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The time-varying volatility of earnings and aggregate precautionary savings*

Lorenzo Pozzi†

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Abstract

Micro data are used for the US over the period $1968-1992$ to estimate time-varying specifications for the conditional variance of earnings of individual households. Specifications estimated are standard and quadratic $ARCH$ and $GARCH$ processes. The cross-sectional mean of the estimated time-varying uncertainty of individual households is found to have a significant impact on aggregate consumption growth implying that earnings uncertainty and precautionary saving motives matter for the aggregate economy. The estimation of a buffer stock model of consumption with time-varying earnings uncertainty provides an estimated precautionary component in aggregate consumption growth. The importance of this component is found to decrease over the sample period, a result which is in line with the existing literature.

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1 Introduction

An important research topic in macroeconomics is the impact of precautionary savings on the aggregate economy. Precautionary savings are additional savings due to increases in the uncertainty about future uninsurable labour income or earnings.[1] If individual consumers or households are increasingly uncertain about future earnings they consume less today because they want to save to avoid low future consumption states if future earnings turn out to be low. Consumption tomorrow versus today increases, i.e. consumption grows faster than under certainty. Stated more formally, consumption growth depends positively on the conditional variance of shocks to consumption and - since earnings affect consumption - on the conditional variance of shocks to earnings. To the extent that this conditional variance is predictable, consumption growth, both at the individual and at the aggregate level, is no longer unpredictable (see Hall, 1978). The incorporation of precautionary saving motives into consumption models invalidates the standard permanent income hypothesis (see Zeldes, 1989). Whether or not uninsured individual earnings risk is important for the aggregate economy (i.e. for aggregate consumption, aggregate saving, and aggregate wealth accumulation) has been subject of controversy. According to Krusell and Smith (1998) precautionary savings have little impact on the aggregate wealth level. Skinner (1988), Zeldes (1989), Deaton (1991), Caballero (1990, 1991), Carroll (1992, 2000), Gourinchas and Parker (2001, 2002), and Parker and Preston (2005) argue however that precautionary savings due to uninsurable earnings risk have a significant impact on the aggregate economy.

This paper investigates the effects of precautionary savings motives on aggregate consumption dynamics by explicitly allowing for time-variation in earnings uncertainty. The empirical analysis is based on a simple consumption model with a large number of consumers who experience consumer-specific uninsurable shocks to earnings and consumption which together with the preference configuration (risk aversion or prudence, impatience) may create buffer stock savings behaviour (see e.g. Carroll 1992, Banks et al. 2001). In the first step of the empirical analysis estimates for the time-varying earnings uncertainty of households in the economy are obtained with earnings data from the Panel Study of Income Dynamics ($PSID$) for individual households in the US over the period $1968 - 1992$. The conditional variance of shocks to earnings is modelled as an autoregressive conditional heteroskedasticity ($ARCH$) process (see Engle, 1982), as a generalized $ARCH$ ($GARCH$) process (see Bollerslev, 1986), as a quadratic $ARCH$ ($QARCH$) process (see Sentana, 1995), and as a generalized quadratic $ARCH$ ($GQARCH$) process. This approach ex-

[1] In the remainder of the paper I abstract from interest rate uncertainty.

ploits and builds on the findings of Meghir and Pistaferri (2004) who show that the earnings shocks of households (both permanent and transitory) can be well described by $ARCH$ type processes. In a second step the cross-sectional mean of the estimated conditional variance of earnings over all households in the sample is used as a regressor in equations of aggregate consumption growth to find out whether average individual earnings uncertainty in the economy affects aggregate consumption dynamics. In a third step the consumption model is estimated by GMM methods. This provides estimates for the structural model parameters and estimates for the precautionary component, i.e. the part of aggregate consumption growth that can be attributed to the precautionary savings motive. The parameter estimates are obtained by minimizing the difference between actual aggregate consumption growth as obtained from the National Income and Product Accounts ($NIPA$) and the average of consumption growth rates of individual consumers as calculated from the model (where a correction method is applied to take into account the difference in aggregation between the $NIPA$ data and the data generated by the model). As such there is no need for consumption data at the level of the individual household to estimate the model. This is an advantage since $PSID$ only reports food consumption which is not an adequate consumption measure to test for precautionary savings while data from the Consumer Expenditure Survey (CEX) are only available from 1980 onward so that the use of these data would inevitably lead to a 50% reduction of the sample size. Since the aim of this paper is an investigation of the impact of time-varying earnings uncertainty on consumption a long enough time dimension is essential.

The approach followed to investigate the impact of precautionary savings on aggregate consumption is somewhat similar in spirit to earlier work by Carroll (1992), Acemoglu and Scott (1994), Hahm and Steigerwald (1999) and, more recently, to the work of Parker and Preston (2005). All these papers empirically investigate the potential presence of a term that reflects precautionary savings in equations for the growth rate of aggregate consumption. Carroll (1992), Acemoglu and Scott (1994), and Hahm and Steigerwald (1999) capture the consumption/earnings risk term in these equations by respectively survey responses to questions regarding unemployment expectations, consumer confidence indicators, and surveyed income forecasts. Parker and Preston (2005) use household data from the CEX to decompose aggregate consumption growth into different sources among which is a component that reflects incomplete markets for consumption insurance. To the best of my knowledge this paper is the first to directly parameterize the individual earnings risk term in consumption Euler equations with $ARCH$ type processes. The paper is also related to the work by Gourinchas and Parker (2002) who use a similar estimation methodology to obtain estimates of the structural model parameters. While they focus on precautionary savings over the life cycle the current paper provides distinct evidence of precautionary savings for aggregate consumption dynamics. And while their structural model is somewhat richer to generate precautionary motives over the life cycle they do not permit time-variation in the conditional variance of earnings shocks.

The results of the paper suggest, first, that the conditional variance of individual earnings can be accurately modelled by $GARCH(1,1)$ or $GQARCH(1,1)$ processes. Second, the cross-sectional mean of the estimated time-varying conditional variances of individual earnings is found to have a significant impact on aggregate consumption growth suggesting that earnings uncertainty and precautionary saving motives matter for the aggregate economy. In line with results previously reported by Storesletten et al. (2004) there is also evidence that individual earnings uncertainty in the US economy is countercyclical. Third, the full estimation of the consumption model provides an estimated precautionary component in aggregate consumption growth. This component is correlated with individual earnings uncertainty, especially during the 1970s, and shows a steady decline over the sample period. A decrease in the importance of precaution for aggregate consumption growth in the US from the 1980s onward has also been reported by Parker and Preston (2005).

The structure of the paper is as follows. In section 2 I present the theoretical framework on which the empirical analysis of the paper is based. In section 3 I use micro data on earnings to estimate different specifications for the conditional variance of individual earnings. In section 4 I calculate the cross-sectional mean of the estimated conditional variance of earnings over all households in the sample, I discuss the properties of this time series, and I conduct a preliminary exploration of the impact of average earnings uncertainty on aggregate consumption growth. In section 5 I estimate the full consumption model to obtain structural parameter estimates and an estimated precautionary component. Section 6 concludes.

2 Theory

In this section the theoretical framework is presented on which the empirical analysis conducted in the paper is based.

2.1 Assumptions and first-order conditions

Consider an economy with a large number of consumers in every period t who are subject to consumer-specific uninsurable shocks to consumption. Each individual consumer i determines individual period t consumption c_{it} by maximizing expected lifetime utility. The instantaneous utility function of the consumers is of the constant relative risk aversion (CRRA) type and is given by $u(c_{it}) = \frac{c_{it}^{1-\rho}}{1-\rho}$ where ρ is the coefficient of relative risk aversion ($\rho > 0$). These preferences imply convex marginal utility. Hence, consumers have a precautionary savings motive. There is one riskless bond in the economy which offers a relatively small risk-free economy-wide interest rate R_t. Consumers' preferences can differ across consumers and can shift through time as well.

These assumptions imply the following set of first-order conditions for all consumers i and all periods t and $t-1$,

$$E_{t-1}\left[(1+R_t)\exp(\mathbf{x}_{it}\boldsymbol{\delta})(c_{it})^{-\rho}\right] = (c_{it-1})^{-\rho} \tag{1}$$

where E_{t-1} is the expectations operator conditional on time $t-1$ information, where $\mathbf{x}_{it}$ is a $1xk$ vector of k deterministic, time-varying, and consumer-specific preference shifters, and where $\boldsymbol{\delta}$ is a $kx1$ vector of parameters (see e.g. Parker and Preston 2005). Under the assumption that individual consumption growth $\Delta \ln c_{it}$ is conditionally normally distributed the first-order conditions take the form,

$$\Delta \ln c_{it} = \frac{1}{\rho}R_t + \frac{1}{\rho}\mathbf{x}_{it}\boldsymbol{\delta} + \rho\frac{1}{2}V_{t-1}\eta_{it} + \eta_{it} \tag{2}$$

where η_{it} is the innovation to individual consumption growth, i.e. $\eta_{it} \equiv \Delta \ln c_{it} - E_{t-1}\Delta \ln c_{it}$, with $E_{t-1}\eta_{it} = 0$, and where $V_{t-1}\eta_{it}$ denotes the variance of innovations to consumption growth conditional on information available to consumers in period $t-1$. The derivation of eq.(2) is presented in appendix A.1. The term $\frac{1}{\rho}R_t$ in eq.(2) reflects intertemporal substitution while the term $\frac{1}{\rho}\mathbf{x}_{it}\boldsymbol{\delta}$ captures shifts in preferences. Upon assuming that $\mathbf{x}_{it}$ is a scalar equal to -1 so that $\boldsymbol{\delta}$ is a scalar as well I obtain the more familiar Euler equation where δ (> 0) captures time preference. Eq.(2) further shows that the conditional variance of consumption growth positively affects consumption growth, i.e. when in period $t-1$ the variance of consumption growth is expected to increase, then period $t-1$ savings rise and period $t-1$ consumption falls. Hence, consumption grows faster between periods t and $t-1$. The strength of this precautionary savings effect depends positively on the magnitude of the coefficient of relative risk aversion ρ.[2]

[2] The term $\rho\frac{1}{2}V_{t-1}\eta_{it}$ can be decomposed as $(1+\rho)\frac{1}{2}V_{t-1}\eta_{it} - \frac{1}{2}V_{t-1}\eta_{it}$ where $(1+\rho)\frac{1}{2}V_{t-1}\eta_{it}$ reflects precautionary savings with the coefficient of relative prudence equal to $1+\rho$ and where $-\frac{1}{2}V_{t-1}\eta_{it}$ reflects Jensen's inequality and the concavity of the logarithm (see Gourinchas and Parker 2001) but this offers no additional insight.

