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Market Power in Water Markets*

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Abstract

We model water markets with market power as multi-market Cournot competition with a river structure. Suppliers are connected through the water balances, which imposes resource constraints, and are connected to heterogeneous water users via a water delivery infrastructure. Our model captures a wide range of specific water market structures. We establish conditions for existence of an equilibrium with market power. Each supplier only serves users with maximal marginal revenues. The monopoly case is similar to the standard one, but three other specific water market structures illustrate that standard intuition has to be revisited. Multiple equilibria may arise for common resources.

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1 Introduction

Next to transaction costs, market power has been identified as an important source of friction in water markets (Saleth, Braden, and Eheart 1991; Lovei and Whittington 1993; Rosegrant and Binswanger 1994; Easter, Rosegrant, and Dinar 1999; Bjornlund and McKay 2002; Jacoby, Murgai, and Rehman 2004; Draper 2008; Krause 2009). As Holland (2006) puts it: *"the owner* [of a water supply project] *might have an incentive to reduce deliveries from the project in order to increase the price of water in the destination market"*. Despite identifying the friction of market power in these references, there is a lack of economic water models suited to study this friction. We aim to provide such a model by extending the multi-market Cournot model to water markets with spatial differentiation that naturally implements the incentive to withhold water. In doing so, we develop a framework for estimating welfare losses due to market power.

We analyse market power in water markets in a setting where agents along a river can extract, sell, and purchase water, constrained only by the unidirectional river flow. Recently, Chakravorty, Hochman, Umetsu, and Zilberman (2009) analysed market power in the extraction, distribution, and end-use of water under different institutional settings. Their model imposes that water is generated at an upstream point source by a single supplier, and is then distributed along a channel to a continuum of identical end-users. In this paper, we take a more general perspective by explicitly modelling the extraction and distribution of water at various points along a river with spatial interdependencies and a given spatial delivery infrastructure, connecting suppliers with heterogeneous end-users or differentiated water markets. Specifically, we assume a setting with multiple suppliers and multiple water users along a river with access to heterogeneous end-users through an infrastructure consisting of direct links between individual suppliers and individual end-users. Given the unidirectional flow of river water and the infrastructure, suppliers are connected with each other through the water balance and are connected to (some of) the end users via the infrastructure. This set-up creates a system of multiple markets for water. The number of potential suppliers to each water market, i.e., end user, is determined by the delivery infrastructure, though strategic considerations on extraction and supply by the suppliers determine which water markets are served. Our model is general enough to cover a wide range of specific water market structures, with heterogeneous agents on both sides, who can make independent decisions on extraction, sale, and purchases of water.

The water market structure described in this paper relates closely to the literature on multi-market oligopolies (Dixit 1984; Bulow, Geanakoplos, and Klemperer 1985; Tirole 1988). For the case of two suppliers and two water users, our model—introduced in Section 2—reduces to a multi-market model. Depending on the delivery infrastructure, each supplier is a monopolist or competes in a duopoly on one or both markets and upstream's water extraction influences downstream's availability and, possibly, the cost of extraction. For any larger number of suppliers, say n and water users, say m, the n suppliers compete on some markets but not necessarily all m. Our model relates to the literature on Cournot-Walras competition (Gabszewicz and Vial 1972), in the sense that water extracting agents maximize their profits by choosing water extraction levels and markets where to sell it, taking into account the (indirect) price effects and extractions by their competing water suppliers. Our paper, however, provides additional structure to the Cournot-Walras model because of the sequential structure of the water resources (mimicking the direction of river flow), the infrastructure to serve markets, and money and water differentiated across m locations as $m+1$ goods, in combination with the quasi-linear utility functions of a user's own water consumption and money. This structure enables us to overcome several technical issues involved in applying the Cournot-Walras equilibrium concept.

There is a major difference, however, with most of the literature: suppliers are resource or capacity constrained. A supplier's constraint that is binding increases the mark-up, measured as the Lerner index with respect to the supplier's own marginal costs, on each potential market that this supplier might supply. The reason is that being physically forced to withhold water increases the market price on all potential output markets and simultaneously reduces the supplier's marginal costs. We prove existence of a market equilibrium with market power. Our model also allows to measure welfare losses due to market power and we illustrate this in a numerical implementation.

In the next section we introduce a somewhat simplified version of our model, for which we derive general results in Section 3. To demonstrate the richness of our model, and also in order to obtain sharper results, we provide further insights on specific water market structures in Section 4, including monopoly, the case of water utilities as local monopolies, duopoly in a river setting, and the setting of gravity-driven trade. Possible extensions to even more general river structures, conveyance losses, operating and maintenance costs and capacity constraints are discussed in Section 5. In Section 6 we provide some concluding remarks.

2 The model

We consider a spatially distributed water body from which water is extracted at several locations by water suppliers who sell their water to water users or water markets through some given infrastructure. The water body may be a river, an irrigation infrastructure, a lake or an aquifer. The infrastructure can be thought of as pipelines, irrigation canals, (national) water carriers or water deliveries by trucks. For ease of exposition, we think of the water body as the main flow of a river with a single supplier per location. Water market structures with a richer river structure, more than one supplier per location such as a common pool, or weaker spatial interdependence such as unconnected aquifers, are discussed in Section 5.

Figure 1 illustrates the model. The left-hand side of this figure represents the river as a directed line graph where location 1 is upstream from location 2, location 2 upstream of location 3 etc. up to location n ($n \geq 1$), similar as in e.g. Ambec and Sprumont (2002) and Ambec and Ehlers (2008). Water supplier i extracts y_i of water at location i and nowhere else. All extractions are stacked into the vector y, and all but i's extractions are stacked in y_{-i}. The right-hand side of Figure 1 illustrates how the infrastructure connects water suppliers

