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TI 11-022 / DSF 8

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CDOs and the Financial Crisis: Credit Ratings and Fair Premia

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Abstract

This paper uses the market-standard Gaussian copula model to show that fair spreads on CDO tranches are much higher than fair spreads on similarly-rated corporate bonds. It implies that credit ratings are not sufficient for pricing, which is surprising given their central role in structured finance markets. Tranche yield enhancement is attributed to a concentration of collateral bonds' risk premia in spreads of non-equity tranches. This illustrates limitations of the rating methodologies, which are solely based on estimates of real-world payoff prospects and thus do not capture risk premia. We also show that payoff prospects and credit quality of CDO tranches are characterized by low stability. If credit conditions deteriorate, then prices and ratings of CDO tranches are likely to fall substantially further than prices and ratings of corporate bonds. Default contagion exacerbates the pace and severity of changes for CDO tranches.

Keywords: Collateralized debt obligations, Credit ratings, Fair premia, Structured finance, Rating agencies
JEL classification: C52, G01, G11

1. Introduction

There is no doubt that structured credit products have only developed so rapidly because they offered higher coupons relative to equally-rated corporate bonds. Yield enhancement on structured securities was particularly appealing to investors who assumed that credit ratings represent a universal and robust indication of

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payoff prospects across different asset classes. For example, the premise that the highest rating grade is a guarantee of very low default risks has encouraged many institutional investors to add triple-A securitized tranches to their portfolios. The frailty of such a rating-based approach has only become evident in 2007-2008 when the mounting losses associated with subprime mortgages eventually led to the collapse of the structured finance markets.

The main result in this paper is that CDO-structuring concentrates risk premia inherent in spreads of the underlying bonds, which provides a clear-cut explanation for the yield enhancement on CDO tranches. Finance theory indicates that credit ratings, which measure only pure default risk, cannot fully account for fair premia due to risk aversion of investors. Strictly speaking, credit ratings are based on expected losses or default probabilities calculated under the physical measure. In contrast, fair premia are closely related to expected losses calculated under the risk-neutral measure that is derived from (significantly) higher market-implied default probabilities. We show that CDO structuring results in high sensitivity of expected tranche losses to default probabilities of the underlying bonds. Therefore CDO tranches can be tailored to have very low real-world expected losses, while having much higher risk-neutral expected losses. This means that CDO tranches can qualify for high credit ratings, while offering significant yield enhancement relative to similarly-rated bonds.

In modeling credit ratings and fair premia of CDO tranches we rely on the market standard one-factor Gaussian copula model. As a stylized example, we consider a portfolio of hundred BBB- bonds[1] with a market spread of 111.95 bps that decomposes into 53.06 bps of pure default risk compensation and 58.89 bps of risk premium. By securitizing this bond portfolio, we create a mezzanine CDO tranche of the same BBB- credit quality and hence roughly the same spread to compensate for pure default risk, but with a much higher total fair spread of 320.69 bps. That is an almost three-fold increase in the total spread due to a five-fold multiplication of the risk premia of the underlying bonds, which illustrates that tranche premia cannot be derived on the basis of similarly-rated bonds. We further re-securitize the BBB- rated tranches to create a CDO-squared[2] with even higher spreads. For example, under realistic assumptions we construct a CDO-squared tranche with a BBB- rating and a fair spread of 795.71 bps.

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[1]Whenever we discuss a credit rating without indicating the rating agency, e.g. a BBB- bond, we always refer to the S&P rating.

[2]A CDO-squared is a CDO-type security backed by a collateral pool consisting of tranches from other CDO deals.

Our results on the tranche yield enhancement demonstrate that the current rating system can be gamed if it is used for pricing purposes. Producing CDOs allows for boosting premia on highly rated securities. This creates vast possibilities for rating arbitrage, which made the structured finance industry so profitable. The excess tranche spreads can be distributed between CDO investors and issuers. The investors are able to increase their returns on highly rated portfolios, while the issuers are compensated for their efforts and risks associated with originating and structuring CDOs. These results complement the paper by Brennan et al. (2009) who propose an analytical model based on the CAPM and the Merton model to analyze the gains of an investment banker selling CDO tranches at the spreads of equally-rated corporate bonds.

Finally, we discuss reasons why structured securities are likely to perform poorly during unfavorable market conditions. The key to this analysis lies in high sensitivity of tranche payoffs to default probabilities of the underlying bonds. It follows that a rise in default probabilities estimates of the underlying bonds, which is typical for a deterioration in credit conditions, affects tranche ratings and prices more heavily than ratings and prices of corporate bonds. We find that default contagion is a crucial factor exacerbating the pace and severity of changes for CDO tranches.

The rest of this paper is organized as follows. Section 2 discusses the background of the structured finance markets. Section 3 explains the modeling approach and assumptions. In Section 4 we present our findings on the CDO yield enhancement and in Section 5 we analyze the sensitivity of CDO tranche payoffs. In Section 6 we discuss the stability of tranche ratings and prices. Section 7 concludes.

2. Background

Structured finance transforms lower quality assets into securitized tranches that are better suited for investors' risk appetite. Pooling assets into large well-diversified portfolios allows for a substantial reduction of idiosyncratic risks. Subsequent prioritization of cash flows associated with an underlying portfolio creates several securities (tranches) of varying credit quality. Tranche investors bear the credit losses incurred by the underlying portfolio within pre-agreed limits and in return they receive premium payments. Most of the credit risk is concentrated in the first-loss (equity) tranche. More senior tranches are characterized by higher credit quality compared to the average quality of the collateral pool. In practice, highly rated AA and AAA tranches constitute about 60% of the volume of securitized portfolios rated by Fitch (2007). The ability of structured finance to produce such large volumes of highly rated tranches was particularly successful to meet the large market demand for very safe securities originating from institutional investors such as pension funds or money-market funds.

Benmelech and Dlugosz (2009) analyze the practice of rating CDOs[3] and they use the term 'alchemy' to describe the apparent disparity between the credit quality of CDO tranches and the credit quality of their underlying collaterals. Equally intriguing is that highly rated tranches offer a significant yield enhancement relative to similarly-rated bonds. For example, in the run-up to the financial crisis, triple-A structured securities provided as much as 50 bps in case of CDO-squareds.[4] Such attractive coupons were not common for triple-A assets in the corporate bond universe. The originators of CDOs have attributed the tranche yield enhancement to the 'leveraging' and 'correlation risk' created by prioritizing tranche payoffs (ABC of CDOs, 2004). However, most likely the implications of these terms were not fully understood by investors. Crouhy et al. (2008) point out that "the argument could be made that as the yields on structured instruments exceeded those on equivalently rated corporations, the market knew they were not of the same credit and/or liquidity risk. But investors still misjudged the risk".

Our focus on credit ratings is motivated by their predominant importance in the structured finance markets. Credit ratings have been essential because the complexity of securitized products limited the ability of unsophisticated investors to conduct independent risk assessment (Crouhy et al., 2008). There is also a growing consensus in academic literature that investors relied heavily on credit ratings not only for risk management, but also to infer fair premia; for a discussion we refer to Krahnen and Wilde (2008), Coval et al. (2009a), Brennan et al. (2009), Crouhy et al. (2008) and Firla-Cuchra (2005). For example, according to Krahnen and Wilde (2008) "Ratings are used almost universally by investors, bankers, supervisors, and regulators as the relevant risk metric. The familiarity of markets with these letter ratings has probably encouraged investors to add these instruments to their portfolios, and has helped to establish the market for various ABS [CDO] products in the first place".

The rating agencies have been ambiguous about the meaning of credit ratings. On the one hand, they advertised credit ratings as "a uniform measure of credit

[3] Benmelech and Dlugosz (2009) focus on collateralized loan obligations (CLOs), which are CDOs backed by portfolios of loans.

[4] We compare the spreads on a few dozen CDO tranches rated by S&P in 2006 (from S&P Ratings Direct database).

quality globally and across all types of debt instruments" (S&P, 2007).[5] The same document further reads, "In other words, an 'AAA' rated corporate bond should exhibit the same degree of credit quality as an 'AAA' rated securitized debt issue.". On the other hand, the agencies asserted that credit ratings are merely "opinions about a relative creditworthiness of a security" (S&P, 2009). Similarly, the rating agencies have indicated that credit ratings are not sufficient for pricing, but they did not explain fundamental differences in risks between like-rated bonds and securitized assets. For illustration, an S&P document explaining the "meaning behind structured finance ratings" stated: "We recognize that the global capital markets may not always price similarly rated debt types the same, all things being equal. This is also true when comparing different securitized issues. Such differences may be based on both credit and non-credit or market considerations, including perceived prepayment risks based on asset or structural characteristics; seller/servicer characteristics; the asset class' historical track record; the availability of historical performance data; and market liquidity considerations, including the depth of secondary markets in certain sectors or markets." (S&P, 2007).

