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Sovereign Default and the Stability of Inflation Targeting Regimes

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Abstract

We analyse the impact of interactions between monetary and fiscal policy on macroeconomic stability. We find that in the presence of sovereign default beliefs a monetary policy, which aims to stabilize inflation through an active interest rate policy, will destabilize the economy if the feedback from debt surprises back to the primary surplus is too weak. This result, which relies on endogenous changes in the default premium, is at odds with the results in an environment without default risk, where an active monetary policy guarantees macroeconomic stability. The results are highly relevant for the design of fiscal and monetary policy in emerging markets where sovereign credibility is not well established. Recent debt developments in Western Europe and in the US suggest these results might become relevant for more mature financial markets too.

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1. Introduction

Can inflation targeting itself become a source of macroeconomic instability? In the last two decades inflation targeting has become the preferred modus operandi of central bankers across the world, with wide support from the academic community. The practice has rapidly gained ground not only in developed countries but also among emerging market economies (IMF, 2005, OECD, 2008). Yet some have argued that it might be an unsuitable strategy for countries with high sovereign debt[2] and as yet shaky reputations as inflation fighters. Recent post-credit-crisis developments have shown that such questions have become relevant in more mature financial markets too, as rescue and stimulus packages have led to rapid increases in deficits and debt levels in Western Europe and the USA.

The implementation of inflation targeting is premised on the assumption that high real rates slow down inflation; in that case a mean reversion to the inflation target is likely to be a stable process. But if for any reason high real rates do not slow down inflation, the possibility of an unstable process can emerge. For example if high real rates and the ensuing increase in debt service burden lead to higher default fears, capital outflows, and pressure on the exchange rate, a perverse impact on inflation cannot be excluded. If the active interest rate policy would be maintained nevertheless, such a perverse effect can clearly become an element of instability. Blanchard (2005) has suggested this possibility for Brazil. Recent crisis experiences in Turkey have led to similar fears as argued by Budina and van Wijnbergen (2007) and confirmed by Kirchner and Rieth (2010), who provide Bayesian estimations of this paper's model for Turkey. An analysis of the potentially destabilizing impact of such interactions thus seems of particular relevance for economies where the reputation of fiscal solidity is not well established, be they emerging market economies or more mature countries in the aftermath of the credit crisis, like Greece in 2009.

The literature on inflation targeting is too large to survey even in summary; an overview is given by Svensson (2005). At the heart of its theoretical foundation is the idea that the central bank should minimize fluctuations in inflation and the output-gap, which are costly because of the existence of price rigidities (see Svensson and Woodford, 2005). While the central idea is in principle not related to any particular policy instrument, inflation targeting is commonly associated with the use of interest rate feedback rules. The idea is that a central bank should adjust this policy instrument in response to an increase in expected

[2] See Blanchard, 2005, or Sims, 2004.

inflation in a way that reduces aggregate demand enough to stabilize inflation around its target value. The consensus view from that literature is that macroeconomic stability will be assured as long as interest rates are set according to the Taylor principle, i.e., respond to inflation by more than one for one (see Woodford, 2003). Feedback effects of debt service costs on the default probability, and the possibility of emerging stability problems, have not been considered in this literature.

These feedback effects (from debt service costs to default risk premia and from there back to debt service costs) are at the core of this paper. We set up a mostly standard model of a small open economy with a floating exchange rate and perfect international capital markets, where a rigidity in domestic producer prices is the main macroeconomic distortion. This implies that the central bank should stabilize domestic producer prices instead of the CPI (Gali and Monacelli, 2005). The government follows a tax rule like in Bohn (1998), with at least some feedback from higher debt levels on taxation. Such a rule guarantees intertemporal solvency in the sense of Bohn (1998), but may imply rates of taxation that are perceived as politically infeasible (a so-called fiscal limit, in the language of Davig and Leeper (2010) and Davig et al. (2010)). We consider sovereign default as the outcome of a preemption game between governments facing such a fiscal limit and speculators, which is known to lead to randomized default decisions as an optimal mixed strategy (see Benabou (1989) or Pastine (2002)). Speculators rationally incorporate the Bayesian decision rule of the government as their prior on the distribution of the fiscal limit and the associated probability of default. The resulting perceived probability of default is increasing in real government debt, which commands overwhelming empirical support (see Edwards, 1984, Eichengreen and Mody, 2000, Ferucci, 2003, Manganelli and Wolswijk, 2009, Codogno et al., 2003, Bernoth et al., 2006, Akitobi and Stratmann, 2008, or Schuknecht et al., 2008). We assume an independent monetary authority that follows a simple inflation targeting rule.

Within this environment, we analyze the stability implications of a standard interest rate rule by which the nominal interest rate is increased in response to changes in (domestic producer price) inflation. We show that in the absence of a sufficient feedback from debt surprizes on the primary surplus, an active interest rate policy will render a stable equilibrium impossible. Only when there is a sufficiently strong feedback from higher debt levels on higher primary surpluses (in our context higher taxes) stability can be restored. The more crisis-prone the country is (in a manner defined in the paper), the stronger that feedback needs to be. If, however, the debt feedback is not strong enough, a non-exploding debt path requires a passive interest rate policy. This result resembles Leeper's (1991) conditions for stable local equilibria with stationary public debt.[3] Since tax rates respond positively to increases in public debt, so-called non-Ricardian fiscal policies are ruled out (Bohn 2007). However, Ricardian equivalence does not apply here, since changes in real debt alter default expectations and thereby the effective rate of return on government bonds. If Ricardian equivalence would apply, a non-stationary debt sequence can be consistent with stationary sequences for the equilibrium allocation as long as it does not violate the household's transversality condition. Here, however, a stable equilibrium can only exists if the debt sequence is also stable, due to its non-neutrality. Our analysis shows that if real debt affects default expectations, stability under a tax policy which responds too weakly to public debt, requires a passive monetary policy, even if fiscal policy is "Ricardian".

We find that the conditions for macroeconomic stability do not depend on the openness of the economy. When higher interest rates raise public debt and the perceived default probability, the fall in the effective real rate of return does not only affect the exchange rate but can also reduce domestic savings. Hence, an inverse response of inflation to an increase in interest rates is also possible in an economy which is less open and where public debt is mainly held by domestic households. Our analysis further suggests that the destabilizing effect of active interest rate policies is also relevant in the case where the government issues debt that is denominated in foreign currency.[4]

It should be noted that the analysis in this paper does not imply that inflation targeting is per-se a source of macroeconomic instability under a weak fiscal policy and fears of sovereign default. Instead, the results described above only apply to the case where the central bank aims at implementing an inflation targeting policy by setting the interest rate. If however an inflation targeting policy is implemented via contingent money supply adjustments, the fiscal policy stance is less crucial (see Schabert and van Wijnbergen, 2006).

In the final part of the paper, we demonstrate that monetary and fiscal policy interactions are not only relevant for the existence of a stable and unique equilibrium, but affect macroeconomic volatilities as well. In particular, we find that higher feedback from debt on the primary surplus can improve the inflation-to-output trade-off faced by the central bank. Our results therefore provide formal backing for the claim often heard from Central Bankers, that loose fiscal policy reduces the leeway a central bank has in pursuing its anti-inflation goals.

[3] Leith and Wren-Lewis (2000) derive similar conditions for an overlapping generations model where fiscal policy also matters for equilibrium determination.

The remainder of this paper is organized as follows. Section 2 develops the model. In section 3 we analyze macroeconomic stability in the presence of endogenous default premia. In section 4 we examine the impact of fiscal policy on the central bank's inflation-to-output trade-off. Section 5 concludes.

2. A small open economy model

In this section we present a model of a small open economy that is mostly standard[5] except for the treatment of sovereign default. Domestic and foreign households have access to a complete set of contingent claims on foreign currency and to domestic currency denominated public debt. For simplicity we neglect holdings of money in this section and assume that the economy is cashless[6], without loss of generality. Nominal (real) variables are denoted by large (small) letters.

2.1 The Public sector

The public sector consists of two parts, the government and an independent central bank. The government levies lump-sum taxes $P_t \tau_t$ on domestic households (P_t denotes the price level of the aggregate consumption good), purchases goods g_t, and issues one-period discount bonds B_t. Domestic government debt is internationally traded and either held by domestic households $(B_{H,t})$ or by foreign households ($B_{F,t}$): $B_t = B_{H,t} + B_{F,t}$. At the beginning of each period *t* the government issues new bonds B_t to finance purchases of goods and outstanding debt obligations. The domestic currency price $1/R_t$ of government bonds is set by the central bank (see below), where each unit of debt B_{t-1} issued in *t*-1 leads to a promised payoff of one unit of the domestic currency in period *t*.

Following Bohn (1998), we assume that the government follows a simple fiscal rule for its core tax policy $\tilde{\tau}_t$. These taxes are raised in a lump-sum way up to a fraction $\kappa > 0$ of the outstanding stock of debt in excess of a (possibly time varying) target level B^*_{t-1}:

$$(1) \quad P_t \tilde{\tau}_t = \kappa \cdot \left(B_{t-1} - B^*_{t-1} \right) \text{ where } \kappa \in (0,1].$$

We account for the possibility of sovereign default and its role for macroeconomic stability

[4] Details on the conditions for macroeconomic stability for the indexed debt case are available upon request from the authors.