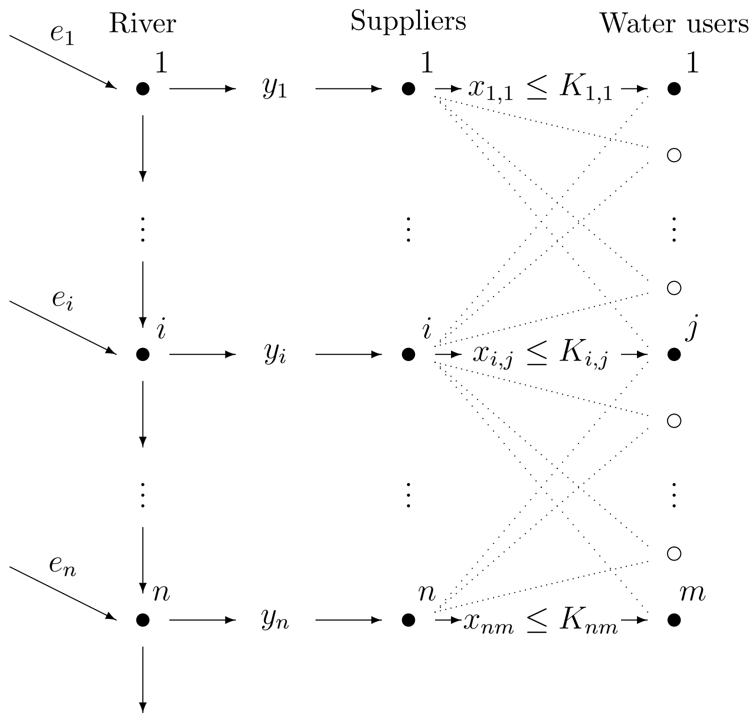

Figure 1: Suppliers along a 'river' extract and deliver water to water users.

and m water users ($m \geq 1$), who are not necessarily tight to locations. The infrastructure consists of bilateral links, denoted i, j, with a maximum capacity, denoted $K_{i,j} \geq 0$. In order to obtain sharper results, we restrict capacity $K_{i,j}$ to either 0 or ∞. Capacity $K_{i,j} = 0$ means that supplier i and water user j cannot trade. Similar, $K_{i,j} = \infty$ means that the trade between supplier i and water user j is unlimited by capacity. So, supplier i trades $0 \leq x_{i,j} \leq K_{i,j}$ of water with water user j. We put all $x_{i,j}$ in the $n \times m$ matrix X. The i-th row sum of X, denoted as x_i, consists of the total supply or deliveries by supplier i to water users, i.e., $x_i = x_{i,1} + \ldots + x_{i,m} = \sum_{j=1}^{m} x_{i,j}$. Similar, the j-th column sum of X, denoted as x_j, is the total water delivered to water user j, i.e., $x_j = x_{1,j} + \ldots + x_{n,j} = \sum_{i=1}^{n} x_{i,j}$.

Water balances are modelled as follows. The physical water resources at location i are equal to $e_i \geq 0$, and all resources are stacked into the vector e. These resources may include tributaries, river diversions, wells, groundwater aquifers, etc. Together with the inflow from upstream, water supplier i extracts y_i from the available water resources, and the remaining water at location i runs off to the adjacent downstream location. The water balance at location 1 dictates $y_1 \leq e_1$ with run-off $e_1 - y_1 \geq 0$. Consequently, the water balance at location 2 dictates $y_2 \leq e_2 + e_1 - y_1$, with run-off $e_1 + e_2 - y_1 - y_2 \geq 0$. Similarly, the water balance at location 3 dictates $y_3 \leq e_3 + e_1 + e_2 - y_1 - y_2$ etc. The key observation is that we can rewrite these water balances in matrix notation, which for $n = 3$ would imply

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \leq \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{1}$$

In general, water balances are modelled using the $n \times n$ matrix R whose elements are $R_{\hat{\imath},i} = 0$ or 1 (e.g. the 3×3 matrix in (1)), such that $R_{\hat{\imath},i} = 0$ for i's upstream location $\hat{\imath} < i$ and $R_{\hat{\imath},i} = 1$ for i's own or downstream location $\hat{\imath} \geq i$. Then, water balances are given by

$$R \cdot y \leq R \cdot e. \tag{2}$$

The i-th row of R is denoted R_i, and $R_i \cdot y \leq R_i \cdot e$ is location i's water balance.

Depending on the water rights regime, suppliers may not have the right or the possibility to extract all available water resources. Individual extraction rights (or legal entitlements) can be modelled in two ways. One way would be to define $\bar{y}_i$ as supplier i's right to extract an amount $y_i \leq \bar{y}_i$, and stack these rights in the vector $\bar{y}$. Then, adding $y \leq \bar{y}$ to the model takes into account feasible legal extractions. However, in case all rights are feasible in the water balances, we might reinterpret our model as follows, which is the second way to model rights. Given n suppliers, we modify R such that $R_{\hat{\imath},i} = 0$ for $\hat{\imath} \neq i$ and $R_{\hat{\imath},i} = 1$ for $\hat{\imath} = i$ because other suppliers are legally not allowed to extract others' unused water resources. Then, it is without loss of generality to set $e_i = \bar{y}_i$ and take R equal to the $n \times n$ identity matrix I, so that (1) is equivalent to $y \leq \bar{y}$. With this reinterpretation in mind, we forego adding $y \leq \bar{y}$ to our model.

Water user j has the quasi-linear utility function $u_j(x_j, m_j) = b_j(x_j) + m_j$, where for technical convenience b_j is a thrice continuously-differentiable benefit function and m_j is monetary wealth. We write $b_j'(x_j)$, $b_j''(x_j)$ and $b_j'''(x_j)$. We impose the following assumption, which is standard in consumer theory.

Assumption 1 *For each $j = 1, \ldots, n$, utility function $u_j(x_j, m_j)$ is strictly quasi-concave.*

This assumption has two implications. First, with price-taking behaviour, water demand satisfies the Law of Demand. This law states that the price of water and the quantity consumed are inversely related. Or, if water consumption goes up the marginal willingness to pay for water has to go down and vice versa. Second, the quasi-linear utility function is strictly quasi-concave if and only if $b_j(x_j)$ is a strictly concave function, which follows directly from Crouzeix and Lindberg (1986). So, the willingness-to-pay for a marginal increase in x_j, which is $b'_j(x_j)$, is decreasing in x_j because its derivative $b''_j(x_j)$ is negative. Obviously, this is in accordance with the Law of Demand. In addition, no other assumption on the relation between price and water demand was found in the water literature.