Credit ratings are an assessment of a security's credit quality. In case of corporate bonds, the rating process depends heavily on both qualitative and quantitative components. Bonds are categorized into a number of grades according to their relative payoff prospects. These rating grades are not meant to represent precise estimates of default probabilities. Actual default performance of bonds typically varies between years. For example, BBB bonds rated by S&P have an average annual default rate of 0.26% with a standard deviation of 0.27% (based on 1985-2009 period), see S&P (2010). The rating agencies also publish average cumulative bond default probabilities for all rating grades, which are more stable than annual default rates. For example, a triple-A bond rated by S&P has a 10-year historical default probability of 0.36% (S&P, 2005). Such statistics give investors an intuitive meaning to the 'relative ranking of payoff prospects' implied by credit ratings.

The rating methodologies for structured securities are based on the principle that their credit ratings should be comparable to bond ratings. The S&P's methodology aims to ensure that CDO tranches have the same cumulative (real-world) default probabilities as equally-rated bonds, while the Moody's methodology aims to match (real-world) expected losses. The rating agencies use quantitative models

[5]This quote and a broader discussion about the meaning of credit ratings is in Ashcraft and Schuermann (2008).

to estimate default probabilities or expected losses of CDO tranches. The values of these risk measures are then mapped into letter-grade ratings according to pre-specified bounds corresponding to different rating categories. For example, a tranche qualifies for the triple-A rating by Standard and Poor's if its 10-year default probability is equal or less than 0.36%, which is the historical (cumulative) default probability of triple-A corporate bonds over the same time horizon. Similarly, Moody's would assign the triple-A rating if tranche expected loss is equal or less than 0.0055% over a 10-year period, which is the historical loss on equally-rated bonds.[6]

3. Model and assumptions

3.1. Modeling approach

In this part we introduce the market standard method for modeling CDOs. We first discuss how a CDO structure allocates losses incurred on the underlying assets to the tranches. We then explain how defaults of the collateral bonds are modeled. We also define tranche credit ratings and fair premia. Last, we discuss how the model is implemented via Monte Carlo simulations.

We construct a CDO backed by a collateral pool consisting of $i = 1, ..., n$ bonds with each bond i having a notional N_i. The total notional of the portfolio is thus equal to $N_{total} = \sum_{i=1}^{n} N_i$. The CDO's maturity time is T. Default times of the obligors are denoted by $\tau_1, \tau_2, ..., \tau_n$ and the corresponding recovery rates are denoted by R_i. The cumulative loss on the collateral pool up to time t is given by:

$$L(t) = \sum_{i=1}^{n} N_i(1 - R_i)\mathbf{1}_{\tau_i < t}, \tag{1}$$

where **1** is the indicator function defined as usual.

The CDO structure splits the total portfolio risk into several tranches with each tranche being defined by its attachment point K_L and its detachment point K_U. Tranche investors cover the portfolio losses exceeding K_L, but limited to the tranche notional $K_U - K_L$. The lower attachment point is also referred to as the tranche subordination level. For example, if the total portfolio notional is $100 million and the tranche attachment and detachment points are equal to,

[6]For details on the S&P rating methodology and the default probability benchmarks for other rating grades see S&P (2002) and S&P (2005). For details on the Moody's methodology and the expected loss benchmarks see Moody's (2005) and Moody's (2007).

respectively, \$3 and \$7 million, then the cumulative portfolio losses between \$3 and \$7 million are passed on as losses to the tranche investors. Formally, the CDO tranche losses up to time t are given by:

$$L(K_L, K_U, t) = \min\left[\max\left(L(t) - K_L, 0\right), K_U - K_L\right]. \tag{2}$$

The key challenge in rating and pricing structured securities lies in modeling defaults of the collateral assets. The appropriate modeling framework must capture not only the univariate risk properties of the underlying assets, but also dependence between defaults of these assets. That is because tranche payoffs are linked to the portfolio loss rate.

The univariate risk properties of the underlying assets are summarized by the cumulative distribution functions of their default times τ_i:

$$F_i(t) = Pr(\tau_i < t) = 1 - S_i(t), \tag{3}$$

where $S_i(t)$ is the survival function to time t. The specification of $F_i(t)$ or $S_i(t)$ depends on the purpose of modeling as it can reflect probabilities either under the physical measure or under the risk-neutral measure. The survival functions together with the recovery rates give all security-specific information needed for analyzing expected cash flows on single-name securities.

Default dependence is modeled using copulas. Let us introduce a series of random variables:

$$V_i = \Phi^{-1}\left(F_i(\tau_i)\right), \tag{4}$$

where $\Phi(\cdot)$ is the cumulative distribution function of the standard normal distribution. To account for default dependence we assume pair-wise correlations between different V_i, so that the V_i jointly follow a multivariate standard normal distribution with a specified correlation matrix. This corresponds to the Gaussian copula approach. For modeling CDOs, the market typically assumes a one-factor model such that the correlations between all collateral assets are due to their exposure to a single common factor. In such case the V_i can be expressed as:

$$V_i = \sqrt{\rho_i}Y + \sqrt{1 - \rho_i}X_i, \tag{5}$$

where $Y \sim N(0, 1)$ is the common (systemic) factor, $X_i \sim N(0, 1)$ is the idiosyncratic (obligor-specific) factor and $\rho_i \in (0, 1)$ is the parameter controlling the correlations. It is typical to interpret $V_i \sim N(0, 1)$ as the scaled asset value of obligor i, which is in line with the Merton approach of default modeling. The market standard is to assume that all ρ_i are equal to a common ρ, which simplifies the correlation structure. In such case we can interpret $\rho \in [0,1]$ in Eq. 5 as the asset value correlation between any two obligors in the collateral portfolio.

For modeling CDO-squareds we assume a slightly more complicated correlation structure, which is captured by the two-factor model. We consider a CDO-squared with a collateral pool composed of $j = 1, ..., K$ underlying CDO tranches. In turn, each of the underlying CDO tranches is backed by a portfolio of bonds indexed $i = 1, ..., n$. Then the first factor $Y \sim N(0,1)$ is the common factor that drives the asset values of all obligors. The second factor $Z_j \sim N(0,1)$ is specific to the reference portfolio of each underlying CDO j. This means that the credit risk of the underlying tranches is partly driven by tranche-specific factors, which is a source of additional diversification. The scaled value of obligor i belonging to the reference portfolio of the underlying CDO j is denoted as $V_{i,j}$ and can be expressed as:

$$V_{i,j} = \sqrt{\alpha\rho_i}Y + \sqrt{(1-\alpha)\rho_i}Z_j + \sqrt{1-\rho_i}X_{i,j}, \qquad (6)$$

where ρ_i is the parameter controlling the asset value correlations between obligors. Parameter $\alpha \in [0,1]$ determines the relative exposure to the common factor Y and the CDO-specific factor Z_j. If all ρ_i are equal to a common ρ, then ρ gives the asset value correlation between any two obligors belonging to the same CDO collateral pool, while $\alpha\rho$ is the asset value correlation between any two obligors belonging to collateral pools of different underlying CDOs.

3.1.1. Rating Measures

Standard and Poor's ratings are based on tranche default probability, while Moody's ratings are based on tranche expected loss. Tranche default probability is the likelihood that the cumulative portfolio loss exceeds the subordination level of the tranche until maturity time T:

$$PD_{tranche} = \mathbb{E}^P\left(\mathbf{1}_{L(T)>K_L}\right), \qquad (7)$$

where P is the physical default probability measure. Tranche expected loss is defined as the loss on the tranche notional until maturity:

$$EL_{tranche} = \frac{\mathbb{E}^P L(K_L, K_U, T)}{K_U - K_L}. \qquad (8)$$

We emphasize that credit ratings are determined under the physical measure. The physical measure captures the actual (real-world) default probabilities and it is typically estimated from historical data on default frequencies. The physical measure is the appropriate choice for modeling credit ratings because they are meant to reflect real-world payoff prospects of a security (i.e. real-world default probability or expected loss).

3.1.2. Fair Premia

Holders of a tranche incur losses if the portfolio loss rate exceeds the subordination level of the tranche. The series of cash flows equal to the tranche losses associated with credit events is called the default leg. The present value of the default leg is calculated as:

$$V_{default} = \mathbb{E}^{Q} \int_0^T B(0,t)\, dL(K_L, K_U, t), \tag{9}$$

where Q is the risk-neutral measure and $B(0,t)$ is the discount factor for the time interval $(0,t)$.

In return for taking on default risk, tranche investors receive premium payments based on the running spread s. The present value of the premium leg is given by:

$$V_{premium}(s) = \mathbb{E}^{Q}\left[\sum_{i=1}^{qT} B\left(0, \frac{i}{q}\right) \frac{s}{q}\left((K_U - K_L) - L\left(K_L, K_U, \frac{i}{q}\right)\right)\right], \tag{10}$$

where q is the frequency of coupon payments (e.g. $q = 4$ for quarterly payments).

Determining the fair spread is equivalent to finding the level of tranche spread, s^*, that equates the default leg and the premium leg. Since the default leg, as given by Eq. 9, is linear as a function of s, the fair tranche spread equals:

$$s^* = \frac{V_{default}}{V_{premium}(s = 1)}. \tag{11}$$

At CDO origination, tranche spreads are typically set equal to the fair spread level such that both sides of the contract have zero value.