[5] cf for example Gali and Monacelli (2005).

using a deliberately simple model. If the core tax policy $\tilde{\tau}_t$ requires a level of taxation in excess of a level deemed politically inacceptable by the government, as is unintentionally possible in this stochastic setup, it defaults on its debt obligations for that period rather than seeing taxes rise to politically unacceptable levels. This implies a "fiscal limit" $\overline{T}$ in the language of Davig and Leeper (2010) and Davig et al. (2010). Define a default indicator Δ_t: if Δ_t equals 1, there is a sovereign default (i.e. B_{t-1} goes unpaid), if $\Delta_t = 0$, debt is serviced as scheduled: thus $\Delta_t = 1$ (0) when core tax policy $\tilde{\tau}_t$ exceeds (falls short of) $\overline{T}$.

We consider the decision to default (or, equivalently, the decision what, given realisations of stochastic shocks, constitutes a politically unacceptable level of taxation), as a preemption game between the government and speculators. Such a game between the government and speculators has a mixed strategy outcome, as shown in Benabou (1989) or Pastine (2002). Thus Δ_t is chosen randomly[7]: The Bayesian outcome implies a distribution for Δ_t, or, equivalently, a distribution for $\overline{T}$ with the decision rule for Δ_t as stated above (default whenever core tax policy would imply $\tilde{\tau}_t > \overline{T}$). We parameterize this Bayesian rule by a probability density function $f(\overline{T})$ for $\overline{T}$. For sake of generality we do not impose any restriction on *f* other than that it is a proper pdf.

The true value of Δ_t is unknown until the moment debt service on B_{t-1} comes due. We furthermore assume that the gains due to default are handed out in lump sum fashion, specifically not proportional to the holdings of B_{t-1}. Core tax policy $\tilde{\tau}_t$ equals the level of taxes that would have resulted if no default would have occurred. And τ_t denotes the actual level of taxes, i.e. net of the lump sum hand outs that take place after a default:

$$(2) \quad P_t \tau_t = P_t \tilde{\tau}_t - \Delta_t B_{t-1}.$$

Thus, the ex post public sector budget constraint is:

$$(3) \quad B_t R_t^{-1} + P_t \tau_t = P_t g_t + \left(1 - \Delta_t\right) B_{t-1}, \quad \text{where } B_t = B_{H,t} + B_{F,t}.$$

Suppose, for simplicity, that $B^*_{t-1} = 0$. With (3), and (1), public debt evolves according to $B_t / R_t = P_t g + \left(1-\kappa\right) B_{t-1}$, such that nominal debt grows with a rate that is smaller than the nominal interest rate. It can easily be shown that this in principle guarantees intertemporal

6 See Woodford (2003) for a discussion of this approach.

7 In Davig et al. (2010a) the fiscal limit is given exogenously; there the government randomizes its adjustment policies once stochastics threaten to drive deficits over that fiscal limit.

government solvency, i.e. $\lim_{k\to\infty}\left(B_k / R_k\right)\prod_{i=1}^{k} R_{i-1}^{-1} = 0$ [8], for any finite initial value B_{-1} (see also Bohn (1992). Hence, fiscal policy is consistent with the households' unwillingness to support a government Ponzi-game (i.e. satisfies the transversality condition) and satisfies the preconditions for Ricardian equivalence to hold.[9] Alternatively, this condition can be seen as a capital market participation constraint without which the government could not place its debt. However, public debt will be non-neutral for the equilibrium allocation given that the investors' rationally perceived default probability depends on the stock of outstanding debt (see below). Since we are interested in the impact of structural interactions of monetary and fiscal policy on macroeconomic stability, exogenous government goods purchases are irrelevant for the analysis; we therefore assume $g_t = g$.

The central bank controls the nominal interest rate R_t on government bonds. We assume that the central bank sets R_t in a state contingent way, according to a simple feedback rule, i.e. it sets the nominal interest rate on government bonds contingent on changes in domestic producer price inflation $\pi_{H,t}$:

$$(4)\quad R_t - \overline{R} = R(\pi_{H,t} - \overline{\pi}_H), \qquad R' \geq 0,\ R_t > 1,$$

where the central bank sets the target inflation rate $\overline{\pi}_H$ and considers an average interest rate $\overline{R}$ that is consistent with the steady state given their inflation target $\overline{\pi}_H$. Gali and Monacelli (2005) show that for the special case of unit intra- and intertemporal substitution elasticities, when imperfectly set domestic producer prices are the main distortion, monetary policy should indeed aim at stabilizing the domestic price inflation rate, not the CPI inflation rate.[10]

1.2 The Private sector

Investors' beliefs

Investors, i.e., domestic and foreign households, expect sovereign defaults to occur with a certain probability. According to their beliefs, defaults occur when servicing the debt would require the politically infeasible level of taxation $\overline{T}$ (the so-called "fiscal limit" in the language of Davig et al. (2010a,b). Lenders do not know $\overline{T}$, but have a prior on its distribution summarized by $f(\overline{T})$. Since we assume rational expectations, this pdf is identical

[8] As long as B* grows at less than the nominal rate of interest, this also holds for non-zero and possibly time-varying target levels of debt B*.

[9] Hence, "non-Ricardian" policy regimes are ruled out. See Kocherlakota and Phelan (1999) for an overview and Buiter (2002) or Niepelt (2004) for critical assessments of these regimes.

to the pdf characterizing the Bayesian game played by the government when setting the default indicator Δ_t. Given that tax revenues $\tilde{\tau}_t$ are set according to (3), the probability of default then equals the probability that the tax rule implies a level of $\tilde{\tau}_t$ exceeding $\overline{T}$:

(5)
$$\delta_t = \int_0^{\tilde{\tau}_t} f(\overline{T})d\overline{T} = \int_0^{k(B_{t-1}-B^*_{t-1})/P_t} f(\overline{T})d\overline{T}$$

For a differentiable distribution function $f(\overline{T})$ the impact of real debt on the default rate satisfies:

(6)
$$\frac{\partial \delta_t}{\partial (B_{t-1}/P_t)} = \kappa f(\kappa(B_{t-1} - B^*_{t-1})/P_t) > 0$$

Thus, the perceived default probability strictly increases with the real value of beginning of period debt. For the local analysis of the model we will repeatedly use the product of the default elasticity with respect to the real value of public debt B_{t-1}/P_t at the steady state with the ratio $\delta/(1-\delta)$, which we call Φ : $\Phi = \delta' \frac{b}{\pi} \frac{1}{1-\delta} > 0$. For simplicity we will refer to the latter as the default elasticity. It should be obvious that δ_t equals the investors' expectation of the value of Δ_t.

Domestic households

Assume a continuum of infinitely lived domestic households, with identical asset endowments, time endowments, and preferences. Their consumption basket c_t is an aggregate of domestically produced goods c_H and foreign goods c_F; $c_t = \gamma c_{H,t}^{1-\vartheta} c_{F,t}^{\vartheta}$, where $0 \le \vartheta \le 1$ and $\gamma = [\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}]^{-1}$. This leads to the standard share equations

$$c_{H,t} = (1-\vartheta)\left(\frac{P_{H,t}}{P_t}\right)^{-1} c_t, \qquad c_{F,t} = \vartheta\left(\frac{P_{F,t}}{P_t}\right)^{-1} c_t,$$

where $P_{H,t}$ and $P_{F,t}$ are the price indices of the domestically produced and foreign consumption goods, respectively. ϑ is the import share, our preference for the other country's goods. The price index of the aggregate consumption good (CPI) is:

[10] Moreover, stabilizing the CPI raises the likelihood of equilibrium multiplicity (see De Fiore and Liu, 2005).

$$P_t = P_{H,t}^{1-\vartheta} P_{F,t}^{\vartheta}.$$

Contemporaneous utility u_t of a representative domestic household rises with aggregate consumption and with leisure l_t, where $l_t \in [0,1]$ and $n_t = 1 - l_t$ is the working time. Its objective is to maximize expected utility of consumption and leisure over time:

$$(7)\quad \max E_0 \left[\sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} + \zeta \frac{(1-n_t)^{1-\sigma_l}}{1-\sigma_l} \right) \right], \quad \sigma > 0, \sigma_l \geq 0 ,$$

with $\beta \in (0,1)$ the time preference discount factor. The household earns labor income $P_t w_t n_t$, pays taxes $P_t \tau_t$, and receives profits from monopolistically competitive firms indexed with $i \in [0,1]$.