2.2 The earnings process

Eq.(2) incorporates precautionary savings measured through the conditional variance of innovations to consumption growth. To measure the importance of precautionary savings for consumption through earnings instead of consumption I assume the following process for log individual labour income or earnings (see e.g. Ludvigson and Michaelides 2001),

$$\ln y_{it} = \ln y_{it}^{p} + v_{it} \tag{3}$$

where $\ln y_{it}^{p}$ is the permanent component and v_{it} is the transitory component. The transitory individual shock v_{it} is a white noise innovation. It is i.i.d across consumers. The permanent component $\ln y_{it}^{p}$ follows a martingale process with a stochastic drift Λ_t that is common to all consumers, i.e.

$$\ln y_{it}^{p} = \Lambda_t + \ln y_{it-1}^{p} + \mu_{it} \tag{4}$$

where μ_{it} is the permanent individual shock to earnings which is white noise and i.i.d across consumers. The shocks μ_{it} and v_{it} are mutually independent. From eq.(4) I obtain $\Delta \ln y_{it}^{p} = \Lambda_t + \mu_{it}$. Writing eq.(3) in first differences gives $\Delta \ln y_{it} = \Delta \ln y_{it}^{p} + v_{it} - v_{it-1}$. Combining both results then gives,

$$\Delta \ln y_{it} = \Lambda_t + \mu_{it} + v_{it} - v_{it-1} \tag{5}$$

I assume that consumers cannot separate the permanent shock μ_{it} and the transitory shock v_{it} (see e.g. Deaton 1992, p.108-109).[3] Eq.(5) can now be written as,

$$\Delta \ln y_{it} = \Lambda_t + \varepsilon_{it} - \theta \varepsilon_{it-1} \tag{6}$$

where ε_{it} is the individual innovation to earnings which is white noise with unconditional variance σ_{ε}^2. It is i.i.d across consumers. It is not assumed to be i.i.d in time however as in section 3.2 I assume that the conditional variance of ε_{it} is time-varying and depends on time $t-1$ information.[4] The parameter θ $(0 \leq \theta \leq 1)$ is close to 0 when the unconditional variance of the permanent shock μ_{it} is large relative to the unconditional variance of the transitory shock v_{it} and

[3] This assumption is made to facilitate the estimation of a conditional variance series for the earnings process with a micro panel. While the parameters of the process (both for the mean and the variance) can be identified if the earnings process consists of a separate transitory and permanent shock (see e.g. the GMM approach of Meghir and Pistaferri 2004) it is not feasible to estimate separate time series for the time varying conditional variance of the shocks μ_{it} and v_{it}. For the conditional variance of ε_{it}, on the other hand, a time series can be obtained.

[4] Hence the conditional variances of the innovations μ_{it} and/or v_{it} also depend on time $t-1$ information.

close to 1 in the opposite case. Hence the quantity $1-\theta$ measures the degree of persistence of the individual innovation to earnings ε_{it}. Note that since ε_{it} is i.i.d across consumers, aggregate earnings growth $\Delta \ln Y_t$ will be equal to Λ_t, i.e. $\Delta \ln Y_t = E(\Delta \ln y_{it}) = \Lambda_t$ where E is the cross-sectional mean operator. I assume that log aggregate earnings follows a martingale with drift κ so that $\Delta \ln Y_t = \Lambda_t = \kappa + \pi_t$ where π_t is the aggregate shock to earnings which is white noise with unconditional variance σ_π^2 and which is independent of ε_{it}.[5] Eq.(6) becomes,

$$\Delta \ln y_{it} = \kappa + \pi_t + \varepsilon_{it} - \theta\varepsilon_{it-1} \tag{7}$$

2.3 Individual consumption growth

With an earnings process given by eq.(7) the innovation to consumption growth η_{it} can be approximated by,

$$\eta_{it} = [\pi_t + (1-\theta)\varepsilon_{it}]\frac{y_{it-1}}{c_{it-1}} \tag{8}$$

(see appendix A.2). From this I calculate $V_{t-1}\eta_{it} = \left(V_{t-1}\pi_t + (1-\theta)^2 V_{t-1}\varepsilon_{it}\right)\left(\frac{y_{it-1}}{c_{it-1}}\right)^2$. Substituting both results into eq.(2) gives

$$\Delta \ln c_{it} = \frac{1}{\rho}R_t + \frac{1}{\rho}\mathbf{x}_{it}\boldsymbol{\delta} + \rho\frac{1}{2}\left[V_{t-1}\pi_t + (1-\theta)^2 V_{t-1}\varepsilon_{it}\right]\left(\frac{y_{it-1}}{c_{it-1}}\right)^2 + [\pi_t + (1-\theta)\varepsilon_{it}]\left(\frac{y_{it-1}}{c_{it-1}}\right) \tag{9}$$

The part of consumption growth that stems from the precautionary saving motive - i.e. the third term on the RHS of eq.(9) - depends on the parameter of risk aversion, on the conditional variance of both earnings shocks (individual and aggregate), on the persistence of the shock ε_{it}, and on the squared ratio of earnings to consumption which captures the fact that, under CRRA preferences, consumers with less wealth and hence lower levels of consumption are more responsive to changes in earnings risk (see also Banks et al. 2001). From the equation it is also clear that earnings shocks only affect consumption and consumption risk to the extent that they are persistent. The aggregate shock π_t is fully persistent but the individual earnings shock ε_{it} is not. The degree of persistence $1-\theta$ therefore determines the impact of ε_{it} and $V_{t-1}\varepsilon_{it}$ on consumption growth.[6] Note finally that since $\Delta \ln c_{it}$ (and therefore the innovation to consumption growth η_{it}) is assumed to be conditionally normally distributed, the earnings shocks π_t and ε_{it} must also be conditionally normally distributed.

[5] It is therefore also independent of μ_{it} and v_{it}.

[6] To the extent that individual shocks are transitory (i.e. when $\theta = 1$) they have no impact on consumption since they are smoothed away through savings or dissavings/borrowing. As a result, they have no impact on consumption risk and on precautionary savings.

2.4 Aggregate consumption growth

Aggregate consumption growth $\Delta \ln C_t$ is defined as $\Delta \ln C_t = E(\Delta \ln c_{it})$ where E is the cross-sectional mean operator and where $\Delta \ln c_{it}$ is given by eq.(9). In section 4.1 I discuss how to correct for the fact that aggregate consumption growth based on $NIPA$ data, i.e. $\Delta \ln C_t^{NIPA}$, is calculated differently than $\Delta \ln C_t$. The precautionary component of aggregate consumption growth (i.e. the part of aggregate consumption growth that can be attributed to the precautionary savings motive) is the third term on the RHS of the cross-sectional mean of eq.(9). It is given by $\Omega_t \equiv \rho \frac{1}{2} V_{t-1} \pi_t E\left[\left(\frac{y_{it-1}}{c_{it-1}}\right)^2\right] + \rho \frac{1}{2}(1-\theta)^2 E\left[V_{t-1}\varepsilon_{it}\left(\frac{y_{it-1}}{c_{it-1}}\right)^2\right]$. Note that the first term of Ω_t is expected to be rather irrelevant empirically since the magnitude of the variance of the aggregate earnings shock $V_{t-1}\pi_t$ is too small to have a discernable impact on aggregate consumption growth under plausible values for the risk aversion parameter ρ (see e.g. Gourinchas and Parker, 2001). Hence, I am mainly interested in the impact of the second term on aggregate consumption growth which consists of the cross-sectional mean of individual earnings uncertainty (weighted by the square of the individual earnings to consumption ratio).

3 The conditional variance of earnings

In this section micro data on earnings are used to estimate different specifications for the conditional variance of individual earnings.

3.1 Data

Data for individual earnings are taken from Meghir and Pistaferri (2004).[7] They use data from the $PSID$ over the period $1968-1993$. The data refer to the period $1967-1992$.[8] Earnings are defined as the labour portion of money income from all sources. Male heads aged between 25 and 55 are selected if they have at least 9 years of successive usable earnings data. There are data for, in total, 2069 individuals ($N = 2069$). For every cross-section the series for nominal earnings is put in real prices of 1983 by deflating it by the aggregate price index for expenditures on non-durable and

[7] For the data used in Meghir and Pistaferri's paper: see supplemental material section (volume 72, 2004) of the site http://www.econometricsociety.org. The variable used for nominal individual earnings is "laby" which can be found in their STATA dataset file "Arch". Further details on the construction of the dataset can be found in the appendix to their paper.

[8] As is common practice 1992 (the 1993 survey year) is the last year used since the methodology to conduct the surveys changed drastically afterwards. Moreover, the survey became bi-annual from 1997 onward.

services (excluding shoes and clothing) which is obtained from the $NIPA$. For the growth rates in real earnings there are minimally 8 observations per cross section and maximally 25 and the sample period is $1968 - 1992$ ($T = 25$). Just like the number of consecutive periods differs per individual (T_i where $i = 1, ..., 2069$ with $8 \leq T_i \leq 25$), the number of cross sections differs per period (N_t where $t = 1968, ..., 1992$ with $746 \leq N_t \leq 1491$). In total there are 31631 observations for log real earnings while there are 29562 observations for the growth rate in real earnings ($TOT = 29562$).

In figure 1 I present the histogram of $\ln(y_{it})$. The distribution of log earnings is skewed to the left and is characterized by kurtosis in excess of the kurtosis implied by a normal distribution. The presence of a permanent component in log earnings, as modelled in section 2, can be inferred from the autocorrelation function of $\ln(y_{it})$. It is presented in figure 3. As can be seen in the figure, the autocorrelation function of log earnings dies out very slowly which is strongly suggestive of the presence of a permanent (i.e. non-stationary) component. As a result, I shift the focus of my analysis to the growth rates in earnings. In figure 2 I present the histogram of $\Delta \ln(y_{it})$. The unconditional distribution of the growth rate in earnings is not particularly skewed but it is characterized by very high excess kurtosis. In the next section I model the conditional variance of the growth rate in earnings through $GARCH$ processes (both standard and quadratic) to capture the excess kurtosis in the unconditional distribution of $\Delta \ln(y_{it})$ while maintaining the assumption that the conditional distribution of $\Delta \ln(y_{it})$ is normal. In figure 4 I present the correlogram of $\Delta \ln(y_{it})$. It strongly suggests that $\Delta \ln y_{it}$ follows an $MA(1)$ process since the autocorrelation function cuts off after lag 1 while the partial correlation function damps out more slowly. As such in the next section I model the growth rate in earnings as an $MA(1)$ process.

3.2 Estimating the conditional variance of earnings

With the micro data on earnings described in the last section I estimate the following system,

$$\Delta \ln \widetilde{y}_{it} = \varepsilon_{it} - \theta \varepsilon_{it-1} \tag{10}$$

$$V_{t-1}\varepsilon_{it} = \sigma_{\varepsilon}^2(1 - \alpha - \beta) + \alpha \varepsilon_{it-1}^2 + \beta V_{t-2}\varepsilon_{it-1} + \gamma \varepsilon_{it-1} \tag{11}$$

The variable $\Delta \ln \widetilde{y}_{it} = \Delta \ln y_{it} - \Delta \ln Y_t$ is the growth rate in purely individual earnings, i.e. it is obtained after removing aggregate earnings growth with the use of time fixed effects (see e.g. Abowd and Card 1989). Note that a preliminary test rejects the redundancy of time fixed effects.[9]

[9] A likelihood ratio test compares the likelihood of estimating $\Delta \ln y_{it}$ on a set of time dummies with the likelihood

The $MA(1)$ process for $\Delta \ln \widetilde{y}_{it}$ presented in eq.(10) is frequently used in the literature to describe earnings at the micro level (see e.g. MaCurdy 1982, Abowd and Card 1989, and Deaton 1992). No cross section fixed effects are added since a preliminary test cannot reject their redundancy.[10] As noted in section 2 the shock ε_{it} is a white noise error with unconditional variance σ_ε^2. It is assumed to be conditionally normally distributed and i.i.d. across consumers. The $MA(1)$ parameter θ ($0 \leq \theta \leq 1$) reflects the persistence of the earnings shock. In eq.(11) I specify the conditional variance of ε_{it} as a generalized quadratic $ARCH$ process, i.e. a $GQARCH(1,1)$ process (see Sentana 1995). This is a very general process that nests the $ARCH(1)$ process (Engle 1982), the $GARCH(1,1)$ process (Bollerslev 1986), and the $QARCH(1)$ process (Sentana 1995) as special cases. In particular when $\beta = \gamma = 0$ the specification is $ARCH(1)$, when $\gamma = 0$ the specification is $GARCH(1,1)$, and when $\beta = 0$ the specification is $QARCH(1)$. Note that while the shock ε_{it} is assumed to be conditionally normally distributed, its unconditional distribution is only normal if $\alpha = \beta = \gamma = 0$, i.e. when ε_{it} is i.i.d in time. When the conditional variance $V_{t-1}\varepsilon_{it}$ is either $ARCH(1)$, $QARCH(1)$, $GARCH(1,1)$, or $GQARCH(1,1)$ the unconditional distribution of ε_{it} is not normal, i.e. it has excess kurtosis. Hence, modelling the conditional variance of ε_{it} with such a process helps to explain the observed excess kurtosis in annual earnings reported in section 3.1. Also, it can be shown that excess kurtosis is higher in quadratic $ARCH$ specifications compared to standard $ARCH$ models (see Sentana 1995). Moreover, the quadratic specifications (i.e. when $\gamma \neq 0$) are interesting because they allow for asymmetric effects on the variance stemming from positive versus negative earnings shocks. In particular, when $\gamma < 0$ negative earnings shocks have a larger impact on earnings uncertainty than positive shocks which supports economic intuition. To ensure positivity of $V_{t-1}\varepsilon_{it}$ the following parameter restrictions are imposed: $\alpha > 0$, $\beta > 0$, $\alpha + \beta < 1$, and $\gamma^2 \leq (4\sigma^2(1-\alpha-\beta)\alpha)$ (see Sentana 1995). The condition $\alpha + \beta < 1$ also serves to ensure covariance stationarity of ε_{it}^2.

