



TI 2010-098/2

Tinbergen Institute Discussion Paper

Labor Market Matching under Imperfect Information

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Labor Market Matching under Imperfect Information*

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May 13, 2011

Abstract

Recent research has shown that the standard labor matching model has difficulties in reproducing the co-movement patterns observed in US data. This is due to the fact that the standard model lacks sufficient propagation of shocks. This paper shows that refining the informational structure of the model leads to improvements along this dimension: when agents cannot separately identify persistent and transitory technology shocks on impact (so that they must solve a signal extraction problem), shocks are propagated. Under this specification the standard matching model even manages to make recoveries initially jobless, as in the data.

Key words: imperfect information, labor market matching, signal extraction, jobless growth

JEL-classification: D80, E32, J63, J64

1 Introduction

The literature on labor market matching models has recently been extended with some notable contributions that highlight the difficulties the standard model has in replicating the co-movement patterns of variables observed in US data. In particular, Fujita and Ramey (2007) and Hagedorn and Manovskii (forthcoming) point out that

*I thank Liam Graham, Wouter den Haan and Sweder van Wijnbergen, as well as audiences at the University of Amsterdam, the NAKRE Research Day 2010 and the 25th EEA-meeting in Glasgow for useful comments and discussions. Any remaining errors are of course mine.

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the contemporaneous correlations of key labor market variables with productivity are too high in the model. They attribute this failure to a lack of shock propagation: in the standard model, firms adjust labor input immediately after a shock has hit, while the data suggest a much more gradual response.

This suggests that vacancies are too sensitive to productivity shocks in the model. To overcome this difficulty, Fujita and Ramey (2007) introduce a sunk cost for the creation of vacancies. This turns vacancies into a predetermined variable, as a result of which they no longer peak on impact of the shock. As shown in their paper, this modification manages to increase the propagation of shocks substantially. To what extent sunk vacancy creation costs play a role in reality however remains to be seen as there is little direct empirical evidence on this.

Moreover, as noted in Hagedorn and Manovskii (forthcoming), the sunk vacancy creation cost model also generates a near-one correlation between labor market tightness and productivity, which is higher than the one observed in US data.¹ Consequently, these authors take a different approach in solving the aforementioned problems and introduce a lag in vacancy posting, so that vacancies created today only enter the labor market (that is: the matching function) k periods from now.² They motivate this lag by noting that it may take "firms time to infer that an aggregate productivity change has occurred" (p. 4).

This paper is similar in nature, but uses a finer informational structure to capture the notion that firms may need some time to realize that productivity has changed. In particular, instead of introducing an *ad hoc* lag in vacancy postings, this paper models the signal extraction process that is associated with this type of problem.

In a standard matching setup, it is always assumed that the nature of shocks is fully clear as soon as they materialize: agents know the model structure and to the extent that the model contains both persistent and transitory shocks, agents immediately observe which of the two has hit their economy. Unfortunately, things are less clear in reality: there, recessions and expansions generally cannot be identified in real time (there exists an announcement lag because there is uncertainty on the contemporaneous state of the economy; *cf.* Orphanides and Van Norden 2002), nor do they come with clear information on their exact length and fierceness. In modeling terms this implies that people are imperfectly informed on the extent to which their economy has been hit by a persistent, or a transitory shock. Only by

¹Using productivity data based on the Current Population Survey, Hagedorn and Manovskii find a correlation between labor market tightness and productivity of 0.703. If they base themselves on data from the Current Employment Statistics, this correlation is even lower (about 0.45).

²Given the close analogy to the treatment of capital in Kydland and Prescott (1982), they refer to this as "time-to-build".

observing tomorrow's state, agents are able to extract some additional information on the nature of today's shock, which they can subsequently use to update their beliefs about the contemporaneous state of the economy.³ Consequently, with respect to taking decisions with some element of irreversibility in them (such as hiring and firing decisions), there exists an option value to waiting, as waiting provides you with valuable additional information (see Willems and Van Wijnbergen 2009, who analytically show that this option value exists in a labor market setting).

The contribution of this paper is intended to be twofold: first, it demonstrates how a standard labor matching model can be solved under the assumption of imperfect information by employing the Kalman filter. Second, it shows that once one allows for this feature, the model performs much better in propagating productivity shocks.

This paper is structured as follows. First, Section 2 describes the standard matching model, after which Section 3 explains how the model can be solved under imperfect information. Section 4 then discusses the calibration, after which Section 5 shows that the introduction of the so-called "persistent-transitory shock confusion" improves the ability of the standard matching model to propagate productivity shocks. Section 6 checks the robustness of this result and Section 7 concludes.

2 Model description

Let there be a unit mass of individuals available for a job and an infinite mass of small firms, indexed by i . The labor market is subject to matching frictions of the Diamond-Mortensen-Pissarides type as described in Pissarides (2000, Chapter 1). Each period, matches are formed via the following aggregate matching function:

$$m(u_t, v_t) = Au_t^\mu v_t^{1-\mu} \quad (1)$$

The number of matches formed thus depends on the number of unemployed workers, u_t , and the total number of vacancies posted, $v_t = \int v_{i,t} di$. Here, $v_{i,t}$ represents the number of vacancies posted by firm i , while the scalar A in equation (1) captures the efficiency of the matching technology.