Households have access to nominally state contingent claims, which are internationally traded. Let $\Gamma_{t,t+1}$ denote the stochastic discount factor for a one-period ahead nominal pay-off, i.e., the period t price of one unit of *foreign* currency in a particular state of period $t+1$ normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the time t domestic currency price of a random payoff D_{t+1} in period $t+1$ is given by $E_t[S_t \Gamma_{t,t+1} D_{t+1}]$, where S_t is the nominal exchange rate. The household maximizes lifetime utility (7) subject to the *perceived* budget constraint, which takes into account default beliefs ($\delta_t \geq 0$),

$$(8)\, E_t[S_t \Gamma_{t,t+1} D_{t+1}] + (B_{H,t} / R_t) \leq S_t D_t + (1-\delta_t) B_{H,t-1} + P_t w_t n_t - P_t c_t - P_t \tau_t + \Sigma_t,$$

and a no-Ponzi-game condition, taking prices, taxes, dividends, the default probability and the initial wealth endowment D_0 and $B_{H,-1}$ as given. Σ_t collects firms' profits. The first order conditions corresponding to the solution of the constrained maximization problem are:

$$\zeta (1-n_t)^{-\sigma_l} = w_t c_t^{-\sigma},$$

$$(9)\quad \beta E_t \left\{ (1-\delta_{t+1}) c_{t+1}^{-\sigma} \pi_{t+1}^{-1} \right\} = c_t^{-\sigma} / R_t,$$

$$\beta S_{t+1} c_{t+1}^{-\sigma} \pi_{t+1}^{-1} = S_t c_t^{-\sigma} \Gamma_{t,t+1},$$

where π_t denotes the gross inflation rate $\pi_t = P_t / P_{t-1}$. The first equation equates the marginal disutility of work to the marginal utility of the consumption it permits; the other two equations equate the intertemporal terms of trade using both available assets to the trade off between (marginal utility of) consumption today and consumption tomorrow. Further, the budget constraint holds with equality and the transversality condition is satisfied:

$$(10)\quad \lim_{k \to \infty} E_t (S_{t+k} \Gamma_{t+k,t+1+k} D_{t+1+k} + B_{H,t+k} / R_{t+k})(S_t / S_{t+k+1}) \Gamma_{t,t+1+k} = 0$$

Of course the possibility of trading over time using two different assets gives rise to an arbitrage condition between the returns of those two assets:

$$(11)\quad 1/R_t = E_t\left\{(1-\delta_{t+1})(S_t/S_{t+1})\Gamma_{t,t+1}\right\}.$$

Hence, higher expected default probabilities lead investors to demand a higher interest rate on government bonds.

We assume that preferences of foreign households exhibit the same qualitative structure as domestic households. Hence, their demand for domestically produced consumption goods $c_{H,t}^*$ and foreign consumption goods $c_{F,t}^*$ satisfies $c_{H,t}^* = \vartheta^*\left(P_t^*/P_{H,t}^*\right)c_t^*$ and $c_{F,t}^* = (1-\vartheta^*)\left(P_t^*/P_{F,t}^*\right)c_t^*$, where $\vartheta^* \in (0,1)$ and c_t^* is aggregate foreign consumption. We assume there is a strong home country bias in consumption: $\vartheta^* << 1-\vartheta$ or the Laursen-Metzler condition $1-\vartheta-\vartheta^* > 0$ such that the import shares add up to less than one.

Foreign households also have access to a complete set of contingent claims and they can hold domestic public debt B_F, which is denominated in domestic currency. We assume that the instantaneous utility function of foreign households is similar to the one of domestic households and that they have the same discount factor β. Their first order conditions for investments in both assets are given by

$$(12)\quad \begin{aligned} \beta E_t\left\{[1-\delta_{t+1}]\left(c_{t+1}^*\right)^{-\sigma^*}\left(\pi_{t+1}^* S_{t+1}\right)^{-1}\right\} &= \left(c_t^*\right)^{-\sigma^*}\left(R_t S_t\right)^{-1}, \\ \beta\left(c_{t+1}^*\right)^{-\sigma^*}\left(\pi_{t+1}^*\right)^{-1} &= \left(c_t^*\right)^{-\sigma^*}\Gamma_{t,t+1}, \end{aligned}$$

where $\pi_t^* = P_t^*/P_{t-1}^*$ and σ^* is the inverse of the foreign households' intertemporal elasticity of substitution.

Further, the price $1/R_t^*$ of a risk-free one-period discount bond F_t, which pays one unit of foreign currency in period $t+1$, has to satisfy $1/R_t^* = E_t[\Gamma_{t,t+1}]$. Thus, (9) and (12) imply the following pricing conditions:

$$(13)\quad \beta E_t\left\{\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{q_{t+1}}{q_t}\frac{1}{\pi_{t+1}^*}\right\} = \frac{1}{R_t^*},\quad \beta E_t\left\{\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma}\frac{1}{\pi_{t+1}^*}\right\} = \frac{1}{R_t^*},$$

where q_t is the real exchange rate defined as $q_t = \frac{S_t P_t^*}{P_t}$ and P_t^* is the foreign consumption price index.

Firms and Domestic Production

The production sector consists of two parts. Firstly, intermediate production is conducted by a continuum of monopolistically competitive firms, each producing a differentiated good being indexed on $i \in [0,1]$. Their technology is linear in labor, $y_{H,it} = n_{i,t}$, where $n_t = \int_0^1 n_{it} di$. Secondly, there are perfectly competitive firms producing the domestic consumption good $y_{H,t}$ by combining the differentiated intermediate goods as inputs: $y_{H,t}^{\frac{\varepsilon-1}{\varepsilon}} = \int_0^1 y_{H,it}^{\frac{\varepsilon-1}{\varepsilon}} di$, where $\varepsilon > 1$ (will be allowed to vary stochastically in section 4). Firm i sets the price for the intermediate good $y_{H,it}$ in home currency $P_{H,it}$. The final good producer's cost minimizing demand is $y_{H,it} = \left(P_{H,it} / P_{H,t}\right)^{-\varepsilon} y_{H,t}$, implying $P_{H,t}^{1-\varepsilon} = \int_0^1 P_{H,it}^{1-\varepsilon} di$ for the price index of home produced goods.

The price setting decision of an intermediate domestic producer is based on Calvo (1983) and Yun (1995). A fraction $\phi \in (0,1)$ of firms is assumed to adjust their prices with the steady state rate of domestic producer price inflation $\overline{\pi}_H$, where $\pi_{H,t} = P_{H,t} / P_{H,t-1}$, such that $P_{H,it} = \overline{\pi}_H P_{H,it-1}$ and there is price dispersion in the long-run. In each period a fraction $1-\phi$ of randomly selected firms sets new prices $\widehat{P}_{H,it}$ in order to maximize the expected sum of discounted future dividends $\left(P_{H,it} - P_{H,t} mc_{H,t}\right) y_{H,it}$:

$$\max_{\widehat{P}_{H,it}} E_t \sum_{s=0}^{\infty} \phi^s S_t \Gamma_{t,t+s} (\widehat{P}_{H,it} y_{H,it+s} - P_{H,t+s} mc_{H,t+s} y_{H,it+s})$$

(14) *subject to*

$$y_{H,it+s} = (\overline{\pi}_H^s \widehat{P}_{H,it})^{-\varepsilon} P_{H,t+s}^{\varepsilon} y_{H,t+s}$$

where mc_H denotes real marginal costs. Suppose there exists a steady state where home prices grow at the rate $\overline{\pi}_H$, while all real variables are constant, e.g. $\overline{mc}_H = MC_H / P_H = (\varepsilon - 1)/\varepsilon$. Then one can derive the following marginal cost based Phillips curve (see Yun, 1995)

$$(15) \quad \hat{\pi}_{H,t} = \chi \hat{mc}_{H,t} + \beta E_t \hat{\pi}_{H,t+1},$$

where $\chi = (1-\phi)(1-\beta\phi)\phi^{-1} > 0$ and $\hat{x}_t$ denotes the percent deviation of a generic variable x_t from its steady state value x: $\widehat{x}_t = \log(x_t) - \log(x)$. Finally, labor demand in a symmetric equilibrium satisfies

$$(16) \quad w_t = \frac{P_{H,t}}{P_t} mc_{H,t}.$$

1.3 Market clearing

The home country is assumed to be small in the sense that its exports are negligible in the foreign price indices. The foreign producer price level $P_{F,t}^{*}$ is then identical to the foreign consumption price index P_t^*, $P_t^* = P_{F,t}^*$. The law of one price holds (separately) for each good such that $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$, where $P_{H,t}^*$ is the price of home produced goods expressed in foreign currency. Thus, we get the following relation between the real exchange rate and the relative price ratio $P_{H,t} / P_t$: $q_t = \left(P_{H,t} / P_t\right)^{1-1/\vartheta}$ implying for CPI inflation

$$(17)\quad \pi_t = \pi_{H,t} \left(q_t / q_{t-1}\right)^{\frac{\vartheta}{1-\vartheta}} \quad \forall t \geq 1 \ .$$

In equilibrium, the goods market for domestically produced final goods clears,

$$y_{H,t} = c_{H,t} + c_{H,t}^* + g \ .$$

Substituting in demand functions and using $q_t = \left(P_{H,t} / P_t\right)^{1-1/\vartheta}$ yields:

$$(18)\quad y_{H,t} = (1-\vartheta) q_t^{\frac{\vartheta}{1-\vartheta}} c_t + \vartheta^* q_t^{\frac{1}{1-\vartheta}} c_t^* + g,$$

We assume that internationally traded default-risk-free assets consists of discount bonds F_t, priced with $1/R^*$. Further, domestic public debt is held by foreign investors B_{Ft}, such that the non-interest current account surplus satisfies:

$$P_{H,t} y_{H,t} - P_t c_t - P_t g \leq S_t \left\{ \left[(F_t / R_t^*) - F_{t-1} \right] \right\} - \left(B_{F,t} / R_t\right) + \left[1 - \delta(t)\right] B_{F,t-1} \ ,$$

or in real terms,

$$(19)\quad q_t \left(f_t / R_t^*\right) - \left(b_{F,t} / R_t\right) = q_t^{\vartheta/(\vartheta-1)} y_{H,t} - c_t - g + q_t \left(f_{t-1} / \pi_t^*\right) - \left(1 - \delta_t\right)\left(b_{F,t-1} / \pi_t\right),$$

where f_t denotes the real value of F_t, $f_t = F_t / P_t^*$.