For the purpose of calculating fair spreads we use the risk-neutral measure, which is implied by market information. Risk-neutral default probabilities are typically much higher than their physical counterparts because they incorporate risk premia. The risk-neutral measure can be derived from the term structure of CDS spreads of the collateral bonds given the recovery rate assumptions. The recovery rates are assumed exogenous to the model; for example they can be based on the rating agencies' studies of historical data or other market estimates.

3.1.3. Implementation

The aforementioned tranche statistics are most easily calculated using Monte Carlo simulations. In each simulation run we draw realizations of the random variables Y and X_i from independent standard normal distributions. Next, we compute default times τ_i of the underlying assets by using formulas (5) or (6) and the inverse of formula (4):

$$\tau_i = F_i^{-1}\left(\Phi\left(v_i\right)\right). \tag{12}$$

Once the default times and the corresponding recoveries are determined for all simulation runs, the calculation of tranche default probabilities, expected losses and fair spreads using formulas (7), (8) and (11) is straightforward.

In addition to the fair tranche spreads calculated under the risk-neutral measure Q, we also calculate tranche spreads under the physical measure P. This gives the spreads compensating for default risk in the real-world (pure default risk). Similarly, we also calculate tranche default probabilities and expected losses under the risk-neutral measure Q instead of the physical measure P.

3.2. Manufacturing structured assets

Manufacturing structured assets can be decomposed into two steps. The first step is to select the collateral portfolio. The second step is the structuring process. We first discuss how we produce a stylized CDO and then we turn to the CDO-squared case.

3.2.1. CDO collateral portfolio

We choose a homogeneous collateral portfolio of one hundred bonds with a maturity of 10 years. Each bond has a default probability of 10% until maturity, which results in a BBB- rating by Standard and Poor's.[7] We make a simplifying assumption that the survival functions of the underlying bonds have an exponential form, $S_i(t) = e^{-t\lambda_i}$, with a constant default intensity parameter λ_i.[8] The intensity parameter is calibrated by equating the assumed default probability until maturity (e.g. $p_i = 10\%$) to default probability implied by the exponential survival function: $p_i = 1 - e^{-T\lambda_i}$.[9]

We assume that the collateral bonds have random recovery rates drawn from

[7] A BBB- bond rated by S&P has a 10-year default probability between 5.88% and 10.64% according to the S&P benchmark tables (S&P, 2005).

[8] The exponential (real-world) survival functions are a good approximation when compared to exact survival functions provided by S&P (2005).

[9] The intensity parameter for the risk-neutral default probability is calibrated similarly.

a Beta distribution with a mean of 50% and a standard deviation of 20%, which implies that the bond expected loss is 5%. Hence, according to the Moody's criteria the bonds qualify for the BB+ rating.[10] (For ease of comparison we convert the Moody's rating convention 'Aaa, Aa1, Aa2, ...' to the S&P rating convention 'AAA, AA+, AA, ...'). Actual CDO deals typically have collateral bonds with mixed ratings ranging from C to AA. However, Benmelech and Dlugosz (2009) examine data on CDO securities and they find that most bonds in collateral pools with a B weighed average credit quality are in fact B rated. Therefore, the assumption that all collateral bonds have the same credit rating is reasonably realistic.

The ability to produce highly-rated tranches is critically dependent on the joint default behavior of their collateral assets. According to the Standard and Poor's rating assumptions, the asset value correlation between corporate obligors belonging to different industry sectors is equal to 5%, while the correlation within an industry sector is 15% (for U.S. bonds), see S&P (2005). We set the asset value correlation to 12.5%, which is a realistic average correlation if the bonds belong to several industries.

We also assume that the market-implied default probability of each of the collateral bonds is equal to 20% until maturity of 10 years, which is double the physical probability. This is equivalent to a market spread of 111.95 bps on the collateral bonds.[11] Such assumptions are in line with the studies on the relationship between physical and risk-neutral default probabilities in the corporate bonds market. The literature suggests that risk-neutral default probabilities are 2 to 5 times higher than their physical counterparts for BBB rated bonds, see Berndt et al. (2005), Driessen (2005), Delianedis and Geske (2003) or Hull et al. (2005). Therefore, our choice of the multiplier is equal to the lower bound of estimates given in the literature. In the context of this study it is a very cautious assumption because if we had chosen a higher multiplier, the results of this paper would have been stronger (i.e. the tranche yield enhancement would have been higher).

3.2.2. CDO structuring

The capital structure of the CDO is chosen for the purpose of minimizing the cost of financing of the underlying debt. Given investors' reliance on credit ratings, the cost of financing of a tranche is decreasing in its credit quality. This leads to clear incentives to maximize the volume of tranches with as good ratings

[10] A BB+ bond rated by Moody's has a 10-year expected loss between 3.25% and 5.17% (Moody's, 2007).

[11] We also assume a fixed discount factor of 2% per annum.

as possible. The market practice is to look at the criteria of the rating agencies and to produce tranches that just qualify for their credit ratings. Therefore, the structuring of CDO tranches is strongly interrelated with the rating process.

We first describe the junior mezzanine tranche, which is tailored to be identical in terms of credit quality to the underlying corporate bonds. Not only do we ensure that this tranche has the same credit ratings as the underlying bonds, but in addition we impose a stronger condition that the tranche has exactly the same expected loss and default probability as the bonds. This is very convenient for our further analysis of the rating-premia relationship, but it slightly departs from the typical structuring process. The lower attachment point of the junior mezzanine tranche is chosen as a 90% quantile of the real-world portfolio loss distribution such that the 10-year tranche default probability is equal to 10%. Next, we fix the upper attachment point such that the tranche expected loss is equal to 5%. For our portfolio this implies that the lower and upper attachments points of the tranche are 9.90% and 14.75%, respectively. The tranche receives a BBB- rating from S&P and a BB+ from Moody's, which follows from the obtained tranche default probability and expected loss.

The more senior tranches are tailored in line with the market practice of maximizing the size of tranches with the highest ratings. While we report ratings of both S&P and Moody's, the structuring for these more senior tranches is based solely on the S&P criteria. For the S&P methodology, which measures tranche default probability, only lower tranche attachment points matter for determining credit ratings. Therefore, we first choose the subordination level of the super-senior AAA tranche as a quantile of the real-world portfolio loss distribution such that the tranche meets the benchmark 10-year default probability of 0.36%.[12] Similarly, we construct the senior AA tranche by choosing its lower attachment point such that the tranche meets the 10-year default probability target of 0.87%. The upper attachment point of the AA tranche is the lower attachment of the super-senior tranche. For our portfolio, these two tranches have subordinations of 17.08% and 19.45%, respectively.

We also obtain two other tranches, which have both attachment and detachment points implied by the tranches defined so far. The first one is the unrated equity tranche, which is at the bottom of the capital structure. Its lower attach-

[12] Starting from 2005, S&P uses different default probability benchmarks for CDO tranches, which are no longer based on historical bond performance. After the change, senior CDO tranches have higher target default probabilities that correspond to historical tranche performance; however, we still use the corporate bond benchmarks to preserve direct comparability of ratings.

ment is 0% and its upper attachment is given by the subordination level of the junior mezzanine tranche, i.e. 9.90%. Another tranche is in between the junior mezzanine tranche and the senior tranche, so it goes from 14.75% to 17.08% of the CDO notional. It is rated A- by S&P and BBB+ by Moody's.

Table 1 presents the results of structuring and rating in columns (1)-(5). These results are obtained under the physical measure, which is appropriate for modeling credit ratings. We can see how CDO prioritization of cash flows adjusts the risks of tranches. The default probabilities and expected losses of the tranches are decreasing with tranche seniority. Most of the credit risk is contained in the equity tranche, which absorbs all portfolio losses up to the limit of 9.90% of the CDO notional. The super-senior AAA tranche has roughly 80% of the deal notional. The remaining columns of Table 1 present the risk-neutral results, which are relevant for tranche pricing as discussed in the next section.

A similar CDO structuring exercise was done by Krahnen and Wilde (2008) who use (Moody's) historical default rates to determine tranche subordination levels as quantiles of the portfolio loss distribution. The practice of choosing tranche subordinations to just meet the rating criteria is common in the market for CDOs backed by non-synthetic assets. A different approach is used in the markets for synthetic CDS index tranches where tranche subordination levels are standardized and pre-defined. For example, the iTraxx Europe index has six tranches with consecutive subordinations of 0%, 3%, 6%, 9%, 12%, and 22%.

3.2.3. Creating a CDO-squared

We also analyze CDO-squared securities, which have incurred particularly large losses during the financial crisis. CDO-squareds are created by re-securitizing CDO tranches for which there is limited market demand. These are typically tranches rated A+ or lower.

CDO-squared tranches are rated according to the same principles as CDOs. The market standard is to use the 'the bottom up' approach, which derives the cash-flows on the underlying tranches directly from the performance of their collateral bonds. This approach accounts for specific characteristics (e.g. credit quality) and overlap among the collaterals of the underlying CDO tranches.