Supplier i's extraction costs depend upon his own extraction y_i and the extraction of the suppliers upstream of i (as these upstream extractions determine the amount of remaining water $\sum_{k=1}^{i-1}(e_k - y_k)$ available for extraction by i. Formally, the total costs of extracting $y_i = x_i$ are

$$c_i(y_i; y_{-i}) = c_i(x_{i,1} + \ldots + x_{i,m}; y_{-i}) = c_i\left(\sum\nolimits_{j=i}^{n} x_{i,j}; y_{-i}\right),$$

where $c_i(\cdot; y_{-i})$ is more convenient than $c_i(\cdot; y_1, \ldots, y_{i-1})$. The costs function $c_i(y_i; y_{-i})$ is increasing and differentiable in all $y_1, \ldots, y_i$ on the relevant range and convex in $y_1, \ldots, y_i$, and independent of $y_{i+1}, \ldots, y_n$.[1] Each water supplier maximizes its profit. We assume quantity competition among water suppliers. An equilibrium with market power is a social equilibrium of Debreu (1952), which is the appropriate extension of the Cournot-Nash equilibrium for water markets.

Our model is general enough to include a wide range of possible water market structures from the literature. We will examine these in detail in Section 4. In the next section we first assess how market power applies in a spatially distributed water body with water balances and market structure as described above.

[1] Our model does not allow for increasing returns to scale in water extraction and distribution, although this is a relevant feature in some water market settings.

3 Equilibrium with market power

By specifying bilateral deliveries from supplier i to water user j, we obtain a multi-market oligopoly with n suppliers that are geographically differentiated and m water users representing differentiated markets. Suppliers are resource constrained with externalities of extraction due to the river, and they face exogenous trade restrictions through the infrastructure. In this section, we first establish market-clearing prices, then we show existence of equilibria. Subsequently we introduce the suppliers' maximization problems and show how these can be solved.

3.1 Market-clearing prices

Multi-market quantity competition implies that each market faces its own uniform market price that clears the market, see e.g. Bulow et al. (1985). This makes each market's price dependent upon this market's supply. For consumers who buy positive amounts of the good, the market-clearing price is equivalent to their marginal willingness to pay. In this subsection, we translate these standard notions to our water market.

Recall that we treat water user j as being equivalent to a differentiated water market, and we denote this user's water price as p_j. A price-taking water user j chooses his optimal consumption $x_j \geq 0$ such that $\max_{x_j \geq 0} b_j(x_j) - p_j x_j$. It therefore must hold that the market-clearing price p_j at market $j = 1, \ldots, m$ that clears a total supply equal to x_j is given by

$$p_j = b'_j(x_j) = b'_j(x_{1,j} + \ldots + x_{n,j}), \tag{3}$$

Each price function is continuous in total supply. Note that the Law of Demand holds for each water user (or market). Equilibrium prices are differentiated per water user, depending on both the relative price elasticities of demand, and the feasibility of supplying each water user as constrained by the river structure and the delivery infrastructure.

Multi-market quantity competition can be regarded as a special case of the Cournot-Walras model in Gabszewicz and Vial (1972). In this model, producers decide on output before consumers purchase goods. Purchasing is modelled as a General Equilibrium model of an exchange economy with given output levels set by the producers in the first stage. The producers' sales prices are therefore derived from a General Equilibrium that describes the market behaviour by price taking consumers after the producers have set their production levels. In general, General Equilibrium prices form a multifunction or correspondence of production with little to none mathematical properties. This hampers application of the Cournot-Walras equilibrium. In case of differentiated water markets, there are in essence two goods per market: water and money. The water user at each differentiated market can be seen as the representative consumer with a concave utility function. By the equivalence of General Equilibria and welfare optima, as first established by (Negishi 1960), the General Equilibrium in each water market is described by a strictly-convex program for the representative water user at this location. Hence, the vector of General Equilibrium prices $(p_i, 1)$ at location i, where money is the numéraire, is a continuous function in all parameters, in particular the water supply to this location. Our model structure of one representative consumer per market resolves a major barrier for application of the Cournot-Walras model.

3.2 Conditions for existence of equilibria

Given the market-clearing prices as a continuous function of local supply, the extraction decisions by the suppliers can be determined. Continuity of the suppliers' profit functions, however, is not sufficient to guarantee existence of an equilibrium and in this subsection we state each suppliers' decision problem, and impose and discuss sufficient conditions for existence of equilibria.

Given the market-clearing prices in (3) expressed as a continuous function of local supply, the extraction decision by supplier $i = 1, \ldots, n$ is given by

$$\begin{array}{ll} \max_{y_i, x_{i,1}, \ldots, x_{i,m} \geq 0} & \sum_{j=1}^{m} b'_j \left(x_{1,j} + \ldots + x_{i,j} + \ldots + x_{n,j}\right) x_{i,j} - c_i \left(y_i, y_{-i}\right), \\ \text{s.t.} & R_i \cdot y \leq R_i \cdot e, \qquad (\beta_i) \\ & x_{i,1} + \ldots + x_{i,m} \leq y_i, \\ & x_{i,j} \leq K_{i,j}, \qquad (\gamma_{i,j}) \end{array} \tag{4}$$

where β_i denotes the shadow price on the available resources and $\gamma_{i,j}$ denotes the shadow price on capacity of the link i, j. The shadow price β_i indicates how much additional marginal profit supplier i can obtain if it could develop new water resources to increase e_i, or from receiving more water from upstream. The other shadow price, $\gamma_{i,j}$, indicates the marginal profit supplier i can obtain from a marginal expansion of the capacity constraint with water user j that would allow him to increase his deliveries to this water user. Since extraction is costly, the water supplier will not extract more water than he can sell, and so, the second constraint will be binding.

