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# How to select instruments supporting R&D and innovation by industry

Marcel J.L. de Heide and Amit Kothiyal

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**Abstract**

We present a theoretical framework which allows for the comparison of the effectiveness of tax measures, loans and funding, in supporting industry-oriented research.

We estimate for each of the instruments the exact contribution required by a firm to decide on investing in R&D, given the costs and probability of success of the project, and the foreseen change in profit following successful implementation of the research results. We apply Prospect Theory to analyze the risk attitude of the firm.

By comparing the contribution required, we identify the instrument which is most effective, and therefore preferred by a government.

Our analysis indicates that there exists a critical value for the probability of success of the project for which the modality of the most effective instruments changes. For a probability of success smaller than the critical value, a tax measures offering support only in case of successful completion of the project is preferred. For a probability higher than the critical value, a loan is most effective. The value of the critical probability depends on the perception of risk and loss aversion of the firm involved in the research.

JEL Code: D04, D81, O38

## 1 Introduction

The global economy is showing signs of recovery after an unprecedented financial crisis initiated one of the most virulent recession in decades [OECD, 2010a]. These turbulent times have indicated again the importance of policies aimed at creating stable and sustainable economic growth. Countries need to increase their labour productivity levels by strengthening their research and innovation capacity. This implies that remaining competitive in the future requires not only intervening in the functioning of (financial) markets.

Incited by institutions like the OECD and the European Commission, countries have acknowledged the relevance of long-term innovation driven growth strategies. They have consequently implemented specific interventions aimed at increasing public as well as private expenditure on research [European-Commission, 2010] [OECD, 2010b]. In general, a government has several *instruments* at its disposal encouraging firms to invest in research. Within the framework of this paper, we focus on the following modalities of support: *funding* (e.g. subsidies or in kind contributions), *tax schemes*, and *loans*. Almost all industrialized countries have implemented these types of specific and dedicated measures supporting industry-oriented research.[1]

In order not to impede future economic recovery, most countries have decided to further rationalize their national budgets. The consequential retrenchments of public expenditure requires them to consider the *effectiveness* of the instruments mentioned above. In other words, and adapted from [Flanagan, 2010], countries are looking for: "...the cheapest one (i.e. measure) to implement, which least distorts the market whilst still achieving its objective".

In this paper, we present a theoretical framework that allows for a comparison of the *effectiveness* of instruments. Effectiveness within the framework of this paper refers to: "the ability to change the behavior of firms regarding investment in research, in comparison to the costs involved in the intervention".

Analysis of the literature indicates that little is known about the effectiveness of a single instruments in comparison to other tools [Boekholt et al. , 2006]. It is even argued that "...from a purely logical and technical point of view, policy tools appear to be perfectly interchangeable" [Landry & Varone, 2005]. Policy evaluation seems to focus on individual measures, and not on a set of instruments [Flanagan, 2010]. Some empirical studies address the trade-off between instruments. In [Guellec & De La Potterie, 2003] for example it is concluded, on the basis of an analysis of business R&D and government funding in seventeen OECD countries, that direct government funding and tax incentives have a positive effect on business financed R&D. Both measures appear to be substitutes: the increase in one reduces the effect of the other. Evaluation of the Dutch fiscal measure WBSO[2] indicated, on the basis of results of a questionnaire, that firms and governments prefer tax-deductions over other forms of financial support, because of the simplicity of application, and the fact that a large part of the infrastructure required for the policy delivery is available [Brouwer et al. , 2002]. Theoretical research on policy formulation builds mainly on multi-stage strategic investment games from industrial organization. Relevant papers such as [Hinloopen, 1997] and [Hinloopen, 2000] compare subsidizing with measures supporting collaboration in research. And in [Spencer & Brander, 1983] and [Inci, 2008] the use of subsidies is assessed in combination with generic tax measures.

We adopt a series of assumption with which we deviate from the existing theoretical research on policy formulation, as to reflect more adequately the current practice concerning industry-oriented research, and the modality of support allowed under the state aid rules [European-Commission, 2006]. We embrace a

[1] For an overview, check: www.proinno-europe.eu, or www.cordis.europa.eu/erawatch, or www.oecd.org.

[2] WBSO: "Wet Bevordering Speur- en Ontwikkelingswerk" or "R&D Work Stimulation Act".

problem driven innovation model, in which R&D is conducted within the framework of a predefined project, originating from an idea addressing a specific issue. We assume that the accompanying costs of the project are predetermined, and the outcome of the R&D process is foreseen. Successful execution of a research project involves a certain probability of failure. Only a subset of the population of firms will conduct research. Intervention is directed towards those involved with the idea and the related R&D. The prevailing literature on the contrary applies a linear, research driven innovation model. The impact of R&D on profit is considered to be a function of the investment research. All firms of the population are assumed to be involved in innovation, induced by generic instruments.

