TI 2011-024/1
Tinbergen Institute Discussion Paper

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Keeping Out Trojan Horses: Auctions and Bankruptcy in the Laboratory*

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February 8, 2011

Abstract

If a government auctions the right to market a good, continuity is likely to be of significant importance. In a laboratory experiment, we compare the effects of bidders' limited liability in the first-price sealed-bid auction and the English auction in a common value setting. Our data strongly reject our theoretical prediction that the English auction leads to less aggressive bids and fewer bankruptcies than the first-price sealed-bid auction. χ-cursedness gives a robust explanation of our experimental observations, in contrast to risk aversion and asymmetric equilibria.

Keywords and Phrases: Auctions, Bankruptcy, Laboratory Experiment

JEL Classification Numbers: C91, D44, L41

*For valuable comments, we thank Susan Athey, Gary Charness, Marcus Cole, Simon Gächter, Charley Holt, Audrey Hu, Thomas Kittsteiner, Dan Levin, Theo Offerman, Marion Ott, Sarah Parlane, Tim Salmon, and participants at conference and seminar presentations at the University of Amsterdam, the University of Nottingham, NAKE 2010, M-BEES 2010, CEDEX 2010, ESA 2010, and EARIE 2010. This project was supported by Dutch National Science Foundation grant NWO-VICI 453-03-606.

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1 Introduction

Confronted with a large wooden horse outside their gate, the Trojans discussed how to deal with it. Some, like the soothsayer Cassandra, advised destruction. Her father, King Priam, decided otherwise, which had the well-known dire consequences for Troy. Nowadays, governments may be confronted with a similar situation when auctioning the right to market a good: The bids may look very attractive from the onset, but the auction can turn into a nightmare if the winner goes bankrupt.

Indeed, a license auction or a procurement procedure can hardly be considered a success if the winning bidder defaults on its obligations. If the winner of a license auction files for bankruptcy, market power of the remaining competitors will increase, potentially at the cost of consumers. This situation may last for several years if the licenses are tied up in bankruptcy litigation. If the winner of a procurement procedure goes bankrupt, the delivery of goods and services may be considerably delayed and the procuring organization may have to buy those for a higher price from a different supplier.

The problem of defaulting bidders is not only of academic interest. In the 1996 C-block auction by the Federal Communications Commission in the US, all major bidders went bankrupt (Zheng 2001). Additionally, in the construction industry in the US between 1990 and 1997, 80,000 contractors filed for bankruptcy. The liabilities for public and private clients are estimated to lie above \$21 billion (Calveras et al. 2004).

Firms on the edge of bankruptcy may find it attractive to enter an auction for the following two reasons. First, they bid for 'options on prizes' rather than on 'prizes'. If the object turns out to be more valuable than expected, they make a nice profit. However, if it turns out to lead to losses, the firms will default, which they probably would have done in any case if they had not participated in the auction (Klemperer 2002, Board 2007). Second, they have an advantage over financially healthy firms because the latter have to take the downward risks of the project into account and are therefore willing to bid less aggressively than underfinanced firms (Zheng 2001, Klemperer 2002).

In this paper, we examine what an auctioneer can do to prevent bidders from going bankrupt. In particular, we answer the following question using a laboratory experiment: How do first-price and second-price auctions perform in terms of the likelihood of bankruptcy? This question is particularly interesting because license auctions tend to be typically second-price auctions, while procurement auctions are usually of the first-price type. If one of the two auction types tends to be less sensitive to ex post bankruptcy, the government may have a reason to switch to the other auction type.[1]

The literature answers our research question only partially. In theory, in settings with (stochastic) private values, the probability of bankruptcy in second-price auctions is higher than in first-price auctions (Parlane 2003, Engel and Wambach 2006, and Board 2007). The intuition is the following. Bidders like taking risks if they are limitedly liable because they are not hurt as much by the downside risk as bidders with sufficient resources. Because the dispersion of the equilibrium price in second-price auctions is larger than in first-price auctions, bidders are willing to bid higher in second-price auctions. As a consequence, it is more likely that bankruptcies arise in second-price auctions than in first-price auctions.

Common value auctions with limitedly liable bidders have hardly been studied theoretically. For settings with unlimited liability, it is well known that in common value auctions, second-price auctions result in higher equilibrium prices than first-price auctions (Milgrom and Weber 1982). Therefore, second-price auctions may be more sensitive to bankruptcy. However, bidders can take information into account contained in others' bids in second-price auctions but not in first-price auctions. So, if this information relates to the value of the object, bidders may bid cautiously in case of 'bad news' resulting in a low probability of bankruptcy. Therefore, second-price auctions may perform better than first-price auctions in terms of bankruptcy. In our setting, this is indeed the case.

Our paper relates to the experimental literature on common value auctions and winner's curse.[2] Levin et al. (1996) find that the first-price sealed-bid auction (FP) and the English auction (EN) do not differ systematically in terms of average revenue unless the uncertainty about the common value is relatively small.[3] Although their experimental design was not aimed

[1] In practice, there are several other mechanisms than standard auctions that may perform well in terms of preventing bankrupt bidders, including cash auctions (as opposed to debt auctions) (Rhodes-Kropf and Viswanathan 2000), surety bonds (Calveras et al. 2004), multi-sourcing (Engel and Wambach 2006), and the 'average bid auction' (Decarolis 2010). Burguet et al. (2009) study optimal procurement auctions for settings with limitedly liable contractors.

[2] See Kagel and Levin's (2002) book for an excellent overview.

[3] In affiliated signals common value settings, overbidding relative to the risk neutral Nash equilibrium is

at studying limited liability, it has some features of it. Subjects interacted in a series of auctions. Profits were added to and losses were subtracted from their starting capital. When their cash balance exhausted, they were declared bankrupt and they had to leave the experiment. It turned out that some students indeed went bankrupt.[4]

Roelofs (2002) and Saral (2009) study the effect of limited liability on bidding behavior in the laboratory. Roelofs observes that in the first-price sealed-bid auction, bidders increase their bid if default is possible compared to a situation where it is not. Saral analyzes bidding in second-price auctions under unlimitedly liability and two types of limited liability: market-based limited liability (inter-bidder resale following the auction) and statutory limited liability (a bidder pays a penalty if she makes a loss). She finds that bids are lower under unlimited liability than under market-based limited liability and statutory limited liability with a low default penalty. In the case of a high default penalty, the average bid does not differ between statutory limited liability and unlimited liability. Neither Roelofs nor Saral study the relative performance of standard auctions, which is the target of our study.

We examine bidding under limited liability in FP and EN. We do so in a laboratory experiment in an independent private signals common-value setting. In section 2, we present our experimental design and hypotheses. Our model is a three-bidder wallet game (Klemperer 1998). Subjects are limitedly liable in the same way as in Saral's (2009) statutory limited liability regime. In our design, subjects always go bankrupt if they win the auction for a price exceeding the object's value. In the case of bankruptcy, subjects do not leave the experiment, but they pay a fine from their starting capital that is large enough to guarantee that it will never be exhausted. This set-up makes it relatively easy to derive the Nash equilibria and construct hypotheses on the basis of those. We show that EN has a symmetric equilibrium in which none of the bidders goes bankrupt. The equilibrium of FP is analytically not solvable, but we numerically derive that bidders bid more aggressively than in EN resulting in both

commonly observed in both FP (Kagel and Levin 1986, Dyer et al. 1989, Lind and Plott 1991, Levin et al. 1996) and EN (Levin et al. 1996). Levin et al. (1996) find that in FP, the average winning bid exceeds the equilibrium winning bid significantly more than in EN. The average winning bids do not differ because the equilibrium winning bid in EN is higher than in FP.

