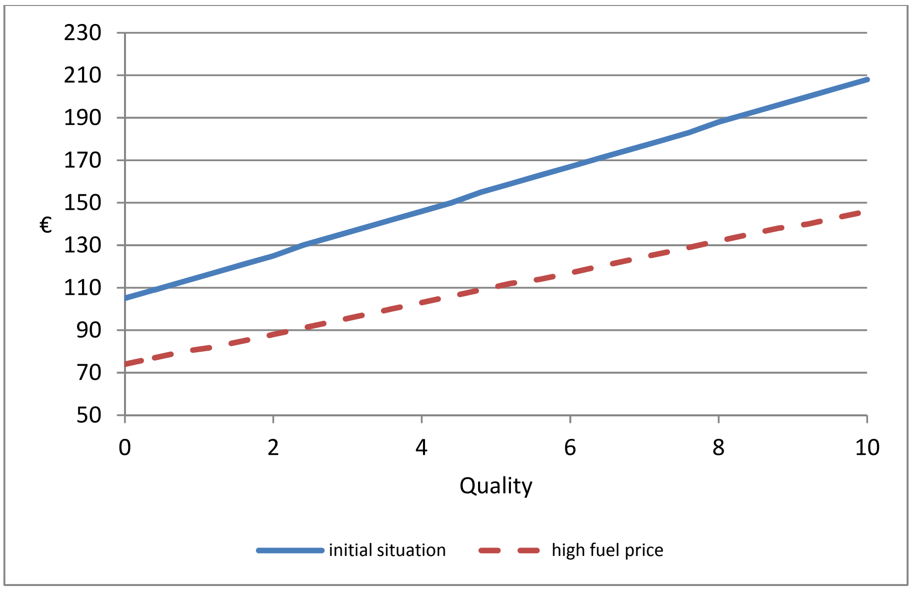


Figure 9. Marginal fixed costs (fuel spending quality) before and after a change in the fuel price

The fixed cost function corresponding to the market equilibrium with the kilometer charge is again almost parallel to the original one. It is not shown to save space.

Finally, some numerical results are summarized in Table 4 for drivers at three income levels. Interpretation is as before. Note that the taxes have rather limited effects on demand and on fuel use. Variable user costs do increase substantially and fixed costs decline, especially for the highest quality cars.

Table 5. Implications for drivers with low, average and high income (fuel-using quality)

y	15000	45000	75000
Reference situation			
Quality m	0	5	10
Kilometer demand q	9,873	14,772	19,632
fuel efficiency	0.075	0.125	0.175
total fuel use	740	1,846	3,436
total variable cost	1,111	2,770	5,153
fixed cost	5,796	6,450	7,362
marginal fixed cost	105	157	208
total cost	6,907	9,219	12,515
Increase in fuel tax			
Quality m	0	5	10
Kilometer demand q	9,695	14,482	19,242
fuel efficiency	0.075	0.125	0.175
total fuel use	727	1,810	3,367
total variable cost	1,309	3,259	6,061
fixed cost	5,576	6,036	6,677
marginal fixed cost	74	110	146
total cost	6,885	9,294	12,738
compensating variation ex ante	-220	-549	-1,022
compensating variation ex post	0	-135	-337
Introduction kilometer tax			
Quality m	0	5	10
Kilometer demand q	9,576	14,477	19,340
fuel efficiency	0.075	0.125	0.175
total fuel use	718	1,810	3,384
total variable cost	1,436	3,257	5,802
fixed cost	5,432	6,069	6,966
marginal fixed cost	102	153	205
total cost	6,868	9,327	12,768
compensating variation ex ante	-365	-549	-732
compensating variation ex post	0	-169	-336

5. Extensions

The model developed in this paper was deliberately kept simple so as to bring out the most relevant insights in the most transparent way. In this section, we briefly suggest a number of obvious extensions. First, in reality, of course, people care for a variety of car characteristics. Although generalization to multiple characteristics is conceptually straightforward, the model becomes substantially more complicated to handle. Most importantly, extending the theory for multiple characteristics does not lead to extra insights. We did extend both the theory and the numerical model by looking at two-dimensional quality, allowing both characteristics to provide direct consumer utility. The theory did not change our main qualitative findings in any way. The results of dealing with two quality attributes in the numerical model are reported in Appendix 4. Second, it seems desirable to allow for taste differences apart from income differences. This can be done by re-specifying the parameter α, which has hitherto been considered as a scalar constant as the sum of a deterministic and a random term: $\alpha = \bar{\alpha} + \varepsilon$, where ε has mean zero. It is then possible to rewrite the demand function as:

$$q^{SR} = \bar{\alpha} + \beta(\pi e(l) + \tau) + \gamma\left(y + \frac{\varepsilon}{\gamma} - f(k) - t(k)\right).$$

We can now interpret $y' = y + \frac{\varepsilon}{\gamma}$ as income that is augmented for differences in the taste for car kilometers. If the simultaneous distribution of income y and the random variable ε is known, we can derive the distribution of y' and proceed just as we did before to derive the equilibrium fixed cost function. Finally, an obvious extension is to allow the relations between costs and quality to be nonlinear.

6. Conclusion

A recent empirical literature (see, e.g., Busse et al. (2009)) suggests that fuel price changes have large effects on the prices of second hand cars, which suggests that a market level analysis is needed to get a complete picture of the implications of fuel price changes and, by implication, of other changes in variable user costs. This paper attempts to provide a better understanding of the impact of fuel price changes and different variable user taxes on the car market.. We distinguish cars that differ in terms of two quality characteristics. The first one (say, better fuel technology) improves fuel efficiency, the second one (for example, engine power)) increases fuel use. Moreover, we focus on fuel taxes and kilometer charges as the main variable tax instruments.