Perfect international risk sharing

Recall that domestic and foreign households are assumed to have access to a complete set of contingent claims, so risk is fully shared internationally. The domestic and foreign first order conditions (equations (9) and (12)) imply that the consumption growth rates are related:

$$(20)\quad \left(c_{t+1} / c_t\right)^{\sigma} = \left(c_{t+1}^* / c_t^*\right)^{\sigma^*} \left(q_{t+1} / q_t\right) \quad \forall t \geq 0 \ .$$

This equilibrium condition on the growth rates of c_t, c_t^*, and q_t determines the relation between their levels up to a constant ξ,

$$c_t^{\sigma} = \xi q_t \left(c_t^*\right)^{\sigma^*} ,$$

where the constant ξ can be pinned down by initial asset endowments and intertemporal solvency (i.e. the intertemporal budget constraint).

1.4 *Equilibrium*

Throughout the analysis, foreign macroeconomic variables (starred variables) are independent from domestic variables, i.e., they are exogenously determined. To simplify the analysis we assume that aggregate foreign consumption is constant, $c_t^* = c^*$, which implies that foreign monetary policy is conducted in a way that is consistent with a constant real interest rate $R_t^* / E_t \pi_{t+1}^* = \beta$ (see the second equation in (12)). The real exchange rate then acts as a shockabsorber, maintaining consistency with (20).

Households are fully rational, including their belief that there exists some maximum politically feasible tax rate, the level of which is governed by the pdf following from the Bayesian strategy played by the government in its preemption game against speculators. This pdf gives rise to positive perceived default probabilities. In equilibrium the domestic goods market equilibrium (18), the current account equation (19), the first order conditions of domestic and foreign households and firms must be satisfied for given domestic monetary and fiscal policies and given sequences for the starred (exogenous) variables as well as initial asset endowments F_{-1}, $B_{H,-1}$, and $B_{F,-1}$ and price levels in t=–1. In equilibrium, domestic households are indifferent between holding internationally traded risk-free private securities and domestic public debt. Since we are particularly interested in the role of foreign debt, we assume that domestic public debt is solely held by foreign investors,

$$B_{H,t} = 0 \Leftrightarrow b_t = b_{F,t}.$$

while domestic households only hold internationally traded risk-free securities. The sequence of internationally traded risk-free securities $\{f_t\}_{t=0}^{\infty}$ can separately be determined from (19) for given initial values F_{-1}^* and P_{-1}^*. The equilibrium is described in more detail in Appendix A.1. If there would be no risk of default $(\delta_t = 0)$ or if the risk premium would be independent of the level of public debt $(\delta' = 0)$, then the sequence of foreign holdings of government bonds b_F would also be irrelevant for the equilibrium allocation. We briefly come back to this case below.

To derive the stability properties under different stances of fiscal and monetary policy, the equilibrium conditions are log-linearized at the steady state (see Appendix A.1). Thus the stability results are locally valid around the long-run equilibrium.

Steady state

In steady state q is constant; this implies $\pi = \pi_H$ (cf (17)), and $1/\beta = R^* / \pi^*$. The long run world real interest rate is equal to the foreign and domestic (gross) rate of time preference. Using the first of the households' first order conditions (9) to substitute out n from the commodity market clearing equation (18), we get an equilibrium relation between domestic consumption c, foreign consumption c^*, and the real exchange rate q. Combined with $c_t^{\sigma} = \xi q_t \left(c_t^*\right)^{\sigma^*}$, the steady state levels of c and q can be determined as a function of ξ, preference parameters, the mark-up, and an exogenous level c^*. Domestic output y_H and – given that there is no long-run price dispersion – hours worked n then follows from the commodity market equilibrium condition. Hence, neither changes in monetary and fiscal policy nor in the perceived default probability will affect the long-run real allocation and thus steady state utility of domestic households.

Since we have no steady state growth, a steady state requires a constant real value of public debt in terms of the aggregate consumption good.[11] Since we focus on the case where the domestic government is indebted, we only consider cases where the steady state satisfies $b_F > 0$. The long-run inflation rate equals the target $\pi = \overline{\pi}_H$. We further assume that the interest rate rule (4) is consistent with the steady state, i.e., that the steady state interest rate equals the average interest rate $R = \overline{R}$. Investors' beliefs are also consistent with a long-run equilibrium and satisfy $\pi = R\beta\left(1-\delta\right)$. Thus, changes in δ affect the long-run equilibrium interest rate R for a given inflation target. The steady state value of real debt is consistent with its target level (see (1)).

3. Debt, Deficits and Macroeconomic Stability

In this section we examine the impact of public debt on macroeconomic stability for the case where the sovereign default premium rises with debt. We restrict our attention to the case of positive steady state debt levels. In the main part of this section, we analyze the case where public debt is entirely held by foreign investors. We then consider the limiting cases where the economy is closed and where debt is neutral.

3.1 *The Blanchard effect*

Since we want to assess the stability implications of fiscal-monetary policy regimes, it suffices to focus on the structural part of the economy. The deterministic versions of the equilibrium conditions can be reduced to a set of conditions for $b_{F,t}$, q_t, and R_t and $\pi_{H,t}$. In a neighborhood of the steady state the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions (see Appendix A.1). An equilibrium is then defined as follows: An equilibrium is a set of sequences $\{\widehat{\pi}_t,\ \widehat{b}_{F,t},\ \widehat{q}_t,\ \widehat{R}_t\}_{t=0}^{\infty}$ for $\delta \in (0,1)$ and $\Phi \in (0,1)$ that converge to the steady state (b_F, q, π, R) and satisfy

$$(a)\ \frac{1-\Phi\vartheta}{1-\vartheta}\left(\widehat{q}_{t+1}-\widehat{q}_t\right)=\widehat{R}_t-\left(1-\Phi\right)\widehat{\pi}_{H,t+1}-\Phi\widehat{b}_{F,t},$$

$$(b)\ \widehat{\pi}_{H,t}=\beta\widehat{\pi}_{H,t+1}+\psi\widehat{q}_t, \qquad \psi=\chi\left[\frac{\sigma_n}{n}\left(\frac{(1-\vartheta)c}{\sigma}+\frac{(1-\vartheta)c\vartheta+\vartheta^*c^*}{1-\vartheta}\right)+\frac{1}{1-\vartheta}\right]>0$$

(21) $$(c)\ \widehat{b}_{F,t}=\widehat{R}_t+\Lambda\widehat{b}_{F,t-1}-\Lambda\left(1-\vartheta\right)\widehat{\pi}_{H,t}-\Lambda\vartheta\widehat{R}_{t-1}, \quad \Lambda=\frac{1-\kappa}{\beta\left(1-\delta\right)}\frac{1}{1-\Phi\vartheta}>0$$

$$(d)\ \widehat{R}_t=\rho_\pi\widehat{\pi}_{H,t}$$

(where $\sigma_n=\sigma_l n/(1-n)$) given $b_{F,-1}$.

Relation (21)d is the central bank's reaction function. The equilibrium relation (21)a, which originates in the asset pricing condition for public debt, relates the real interest rate to the change in the real exchange rate in an almost conventional way. A higher (home) real interest rate requires a future real depreciation to be consistent with asset market equilibrium, at least for sufficiently small values for Φ. In standard overshooting fashion, a future real depreciation requires an instantaneous real appreciation up front. The implied real appreciation ($\widehat{q}_t \downarrow$) leads to a decline in aggregate (domestic and foreign) demand for domestically produced goods (see 18). As a consequence, domestic producers tend to lower their prices, as can be seen from the aggregate supply relation (21)b. At the same time a rise in the nominal interest rate tends to raise real public debt $\widehat{b}_{F,t}$ (measured in units of the aggregate domestic consumption good) for a predetermined value of beginning-of-period real debt (see (21)c).

A rise in real debt $\widehat{b}_{F,t}$, however, tends to lower its expected total return, since it raises default expectations. This can be seen from the RHS of (21)a, which decreases with $\widehat{b}_{F,t}$. How the rise in public debt affects the previously described chain of events crucially depends on monetary policy, because that determines the initial interest rate rise, and on fiscal policy,

[11] Note that without growth a constant level of debt also implies a constant steady state debt-output ratio.

which determines the issuance of new debt.