Existence of equilibria is complicated by the presence of extraction externalities and insufficient structure to the suppliers' profit functions. First, in contrast to quantity competition with (possibly) an exogenous capacity constraint, extraction y_i is confined to an interval whose upper bound $e_i + R_{i-1} \cdot (e - y)$ depends upon upstream extractions, where $R_{i-1} \cdot (e - y) = 0$ for $i = 1$. This means that the standard assumption underlying Nash's seminal existence theorem, which states that the outcome space is the Cartesian product of each player's strategy space, is not met.[2] This issue can be overcome by resorting to the weaker notion of a social equilibrium, as proposed by Debreu (1952), that allows for games defined on any convex polyhedron of the traditional Cartesian product. The sufficient conditions for existence of a social equilibrium also requires quasi-concavity of the player's utility functions, which is the second complicating issue. Application of the necessary and sufficient conditions for quasi-concavity in Crouzeix and Lindberg (1986) do not translate into simple conditions on the primitives of our model. The results in Hahn (1962) do not apply either, because these are derived for producers that produce a single output. To the best of our knowledge, there is no existence result for multi-product oligopoly models, except Laye and Laye (2008) for quadratic benefit functions and in the absence of the river structure.

In our model, supplier $i = 1, \ldots, n$ produces (possibly) multiple outputs. Given arbitrary convex cost functions, which include constant marginal costs, we impose as a sufficient condi-

[2]In a standard normal form, player $i = 1, \ldots, n$ has a strategy set S_i that defines the set of outcomes S as the Cartesian product $S_1 \times \ldots \times S_N$, see Mas-Colell, Whinston, and Green (1995).

tion the stronger strict concavity of each supplier's revenue function. Due to the summation of product forms $b'_j(x_{i,j}+r_j)\,x_{i,j}$, where $r_j = x_j - x_{i,j} \geq 0$ denotes the other deliveries to water user j, the Hessian of supplier i's revenue function with respect to $x_{i,1},\ldots,x_{i,m}$ is a diagonal matrix and, therefore, concavity of the revenue function is equivalent to concavity of each product form. The product form $b'_j(x_{i,j}+r_j)\,x_{i,j}$ is strictly concave in $x_{i,j}$ on the relevant domain when for all $x_{i,j}+r_j \in [0, R_i \cdot e]$:

$$b'''_j(x_{i,j}+r_j) < \frac{2\left|b''_j(x_{i,j}+r_j)\right|}{x_{i,j}}. \tag{5}$$

The latter condition on the model's primitive b_j seems relatively unrestricted for small $x_{i,j}$. It is always satisfied for quadratic benefit functions and $b_j(x_j) = \frac{1}{\alpha_j}(x_j)^{\alpha_j}$, $\alpha_j \in (0,1)$.[3]

The above discussion on existence of equilibria with market power is formalised in the following result. All proofs are deferred to the Appendix.

Proposition 2 *Under* (5) *for all* $i = 1,\ldots,n$ *and* $j = 1,\ldots,m$ *there exists an equilibrium with market power.*

To summarise, we impose strict concavity on each product form $b'_j(x_{i,j}+r_j)\,x_{i,j}$ and this sufficient condition allows relative straightforward verification in terms of second and third derivatives of each benefit function. In the absence of extraction externalities on costs and available resources, quadratic profit functions are sufficient for uniqueness of the equilibrium, see Laye and Laye (2008). In Section 5.1, however, we report multiplicity of equilibria in a duopoly with extraction from a common pool. For this reason we forego investigating conditions for uniqueness. Nevertheless, the strict concavity of profit functions does allow a first-order approach in characterizing equilibria, to which we turn next.

[3]Differentiation with respect to x_j, and making use of $x_{i,j} \leq x_{i,j}+r_j = x_j$, yields

$$b'''_j(x_j) = (\alpha_j - 1)(\alpha_j - 2)\,x_j^{\alpha_j - 3} = \frac{(2-\alpha_j)(1-\alpha_j)\,x_j^{\alpha_j-2}}{x_j} < \frac{2\,|\alpha_j - 1|\,x_j^{\alpha_j-2}}{x_{i,j}} = \frac{2\left|b''_j(x_{i,j}+r_j)\right|}{x_{i,j}}.$$

3.3 Profit-maximizing supply

The typical approach to solve for equilibria is to derive intersection points of the profit-maximizing supply (or best-response) functions. For a given supplier i, the derivation of these functions is rather involved. Instead, we characterise optimal supply by supplier i to water user j based on the difference in water price, marginal willingness to pay and marginal extraction costs.

Proposition 3 *For $K_{i,j} = \infty$, supplier i's optimal supply to water user j is given by*

$$\begin{cases} x_{i,j} = 0, & \textit{if either } b'_j(r_j) \leq c'_i(\sum_{\hat{\jmath} \neq j} x_{i,\hat{\jmath}}; y_{-i}), \\ & \textit{or } b'_j(r_j) > c'_i(\sum_{\hat{\jmath} \neq j} x_{i,\hat{\jmath}}; y_{-i}) \textit{ and } \sum_{\hat{\jmath} \neq j} x_{i,\hat{\jmath}} + R_{i-1} \cdot y = R_i \cdot e, \\ x_{i,j} > 0, & \textit{if } b'_j(r_j) > c'_i(\sum_{\hat{\jmath} \neq j} x_{i,\hat{\jmath}}; y_{-i}) \textit{ and } \sum_{\hat{\jmath} \neq j} x_{i,\hat{\jmath}} + R_{i-1} \cdot y < R_i \cdot e. \end{cases}$$

Moreover, $x_{i,j} > 0$ implies $b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j} \geq c'_i(x_i; y_{-i})$ independent whether optimal extraction is constrained, but equality holds when it is unconstrained. For $K_{i,j} = 0$, the capacity constraint $x_{i,j} \leq K_{i,j}$ restricts profits if and only if $b'_j(r_j) > c'_i(\sum_{\hat{\jmath} \neq j} x_{i,\hat{\jmath}}; y_{-i})$.

Proposition 3 demonstrates that supplier i never supplies water to a user whose marginal willingness to pay is less than this supplier's marginal costs. Otherwise, profitable supply will in principle occur, unless supplier i is constrained and considers trading with other water users as more profitable. An interesting feature that appears only indirectly in Proposition 3 is that upstream extraction affects supplier i in three different ways: (i) by the physical availability of water $R_i \cdot e - R_{i-1} \cdot y$; (ii) by the effect on extraction costs $c'_i(x_i; y_{-i})$; and (iii) by the effect on marginal willingness to pay $b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j}$ through upstream suppliers' deliveries. We will take up this issue in detail in Section 5 for a duopoly case.