In a first step, we studied the effect of variable user taxes on individual behavior. We found substantial qualitative differences in the implications of a fuel tax and a kilometer charge at the individual levels. . For example, a higher kilometer tax reduces the demand or fuel efficient cars, whereas an increase in the fuel tax induces households to choose cars of better fuel efficiency, provided that the demand for car use is inelastic. Looking at quality characteristics that raise variable user costs (engine size, acceleration speed,…), we found that higher fuel taxes plausibly induce households to demand cars of lower quality. In a second step, the models were embedded into an equilibrium model for the car market, using a simple assignment model in which higher income households choose better cars. Equilibrium at the market level could be described by a 'hedonic' function that relates fixed car costs to quality. We show that changes in fuel taxes or kilometer charges cause large disruptions in the original market equilibrium that call for substantial changes in car prices to restore equilibrium for all car makes. In general, these changes in fixed car costs compensate partly for the changes in the variable costs that result from the change in the fuel tax or the kilometer charge, although with fuel saving quality we find that the fixed cost of the highest quality cars may increase.

The analysis of this paper has clearly shown that studying the implications of taxation at the household level can be highly inadequate, as large adjustments may take place at the market level. Future work could address the connection between the approach adopted here and a small literature that takes an asset market approach to the study of the car market (Kahn (1986), Engers et al. (2009)). The idea is that the car is an investment good whose value is determined by that of the future services (car miles driven) to be derived from it. When variable car costs change, the value of these services change as well and this has implications for the price of the car itself, as is the case in our analysis. However, this approach seems to ignore the impact of the existing stock of cars on price formation that is central in the analysis of the present paper, and thus seems more suitable for a long run analysis.

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Appendix 1. Extension for two car attributes

Conceptually, the model easily extends to cover multiple car attributes. Not surprisingly, however, the analytics become cumbersome and the results are much less transparent as we have to take into account the interaction of the different car attributes. In this appendix, we illustrate the extension of the theoretical model for the case of two car attributes. As before, one attribute (*k*) is interpreted as better fuel technology; it reduces variable cost per kilometer and does not generate intrinsic utility. The second one (*m*) is a characteristic (such as size) that raises variable cost*;* it does have intrinsic utility. Both attributes raise fixed annual costs.

First, consider the choice of car use for a car of given qualities. The individual solves:

$$\underset{x,q}{Max} \quad u(x,q)\ s.t. \quad x+\left[\pi e(k,m)+\tau\right]q = y - f(k,m)$$

Here, as before, $\frac{\partial e}{\partial k}<0, \frac{\partial e}{\partial m}>0$ and $\frac{\partial f}{\partial k}>0, \frac{\partial f}{\partial m}>0$. To facilitate the interpretation, further restrictions will be placed on the functions *e(.)* and *f(.)* below.

The solution to this optimization problem yields the following indirect utility function

$$v\left\{y-f(k,m), \pi e(k,m)+\tau, m\right\}$$

This indirect utility function is the starting point of our invstigation of the optimal choice of the two quality characteristics *k* and *m*. The first-order conditions describing optimal attribute choices can be written as:

$$-\frac{\partial v}{\partial y}\frac{\partial f}{\partial k}+\frac{\partial v}{\partial p}\pi\frac{\partial e}{\partial k}=0 \tag{A1}$$

$$-\frac{\partial v}{\partial y}\frac{\partial f}{\partial m}+\frac{\partial v}{\partial p}\pi\frac{\partial e}{\partial m}+\frac{\partial v}{\partial m}=0 \tag{A2}$$

We assume the second-order conditions to be satisfied.

To analyze the effect of taxes on optimal quality choices, we first use Roy's theorem to rewrite (A1)-(A2) as follows:

$$\begin{aligned} q\pi\frac{\partial e}{\partial k}+\frac{\partial f}{\partial k}=0 \\ q\pi\frac{\partial e}{\partial m}+\frac{\partial f}{\partial m}+B=0 \end{aligned} \tag{A3}$$

where $B = \frac{\frac{\partial v}{\partial m}}{\frac{\partial v}{\partial y}}$ is, as before, the intrinsic willingness to pay for attribute *m*. Defining

$$Q_k = q\pi \frac{\partial e}{\partial k} + \frac{\partial f}{\partial k}$$

$$Q_m = q\pi \frac{\partial e}{\partial m} + \frac{\partial f}{\partial m} + B$$

and totally differentiating system (A3), we obtain:

$$\begin{bmatrix} Q_{kk} & Q_{km} \\ Q_{mk} & Q_{mm} \end{bmatrix} \begin{bmatrix} dk \\ dm \end{bmatrix} = \begin{bmatrix} -Q_{k\pi} d\pi - Q_{k\tau} d\tau \\ -Q_{m\pi} d\pi - Q_{m\tau} d\tau \end{bmatrix} \quad \text{(A4)}$$

Finally, solving (A4) by Cramer's rule yields the effects of taxes on optimal quality choices:

$$\frac{dk}{d\pi} = \frac{1}{\Delta}\left(Q_{km}Q_{m\pi} - Q_{mm}Q_{k\pi}\right); \quad \frac{dm}{d\pi} = \frac{1}{\Delta}\left(Q_{mk}Q_{k\pi} - Q_{kk}Q_{m\pi}\right) \quad \text{(A5)}$$

$$\frac{dk}{d\tau} = \frac{1}{\Delta}\left(Q_{km}Q_{m\tau} - Q_{mm}Q_{k\tau}\right); \quad \frac{dm}{d\tau} = \frac{1}{\Delta}\left(Q_{mk}Q_{k\tau} - Q_{kk}Q_{m\tau}\right) \quad \text{(A6)}$$