In order to assess the willingness of the firm to invest in this research project with an associated risk, we apply Prospect Theory as introduced in [Kahneman & Tversky, 1979]. The willingness to invest is defined by the valuation of the potential gains and losses associated with the implementation of the project, and the weighing of the risk involved in conducting the research.

We argue that if a firm is not willing to invest, a government has the possibility to intervene, and implement measures such that the firm will alter its behavior, and decide to conduct the R&D. We calculate for each instrument the exact amount of support required by the firm from the government to implement the project. We identify the most effective instrument by comparing the expected earnings resulting from the intervention for a government. We apply Expected Value theory to describe the behavior of a government concerning valuation of its earnings

The choice for behavioral model for the actors is motivated as follows: In decision analysis, Expected Utility theory (EU) is considered to be the right normative model for decision under uncertainty [Bleichrodt et al. , 2001]. We therefore apply EU to analyze the behavior of a government. But as the expected contribution required to initiate the research is relatively small for a government, we assume that its utility function can be well approximated by a linear function. EU consequently reduces in our case to Expected Value theory, in line [Arrow, 1962]. Prospect Theory (PT) on the other hand does a much better job than EU in predicting the behavior of individuals and small entities (e.g. SMEs) in risky situations [Kahneman & Tversky, 1979]. We therefore analyze the behavior of the firm under uncertainty using PT.

Our analysis indicates that there exists a critical value for the probability of success of the project for which the modality of the most effective instruments changes. For a probability of success smaller than the critical value, a tax measures offering support only in case of successful completion of the project is preferred. For a probability higher than the critical vale, a loan is most effective. The value of the critical probability depends on the perception of risk and loss aversion of the firm involved in the research. Our paper shows furthermore that the current approach towards support for industry-oriented research, as defined by [European-Commission, 2006], results in the implementation of measures such that they are not effective.

The next sections describe the model used, main results, and conclusions and recommendations with emphasis on the implications for policy formulation. The annex includes the technical proofs.

# 2 Model for investment in R&D

## 2.1 Investment decision on research by a firm

We assume that the industry consist of two firms: $I = \{i, k\}$, with initial corresponding profit $\{\pi_i^0, \pi_k^0\}$. Firm $k$ now has an idea to increase its profit. Successful translation of this idea into a market application requires conducting a research project with costs $K$, and probability of success $p \in [0, 1]$. The project costs $K$ are predefined, and budgeted according to foreseen allocation of resources. The probability $p$ depends on the current level of knowledge and state of the art in the research field of the project. If the required knowledge is not available, and lies beyond the current state of the art $p$ will be close to 0. If the required knowledge is available $p$ will be close to 1. Successful implementation of the research results will lead to a change in profit according to $\{\pi_i^-, \pi_k^+\}$, with $\pi_i^- \leq \pi_i^0$ and $\pi_k^+ > \pi_k^0 + K$.[3] If the research project fails, the corresponding profit equals $\left\{\pi_i^0, \pi_k^0 - K\right\}$.

Investment in the project involves a risk, and we assume that Prospect Theory [Kahneman & Tversky, 1979] describes the behavior of the firm in this situation. Within the framework of Prospect Theory (PT), losses and gains relative to some predefined *reference point* are the carriers of value. It is a well known empirical phenomenon that "losses loom larger than gains" [Kahneman & Tversky, 1979]. PT models this phenomenon by weighting the losses relative to gains with a *loss aversion parameter* $\lambda > 1$.

Also in Prospect Theory, probabilities are distorted by a *probability weighting function*, which over-weights low probabilities and under-weights high probabilities. [Tversky & Kahneman, 1992] propose the following family of weighting functions:

$$w(p) = \frac{p^\beta}{\left(p^\beta + (1-p)^\beta\right)^{\frac{1}{\beta}}}. \tag{1}$$

In this paper we assume the family of probability weighting functions with *parameter* $\beta \in [0.28, 1)$. On this interval the lower the value of $\beta$ the more the distortion in probabilities (e.g. [Tversky & Kahneman, 1992] estimated $\beta = 0.61$, see Figure 1).[4]

In Prospect Theory, gains and losses are furthermore evaluated by means of a utility function. In Expected Utility theory, the curvature of a utility

[3] Our model holds under different market structures. As an example, consider two firms competing in a market on price (i.e. Cournot competition). Now assume that firm $k$ has an idea that will change the marginal cost of production. Successful implementation of the research results will increase the pay-off for the innovating firm, while decreasing the profit for the other firm.