As suggested by Blanchard (2005) the negative feedback from public debt to its return, which originates in sovereign default expectations, may cause destabilizing dynamics. To get an intuition for this, suppose that inflation exceeds its steady state value due to some (unspecified) temporary fundamental shock. The central bank, which aims to stabilize inflation ($\rho_\pi > 0$), will then raise the nominal interest rate. The rise in the nominal interest rate can then cause an increase in real debt $\widehat{b}_{F,t}$, if the feedback coefficient κ is small. The perceived default probability will then go up, which reduces the foreign households' willingness to invest in public debt.

The associated real depreciation ($\hat{q}_t \uparrow$, see LHS of (21)a) then exerts an upward pressure on domestic prices through different channels. A rise in the real exchange rate q_t directly raises aggregate consumption $\hat{c}_t$, as implied by (see (20)), which tends to increase the demand for home goods. In addition, expenditure switching of domestic and foreign households in response to the exchange rate change further increases the demand for domestically produced goods. This adds to the price pressure as producers incur higher marginal costs at higher output levels. Moreover, households will demand a higher nominal wage, since the price level of aggregate consumption will rise due to higher prices of imported goods. Hence, domestic producers will unambiguously raise their prices in response to the real depreciation (see (21)b), which reinforces the initial rise in inflation. Due to all these channels, a rise in the nominal interest rate can actually lead to *higher* inflation if κ is small and ρ_π is high.

The interaction of monetary and fiscal policy is decisive for the economy to evolve in a stable way. The system (21) features two predetermined (sluggish) variables, such that a stable set of equilibrium sequences requires two stable eigenvalues. The necessary condition for the existence of a determinate equilibrium (i.e. stable and unique equilibrium) is given in the following proposition.

Proposition 1 *Suppose that taxes are raised according to* (1) *for* $\kappa \in (0,1]$, *monetary policy satisfies* $\widehat{R}_t = \rho_\pi \widehat{\pi}_{H,t}$, *and that* $\delta \in (0,1)$ *and* $\Phi \in (0,1)$.

1) *When* $\kappa < \overline{\kappa}$, *there is a stable and unique equilibrium only if* $\rho_\pi < 1$.
2) *When* $\kappa > \overline{\kappa}$, *there is a stable and unique equilibrium only if* $1 < \rho_\pi$ *where* $\overline{\kappa} = 1 - \beta(1-\delta)(1-\Phi)$.

See appendix A.2 for the proof. Proposition 1 shows that the existence of a stable set of equilibrium sequences depends on the particular monetary and fiscal policy stance, measured by the feedback parameters ρ_π and κ.[12]

The main result summarized in Proposition 1 is that a monetary policy which aims to stabilize inflation through an active interest rate policy ($\rho_\pi > 1$) will destabilize the economy if the feedback from debt surprises back to the primary surplus is too weak. In particular if $\kappa < \overline{\kappa}$, an active interest rate policy will destabilize the economy in the presence of default risk[13]. This property is clearly at odds with the main principle (the Taylor-principle) known from many models of closed and open economies, which demands monetary policy to react actively, i.e. by more than one for one, to changes in (domestic producer price) inflation $\rho_\pi > 1$ in order to ensure macroeconomic stability (see Woodford, 2003, and Gali and Monacelli, 2005, for example). Thus when public debt is associated with a default risk premium, which in turn is influenced by the level of public debt, a feedback smaller than one-for-one is required from inflation to the nominal interest rate, if the feedback from debt to taxes κ is small, $\kappa < \overline{\kappa}$.

Notably, these stability conditions closely relate to the stability conditions in Leeper (1991), where sovereign default is not considered while equilibria are nevertheless restricted to exhibit stationary debt sequences. The difference between his conditions and ours are the default rate and its elasticity Φ, which both tend to increase the threshold for the fiscal feedback κ. Thus, fiscal policy has to be more responsive to changes in real debt in order to allow monetary policy to stabilize inflation.

To see the intuition for this result, consider the case where some (unspecified) temporary shock leads to a rise in public debt. Since expected default rises, investors are less willing to hold domestic public debt. The associated depreciation (see 22a), leads to a rise in the demand for domestic goods and thus to an upward pressure on inflation (see 22b). If the central bank aggressively raises the nominal interest in response to higher expected inflation, $\rho_\pi > 1$, debt service costs will rise strongly, and will for small $\kappa's$ lead to an even further increase in real debt and thus to unstable debt dynamics. If, however, the interest rate response

[12] Uniqueness can be ensured if the fiscal feedback coefficient κ is not too large, i.e. if (but not only if) $\kappa < 1 + (1-\delta)\left[(1-\vartheta)(1-\Phi)\psi - (2\beta - 1)(1-\Phi\vartheta)\right]$ or $\kappa < 1 - \beta^2(1-\delta)(1-\Phi\vartheta)/(\psi\rho_\pi(1-\vartheta)+1)$. These two conditions guarantee that there is not more than one stable eigenvalue. Applying reasonable parameter values (see section 4), we find that these restrictions are very unlikely to be binding.

[13] It should be noted that a tax rule tied to the interest inclusive deficit effectively has a feedback rule in with coefficient $\kappa(R_t\text{-}1)$. This explains the somewhat weaker results in Schabert and van Wijnbergen (2006).

is moderate, $\rho_\pi < 1$, the real value of public debt can decrease due to the revaluation by a higher domestic price level.

For a high feedback coefficient $\kappa > \overline{\kappa}$, active monetary policy $\rho_\pi > 1$ will not destabilize the economy. A temporary rise in real debt will again tend to raise expected default and inflation, but with the higher feedback coefficient κ tax revenues will be sufficiently high to eventually lower future real debt. Forward-looking price setters and investors realise the fiscal stance, and will therefore not raise prices and will not demand a higher default premium. The feedback from debt to the rate of return will then not blur the logic underlying the Taylor-principle. With a sufficiently strong feedback from debt surprises to the primary surplus, an active policy will lead to macroeconomic stability.

3.2 *Two extreme cases: a closed economy and debt neutrality*

Let us first consider the closed economy version of the model, $\vartheta = 0$, where public debt is held by domestic households, $b_t = b_{H,t}$, and CPI inflation equals PPI inflation, $\pi_t = \pi_{H,t}$. In this case, the model can be reduced to a set of three equilibrium conditions, i.e. an aggregate demand condition, an aggregate supply condition and the government budget constraint (see appendix B). The stock of public debt is then still non-neutral due to its impact on the perceived default probability. Under a weak feedback from debt to taxes, an increase in the real interest rate tends to increase public debt and thereby default expectations, like in the open economy case. While the associated decline in the effective rate of return on domestic debt leads to a depreciation in the open economy case, it also tends to reduce domestic households' willingness to save. As a consequence, aggregate demand and prices tend to increase, which implies that interest rates set according to an active feedback rule can destabilize public debt and other macroeconomic aggregates.

It can be shown that the existence of a destabilizing feedback from interest rates to public debt gives rise to the same type of monetary and fiscal policy interactions as in the open economy. Specifically, we show in Appendix B that the stability conditions in proposition 1 exactly apply also for the case where the economy is closed and public debt is held by domestic households. We can therefore conclude that the destabilizing effect suggested by Blanchard is not only relevant for a small open economy, but should also be taken into account for large developed economies (which can typically be modelled as a closed economy).

To relate our findings to existing results in the literature, we consider the second “extreme” case, where public debt is not perceived to be risky, such that $\Phi = 0$. Notably, this case cannot be assessed by analyzing the limiting case $\Phi \to 0$, since the model exhibits a discontinuity at $\Phi = 0$. The equilibrium allocation is for $\Phi = 0$ independent of fiscal policy, i.e. there are infinitely many sequences of debt and taxes that are consistent with equilibrium. Since the level of public debt does not affect its rate of return, consumption growth depends solely on the interest rate $\widehat{R}_t$, which is set by the central bank, and on inflation $\widehat{\pi}_{H,t+1}$: $\widehat{c}_{t+1} - \widehat{c}_t = (1-\vartheta)\sigma^{-1}(\widehat{R}_t - \widehat{\pi}_{H,t+1})$. Given that condition (21)b can be written as $\widehat{\pi}_{H,t} = \psi\sigma^{-1}\widehat{c}_t + \beta\widehat{\pi}_{H,t+1}$, the equilibrium allocation can be determined independently from fiscal policy and, therefore, in an entirely forward-looking way, like in Gali and Monacelli (2005). Equilibrium stability and uniqueness then requires interest rate policy to be active $\rho_\pi > 1$, as shown in Gali and Monacelli (2005).

Hence the stark contrast between Proposition 1 and the traditional principles of stabilizing interest rate policies in models with risk-free debt is solely due to the existence (and not the size) of default expectations. When the tax feedback coefficient is sufficiently large, the central bank can apply an active interest rate policy to stabilize inflation via the conventional Fischer effect: the depressing effect of high real rates on aggregate demand for domestic goods is high enough to slow inflation down. Without such a feedback, or too weak a feedback, the interaction between default fears and exchange rate depreciation triggers an upward shift in inflation and the dynamics become unstable if active interest rate rules are implemented nevertheless.