Here

$$\Delta = Q_{kk}Q_{mm} - Q_{km}Q_{mk}$$

is the determinant associated with the matrix on the left-hand side of (A4). The second order conditions are easily shown to imply

$$Q_{kk} > 0, Q_{mm} > 0, \Delta > 0$$

Consider the effects of a fuel tax on the choice of car attributes, as given by (A5). Straightforward algebra shows:

$$Q_{km} = Q_{mk} = q\pi \frac{\partial^2 e}{\partial k \partial m} + \pi \frac{\partial e}{\partial k}\frac{dq}{dm} + \frac{\partial^2 f}{\partial k \partial m} \quad \text{(A7)}$$

$$Q_{k\pi} = q\frac{\partial e}{\partial k}\left[1 + \frac{dq}{d\pi}\frac{\pi}{q}\right] \quad \text{(A8)}$$

$$Q_{m\pi} = \frac{\partial e}{\partial m} q\left(1 + \frac{dq}{d\pi}\frac{\pi}{q}\right) - \frac{\partial B}{\partial \pi} \quad \text{(A9)}$$

In these expressions, $\frac{dq}{dm}, \frac{dq}{d\pi}$ capture the total impact of a better quality car (in the sense of a higher *m*) and of a fuel tax increase on kilometer demand, respectively. Substituting (A7)-(A8)-(A9) in (A5), it is obvious that -- in general – the effect of a fuel tax increase on car attributes is ambiguous. It not only depends in a complex way on the effect of fuel technology and fuel taxes on demand, but also on the interaction between the two car attributes in the fixed cost and fuel efficiency functions (*f(k,m)* and *e(k,m)*, respectively).

Interpretation is facilitated by first considering some simplifying assumptions. As before, let demand for kilometers be inelastic. Further assume that the overall effect of an increase in quality attribute *m* is to reduce demand for kilometers. On the one hand, an increase in *m* raises fixed as well as variable cost; both effects reduce demand. On the other hand, driving in a larger or better quality car has a positive demand effect, independent of fixed and variable costs. Assuming the first effect dominates, we have $\frac{\partial q}{\partial m} < 0$. Moreover, make the realistic assumption that the intrinsic value of a larger and more consuming car declines at higher fuel prices, so that $\frac{\partial B}{\partial \pi} < 0$. Finally, let both *f(.)* and *e(.)* be linear in the quality indicators.

Using these assumptions in (A7)-(A8)-(A9) we immediately find

$$Q_{km} > 0, Q_{k\pi} < 0, Q_{m\pi} > 0$$

It then unambiguously follows from (A5) that

$$\frac{dk}{d\pi} > 0 \text{ and } \frac{dm}{d\pi} < 0. \qquad \text{(A10)}$$

A higher fuel price raises fuel efficiency, but it reduces size or similar quality attributes that raise variable costs.

Of course, our assumptions were quite extreme, and in general the opposite signs are theoretically possible. Crucial in this respect is the sign of Q_{km}, as defined in (A7): as long as this is positive, (A10) will hold. Note that Q_{km}>0 if (i) the impact of an increase in a quality characteristic *m* that raises both fixed and variable costs indeed reduces demand, and (ii) the interaction effects between the two car attributes in the fixed cost function *f(.)* and the fuel use function *e(.)* are not strongly negative. But even Q_{km}<0 is not a sufficient condition for changing the signs of (A10); it has to be sufficiently largely negative to reverse the signs of the tax effects given in (A10). Plausibly, therefore, (A10) is likely to hold under quite general conditions.

Similar reasoning can be used to study the effect of a higher kilometer tax. We find, under the assumptions made, that a kilometer tax reduces the demand for fuel efficiency and plausibly raises the demand for quality characteristics *m* such as weight or size, unless the effect of the tax on intrinsic utility *B* is large.

Appendix 2. Initial condition

An initial condition is straightforward to derive from two other requirements of a market equilibrium. One is that a household with the critical income should be indifferent between having a car and using public transport and that all car owning households, including those with the critical income, chose their optimal car quality. The first requirement says that we must have:

$$v^{SR}\left(y^c - f\left(k^{min}\right) - t\left(k^{min}\right), \pi e\left(k^{min}\right) + \tau\right) = v(y^c, p^{pt}, m^{pt})$$

This equation can be solved for $f\left(k^{min}\right)$. The second implies that the first order condition must be valid at the critical income:

$$\frac{\partial f}{\partial k} = -\frac{\partial t}{\partial k} - q^{SR}\left(y^c - f\left(k^{min}\right) - t\left(k^{min}\right), \pi e\left(k^{min}\right) + \tau\right)\pi \frac{\partial e}{\partial k}$$

This gives us the value of $\frac{\partial f}{\partial k}$ for $k = k^{min}$. Knowledge of the value and the first derivative of the fixed cost function at $k = k^{min}$ gives us the initial condition from which we can find the whole fixed cost function *f(k)*. Noting that the solution of (9) clearly depends on the tax parameters, it therefore tells us how the fixed cost function will change in order to restore market equilibrium after a tax change.

Appendix 3. Solution method of the differential equation

In the text we pointed out that assuming

$$G(y) = \frac{y - y^{min}}{y^{max} - y^{min}}, \quad H(k) = \frac{k - k^{min}}{k^{max} - k^{min}}.$$

transforms differential equation (9) into:

$$\frac{\partial f}{\partial k} = -\frac{\partial t}{\partial k} - (\alpha + \beta p(k) + \gamma \left(y^* + \left(k - k^{min}\right)\frac{(y^{max} - y^{min})}{(k^{max} - k^{min})} - f(k) - t(k)\right)\pi\theta_1$$

In fact, this can be solved in closed form for a value of $f\left(k^{min}\right)$ that still has to be determined as:

$$f(k) = A(\pi, \tau) + B(\pi)k + C\left(\pi,\ \tau, f\left(k^{min}\right)\right)e^{\gamma\left(k - k^{min}\right)(\pi\theta_1)}$$

where *A*, *B* and *C* are functions of the parameters of the model. Although these functions can be written down, it is not easy to derive transparent comparative static results from the formulas. Therefore, we use a simulation approach.