[4] For $\beta < 0.28$ the probability weighting functions are not strictly increasing; for $\beta > 1$ they over-weight small probabilities rather than under-weight them; and for $\beta = 1$ the function is linear (i.e. no distortion of the probabilities).

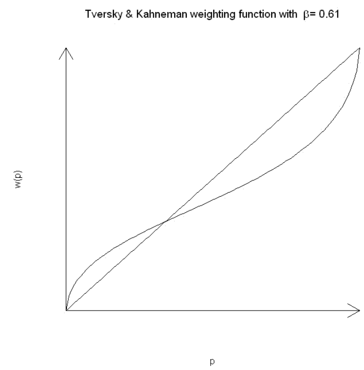


Figure 1: Weighting function

function is solely used to analyze risk attitude. But in PT risk attitude largely depends upon probability distortion function and loss aversion. So for the sake of simplicity, we assume the utility function to be piecewise linear within the framework of our paper.

The outcome of the investment in the research project as described in this paper can be represented as a binary prospect yielding $(\pi_k^+ - K)$ with probability $p$, and $(\pi_k^0 - K)$ with probability $(1-p)$. We adopt a notation which represents this as $(\pi_k^+ - K)_p(\pi_k^0 - K)$. The decision problem for the firm is to choose between this prospect or the old profit $\pi_k^0$. The natural reference point for this decision problem is $\pi_k^0$.

The value function, which shows the result of the aggregation of gains and losses of the firm, is consequently given by:

$$V = w(p)\left((1-f)\left(\pi_k^+ - \pi_k^0\right) - K\right) + \lambda w(1-p)\left(-K\right). \tag{2}$$

In our model, we assume a fraction $f$ of tax collected by the government on the profit. The value function shows the gain as the profit after successful implementation of the research result, minus the project costs, relative to the reference point. The loss reflects the other outcome of the binary prospect relative to the reference point. Both gain and loss are weighted, but only the latter includes the loss aversion parameter.

The firm will decide on investing in the research project if and only if the result of the value function is not less than zero.

## 2.2 Contribution to the research by the government

If the result of the value function is negative, a government has the possibility to intervene, and implement policy instruments such that firm $k$ will consider altering its behavior, and decide on conducting the R&D.[5] The rationale for government intervention supporting basic research has been established by [Nelson, 1959] and [Arrow, 1962]. They argue that the difficulty of selecting potentially marketable research and the uncertainty with respect to its successful outcome limits incentives of industry to conduct R&D. The indivisible and non-excludable character of this type of research, and the fact that its results are almost freely available in scientific publications further enhances the tendency to under-invests in R&D. This market failure argument has been further sophisticated by [Jaffe, 1996] as a rationale for supporting pre-competitive industry oriented research. In his work he identifies additional types of spill-overs that negatively affect decisions of firms concerning investments in research. These types refer to appropriability of knowledge (for example through imitation), the benefits to users of the innovation not captured in its price, and network spill-overs (when successful innovation relies upon developments in related technologies.

We argue that a government should select the instrument that maximizes its expected earning $E_y$. The earnings are defined by the taxation on firms with a fraction $f$ on the profit, and the expected contribution $I$ corresponding to a specific instrument, required to initiate the research:

$$E_y = pf\left(\pi_k^+ + \pi_i^-\right) + (1-p)\, f\left(\pi_i^0 + \pi_k^0\right) - I \tag{3}$$

If the expected government earning of a certain Instrument A is more than that of Instrument B then we say Instrument A *performs better* than Instrument B.

## 2.3 Instruments

We identify four different modalities of support for firms conducting research. The contribution is provided in different stages of the project, depending on success or failure of the outcome.

### 2.3.1 Funding

A government can contribute to the project costs by offering direct funding (e.g. subsidies and grants), or indirect funding (e.g. as access to research infrastructure, vouchers, rebate on social insurance, and advice). The first type of funding involves an actual transfer of funds, which are not recoverable by the government from the beneficiary. With the latter, a government tries to compensate for the costs by providing in kind resources. We assume that Funding is allocated unconditionally, i.e. in case of success or failure.