Using Proposition 3, the level of market power of supplier i on water user j can be expressed using the Lerner index:[4]

$$\frac{p_j - c_i'}{p_j} \equiv \frac{b_j' - c_i'}{b_j'} \geq \left[-\frac{\partial b_j'}{\partial x_j} \cdot \frac{x_j}{b_j'} \right] \cdot \frac{x_{i,j}}{x_j} > 0,$$

where the term between square brackets is the inverse of the price elasticity of demand and the second term is i's market share on market j. A Lerner Index equal to the lower bound can only be attained in case supplier i is unconstrained in his supply to water user j, and otherwise there will be a gap. In other words, the Lerner index is higher for boundary solutions than for interior solutions. To see this, consider an incremental increase of the resource, which is of course impossible when the supplier is at his maximal extraction. Such increase, if it could somehow be realised, would simultaneously decreases the price and (weakly) increase marginal costs due to the convexity of the cost function. Therefore, such incremental increase in supply would decrease the Lerner index.

Interior equilibria provide more flexibility to gain rents from market power by either increasing or decreasing supply. Suppliers with maximal extraction can only consider under-supply in order to achieve higher prices, but would rather prefer an increase of total extraction. This implies that these suppliers exercise of market power is restricted to redistributing their supplies over water users. Hence, full extraction with market power implies full extraction in the competitive equilibrium, but deliveries to water users may differ.

Further insights on optimal supply are obtained by comparing i's supply to two water users, j and $\hat{\jmath}$. The intuition of the following result generalises naturally to larger cases.

Proposition 4 *Let $K_{i,j} = K_{i,\hat{\jmath}} = \infty$. Whenever the optimal supply $x_{i,j}$ and $x_{i,\hat{\jmath}}$ by supplier i to both water users j and $\hat{\jmath}$ is positive, it also holds that*

$$b_j'(x_{i,j} + r_j) + b_j''(x_{i,j} + r_j)\, x_{i,j} = b_{\hat{\jmath}}'(x_{i,\hat{\jmath}} + r_{\hat{\jmath}}) + b_{\hat{\jmath}}''(x_{i,\hat{\jmath}} + r_{\hat{\jmath}})\, x_{i,\hat{\jmath}} \geq c_i'(x_i, y_{-i}),$$

[4]Since $p_j = b_j'(x_j)$ and $\frac{\partial b_j'(x_j)}{\partial x_j} = \frac{\partial b_j'(x_j)}{\partial x_{i,j}}$, it follows from Proposition (3) that

$$p_j - c_i' \geq -\frac{\partial b_j'(x_j)}{\partial x_j} \cdot x_{i,j} \Longrightarrow \frac{p_j - c_i'}{p_j} \geq -\frac{\partial b_j'(x_j)}{\partial x_j} \cdot \frac{x_j}{p_j} \cdot \frac{x_{i,j}}{x_j}.$$

with equality when optimal extraction is unconstrained. In case the optimal supply by supplier i is such that $x_{i,j} > 0$ and $x_{i,\hat{\jmath}} = 0$, it holds that $b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j} \geq b'_{\hat{\jmath}}(r_{\hat{\jmath}})$.

Proposition 4 demonstrates that supplier i maximizes his profits by serving only those water users with the highest marginal willingness to pay among the water users he can supply. His supply is such that the marginal willingness to pay is equated among water users supplied. This result is independent of whether the supplier is resource constrained or not. From the proofs of Proposition 3 and 4 it follows that shadow price β_i is equal to the largest difference between marginal revenues and marginal costs of water users supplied.

Remark 5 *An interior equilibrium presupposes that for all bilateral links with $K_{i,j} = \infty$ we have $x_{i,j} > 0$. This equilibrium-feature is supported by Bulow et al. (1985) who show that in multi-market oligopolies, firms' profits may be sub-optimal because they cannot commit to serve only one or a few markets. The same lack of commitment obstructs the suppliers' profits in our setting. This effect may therefore mitigate the possible rents from market power to some extent. Obviously limited capacity of infrastructure imposes partial commitment, because water cannot be supplied freely in case $K_{i,j} = 0$ (or any small positive $K_{i,j}$ as in Section 5.1). Note that limited water resources also provide partial commitment in the sense that a supplier is committed to deliveries at his maximal extraction, but then still the limited possibility to strategically substitute between water markets remains, as illustrated in Section 4.4.*

4 Specific cases

Our model is general enough to include a wide range of possible water market structures from the literature. In this section, we demonstrate how four of such specific structures can be implemented. Also, we present sharper results for these structures.

4.1 River setting: monopoly

Consider a water distribution infrastructure with a single upstream supplier and many downstream water users as in Chakravorty et al. (2009). This water market structure is modelled by taking $n = 1$ and $K_{1,j} = \infty$ for all water users j. Conveyance losses, as in Chakravorty et al. (2009), can be included as discussed in Section 5.2, but are left out here for simplicity. The monopoly case is a multi-product monopoly with independent demands and dependent costs as in e.g. Tirole (1988), but novel is the resource constraint. After substitution of y_i and dropping both $r_j = 0$ and y_{-1} from our notation, we obtain the following result that is stated without a formal proof.

Proposition 6 *For the monopoly case $n = 1$, the equilibrium with market power is the unique solution to*

$$\begin{array}{ll} \max_{x_{1,1},\ldots,x_{1,m}\geq 0} & \sum_{j=1}^{m} b_j'(x_{1,j})\, x_{1,j} - c_1(x_{1,1} + \ldots + x_{1,m}), \\ s.t. & x_{1,1} + \ldots + x_{1,m} \leq e_1. \qquad (\beta_i) \end{array}$$

In essence, the monopolist solves a strictly convex program, for which it is known that a unique profit maximum exists. Moreover, the unique monopoly is continuous in the exogenous parameters, such as the resource e_1, and parameters of the benefit functions and the cost function. Propositions 3 and 4 then imply that the monopolist serves only those water users with the highest marginal willingness to pay among the water users he can supply, and his supply is such that the marginal willingness to pay is equated among water users served. The monopolist's supply is less than or equal to the competitive equilibrium extraction independent whether the resource constraint is binding. In case the equilibrium extraction is interior, i.e., $0 < y_1 < e_1$, then the monopolists always under-supplies the water users.