Appendix 4. Solving the numerical model with two quality aspects

<u>*Specification of the model*</u>

Consider the household problem:

$$\max_{x,q,k,m} u(x,q;k,m) \quad s.t. \quad x + [f(k,m) + t(k,m)] + [p(\pi, k.m) + \tau]q = y$$

Apart from dealing with k and m jointly, we have allowed both attributes to yield direct utility. This allows, for instance, 'green' tastes in which the consumers derives utility from driving a more fuel efficient cars, independent of the consequences of this behavior for fixed and variable costs. It also covers the case in which improved fuel efficiency is related to the 'newness' of a car, which can be defined as the difference between some maximum age and the actual age of the car.[18] Newer cars of a particular make often embody better technology, including improved fuel efficiency.

We only discuss the simulation results of a specific version of the model in which quality, now denoted as *l*, is an index of *k* and *m* . Specifically, short-run (indirect) utility is:

$$v^{SR} = \left(\alpha\gamma + \beta + \beta\gamma(\pi e(l) + \tau) + \gamma^2\big(y - f(l)\big) + \delta\gamma m\right)\frac{1}{\gamma^2}e^{-\gamma\left((\pi e(l)+\tau)+(\delta/\beta)l\right)}$$

Here l is:

$$l = \mu_1 k + \mu_2 m.$$

Moreover, we reformulate the fuel use function as:

$$e(k,m) = \theta_0 + \theta_1 k + \theta_2 m$$

with $\theta_1 < 0$ and $\theta_2 > 0$.

The first-order conditions for optimal quality choice are now:

$$\frac{\partial f}{\partial k} = q^{SR}\left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right), \qquad \text{(A3.1)}$$

$$\frac{\partial f}{\partial m} = q^{SR}\left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right) \qquad \text{(A3.2)}$$

---

[18] Equivalently, newness can be interpreted as the (expected) remaining lifetime of the car.

Solution of the model

To solve the model, we observe from (A3.1)-(A3.2) that the partial derivatives of the fixed cost function *f* with respect to *m* and *k* are proportional. It follows that we can regard *f* as a function of a single variable, that we denote as *z*:

$$z = \left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right)m + \left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)k.$$

Intuitively the variable *z* can itself be interpreted as a kind of quality index. We assume throughout that $\left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right) > 0$ and $\left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right) > 0$, which means that *z* is increasing in both quality aspects. Note, however, that this index refers to the consumer's preference for quality as well as to the impact of quality on variable cost and that it therefore also depends on the fuel price. Put differently, if the fuel price changes the quality index *z* will in general also change.

We can also write demand for kilometers as a function of *z*. The utility function is still defined as in section 3.2, but we now with *l* as defined above substituted for *m*. It is not difficult to verify that we can write the demand function as:

$$q^{SR} = \alpha + \beta(\pi(\theta_0 + \theta_1 m + \theta_2 k) + \tau) + \gamma\big(y - f(z)\big) + \delta(\mu_1 m + \mu_2 k)$$
$$= \alpha + \beta(\pi\theta_0 + \tau) - \beta z + \gamma\big(y - f(z)\big)$$

Using the new variable *z*, we can reformulate the condition for optimal quality choice as:

$$\frac{\partial f}{\partial z} = \alpha + \beta(\pi\theta_0 + \tau) - \beta z + \gamma\big(y - f(z)\big).$$

It is not difficult to verify that the short run indirect utility function can also be written as a function of *z*, and that it is increasing in *z*. The same reasoning that we used above can be used to show that both quality aspects are normal goods. It follows that in equilibrium the households with the highest incomes will use the cars with the highest *z*.

If we know the car stock, i.e. the distribution of *m* and *k*, we can derive the distribution of *z* from that of *m* and *k*. For the purposes of our simulation exercises, we assume these variables to be uniformly distributed over intervals $[m^{min}, m^{max}]$ and $[k^{min}, k^{max}]$:

$$H(m,k) = \frac{\left(m - m^{min}\right)\left(k - k^{min}\right)}{(m^{max} - m^{min})(k^{max} - k^{min})}.$$

The associated density is $h(m,k) = 1/\left(m^{max} - m^{min}\right)\left(k^{max} - k^{min}\right)$.

To derive the distribution of *z*, we introduce four special values of this variable:

$$z^{min} = \left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right)m^{min} + \left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)k^{min}$$

$$\hat{z} = \left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right)m^{min} + \left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)k^{max}$$

$$\hat{\hat{z}} = \left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right)m^{max} + \left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)k^{min}$$

$$z^{max} = \left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right)m^{max} + \left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)k^{max}$$

These four values refer to the corners of the support of *m* and *k,* as is shown in Figure A1. The rectangle in this Figure delineates the boundary of the support. It is clear from the definitions given above that $z^{min} < z^{max}$ and that $\hat{z}$ and $\hat{\hat{z}}$ are in between. These four points refer to the corners of the support of (k_1,k_2), as shown in Figure 7. The rectangle drawn in this Figure shows the support of *H*. Figure A1 also shows the 'indifference curves' related to $\hat{z}$ and $\hat{\hat{z}}$. These lines give all the feasible combinations of the two quality aspects that lead to the values $\hat{z}$ and $\hat{\hat{z}}$, respectively. It is not difficult to verify that these indifference curves of *z* are straight lines with the same slope and that they refer to higher values of *z* if we move to the right and upwards. In Figure A1, $\hat{z} > \hat{\hat{z}}$, but this is not necessarily the case. If the slope $-\left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right) / \left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)$ of the indifference curves were larger in absolute value, the indifference curve starting from $\hat{z}$ could have crossed the lower boundary of the support to the left of $\hat{\hat{z}}$, and in that situation the indifference curve starting from $\hat{\hat{z}}$ would have crossed the upper boundary to the right of $\hat{\hat{z}}$.