Every period, all matched worker-firm pairs produce z_t . The latter is assumed to be the product of a persistent component (ϕ_t) and a purely transitory one (ψ_t):

$$z_t = \phi_t \cdot \psi_t, \quad (2)$$

³This type of uncertainty particularly showed itself in 2010. At that time, there seemed to be great uncertainty on whether the recovery to the "Great Recession" was going to persist, or whether it was just going to be a temporary revival (*cf.* Warren Buffett arguing the former (*The Guardian*, Sept 14 2010), while Nouriel Roubini saw a double dip coming up (*The Guardian*, Sept 16 2010)). Only the future reader of this paper will be able to tell which case has materialized.

with

$$\phi_t = \phi_{t-1}^{\rho_\phi} \cdot \exp(\varepsilon_{\phi,t}) \quad (3)$$

$$\psi_t = \exp(\varepsilon_{\psi,t}), \quad (4)$$

where $\rho_\phi \in (0, 1)$ and $\varepsilon_{s,t} \sim \mathcal{N}(0, \sigma_s^2)$, $s \in \{\phi, \psi\}$.⁴

By defining θ as the labor market tightness parameter

$$\theta_t \equiv \frac{v_t}{u_t}, \quad (5)$$

one can write the probability that a vacancy is filled as:

$$q(\theta_t) \equiv \frac{m(u_t, v_t)}{v_t} = m\theta_t^{-\mu} \quad (6)$$

Equivalently, the probability that an unemployed worker finds a job equals:

$$p(\theta_t) \equiv \frac{m(u_t, v_t)}{u_t} = m\theta_t^{1-\mu} = \theta_t q(\theta_t) \quad (7)$$

Every period, an exogenous fraction ρ_x of all matches is assumed to be destroyed. Consequently, employment for each firm i evolves according to:⁵

$$n_{i,t} = (1 - \rho_x) [n_{i,t-1} + v_{i,t}q(\theta_t)], \quad (8)$$

where all firms take the matching probability $q(\theta_t)$ as given.

Total output of firm i is given by:

$$y_{i,t} = z_t n_{i,t-1} \quad (9)$$

As the labor force is normalized to 1, the unemployment rate equals:

$$u_t = 1 - n_{t-1}, \quad (10)$$

⁴One could interpret the transitory shock ψ_t as a truly transitory productivity shock or as statistical noise of which agents only find out *ex post* that it has not materialized (see *e.g.* Bomfim 2001 for an example of the latter interpretation). Although these two interpretations are not mutually exclusive for large parts of this paper, this distinction does for example matter when the model is going to be confronted with the data. To minimize deviations from the standard model (which has only one shock, namely the persistent one), I will follow Bomfim (2001) in the data confrontation step and interpret ψ_t as statistical noise and use the persistent productivity shock ϕ_t to calculate correlations with.

⁵Note that my notation is such that all variables carry the subscript that indicates the time period in which they were decided upon. Hence, n_t is a choice variable in period t , while it is a state variable for period $t + 1$.

with $n_{t-1} = \int n_{i,t-1} di$.

Knowing that per period real vacancy posting costs equal γ , each firm i sets its next period's level of employment ($n_{i,t}$) via the number of posted vacancies ($v_{i,t}$) so as to maximize the discounted stream of expected real profits:

$$\Pi_{i,0} = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \beta^t [y_{i,t} - w_t n_{i,t-1} - \gamma v_{i,t}] \right\},$$

subject to production function (9) and the law of motion for employment (8).

The firm's first order conditions then read:

$$\begin{aligned} \partial n_{i,t} &: \lambda_t = \beta \mathbb{E}_t \{ z_{t+1} - w_{t+1} \} + \beta (1 - \rho_x) \mathbb{E}_t \{ \lambda_{t+1} \} \\ \partial v_{i,t} &: \frac{\gamma}{q(\theta_t)} = (1 - \rho_x) \lambda_t \end{aligned}$$

Here, λ_t is the time t Lagrange multiplier on the employment constraint. It represents the time t value of having a match for the firm, which is equal to the discounted expected value of next period's profit, augmented with the expected continuation value into period $t+2$. We can combine the first-order conditions to yield:

$$\frac{\gamma}{q(\theta_t)} = (1 - \rho_x) \beta \mathbb{E}_t \left\{ \left[z_{t+1} + \frac{\gamma}{q(\theta_{t+1})} - w_{t+1} \right] \right\} \quad (11)$$

This is the job creation condition and states that, at the optimum, the expected cost of hiring a worker should equal the expected value of a match.

By now, we only need to find an expression for the real wage rate, w_t . Under the assumption that workers and firms Nash-bargain on the matching surplus (with bargaining weights ω and $(1 - \omega)$, respectively) we get that:

$$w_t = b + \omega [z_t - b + \gamma \theta_t] \quad (12)$$

Hence, workers receive a real wage that is equal to their outside option b , augmented with a fraction ω of the value of their marginal productivity in excess of b , plus the average vacancy posting cost an employer saves as long as a match is sustained.

3 Solution method

After dropping the i -subscripts by symmetry, the equilibrium for this model is defined as:

Definition 1 A set of sequences for $m_t, \theta_t, q(\theta_t), p(\theta_t), y_t, n_t, u_t, v_t, w_t, z_t, \phi_t, \psi_t$ satisfying equations (1)-(12) augmented with initial values for productivity (ϕ_0 and ψ_0) and an initial level of employment (n_0).