I estimate the system in eqs.(10)-(11) by maximum likelihood (ML). Under the assumption that ε_{it} is conditionally normally distributed the maximum likelihood estimator is consistent and asymptotically normally distributed. As noted by Arellano (2003, p.23-27) to obtain consistent estimates with maximum likelihood in a panel data context it is important to avoid incidental parameter problems. This means that the likelihood should not depend on a subset of parameters

of estimating $\Delta \ln y_{it}$ on a constant only. The estimation method is panel OLS. The null hypothesis of redundant time fixed effects is convincingly rejected (probability 0.000)

[10] A likelihood ratio test compares the likelihood of estimating $\Delta \ln y_{it}$ on a set of cross section dummies with the likelihood of estimating $\Delta \ln y_{it}$ on a constant only. The estimation method is panel OLS. The null hypothesis of redundant cross section fixed effects cannot be rejected (probability 0.9999).

whose number increases with the sample size. With respect to such incidental parameters, first, as noted above, I find that cross section fixed effects in eq.(10) can be neglected since their redundancy cannot be rejected. Second, there is no need to estimate a potentially large number of correlations between cross sections since aggregate labour income growth has been removed from $\Delta \ln y_{it}$. Hence, the variable $\Delta \ln \widetilde{y}_{it}$ will not be correlated across consumers. Note that I calculate the likelihood under the assumption that the cross sections are independent (see e.g. Cermeno and Grier, 2006).[11]

The results from estimating eqs.(10)-(11) are presented in table 1. Results are presented for five different specifications of $V_{t-1}\varepsilon_{it}$: a constant (the i.i.d case), an $ARCH(1)$ process, a $QARCH(1)$ process, a $GARCH(1,1)$ process, and a $GQARCH(1,1)$ process. The results show that $(G)ARCH$ type effects are present in individual earnings which supports and extends previous findings reported by Meghir and Pistaferri (2004) even though the results are not directly comparable.[12] From the Akaike information criterion (AIC) for model comparison it should be noted that the $GQARCH(1,1)$ process is preferred over all other specifications. The parameter γ in the $QARCH$ and $GQARCH$ specifications has a negative sign and is significant which suggests that negative shocks in earnings increase uncertainty more than positive shocks even though the magnitude of the effect is quite small. More interestingly, the estimations suggest that the conditional variance of earnings is better captured by a $GARCH$ or $GQARCH$ process than by an $ARCH$ or $QARCH$ process. The sum of the estimates for α and β implies that the conditional variance of earnings is quite persistent. For all four time-varying specifications an estimated time series $\widehat{V}_{t-1}\varepsilon_{it}$ is obtained which will be used in the next sections to estimate the precautionary component and its impact on aggregate consumption growth.

[11] The log likelihood is given by $-\frac{1}{2}(TOT)\ln(2*pi) - \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T_i}\ln(V_{t-1}\varepsilon_{it}) - \frac{1}{2}\sum_{i=1}^{N}\sum_{t=1}^{T_i}\frac{[\Delta \ln \widetilde{y}_{it}+\theta\varepsilon_{it-1}]^2}{V_{t-1}\varepsilon_{it}}$ where $TOT = 29562$, where $N = 2069$, where $8 \leq T_i \leq 25$, and where $V_{t-1}\varepsilon_{it}$ is given by eq.(11). Note that I set the initial values ε_{i0}^2 and V_0 equal to σ^2, and I set the initial value ε_{i0} equal to 0.

[12] One difference is that Meghir and Pistaferri estimate $ARCH$ processes separately for the transitory and the permanent shock to individual earnings.

4 The conditional variance of earnings and aggregate consumption growth

As seen in section 2.4 the precautionary component in aggregate consumption growth Ω_t depends crucially on (the cross sectional mean of) the conditional variance of earnings $V_{t-1}\varepsilon_{it}$. Before conducting a full estimation of the model in section 5 this section carries out a preliminary exploration of the impact of the cross-sectional mean of the estimated series $\widehat{V}_{t-1}\varepsilon_{it}$ on aggregate consumption growth with the use of simple ordinary least squares (OLS) regressions.

4.1 Aggregate consumption growth

This paper focuses on the impact of precaution - measured through the conditional variance of earnings - on aggregate consumption growth. Hence, data for consumption are needed. Data used for C_t are real per capita expenditures on non-durables and services (excluding shoes and clothing). This series is put in real prices of 1983 by deflating it by the aggregate price index for expenditures on non-durable and services (excluding shoes and clothing). These data are obtained from the $NIPA$. There is both an advantage and a disadvantage related to using aggregate consumption data from the $NIPA$. The advantage of using $NIPA$ data is that aggregate data are available for a long enough time span and hence also for the period $1968-1992$ for which we have usable earnings data from the $PSID$. Alternatively, an aggregate series for consumption could be constructed from micro data. The $PSID$ however only reports food consumption which is not an adequate consumption measure to test for precautionary savings. Therefore some papers have combined the use of earnings data from the $PSID$ with the use of consumption data from the CEX to study precautionary savings (see Gourinchas and Parker 2002, Blundell et al. 2008). But data from the CEX are only available from 1980 onward so that the use of these data would inevitably lead to a 50% reduction of the sample size. Since the aim of this paper is an investigation of the impact of time-varying earnings uncertainty on consumption a long enough time dimension is essential. The disadvantage of using $NIPA$ data however is that National Accounts aggregate consumption data are only available as arithmetic means of individual consumption levels. The growth rate in aggregate consumption is then, by necessity, calculated as the difference in the logs of the arithmetic means of individual consumption levels, i.e. as $\Delta \ln E(c_{it})$ where E is the cross-sectional mean operator. In the consumption model presented in section 2.4 however, aggregate consumption growth is obtained as the mean of the individual consumption growth rates. The growth rate in aggregate consumption is then calculated as the difference in the logs of the geometric means of individual consumption levels. The latter equals the difference in the arithmetic means of the logs of individual consumption levels, i.e. $\Delta E(\ln c_{it})$ which equals $E(\Delta \ln c_{it})$. The difference between both approaches is known as Theil's entropy measure (see Attanasio and Weber 1993) which I denote by Δ_t. It can be defined as

$$\begin{aligned} \Delta_t &\equiv \Delta \ln E(c_{it}) - \Delta E(\ln c_{it}) \\ &\equiv \Delta \ln E(c_{it}) - E(\Delta \ln c_{it}) \end{aligned} \tag{12}$$

Attanasio and Weber (1993) approximate Δ_t by $\Delta_t \approx \frac{1}{2}\Delta M_t^2 + \frac{1}{6}\Delta M_t^3 + \frac{1}{24}\Delta M_t^4$ where M_t^j is the period t centralized j-th moment of the cross-sectional distribution of $\ln c_{it}$. Under the assumption that the third and fourth moments of the cross-sectional distribution of $\ln c_{it}$ change little from one period to the next then $\Delta_t \approx \frac{1}{2}\Delta V_t$ where V_t is the cross-sectional variance of $\ln c_{it}$.[13] With these results I can partially correct the $NIPA$ based growth rate in aggregate consumption by calculating aggregate consumption growth as $\Delta \ln C_t = \Delta \ln C_t^{NIPA} - \frac{1}{2}\Delta \widehat{V}_t$ where $\Delta \ln C_t^{NIPA}$ is the growth rate in aggregate consumption as calculated from $NIPA$ aggregate consumption data and $\widehat{V}_t$ is the estimated variance of the cross-sectional distribution of $\ln c_{it}$. The latter has been calculated with the use of CEX data over the period $1980-1992$ by Blundell et al. (2008). They find that this variance increases with about 0.01 units over the period $1981-1985$ and remains fairly stable afterwards. Hence I set $\Delta \widehat{V}_t = 0.01$ for the period $1981-1985$ and $\Delta \widehat{V}_t = 0$ for the period $1986-1992$. For the period before 1980 there are no available data so I assume $\Delta \widehat{V}_t = 0$ also over the period $1968-1980$. Figure 5 shows $\Delta \ln C_t^{NIPA}$ - aggregate consumption growth as calculated from $NIPA$ data - as well as $\Delta \ln C_t$ - aggregate consumption growth partially corrected for Theil's entropy.

4.2 The cross-sectional mean of $V_{t-1}\varepsilon_{it}$

The cross-sectional means of the time series estimated for the four different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ ($ARCH$, $QARCH$, $GARCH$, $GQARCH$) in section 3.2 are presented in figure 6. As can be seen from this figure the evolution is very similar across the four cases. In particular the difference between $(G)ARCH$ on the one hand and its quadratic counterpart $(G)QARCH$ on the other hand is negligible. Three features, common to all series, are worth

[13] While, as Attanasio and Weber (1993) show, this assumption does not hold in quarterly data, the evolution of the cross-sectional moments in annual data is expected to be much smoother (as evidenced by a comparison between the evolution of the cross-sectional variance in annual data versus quarterly data).

mentioning. First, the series are quite persistent (i.e. mean reversion is rather slow). This makes sense given the high persistence found in $\widehat{V}_{t-1}\varepsilon_{it}$ as reported in section 3.2. Second, the series fall during the eighties so that the reduction in the cross-sectional mean of individual earnings uncertainty may provide an explanation for the decrease in the US personal saving rate observed during that period (see also Parker and Preston 2005). Third, the humps and bumps in the series follow a pattern similar to the one observed in the aggregate unemployment rate. When regressing the cross-sectional mean of the conditional variances on the 1-period lagged aggregate unemployment rate and a constant I find a significant positive effect, i.e. the null hypothesis of insignificance is rejected with a p-value lower than 2.5% for all specifications used for $V_{t-1}\varepsilon_{it}$.[14] This provides support for studies that argue that individual earnings risk in the US economy is countercyclical (see e.g. Storesletten et al. 2004).