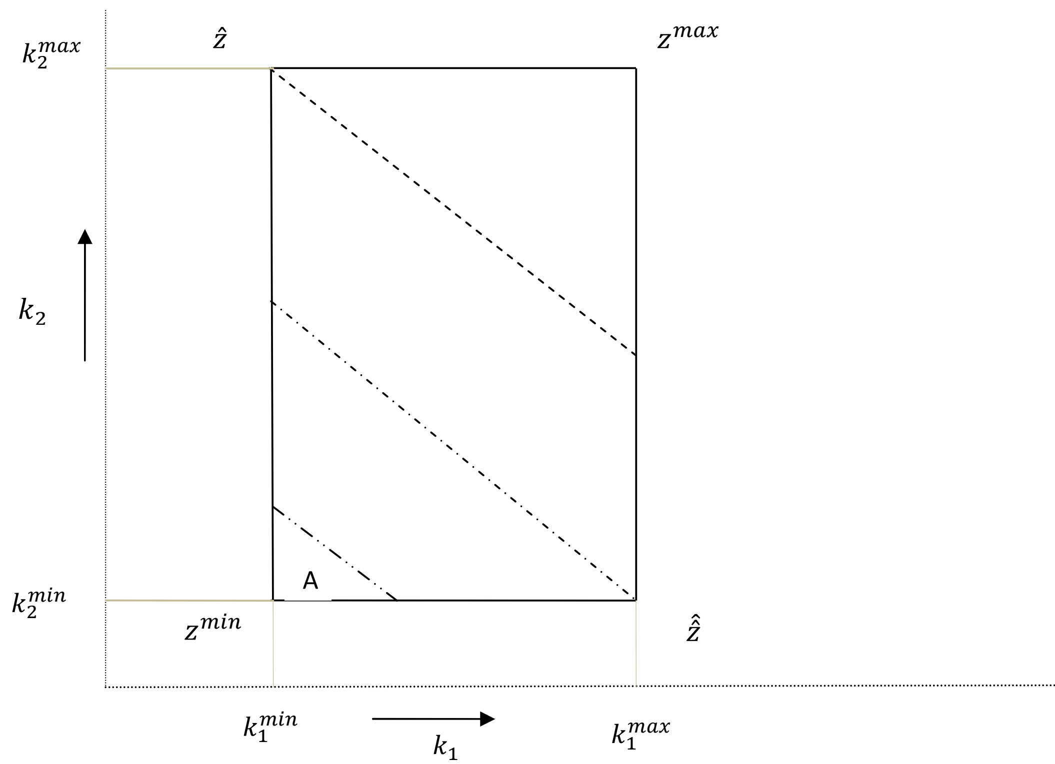


Figure A1. The distribution of the two quality aspects

Let $\widetilde{H}(z)$ denote the distribution of z and $\tilde{h}(z)$ the associated density.[19] Define $\bar{z} = \min\ \{\hat{z}, \hat{\hat{z}}\}$ and $\bar{\bar{z}} = \max\ \{\hat{z}, \hat{\hat{z}}\}$. For $z < \bar{z}$, $\widetilde{H}(z)$ is proportional to the surface of the triangle below the indifference curve associated with z on the support of k_1 and k_2. In Figure A1, an example is given by A. We can derive:

$$\widetilde{H}(z) = \frac{1}{2}\left(\frac{z-\left(-\frac{\delta}{\beta}\mu_1-\pi\theta_1\right)k_1^{min}}{\left(-\frac{\delta}{\beta}\mu_2-\pi\theta_2\right)} - k_2^{min}\right)\left(\frac{z-\left(-\frac{\delta}{\beta}\mu_2-\pi\theta_2\right)k_2^{min}}{\left(-\frac{\delta}{\beta}\mu_1-\pi\theta_1\right)} - k_1^{min}\right)$$

$$= \frac{1}{2}\frac{\left(z-z^{min}\right)^2}{\left(-\frac{\delta}{\beta}\mu_2-\pi\theta_2\right)\left(-\frac{\delta}{\beta}\mu_1-\pi\theta_1\right)}$$

and for the density we derive through differentiation:

$$\tilde{h}(z) == \frac{z-z^{min}}{\left(-\frac{\delta}{\beta}\mu_1-\pi\theta_1\right)\left(-\frac{\delta}{\beta}\mu_2-\pi\theta_2\right)}$$

This shows that $\tilde{h}(z)$ is linearly increasing for $z^{min} \leq z \leq \bar{z}$, with $\tilde{h}\left(z^{min}\right) = 0$.

Following the same reasoning, it can be shown that the density of z is constant for $\bar{z} \leq z \leq \bar{\bar{z}}$, and linearly decreasing for $\bar{\bar{z}} \leq z \leq z^{max}$, with $\tilde{h}(z^{max}) = 0$. The density of z is symmetric on its support $\left[z^{min}, z^{max}\right]$.