The model can be solved by log-linearizing these equations around the deterministic steady state. In the full information case - where the agents in the model contemporaneously observe the decomposition of the aggregate productivity shock (z_t) into its persistent (ϕ_t) and transitory (ψ_t) component - we can define the state of the system as $X_{t-1} \equiv \begin{bmatrix} \hat{n}_{t-1} & \hat{\phi}_{t-1} & \hat{\psi}_{t-1} \end{bmatrix}'$ (where a hat above a variable indicates a log-deviation from the variable's steady state). Subsequently, we can write the model in state space form:

$$X_t = F X_{t-1} + H \varepsilon_t,$$

where $\varepsilon_t \equiv \begin{bmatrix} \varepsilon_{\phi,t} & \varepsilon_{\psi,t} \end{bmatrix}'$, $\mathbb{E}[\varepsilon_t \varepsilon_t'] = \Sigma_{\varepsilon\varepsilon} = \begin{bmatrix} \sigma_\phi^2 & 0 \\ 0 & \sigma_\psi^2 \end{bmatrix}$, and the dynamic choice variable \hat{v}_t obeys:

$$\hat{v}_t = \mathbf{g}' X_{t-1}$$

One can then solve the model by standard methods, such as Blanchard and Kahn (1980).

When information is imperfect, the solution method is slightly more involved however.⁶ Here, the informational imperfection consists of the fact that agents can only see *the sum* of the persistent and transitory component of productivity. That is: next to employment n_{t-1} , they only observe z_t . Consequently, they are unable to identify ϕ_t and ψ_t contemporaneously. Only as more information arrives over time, they can form an idea on what type of shock hit them in the past by solving the signal extraction problem.

They do this in the following way. Define the exogenous state vector as $Z_t \equiv \begin{bmatrix} \hat{\phi}_t & \hat{\psi}_t \end{bmatrix}'$. As these two variables are not separately identified, agents need to form a contemporaneous estimate of this state. If we refer to this estimate as $Z_{t|t}$, the system can be represented as:

$$\begin{bmatrix} Z_t \\ Z_{t|t} \end{bmatrix} = \begin{bmatrix} W_{11} & \mathbf{0}_{2 \times 2} \\ W_{21} & W_{22} \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} V_{11} \\ V_{12} \end{bmatrix} \varepsilon_t$$

with $W_{11} = \begin{bmatrix} \rho_\phi & 0 \\ 0 & 0 \end{bmatrix}$

⁶See for example Nimark (2008) and Baxter *et al.* (2011) for general accounts of solving models under informational imperfections.

However, to solve the system completely, we also need to find the law of motion for the estimate of the imperfectly observed state ($Z_{t|t}$), *i.e.* we have to solve for matrices W_{21} and W_{22} . As the problem is linear, the optimal way to update the state estimate is through the Kalman filter, which the agents in this model are assumed to do.

The Kalman filter updating equation reads:⁷

$$Z_{t|t} = (W_{11} + W_{12}) Z_{t-1|t-1} + K [Y_t - D (W_{11} + W_{12}) Z_{t-1|t-1}],$$

where K is the Kalman gain matrix. It is given by:

$$\begin{aligned} K &= PD'(DPD')^{-1} \\ P &= (W_{11} + W_{12}) (P - PD'(DPD')^{-1}DP) (W_{11} + W_{12})' + V_{11}\Sigma_{\varepsilon\varepsilon}V_{11}' \end{aligned}$$

In the above equations, D represents a selection matrix that determines which variables the agents in the model are able to observe. In this case, the exogenous, observable state Y_t is thus given by $Y_t = DZ_t = \widehat{z}_t$.

The evolution of Y_t occurs according to:

$$Y_t = DW_{11}Z_{t-1} + DW_{12}Z_{t-1|t-1} + DV_{11}\varepsilon_t$$

Plugging this into the Kalman filter updating equation (and using that $W_{12} = \mathbf{0}_{2 \times 2}$) yields:

$$Z_{t|t} = W_{11}Z_{t-1|t-1} + K [DW_{11}Z_{t-1} + DV_{11}\varepsilon_t - DW_{11}Z_{t-1|t-1}]$$

From this equation, we can read the matrix expressions for W_{21} and W_{22} . Hence, with imperfect information the evolution of the imperfectly observed state is described by:

$$\begin{bmatrix} Z_t \\ Z_{t|t} \end{bmatrix} = \begin{bmatrix} W_{11} & \mathbf{0}_{2 \times 2} \\ KDW_{11} & W_{11} - KDW_{11} \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-1|t-1} \end{bmatrix} + \begin{bmatrix} V_{11} \\ KDV_{11} \end{bmatrix} \varepsilon_t$$

At this stage, one should note that certainty equivalence applies (see Baxter *et al.* 2011), which means that:

$$\widehat{v}_t = \mathbf{g}'X_{t-1|t-1},$$

where $X_{t-1|t-1} \equiv [\widehat{n}_{t-1} \ Z_{t-1|t-1}]'$ (recall that the level of employment is assumed to be perfectly observable, as a result of which $\widehat{n}_{t|t} = \widehat{n}_t, \forall t$). Here, certainty

⁷See Anderson and Moore (1979) for a derivation of the Kalman filter.