4.3 Regressing aggregate consumption growth on the cross-sectional mean of $V_{t-1}\varepsilon_{it}$

With the estimated time series $\widehat{V}_{t-1}\varepsilon_{it}$ in hand it is possible to conduct regressions of the form,

$$\Delta \ln C_t = a_0 + a_1 \left[\frac{1}{N_t} \sum_{i=1}^{N_t} \widehat{V}_{t-1}\varepsilon_{it} \right] + \omega_t \tag{13}$$

where a_0 denotes a constant, where ω_t is an error term, and where the regressor $\frac{1}{N_t}\sum_{i=1}^{N_t} \widehat{V}_{t-1}\varepsilon_{it}$ is the sample counterpart of the population moment $E\left[\widehat{V}_{t-1}\varepsilon_{it}\right]$ where N_t is the number of cross sections in the sample in period t (where $t = 1968, ..., 1992$). I estimate eq.(13) with OLS over the period $1968 - 1992$. Consistent estimates for a_0 and a_1 are obtained if there is no autocorrelation in ω_t so that there is orthogonality between the regressor $\frac{1}{N_t}\sum_{i=1}^{N_t} \widehat{V}_{t-1}\varepsilon_{it}$ and the error term ω_t. In tables 2 and 3 I report the results of estimating eq.(13) with four specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ (i.e. $ARCH$, $QARCH$, $GARCH$, and $GQARCH$) used to calculate the cross-sectional means.[15] In table 2 no correction for Theil's entropy is imposed on aggregate consumption growth data (see section 4.1). In table 3 I do impose a correction for Theil's entropy

[14] This result is obtained using simple OLS regressions with a Newey-West correction on the standard errors. Data for the annual unemployment rate are taken from the US Bureau of Labor Statistics. The results are not reported but are available from the author upon request.

[15] When ε_{it} is i.i.d in time then $V_{t-1}\varepsilon_{it} = \sigma_\varepsilon^2$ and the cross-sectional mean of $V_{t-1}\varepsilon_{it}$ becomes a part of the constant so that identifying the impact of earnings uncertainty on aggregate consumption growth in this case is not feasible.

on aggregate consumption growth data. I further report the R^2 of the regression, the Akaike information criterion (AIC) for model comparison, and the Durbin Watson (DW) test statistic.

From both tables I conclude, first, that the estimates for a_1 are highly significant.[16] The significant impact of the estimated cross-sectional mean of individual earnings uncertainty on aggregate consumption growth over the sample period suggests that precaution is a relevant determinant of aggregate consumption growth (see also Parker and Preston 2005). Second, the R^2 of the regressions suggests that individual earnings uncertainty explains between 20% and 30% of the variance of aggregate consumption growth over the sample period. Third, the regressions conducted with Theil's entropy correction suggest that individual earnings uncertainty is somewhat less important for aggregate consumption growth when a theoretically more adequate measure for aggregate consumption growth is used. The differences between the results reported in tables 2 and 3 are rather modest however. In figures 7 and 8 I compare actual aggregate consumption growth (without Theil's entropy correction in figure 7, with Theil's entropy correction in figure 8) with aggregate consumption growth as fitted from the estimation of eq.(13) (for the preferred $GQARCH$ case). The figure supports the findings reported in tables 2 and 3, i.e. that the cross-sectional mean of the estimated individual earnings uncertainty explains an important fraction of the variance of aggregate consumption growth.

5 Estimation of the model

While the estimation of eq.(13) provides preliminary evidence that individual earnings uncertainty has a significant impact on aggregate consumption growth it does not constitute a complete estimation of the model. The reason is that important additional determinants of aggregate consumption growth (i.e. the ratio of earnings to consumption which interacts with the conditional variance of earnings, the interest rate, and aggregate uncertainty) are not included in eq.(13). In this section I fully estimate the model which can be summarized by

$$\Delta \ln C_t = E(\Delta \ln c_{it}) \tag{14}$$

[16] Note that the reported standard errors are not corrected for the fact that the regressor $\frac{1}{N_t}\sum_{i=1}^{N_t} \widehat{V}_{t-1}\varepsilon_{it}$ in eq.(13) is generated from the first stage estimation reported in section 3.2. From table 1 it is clear that the first stage estimation of the individual earnings process is rather precise so that little impact is expected on the second stage estimated standard errors. In the next section (section 5 below) a full-blown estimation of the model is conducted and the second stage standard errors are corrected for first stage estimation.

where E is the cross-sectional mean operator and where $\Delta \ln c_{it}$ is given by eq.(9) which is repeated here for convenience,

$$\Delta \ln c_{it} = \frac{1}{\rho} R_t + \frac{1}{\rho} \mathbf{x}_{it} \boldsymbol{\delta} + \rho \frac{1}{2} \left[V_{t-1} \pi_t + (1-\theta)^2 V_{t-1} \varepsilon_{it} \right] \left(\frac{y_{it-1}}{c_{it-1}} \right)^2 + \left[\pi_t + (1-\theta) \varepsilon_{it} \right] \left(\frac{y_{it-1}}{c_{it-1}} \right) \quad (15)$$

Estimates need to be obtained for the structural preference parameters ρ and $\boldsymbol{\delta}$ as well as for the precautionary component in aggregate consumption growth which I denoted previously by $\Omega_t \equiv \rho \frac{1}{2} V_{t-1} \pi_t E \left[\left(\frac{y_{it-1}}{c_{it-1}} \right)^2 \right] + \rho \frac{1}{2} (1-\theta)^2 E \left[V_{t-1} \varepsilon_{it} \left(\frac{y_{it-1}}{c_{it-1}} \right)^2 \right]$.

5.1 Estimation method

I rewrite eq.(14) as,

$$E\left[h(\Upsilon)\right] = 0 \quad (16)$$

where $h(\Upsilon) = \left[\begin{array}{ccccc} h_{i1}(\Upsilon) & ... & h_{it}(\Upsilon) & ... & h_{iT}(\Upsilon) \end{array} \right]'$ with $h_{it}(\Upsilon) = \Delta \ln c_{it}(\Upsilon) - \Delta \ln C_t$ for $t = 1, ..., T$ - or, interchangeably, for $t = 1968, ..., 1992$. Eq.(14) thus constitutes a set of $T = 25$ population moment conditions which are assumed to hold at the true parameter vector Υ. The Generalized Method of Moments (GMM) approach by Hansen (1982) suggests to replace the population moment conditions in eq.(16) by the sample moment conditions $g_t(\Upsilon) = \frac{1}{N_t} \sum_{i=1}^{N_t} h_{it}(\Upsilon)$. In the general case where the number of moment conditions exceeds the number of parameters a GMM estimator $\widehat{\Upsilon}$ is obtained from

$$\widehat{\Upsilon} = \arg \min_{\Upsilon} g(\Upsilon)' W g(\Upsilon) \quad (17)$$

where $g(\Upsilon) = [g_1(\Upsilon)...g_T(\Upsilon)]'$ and where W is a TxT weighting matrix. Under some regularity conditions the GMM estimator is consistent and asymptotically normally distributed. It is asymptotically efficient if $W = S^{-1}$ where S is the variance-covariance matrix of the moment conditions, i.e. $S = E[h(\Upsilon)h(\Upsilon)']$. This matrix can only be estimated with an initial consistent estimate of the parameter vector. Such a first step consistent GMM estimate $\widehat{\Upsilon}_{(1)}$ is obtained by setting $W = I$ (i.e. the identity matrix) in eq.(17). When estimating eq.(17) with $W = \widehat{S}^{-1}$ the second step GMM estimate $\widehat{\Upsilon}_{(2)}$ is obtained. When presenting the results I report both the first step and second step GMM estimates. I also report the test for overidentifying restrictions which tests the validity of the moment conditions for both the first step and second step GMM estimators. The calculation of this test statistic is presented in appendix C.1. The calculation of the first step and second step asymptotic standard errors of the GMM estimator is presented in appendix C.2.

Two complications must be addressed.

The first complication arises from the fact that individual consumption growth $\Delta \ln c_{it}$ in eq.(15) depends on lagged individual consumption c_{it-1} which is unobserved.[17] While in general c_{it-1} can be calculated from the growth rate $\Delta \ln c_{it-1}$ an initial value c^0_{it-1} is still needed where $t-1$ pertains to different years for different consumers since the panel is unbalanced and the sample starts in different years for different individuals. The best guess is $c^0_{it-1} = E(c_{it-1})$. Hence, I assume that the initial value of consumption for individual i for whom the sample starts in period t equals the mean of the cross-sectional distribution of consumption in period $t-1$. Of course, the period $t-1$ mean of the cross-sectional distribution of consumption equals aggregate per capita consumption in period $t-1$, i.e. $E(c_{it-1}) = C_{t-1}$.[18] [19] With repeated backward substitution it is then possible to get rid of past values of c_{it} in eq.(15) so that, in the end, $\Delta \ln c_{it}$ does not depend on past values of c_{it} but only on one past value of C_t which is observed. As a result the moment conditions g_t are known deterministic functions of observed data and parameters only and GMM can still be applied.

The second complication is that, mainly due to computational complexity, not all the parameters in the parameter vector Υ can be estimated simultaneously. In particular, the parameters of the earnings process are estimated in a first stage. I consider the following three subvectors of parameters. The first subvector Φ contains the parameters of the individual earnings process $\Delta \ln \widetilde{y}_{it} = \Delta \ln y_{it} - \Delta \ln Y_t = \varepsilon_{it} - \theta \varepsilon_{it-1}$, i.e. the parameters of the system given by eqs.(10)-(11). This vector is $\Phi = (\begin{array}{ccccc} \theta & \sigma^2_\varepsilon & \alpha & \beta & \gamma \end{array})'$. Estimation of these parameters has been detailed in section 3.2. From the estimated parameters I obtain the estimated time series $\widehat{\varepsilon}_{it}$ and $\widehat{V}_{t-1}\varepsilon_{it}$. The second subvector Γ contains the parameters of the aggregate earnings process $\Delta \ln Y_t = \Lambda_t = \kappa + \pi_t$. This vector is $(\begin{array}{cc} \kappa & \sigma^2_\pi \end{array})'$. Estimation of this vector is discussed in appendix B. It should be noted that no time-variation was detected in the (conditional) variance of the aggregate earnings shock π_t so that only the unconditional variance σ^2_π is estimated. From the estimated parameters I obtain the estimated time series $\widehat{\pi}_t$. The third subvector Ψ contains the structural preference parameters from the consumer optimization problem. This vector is $\Psi = (\begin{array}{cc} \rho & \boldsymbol{\delta} \end{array})'$. These parameters still

[17] The limitations of using micro data for c_{it} in our set-up have been discussed in the introduction and in section 4.1.

[18] The error that this initialization introduces in individual consumption growth rates over the periods after initialization is purely idiosyncratic and can be expected to disappear upon aggregation across consumers.

[19] As discussed in section 4.1 the use of $NIPA$ data for $\Delta \ln C_t$ to test the condition $\Delta \ln C_t = E(\Delta \ln c_{it})$ can be problematic. This is not so when using $NIPA$ data for C_t in the case $C_t = E(c_{it})$ however as both the $NIPA$ data and the moment condition in this case are based on arithmetic means of c_{it}.

need to be estimated. Since the parameters Φ and Γ are estimated in a first stage, the parameter vector Υ in the above GMM set-up can be written as $\Upsilon = (\Psi|\widehat{\Phi}, \widehat{\Gamma})$. The implication is that $\Delta \ln c_{it}$ used in the construction of the moment conditions g_t depends on a number of previously estimated parameters and time series, i.e.

$$\Delta \ln c_{it} = \frac{1}{\rho} R_t + \frac{1}{\rho} \mathbf{x}_{it} \boldsymbol{\delta} + \rho \frac{1}{2} \left[\widehat{\sigma}_\pi^2 + (1-\widehat{\theta})^2 \widehat{V}_{t-1} \varepsilon_{it} \right] \left(\frac{y_{it-1}}{c_{it-1}} \right)^2 + \left[\widehat{\pi}_t + (1-\widehat{\theta}) \widehat{\varepsilon}_{it} \right] \left(\frac{y_{it-1}}{c_{it-1}} \right) \quad (15')$$

where $\widehat{}$ indicates that variables or time series are obtained from prior estimation. Since Φ and Γ are consistently estimated in the first stage this does not affect the consistency and asymptotic normality characteristics of the GMM estimator used to estimate Ψ. However, the asymptotic standard errors calculated for $\widehat{\Psi}$ need to be corrected for the fact that $\widehat{\Phi}$ and $\widehat{\Gamma}$ are not known with certainty but are estimated. The calculation of these corrected standard errors is presented in appendix C.3.