We conclude this subsection with an example that will return as the illustrating example throughout this section. The technical details are deferred to the Appendix. Consider a monopolist with resources e_1 and cost function $c_1(y_1) = -c_1 \ln(e_1 - y_1)$, where $c_1 > 0$. So, the extraction costs go to infinity as y_1 goes to e_1, and hence it is without loss to assume $y_1 < e_1$. For explanatory reasons, the monopolist serves a single market with benefit function

$b_1(x_1) = x_1(2 - x_1)$ so that the market's satiation point is 1.[5] According to Proposition 6, the monopoly supply (or extraction) solves

$$x_1 = \arg\max_{x_1 \in [0,e_1)} b_1'(x_1)x_1 + c_1 \ln(e_1 - x_1) = \arg\max_{x_1 \in [0,e_1)} 2x_1(1 - x_1) + c_1 \ln(e_1 - x_1),$$

from which we obtain the optimal monopoly supply

$$x_1 = \begin{cases} \frac{1}{2}\left(\frac{1}{2} + e_1 - \sqrt{\left(e_1 - \frac{1}{2}\right)^2 + c_1}\right) < e_1, & \text{if } c_1 < 2e_1, \\ 0, & \text{if } c_1 \geq 2e_1. \end{cases} \tag{6}$$

The water user is charged his monopoly water price $p_1(x_1) = b_1'(x_1) = \frac{3}{2} - e_1 + \sqrt{\left(e_1 - \frac{1}{2}\right)^2 + c_1}$.

In order to study under-development of resources, we also derive the competitive equilibrium. The associated supply (or extraction) is Pareto efficient, and therefore solves

$$x_1^* = \arg\max_{x_1 \in [0,e_1)} b_1(x_1) + c_1 \ln(e_1 - x_1) = \arg\max_{x_1 \in [0,e_1)} x_1(2 - x_1) + c_1 \ln(e_1 - x_1).$$

Its solution is

$$x_1^* = \begin{cases} \frac{1}{2}\left(1 + e_1 - \sqrt{(e_1 - 1)^2 + 2c_1}\right) < e_1, & \text{if } c_1 < 2e_1, \\ 0, & \text{if } c_1 \geq 2e_1. \end{cases} \tag{7}$$

Comparing (6) and (7), we observe for $c_1 < 2e_1$ that we have $x_1^* > x_1$, i.e., the resource is under-developed when compared to the competitive equilibrium extraction.

For later reference, we also investigate the boundary case of constant marginal costs, i.e., $c_1 = 0$. Substitution of $c_1 = 0$ into (7) yields the market's satiation point $x_1^* = 1$ due to the zero extraction costs, and in case $e_1 < 1$, maximal extraction is Pareto efficient with the market clearing price $b_1'(e_1) > 0$. Substitution of $c_1 = 0$ yields the monopoly supply $x_1 = \frac{1}{2}$, which is interior whenever $e_1 > \frac{1}{2}$. So, for $e_1 \leq \frac{1}{2}$, the equilibrium supplies in both the monopoly and competitive case are e_1, for $\frac{1}{2} < e_1 \leq 1$ the monopoly supply of $\frac{1}{2}$ is less than the competitive equilibrium supply of e_1, and for $e_1 > 1$ the monopoly supply of $\frac{1}{2}$ is half the competitive equilibrium supply of 1.

[5] This example can be modified to allow for m identical water users and resources $m \cdot e_1$ instead of e_1.

4.2 River setting: water utilities as local monopolies

Consider a water distribution infrastructure with several suppliers that each are exclusive suppliers for one or more water users. This infrastructure captures the setting of several cities that each have a municipal utility that provides its local inhabitants with water. Water that is not used by upstream utilities runs off to downstream cities according to the water balance. Such water market structure is modelled by partitioning the water users over water suppliers such that supplier i is connected to $m_i \geq 1$ users, being any j between $\underline{m}_i = m_1 + \ldots + m_{-i} + 1$ and $\overline{m}_i = m_1 + \ldots + m_{-i} + m_i$. In addition, exclusive supply requires that $K_{i,j} = \infty$ for all j between $m_1 + \ldots + m_{-i} + 1$ and $m_1 + \ldots + m_{-i} + m_i$, while $K_{i,j} = 0$ for all other j.

There are similarities to the monopoly setting of Section 4.1, but a crucial difference is that i's extraction and supply decisions may be affected by upstream extraction through resource availability and the cost of extraction, though never do these affect the water prices and demand for supplier i. Hence, this water market structure can be solved recursively, starting upstream with supplier 1. Obviously, supplier 1's decision is characterized by Proposition 6. Given the monopolist's optimal extraction y_1, we proceed to supplier 2. His water balance dictates that $y_2 \leq e_2 + e_1 - y_1$ and again, this supplier's extraction decision is characterized by Proposition 6, and so on. This implies the following result, which we state without a formal proof.

Proposition 7 *For the local-monopoly case, the equilibrium with market power is the unique recursive solution to: For $i = 1, \ldots, n$, solve*

$$\begin{array}{ll} \max_{x_{i,\underline{m}_i},\ldots,x_{i,\overline{m}_i}\geq 0} & \sum_{j=\underline{m}_i}^{\overline{m}_i} b'_j\left(x_{i,j}\right) x_{i,j} - c_i\left(x_{i,\underline{m}_i} + \ldots + x_{i,\overline{m}_i}, y_{-i}\right), \\ s.t. & x_{i,\underline{m}_i} + \ldots + x_{1,\overline{m}_i} \leq e_i + R_{i-1}\cdot(e-y), \qquad (\beta_i) \end{array}$$

set $y_i = x_{i,\underline{m}_i} + \ldots + x_{1,\overline{m}_i}$ and compute $R_i \cdot (e-y)$. If $i < n$, go to $i+1$, otherwise stop.

The key insight is that every water user is charged the monopoly price set by its exclusive supplier given the inflow from upstream. The solution is unique, because it is the solution to a finite sequence of strictly convex programs. Moreover, supplier i's unique monopoly supply is continuous in the exogenous parameters, such as the resource e_1 and upstream extractions y_1, y_2 up to y_{i-1}. Propositions 3 and 4 then imply that supplier i serves only those water users with the highest marginal willingness to pay among the water users he can supply, and his supply is such that the marginal willingness to pay is equated among water users served. Supplier 1's extraction is always less than the competitive equilibrium extraction in case the equilibrium extraction is interior, i.e., $0 < y_1 < e_1$. However, for supplier 2 this is less trivial, as illustrated in the following.