[4] Lind and Plott (1991) created an environment that mimicked unlimited liability more closely than in Levin et al.'s (1996) experiment: The subjects earned funds in private value auctions which substantially reduced the likelihood of bankruptcy. Moreover, if they still went bankrupt, they would work off losses by doing jobs like photocopying for the department.

higher expected revenue and a strictly positive probability of bankruptcy.

Section 3 contains our experimental results. We observe that in both auctions, subjects bid more aggressively and, in turn, go bankrupt more often than predicted by theory. Moreover, FP does not raise more money than EN and both auctions perform equally well in terms of frequency of bankruptcy. These results remain valid when comparing the experimental outcomes with the outcomes in settings in which subjects had to cover their losses.

In Section 4, we check whether our data are consistent with risk aversion, asymmetric equilibria, and Eyster and Rabin's (2005) χ-cursedness. We argue that χ-cursedness gives a robust explanation of where our experimental observations differ from our initial theoretical results, in contrast to risk aversion and asymmetric equilibria. Section 5 concludes.

2 Experimental Design and Hypotheses

2.1 Procedures and Parameters

We ran our experiment at the Center for Research in Experimental Economics and political Decision making (CREED) at the University of Amsterdam. From the student population, 144 undergraduates were publicly recruited and split into 4 groups of 36 students, one group for each treatment. Each session consisted of 4 parts of 12 rounds. Subjects read the computerized instructions at the start of each part. The instructions of part 1 and 2 included test questions to check the subjects' understanding of the instructions. Because parts 3 and 4 were equal to parts 1 and 2 respectively, we did not ask test questions for those parts.[5] Each session took about 2 hours and participants earned on average 19.28 euro. Earnings were noted during the experiment in experimental 'francs', having an exchange rate of 100 francs for 3.50 euros. The experiment and the instructions were programmed within the AJAX framework in JavaScript and PHP Script.

Two treatments consisted of English auctions and two consisted of first-price sealed-bid auctions. All sessions alternated 2 parts in which participants were limitedly liable with 2 parts where they were unlimitedly liable. We included rounds with unlimited liability so that we could identify the effect of limited liability on bidding behavior. Subjects got a starting capital of 50 [150] francs before the beginning of each part in the case of [un]limited liability. To control

[5] The instructions and test questions can be found at www.sanderonderstal.com/InstructionsTrojanhorses.pdf.

for order effects, we ran the parts in half of the treatments in an ULUL sequence (unlimited, limited, unlimited, and limited) and the other half in a LULU sequence. The first two parts of every session were meant to give the participants the opportunity to gain experience. For the duration of each session, the group of participants was randomly split in fixed matching groups of 6 out of which for all rounds, 2 bidding groups of 3 bidders each were randomly chosen by the software, resulting in the four treatments in table 1.

The subjects interacted in the three-bidder wallet game (Klemperer 1998). Before the auction, the three bidders $i \in \{1, 2, 3\}$ were each presented a private signal θ_i, randomly and independently drawn from a uniform distribution on $[0, 100]$. We kept draws constant across treatments for the sake of comparability of the results. The value of the object was the sum of the three private signals:

$$v = \theta_1 + \theta_2 + \theta_3.$$

In FP, subjects independently entered a bid between 0 and 300. The highest bidder won and paid a price equal to his own bid. EN consisted of two phases. In phase 1, the price started at zero and was increased by one every 1/6 second. The first phase ended as soon as a subject quit the auction by pressing a 'stop' button. Before the start of the second phase, the other participants were informed that one of the bidders stepped out and the level of her bid. After 5 seconds, the price was increased again until one of the two remaining bidders dropped out. The remaining bidder won the object for the price at which the second highest bidder quit. All bidders automatically stepped out at a price of 300 when they had not quit beforehand. In both auctions, ties were resolved randomly. Between rounds, subjects were informed about the true value of the object but not about the signals or bids of others.

Table 1: Summary of treatments

Auction	Order of Liability Regimes	# Sessions	# matching groups
EN	ULUL	2	6
	LULU	2	6
FP	ULUL	2	6
	LULU	2	6

Notes: U [L] stands for 'unlimited liability' ['limited liability']

The payoffs for each round were as follows. In the limited liability regime, bidder i's utility is given by

$$U_i^\ell(v,p,w) = \begin{cases} v-p & \text{if } w=i \text{ and } v \geq p \\ -c & \text{if } w=i \text{ and } v<p \\ 0 & \text{if } w \neq i \end{cases}$$

where $w \in \{1,2,3\}$ denotes the winner of the auction, p the price the winner pays, and $c>0$ bankruptcy costs. In the experiment, $c=4$. Note that the 50 francs endowment at the start of each part of 12 rounds ensured that subjects always obtained positive earnings. This model captures a situation where the winning bidder goes bankrupt if she makes a loss, in which case she incurs some (fixed) bankruptcy costs instead of the loss. Notice that these costs can be higher than the loss. For example, if the price exceeds value by 3, the incurred loss equals 4 instead of 3.

In the unlimited liability regime, payoffs are

$$U_i^\infty(v,p,w,s) = \begin{cases} \max(v-p,-s) & \text{if } w=i \\ 0 & \text{if } w \neq i \end{cases}$$

where s denotes the total score of the participant i before the start of that round, i.e., the payoffs in this part up to the current round including the endowment at the start of the part. Therefore, under the unlimited liability regime the total score of a participant also could never become negative. By choosing the 150 francs endowment, we feel that we found a good balance between mimicking a setting with truly unlimited liability (which requires an extremely high starting capital) and giving subjects sufficient incentives to earn money on top of the endowment (which favors a low starting capital).[6]

[6]In parts 3 and 4, 3 out of the 144 participants in some round, had accumulated losses exceeding their endowment. Of these participants, one took part in FP and two in EN. The fact that subjects did not have to cover losses above their endowment may have induced them to bid more aggressively relative to a setting with truly unlimited liability. Note that this is unfavorable to our hypothesis that bidders bid at least as aggressively under limited liability as under unlimited liability.

2.2 Hypotheses

The equilibrium strategies can be straightforwardly derived from the literature.[7] The symmetric Bayesian Nash equilibrium of EN with unlimited liability is given by

$$\begin{aligned} B_E^1(\theta) &= 3\theta, \text{ and} \\ B_E^2(\theta, \tilde{B}_E^1) &= 2\theta + \frac{\tilde{B}_E^1}{3} \end{aligned} \tag{1}$$

where B_E^φ is the price at which a bidder steps out of the auction in phase $\varphi = 1, 2$ of the auction and $\tilde{B}_E^1$ is the price at which the lowest bidder leaves the auction. It is readily verified that the winning bidder will always make a positive profit in equilibrium so that the equilibrium under unlimited liability is also an equilibrium in the case of limited liability. Let $\theta^{(k)}$ denote the k-th highest value from $\{\theta_1, \theta_2, \theta_3\}$, $k = 1, 2, 3$. In equilibrium, expected revenue equals

$$R_E^\infty = R_E^\ell = E\left\{B_E^2(\theta^{(2)}, \tilde{B}_E^1(\theta^{(3)}))\right\} = 125.$$

The unique equilibrium of FP with unlimited liability is given by

$$B_F(\theta) = \frac{5}{3}\theta. \tag{2}$$

If bidders are unlimitedly liable, expected revenue[8] in FP equals

$$R_F^\infty = E\left\{B_F(\theta^{(1)})\right\} = 125.$$

Therefore, expected revenue for FP and EN is the same and revenue equivalence holds in the unlimited liability case, which is not surprising in view of Myerson's (1981) revenue equivalence theorem.