We choose a CDO-squared collateral pool composed of thirty mezzanine BBB- tranches. Each BBB- tranche comes from the stylized CDO deal described in the previous subsection. We further assume that the underlying BBB- tranches reference portfolios of different bonds, so there is no overlap among their collateral portfolios. We assume that the asset value correlation between two obligors be-

Table 1: CDO tranche risk statistics, ratings and premia.

This table reports the results of CDO structuring, rating and tranche pricing. The first two columns summarize the capital structure of the CDO. Next, the table reports 10-year horizon tranche default probabilities, expected losses and annualized spreads calculated under the physical measure and the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The 'Physical measure' part of the table is related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk. From the risk-neutral results, the most important is column (8), which gives the fair (market) spread. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordination	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0.00%	98.33% 'NR'	47.50% 'NR'	636.54	99.90%	78.53%	1475.40
tranche 2 junior mezz.	9.90%	10.00% 'BBB-'	5.00% 'BB+'	48.25	44.64%	30.24%	320.69
tranche 3 senior mezz.	14.75%	1.97% 'A-'	1.35% 'BBB+'	12.76	18.43%	14.55%	143.83
tranche 4 senior	17.08%	0.87% 'AA'	0.58% 'A'	5.43	11.09%	8.46%	81.81
tranche 5 super-senior	19.45%	0.36% 'AAA'	0.01% 'AA+'	0.10	6.21%	0.27%	2.52
corporate bond	n.a	10.00% 'BBB-'	5.00% 'BB+'	53.06	20.00%	10.00%	111.95

longing to the same CDO collateral pool is equal to 12.5%, while the asset value correlation between two obligors belonging to the collaterals of different underlying CDOs is set to 3.5%. This is equivalent to assuming that in Eq. 6 parameter $\rho = 12.5\%$ and $\alpha\rho = 3.5\%$. We thus assume additional diversification at the level of the underlying tranches, which is critical for the ability to produce highly rated CDO-squared tranches. A similar approach was used by Hull and White (2010) in the analysis of ABS CDOs. In practice, such diversification can be achieved by selecting tranches backed by collateral pools that are well-diversified in terms of industry concentration and geographic location.[13] In addition, collateral pools of CDO-squareds very often include some tranches of asset backed securities (e.g. RMBS or ABS).

We choose the capital structure of the CDO-squared to ensure that its tranches have similar credit quality to the corresponding tranches of the stylized CDO. For this purpose we apply the same structuring scheme as in the CDO case, but to the CDO-squared collateral pool. The results of structuring for the CDO-squared are reported in columns (1) – (5) of Table 2. Due to the assumed structuring process, the corresponding CDO and CDO-squared tranches have the same credit ratings from both S&P and Moody's. In particular, the junior mezzanine tranche of the CDO-squared is tailored to have a default probability of 10% and an expected loss of 5%, which results in a BBB- rating by S&P and a BB+ rating by Moody's. We also produce two CDO-squared tranches with AAA and AA ratings by S&P, which just meet the default probability benchmarks of 0.87% and 0.36% required for these rating categories. Finally, we obtain the senior mezzanine tranche and the equity tranche, which have the attachment and detachment points implied by the other tranches of the CDO-squared.

4. Credit ratings and fair premia

Having created a stylized CDO and CDO-squared, we turn to analyzing the relation between credit ratings and fair premia. Keeping the capital structure fixed, we recalculate tranche default probabilities, expected losses and spreads under the assumption that the market-implied default probabilities of the underlying bonds are equal to 20% until maturity. The obtained results are reported in columns (6)-(8) of Tables 1 and 2. It is seen that the transition from the physical to the risk-neutral measure corresponds to a huge increase in the default probabilities, expected losses and spreads for all tranches. The magnitude of the increase for the CDO-squared is much higher than for the CDO.

We first analyze the results for the junior mezzanine tranches as they allow us to directly compare the rating-premia relationship between corporate bonds, CDOs and CDO-squareds. That is because the junior mezzanine tranches and the collateral bonds have the same (real-world) default probabilities and (real-world)

[13]For example, S&P assumes a correlation of 0% between two corporate bonds belonging to different industry sectors and different regions (regions are defined as Asia, Europe etc.), see S&P (2005).

Table 2: CDO-squared tranche risk statistics, ratings and premia.

This table reports the results of CDO-squared structuring, rating and tranche pricing. The first two columns summarize the capital structure of the CDO-squared. Next, the table reports 10-year horizon tranche default probabilities, expected losses and annualized spreads calculated under the physical measure and the risk-neutral measure. The physical measure corresponds to the assumption of 10% default probability of the underlying bonds (over a 10-year horizon), whereas the risk-neutral measure corresponds to default probability of 20%. The 'Physical measure' part of the table is related to the rating process, so in columns (3) and (4) we also report credit ratings by S&P and Moody's. Column (5) reports the spreads compensating for pure default risk. From the risk-neutral results, the most important is column (8), which gives the fair (market) spread. The last row of the table shows the statistics for the underlying corporate bonds.

		Physical measure (PD=10%)			Risk-neutral measure (PD=20%)		
Tranche	Tranche subordination	Default probability & S&P rating	Expected loss & Moody's rating	Spread (bps)	Default probability	Expected loss	Fair spread (bps)
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0%	77.88% 'CCC-'	32.46% 'CCC'	338.84	99.62%	91.87%	1498.01
tranche 2 junior mezz.	13.27%	10.00% 'BBB-'	5.00% 'BB+'	46.89	80.60%	68.16%	795.71
tranche 3 senior mezz.	24.92%	2.07% 'A-'	1.38% 'BBB+'	12.86	55.67%	49.27%	520.66
tranche 4 senior	31.25%	0.87% 'AA'	0.58% 'A'	5.36	43.09%	37.50%	379.86
tranche 5 super-senior	37.50%	0.36% 'AAA'	0.04% 'AA+'	0.40	32.21%	7.61%	71.83
corporate bond	n.a	10.00% 'BBB-'	5.00% 'BB+'	53.06	20.00%	10.00%	111.95

expected losses, which implies identical credit quality and the same credit ratings.

From a market pricing perspective, the most important observation based on Table 1 is that under the assumption that the underlying portfolio of BBB- bonds has a fair spread of 111.95 bps, the similarly-rated CDO tranche has a fair spread of 320.69 bps. Hence the fair spread is almost three times as high. From Table 2 we see that the corresponding BBB- CDO-squared tranche has a fair spread of 749.52 bps, which is roughly 7 times higher than the spread on the similarly-rated underlying bonds.

We argue that the yield enhancement on tranches is attributable to concentration of risk premia inherent in spreads of the underlying bonds. Fair spreads on credit-sensitive instruments consist of compensation for default risk in the real-world (pure default risk) and additional risk premia. Only pure default risk is closely related to credit ratings, while risk premia compensate investors for the uncertainty about securities' payoffs. The compensation for pure default risk can be read in column (5) of Table 1 or Table 2. In turn, the risk premia are calculated by subtracting the compensation for pure default risk from the total fair spread reported in column (8). For the corporate bonds we have assumed a fair spread of 111.95 bps, which can be decomposed into 53.06 bps of pure default risk compensation and 58.89 bps of risk premia. For the CDO and CDO-squared junior mezzanine tranches, the compensation for pure default risk is slightly lower compared to that of the underlying corporate bonds. In contrast, the tranche risk premia are much higher and equal to 272.44 bps and 748.82 bps, respectively. Relative to the similarly-rated corporate bonds benchmark, the risk premia are thus multiplied by a factor of almost 5 for the CDO tranche and by a factor of 13 for the CDO-squared tranche.

The foregoing analysis demonstrates that fair spreads on structured securities are much higher than fair spreads on similarly-rated corporate bonds, which means that credit ratings are by far insufficient for pricing. That is because CDO-structuring concentrates risk premia in non-equity tranches, while the rating methodologies capture solely pure default risk. In other words, structured finance allows for producing securities that have low pure default risk and thus obtain high credit ratings, but have inherently high risk premia. On the one hand, it creates opportunities for rating arbitrage, which can be attractive to investors. On the other hand, investors who overly rely on credit ratings for inferring fair spreads are likely to accept insufficient risk compensation on structured products.

In Table 3 we investigate the yield enhancement on the more senior tranches. For this purpose, it is convenient to create hypothetical corporate bonds, which have the same credit quality as the respective tranches, i.e. identical (real-world) default probabilities and (real-world) expected losses. We call these bonds *risk-equivalent* to the respective tranches.[14] To determine fair spreads on the risk-equivalent bonds we assume that their risk-neutral default probabilities are double the historical probabilities regardless of credit quality (robustness to this assumption is explained further down). In columns (1)-(5) of Table 3 we report tranche ratings and fair premia (previously included in Tables 1 and 2), while column (6) reports the spreads on the risk-equivalent corporate bonds. Note that the corre-

[14]Since the expected losses and default probabilities of the corresponding CDO and CDO-squared tranches are different, we construct risk-equivalent bonds separately for the CDO and CDO-squared. In most cases the differences are small.