3.3 *Successful inflation stabilization and sovereign default risk*

We demonstrated that an active interest rate policy leads to undesirable outcomes in the presence of sovereign default risk if not supported by a sufficiently strong fiscal policy response to debt. Of course this does not imply that inflation stabilization is infeasible or unwanted in those circumstances, but that the central bank should not use an interest rate on debt that is associated with a default risk premium as its instrument. But there are alternative monetary policy instruments that sidestep the problems caused by the endogeneity of default premia on debt. Taylor (2002) already conjectured this for environments with high and variable risk premia: "Thus, policy makers in emerging market economies might want to give greater consideration to policy rules with monetary aggregates, even if rules with the interest rate become the preferred choice." (Taylor, 2002, p. 445).

In line with Taylor’s suggestion, we show in Schabert and van Wijnbergen (2006) that the central bank can safely control inflation through a money rule independent of interest rates or actual inflation. In this way it implements a stable and uniquely determined equilibrium characterized by a stabilized inflation sequence. Their analysis of course neglects problems like those stemming from for example money demand instability. But it shows that money supply based inflation stabilization policy is feasible even with risky public debt, without the stability problem plaguing interest rate rules in such circumstances.

4. Debt stabilization and macroeconomic fluctuations: a numerical example

We have shown that at least some degree of debt stabilization is necessary for a non-explosive equilibrium under an interest rate rule to exist. The government has to raise taxes by a sufficiently high amount in response to debt surprizes, i.e. κ has to be sufficiently high, to allow successful stabilization of inflation and aggregate demand by using an active interest rate policy $\rho_\pi > 1$. This result seems to suggest that, as long as $\kappa > \overline{\kappa}$, fiscal policy is irrelevant for the stabilization of macroeconomic aggregates. Yet, this would overlook the impact of (the time path of) public debt on the effectiveness of interest rate adjustments through its impact on perceived default probabilities.

In this section we demonstrate that fiscal policy matters for monetary stabilization policy even if condition 2) in proposition 1 is satisfied. For this, we use a numerical example that is intended to show how the public debt dynamics alter the central bank's ability to reduce macroeconomic fluctuations. This possibility has been shown by Linnemann and Schabert (2010), in an environment where public debt non-neutral by providing transaction services. The parameter values are therefore chosen in the first place to clarify the role of debt stabilization and to isolate the effects from changes in the policy parameter κ and of the aggressiveness of interest rate policy as measured by ρ_π. For this exercise we set non-policy parameter and steady state values equal to standard values (with periods interpreted as quarters).

The discount rate is set equal to 0.9923 to match a reasonable risk-free long-run interest rate (see below), the elasticity of intertemporal substitution to 0.5 ($\sigma = \sigma_n = 2$), the domestic import share $\vartheta = 0.5$ and the foreign import share $\vartheta^* = 0.01$. We set the relative size of the foreign country to $c^*/c = 20$ to get close to the small country assumption. The government share equals $(g/y) = 0.3$, and the fraction of non-price adjusting firms $\phi = 0.8$, while the preference parameter ζ and initial endowments (and thus ξ) are chosen to get working time $n = 0.5$ and the real exchange rate $q = 1$ in steady state. To examine whether the average size and the elasticity of the perceived default probability matter even at relatively small values, we set them equal to $\delta = 0.005$, implying a plausible annualized premium of about 2%, and $\Phi = 0.01$, for simplicity. We further assume that the central bank aims at zero inflation in the long-run ($\pi = 1$). The long-run nominal interest rate on public debt then equals $R = 1.03$ (implying an annual interest rate of 5.3%). We vary the policy parameters $\kappa > \overline{\kappa}$ and ρ_π within a reasonable range around the benchmark values, $\rho_\pi = 1.5$ and $\kappa = 0.1$, the latter choices imply a steady state debt-to-output ratio of 86%. Note that monetary and fiscal policy will always satisfy condition 2) in proposition 1, so a non-explosive equilibrium is guaranteed to exist at these parameter values.

We consider uncertainty in form of exogenous changes in the elasticity $\varepsilon > 1$, which gives rise to a standard cost push shock z_t to aggregate supply constraint. This leads to a non-trivial problem for a central bank aiming to minimize welfare costs due to imperfect price adjustments. In particular, we assume that z_t in $\widehat{\pi}_{H,t} = \beta \widehat{\pi}_{H,t+1} + \psi \widehat{q}_t + \widehat{z}_t$ satisfies $\widehat{z}_t = \rho \widehat{z}_{t-1} + \varepsilon_t$ with $\rho = 0.9$ and ε_t is i.i.d. with $E_{t-1}\varepsilon_t = 0$ and $\text{var}(\varepsilon_t) = 0.01$.

Table 1 shows unconditional variances of producer price inflation $\hat{\pi}_{H,t}$, domestic output $\hat{y}_{H,t}$ and real debt $\hat{b}_t$ for several values ρ_π and κ. The table shows the key result: a higher fiscal feedback coefficient tends to lower all three variances. The debt variance is most strongly affected by higher $\kappa's$, but inflation and output variances are also reduced, be it to a much smaller extent. Of interest is the fact that a higher κ lowers both the variance of inflation *and* output, thereby improving the trade-off between inflation and the output-gap.

In contrast, higher values for the inflation feedback ρ_π of interest rate policy lower the inflation variance, but at the expense of higher output variance. At the same time, the debt sequences become more volatile, since more pronounced interest rate adjustments tend to increase variations in debt servicing costs. Overall, debt variations have a relatively minor impact on inflation and output fluctuations due to the small value for the default elasticity Φ, while a higher fiscal feedback coefficient facilitates macroeconomic stabilization by lowering both output and inflation variance.

Table 1 Unconditional variances (benchmark values $\rho_\pi = 1.5$ and $\kappa = 0.1$)

	$\kappa = 0.05$	$\kappa = 0.1$	$\kappa = 0.2$	$\rho_\pi = 1.25$	$\rho_\pi = 1.5$	$\rho_\pi = 1.75$
$\Phi = 0.01$						
$\text{var}(\hat{\pi}_{H,t})$	0.103	0.100	0.097	0.243	0.100	0.054
$\text{var}(\hat{y}_{H,t})$	1.116	1.111	1.107	0.914	1.111	1.210
$\text{var}(\hat{b}_t)$	36.10	2.60	0.30	2.48	2.60	2.65
$\Phi = 0.02$						
$\text{var}(\hat{\pi}_{H,t})$	0.1117	0.108	0.102	0.265	0.108	0.059
$\text{var}(\hat{y}_{H,t})$	1.126	1.115	1.105	0.917	1.115	1.213
$\text{var}(\hat{b}_t)$	58.42	3.05	0.319	2.873	3.05	3.134

These effects are more accentuated at higher values of Φ as the lower half of Table 1 shows: the reduction in output variance is slightly over 2%, twice as large as in the upper half of Table 1 when κ goes from 0.05 to 0.20. Inflation variance is reduced by 6% for Φ=0.01 but by almost 13% when Φ=0.02. The impact of varying ρ_π is similar for both the lower and the higher value of Φ, as can be seen by comparing the lower and upper blocks of Table 1.

The true value of Φ depends on investors' beliefs, which are of course an empirical matter. But it is clearly possible, by judiciously choosing a positive κ in combination with an active interest rate policy $\rho_\pi > 1$, to lower inflation variance substantially without having to accept higher output variance in return. But running an active interest rate policy without sufficiently high κ will lead to instability (see proposition 1).

5. Conclusion

Inflation targeting based on interest rate control has become the preferred modus operandi of Central Banks around the world. Yet concerns have emerged about the wisdom of applying this framework in an environment where doubts about the willingness to pay out on debt service obligations are persistent and increasing in measures of indebtness of the government involved. Taylor (2002) expresses similar concerns: "*nominal interest rates are a less appropriate instrument in cases where risk premia can be high and variable*" (Taylor, 2002,

p. 444). Yet the formal literature on inflation targeting has not addressed this issue.

Nevertheless the issues are real. If fears of debt default are positively correlated with the debt service burden, unstable cycles are a possibility as we show in this paper. In that case, higher interest rates lead to an increased debt service burden, hence to higher fears of debt default, in an open economy to real exchange rate depreciation and higher domestic goods prices, which in turn call for higher interest rates under a Taylor rule.

We examine this mechanism in a dynamic general equilibrium model of a small open economy where a goods price rigidity provides a rationale for inflation stabilization. The model used is mostly standard, except for the introduction of default probabilities on public debt. We derive default probabilities and the associated default premium in interest rates endogenously to the model and based on full rationality of debt holders in capital markets.

We show that stable equilibrium dynamics cannot be guaranteed when interest rates are raised aggressively in response to higher inflation. In fact we obtain a very strong result: Unless there is a sufficiently strong feedback from higher debt to higher (primary) surpluses on fiscal account, active interest rate policy will *always* result in unstable dynamics. This provides formal support for the view often expressed by central bankers, that their leeway on monetary policy is much reduced when there is insufficient back up from fiscal policy.

If no such fiscal support is forthcoming, central banks are not powerless: the central bank is still able to stabilize inflation, if it does not use the interest rate as its instrument. An inflation targeting policy based on money supply rules can safely be implemented even in the presence of endogenous default fears. And it is clearly possible, by choosing a sufficiently positive feedback of rising debt levels to higher primary surpluses, in combination with an active interest rate policy, to lower inflation variance substantially without having to accept higher output variance in exchange. But active, interest rate rule based, inflation targeting without such fiscal stringency equally clearly will lead to instability.