We conclude this subsection by extending the model of the example in Section 4.1. The purpose is to consider a river with $n = 2$ local monopolies who each serve a single water user, that is $\underline{m}_1 = \overline{m}_1 = 1$, $\underline{m}_2 = \overline{m}_2 = 2$ and supplier $i = 1, 2$ exclusively serves water user $j = i$. As in the example in Section 4.1, upstream (local) monopolist 1 has resources e_1 and cost function $c_1(y_1) = -c_1 \ln(e_1 - y_1)$, where $c_1 > 0$. Similar, downstream monopolist 2 has resources $e_1 + e_2 - y_1$ and cost function $c_2(y_2, y_1) = -c_2 \ln(e_1 + e_2 - y_1 - y_2)$, where $c_2 > 0$. Water user j has a benefit function $b_j(x_j) = x_j(2 - x_j)$, with satiation point equal to 1. Since by definition $x_1 = x_{1,1}$ and $x_2 = x_{2,2}$, we perform the analysis in x_1 and x_2. Application of Proposition 7 means we first solve monopolist 1's optimal supply, which yields (6) and water availability $e_1 + e_2 - x_1$. Then, we solve

$$x_2 = \arg \max_{x_2 \in [0, e_1 + e_2 - x_1)} 2x_2(1 - x_2) + c_2 \ln(e_1 + e_2 - x_1 - x_2),$$

which is straightforward if we substitute $e_1 + e_2 - x_1$ for e_1 and c_2 for c_1 in (6). So, monopolist 2's optimal supply is given by

$$x_2 = \begin{cases} \frac{1}{2}\left(\frac{1}{2} + e_1 + e_2 - x_1 - \sqrt{\left(e_1 + e_2 - x_1 - \frac{1}{2}\right)^2 + c_2}\right), & \text{if } c_2 < 2(e_1 + e_2 - x_1), \\ 0, & \text{if } c_2 \geq 2(e_1 + e_2 - x_1). \end{cases} \tag{8}$$

In the presence of externalities, the competitive equilibrium is Pareto inefficient because the upstream market does not take into account its negative effects for the downstream market. We exploit this fact by sequentially solving the competitive equilibrium as well, starting with the upstream market. Repeating the arguments of the example in Section 4.1, we arrive at the same x_1^* as in (7), and performing the same substitution as for the monopolist, we arrive at the competitive equilibrium supply

$$x_2^* = \begin{cases} \frac{1}{2}\left(1 + e_1 + e_2 - x_1^* - \sqrt{\left(e_1 + e_2 - x_1^* - 1\right)^2 + 2c_2}\right), & \text{if } c_2 < 2\left(e_1 + e_2 - x_1^*\right), \\ 0, & \text{if } c_2 \geq 2\left(e_1 + e_2 - x_1^*\right). \end{cases} \tag{9}$$

Comparing (8) and (9) becomes ambiguous because of the following reasons. For identical resource availability, and hence identical cost functions, the monopolist under-develops the resource compared to the competitive equilibrium. However, by $x_1^* > x_1$ we have that monopolist 2's cost function lies below the cost function in the competitive equilibrium, i.e., $-c_2 \ln\left(e_1 + e_2 - x_1 - x_2\right) < -c_2 \ln\left(e_1 + e_2 - x_1^* - x_2\right)$. A monopolist with the low cost function would supply a larger amount than a monopolist with the high cost function, and this increased monopoly supply is opposite to the earlier standard contraction of supply in a monopoly. Of course, the ambiguity cannot arise in the boundary case of constant marginal costs, i.e., $c_2 = 0$, and this insight generalizes to all cost functions $c_2\left(y_2, y_1\right)$ that are independent of resource availability, i.e., independent of y_1.

4.3 River setting: duopoly and extraction costs

The monopoly and local monopoly cases of Sections 4.1 and 4.2 are special cases in which there is no strategic interaction between the water suppliers. In this subsection, we consider such interaction by considering the duopoly case. As mentioned just after Proposition 3, extractions matter in both the physical resource availability and the cost functions. In this subsection, we investigate each in isolation by first assuming constant marginal cost of extraction, and second, by assuming the costs of fully depleting the resource are prohibitively large.

We will do so by extending the example of Section 4.1 to the situation where two suppliers supply one water user, with constant marginal extraction costs.[6] This captures situations in which depleting the entire resource would involve relatively low extraction costs, which we

[6]In the Appendix, we show that positive constant marginal costs yield qualitatively similar results.

set equal to zero for expository simplicity. So, we take $n = 2$ and $m = 1$, with supplier $i = 1$ upstream of supplier 2, and $K_{1,1} = K_{2,1} = \infty$. Specifically, the water body is a river with $e_1 + e_2 \in (0, 1)$, extraction costs $c_1(y_1) = c_2(y_2; y_1) = 0$, and one water user with benefit function $b_1(x_1) = x_1(2 - x_1)$. As in the monopoly example, the competitive equilibrium water demand $x_1^* = 1 > e_1 + e_2$ is not feasible and location 1's marginal willingness to pay for an incremental increase of x_1 is equal to $b_1'(x_1)$, i.e., the market clearing price $p_1(x_1)$. Because each supplier only delivers to a single water user, we substitute y_i for $x_{i,1}$ and perform our analysis in y_1 and y_2.