Table 3: Comparison of fair spreads on tranches and risk-equivalent bonds.

In columns (1)-(5) we summarize tranche ratings and fair spreads reported previously in Tables 1 and 2. In column (6) we report fair spreads on corporate bonds that are risk-equivalent to the respective tranches (separately for CDO / CDO-squared). The risk-equivalent bonds are defined as having the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. To calculate fair spreads on these bonds we assume that their risk-neutral default probabilities are double the physical probabilities.

Tranche	S&P rating	Moody's rating	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	'BBB-'	'BB+'	320.69	795.71	111.95 / 111.95
tranche 3 senior mezz.	'A-'	'BBB+'	143.83	520.66	28.03 / 28.23
tranche 4 senior	'AA'	'A'	81.81	379.86	11.62 / 11.83
tranche 5 super-senior	'AAA'	'AA+'	2.52	71.83	0.22 / 0.83

sponding CDO and CDO-squared tranches as well as their risk-equivalent bonds have the same credit ratings from both S&P and Moody's.

The main message from Table 3 is that fair premia on all non-equity CDO tranches are much higher than fair premia on risk-equivalent bonds. The magnitude of the yield enhancement critically depends on whether the tranche belongs to the CDO or CDO-squared (being much higher for the latter) and it also varies with tranche seniority. For example, the spread on the super-senior CDO tranche is equal to 2.52 bps, while the spread on the corresponding risk-equivalent AAA bond is equal to 0.22 bps. For the CDO-squared, the spread on the super-senior tranche is as high as 71.83 bps, while its risk-equivalent bond yields 0.83 bps.

An argument can be made that the spreads on highly rated risk-equivalent bonds reported in Table 3 are underestimated. These bond spreads are calculated under the assumption that risk-neutral default probabilities of all bonds are double the physical probabilities; however, our results are fairly robust to this assumption. The market evidence suggests that for highly rated bonds the ratio of risk-neutral to physical default probabilities can be much higher. For example, Hull et al. (2005) report a ratio of 16.8 for AAA bonds. If we had assumed such ratio in Table 3, then the AAA bond, which is risk-equivalent to the super-senior CDO tranche, would have a spread of 1.88 bps instead of 0.22 bps. That is still lower than the fair tranche spread of 2.52 bps. For the super-senior CDO-squared tranche, the same ratio would result in the fair spread on its risk-equivalent bond being 7.01 bps, which is much lower than the tranche spread of 71.83 bps.

In Table A.I (in the appendix) we replicate the results from Table 3 under the assumption of 180.72 bps market spread on the collateral BBB- bonds, i.e. a higher bond risk premium as suggested by Hull et al. (2005).[15] We find that higher risk premia of the underlying bonds results in greater yield enhancement on tranches, e.g. the fair spreads on the super-senior CDO and CDO-squared tranches are equal to 13.46 bps and 428.03 bps, respectively.

The documented large differences in fair spreads between similarly-rated tranches and bonds create opportunities for rating arbitrage, which means that excess spreads can be distributed between tranche investors and CDO issuers. This explains why structured finance securities can be so appealing to both originators and investors. Since investors are typically risk averse and CDO tranches are tailored to their risk appetites, the total risk compensation paid on the tranches can be lower than the total spread received on the collateral portfolio. In other words, the risk-return profiles of the tranches can be attractive to investors at the spread levels, which are below the model-implied fair spreads. In this sense the ability of financial engineering to tailor the risks of tranches creates value. The remaining share of the yield can then be allocated to CDO issuers compensating them for the work and risks associated with their part of the structured finance activities. These risks arise because very often the originators are unable to sell the total notional of all CDO tranches and have to retain and hedge the remaining risks. For example, in the market for synthetic CDOs, single tranche issues were very popular, so the originating banks were only partly securitizing the underlying CDS portfolios. There are also reputational risks, which can lead financial institutions to even bail-out their CDOs as was in the case of Bear Stearns.

The analysis done in this paper assumes that credit ratings and fair spreads are accurate and unbiased for all securities under consideration. This follows from our theoretical approach where tranche risk measures and spreads are calculated using the market-standard models on the basis of (realistic) assumptions. We are not concerned about a possible divergence between the true-world and the models because the analysis is limited to the stylized setting. Since we use the same

[15]The market spread of 180.72 bps is equivalent to assuming that risk-neutral default probability of each of the underlying BBB- bonds is triple the physical default probability of 10%. That is motivated by Hull et al. (2005) who find that the ratio of risk-neutral to physical default probabilities is equal to 5.1 for BBB bonds and 2.1 for BB bonds.

assumptions for both rating and pricing securities, our results are consistent and they illustrate fundamental problems with the rating methodologies. We find that even if the rating agencies correctly estimate the real-world default probabilities and/or expected losses of CDO tranches, these two risk measures do not imply much about the level of fair spreads.

Several recent studies attribute the failings of credit ratings to mistakes made by the rating agencies, for example, pointing out to overly optimistic rating assumptions or failure to account for parameter uncertainty (e.g. Coval et al., 2009b). Undoubtedly, during the financial crisis, credit ratings of structured instruments have failed badly as an indication of payoff prospects. Structured products experienced many more downgrades than corporate bonds and their downgrades were also very severe, particularly in case of triple-A tranches (see Benmelech and Dlugosz (2010) for a discussion on performance of CDO ratings). We argue that the problems with the current rating methodologies go beyond difficulties in implementation of otherwise correct methodologies. The key challenge comes from the fact that the rating methodologies do not capture risk premia, which are (typically) much higher for CDO tranches than for similarly-rated corporate bonds. While the standard rating approach has proven to be adequate for corporate bonds, structured securities might require a different rating approach because they are specifically tailored by originators to maximize the yield enhancement on tranches. Therefore, minor changes to the current rating methodologies (such as introducing more cautious parameter estimates) are likely to be insufficient to ensure robustness of credit ratings.

Throughout the paper we interpret the spreads computed under the risk-neutral measure as the market fair spreads. These spreads are calculated using the Gaussian copula model with the same correlation parameter for all tranches. The market practice for pricing is to use different correlation parameters for different tranches. For example, in the market for CDS index tranches, which are traded by professional parties, the one-factor model is the usual method of choice, but it is used along with the base correlation curve. This curve summarizes the correlations that the market uses for pricing different tranches. The use of the base correlation curve can be interpreted as a way to correct for limitations of the Gaussian copula default dependence structure, market appetite for risk or tranche exposure to systemic risk. Thus our approach departs from the actual market pricing, however, it captures the economics of CDO yield enhancement and rating arbitrage.

5. Sensitivity analysis

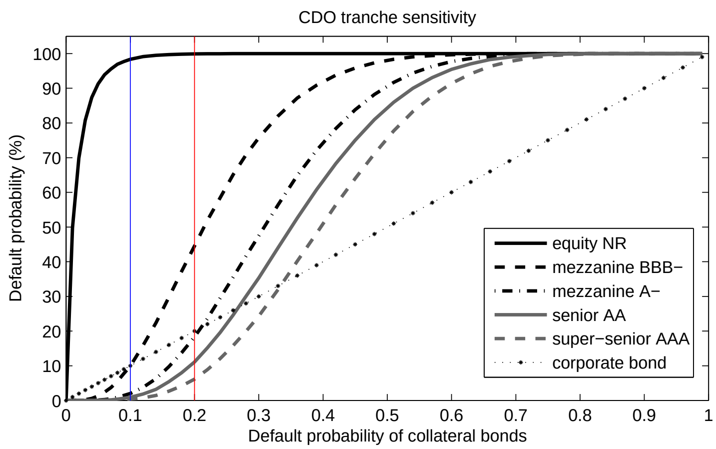
In this section we examine the sensitivity of tranche payoffs to default probabilities of the underlying bonds. The sensitivity analysis provides a clear-cut explanation of the yield enhancement on tranches and illustrates their associated risk properties. The focus is on the sensitivity of tranche default probabilities and tranche expected losses because these risk measures determine credit ratings. In addition, expected tranche losses are closely related to tranche premia. It follows from Eq. 8 and Eq. 9 that the tranche expected loss is equal to the tranche default leg rescaled by the tranche notional and corrected for discounting. Thus by analyzing expected losses we are able to make inference about spreads. Fair spreads are implied by the risk-neutral level of expected losses, i.e. at the collateral default probability equal 20%, while compensation for default risk in the real-world is determined by expected losses under the physical measure, i.e. at the collateral default probability equal 10%.

Figure 1 presents the sensitivity results for CDO tranches. To benchmark tranche sensitivities we also plot curves representing the underlying portfolio of bonds. To facilitate inference, we mark two levels of collateral default probabilities: the real-world level is marked by a vertical line at value 0.1, while the risk-neutral level is marked by a vertical line at value 0.2. These two vertical lines cross the CDO tranche curves at the values corresponding to the results of Table 1.