There are many questions inviting future research. Are countries with heavily indexed debt structures more vulnerable to this problem than countries without indexed debt, as Schabert and van Wijnbergen (2006) suggest? What about nominal deficit targets, do they imply a sufficiently strong feedback from debt levels (through interest payments) to implicit primary surplus targets to put to rest instability fears? Does the choice of exchange rate regime matter for this debate? Whatever the answer to these questions, it is clear that in crisis-prone environments, monetary policy cannot be seen in separation from fiscal policy. There is much to be said in favor of recommending feedback rules calling for higher primary surpluses when debt levels are increasing, to complement inflation targeting through active Taylor rules.

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Appendix A

A.1 Equilibrium

A rational expectations equilibrium is a set of sequences $\{mc_{H,t}, w_t, \tau_t, \pi_t, \pi_{H,t}, c_t, n_t, q_t, z_t, y_{H,t}, R_t, R_t^*, f_t, b_{H,t}, b_{F,t}\}_{t+0}^{\infty}$ satisfying

$$\zeta(1-n_t)^{-\sigma_l} = w_t c_t^{-\sigma},$$

$$\beta E_t\left\{\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{q_{t+1}}{q_t}\frac{1}{\pi_{t+1}^*}\right\} = \frac{1}{R_t^*},$$

$$\beta E_t\left\{(1-\delta_{t+1})\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{1}{\pi_{t+1}}\right\} = \frac{1}{R_t},$$

$$c_t^{\sigma} = \xi q_t\left(c_t^*\right)^{\sigma^*},$$

$$w_t = \left(q_t z_t\right) mc_{H,t},$$

$$z_t^{\vartheta-1} = q_t,$$

$$\left(b_{H,t}+b_{F,t}\right)/R_t = g+\tau_t+\left(b_{H,t-1}+b_{F,t-1}\right)\pi_t^{-1},$$

$$\tau_t = \kappa\left[\left(b_{t-1}/\pi_t\right)-\left(b^*/\pi\right)\right]$$

$$y_{H,t} = (1-\vartheta)\left(z_t^{\vartheta}\right)^{-1}c_t+\vartheta^*\left(z_t\right)^{-1}c_t^*+g,$$

$$\pi_t = \pi_{H,t}\left(q_t/q_{t-1}\right)^{\frac{\vartheta}{1-\vartheta}},$$

$$\left[\frac{b_{F,t}}{R_t}-\left(1-\delta_t\right)\frac{b_{F,t-1}}{\pi_t}\right]+z_t^{\vartheta}y_{H,t} = c_t+q_t\left[\frac{f_t}{R_t^*}-\frac{f_{t-1}}{\pi_t^*}\right],$$

the aggregate supply constraint and aggregate domestic production (which in linearized form read $\widehat{\pi}_{H,t} = \chi \widehat{mc}_{H,t}+\beta E_t\widehat{\pi}_{H,t+1}$ and by $\widehat{y}_{H,t} = \widehat{n}_t$), the transversality condition, and a monetary policy for given sequences $\{c_t^*, \pi_t^*\}_{t=0}^{\infty}$ satisfying $\beta E_t\{\left(c_t^*/c_{t+1}^*\right)^{\sigma}/\pi_{t+1}^*\} = 1/R_t^*$, initial asset endowments F_{-1} , $B_{H,-1}$, and $B_{F,-1}$, and an initial price level $P_{H,-1}$.

In a neighborhood of the steady state the equilibrium sequences are approximated by the solutions to the linearized equilibrium conditions. Due to the availability of lum-sum taxes, Ricardian equivalane applies, such that the equilibrium allocation is consistent with infinitely many pairs of sequences for domestic and foreign debt holdings. Here, we focus on the case where domestic households' holdings of public debt equals zero, $B_{H,t} = 0$. The equilibrium can be defined as follows (where $\widehat{x}_t$ denotes the percent deviation of a generic variable x_t from its steady state value x: $\widehat{x}_t = \log x_t - \log x$):

Definition *A rational expectations equilibrium for* $B_{H,t} = 0$ *and* $\widehat{c}_t^* = 0$ *is a set of sequences*

$\{\widehat{w}_t\ ,\ \widehat{\pi}_t, \widehat{\pi}_{H,t}\ ,\ \widehat{c}_t\ ,\ \widehat{q}_t, \widehat{y}_{H,t}\ ,\ \widehat{R}_t,\ \widehat{b}_{F,t}\}_{t=0}^{\infty}$ *satisfying*

(i) $$\hat{w}_t - \sigma\hat{c}_t = \sigma_n \hat{y}_{H,t},$$

(ii) $$\Phi\hat{b}_{F,t} - \Phi E_t\hat{\pi}_{t+1} = \hat{R}_t - E_t\hat{\pi}_{t+1} - \sigma(E_t\hat{c}_{t+1} - \hat{c}_t),$$

(iii) $$\sigma\hat{c}_t = \hat{q}_t$$

(iv) $$\hat{\pi}_{H,t} = \chi\left(\hat{w}_t + \left[\vartheta/(1-\vartheta)\right]\hat{q}_t\right) + \beta E_t\hat{\pi}_{H,t+1}$$

(v) $$\hat{y}_{H,t} = [(c/n)\vartheta + \vartheta^*\left(c^*/n\right)/(1-\vartheta)]\hat{q}_t + (1-\vartheta)(c/n)\hat{c}_t$$

(vi) $$\hat{\pi}_t = \hat{\pi}_{H,t} + \left[\vartheta/(1-\vartheta)\right]\left(\hat{q}_t - \hat{q}_{t-1}\right).$$

(*where* $\sigma_n = \sigma_l n/(1-n)$ *and* Φ *denotes the default elasticity times* $\delta/(1-\delta)$ *with respect to the real value of public debt* $B_{F,t-1}/P_t$ *at the steady state*: $\Phi = \delta' \frac{b_F}{\pi}\frac{1}{1-\delta} > 0$), *the transversality condition, and monetary and fiscal policy characterized by*

$$\widehat{b}_{F,t} = \widehat{R}_t + \frac{1-\kappa}{\beta(1-\delta)}\widehat{b}_{F,t-1} - \frac{1-\kappa}{\beta(1-\delta)}\widehat{\pi}_t,$$

$$\widehat{R}_t = \rho_\pi E_t\widehat{\pi}_{H,t},$$

where $\rho_\pi = R'\pi_H/R \geq 0$ *and* $\eta = \frac{1-\kappa}{1+\kappa(R-1)} \in (0,1)$ *for given initial values* $\widehat{b}_{F,-1}$ *and* $\widehat{q}_{-1}$.

Eliminating $\widehat{w}_t$ and $\widehat{y}_{H,t}$ with (i) and (v), the aggregate supply constraint (iv) can be rewritten as $\widehat{\pi}_{H,t} = \chi\left[\left(\sigma_n\alpha + \frac{\vartheta}{1-\vartheta}\right)\widehat{q}_t + \left(\sigma_n\frac{(1-\vartheta)c}{n} + \sigma\right)\widehat{c}_t\right] + \beta E_t\widehat{\pi}_{H,t+1}$, where $\alpha = \frac{(1-\vartheta)c\vartheta + \vartheta^* c^*}{n(1-\vartheta)}$. Further, eliminating $\widehat{\pi}_t$ and $\widehat{c}_t$ with (iii) and (vi), the set of equilibrium conditions can be reduced to the following system in $\{\widehat{\pi}_{H,t}\ ,\ \widehat{q}_t, \widehat{R}_t,\ \widehat{b}_{F,t}\}_{t=0}^{\infty}$:

(vii) $$-\Phi\hat{b}_{F,t} = -\hat{R}_t + (1-\Phi)E_t\hat{\pi}_{H,t+1} + \frac{1-\Phi\vartheta}{1-\vartheta}\left(E_t\hat{q}_{t+1} - \hat{q}_t\right)$$

(viii) $$\hat{b}_{F,t} = \hat{R}_t + \frac{1-\kappa}{\beta(1-\delta)}\frac{1}{1-\Phi\vartheta}\hat{b}_{F,t-1} - \frac{1-\kappa}{\beta(1-\delta)}\frac{1-\vartheta}{1-\Phi\vartheta}\hat{\pi}_{H,t} - \frac{1-\kappa}{\beta(1-\delta)}\frac{\vartheta}{1-\Phi\vartheta}\hat{R}_{t-1}$$

(ix) $$\hat{\pi}_{H,t} = \chi\left[\frac{\sigma_n}{\sigma}\frac{(1-\vartheta)c}{n} + 1 + \sigma_n\alpha + \frac{\vartheta}{1-\vartheta}\right]\hat{q}_t + \beta E_t\hat{\pi}_{H,t+1}$$

(x) $$\hat{R}_t = \rho_\pi E_t\hat{\pi}_{H,t}$$

Under certainty, we thus end up with the system (21).