equivalence implies that even though firms know that they only have an imperfect estimate of the true state, their actions are still as if they have full information. Using this, one can write the solution to the system under imperfect information as:

$$\begin{bmatrix} Z_t \\ Z_{t|t} \\ \hat{n}_t \end{bmatrix} = \begin{bmatrix} W_{11} & \mathbf{0}_{2 \times 2} & \mathbf{0}_{2 \times 1} \\ KDW_{11} & W_{11} - KDW_{11} & \mathbf{0}_{2 \times 1} \\ \mathbf{0}_{1 \times 2} & Q_{32} & f_1 \end{bmatrix} \begin{bmatrix} Z_{t-1} \\ Z_{t-1|t-1} \\ \hat{n}_{t-1} \end{bmatrix} + \begin{bmatrix} V_{11} & \mathbf{0}_{2 \times 3} \\ KDV_{11} & \mathbf{0}_{2 \times 3} \\ \mathbf{0}_{1 \times 2} & \mathbf{0}_{1 \times 3} \end{bmatrix} \begin{bmatrix} \varepsilon_t \\ \mathbf{0}_{3 \times 1} \end{bmatrix},$$

where:

$$Q_{32} = [f_2 \quad f_3]$$

4 Calibration

4.1 Labor market

The model is calibrated at a quarterly frequency. I set the discount factor β equal to its standard value of 0.99. The matching elasticity parameter μ is set at 0.5, which is in line with the empirical evidence surveyed in Petrongolo and Pissarides (2001). To abstract from any externalities, I assume that the Hosios-condition holds and set the bargaining weight for workers (ω) equal to 0.5 as well. As in Fujita and Ramey (2007), the steady state unemployment rate is set equal to 0.08.

Following Den Haan *et al.* (2000), I set the steady state firm matching rate $q(\theta)$ equal to 0.7. The steady state job finding probability $p(\theta) = \theta q(\theta)$ is set at 0.6 to match the average duration of unemployment of 1.67 quarters (Cole and Rogerson, 1999).

To solve the so-called "Shimer (2005)-puzzle" (in short, the lack of shock amplification in the standard matching model⁸) - which is not the aim of this paper - I follow the solution offered by Hagedorn and Manovskii (2008) and pick a rather high value for b .⁹ In particular, I set b equal to 0.9 (so as to bring the standard deviation of tightness in the benchmark model closer to its empirical counterpart).

Finally, the value of the vacancy posting cost parameter γ is obtained via the steady state job creation condition and equals 0.0974 for the above parameterization.

⁸More precisely, the Shimer-puzzle refers to the fact that the matching model fails to reproduce the volatility of labor market tightness (θ) observed in reality. Shimer (2005) shows that this ratio is ten times more volatile in US data than in the model for standard calibrations.

⁹Note that the aim of this paper is to increase the model's ability to *propagate* shocks, which is different from the amplification issue. Also see the discussion in Fujita and Ramey (2007).

4.2 Shocks

Following the RBC-literature, the process for the persistent component of productivity ($\widehat{\phi}_t$) is assumed to follow an $AR(1)$ with its persistence parameter ρ_ϕ equal to 0.95 and a standard deviation σ_ϕ of 0.008.

The parameter that remains to be calibrated (σ_ψ) is a crucial one. This is the standard deviation of the transitory noise component of the productivity shock. Consequently, $\eta \equiv \sigma_\phi^2/\sigma_\psi^2$ can be defined as the signal-to-noise ratio. In line with Bomfim (2001)¹⁰ and based upon more recent work by Blanchard *et al.* (2009, who have estimated η to lie close to unity), I set the signal-to-noise ratio in my benchmark calibration equal to one. In Section 6, I carry out some robustness checks by discussing the results for different signal-to-noise ratios.

5 Model dynamics under imperfect information

5.1 Propagation of shocks

Because agents only observe the sum of the persistent and transitory component of aggregate productivity, they can only find out what type of shock hit them in the past as more information arrives over time. Figure 1 depicts this signal extraction process in response to a persistent, one standard deviation, positive productivity shock for the benchmark calibration.

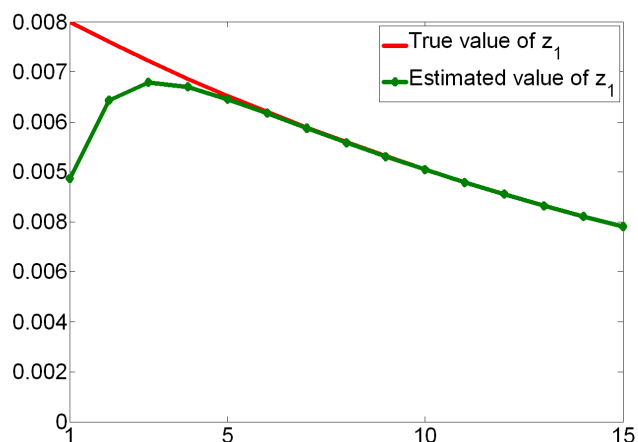


Figure 1: True and estimated value of a persistent productivity shock

¹⁰Bomfim (2001) in turn based himself upon Mankiw *et al.* (1984, who reported an η of 0.56) and upon Diebold and Rudebusch (1991, who estimated η to equal 1.3)

As can be seen from the figure, agents initially place some weight on the possibility that the shock is just transitory. However, as time passes, agents note that productivity is high for several periods, and they start to put more weight on the possibility that they have been hit by a persistent shock. For this benchmark calibration, the difference between the true and estimated shock becomes negligibly small after three quarters. This seems reasonable given the fact that NBER business cycle turning points are announced with an average lag of about four quarters.¹¹

Figure 2 shows what this persistent-transitory confusion implies for the impulse-response functions (IRFs) of our key variables. As the figure shows, imperfect information increases the propagation of shocks by changing the response of vacancies. Their impact response is much more muted, while the imperfect information model also manages to reproduce the hump-shaped response of vacancies found in the data (see Fujita and Ramey 2007, Ravn and Simonelli 2008 and Fujita 2011).