Panel A of Figure 1 explores the sensitivity of tranche default probabilities to changes in default probabilities of the collateral bonds. It is seen that the sensitivity of the tranche default probabilities is generally higher than the corresponding sensitivity of the collateral bonds. The lowest sensitivity is attributable to the super-senior tranche, which in relative terms is still much higher than the sensitivity of the collateral bonds' default probability. In particular, if the collateral default probability changes from 10% to 20%, then the default probability of the super-senior tranche increases from 0.36% to 6.21% (see Table 1). It means that the tranche default probability increases 17 times when the bond default probability doubles. In Panel B we present the sensitivity of expected tranche losses and we observe a qualitatively similar result. Only the expected loss of the super-senior tranche seems fairly insensitive to a modest rise in the collateral default probability. However, the relative (percentage) increase in the expected loss of the super-senior tranche is very large in the 10-20% interval of the collateral default probability. Table 1 shows that the expected tranche loss increases 27 times (from 0.01% to 0.27%) when the collateral default probability doubles (from 10% to 20%).

The results from Figure 1 indicate that the key to understanding the mechanics of the yield enhancement on tranches lies in high sensitivity of expected tranche losses to default probability of the underlying bonds. To illustrate the ar-

Panel A: Sensitivity of CDO tranche default probabilities



Panel B: Sensitivity of CDO expected tranche losses

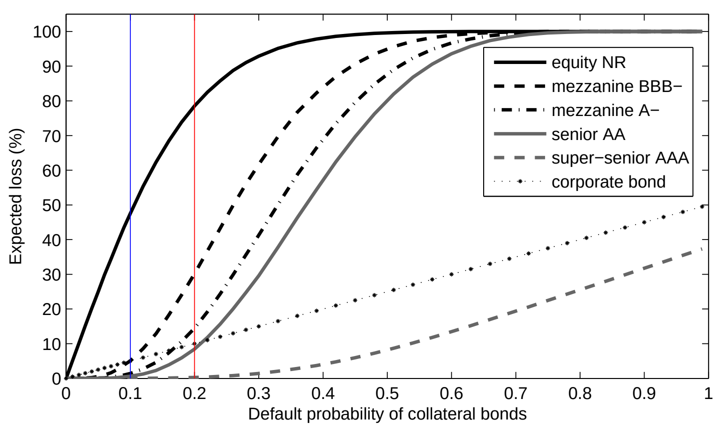


Figure 1: Sensitivity of payoff prospects of CDO tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical lines at value ‘0.1’ correspond to the real-world level of collateral default probabilities, whereas the ‘0.2’ lines correspond to the risk-neutral level. For ease of comparison we add curves representing the collateral bonds.

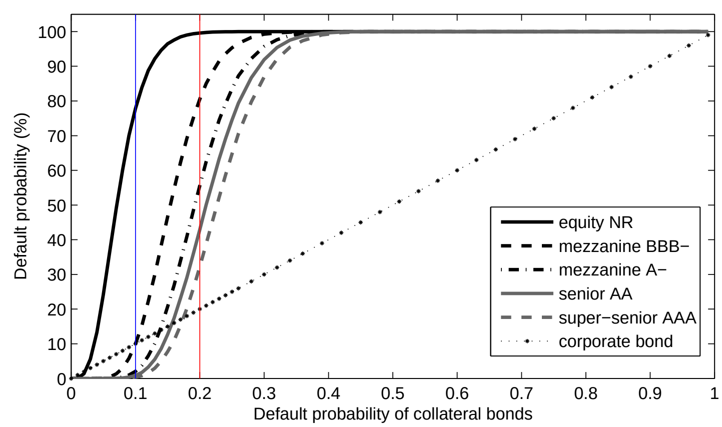
gument let us first consider the junior mezzanine tranche and the corporate bond. Clearly, these two securities have equal expected losses at the real-world level of the collateral default probability ('0.1' line). In contrast, the expected tranche loss at the risk-neutral level of the collateral default probability ('0.2' line) is much higher than the expected bond loss, which is because the curve of the expected tranche loss is steeper than the curve of the expected bond loss in the 10-20% interval of the collateral default probability. Consequently, the fair spread on the junior-mezzanine tranche is much higher than the fair spread on the corporate bond. Similar reasoning applies to the case of the more senior tranches. These tranches cannot be directly compared to the collateral bonds as they are of different credit quality. However, the yield enhancement on the more senior tranches is similarly driven by the difference between the real-world and the risk-neutral levels of expected losses. This difference strongly depends on the sensitivity of expected tranche losses.

The yield enhancement mechanics for the CDO-squared tranches is the same as in the CDO case. Figure 2 depicts the sensitivities of CDO-squared tranche payoffs and they are evidently much higher relative to the CDO tranches considered in Figure 1. It is seen that the non-equity CDO-squared tranches are structured to meet very low default probability and expected loss benchmarks that are required for their rating categories. However, as soon as the default probability of the collateral bonds increases beyond the 10% real-world level, we note a huge increase in both risk measures of the tranches. Particularly, the largest increments in the tranche default probabilities and expected losses are visible in the 10-20% interval of the collateral default probability, which explains the magnitude of the yield enhancement on the CDO-squared tranches.

The tranche yield enhancement implied by the difference between the real-world and the risk-neutral levels of expected tranche losses is determined by two factors. The first factor corresponds to the size of risk premia of the underlying assets. The higher the risk premia, the larger the yield enhancement. An intuition for this result comes from Figure 1 and Figure 2 where risk-premia determine the relative position of the risk-neutral level of collateral default probabilities. The second factor is the sensitivity of expected tranche losses to default probabilities of the collateral bonds. The higher the sensitivity, the larger the yield enhancement. We will analyze how the magnitude of this sensitivity can be related to the collateral portfolio diversification.

Risk premia of corporate bonds belonging to the same rating category can be very different. That is because risk premia depend on the difference between physical and risk-neutral default probabilities of bonds. Physical default probabilities are typically assumed on the basis of credit ratings, so by construction they are equal for similarly-rated bonds. In contrast, risk-neutral default probabilities represent market assessment (pricing) of risk. This assessment is captured by

Panel A: Sensitivity of CDO-squared tranche default probabilities



Panel B: Sensitivity of CDO-squared expected tranche losses

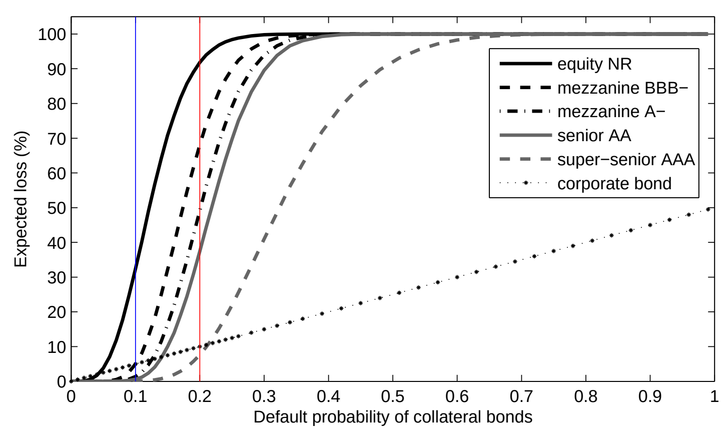


Figure 2: Sensitivity of payoff prospects of CDO-squared tranches to default probabilities of the collateral bonds. Panel A presents sensitivity of tranche default probabilities, whereas Panel B presents sensitivity of expected tranche losses. The vertical '0.1' lines correspond to the real-world level of collateral default probabilities, whereas the '0.2' lines correspond to the risk-neutral level. For ease of comparison we add curves representing the collateral bonds.

CDS spreads, which are characterized by substantial variation between similarly-rated bonds. A CDO issuer who aims to maximize rating arbitrage on tranches should therefore select collateral bonds with relatively high CDS spreads for their credit ratings. Such practice leads to the adverse selection problems as high CDS spreads indicate higher risks that are not captured by the rating agencies. There is anecdotal evidence that such adverse selection was one of the reasons for the poor performance of CDOs during the financial crisis (Fitch, 2008).

The sensitivity of tranche payoffs can be associated with tranche leverage. Tranches are highly leveraged when their expected payoffs change a lot in response to changes in credit conditions. A comparison of Figures 1 and 2 demonstrates that CDO-squareds are much more leveraged than CDOs. Figure 2 also illustrates how risk properties of CDO-squared tranches differ from the risk properties of bonds. Particularly, the shape of CDO-squared tranche curves indicates that tranche payoffs have little upside potential, while adverse market changes are likely to result in huge losses. Similar asymmetry of payoffs is present in CDOs although it is less pronounced. In contrast, a pool of bonds is characterized by a symmetry in payoff prospects in the sense that the upside potential is of the same magnitude as the downside potential.