In FP, the winner makes a loss with some probability because

$$v - B_F(\theta^{(1)}) = -\frac{2}{3}\theta^{(1)} + \theta^{(2)} + \theta^{(3)} < 0$$

[7] The wallet game is a special case of Milgrom and Weber's (1982) affiliated signals model. Milgrom and Weber derive symmetric equilibria for the English auction and the first-price sealed-bid auction with unlimited liability. These equilibria are presented here. Equilibrium uniqueness follows from a standard argument (see e.g., Bulow et al. 1999).

[8] Note that in our design, the winning bidder actually pays the seller, even if the bidder goes bankrupt. Clearly, in the case of a debt auction, expected revenue for the seller may be different. In such settings, 'expected revenue' should be read as the expected price to be paid by the winner.

for low values of $\theta^{(2)}$ and $\theta^{(3)}$. More specifically,

$$\begin{aligned} P\left\{v - B_F(\theta^{(1)}) < 0 \middle| \theta^{(1)} = \theta\right\} &= P\left\{\theta^{(2)} + \theta^{(3)} < \frac{2}{3}\theta^{(1)} \middle| \theta^{(1)} = \theta\right\} \\ &= P\left\{\theta_1 + \theta_2 < \frac{2}{3}\theta \middle| \theta_1, \theta_2 < \theta\right\} \\ &= \frac{2}{9}. \end{aligned}$$

So, the probability that the winner makes a loss is independent of the winner's signal, which makes sense because the signals for the second and third highest bidder are uniformly distributed between 0 and the highest signal.

With respect to equilibrium bidding in FP in the case of limited liability, we derive the following result.[9]

Proposition 1 *FP has a symmetric Bayesian Nash equilibrium which follows from the following differential equation:*

$$b_F'(\theta) = \frac{10\theta^2 - 4\theta b_F(\theta)}{\theta^2 + 2\theta b_F(\theta) - (b_F(\theta))^2 + 2c\,(b_F(\theta) - \theta)} \tag{3}$$

with boundary condition

$$b_F(0) = 0.$$

Because the differential equation is not solvable analytically, we rely on the fourth order Runge-Kutta method to approximate a solution using signals starting at zero with increments of 0.01.[10] We find that if $c = 4$, revenue in FP is approximately

$$R_F^\ell \approx 137.$$

The probability that the winner makes a loss and goes bankrupt equals about 34%. So, in the case of limited liability, both expected revenue and the probability of bankruptcy is higher in FP than in EN.

Comparing between settings with limited and unlimited liability, we observe that expected revenue remains the same in EN, while it increases in FP. Moreover, according to theory, bidders

[9] We relegated proofs of propositions to the Appendix.

[10] It is readily verified that if $c = 0$, the equilibrium bidding function is $b_F(\theta) = 2\theta$. In this equilibrium, the probability that the winning bidder goes bankrupt is equal to 50% and expected revenue equals 150.

never make losses in EN regardless of their liability. This is in contrast to FP, in which bidders make losses in both liability settings. In particular, winners are expected to go negative more often under limited liability than under unlimited liability. These results allow us to construct the following hypotheses related to our main research questions:

Hypothesis 1 In the case of limited liability, FP raises higher revenue than EN. In FP, bidders incur losses more often than in EN.

Hypothesis 2 For EN, limitation of liability does neither increase revenue nor the probability of overbidding.

Hypothesis 3 For FP, limitation of liability increases both revenue and the probability of overbidding.

3 Experimental Results

We present the results of our experiment in two sections. Section 3.1 deals with differences in revenues and the presence of winners with negative payoffs between auctions and liability regimes. Section 3.2 explores individual bidding behavior including learning and order effects.

3.1 Comparisons over auctions and liability regimes

In this section, we focus on the aggregate results from parts 3 and 4, i.e., we only consider experienced bidders. Figure 1 indicates that average revenue is higher under limited liability than under unlimited liability for both FP and EN. While this was expected for FP, our analysis predicted revenue equivalence for EN. Moreover, in the limited liability regime, revenue in EN is higher than in FP, although the difference between auctions is smaller than the difference between liability regimes. This observation is also in contrast with our theoretical predictions that FP revenue dominates EN in the case of limited liability.