A.2 Proof of proposition 1

In order to prove the claims made in the proposition, the interest rate is eliminated in (21)a-d by substituting in the policy rule $\widehat{R}_t = \rho_\pi\widehat{\pi}_{H,t}$, leading to the following 4×4 system in $\widehat{b}_{F,t}$, $\widehat{\pi}_{H,t}$, $\widehat{q}_t$ and the auxiliary variable $\widehat{x}_t$:

$$\Phi\widehat{b}_{F,t} = \frac{1-\Phi\vartheta}{1-\vartheta}\widehat{q}_t - \frac{1-\Phi\vartheta}{1-\vartheta}\widehat{q}_{t+1} + \rho_\pi\widehat{\pi}_{H,t} - (1-\Phi)\widehat{\pi}_{H,t+1}$$
$$\widehat{b}_{F,t} = \Lambda\widehat{b}_{F,t-1} + \left(\rho_\pi - \Lambda(1-\vartheta)\right)\widehat{\pi}_{H,t} - \Lambda\vartheta\rho_\pi\widehat{x}_{t-1}$$
$$\beta\widehat{\pi}_{H,t+1} = -\psi\widehat{q}_t + \widehat{\pi}_{H,t}$$
$$\widehat{x}_t = \widehat{\pi}_{H,t}$$

($\Lambda = \frac{1-\kappa}{\beta(1-\delta)}\frac{1}{1-\Phi\vartheta}$), which can be rewritten as

$$\begin{pmatrix} \widehat{b}_{F,t} \\ \widehat{q}_{t+1} \\ \widehat{\pi}_{H,t+1} \\ \widehat{x}_t \end{pmatrix} = A \begin{pmatrix} \widehat{b}_{F,t-1} \\ \widehat{q}_t \\ \widehat{\pi}_{H,t} \\ \widehat{x}_{t-1} \end{pmatrix}$$

$$A = \begin{pmatrix} \Phi & \frac{1-\Phi\vartheta}{1-\vartheta} & 1-\Phi & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 0 & \frac{1-\Phi\vartheta}{1-\vartheta} & \rho_\pi & 0 \\ \Lambda & 0 & \rho_\pi - \Lambda(1-\vartheta) & -\Lambda\vartheta\rho_\pi \\ 0 & -\psi & 1 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

The characteristic polynomial of A is given by

$$H(X) = XG(X)$$
$$G(X) = X^3 + X^2\frac{\psi(\vartheta-1)(\Phi-1) - (\Phi\vartheta-1)(\beta+\Lambda\beta+1)}{\beta(\Phi\vartheta-1)} + \Lambda\frac{\psi\rho_\pi(\vartheta-1)-1}{\beta}$$
$$+X\frac{-\psi(\vartheta-1)(\Phi-1)\rho_\pi + (\Phi\vartheta-1)(\Lambda+\Lambda\beta+\Lambda\psi-\Lambda\psi\vartheta+1)}{\beta(\Phi\vartheta-1)}$$

One eigenvalue equals zero and can be assigned to $\widehat{x}_t$. Since there remains one further predetermined variable $\widehat{b}_{F,t-1}$, a unique and stable equilibrium requires $G(X)$ to exhibit exactly two unstable and one stable eigenvalue. To identify the conditions for this, we first examine $G(0)$

$$G(0) = -(1-\kappa)\frac{\psi\rho_\pi(1-\vartheta)+1}{\beta^2(1-\delta)(1-\Phi\vartheta)} < 0$$

which is strictly negative, for δ and Φ not exceeding one. One or three negative stable roots are further ruled out, since $G(-1)$, given by

$$G(-1) = -\frac{\begin{pmatrix} (\rho_\pi+1)\psi(1-\vartheta)\left(-\kappa+1+\beta(1-\delta)(1-\Phi)\right) \\ +2(\beta+1)\left(-\kappa+1+\beta(1-\delta)(1-\Phi\vartheta)\right) \end{pmatrix}}{\beta^2(1-\delta)(1-\Phi\vartheta)} < 0,$$

is strictly negative (given that $\kappa \le 1$). Two or zero positive stable roots are ruled out, if $G(1)$, given by

$$G(1) = -\psi\left(1-\rho_\pi\right)\left(1-\vartheta\right)\frac{\kappa-1+\beta\left(1-\delta\right)\left(1-\Phi\right)}{\beta^2\left(1-\delta\right)\left(1-\Phi\vartheta\right)}$$

is strictly positive. Hence, there are two sets of conditions that lead to $G(1) > 0$: For $\kappa > 1-\beta\left(1-\delta\right)\left(1-\Phi\right)$ monetary policy has to be active, $\rho_\pi > 1$, while for $\kappa < 1-\beta\left(1-\delta\right)\left(1-\Phi\right)$ monetary policy has to be passive, $\rho_\pi < 1$. Then, there exists either one or three stable eigenvalues, from which at least one is positive. This establishes the claims made in the proposition. □

Uniqueness of the a stable local equilibrium, is further guaranteed if $G(0) < -1 \Leftrightarrow$

$$\kappa < 1-\frac{\beta^2\left(1-\delta\right)\left(1-\Phi\vartheta\right)}{\left(\psi\rho_\pi\left(1-\vartheta\right)+1\right)}$$

or if $G''(1) = 2\frac{\kappa-\left(1+\left(1-\delta\right)\left(\left(1-\vartheta\right)\left(1-\Phi\right)\psi-\left(2\beta-1\right)\left(1-\Phi\vartheta\right)\right)\right)}{\beta\left(1-\delta\right)\left(1-\Phi\vartheta\right)} < 0$, which requires

$$\kappa < 1+\left(1-\delta\right)\left[\left(1-\vartheta\right)\left(1-\Phi\right)\psi-\left(2\beta-1\right)\left(1-\Phi\vartheta\right)\right]$$

Both restrictions on κ are sufficient, but not necessary for the existence of a unique stable eigenvalue. (Further details are available upon request from the aurthors.)

Appendix B

Consider a closed economy version of the model, $\vartheta = 0$, where public debt is held by domestic households, $b_t = b_{H,t}$, and PPI inflation equals CPI inflation, $\pi_t = \pi_{H,t}$. The set of linearized equilibrium conditions can then be reduced to the following conditions in real debt, consumption, inflation, and the nominal interest rate

$$\Phi\widehat{b}_{H,t} - \Phi E_t\widehat{\pi}_{t+1} = \widehat{R}_t - E_t\widehat{\pi}_{t+1} - \sigma(E_t\widehat{c}_{t+1} - \widehat{c}_t),$$
$$\widehat{\pi}_t = \chi\left(\sigma+\sigma_n\right)\widehat{c}_t + \beta E_t\widehat{\pi}_{t+1},$$
$$\widehat{b}_{H,t} = \widehat{R}_t + \frac{1-\kappa}{\beta\left(1-\delta\right)}\widehat{b}_{H,t-1} - \frac{1-\kappa}{\beta\left(1-\delta\right)}\widehat{\pi}_t,$$

and $\widehat{R}_t = \rho_\pi\widehat{\pi}_t$. Eliminating the interest rate, the system can be written as

$$\begin{pmatrix}\widehat{b}_{H,t}\\ \widehat{c}_{t+1}\\ \widehat{\pi}_{t+1}\end{pmatrix} = A\begin{pmatrix}\widehat{b}_{H,t-1}\\ \widehat{c}_t\\ \widehat{\pi}_t\end{pmatrix}, \quad A = \begin{pmatrix}\Phi & \sigma & 1-\Phi\\ 1 & 0 & 0\\ 0 & 0 & \beta\end{pmatrix}^{-1}\begin{pmatrix}0 & \sigma & \rho_\pi\\ \Lambda & 0 & \rho_\pi-\Lambda\\ 0 & -\Psi & 1\end{pmatrix}$$

where $\Lambda = \frac{1-\kappa}{\beta(1-\delta)} > 0$ and defining $\Psi = \chi\left(\sigma+\sigma_n\right) > 0$. The characteristic polynomial of A

is

$$F(X) = X^3 + \tfrac{-\Psi-\beta+\Phi\Psi-\Lambda\beta-1}{\beta} X^2 + \tfrac{\Lambda+\Psi\Lambda+\Lambda\beta+\Psi\rho-\Phi\Psi\rho+1}{\beta} X - \Lambda \tfrac{\Psi\rho+1}{\beta} \ ,$$

where

$$F(0) = -\tfrac{\Lambda}{\beta}\left(\Psi\rho_\pi + 1\right) < 0$$

$$F(1) = -\Psi\left(1-\rho_\pi\right)[\Phi - 1 + \tfrac{1-\kappa}{\beta(1-\delta)}]/\beta.$$

$$F(-1) = -[\Psi\left(1+\rho\right)\left(1+\Lambda-\Phi\right) + 2\left(1+\beta\right)\left(1+\Lambda\right)]/\beta < 0$$

Like in the open economy case, there are two sets of conditions that lead to $F(1) > 0$, such that there exist either one or three stable eigenvalues (from which at least one is positive). For $\kappa > 1-\beta\left(1-\delta\right)\left(1-\Phi\right)$ monetary policy has to be active, $\rho_\pi > 1$, while for $\kappa < 1-\beta\left(1-\delta\right)\left(1-\Phi\right)$ monetary policy has to be passive, $\rho_\pi < 1$. These necessary conditions for macroeconomic stability are identical to those presented in proposition 1.