Apart from the security type, CDO versus CDO-squared, the sensitivity of tranche payoffs depends on characteristics of the collateral pool, primarily portfolio diversification. Portfolio diversification can be increased by choosing assets that are less correlated or by increasing the number of assets while keeping the portfolio notional fixed. Higher portfolio diversification means that the probability of large portfolio losses is lower, which given our structuring process results in lower tranche subordination levels for all non-equity tranches. However, the sensitivity graphs presented in Figures 1 and 2 would have been steeper if we had chosen a more diversified portfolio, which means that the fair tranche premia would have been higher. For example, under the assumption of a 5% asset correlation the junior mezzanine tranche has a fair spread of 522.85 bps and the corresponding CDO-squared tranche has a fair spread of 1271.46 bps (see Table A.II in the appendix for results on other tranches). This finding is interesting because a high diversification of the collateral pool is often quoted by the rating agencies as a 'strength' of a CDO issue, while we establish that tranches backed by more diversified portfolios have higher fair spreads than tranches backed by less diversified portfolios.

6. Rating and price stability of CDO tranches

In this section we analyze the stability of tranche ratings and prices. Figures 1 and 2 illustrate that even a highly rated tranche, which is structured to have a minute expected loss under the physical measure, can suffer heavy losses if the realized default rate of the collateral pool exceeds the assumed rate. In a dynamic setting, tranche prices are likely to become depressed even prior to the realization of collateral losses. If credit conditions deteriorate, then market participants should upwardly revise their expectations regarding default rates of the underlying portfolios and accordingly re-price CDO tranches. The re-pricing of CDO tranches should typically be higher than the re-pricing of the underlying bonds due to the high sensitivity of tranche payoffs documented in Section 5. Similarly, an increase in market participants' risk aversion, which leads to an increase in risk premia of the underlying bonds, depresses the value of CDO tranches more severely than the value of bonds.

In case of unfavorable market conditions, tranche ratings can be expected to come under severe stress as well. Hereby, we enumerate key factors likely to cause a deterioration in tranche ratings. Firstly, credit ratings are highly sensitive to credit enhancement levels, which are reduced once defaults hit the underlying pools. Secondly, possible downgrades within collateral pools result in an increase of the rating agencies' estimates of (real-world) default probabilities of the collateral bonds. This affects tranche ratings relatively more than bond ratings as discussed further in this section. Thirdly, the re-pricing of CDO tranches, which is typical during unfavorable market conditions, is likely to trigger tranche downgrades that go beyond what is implied by actual defaults or downgrades of the collateral bonds. That is because the rating agencies will feel the pressure to revise tranche credit ratings in line with the fall in their market prices. Otherwise the disparity between high credit ratings and low market prices would cast a doubt on the reliability of the rating agencies.

To illustrate divergence between the stability of CDO tranches and corporate bonds we analyze a scenario that corresponds to a fairly severe deterioration in credit conditions, i.e. a one notch downgrade of the entire collateral portfolio from BBB- to BB+. We assume that the CDO and CDO-squared are structured and rated under the baseline assumptions discussed in Section 3 and just after deal-closing the estimates of 10-year default probabilities of the underlying bonds increase from 10% to 13%. The tranches are next re-rated by using the revised default probabilities of the collateral bonds, but keeping the tranche subordination levels fixed. Table 4 presents the results for the CDO and CDO-squared.[16]

Table 4 documents a dramatic deterioration in the credit quality of the tranches, particularly for the senior and super-senior tranches.[17] The super-senior CDO tranche is downgraded from the initial rating of AAA to AA- and the corresponding CDO-squared tranche is downgraded as far as to the BBB+ grade. In other words, a one notch downgrade of the collateral pool triggers downgrades of the super-senior tranches by as many as 3 and 7 notches.

An argument can be made that the scenario analyzed in Table 4 is not very realistic as the rating agencies are unlikely to downgrade the entire collateral pool and re-run their rating models using the revised default probabilities. However, we note that a similar deterioration in credit quality of the tranches can occur if a large portion of the collateral bonds are downgraded by more than one notch. Furthermore, to support the analysis based on Table 4 we will show that an increase in collateral default probability from 10% to 13% is very probable if one considers default contagion.

To examine the impact of default contagion we consider a scenario when a single default within the collateral portfolio occurs soon after CDO origination. Let us first assume that an early default does not change the market expectations about default probabilities of the surviving bonds. In such case a single default can only have a limited impact on tranche ratings because the portfolio loss rate increases by about 0.5% (given a 50% recovery rate). This could trigger a one notch downgrade of the tranches if they were tailored to just satisfy the rating criteria.[18]

The impact of a single default is likely to be very different in the presence of credit contagion when an early default signals a low realization of the common economic factor Y in Eq. 5 or Eq. 6. Thus in addition to the reduced credit enhancements, the credit-worthiness of the tranches is affected by the increased likelihood of a market-wide deterioration in credit conditions. In Figure 3 we plot the conditional default probability of the surviving obligors given that the first default in the collateral portfolio occurs at time t. This example is based on the CDO collateral portfolio of 100 bonds described in Section 3. See Appendix B for an explanation on how the conditional default probabilities are calculated.

[16]The values reported in Table 4 correspond to the values at which tranche curves in Figures 1 and 2 would cross vertical lines at 10% and 13% of the collateral default probability.

[17] In this section, for ease of exposition, we consider only S&P ratings.

[18]Nevertheless, tranches are highly sensitive to several defaults within a collateral pool, which follows from the sensitivity analysis of tranche payoffs in Section 5.

Table 4: Impact of a deterioration in collateral credit quality.

The table analyzes the impact of a deterioration in credit conditions, which corresponds to an increase in 10-year default probabilities of the collateral bonds from 10% to 13% (i.e. a one-notch downgrade of the collateral bonds from BBB- to BB+). The first two columns summarize the capital structure of the CDO and CDO-squared. Next, in columns (4) and (5) we report tranche default probabilities under the standard market conditions (i.e. 10% collateral PD). The CDO and CDO-squared are structured and rated under the standard market conditions, so in columns (4) and (5) we also report S&P ratings. In columns (7) and (8) we report tranche default probabilities after the market conditions have deteriorated (i.e. 13% collateral PD). In these columns we also report revised tranche ratings and we calculate by how many notches the tranches are downgraded.

		Standard market conditions (PD = 10%)			Deteriorated market conditions (PD=13%)		
Tranche	Tranche subordination CDO/CDO-sq.	Default probability			Default probability		
		Bond	CDO	CDO-squared	Bond	CDO	CDO-squared
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
tranche 1 equity	0% / 0%		98.33% 'NR'	77.88% 'CCC-'		99.34% 'NR'	92.23% 'NR'
tranche 2 junior mezz.	9.90% / 13.27%	10.00% 'BBB-'	10.00% 'BBB-'	10.00% 'BBB-'	13.00% 'BB+' 1 notch	19.14% 'BB-' 3 notches	29.83% 'B+' 4 notches
tranche 3 senior mezz.	14.75% / 24.92%		1.97% 'A-'	2.07% 'A-'		4.96% 'BBB' 2 notches	10.15% 'BBB-' 3 notches
tranche 4 senior	17.08% / 31.25%		0.87% 'AA'	0.87% 'AA'		2.40% 'A-' 4 notches	5.46% 'BBB' 6 notches
tranche 5 super-senior	19.45% / 37.50%		0.36% 'AAA'	0.36% 'AAA'		1.06% 'AA-' 3 notches	2.87% 'BBB+' 7 notches

Figure 3 shows that a sudden and early default event can lead to a large shock in the default probabilities of the surviving names. If the first default occurs after 1 month, 3 months and 6 months, then the conditional default probabilities of the surviving bonds jump to, respectively, 16.19%, 13.31%, 11.14%. We thus see that a deterioration in credit conditions considered in Table 4 corresponds to a single default after roughly 3 months, which is not an unlikely scenario under stressful credit conditions. If the first default occurs exactly after 1 year, then the conditional default probability of the surviving bonds equals 8.86%, which is approximately a 1% annual default probability over the remaining time until maturity. That is because a single default within 1 year can be expected given the assumed 10-year default probability of the collateral bonds equal to 10%.

Related to the problem of default contagion is the inability of the rating methodologies to capture the relation between the actual default probabilities of the collateral assets and the business cycle. The rating agencies derive default probability estimates of the collateral bonds solely based on their credit ratings. It means that these are fixed 'through-the-cycle' estimates reflecting average historical default frequencies of similarly-rated bonds. Therefore, in economic recessions the actual default probabilities of the collateral bonds are expected to significantly exceed the rating agencies' estimates. That is particularly important because collateral portfolios typically consist of bonds that have low investment-grade ratings (such as BBB).[19] The performance of bonds with low investment grade ratings is well-known to be counter-cyclical with much higher default rates during economic recessions.[20]

The rating agencies have to balance between the stability of bond ratings and

[19]In most cases CDO collateral pools have restrictions on the heterogeneity of credit quality of the underlying assets; e.g. a maximum of 5% of bonds rated CCC or lower with an average credit rating of the poll around BBB.