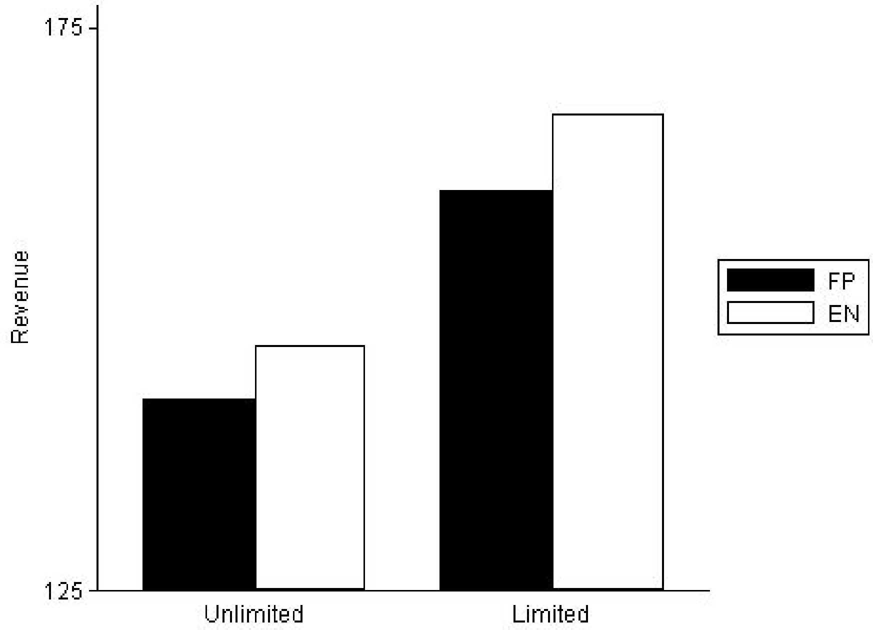

Figure 1: Aggregated revenue over auctions and liability regimes

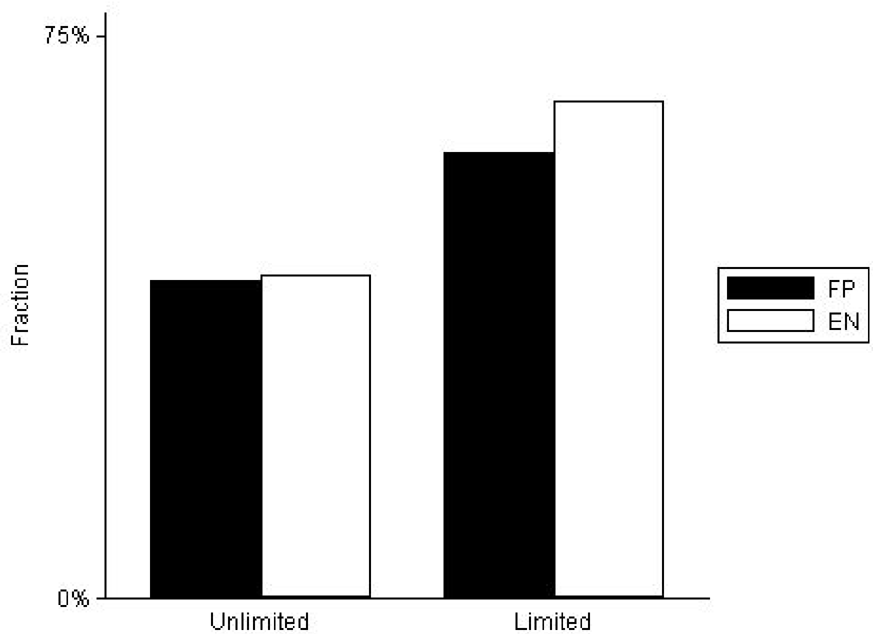

Figure 2: Fraction of winners with negative payoffs

When we aggregate the fraction of winners having negative payoffs (figure 2), the above pattern is confirmed: There is a (slightly) higher frequency of negative payoff in EN than in

FP and substantially more bankruptcies in the case of limited liability than losses in the case of unlimited liability. Furthermore, figure 2 indicates a much higher number of winners scoring a negative payoff than expected.[11]

The results above are made more precise in table 2, using a within-comparison between liability regimes per auction type concerning revenues and the fraction of winning bidders making a loss. The statistical tests are based on aggregate data per matching group. For both auctions, we find a significantly higher revenue and fraction of winners making a loss under the limited liability regime than under the unlimited liability regime.

Table 2: Within-auction comparison of liability types

		Unlimited		Limited		Difference	
Type	Variable	Nash	Realized (s.d.)	Nash	Realized (s.d.)	Nash	Realized (s.d.)
FP	Revenue	120.8	142.0 (6.5)	132.4	160.5 (12.7)	11.6	18.4 (12.6)**
	%Losing	8.3%	42.4% (8.3)	20.8%	59.4% (9.4)	12.5%	17.0% (10.6)**
EN	Revenue	130.0	146.8 (9.7)	130.0	167.4 (9.0)	0	20.6 (7.6)**
	%Losing	0%	43.1% (12.7)	0%	66.3% (10.0)	0%	23.3% (13.8)**

Notes: The unit of observation is the average per matching group, %Losing refers to the fraction of winners with negative payoffs, and s.d. stands for 'standard deviation'. ** indicates statistical significance at the 1% level on the basis of the signed-rank test. The Nash predictions are based on the actual draws of the signals.

Table 3 compares for both auction types the effect of the liability regimes with respect to revenue, fraction of winners with a negative payoff, and the losses made. To make the losses made comparable for limited and unlimited liability regimes, we present for both the difference between the value of the object and the price of the object, ignoring the protection limitation of liability would offer to bidders making a loss. We do not find support for the hypothesis that bidders protected by limited liability bid more aggressively in FP than in EN. On the contrary, EN generates significantly more revenue than FP and also the number of winners going bankrupt is higher, albeit not significantly so. Moreover, using a difference-in-difference approach, all differences cease to be significant. Finally, with respect to losses made, we cannot reject the hypothesis that these are the same for both types of auction, neither on the level of the liability regimes nor with respect to the difference between regimes.

[11] On the basis of the drawn signals, we predict 0% for the EN treatments and 8.3% and 20.8% for unlimited and limited liability respectively in the FP treatments. The realized fractions are clearly higher.

Table 3: Between-auction comparison

Variable	Liability	FP Nash	FP Realized (s.d.)	EN Nash	EN Realized (s.d.)	p-values
Revenue	Unlimited	120.8	142.0 (6.5)	130.0	146.8 (9.8)	0.17
	Limited	132.4	160.5 (12.7)	130.0	167.4 (9.0)	0.03*
	Diff-in-diff	11.6	18.4 (4.5)	0	20.6 (7.6)	0.25
%Losing	Unlimited	8.3%	42.4% (8.3%)	0%	43.1% (12.7%)	0.82
	Limited	20.8%	59.4%(9.4%)	0%	66.3% (10.0%)	0.11
	Diff-in-diff	12.5%	17.0% (10.6%)	0%	23.3% (13.8%)	0.33
Losses Made	Unlimited	10.8	25.9 (7.4)	0	27.4 (8.5)	0.39
	Limited	19.4	37.2 (11.5)	0	37.6 (7.5)	0.56
	Diff-in-diff	8.6	11.3 (11.6)	0	10.2 (9.5)	0.95

Notes: The unit of observation is the average per matching group, %Losing refers to the fraction of winners with negative payoffs, Losses Made are the average losses when the winner has a negative payoff, Diff-in-diff is the outcome of the difference for the auction type between the limited and unlimited regime, and s.d. stands for 'standard deviation'. The p-values emerge from the Mann-Whitney test. * indicates statistical significance at the 5% level.

3.2 Individual behavior

In this section, we study subjects' individual bidding behavior, which serves as a stepping stone to our analysis in Section 4 in which we try to unravel why observed behavior differs from the theoretical predictions. Table 4 shows the fraction of auctions in which bidders with the highest signal win the auction. Observe that this is not a measure of efficiency because we deal with pure common value auctions. The goal here is to check our theoretical prediction that in equilibrium, all participants bid according to the same bid function that is monotonically increasing in their signal. Table 4 shows that on average only in between 60% and 70% of the cases, the bidder with the highest signal wins.

Table 4: Fraction of winners having the highest signal

Auction	Liability	% highest signal wins
FP	Limited	69.79%
	Unlimited	61.81%
EN	Limited	63.54%
	Unlimited	62.85%

To examine bidding behavior in greater detail, we estimated a random effects model with a clustering specification to get robust p-values. We estimated three bidding functions: B_{ijt}^{F} for bidders in FP, and $B_{ijt}^{E_1}$ [$B_{ijt}^{E_2}$] for the first [second] bidder to step out in EN, where ijt indicates bidder i in matching group j in round t:

$$
\begin{aligned}
B_{ijt}^{F} &= \beta^{F} + \beta_{\theta}^{F}\theta_{ijt} + \beta_{L}^{F}L_{ijt} + \beta_{\theta L}^{F}\theta_{ijt}L_{ijt} \\
&\quad + \beta_{Lulu}^{F}Lulu_{ijt} + \beta_{\theta Lulu}^{F}\theta_{ijt}Lulu_{ijt} + \beta_{X}^{F}X_{ijt} + \beta_{\theta X}^{F}\theta_{ijt}X_{ijt} + \alpha_{j}^{F} + \epsilon_{ijt}^{F}, \\
B_{ijt}^{E_1} &= \beta^{E_1} + \beta_{\theta}^{E_1}\theta_{ijt} + \beta_{L}^{E_1}L_{ijt} + \beta_{\theta L}^{E_1}\theta_{ijt}L_{ijt} \\
&\quad + \beta_{Lulu}^{E_1}Lulu_{ijt} + \beta_{\theta Lulu}^{E_1}\theta_{ijt}Lulu_{ijt} + \beta_{X}^{E_1}X_{ijt} + \beta_{\theta X}^{E_1}\theta_{ijt}X_{ijt} + \alpha_{j}^{E_1} + \epsilon_{ijt}^{E_1}, \\
B_{ijt}^{E_2} &= \beta^{E_2} + \beta_{\theta}^{E_2}\theta_{ijt} + \beta_{\tilde{B}^{E_1}}^{E_2}\tilde{B}_{ijt}^{E_1} + \beta_{L}^{E_2}L_{ijt} + \beta_{\theta L}^{E_2}\theta_{ijt}L_{ijt} \\
&\quad + \beta_{Lulu}^{E_2}Lulu_{ijt} + \beta_{\theta Lulu}^{E_2}\theta_{ijt}Lulu_{ijt} + \beta_{X}^{E_2}X_{ijt} + \beta_{\theta X}^{E_2}\theta_{ijt}X_{ijt} + \alpha_{j}^{E_2} + \epsilon_{ijt}^{E_2},
\end{aligned}
$$

where L is a dummy that equals 1 iff liability is limited, $Lulu$ is a dummy which is equal to 1 iff subjects play the LULU sequence, X is a dummy referring to a subjects' experience (1 for parts 3 and 4), and $\tilde{B}^{E_1}$ denotes the price at which the first bidder stepped out in EN. The β's are the parameters of the model.

Table 5 contains the regression results. Observe that the slopes are much lower and the constants much higher than theory predicts.[12] Figure 3 contrasts the theoretical equilibrium bidding function and the estimated one for FP in the case of limited liability. Note that the theoretical equilibrium bidding function is almost linear so that it makes sense to compare it with the estimated bidding function, which we restricted to be linear. Limitation of liability has a strongly significant effect on the constant of the bidding function, but not on the slope. Furthermore, for the bidding function for the lowest bid in EN, there is a higher constant and a higher slope than for FP. In contrast, for the bidding function for the highest bid, the opposite holds true: a lower constant and a lower slope for EN than for FP. The reason can be seen in the regression for the highest bid, participants react strongly to the level at which the first bidder stepped out. Bidding turns out to be quite aggressive in phase 1 of the auction, while in phase 2, bidders step out relatively quickly. Subjects behave as if thinking that in the second phase of EN, they always have the chance to safely step out of the auction. Still, bidders use the information contained in the behavior of the first bidder in that in they quit earlier in the second phase the earlier another bidder stepped out in the first phase.

[12] Those differences are statistically significant according to Wald tests.

Table 5: Estimated bidding functions

	FP	EN	
	Bid	Lowest bid	Winning bid
	Coef (s.e.)	Coef (s.e.)	Coef (s.e.)
Constant	58.76 (4.33)**	73.80 (4.69)**	55.59 (4.42)**
Signal (θ)	0.95 (0.06)**	0.76 (0.13)**	0.60 (0.07)**
Lowest bid ($\tilde{B}^{E_1}$)			0.53 (0.03)**
Limited (L)	16.83 (4.51)**	12.73 (6.37)*	15.36 (4.17)**
Signal*Limited (θL)	-0.06 (0.08)	-0.00 (0.18)	0.06 (0.06)
LULU	-4.28 (6.27)	8.78 (7.99)	-1.20 (5.13)
Signal*LULU ($\theta Lulu$)	0.07 (0.74)	0.05 (0.15)	-0.11 (0.06)
Experienced (X)	-1.23 (3.62)	0.23 (2.92)	-6.09 (2.78)*
Signal*Experienced (θX)	0.11 (0.05)*	0.43 (0.09)**	0.12 (0.06)

Notes: ** [*] indicates statistical significance at the 1% [5%] level, and s.e. stands for '(robust) standard error'.

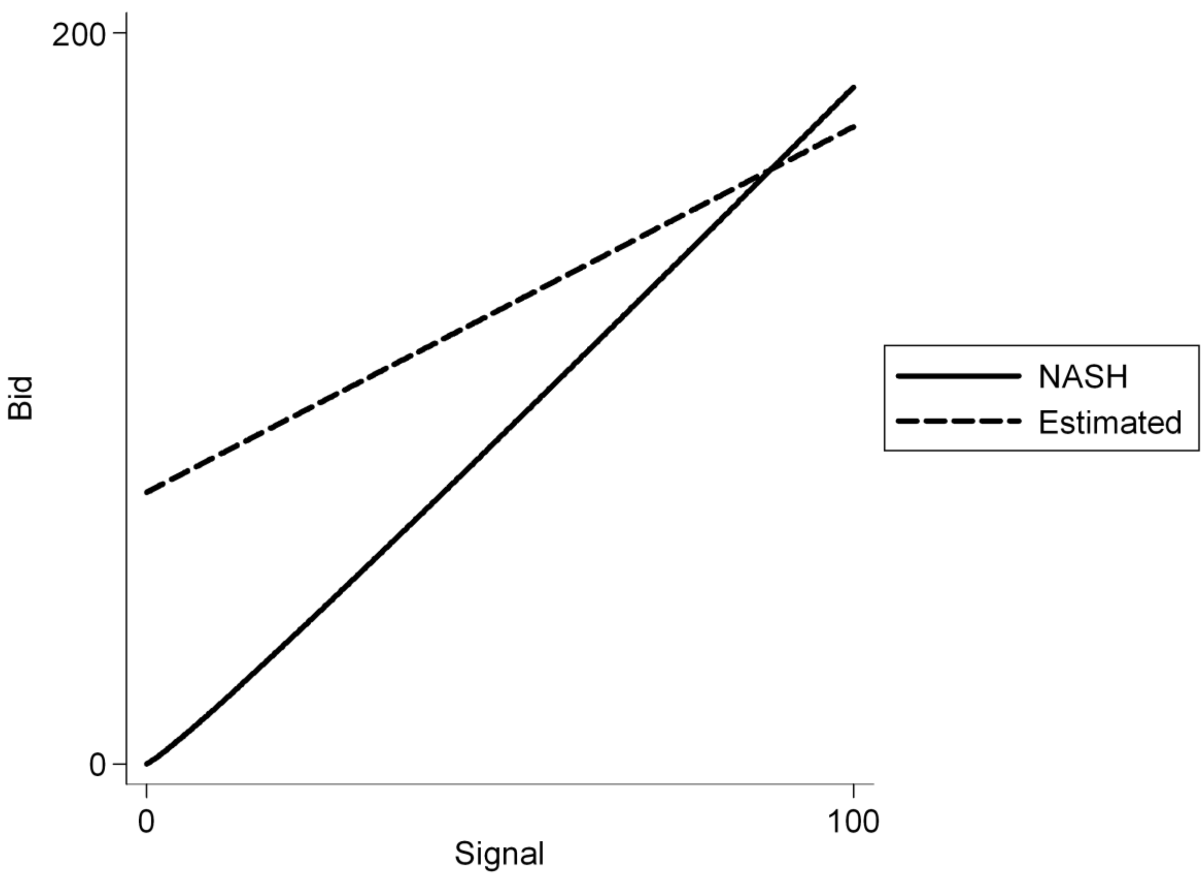

Figure 3: Theoretical and estimated bid function for FP for the case of limited liability.