If $F$ is the minimum amount of funding required to initiate the research project, then the corresponding value function of the firm, as given in (2) be-

[5] In practice, intervention as such is limited: the total government costs (on project level), which include implementation costs and the actual contribution to initiate the research, should never be too high, in that they should not undo the gain in social surplus.

comes:

$$w(p)\left((1-f)\left(\pi_k^+ - \pi_k^0\right) + F - K\right) + \lambda w(1-p)\left(F-K\right) = 0. \tag{4}$$

And the corresponding expected government earnings $E_F$ (3) is:

$$E_F = pf\left(\pi_k^+ + \pi_i^-\right) + (1-p)\,f\left(\pi_i^0 + \pi_k^0\right) - F. \tag{5}$$

#### 2.3.2 Unconditional tax-rebate

Support can also be offered proportional to the expected profit. We first analyze a tax rebate given unconditionally on whether the project succeeds or fails. Let $t$ be the minimum fraction of tax rebate required to initiate the research project, then the value function of the firm is:

$$w(p)\left((1-f)\left(\pi_k^+ - \pi_k^0\right) + t\pi_k^+ - K\right) + \lambda w(1-p)\left(t\pi_k^0 - K\right) = 0, \tag{6}$$

with corresponding expected government earnings $E_t$ denoted by:

$$E_t = pf\left(\pi_k^+ + \pi_i^-\right) + (1-p)\,f\left(\pi_i^0 + \pi_k^0\right) - t\left(p\pi_k^+ + (1-p)\,\pi_k^0\right). \tag{7}$$

#### 2.3.3 Conditional tax-rebate

A tax rebate can be provided also only in case the project is completed successfully. Let $\tau$ be the minimum fraction of conditional tax rebate, then the value function of the firm is:

$$w(p)\left((1-f)\left(\pi_k^+ - \pi_k^0\right) + \tau\pi_k^+ - K\right) + \lambda w(1-p)\left(-K\right) = 0,$$

with corresponding expected government earnings $E_\tau$ denoted by:

$$E_\tau = pf\left(\pi_k^+ + \pi_i^-\right) + (1-p)\,f\left(\pi_i^0 + \pi_k^0\right) - p\tau\pi_k^+.$$

#### 2.3.4 Loans

Loans involves the allocation of funds which have to be reimbursed by the firm to the government in case the research project is implemented successfully. Let $L$ be the minimum amount provided by means of a loan required to initiate the research project, then the value function of the firm is:

$$w(p)\left((1-f)\left(\pi_k^+ - \pi_k^0\right) - K\right) + \lambda w(1-p)\left(L-K\right) = 0,$$

with corresponding expected government earnings $E_L$ denoted by:

$$E_L = pf\left(\pi_k^+ + \pi_i^-\right) + (1-p)\,f\left(\pi_i^0 + \pi_k^0\right) - (1-p)\,L.$$

## 2.4 Main results

By comparing the expected earnings from the government resulting from the implementation of the different instruments, we can assess the performance of measures compared to tools.

**Theorem 1** *For $p < p_c$ the order of instruments in terms of performance from best to worst equals: conditional tax-rebate, unconditional tax-rebate, funding, loan. For $p > p_c$ the above ordering is reversed. For $p = p_c$ all instruments perform equally well. The critical probability of success of the research project for which the disposition changes is given by:*

$$p_c = \frac{1}{1 + \lambda^{\frac{1}{1-\beta}}}.$$

The proof for Theorem 1 is given in the Appendix. The intuition behind the theorem is as follows: . For $p < p_c$ a fiscal measure that allocates support only in case of success provides the highest *actual* transfer of resources to a firm, in comparison to the other tools. In case of such a conditional tax-rebate, this contribution is of course granted only in case the research project is conducted successfully. But the valuation of this support, as defined by the utility function, and the weighing of the probability of success, as stipulated by the weighing function, is such that a firm is willing to take the risk, and conduct the research for an indemnification that results in the lowest *expected* contribution by a government. For a high probability of success for the research project, the reasoning is similar for a loan. In practice, a measure is preferred over other instruments if its corresponding *expected* contribution, allocated by a government to a firm, is lower than that of other tools

**Corollary 2** *For $p < p_c$ a conditional tax-rebate is most effective. For $p > p_c$ a loan is the optimal strategy.*

With the help of our model, we are able to define for each of the instruments the exact contribution required by the firm to alter its behavior. The change in market equilibria causing the expected gain in profit by the firm results from the foreseen implementation of the research results, and not from the transfer of support. The contribution itself is appropriated merely to amend the prospect of the firm such that it will initiate the project. We therefore maintain that we have defined a framework for the selection of instruments, such that they are efficient. We also have defined our model such that all measure induce a similar level of efforts in R&D. Based on our analysis, we therefore conclude that unconditional tax rebates and loans are most effective, as they require the lowest level of contribution to initiate the research project, and therefore maximize government earnings.