5.2 Results

This section discusses the results of estimating eqs.(14)-(15') by GMM as explained in the previous section. This estimation leads to estimates for ρ and $\boldsymbol{\delta}$. From this the precautionary component in aggregate consumption growth can be estimated, i.e. $\widehat{\Omega}_t \equiv \widehat{\rho} \frac{1}{2} \widehat{\sigma}_\pi^2 \left[\frac{1}{N_t} \sum_{i=1}^{N_t} \left(\frac{y_{it-1}}{c_{it-1}} \right)^2 \right] + \widehat{\rho} \frac{1}{2} (1 - \widehat{\theta})^2 \left[\frac{1}{N_t} \sum_{i=1}^{N_t} \left(\widehat{V}_{t-1} \varepsilon_{it} \left(\frac{y_{it-1}}{c_{it-1}} \right)^2 \right) \right]$.

In section 5.2.1 I estimate the model without preference shifters so that no data for $\mathbf{x}_{it}$ are needed. In section 5.2.2 I do estimate the model with preference shifters and hence with data for $\mathbf{x}_{it}$ which are taken from the $PSID$. In both sections the estimates $\widehat{V}_{t-1}\varepsilon_{it}$, $\widehat{\pi}_t$, and $\widehat{\varepsilon}_{it}$ as well as values for the parameters $\widehat{\sigma}_\pi^2$ and $\widehat{\theta}$ are obtained from the first stage estimation detailed in section 3.2.

For $\Delta \ln C_t$ I use real per capita aggregate consumption growth partially corrected for Theil's entropy as discussed in section 4.1. For y_{it} I use individual earnings as described in section 3.1. For R_t I use the real 3 month Treasury Bill rate for the US as provided by International Financial Statistics (IMF).[20]

[20] It is put in real terms with use of the inflation rate as calculated from the deflator for expenditures on non-durables and services (excluding shoes and clothing) with base year 1983.

5.2.1 Specification without preference shifters

Assuming in eq.(15') that $\mathbf{x}_{it}$ is a scalar equal to -1 implies that $\boldsymbol{\delta}$ is a scalar as well. This gives,

$$\Delta \ln c_{it} = \frac{1}{\rho}R_t - \frac{1}{\rho}\delta + \rho\frac{1}{2}\left[\widehat{\sigma}_{\pi}^2 + (1-\widehat{\theta})^2\widehat{V}_{t-1}\varepsilon_{it}\right]\left(\frac{y_{it-1}}{c_{it-1}}\right)^2 + \left[\widehat{\pi}_t + (1-\widehat{\theta})\widehat{\varepsilon}_{it}\right]\left(\frac{y_{it-1}}{c_{it-1}}\right) \quad (15")$$

This is the more familiar expression for consumption growth where δ (> 0) captures time preference. Two parameters need to be estimated, i.e. ρ and δ. The results of estimating eqs.(14)-(15") with four different specifications for $V_{t-1}\varepsilon_{it}$ - an $ARCH(1)$ process, a $QARCH(1)$ process, a $GARCH(1,1)$ process, and a $GQARCH(1,1)$ process - are presented in table 4.[21] I present both the first step and second step GMM estimates. The reported standard errors are corrected for the first stage estimation of the individual and aggregate earnings processes. The degrees of freedom of the reported test for overidentifying restrictions always equal 23 since the number of moment conditions equals $T = 25$ and the number of estimated parameters equals 2. The estimated precautionary component $\widehat{\Omega}_t$ corresponding to each case presented in table 4 is presented in figure 9.

The estimates for the coefficient of relative risk aversion ρ have theoretically plausible values, are significant, and have magnitudes that are in accordance with values reported in the literature (see e.g. Banks et al. 2001). The same is true for the estimates of the degree of time preference δ. The second step GMM estimation which is based on an optimal weighting matrix gives somewhat higher estimates for ρ and lower estimates for δ compared to the first step GMM estimation which equally weighs moment conditions (i.e. time periods). The estimates are quite similar across the different specifications used for $V_{t-1}\varepsilon_{it}$ however. This is also reflected by the precautionary components estimated for each specification used for $V_{t-1}\varepsilon_{it}$ and presented in figure 9. All show a very similar evolution. This evolution is characterized by a steady decline over the period $1969-1992$ (the year 1968 is clearly affected by initialization and should be ignored).[22] A decrease in the importance of precaution for aggregate consumption growth over the period $1982-1997$ is also reported by Parker and Preston (2005) using a decomposition of US aggregate consumption growth based on CEX data. During the 1970s the precautionary component $\widehat{\Omega}_t$ shows a cyclical evolution that can

[21] The reason that the precautionary component is not calculated when ε_{it} is i.i.d in time (i.e. when $V_{t-1}\varepsilon_{it} = \sigma_{\varepsilon}^2$) is that in this case time-variation in the precautionary component depends solely on time-variation in the cross-sectional mean of $\left(\frac{y_{it-1}}{c_{it-1}}\right)^2$. There may be reasons unrelated to precaution why this variable affects aggregate consumption growth.

[22] This decrease is caused by the following mechanism: the estimated values for the parameters ρ and δ are such that consumption tends to grow somewhat faster than income leading to a reduction over time of the ratio $\frac{y_{it-1}}{c_{it-1}}$ which results in a decline of $\widehat{\Omega}_t$.

be linked to the evolution of the cross-sectional mean of $\widehat{V}_{t-1}\varepsilon_{it}$, i.e. the correlation between both is significantly positive (with values between 0.55 and 0.60 depending on the case).[23] During the 1980s this cyclical behaviour is much less pronounced.

The test statistic for overidentifying restrictions is lower for the $GARCH$ and $GQARCH$ specifications used for $V_{t-1}\varepsilon_{it}$ compared to the $ARCH$ and $QARCH$ specifications which suggests that the generalized specifications lead to a better fit of the model. In all cases, however, the overidentifying restrictions are rejected. From a statistical viewpoint this is not surprising since 25 moment conditions are expected to hold with only 2 parameters. Similar rejections have been reported by Gourichas and Parker (2002).

5.2.2 Specification with preference shifters

The estimation of a more elaborate specification with preference shifters allows to control for aggregation biases that may result from an incomplete specification.[24] I estimate eqs.(14)-(15') with two preference shifters included in $\mathbf{x}_{it}$. First, as household characteristics can be assumed to shift the preference for a given level of consumption, following Parkler and Preston (2005), I use household size as the first preference shifter (x_{it}^1). Second, to allow for non-separabilities in the utility function between consumption and hours worked, I add the log of hours worked of the household head as a second preference shifter (x_{it}^2). Other preference shifters were included (e.g. number of children) but were found to be insignificant. Since $\mathbf{x}_{it}$ is a $1x2$ vector, i.e. $\mathbf{x}_{it} = \begin{bmatrix} x_{it}^1 & x_{it}^2 \end{bmatrix}$, then $\boldsymbol{\delta}$ is a $2x1$ estimable parameter vector, i.e. $\boldsymbol{\delta} = \begin{bmatrix} \delta_1 & \delta_2 \end{bmatrix}'$. The estimation results of this extended model are presented in table 5 where again both the first step and second step GMM estimates are reported. The reported standard errors are again corrected for the first stage estimation of the individual and aggregate earnings processes. The degrees of freedom of the reported test for overidentifying restrictions always equal 22 since the number of moment conditions

[23] When compared to the business cycle the precautionary component is somewhat lagging, i.e. it remains high some time after the recession. This can be explained as follows: by the end of the recession in period $t-1$ the variable $V_{t-1}\varepsilon_{it}$ has fallen leading to slower consumption growth and a relatively low consumption level c_{it-1}. Hence, a relatively high ratio $\frac{y_{it-1}}{c_{it-1}}$ in period $t-1$ leads to a relatively high precautionary component $\widehat{\Omega}_t$ in the period after the recession.

[24] There are other potential sources of aggregation bias. First, there may be discrepancies between model based and $NIPA$ based aggregate consumption growth resulting from Theil's entropy. This was discussed and dealt with in section 4.1. Second, there may be measurement error in aggregate consumption data. Applying filtering techniques on aggregate consumption data as in Sommer (2007) I find that classical measurement error, while important in quarterly data, is not an issue in annual data.

equals $T = 25$ and the number of estimated parameters equals 3. The estimated precautionary component $\widehat{\Omega}_t$ corresponding to each case presented in table 5 is presented in figure 10.

The results from table 5 largely confirm the findings reported in table 4. The estimates for the coefficient of relative risk aversion ρ have theoretically plausible values, are significant, and have magnitudes that are in accordance with values reported in the literature. They are somewhat higher compared to those reported in table 4. The estimated precautionary components $\widehat{\Omega}_t$ presented in figure 10 again all show a very similar evolution. Similar to the components presented in figure 9 they show a steady decline. Their cyclical evolution is considerably more outspoken however. To conclude, the finding of significant and plausible risk aversion estimates as well as an important precautionary component in aggregate consumption growth seems to be robust across different specifications of the model.

6 Conclusions

This paper investigates the effects of precautionary savings motives on aggregate consumption dynamics by explicitly allowing for time-variation in earnings uncertainty. In the first step of the empirical analysis estimates for the time-varying earnings uncertainty of households in the economy are obtained with earnings data from the Panel Study of Income Dynamics ($PSID$) for individual households in the US over the period $1968 - 1992$. In a second step the cross-sectional mean of the estimated conditional variance of earnings over all households in the sample is used as a regressor in equations of aggregate consumption growth to find out whether average individual earnings uncertainty in the economy affects aggregate consumption dynamics. In a third step the consumption model is estimated by GMM methods. This provides estimates for the structural model parameters and estimates for the precautionary component, i.e. the part of aggregate consumption growth that can be attributed to the precautionary savings motive.

The results of the paper suggest, first, that the conditional variance of individual earnings can be accurately modelled by $GARCH$ type processes. Second, the cross-sectional mean of the estimated time-varying conditional variance of individual earnings is found to have a significant impact on aggregate consumption growth suggesting that earnings uncertainty and precautionary saving motives matter for the aggregate economy. Third, the estimation of the consumption model provides an estimated precautionary component in aggregate consumption growth. This component is correlated with individual earnings uncertainty, in particular during the 1970s, and shows a steady decline over the sample period.