In the regression, we added the last four variables in table 5 to control for order effects and learning. This turned out not to change the significance and direction of the other coefficients.

We do not observe order effects, but there seems to be some learning. In FP, bidders adapt their bidding behavior albeit in the wrong direction: In parts 3 and 4, they bid more aggressively than in the first two parts, overbidding even more relative to the Nash equilibrium. For EN, we observe experienced bidders to let their bids depend more on their signal than inexperienced ones. However, given that the expected second highest signal equals 50, the net effect of experience on the average winning bid is minimal.

4 Explanation of the main results

In this section, we attempt to explain the differences between our data and the theoretical predictions. In particular, in both auctions and under both liability regimes, bidders tend to overbid relative to the Nash equilibrium. Moreover, we reject the hypothesis that in the case of limited liability, bidding is more aggressive in FP than in EN. We explore risk aversion, asymmetric equilibria, and χ-cursedness as potential explanations in subsections 4.1, 4.2, and 4.3 respectively.

4.1 Risk aversion

To which extent are our data consistent with equilibrium bidding for risk averse bidders? Suppose that all three bidders have the same common utility function u, where u is differentiable, strictly increasing, and strictly concave, with $u(0) = 0$. In EN, equilibrium bidding is not affected by bidders' risk attitudes: In both phases of the auction, bidders drop out at the price at which their payoff would be zero if the remaining competitor(s) dropped out at that price. In FP, the affect of risk aversion is not clear a priori. In the standard symmetric independent private values model, risk averse bidders bid more aggressively than risk neutral ones (Maskin and Riley 1984). However, in the case of a common value, from a bidder's viewpoint, the object's value is stochastic because she does not know the signals of the other bidders. This tends to drive down bids. Holt and Sherman (2000) show that these two effects exactly cancel in a two-bidder wallet game. In equilibrium, risk averse bidders bid as if they were risk neutral. In the case of three bidders, intuitively, the second effect dominates the first: More competition drives up the price so that a risk averse bidder has lower incentives to further increase it while she is more inclined to shade the risk neutral equilibrium bid because she has less information about the common value. The following proposition confirms this intuition.

Proposition 2 *In the case of unlimited liability, for risk averse bidders, the symmetric Bayesian Nash equilibrium of FP has the property that*

$$B_F^r(\theta) < \frac{5}{3}\theta = B_F(\theta).$$

All in all, risk aversion does not seem to be the (sole) reason why subjects tend to overbid in either auction.

4.2 Asymmetric equilibria

Alternatively, subjects may have played different equilibria than the above symmetric equilibria. However, for FP this cannot be the case as the symmetric equilibrium is the unique equilibrium. In contrast, EN has a continuum of asymmetric equilibria as the following proposition by Engelmann and Wolfstetter (2009) shows.

Proposition 3 *In the case of unlimited liability, EN has the following equilibria:*

$$\begin{aligned} B_{E,i}^1(\theta) &= \gamma_i\theta, \\ B_{E,i}^2(\theta, \tilde{B}_E^1, k) &= \delta_i\theta + \frac{\tilde{B}_E^1}{\gamma_k}, \end{aligned}$$

where $B_{E,i}^1(\theta)$ [$B_{E,i}^2(\theta, \tilde{B}_E^1, k)$] denotes the price at which bidder i steps out when no one [bidder $k \in \{1,2,3\}\backslash\{i\}$] has stepped out [at price $\tilde{B}_E^1$], $i = 1,2,3,$ and

$$\begin{aligned} \gamma_i, \delta_i &> 0,\ i = 1,2,3, \\ \gamma_1\gamma_2 &> \gamma_1 + \gamma_2, \\ \gamma_3 &= \frac{\gamma_1\gamma_2}{\gamma_1\gamma_2 - \gamma_1 - \gamma_2},\ \textit{and} \\ \delta_m &= \frac{\delta_n}{1-\delta_n},\ \{m,n\} = \{1,2,3\}\backslash\{k\}. \end{aligned}$$

Corollary 1 *Expected revenue in the symmetric equilibrium (1) of EN is at least as high as in any of the equilibria in Proposition 3.*

The asymmetric equilibria of EN share two properties that are inconsistent with our data. First, the equilibrium price is always below the value of the object so that bidders never make a loss. This implies that the above strategies are also an equilibrium for a setting with limited liability. In other words, asymmetric equilibria cannot explain why bidders bid more aggressively in the case of limited liability compared to the case of unlimited liability. Second, expected revenue in the asymmetric equilibria is always lower than in the symmetric one. This is clearly inconsistent with our observation in the experiment that average revenue is much higher than in the symmetric equilibrium.

Also the explanation that subjects 'miscoordinate' on an asymmetric equilibrium does not seem appealing. Clearly, an asymmetric equilibrium requires bidders to coordinate as to who bids aggressively and who does not. However, we did not find evidence that bidders adapted their strategies over time in the direction of an asymmetric equilibrium. Moreover, even in the case of miscoordination, the first-phase bidding functions should have a zero constant, which we clearly rejected when estimating bidding functions in section 3.2.

We conclude that our data cannot be solely explained by bidders playing asymmetric equilibria.

4.3 Cursed bidders

Finally, subjects may have behaved as 'cursed' bidders in line with Eyster and Rabin's (2005) χ-cursed equilibrium. We start by deriving the χ-cursed equilibrium for the two auctions if bidders are unlimitedly liable.

Proposition 4 *The symmetric χ-cursed equilibrium of EN with unlimited liability is given by*

$$\begin{aligned} B_E^{1,\chi}(\theta) &= 100\chi + \theta\,(3-2\chi)\,, \textit{ and} \\ B_E^{2,\chi}(\theta, \tilde{B}_E^1) &= \left(2\theta + \frac{\tilde{B}_E^1 - 100\chi}{3-2\chi}\right)(1-\chi) + (\theta+100)\,\chi. \end{aligned} \tag{4}$$

Proposition 5 *The symmetric χ-cursed equilibrium of FP with unlimited liability is given by*

$$B_F^{\chi}(\theta) = 100\chi + \left(\frac{5}{3} - \chi\right)\theta. \tag{5}$$

The following corollary shows that the expected revenue for the seller is the same for both auctions, given that all bidders possess the same level of χ-cursedness.

Corollary 2 *In the case of unlimited liability, FP and EN generate the same expected equilibrium if bidders play the symmetric χ-cursed equilibrium. Equilibrium revenues are equal to*

$$R_F^{\infty,\chi} = R_E^{\infty,\chi} = 125 + 25\chi.$$

The estimated coefficients for the bidding function for FP in table 5 indicate that on aggregate, bidding strategies correspond to an average χ-cursedness level of about 0.65. For EN, the estimated bidding functions are less appropriate to estimate the average χ because we only observe the lowest two bids. Average revenue for EN produces a better approximation for the average χ because the bid in the middle determines revenue. Using this, the average χ is about 0.87. Eyster and Rabin (2005) find that the average χ-cursedness level for experienced subjects in Avery and Kagel's (1997) experiment on the two-bidder wallet game equals 0.64. Our estimates seem reasonably close to that. Moreover, subjects may differ in the level of χ-cursedness, which could explain the observation in table 4 that it is not always the bidder with the highest signal who wins. The difference in estimated average χ-cursedness level between EN and FP may be explained by 'auction fever'. To some extent, cursed bidders compete as if bidding in a setting with uncertain private values. In a lab experiment, Ehrhart et al. (2008) show that in an environment with uncertain private values, bidders tend to be affected by auction fever in that they bid higher in ascending auctions than in strategically equivalent sealed-bid auctions.