**Remark 3** *If we assume $\lambda = \beta = 1$ within the framework of our model, Prospect Theory reduces to Expected Value theory, and all instruments perform equally well.*

Without applying Prospect Theory, we find that all instruments require the same level of expected contribution to initiate research. This is in line with what is argued in the literature [Landry & Varone, 2005]. Our framework indicates that there are differences in the effectiveness of instruments, resulting from our assumptions concerning risk aversion and the perception of risk involved in conducting the research by the firm.

# 3 Conclusions and recommendations

Theorem 1 indicates that our results do not depend on initial profit, or a change in pay-off after implementation of the project. This implies that the result holds under different market structures (e.g. differentiated markets under Cournot or Bertrand). We therefore maintain that our result is robust.

The current practice regarding funding of collaborative research foresees in a contribution of 50-80% to the costs of the project, regardless of the risks involved in the R&D. The exact level of funding depends on the type of research conducted (e.g. experimental research, or industry-oriented research close to the market), or actor involved (e.g. SMEs, MNFs) [European-Commission, 2006]. Based on our model we argue that under this legal framework, projects requiring a higher level of support will not be conducted, as the firm with the idea will not receive sufficient funding to make up its willingness to invest in the required research. We therefore argue that under these conditions the measures are not effective as they are not able to change behavior of the firm. We also maintain that projects which have been conducted with the help of support according to state aid rules in practice most likely would have required less contribution than provided. We therefore contend that under these conditions the instruments are not effective as they are not efficient.

Our paper suggests that redefining the approach towards support for industry oriented research, and the legal framework governing it, could improve the effectiveness of policy aimed at strengthening the innovation system. Adopting our model as a basis for policy delivery however requires further analysis of the factors which define the investment decision of firms (i.e. perception of risk and loss aversion), and how they vary (e.g. for type of firm, or sector). As an example, if we assume the perception of risk to be given and constant, we see that the critical probability of success changes with the risk aversion of the firm involved. This implies that if firms within a certain sector are more risk averse than those in other sectors, their corresponding $p_c$ is different (e.g. lower), and as a consequence a loan might be more attractive as a tool to support their research projects.

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# A Proofs

Proof of the Theorem 1 follows trivially from Lemma 4, Lemma 5 and Lemma 6 below. Since proofs of these lemmas are similar, we only provide the proof for Lemma 5.

**Lemma 4** *For $p < p_c$ Conditional tax-rebate performs better than Unconditional tax-rebate, and for $p > p_c$ reverse is true.*

**Lemma 5** *For $p < p_c$ Unconditional tax-rebate performs better than Funding, and for $p > p_c$ reverse is true.*

**Lemma 6** *For $p < p_c$ Funding performs better than a Loan, and for $p > p_c$ reverse is true.*

**Proof.** Funding performs better than Unconditional tax-rebate if and only if $E_F > E_t$, where $E_F$ and $E_t$ are defined according to equation (5) and (7). From (4) and (6) we get:

$$(w(p) + \lambda w(1-p))F = (w(p)\pi_k^+ + \lambda w(1-p)\pi_k^0)t$$

If we let $\theta = \pi_k^+/\pi_i^-$ then from the above equation it follows that:

$$E_F > E_t \iff \frac{w(p)\theta + \lambda w(1-p)}{(p\theta + (1-p))(w(p) + \lambda w(1-p)} < 1$$

With (1) we get that:

$$E_F > E_t \iff p^\beta \theta + \lambda(1-p)^\beta < (p\theta + (1-p))(p^\beta + \lambda(1-p)^\beta)$$

Let $r = p/\,(1-p)$, then:

$$\begin{aligned}
E_F &> E_t \iff \\
r^\beta\theta + \lambda &< (p\theta + (1-p))(r^\beta + \lambda) \iff \\
r^\beta\theta(1-p) - r^\beta(1-p) &< \lambda(p\theta + (1-p) - 1) \iff \\
r^\beta(1-p)(\theta - 1) &< \lambda p(\theta - 1) \iff \\
r^{\beta-1} &< \lambda
\end{aligned}$$

$$E_F > E_t \iff r > \lambda^{\frac{-1}{1-\beta}} \quad \textit{since} \quad \beta < 1$$

$$E_F > E_t \iff p > \frac{1}{1 + \lambda^{\frac{1}{1-\beta}}}$$

For $p > p_c$ Funding performs better than Unconditional tax-rebate, and for $p < p_c$ reverse is true. ■