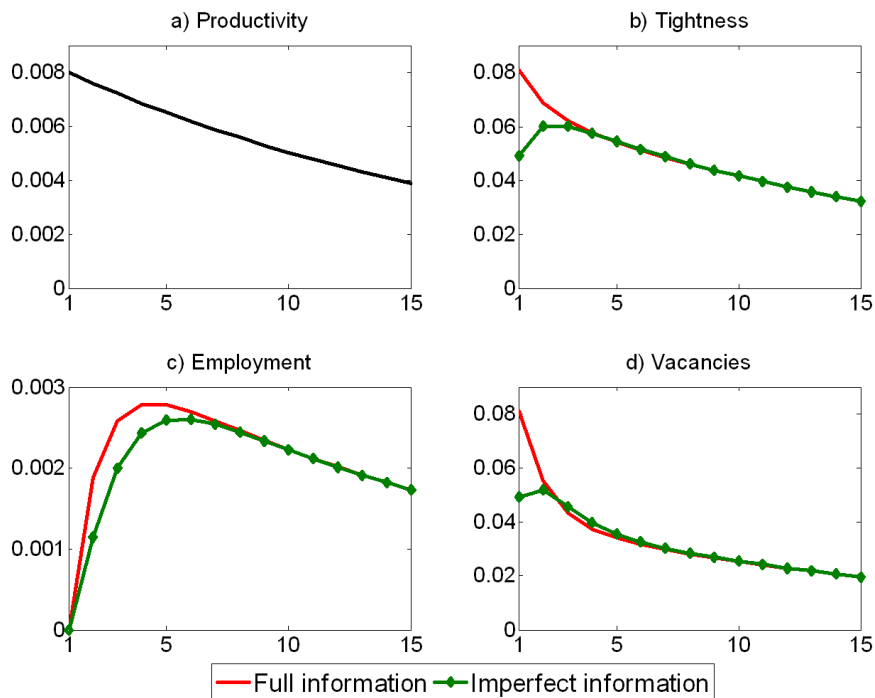


Figure 2: Impulse responses to a persistent productivity shock

This is caused by the fact that matching frictions make labor adjustment costly, as a result of which firms do not want to respond to transitory shocks, while they

¹¹See the announcement dates on <http://www.nber.org/cycles/main.html>.

would like to react to persistent ones. But as employers can only disentangle the two over time, they respond less enthusiastically on impact when they are still uncertain about the nature of a shock. That is: there exists an option value to waiting.

Consequently, tightness obtains a hump-shaped response with a muted reaction on impact, thereby making the adjustment of labor input more lagged as well - both in line with the empirical evidence (see Fujita and Ramey 2007, Figure 4).

To learn more about the model's internal propagation, it is also instructive to look at the IRFs to a purely transitory shock (that is: a shock to ψ). Then, all spillovers to future periods stem from the internal propagation of the model. Figure 3 shows these IRFs. As can be seen from the figure, employers in the full information case show no response to this shock at all: vacancy postings do not move. The reason is that next period (when a fraction of the vacancies posted will result in new jobs, at the earliest) things are as before again, and employers realize this on impact of the shock. With imperfect information on the other hand, vacancy postings do move, as firms then believe that there is a possibility that the shock is in fact going to persist, to which they want to guard themselves by posting additional vacancies. This sets off the dynamics for tightness and employment shown in the figure.

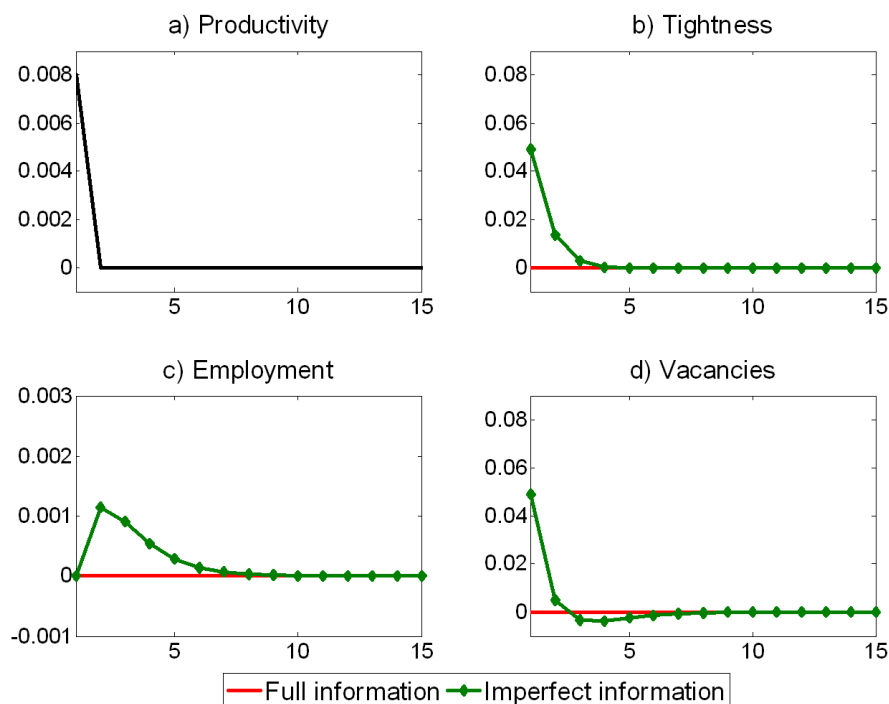


Figure 3: Impulse responses to a transitory productivity shock

Under the benchmark calibration, a one standard deviation transitory shock pushes employment away from its steady state value for about five quarters. So in this setting, purely transitory shocks (potentially even pure noise) can have relatively long-lasting effects on employment; in fact, a series of purely transitory shocks already generates an employment cycle.