For the limited liability setting, our data reject the theoretical prediction that FP yields more revenue and more bankruptcies than EN. Cursedness could offer an explanation here as well. Fully cursed bidders (for whom $\chi = 1$) experience the auction as a pure private value auction because they do not take into account that the fact of winning impacts the expected value for the object. As is well known for (stochastic) private value auctions, in the case of limited liability, expected revenue is higher and the winner is more likely to go bankrupt in EN than in FP (Parlane 2003, Engel and Wambach 2006, and Board 2007). This result also holds true in our setting as the propositions below show. Define

$$\tilde{U}(p, \theta_1) \equiv E_{\theta_2,\theta_3}\left\{\max(0, v - p)\right\} - cP\left\{v < p\right\}$$

as the perceived expected utility of a 1-cursed bidder with signal θ_1 when winning at price p.

Proposition 6 *In the case of limited liability, in the symmetric 1-cursed equilibrium of EN, a bidder with signal θ steps out at $b_E^{\chi=1}(\theta)$ which is implicitly defined by*

$$\tilde{U}(b_E^{\chi=1}(\theta), \theta) = 0.$$

To solve for the bidding function, assume that $b_E^{\chi=1}(\theta_1) > 100 + \theta_1$ for all $\theta_1 \in [0, 100]$. Bidder 1 solves

$$\frac{1}{6,000,000}(200 - p + \theta_1)^3 - \frac{c}{10,000}\left[10,000 - \frac{1}{2}(200 - p + \theta_1)^2\right] = 0.$$

The first [second] term on the left-hand side refers to the situation in which bidder 1 does not go [goes] bankrupt. The resulting bidding function is approximately

$$b_E^{\chi=1}(\theta) \approx \theta + 200 - \sqrt[3]{60,000c} + \sqrt[3]{\frac{c^2}{60,000}} + c \approx 141.9 + \theta.$$

Indeed, $b_E^{\chi=1}(\theta_1) > 100 + \theta_1$, like we assumed. The corresponding expected revenue equals

$$R_E^{\ell,\chi=1} \approx 191.9.$$

Proposition 7 *In the case of limited liability, the symmetric 1-cursed equilibrium of FP follows from the following differential equation:*

$$b_F^{\chi=1\prime}(\theta) = -\frac{2}{\theta}\frac{\tilde{U}(b_F^{\chi=1}(\theta), \theta)}{\tilde{U}_1(b_F^{\chi=1}(\theta), \theta)} \tag{6}$$

with boundary condition

$$\tilde{U}(b_F^{\chi=1}(0), 0) = 0.$$

Numerically, we derive that expected revenue equals approximately

$$R_F^{\ell,\chi=1} \approx 188.1,$$

which is below the revenue of EN. Indeed, the revenue ranking reverses for fully cursed bidders compared to a setting with fully rational bidders. The following corollary formalizes this result.

Corollary 3 *In the case of limited liability, in the symmetric 1-cursed equilibrium, EN raises more money than FP.*

Intuitively, by a continuity argument, we may expect that there is a cursedness level between zero and one for which EN and FP yield the same revenue, like we observe in our data. For FP, the observed revenue is roughly in the middle between the theoretical predictions for fully rational and fully cursed bidders. For EN, the observed revenue is closer to the predicted revenue for fully cursed bidders than the predicted revenue for rational bidders. This observation is in line with the higher estimated χ-cursedness level in the case of unlimited liability for EN than for FP, which may be explained by 'auction fever' as in Ehrhart et al. (2008).

To summarize, χ-cursedness explains our experimental observations quite well, at least on the aggregate level.[13]

5 Conclusion

In a laboratory experiment, we have studied which standard auction is least conducive to bankruptcy. More precisely, we have analyzed the first-price sealed-bid auction and the English auction in the context of a three-player wallet game. Our data strongly reject our theoretical prediction that the English auction leads to less aggressive bids and fewer bankruptcies than the first-price sealed-bid auction. If there is a difference between the two auction formats, it is exactly opposite to what (standard) theory predicts. Our results suggest that for license auctions and procurement procedures, it will not be helpful for governments to run a second-price auction instead of a first-price auction (or the other way around) if they wish to prevent the winner from going bankrupt.

Appendix: Proofs of Propositions

Proof of Proposition 1. Let $\tilde{u}(\theta, \tilde{\theta})$ be the utility of bidder 1 with type θ who bids as if having type $\tilde{\theta}$ 'close' to θ while the other two bidders bid according to the same strictly

[13] Obviously, it could be the case behavior is explained by a mixture of χ-cursedness, risk aversion, and asymmetric equilibria.

increasing bidding function B with $B(\theta) < 2\theta$. Then,

$$\begin{aligned}\tilde{u}(\theta,\tilde{\theta}) &= \int_0^{\tilde{\theta}}\int_0^{\tilde{\theta}} \max\left\{\theta+\theta_2+\theta_3-B(\tilde{\theta}),0\right\} d\frac{\theta_2}{100}d\frac{\theta_3}{100} - \frac{1}{20,000}c\left[B(\tilde{\theta})-\theta\right]^2 \\ &= \frac{1}{60,000}\left[\theta+2\tilde{\theta}-B(\tilde{\theta})\right]^3 - \frac{1}{30,000}\left[\theta+\tilde{\theta}-B(\tilde{\theta})\right]^3 - \frac{1}{20,000}c\left[B(\tilde{\theta})-\theta\right]^2.\end{aligned}$$

The first [second] term on the right-hand side in the first line refers to situations in which bidder 1 does not go [goes] bankrupt. The first-order condition of the equilibrium is given by

$$\left.\frac{\partial\tilde{u}(\theta,\tilde{\theta})}{\partial\tilde{\theta}}\right|_{\tilde{\theta}=\theta} = \frac{\frac{1}{2}\left[3\theta-B(\theta)\right]^2\left[2-B'(\theta)\right]-\left[2\theta-B(\theta)\right]^2\left[1-B'(\theta)\right]-cB'(\theta)\left[B(\theta)-\theta\right]}{10,000} = 0$$

from which differential equation (3) follows.