Summarizing the discussion, we can say that the density of z is:

$$\tilde{h}(z) = \begin{cases} a\left(z - z^{min}\right) & \text{if } z^{min} \leq z \leq \bar{z} \\ a\left(\bar{z} - z^{min}\right) & \text{if } \bar{z} \leq z \leq \bar{\bar{z}} \\ a\left(\left(\bar{z} - z^{min}\right) - (z - \bar{\bar{z}})\right) & \text{if } \bar{\bar{z}} \leq z \leq z^{max} \end{cases}$$

where $a = 1/\left(-\frac{\delta}{\beta}\mu_2 - \pi\theta_2\right)\left(-\frac{\delta}{\beta}\mu_1 - \pi\theta_1\right)$. The distribution is:

$$\widetilde{H}(z) = \begin{cases} \frac{a}{2}\left(z - z^{min}\right)^2 & \text{if } z^{min} \leq z \leq \bar{z} \\ \frac{a}{2}\left(\bar{z} - z^{min}\right)^2 + a\left(\bar{z} - z^{min}\right)(z - \bar{z}) & \text{if } \bar{z} \leq z \leq \bar{\bar{z}} \\ \frac{a}{2}\left(\bar{z} - z^{min}\right)^2 + a\left(\bar{z} - z^{min}\right)(z - \bar{z}) - \frac{a}{2}(z - \bar{\bar{z}})^2 & \text{if } \bar{\bar{z}} \leq z \leq z^{max} \end{cases}$$

[19] In the equations that follow the density and distribution of *z* has not been multiplied by the uniform density of the underlying distribution of the *k*'s. This simplifies the notation, while nothing essential is missing.

The allocation rule can now be written as: $y(z) = G^{-1}\left(G(y^*) + \widetilde{H}(z)\right)$. Like before we assume that the income distribution is uniform. This implies: $y(z) = y^* + \widetilde{H}(z)(y^{max} - y^{min})$. The differential equation we have to solve follows from (9) when we substitute the allocation function for y:

$$\frac{\partial f}{\partial z} = \alpha + \beta\pi\theta_0 - \beta z + \gamma\left(y^* + \widetilde{H}(z)(y^{max} - y^{min}) - f(z)\right)$$

We have to solve this equation for the density function $\widetilde{H}(z)$ that we derived above. Clearly, there are 3 regimes with different solutions. We solve the three corresponding differential equations. We have an initial condition for the first regime that follows from the public transport alternative and is analogous to the one we discussed in the cases of single dimensional quality. The initial condition for the solution of the second regime is simply the value of the solution for the first regime if $z = \bar{z}$. And similar for the third regime.

Figure A2 shows the density and distribution of *z* for fuel prices equal to €1.5 and €1.8. The higher fuel price causes a decrease in the value of *z* for all car makes. As a result, the density and the distribution both shift to the left, implying a lower experienced utility of the cars. Note that the distribution of *z* is not affected by a kilometer charge.

What happens if the fuel price increases while the fixed cost function remains unchanged? We investigated this question in section 3 for the cases with one-dimensional quality. To find the answer to this question in the present two-dimensional case, we return to the first order conditions (12). We recall that the fixed cost function can be written as a function of *z*, and that the first derivatives with respect to quality aspect *m* can therefore be written as: $\frac{df}{dm} = \frac{df}{dz}\frac{dz}{dm}$, (and analogously for *k*) for the original fixed cost function. For the right-hand sides of (12) the new fuel price is relevant. We use superfixes 0 and 1 to distinguish between the original and new fuel prices, respectively. An interior optimum requires:

$$\frac{df}{dz}\left(-\frac{\delta}{\beta}\mu_1 - \pi^0\theta_1\right) = q^{SR}\left(-\frac{\delta}{\beta}\mu_1 - \pi^1\theta_1\right), \tag{A1a}$$

$$\frac{df}{dz}\left(-\frac{\delta}{\beta}\mu_2 - \pi^0\theta_2\right) = q^{SR}\left(-\frac{\delta}{\beta}\mu_2 - \pi^1\theta_2\right). \tag{A1b}$$

However, these two equations are in general incompatible as can easily be verified by taking their ratio:

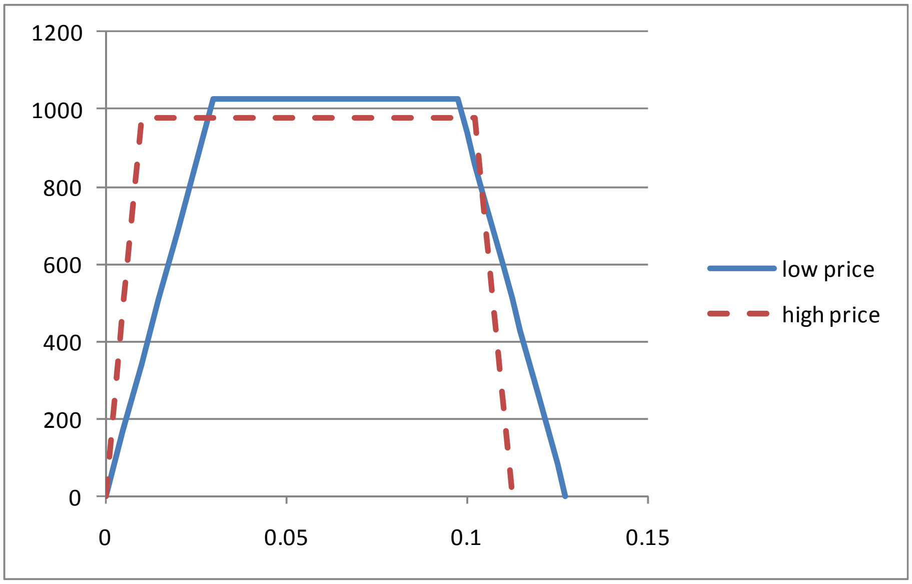
$$\frac{\left(-\frac{\delta}{\beta}\mu_1 - \pi^0\theta_1\right)}{\left(-\frac{\delta}{\beta}\mu_2 - \pi^0\theta_2\right)} \neq \frac{\left(-\frac{\delta}{\beta}\mu_1 - \pi^1\theta_1\right)}{\left(-\frac{\delta}{\beta}\mu_2 - \pi^1\theta_2\right)}.$$