[20]An S&P definition of the BBB ratings states "An obligor rated 'BBB' has adequate capacity to meet its financial commitments. However, adverse economic conditions or changing circumstances are more likely to lead to a weakened capacity of the obligor to meet its financial commitments." (S&P, 2009)

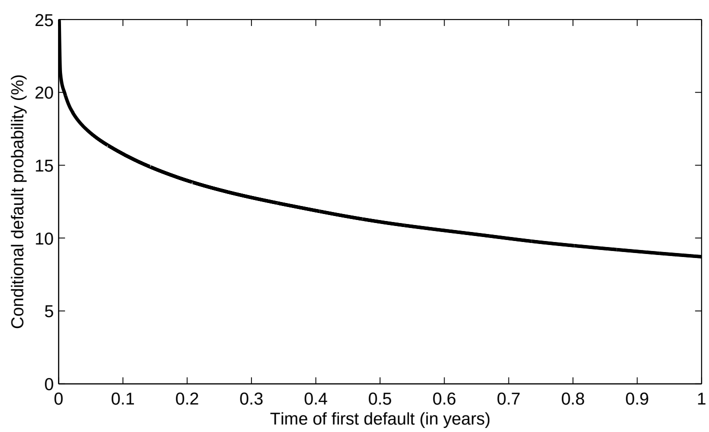


Figure 3: Default probability of the surviving bonds conditional on the information that the first default in the collateral pool occurs at time t within the first year after CDO issuance.

ensuring that highly rated tranches do not incur losses. In contrast, investors in CDO tranches can continuously condition on the information about credit quality of the collateral bonds and accordingly adjust CDS spreads. In Section 5 we have shown that expected losses of CDO tranches are very sensitive to default probabilities of the collateral bonds. Therefore when the market-implied default probabilities increase, the re-pricing of CDO tranches is typically much higher than the re-pricing of corporate bonds.

The re-pricing of CDO tranches has critical implications for rating stability. During favorable market conditions, a CDO tranche is only downgraded when its credit enhancement is reduced due to realized defaults in the collateral portfolio. However, in deteriorating market conditions a large fall in the market price of a tranche might force the rating agencies to take rating actions even prior to the realization of collateral losses. Otherwise the disparity between the falling value of a tranche and its credit rating would cast a doubt on the reliability of the rating agencies. A decision to downgrade a tranche is made by a rating committee and the process is to some extent arbitrary. In particular, the rating agencies do take into consideration market-wide factors. According to Fitch (2008) "The committees may make adjustment to standard assumption, or call for bespoke analysis. In addition, general economic outlook for certain sectors or industries may be taken into account.". When the credit outlook is unfavorable, the agencies can quote deteriorating market conditions and revise their rating assumptions. This happened during the financial crisis when the rating agencies took drastic rating actions with respect to a variety of CDO tranche issues. For example, S&P updated its assumption about the baseline correlations between RMBS tranches from 0.3 to 0.35-0.75, which resulted in large downgrades of ABS CDOs (S&P, 2008).

In economically robust periods, highly rated CDO tranches as well as highly rated bonds perform well. The tranches might be even characterized by higher rating stability because they have low exposure to idiosyncratic risks, but their capability to withstand economic recessions might be unsatisfactory. This follows from the findings of Coval et al. (2009a) who show that default risks of CDO tranches are concentrated in systematically adverse economic states. Moreover, in case of a deterioration in market fundamentals, the prices of CDO tranches are likely to become depressed more severely than the prices of corporate bonds. That is due to high sensitivity of tranche payoffs to default probabilities of the underlying bonds. If a deterioration in credit conditions is permanent or prolonged relative to the time until CDO maturity, then the prices of a CDO are unlikely to recover. That is in line with the findings of Coval et al. (2009c) and Bhansali et al. (2008) who argue that the fall in the prices of CDO tranches during the financial was not due to fire-sales, but was rather a permanent re-pricing corresponding to the deterioration in market fundamentals.

In light of the above discussion, the comparability of the rating stability between corporate bonds and CDO tranches is doubtful, particularly for highly rated tranches. In the corporate bonds market, the highest credit quality can be considered a guarantee of very low default risk and good rating stability. Long-term historical data indicates that obligors who issue AAA bonds perform robustly even in severe economic recessions. Notably very few corporates qualify for the AAA rating, which in case of Fitch is about 1% of its total corporate coverage (Fitch, 2007). In contrast, almost 60% of the volume of CDO tranches rated by Fitch are given AAA ratings (Fitch, 2007). It seems that lower rating stability is the downside of the rating methodologies that allow for issuance of such volumes of highly rated tranches, which in addition offer yield enhancement relative to similarly-rated bonds.

7. Conclusions

This paper analyzes the relation between credit ratings and fair spreads of CDO tranches. Credit ratings are based on real-world expected losses or real-world default probabilities, while fair spreads are implied by risk-neutral expected losses. Therefore credit ratings cannot fully account for fair spreads. Manufactures of CDOs can exploit this gap in the rating methodologies by producing CDO tranches that qualify for high credit rating, while offering significant yield enhancement relative to similarly-rated bonds. We further show that the yield enhancement on tranches is only possible due to high sensitivity of tranche payoffs to default probabilities of the collateral bonds. The sensitivity analysis also indicates that CDO tranches are prone to incur large losses and downgrades during unfavorable market conditions. These losses and downgrades are likely to be significantly higher for CDO tranches than for corporate bonds.

Acknowledgements

I gratefully acknowledge financial support from the Tinbergen Institute and the Netherlands Organization for Scientific Research (NWO) grant 400-09-124. I am grateful for valuable comments and discussions to Ton Vorst and Svetlana Borovkova. Any errors are my own.

Appendix A.

Table A.I: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 30% risk-neutral default probabilities of the underlying bonds.

In columns (1)-(3) we summarize tranche subordination levels and ratings by S&P and Moody's. The structuring and rating is done under the baseline assumptions, so these results are the same as in Tables 1 and 2. To calculate fair spreads we assume that risk-neutral default probabilities of bonds are triple the physical probabilities. In columns (4) and (5) we report fair spreads on tranches, while column (6) reports fair spreads on the corresponding risk-equivalent bonds (for CDO / CDO-squared separately). The risk-equivalent bonds have the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches.

Tranche	Tranche subordination CDO/CDO-squared	Tranche S&P/ Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	9.90% / 13.27%	'BBB-'/'BB+'	784.92	1717.63	180.72 / 180.72
tranche 3 senior mezz.	14.75% / 24.92%	'A-'/'BBB+'	453.97	1407.68	42.49 / 42.75
tranche 4 senior	17.08% / 31.25%	'AA'/'A'	306.75	1228.89	17.83 / 17.34
tranche 5 super-senior	19.45% / 37.50%	'AAA'/'AA+'	13.46	428.03	0.33 / 1.21

Table A.II: Comparison of fair spreads on tranches and risk-equivalent bonds under the assumption of 5% asset correlation.

In columns (1)-(5) we summarize tranche subordination levels, ratings and fair spreads. The structuring and rating is done under the assumption that the asset value correlation is equal to 5% instead of 12.5% used in the baseline case. In column (6) we report fair spreads on corporate bonds that are risk-equivalent to the corresponding tranches (for CDO / CDO-squared separately). The risk-equivalent bonds have the same (real-world) default probabilities and (real-world) expected losses as the corresponding tranches. To calculate fair spreads these bonds we assume that risk-neutral default probabilities are double the physical probabilities.

Tranche	Tranche subordination CDO/CDO-squared	Tranche S&P/ Moody's ratings	Fair spread (bps)		
			CDO	CDO-squared	Corporate bond
(1)	(2)	(3)	(4)	(5)	(6)
tranche 2 junior mezz.	8.45% / 12.52%	'BBB-'/'BB+'	522.85	1271.46	111.95 / 111.95
tranche 3 senior mezz.	11.40% / 22.07%	'A-'/'BBB+'	282.38	1057.71	27.34 / 28.16
tranche 4 senior	12.64% / 27.35%	'AA'/'A'	186.13	906.41	11.46 / 11.09
tranche 5 super-senior	13.96% / 33.41%	'AAA'/'AA+'	4.15	237.00	0.12 / 0.62

Appendix B.

Consider a random vector $(V_1, V_2, ..., V_{100})$, which has a multivariate standard normal distribution with a common pair-wise correlation parameter ρ. The V_i represents the scaled value of obligor i. We want to calculate default probability of the surviving obligors conditional on the information that the first obligor in the collateral pool defaults at a given time t. Without loss of generality (due to symmetry) we calculate the default probability of the first obligor conditional on the default of the 100th obligor at time t. In mathematical terms this probability is given by:

$$Pr\left(V_1 \leq K | V_1 > k^*, V_2 > k^*, ..., V_{99} > k^*, V_{100} = k^*\right), \tag{B.1}$$

where K is the threshold corresponding to a default after 10 years and k^* is the threshold corresponding to a default at time t (e.g. 3 months).

A feature of the multivariate normal distribution is that the conditional distribution of $(V_1, V_2, ..., V_{99})$ given a known value of V_{100} is again normally distributed with adjusted conditional mean and variance matrices. The conditional mean of each V_i (corresponding to the surviving bonds) is equal to ρk^*, the variance is is equal to $1-\rho^2$ and the pair-wise correlation between different V_i is equal to $\rho-\rho^2$.

Using this conditional distribution we can easily calculate the probability given by formula B.1 using simulations.

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