To get a better idea on the impact of informational imperfections on the actual path followed by employment, Figure 4 displays part of a simulated series for \hat{n}_t under the assumption of both perfect and imperfect information, in response to the same path for productivity (displayed in the upper panel of the figure).

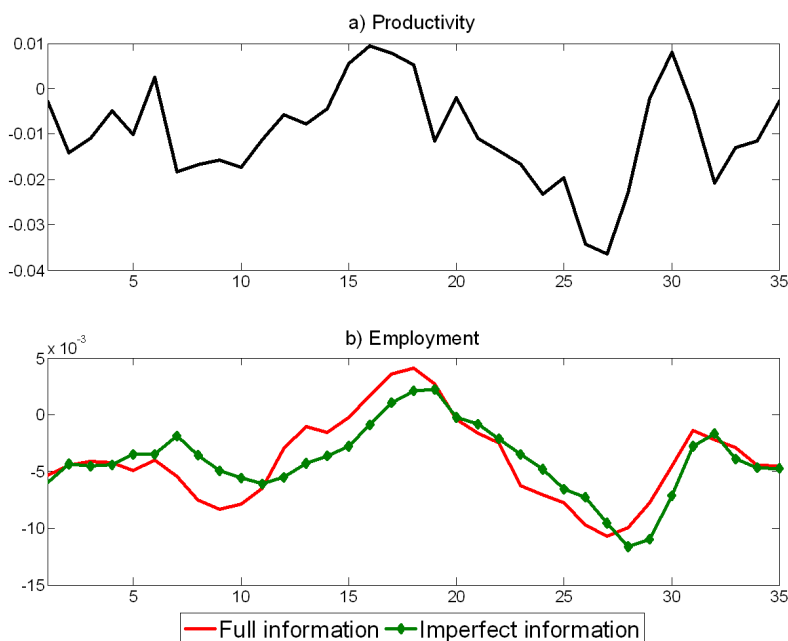


Figure 4: Typical employment paths under full and imperfect information

As can be seen from Figure 4, the model with imperfect information indeed makes labor adjustment more lagged: whereas the series for productivity and employment tend to reach their turning points around the same quarter under full information, this is no longer the case when information is imperfect (a phase shift has occurred). Consequently, the model with informational imperfections even succeeds in generating episodes of jobless growth.¹² This corresponds well with reality where the em-

¹²Look for example at the situation at the end of the sample, where productivity shows a strong increase as of quarter 27, whereas employment under imperfect information only shows a clear increase in the 30th quarter. Under full information, employment troughs in quarter 27 as well, just

ployment cycle also tends to lag the cycles for output and productivity - especially since the 1980s (*cf.* Willems and Van Wijnbergen 2009).

As can be read from Table 1, the introduction of informational imperfections also has its impact on unconditional, HP(1600)-filtered correlations. Although the correlation between unemployment and productivity undershoots its US data equivalent in absolute value compared to the full information solution, the other correlations do move in the desired direction. Most notably, the correlation between vacancies and productivity falls from 0.927 to 0.810, while the correlation between productivity and labor market tightness falls from 1.000 to 0.855. Although this is still above the corresponding value in US data (which equals 0.719 as reported by Hagedorn and Manovskii (forthcoming)), this is a substantial reduction. In particular, it is similar to the reduction that Hagedorn and Manovskii (forthcoming) achieve in a reduced form way via the introduction of a three month planning lag for vacancies.

	$corr(u, \phi)$	$corr(v, \phi)$	$corr(\theta, \phi)$
Full information	-0.654	0.927	1.000
Imperfect information	-0.522	0.810	0.855
Empirical	-0.633	0.719	0.703

Table 1: Correlations of key labor market variables with productivity

5.2 Amplification of shocks

What does the introduction of imperfect information imply for the amplification of shocks? That is: is the model still able to match the volatility of our key labor market variables?

For vacancies, we can actually give an analytical answer to this question, which is done through the following proposition.

Proposition 1. *Under imperfect information the volatility of vacancies is lower than in the full information case.*

Proof. The proof follows Nimark (2008). Recall that vacancies equal $\mathbf{g}'X_{t-1}$ under full information and $\mathbf{g}'X_{t-1|t-1}$ in the imperfect information case. Adding these two

as productivity (and output, which is not displayed in the figure), and there is no jobless growth. A similar situation occurs at the beginning of the sample: there, productivity shows a hesitant recovery as of quarter 7. However, as the persistence of this movement is not immediately clear to the agents in the model with imperfect information, it takes up to quarter 12 until employment increases. With full information, employment increases as of quarter 9 already.

quantities up enables us to start from the following identity:

$$\mathbf{g}'X_{t-1} + \mathbf{g}'X_{t-1|t-1} = \mathbf{g}'X_{t-1} + \mathbf{g}'X_{t-1|t-1}$$

Rearranging yields:

$$\mathbf{g}'X_{t-1} = \mathbf{g}'X_{t-1|t-1} + \mathbf{g}'(X_{t-1} - X_{t-1|t-1}),$$