This means that consumers will in general attempt to move to one of the boundaries of the support of (m,k). Starting from an equilibrium, an increase in the fuel price will imply that the left-hand side of (15a) will become smaller than the right-hand side, whereas the opposite will happen with (15b). The consumer will therefore decrease the value of the first quality aspect and increase the second (for instance by moving to a smaller and newer car), but this will not restore the equilibrium as long as the combination considered is in the interior of the support because the ratio of the two left-hand sides does not become equal to that of the right hand side. Only one condition can be satisfied, and this means that the optimum can only be a corner solution.

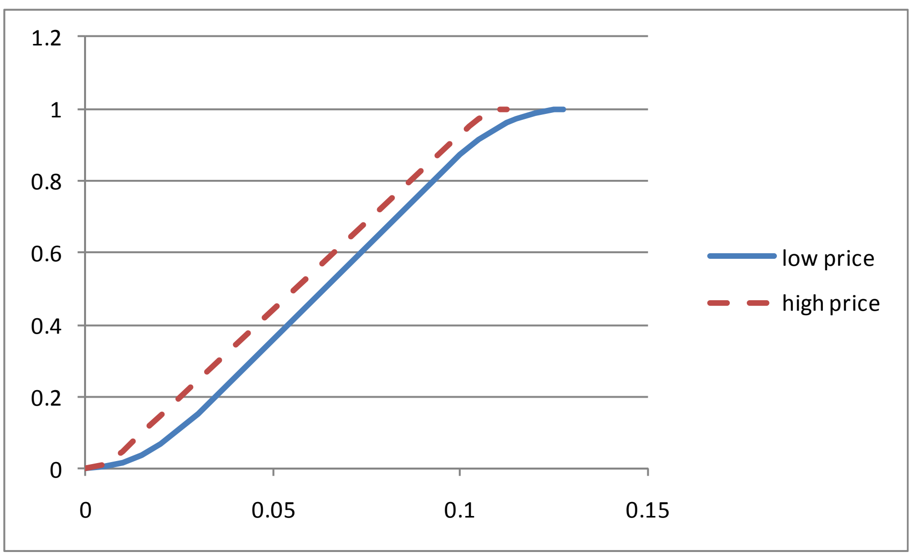
The consequence of a higher fuel price will therefore be that the demand of all consumers who still want to own a car shifts to cars with either a maximum value of k or a minimum value of m. We therefore conclude that in the model with two-dimensional quality a change in the fuel price causes a dramatic shift in demand. A change in the fixed cost function is necessary to restore the market equilibrium.

The new fixed cost function can be written as a function of z as it is defined on the basis of the new fuel price. Apart from a shift in demand by the consumers who want to keep a car (although usually of a different quality), there will be consumers who decide not to own a car anymore.

The consequences of the introduction of a kilometer charge are less drastic. The equilibrium needs only to adapt to the lower demand for kilometers which means that less of both qualities will be demanded. And also in this case, some consumers will decide not to own a car anymore.



a) Density of z



b) Distribution of z

Figure A2 Density and distribution of z with high and low fuel price

Adjustment in the fixed cost function

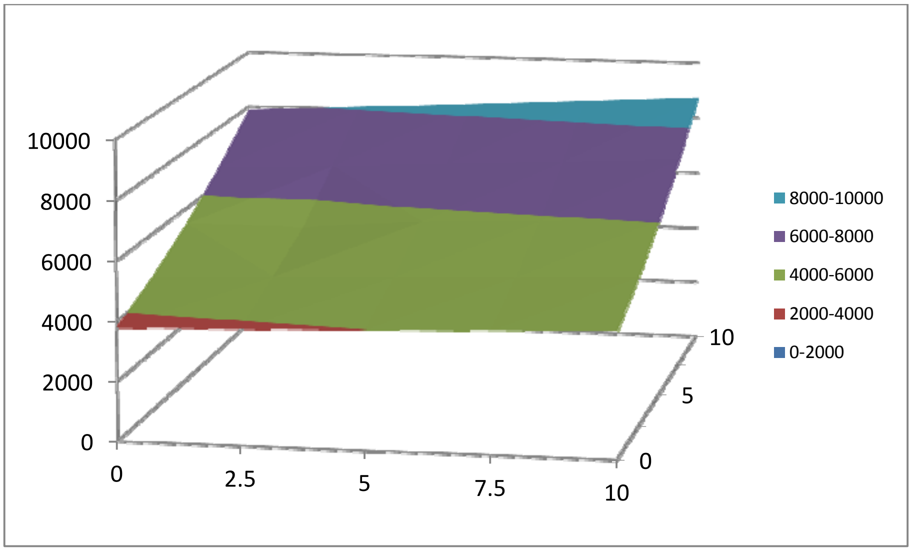
We again use the extended model to study the effects of an increase in the fuel price from €1.5 to €1.8. We kept the same values for most of the parameters; those that are different are given in Table A3.1 below. Quality aspect *m* increases variable cost and has the largest weight in the quality index. Since most quality aspects one can think of (engine power, cabin space, safety) tend to increase fuel use per km, this seems plausible. In comparison with the previous section we have also decreased the value of the parameter θ_2. This also intends to reflect reality in which we don't observe a trade-off between fixed and variable costs of otherwise identical cars (see Mulalic and Rouwendal, 2011). Also in recent years new cars hardly seem more fuel efficient than older ones. Note, however, that the fuel saving quality aspect now does have a direct impact on utility via the weight μ_2. We noted earlier that this can either be interpreted as 'green' preferences or as a taste for newness of cars.