Proof of Proposition 2. Let B be the equilibrium bid function. According to the ranking lemma (see, e.g., Milgrom 2004), the proposition holds true if $B(0) = 0$ and if $B(\theta) = \frac{5}{3}\theta$ implies that $B'(\theta) < \frac{5}{3}$. It is standard that $B(0) = 0$ must hold in a symmetric equilibrium. Moreover, suppose that bidders 2 and 3 bid according to B and that bidder 1 with signal θ bids as if having signal $\tilde{\theta}$. Bidder 1's utility equals

$$\tilde{u}(\theta,\tilde{\theta}) = \int_0^{\tilde{\theta}}\int_0^{\tilde{\theta}} u(\theta+\theta_2+\theta_3-B(\tilde{\theta}))d\frac{\theta_2}{100}d\frac{\theta_3}{100}.$$

The first-order condition of the equilibrium implies that if $B(\theta) = \frac{5}{3}\theta$,

$$\begin{aligned}0 &= 10,000 * \tilde{u}_2(\theta,\theta) \\ &= 2\int_0^{\theta} u(2\theta+\theta_2-B(\theta))d\theta_2 - B'(\theta)\int_0^{\theta}\int_0^{\theta} u'(\theta+\theta_2+\theta_3-B(\theta))d\theta_2 d\theta_3 \\ &= 2\int_0^{\theta} u\left(\frac{1}{3}\theta+\theta_2\right)d\theta_2 - B'(\theta)\int_0^{\theta}\left[u\left(\frac{1}{3}\theta+\theta_2\right)-u\left(\theta_2-\frac{2}{3}\theta\right)\right]d\theta_2 \Rightarrow \\ B'(\theta) &= \frac{2\int_0^{\theta} u\left(\frac{1}{3}\theta+\theta_2\right)d\theta_2}{\int_0^{\theta}\left[u\left(\frac{1}{3}\theta+\theta_2\right)-u\left(\theta_2-\frac{2}{3}\theta\right)\right]d\theta_2} < \frac{5}{3}.\end{aligned}$$

The third equality follows by direct integration and by substituting $B(\theta) = \frac{5}{3}\theta$. The inequality follows because the strict concavity of u implies that

$$\int_0^{\theta}\left[u\left(\frac{1}{3}\theta+\theta_2\right)+5u\left(\theta_2-\frac{2}{3}\theta\right)\right]d\theta_2 < u'(0)\int_0^{\theta}\left[\left(\frac{1}{3}\theta+\theta_2\right)+5\left(\theta_2-\frac{2}{3}\theta\right)\right]d\theta_2 = 0.$$

Proof of Corollary 1. Expected revenue equals

$$E\left\{\min\left(\frac{\delta_n\theta_m}{1-\delta_n},\delta_n\theta_n\right)+\theta_k\right\}\leq E\left\{\delta_n\theta_n+\theta_k\right\}\leq E\left\{\theta_n+\theta_k\right\}\leq E\left\{\theta^{(1)}+\theta^{(2)}\right\}=125=R_E^{\infty},$$

from which the result immediately follows.

Proof of Proposition 4. Suppose both opponents of bidder 1 bid according to (4). Bidder 1 wishes to step out of the auction at a price equal to her (perceived) expected value. If both of her components step out at the same price p, bidder 1 knows that both have signal

$$\theta=\frac{p-\chi}{3-2\chi}.$$

She steps out at price p equal to her perceived expected value, i.e.,

$$v=\theta_1+2(1-\chi)\theta+\chi=\theta_1+2(1-\chi)\frac{p-\chi}{3-2\chi}+\chi=p.$$

It is readily verified that $B_E^{1,\chi}$ in (4) is a solution. Similarly, $B_E^{2,\chi}$ follows by taking into account that bidder 1 updates her beliefs about the signal of the lowest bidder with probability $1-\chi$.

Proof of Proposition 5. Let $\tilde{u}(\theta,\tilde{\theta})$ be the perceived utility of bidder 1 with type θ who bids as if having type $\tilde{\theta}$ while the other two bidders bid according to the same strictly increasing bidding function B. Then,

$$\tilde{u}(\theta,\tilde{\theta})=\tilde{\theta}^2\left[(1-\chi)\left(\theta+\tilde{\theta}\right)+\chi\left(\theta+1\right)-B(\tilde{\theta})\right].$$

The first-order condition of the equilibrium is given by

$$\left.\frac{\partial\tilde{u}(\theta,\tilde{\theta})}{\partial\tilde{\theta}}\right|_{\tilde{\theta}=\theta}=2\theta\left[2\theta\left(1-\chi\right)+\chi\left(\theta+1\right)-B(\theta)\right]+\theta^2\left[(1-\chi)-B'(\theta)\right]=0.$$

It is readily verified that (5) is a solution.

Proof of Proposition 6. Bidder 1 steps out at price p equal to her perceived expected value of winning given that her two opponents bid according to equilibrium. Because bidder 1 is fully cursed, she assumes that the other two bidders' signals are uniformly distributed on $[0,100]$ regardless of her winning the auction and regardless of the price at which an opponent steps out. Therefore, she indeed steps out at a price p which solves $\tilde{U}(p,\theta)=0$.

PROOF OF PROPOSITION 7. Let $\tilde{u}(\theta, \tilde{\theta})$ be the utility of bidder 1 with type θ who bids as if having type $\tilde{\theta}$ while the other two bidders bid according to the same strictly increasing bidding function B. Then,

$$\tilde{u}(\theta, \tilde{\theta}) = G(\tilde{\theta})\tilde{U}(B(\tilde{\theta}), \theta)$$

where

$$G(\theta) \equiv \frac{\theta^2}{10,000}$$

is the distribution function of the higher of two draws from U[0,100]. Equation (6) follows immediately from the first-order condition of the equilibrium:

$$\left.\frac{\partial \tilde{u}(\theta, \tilde{\theta})}{\partial \tilde{\theta}}\right|_{\tilde{\theta}=\theta} = G'(\theta)\tilde{U}(B(\theta), \theta) + G(\theta)\tilde{U}_1(B(\theta), \theta)B'(\theta) = 0.$$

PROOF OF COROLLARY 3. (The proof proceeds along the same lines as Maskin and Riley's (1984) proof of their Theorem 4.) Conditional on a bidder with type θ winning, revenue in EN is given by

$$R_E(\theta) = \int_0^\theta \frac{b_E^{\chi=1}(t)}{G(\theta)} dG(t)$$

where G is the distribution function of the higher of two draws from U[0,100]. Consequently,

$$R'_E(\theta) = \left[b_E^{\chi=1}(\theta) - R_E(\theta)\right] \frac{G'(\theta)}{G(\theta)}.$$

Revenue in FP equals $R_F(\theta) = b_F^{\chi=1}(\theta)$. Therefore,

$$R'_F(\theta) = b_F^{\chi=1\prime}(\theta) = -\frac{\tilde{U}(b_F^{\chi=1}(\theta), \theta)}{\tilde{U}_1(b_F^{\chi=1}(\theta), \theta)} \frac{G'(\theta)}{G(\theta)}.$$

Because $b_E(0) = b_F(0)$, it follows that $R_E(0) = R_F(0)$. According to the ranking lemma (see, e.g., Milgrom 2004), the proposition follows if $R_E(\theta) = R_F(\theta) \Rightarrow R'_E(\theta) > R'_F(\theta)$, which is equivalent to

$$b_E^{\chi=1}(\theta) - b_F^{\chi=1}(\theta) > -\frac{\tilde{U}(b_F^{\chi=1}(\theta), \theta)}{\tilde{U}_1(b_F^{\chi=1}(\theta), \theta)}.$$

Consider the left- and right-hand sides as functions of b_F. For $b_F = b_E$, both sides vanish. The derivative of the right-hand side is equal to $-1 + \frac{\tilde{U}\tilde{U}_{11}}{(\tilde{U}_1)^2} < -1$ whereas the derivative of the left-hand side equals -1. Therefore, because $b_F^{\chi=1}(\theta) < b_E^{\chi=1}(\theta)$, we conclude that the inequality is satisfied.

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