This says that the full information solution is the sum of the solution under partial information and a linear function of the estimation error. Given the optimality of $X_{t-1|t-1}$, $(X_{t-1} - X_{t-1|t-1})$ is orthogonal to $X_{t-1|t-1}$. Consequently, the covariance between these two terms equals zero and we get that:

$$\begin{aligned} \mathbf{g}'\mathbb{E}[X_{t-1}X'_{t-1}]\mathbf{g} &= \mathbf{g}'\mathbb{E}[X_{t-1|t-1}X'_{t-1|t-1}]\mathbf{g} \\ &\quad + \mathbf{g}'\mathbb{E}\left[(X_{t-1} - X_{t-1|t-1})(X_{t-1} - X_{t-1|t-1})'\right]\mathbf{g} \end{aligned}$$

Since $\mathbf{g}'\mathbb{E}\left[(X_{t-1} - X_{t-1|t-1})(X_{t-1} - X_{t-1|t-1})'\right]\mathbf{g}$ is positive definite under imperfect information, it follows that $\mathbf{g}'\mathbb{E}[X_{t-1}X'_{t-1}]\mathbf{g} > \mathbf{g}'\mathbb{E}[X_{t-1|t-1}X'_{t-1|t-1}]\mathbf{g}$, *i.e.* the variance of vacancies in the presence of informational imperfections is lower than that in the full information case. ■

As can be seen from the proof, the reduction in the volatility of vacancies is positively related to the variance of the contemporaneous prediction error of the state (given by $\mathbb{E}\left[(X_{t-1} - X_{t-1|t-1})(X_{t-1} - X_{t-1|t-1})'\right]$). However, the proposition does not tell us anything about the magnitude of this effect for the standard calibration. To get an idea on the latter, I calculated the standard deviations of n , v and θ in a simulated series and compared them with the empirical values of these variables reported by Fujita and Ramey (2007).

	n	v	θ
Full information	0.010	0.154	0.258
Imperfect information	0.010	0.150	0.250
Empirical	0.008	0.131	0.298

Table 2: Standard deviations of key labor market variables

As one can see from Table 2, the introduction of informational imperfections indeed reduces the volatility of our labor market variables. But this effect turns out to be small. The reason is that informational imperfections introduce two opposing

forces that nearly cancel each other for my benchmark calibration: on the one hand, they mute the response to persistent shocks (as agents initially believe that there is a positive probability that they are only of a transitory nature), but on the other hand, they amplify the response to transitory ones by the symmetric argument.¹³ Table 2 shows that, on balance, the former effect slightly outweighs the latter, but that the quantitative impact is very small.

6 Sensitivity to different signal-to-noise ratios

As noted before, the signal-to-noise ratio is a key parameter in any model with informational imperfections of the kind discussed in this paper. But as it is not easy to estimate this parameter, there is considerable uncertainty about its true value. The only thing we probably know is that the value of this parameter is smaller than infinity (which corresponds to the full information case). Therefore, this section checks the robustness of the aforementioned results to different values for η .

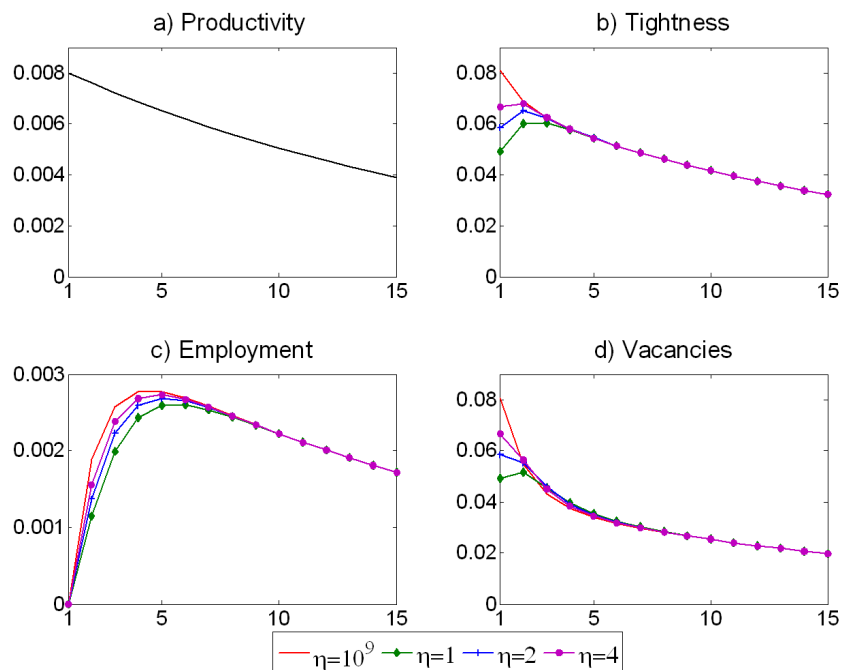


Figure 5: Impulse responses to a persistent productivity shock for higher values of η

¹³The IRFs displayed in Figures 2 and 3 illustrate this point.

Figure 5 shows the IRFs to a persistent productivity shock for our baseline calibration ($\eta = 1$), as well as those for cases in which the signal is more informative (*viz.* $\eta = 2$ and $\eta = 4$).¹⁴ All other parameters are as before and the case where $\eta = 10^9$ represents the full information solution.

Unsurprisingly, the internal propagation of the model is decreasing in the signal-to-noise ratio. More interestingly, the model's ability to produce a hump-shaped response in the tightness parameter survives up to values of η as high 4 (well above the available estimates on this parameter).