Table A3.1. Numerical values for the simulations with two-dimensional quality

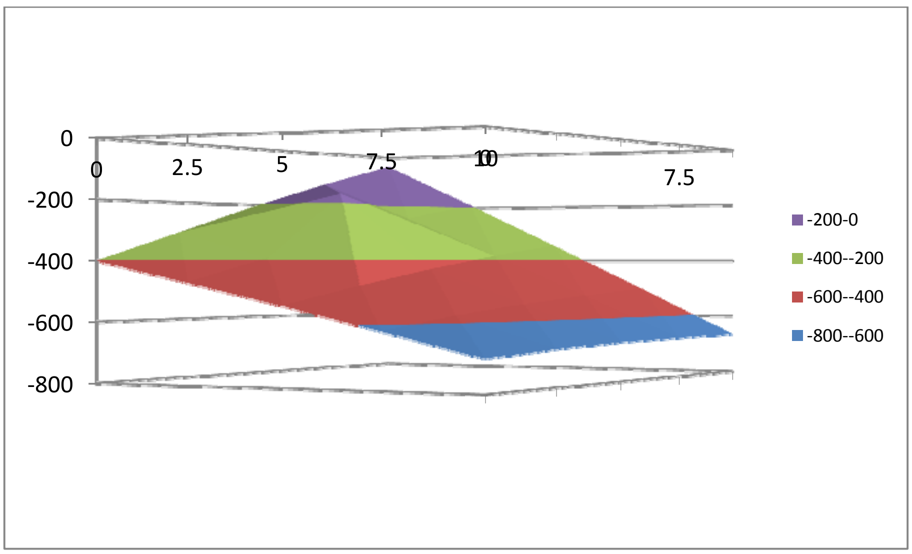
Parameter	**Interpretation**	**Numerical value**
μ_1	Weight of quality aspect *k*	0.25
μ_2	Weight of quality aspect *m*	0.75
θ_0	Intercept of fuel use function	0.15
θ_{11}	Sensitivity of fuel use for *k*	-0.0015
θ_{12}	Sensitivity of fuel use for *m*	0.01
k_1^{max}	Maximum quality *k*	10
k_1^{min}	Minimum quality *k*	0
k_2^{max}	Maximum quality *m*	10
k_2^{min}	Minimum quality *m*	0

The values of the other parameters are the same as in Table 1.

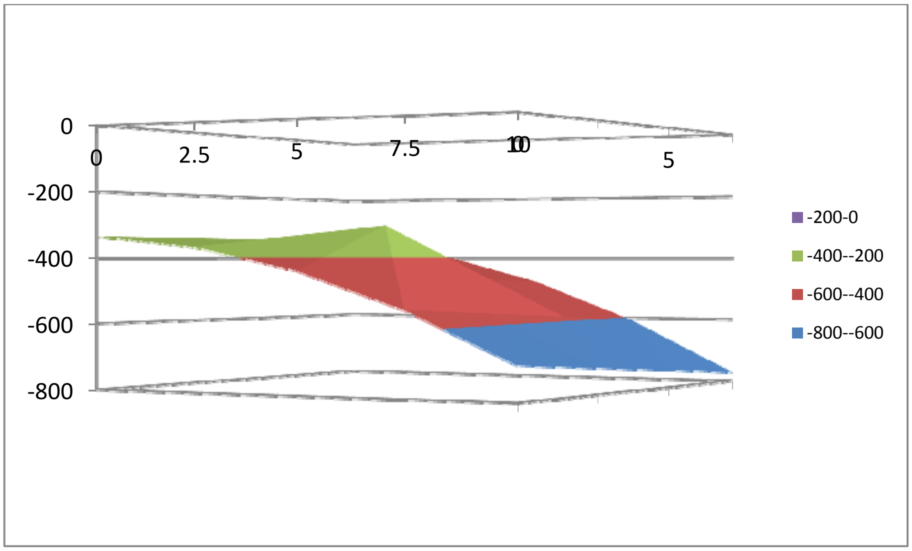
The fixed cost function in the initial situation and the changes in fixed costs that bring the market back to equilibrium after an increase in the fuel price or the introduction of a kilometer charge are shown in Figure A3.1. All three diagrams have fuel using quality *k* on the horizontal axis and fuel saving technology *m* on the backward moving axis. The first diagram has the fixed cost on the vertical axis, the other two the change in fixed cost.



a) The original fixed cost function



b) Impact of an increase in the fuel price



c) Impact of a kilometer charge

Figure A3.1. The equilibrium fixed cost function and the impact of tax changes

Diagram a) shows that the impact of fuel spending technology on fixed cost is much larger than that of fuel saving technology, which is no surprise given their impact on variable cost. Diagram b) shows that for low values of fuel spending quality, the decrease in fixed cost is smaller for more fuel efficient cars, whereas the converse is true if there is a high level of fuel spending quality. The decrease in fixed costs is always larger when fuel spending quality is larger.

Table A3.2 presents the average effects for the average driver.

Table A3.2. Implications for the average driver: two quality aspects

	Reference	Increase in fuel price	Kilometer charge
Kilometrage	15,231	14,719	14,879
Price per km	0.1875	0.225	0.225
Fuel use per km	0.125	0.125	0.125
Total fuel use	1,768	1,697	1,724
Total var. cost	2,652	3,054	3,144
Fixed cost	5,880	5,441	5,385
Total car cost	8,531	8,495	8,529
Comp var *ex ante*		-518	-569
Comp var *ex post*		-79	-74

Note: all figures refer to average effects per car/household.