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Appendix A.1: Derivation of eq.(2)

I write eq.(1) as,

$$(1 + R_t) \exp(\mathbf{x}_{it}\boldsymbol{\delta}) E_{t-1}(z_{it}) = 1 \tag{A1}$$

where $z_{it} = \left(\frac{c_{it}}{c_{it-1}}\right)^{-\rho}$. Note that $\ln z_{it} = -\rho\Delta \ln c_{it}$. If, as I assume, $\Delta \ln c_{it}$ is conditionally normally distributed, then $\ln z_{it}$ is also conditionally normally distributed. This implies that z_{it} is conditionally lognormally distributed. As such I can write,

$$E_{t-1}(z_{it}) = \exp\left[E_{t-1}(\ln z_{it}) + \frac{1}{2}V_{t-1}(\ln z_{it})\right] \tag{A2}$$

Substituting eq.(A2) into eq.(A1) and taking logs of both sides of the resulting equation gives,

$$R_t + \mathbf{x}_{it}\boldsymbol{\delta} + E_{t-1}(\ln z_{it}) + \frac{1}{2}V_{t-1}(\ln z_{it}) = 0 \tag{A3}$$

where I use the approximation $\ln(1 + R_t) \approx R_t$. Substituting $\ln z_{it} = -\rho\Delta \ln c_{it}$ into eq.(A3) and rearranging leads to eq.(2) in the main text.

Appendix A.2: Derivation of eq.(8)

For small values of the interest rate the results by Deaton (1992, p.110) suggest that with the earnings process given by eq.(7) - i.e. $\Delta \ln y_{it} = \kappa + \pi_t + \varepsilon_{it} - \theta\varepsilon_{t-1}$ or equivalently $\Delta y_{it} = (\kappa + \pi_t + \varepsilon_{it} - \theta\varepsilon_{it-1})y_{it-1}$ - the innovation to the *change* in consumption can be approximated by $(\pi_t + (1 - \theta)\varepsilon_{it})y_{it-1}$. Then the innovation to the *growth rate* in consumption can be written as $\eta_{it} = (\pi_t + (1 - \theta)\varepsilon_{it})\left(\frac{y_{it-1}}{c_{it-1}}\right)$.

Appendix B: Estimation of the aggregate earnings process

From section 2 the growth rate in aggregate earnings $\Delta \ln Y_t = \Lambda_t = \Delta \ln y_{it} - \Delta \ln \widetilde{y}_{it}$ is asumed to follow a martingale with drift, i.e.,

$$\Delta \ln Y_t = \kappa + \pi_t \tag{B1}$$

where κ is a constant drift term, and where π_t is the aggregate shock to earnings which is assumed to be *i.i.d.* in time. It has mean zero and variance equal to σ_π^2. The asssumption of *i.i.d.*-ness of π_t is based upon preliminary regressions with $(G)ARCH$ type specifications for the conditional variance $V_{t-1}\pi_t$. These suggest that there is no time-variation in $V_{t-1}\pi_t$. Hence $V_{t-1}\pi_t = \sigma_\pi^2$ and the unconditional distribution of π_t is Gaussian. I use ML to estimate the parameters κ and σ_π^2.[25] A time series for $\Delta \ln Y_t$ is obtained by filtering out the time fixed effects Λ_t from individual earnings growth $\Delta \ln y_{it}$. Data for the latter variable are obtained from the $PSID$ and are discussed in section 3.1. There are two advantages of using this series for $\Delta \ln Y_t$ instead of using data from the $NIPA$. First, with $NIPA$ data earnings (or labour income) have to be inferred from total income and this can be an arbitrary process. Second, and more importantly, $NIPA$ data on aggregate income are only available as arithmetic means of individual income levels. The growth rates in income are then calculated as the difference in the logs of the arithmetic means of individual income levels, i.e. as $\Delta \ln E(y_{it})$ where E is the cross-sectional mean operator. In the model however (see section 2.2) the growth rate in aggregate earnings is obtained as the mean of individual earnings growth rates. The growth rate in aggregate earnings is then calculated as the difference in the logs of the geometric means of individual earnings levels. The latter equals the difference in the arithmetic means of the logs of individual earnings levels, i.e. $\Delta E(\ln y_{it})$ which equals $E(\Delta \ln y_{it})$. As is also discussed in section 4.1 on consumption, the difference between both - $\Delta \ln E(y_{it})$ versus $E(\Delta \ln y_{it})$ - can be substantial and need not be constant.

The obtained estimates for the aggregate earnings process are $\widehat{\kappa} = 0.017$ (with Hessian based standard error 0.006) and $\widehat{\sigma}_\pi^2 = 0.001$ (with Hessian based standard error 0.0003). The magnitude of the variance of the aggregate earnings shock is rather small (e.g. compared to the magnitude of the variance of shocks to individual earnings as reported in table 1) and, as noted in section 2.4, too small to have much of an impact on aggregate consumption growth and precautionary savings. This explains why the paper focuses mostly on the impact of individual earnings uncertainty on aggregate consumption growth.

[25] The log likelihood is given by $-\frac{1}{2}(T)\ln(2 * pi) - \frac{1}{2}\sum_{t=1}^{T}\ln(\sigma_\pi^2) - \frac{1}{2}\sum_{t=1}^{T}\frac{[\Delta \ln Y_t - \kappa]^2}{\sigma_\pi^2}$ where $T = 25$.

Appendix C: Technical details of the GMM estimation

Appendix C.1: Test for overidentifying restrictions (steps 1 and 2)

The general form of the test for overidentifying restrictions - i.e. for any weighting matrix W - is given by,

$$J = \left[\sqrt{\mathbf{N}} \bullet g(\widehat{\Upsilon})\right]' pinv(V_g) \left[\sqrt{\mathbf{N}} \bullet g(\widehat{\Upsilon})\right] \tag{C1.1}$$

where $\sqrt{\mathbf{N}} = \begin{pmatrix} \sqrt{N_1} & ... & \sqrt{N_T} \end{pmatrix}'$ with N_t denoting the number of cross-sections in the sample in period t (where $t = 1, ..., T$ or, interchangeably, $t = 1968, ..., 1992$), where $\bullet$ denotes the Hadamard product operator, where $g(\widehat{\Upsilon}) = \left[g_1(\widehat{\Upsilon})...g_T(\widehat{\Upsilon})\right]'$ contains the sample moment conditions evaluated at $\widehat{\Upsilon}$, where *pinv* denotes pseudo-inversion, and where

$$V_g = \left[I - g(\widehat{\Upsilon}) \left[g(\widehat{\Upsilon})' W g(\widehat{\Upsilon})\right] g(\widehat{\Upsilon})' W\right] \widehat{S} \left[I - g(\widehat{\Upsilon}) \left[g(\widehat{\Upsilon})' W g(\widehat{\Upsilon})\right] g(\widehat{\Upsilon})' W\right]' \tag{C1.2}$$

where $\widehat{S}$ is the estimated variance-covariance matrix of the moment conditions.[26] Note that $J \overset{a}{\sim} \chi^2$ with degrees of freedom equal to the number of moment conditions minus the number of estimated parameters. Note that $\widehat{\Upsilon} = (\widehat{\Psi}|\widehat{\Phi}, \widehat{\Gamma})$, i.e. only Ψ is estimated while Φ and Γ are obtained from prior estimation. As such the number of parameters equals the dimension of Ψ. The first step GMM estimator $\widehat{\Upsilon}_{(1)}$ is obtained with $W = I$ and the corresponding test for overidentifying restrictions is given by substituting $W = I$ and $\widehat{\Upsilon} = \widehat{\Upsilon}_{(1)} = (\widehat{\Psi}_{(1)}|\widehat{\Phi}, \widehat{\Gamma})$ into eqs.(C1.1)-(C1.2) to obtain,

$$J_{(1)} = \left[\sqrt{\mathbf{N}} \bullet g(\widehat{\Upsilon}_{(1)})\right]' pinv(V_g) \left[\sqrt{\mathbf{N}} \bullet g(\widehat{\Upsilon}_{(1)})\right]$$

with

$$V_g = \left[I - g(\widehat{\Upsilon}_{(1)}) \left[g(\widehat{\Upsilon}_{(1)})' g(\widehat{\Upsilon}_{(1)})\right] g(\widehat{\Upsilon}_{(1)})'\right] \widehat{S} \left[I - g(\widehat{\Upsilon}_{(1)}) \left[g(\widehat{\Upsilon}_{(1)})' g(\widehat{\Upsilon}_{(1)})\right] g(\widehat{\Upsilon}_{(1)})'\right]'$$

It should be noted that $\widehat{S}$ is calculated with the first step estimator $\widehat{\Upsilon} = \widehat{\Upsilon}_{(1)}$. The second step GMM estimator $\widehat{\Upsilon}_{(2)} = (\widehat{\Psi}_{(2)}|\widehat{\Phi}, \widehat{\Gamma})$ is obtained with $W = \widehat{S}^{-1}$ in which case eq.(C1.1) simplifies to

$$J_{(2)} = \left[\sqrt{\mathbf{N}} \bullet g(\widehat{\Upsilon}_{(2)})\right]' W \left[\sqrt{\mathbf{N}} \bullet g(\widehat{\Upsilon}_{(2)})\right]$$

where $W = \widehat{S}^{-1}$.

[26] The estimated variance-covariance matrix of the moment conditions $\widehat{S}$ is a TxT matrix where each element $\widehat{S}_{jk}$ (with $j = 1, ..., T$ and $k = 1, ..., T$) is given by $\widehat{S}_{jk} = \frac{1}{N_{j,k}} \sum_{i=1}^{Nj,k} (\Delta \ln c_{ij}(\widehat{\Upsilon}) - \Delta \ln C_j)(\Delta \ln c_{ik}(\widehat{\Upsilon}) - \Delta \ln C_k)$ where $N_{j,k}$ equals the number of cross-sections that are in the panel both at time j and at time k. When $j = k$ then $N_{j,k} = N_j = N_k$.

Appendix C.2: Asymptotic standard errors for GMM estimation (steps 1 and 2)

Since $\widehat{\Upsilon} = (\widehat{\Psi}|\widehat{\Phi}, \widehat{\Gamma})$ - i.e. only Ψ is estimated while Φ and Γ are obtained from prior estimation - the estimated asymptotic variance-covariance matrix of the GMM estimator $\widehat{\Upsilon}$ is given by,

$$\widehat{V}_{\Psi} = \left[(g_{\Psi})'W(g_{\Psi})\right]^{-1}(g_{\Psi})'W\widehat{S}W(g_{\Psi})\left[(g_{\Psi})'W(g_{\Psi})\right]^{-1} \tag{C2.1}$$

where $\widehat{V}_{\Psi}$ has a dimension equal to the number of elements in Ψ, where $\widehat{S}$ is the estimated variance-covariance matrix of the moment conditions, and where $g_{\Psi} = \left(\sqrt{\mathbf{N}}_{\Psi}\right) \bullet \left(\frac{\partial g(\widehat{\Upsilon})}{\partial \Psi'}\right)$ with $g(\widehat{\Upsilon})$ containing the sample moment conditions evaluated at $\widehat{\Upsilon}$, with $\bullet$ being the Hadamard product operator, and with $\sqrt{\mathbf{N}}_{\Psi}$ being a matrix with number of columns equal to the number of elements in Ψ and with each column equal to the vector $\sqrt{\mathbf{N}}$. For example, if Ψ contains 2 parameters, ρ and δ, then $\frac{\partial g(\widehat{\Upsilon})}{\partial \Psi'} = \begin{bmatrix} \frac{\partial g_1}{\partial \rho} & ... & \frac{\partial g_T}{\partial \rho} \\ \frac{\partial g_1}{\partial \delta} & ... & \frac{\partial g_T}{\partial \delta} \end{bmatrix}'$ and $\sqrt{\mathbf{N}}_{\Psi} = \begin{bmatrix} \sqrt{N_1} & ... & \sqrt{N_T} \\ \sqrt{N_1} & ... & \sqrt{N_T} \end{bmatrix}'$. The first step variance-covariance matrix $\widehat{V}_{\Psi(1)}$ is given by substituting $W = I$ and $\widehat{\Upsilon} = \widehat{\Upsilon}_{(1)} = (\widehat{\Psi}_{(1)}|\widehat{\Phi}, \widehat{\Gamma})$ into eq.(C2.1) to obtain,

$$\widehat{V}_{\Psi(1)} = \left[(g_{\Psi})'(g_{\Psi})\right]^{-1}(g_{\Psi})'\widehat{S}(g_{\Psi})\left[(g_{\Psi})'(g_{\Psi})\right]^{-1}$$

where $g_{\Psi} = \left(\sqrt{\mathbf{N}}_{\Psi}\right) \bullet \left(\frac{\partial g(\widehat{\Upsilon}_{(1)})}{\partial \Psi'}\right)$. It should be noted that $\widehat{S}$ is calculated with the first step estimator $\widehat{\Upsilon} = \widehat{\Upsilon}_{(1)}$. The second step variance-covariance matrix $\widehat{V}_{\Psi(2)}$ is given by substituting $W = \widehat{S}^{-1}$ and $\widehat{\Upsilon} = \widehat{\Upsilon}_{(2)} = (\ \widehat{\Psi}_{(2)}|\widehat{\Phi}, \widehat{\Gamma})$ into eq.(C2.1) to obtain,

$$\widehat{V}_{\Psi(2)} = \left[(g_{\Psi})'W(g_{\Psi})\right]^{-1}$$

where $W = \widehat{S}^{-1}$ and where $g_{\Psi} = \left(\sqrt{\mathbf{N}}_{\Psi}\right) \bullet \left(\frac{\partial g(\widehat{\Upsilon}_{(2)})}{\partial \Psi'}\right)$.