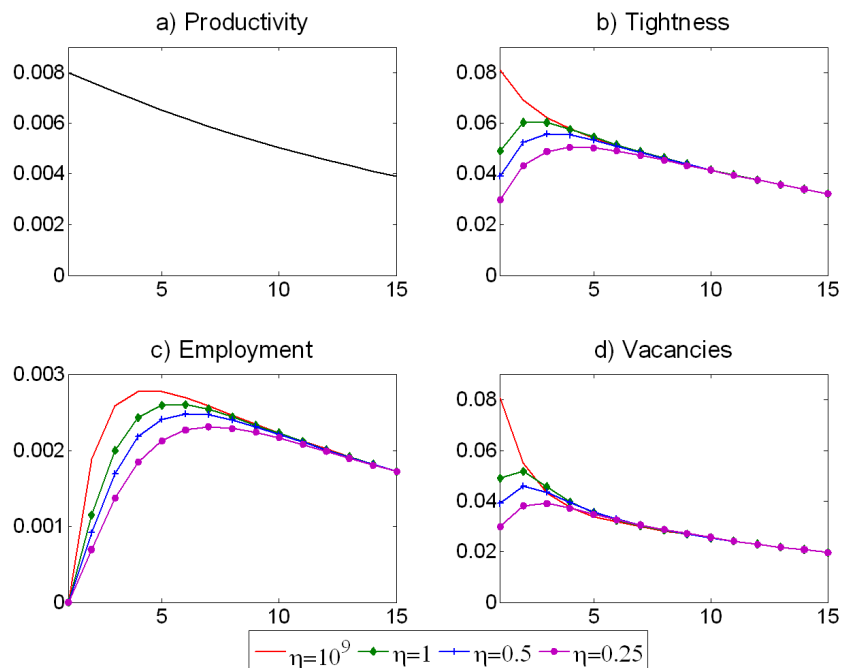


Figure 6: Impulse responses to a persistent productivity shock for lower values of η

There are however also good reasons to believe that η is in fact smaller than one, especially in particular circumstances. Mankiw *et al.* (1984) already reported a signal-to-noise ratio as low as 0.56, while all aforementioned estimates are essentially averages over both the booms and recessions that were included in the samples underlying the various studies. But as shown in Van Nieuwerburgh and Veldkamp (2006) and Bloom *et al.* (2010), uncertainty tends to evolve countercyclically. Consequently, η is likely to be smaller than 1 in and around recessions, when labor

¹⁴The increase in the signal-to-noise ratio is established by decreasing the variance of the transitory shock σ_2^2 ; to keep the analogy with the RBC-literature, σ_1^2 is kept at 0.008^2 .

adjustment is particularly important.¹⁵ Therefore, Figure 6 shows the responses of the system to a persistent productivity shock for $\eta = 0.5$ and $\eta = 0.25$ along with the full information solution and the benchmark case where η equals one.

Note that the model's ability to propagate shocks increases quite rapidly in η . Combining the observation that the propagation of shocks is increasing in the degree of uncertainty with the finding that uncertainty tends to be high at the end of recessions (*cf.* Bloom *et al.* 2010) yields an explanation for the fact that labor adjustment tends to occur so slowly in the early stages of recoveries (leading to cases of jobless growth that are even more severe than those displayed in Figure 4).

7 Conclusion and directions for future research

This paper studies the behavior of the standard labor market matching model under imperfect information. Agents in the model are imperfectly informed on the extent to which their economy has been hit by transitory and/or persistent shocks, as they are not separately identified on impact. Only by solving a signal extraction problem, agents are able to disentangle the two over time.

It is shown that the introduction of this persistent-transitory shock confusion improves the standard matching model's ability to propagate shocks: vacancies now show a persistent and hump-shaped response, as a result of which labor adjustment is much more gradual and recoveries are initially jobless (as observed in US data).

An important next step along this line of research could be the introduction of informational imperfections in a full dynamic stochastic general equilibrium matching model. At least since Cogley and Nason (1995) we know that the standard RBC-model lacks internal propagation as well. Andolfatto (1996) shows that augmenting the RBC-model with labor market matching frictions yields some propagation of shocks, but as argued in Den Haan *et al.* (2000), it still falls short of its empirical counterpart. Potentially, informational imperfections can improve the model's performance in this setting as well.

Moreover, enriching the imperfect information matching model with an intensive margin (hours and/or effort), which is not characterized by irreversibilities, may increase the amplification of shocks in the model. This relates to earlier work by Burnside *et al.* (1993), where firms must choose the size of the labor force before they can observe the state of the economy. After observing the realization of the shock, they can only vary work intensity as a result of which effort turns procyclical

¹⁵In addition, there is also evidence that η has gone down since the 1980s - see Willems and Van Wijnbergen (2009).

(which amplifies the shocks). As Burnside *et al.* do not model labor adjustment costs explicitly, their model is basically a reduced-form representation of a model in which it is infinitely costly to make within quarter adjustments on the extensive margin. Informational imperfections as in the present paper could have a similar effect (as they also make firms less eager to adjust the extensive margin on impact of a shock, due to the option value of waiting) and may hence contribute to the amplification of shocks in DSGE-models.

Finally, it would also be interesting to investigate how the model behaves when the signal-to-noise ratio fluctuates countercyclically over time, as it seems to do in reality. Quite possibly, this could replicate the business cycle asymmetries in labor adjustment as for example reported by Acemoglu and Scott (1994).

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