Appendix C.3: Corrected asymptotic standard errors for two stage estimation

The asymptotic standard errors for $\widehat{\Psi}$ as calculated from the estimated asymptotic variance-covariance matrices $\widehat{V}_{\Psi(1)}$ or $\widehat{V}_{\Psi(2)}$ in Appendix C.2 are not corrected for the fact that the parameters $\widehat{\Phi}$ and $\widehat{\Gamma}$ are not known with certainty but are obtained from prior estimation. Following Gourinchas and Parker (2002, Appendix B, equation 17) the following "corrected" variance-covariance matrix of the moment conditions for the GMM estimator $\widehat{\Psi}$ can be used,

$$\widehat{S} = \widehat{S}^{*} + g_{\Phi}\widehat{V}_{\Phi}g'_{\Phi} + g_{\Gamma}\widehat{V}_{\Gamma}g'_{\Gamma} \tag{C3.1}$$

The matrix $\widehat{S}^*$ is the estimated "uncorrected" variance-covariance matrix of the moment conditions.

The first correction is given by the term $g_\Phi \widehat{V}_\Phi g'_\Phi$ and is used to correct for the prior estimation of the parameters of the individual earnings process $\Phi = (\ \theta \quad \sigma^2_\varepsilon \quad \alpha \quad \beta \quad \gamma\ )'$ as detailed in section 3.2. The matrix g_Φ is given by $g_\Phi = \left(\sqrt{\mathbf{N}}_\Phi\right) \bullet \left(\frac{\partial g(\widehat{\Upsilon})}{\partial \Phi'}\right)$ where $g(\widehat{\Upsilon})$ contains the sample moment conditions evaluated at $\widehat{\Upsilon}$, where $\bullet$ is the Hadamard product operator, and where $\sqrt{\mathbf{N}}_\Phi$ is a matrix with number of columns equal to the number of elements in Φ and with each column equal to the vector $\sqrt{\mathbf{N}}$. The matrix $\widehat{V}_\Phi$ is the estimated asymptotic variance-covariance matrix for the maximum likelihood estimate $\widehat{\Phi}$ obtained in the first stage.

The second correction is given by the term $g_\Gamma \widehat{V}_\Gamma g'_\Gamma$ and is used to correct for the prior estimation of the parameters of the aggregate earnings process $(\ \kappa \quad \sigma^2_\pi\ )'$ as detailed in Appendix B. The matrix g_Γ is given by $g_\Gamma = \left(\sqrt{\mathbf{N}}_\Gamma\right) \bullet \left(\frac{\partial g(\widehat{\Upsilon})}{\partial \Gamma'}\right)$ where $g(\widehat{\Upsilon})$ contains the sample moment conditions evaluated at $\widehat{\Upsilon}$, where $\bullet$ is the Hadamard product operator, and where $\sqrt{\mathbf{N}}_\Gamma$ is a matrix with number of columns equal to the number of elements in Γ and with each column equal to the vector $\sqrt{\mathbf{N}}$. The matrix $\widehat{V}_\Gamma$ is the estimated asymptotic variance-covariance matrix for the maximum likelihood estimate $\widehat{\Gamma}$ obtained in the first stage.

In Gourinchas and Parker's correction the first stage and second stage estimators are both GMM estimators. It is therefore important to note that the first stage ML estimator for Φ (respectively Γ) is equivalent to an exactly identified GMM estimator, i.e. maximization of the log likelihood with respect to Φ (respectively Γ) leads to a number of first-order conditions which are basically moment conditions. The number of moment conditions equals the number of parameters in Φ (respectively Γ). Furthermore, it can be shown that the estimated asymptotic variance-covariance matrix of the ML estimator is identical to that of the equivalent exactly identified GMM estimator.

Finally it should be noted that $\widehat{S}$ is calculated with the first step estimator $\widehat{\Upsilon} = \widehat{\Upsilon}_{(1)}$. As noted before the "uncorrected" $\widehat{S}^*$ is calculated with the first step estimator $\widehat{\Upsilon} = \widehat{\Upsilon}_{(1)}$ and this also holds for the matrices g_Φ and g_Γ. To obtain the "corrected" tests for overidentifying restrictions and the "corrected" asymptotic standard errors for the first and second step GMM estimator of Ψ it then suffices to use $\widehat{S}$ as defined in eq.(C3.1) into the expressions for $J_{(1)}$ and $J_{(2)}$ derived in Appendix C.1 and into the expressions for $\widehat{V}_{\Psi(1)}$ and $\widehat{V}_{\Psi(2)}$ derived in Appendix C.2.

Tables and figures

Table 1. Panel maximum likelihood estimation of eqs.(10)-(11) for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968 - 1992)$

	Constant	*ARCH*	*QARCH*	*GARCH*	*GQARCH*
θ	0.391	0.345	0.340	0.341	0.330
	(0.006)	(0.008)	(0.008)	(0.009)	(0.008)
σ_ε^2	0.099	0.166	0.165	0.137	0.135
	(0.001)	(0.005)	(0.005)	(0.003)	(0.003)
α	-	0.716	0.712	0.400	0.392
		(0.009)	(0.009)	(0.009)	(0.009)
β	-	-	-	0.491	0.497
				(0.011)	(0.011)
γ	-	-	-0.076	-	-0.066
			(0.004)		(0.003)
AIC	0.517	0.246	0.236	0.183	0.165

Notes: Hessian based standard errors between brackets. AIC denotes the Akaike information criterion of model comparison (lower values of AIC are preferred).

Table 2. Ordinary least squares estimation of eq.(13) for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968-1992)$ - Results without Theil's entropy correction on $\Delta \ln C_t$

	$ARCH$	$QARCH$	$GARCH$	$GQARCH$
a_0	-0.015	-0.015	-0.028	-0.028
	(0.010)	(0.010)	(0.013)	(0.013)
a_1	0.308	0.311	0.459	0.467
	(0.074)	(0.075)	(0.106)	(0.111)
R^2	0.231	0.232	0.285	0.285
AIC	-5.990	-6.017	-6.089	-6.090
DW	1.529	1.537	1.645	1.658

Notes: Newey-West standard errors between brackets. AIC denotes the Akaike information criterion of model comparison (lower values of AIC are preferred). DW denotes the Durbin Watson test statistic.

Table 3. Ordinary least squares estimation of eq.(13) for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968-1992)$ - Results with Theil's entropy correction on $\Delta \ln C_t$

	$ARCH$	$QARCH$	$GARCH$	$GQARCH$
a_0	-0.011	-0.012	-0.023	-0.024
	(0.011)	(0.011)	(0.014)	(0.014)
a_1	0.269	0.272	0.408	0.419
	(0.078)	(0.079)	(0.114)	(0.118)
R^2	0.174	0.175	0.222	0.226
AIC	-5.930	-5.932	-5.990	-5.995
DW	1.367	1.374	1.463	1.479

Notes: Newey-West standard errors between brackets. AIC denotes the Akaike information criterion of model comparison (lower values of AIC are preferred). DW denotes the Durbin Watson test statistic.

Table 4. GMM estimation of eqs.(14)-(15") - model without preference shifters - for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968-1992)$

	$ARCH$	$QARCH$	$GARCH$	$GQARCH$
First step GMM estimates				
ρ	1.618	1.611	1.657	1.622
	(0.201)	(0.195)	(0.247)	(0.247)
δ	0.053	0.052	0.050	0.049
	(0.008)	(0.008)	(0.008)	(0.008)
$Test$	439.2	443.9	414.9	417.6
Second step GMM estimates				
ρ	1.746	1.746	1.817	1.792
	(0.093)	(0.091)	(0.098)	(0.093)
δ	0.042	0.041	0.036	0.035
	(0.006)	(0.006)	(0.006)	(0.006)
$Test$	445.1	450.2	420.1	423.1

Notes: Asymptotic standard errors between brackets. The standard errors are corrected for the first stage estimation of the individual and aggregate earnings processes as detailed in appendix C.3. Test denotes the test statistic of the test for overidentifying restrictions. With degrees of freedom equal to 23 (i.e. 25 moment conditions minus 2 estimated parameters) the critical value at 5% is 35.2.

Table 5. GMM estimation of eqs.(14)-(15') - model with preference shifters - for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968 - 1992)$

	$ARCH$	$QARCH$	$GARCH$	$GQARCH$
First step GMM estimates				
ρ	1.804	1.807	1.839	1.804
	(0.238)	(0.235)	(0.312)	(0.310)
δ_1	-0.019	-0.020	-0.017	-0.018
	(0.009)	(0.009)	(0.011)	(0.012)
δ_2	0.002	0.002	0.002	0.002
	(0.004)	(0.004)	(0.005)	(0.005)
$Test$	417.7	419.7	368.6	361.0
Second step GMM estimates				
ρ	1.921	1.937	2.196	2.220
	(0.133)	(0.134)	(0.174)	(0.171)
δ_1	-0.024	-0.025	-0.033	-0.036
	(0.006)	(0.006)	(0.007)	(0.007)
δ_2	0.005	0.005	0.009	0.010
	(0.002)	(0.002)	(0.003)	(0.003)
$Test$	426.2	428.9	389.3	386.8

Notes: Asymptotic standard errors between brackets. The standard errors are corrected for the first stage estimation of the individual and aggregate earnings processes as detailed in appendix C.3. Test denotes the test statistic of the test for overidentifying restrictions. With degrees of freedom equal to 22 (i.e. 25 moment conditions minus 3 estimated parameters) the critical value at 5% is 33.9.

Figure 1: The unconditional distribution of $\ln(y)$

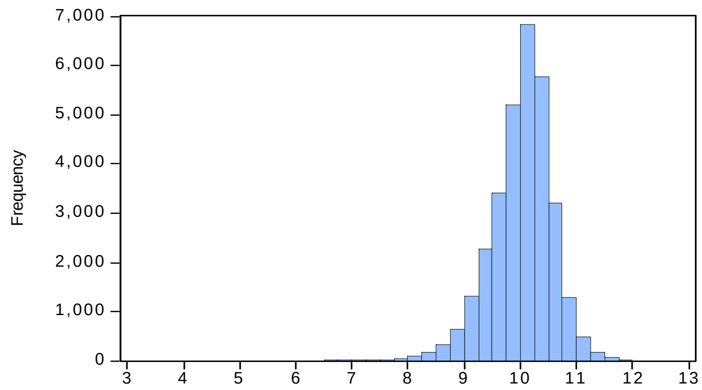


Figure 2: The unconditional distribution of $\Delta \ln(y)$

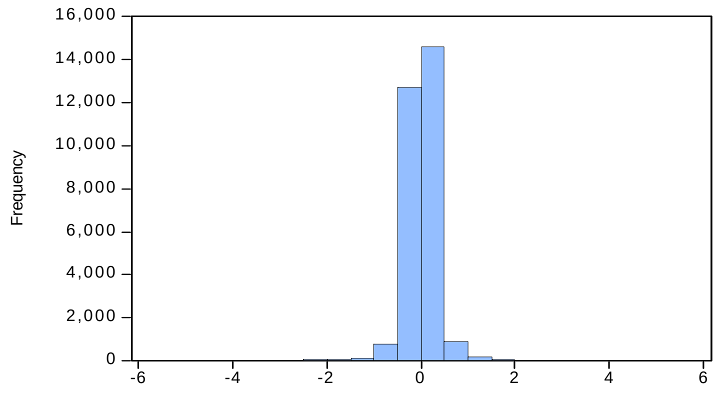
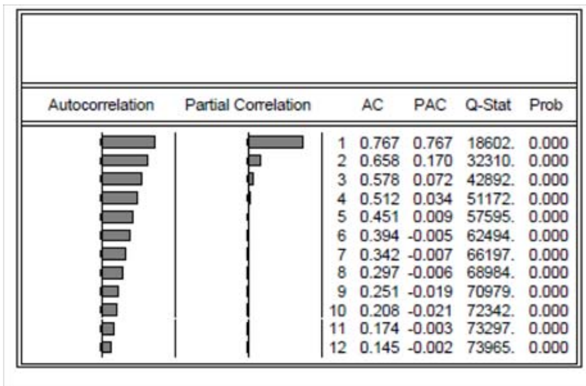
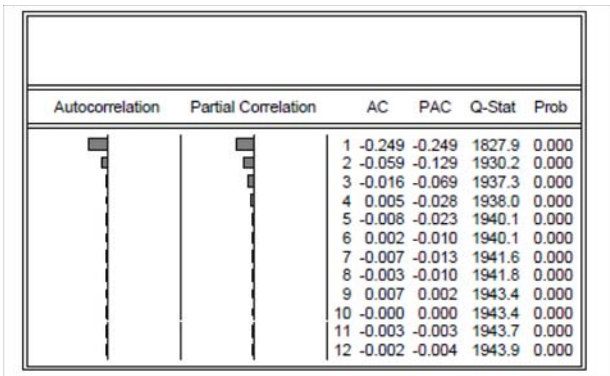


Figure 3: Autocorrelations and partial correlations of $\ln(y)$ at lag lengths 1-12



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.767	0.767	18602.	0.000
		2	0.658	0.170	32310.	0.000
		3	0.578	0.072	42892.	0.000
		4	0.512	0.034	51172.	0.000
		5	0.451	0.009	57595.	0.000
		6	0.394	-0.005	62494.	0.000
		7	0.342	-0.007	66197.	0.000
		8	0.297	-0.006	68984.	0.000
		9	0.251	-0.019	70979.	0.000
		10	0.208	-0.021	72342.	0.000
		11	0.174	-0.003	73297.	0.000
		12	0.145	-0.002	73965.	0.000

Figure 4: Autocorrelations and partial correlations of $\Delta\ln(y)$ at lag lengths 1-12



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.249	-0.249	1827.9	0.000
		2	-0.059	-0.129	1930.2	0.000
		3	-0.016	-0.069	1937.3	0.000
		4	0.005	-0.028	1938.0	0.000
		5	-0.008	-0.023	1940.1	0.000
		6	0.002	-0.010	1940.1	0.000
		7	-0.007	-0.013	1941.6	0.000
		8	-0.003	-0.010	1941.8	0.000
		9	0.007	0.002	1943.4	0.000
		10	-0.000	0.000	1943.4	0.000
		11	-0.003	-0.003	1943.7	0.000
		12	-0.002	-0.004	1943.9	0.000

Figure 5: Aggregate consumption growth with and without Theil's entropy correction (1968−1992)

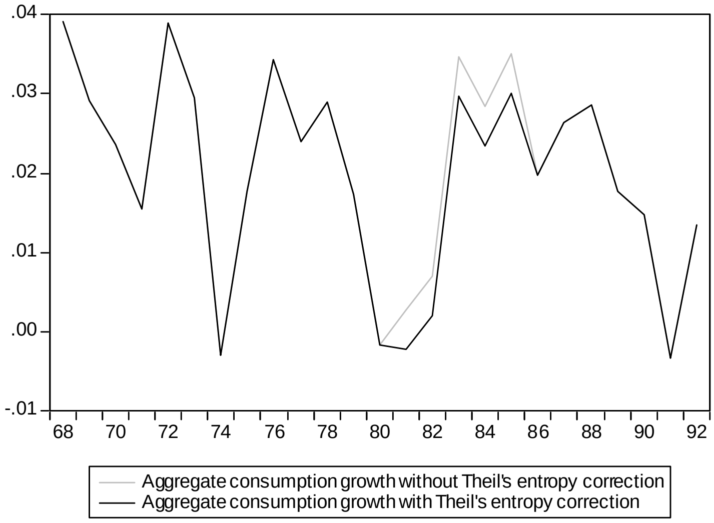


Figure 6: The cross-sectional mean of $\widehat{V}_{t-1}\varepsilon_{it}$ for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968 - 1992)$

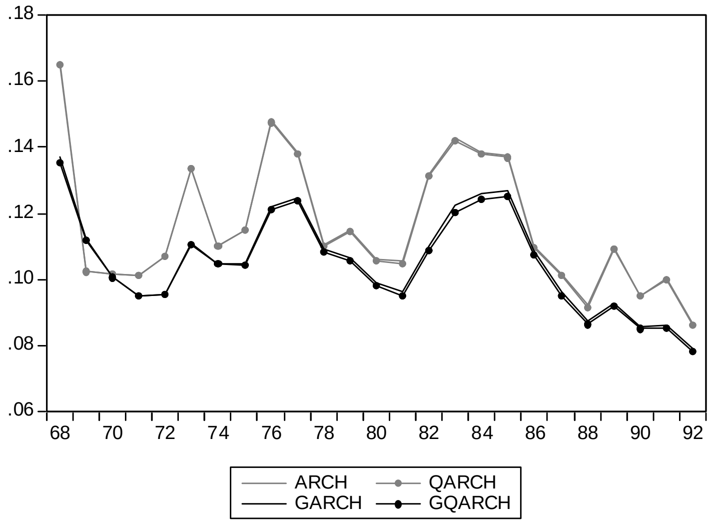


Figure 7: Fitted and actual aggregate consumption growth from estimation of eq.(13) with a $GQARCH$ specification used for $V_{t-1}\varepsilon_{it}$ $(1968-1992)$ - Results without Theil's entropy correction on $\Delta \ln C_t$ (see table 2)

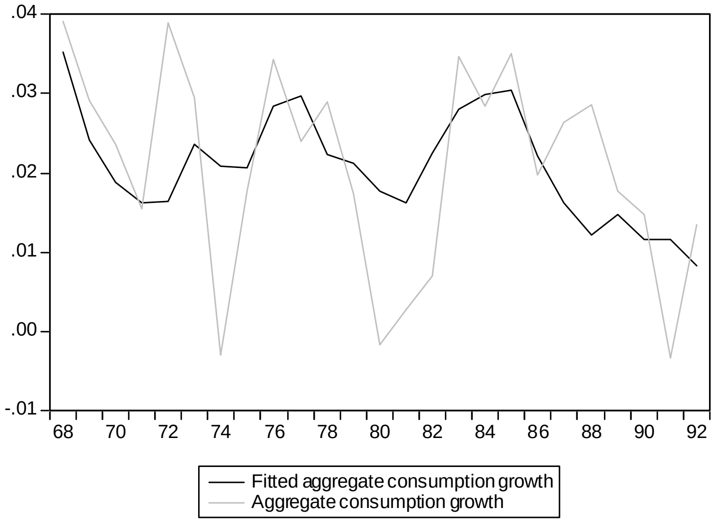


Figure 8: Fitted and actual aggregate consumption growth from estimation of eq.(13) with a $GQARCH$ specification used for $V_{t-1}\varepsilon_{it}$ $(1968-1992)$ - Results with Theil's entropy correction on $\Delta \ln C_t$ (see table 3)

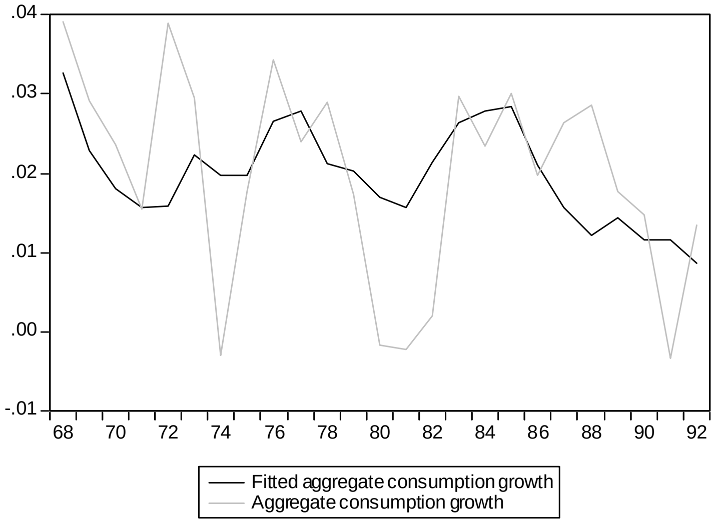


Figure 9: The estimated precautionary component $\widehat{\Omega}_t$ for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968-1992)$ - Results from the model without preference shifters (table 4, second step GMM estimates)

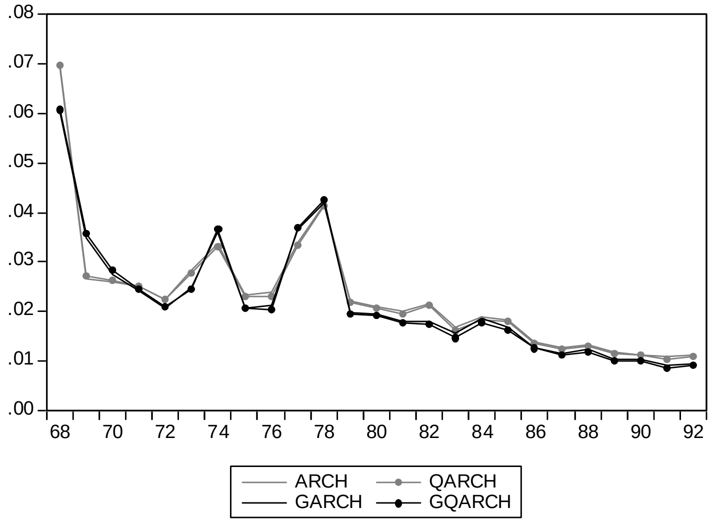


Figure 10: The estimated precautionary component $\widehat{\Omega}_t$ for different specifications of the conditional variance $V_{t-1}\varepsilon_{it}$ $(1968-1992)$ - Results from the model with preference shifters (table 5, second step GMM estimates)