Solving for equilibria with market power requires to derive the best-response functions and determine the intersection points. From solving each supplier's profit-maximization problem we obtain the best-response functions in terms of extraction:

$$R_1(y_2) = \min\left\{\tfrac{1}{2} - \tfrac{1}{2}y_2, e_1\right\} \qquad \text{and} \qquad R_2(y_1) = \min\left\{\tfrac{1}{2} - \tfrac{1}{2}y_1, e_1 + e_2 - y_1\right\},$$

where we defer details to the Appendix. The best-response function for supplier 1 is similar to the best-response function in the modified Cournot model with exogenous capacity constraints. However, supplier 2's capacity is endogenous, and therefore his best-response function is different from the earlier mentioned modified Cournot model. The expressions for the best-response functions imply that we need to distinguish four distinct cases, labelled A to D. Solving all these cases yields the following equilibrium values

$$\begin{cases} y_1 = e_1, & y_2 = e_2, & \text{if } 2e_1 + e_2 < 1 & \text{and } e_1 + 2e_2 < 1, & \text{(A)} \\ y_1 = e_1, & y_2 = \frac{1}{2} - \frac{1}{2}e_1, & \text{if } e_1 < \frac{1}{3} & \text{and } e_1 + 2e_2 \geq 1, & \text{(B)} \\ y_1 = 1 - e_1 - e_2, & y_2 = 2e_1 + 2e_2 - 1, & \text{if } 2e_1 + e_2 \geq 1 & \text{and } e_1 + e_2 < \frac{2}{3}, & \text{(C)} \\ y_1 = \frac{1}{3}, & y_2 = \frac{1}{3}, & \text{if } e_1 \geq \frac{1}{3} & \text{and } e_1 + e_2 \geq \frac{2}{3}. & \text{(D)} \end{cases}$$

Figure 2 illustrates these cases in the (e_1, e_2)-space. Maximal aggregate extraction equals the resources available in cases A and C. In these two cases, the equilibrium with market power coincides with the competitive equilibrium. In case D, however, we have the standard unconstrained Cournot duopoly outcome with its classic under-supply. For the remaining case B, we have that the maximal extraction $\frac{1}{2} + \frac{1}{2}e_1$ is strictly less than $e_1 + e_2$ whenever $e_1 + 2e_2 > 1$. Because also $y_1 + y_2 = \frac{1}{2} + \frac{1}{2}e_1 < \frac{2}{3}$ this implies that the aggregate supply in

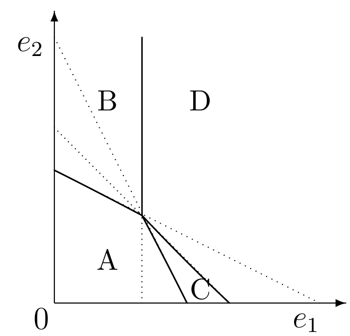

Figure 2: The solid lines partition the (e_1, e_2)-space into the four areas where each case holds.

case B is less than in the competitive equilibrium. We compare the duopoly outcome with the monopoly of Section 4.1 for the special case $e_1 > 0$ and $e_2 = 0$. Recall that the monopoly is inefficient for $e_1 > \frac{1}{2}$, whereas the duopoly supplies the entire resource e_1 whenever $e_1 \leq \frac{2}{3}$. So, the duopoly is less harmful for the water user, even in case supplier 2 does not have any resource on his own.

In cases A and B, the available resources are so scarce that each individual supplier maximally extracts his own water resource. In case C, however, whenever $2e_1 + e_2 > 1$ supplier 1 extracts less than his available resource e_1, and this supplier's unused water will be extracted by supplier 2 who exhausts his available resource, i.e., $y_1 < e_1$ and $y_2 = e_1 + e_2 - y_1 > e_2$.

As a second case, we investigate the case in which depleting the entire resource would involve relatively high extraction costs. For expository reasons, we consider the same non-linear extraction costs $c_1(y_1) = -c_1 \ln(e_1 - y_1)$ and $c_2(y_2, y_1) = -c_2 \ln(e_1 + e_2 - y_1 - y_2)$ as in Section 4.2. These cost functions feature that less inflow from upstream shifts downstream's costs of extraction upward, as mentioned under (ii) following Proposition 3. The system of first-order conditions from which the equilibrium with market power must be computed reduces to a polynomial of degree four in y_1. Such polynomial does not allow a closed-form solution. Therefore, one must resort to numerical methods to solve for equilibria. In principle, there can be four roots. A unique equilibrium would imply that all except one will be infeasible. As a numerical example, we take $e_1 = e_2 = 1$ and consider $c_1 = c_2 = \frac{1}{4}$

Values for c_1, c_2	The equilibrium (y_1, y_2)
$c_1 = \frac{1}{4}, c_2 = \frac{1}{4}$	$(0.251, 0.330)$
$c_1 = \frac{1}{4}, c_2 = \frac{1}{2}$	$(0.276, 0.276)$

Table 1: Unique equilibria with market power for two combinations of c_1 and c_2.

as well as $c_1 = \frac{1}{4}$ and $c_2 = \frac{1}{2}$. The numerical method is applied to the system of first-order conditions and is implemented in MATLAB. We obtained all four numerical solutions for (y_1, y_2) for each choice of the parameters, but in both cases three were infeasible. Therefore, there is a unique equilibrium with market power that is reported in Table 1. From this table, we observe that when supplier 2's cost parameter increases from $\frac{1}{4}$ to $\frac{1}{2}$, the suppliers become symmetric. By increasing c_2, the marginal costs for supplier 2 go up and he extracts less, which is standard economic intuition. By extracting less, supplier 1 increases his production and thereby decreases the flow $e_1 + e_2 - y_1$ to downstream and this increases supplier 2's cost function, which is a novel effect. By strategic substitution of the classic Cournot duopoly, supplier 2 reacts by reducing his extraction once more, to which supplier 1 reacts by further increasing his production and reducing 2's inflow further etc. until the new equilibrium is reached.

4.4 Private resources and gravity-driven water trade

In this subsection, we investigate suppliers that are unconnected according to the river water balances so that under-development of water resources does not cause run-off to downstream suppliers. This setting is also relevant whenever suppliers pump groundwater from aquifers or extract from other human-controlled reservoirs. Recall from Section 2 that our model can be easily accommodated for water-rights regimes. This is appropriate for water market structures where suppliers can supply water users at other locations, but are hampered by capacity or (physical) trade restrictions, as is the case in e.g. US and Australian water markets (Weber 2001; Bjornlund 2004; Brennan 2006; Chong and Sunding 2006). In many such markets, a gravity-driven infrastructure facilitates trade from upstream to downstream, but excludes opposite water flows.

In all these situations, either suppliers are unconnected or each supplier has his private water resource e_i. By Section 2, water balances are adjusted such that the matrix R—see (1)—is the identity matrix of size n. Although we could allow for an arbitrary number of water users, we equate the number of users m to the number of suppliers with the interpretation that there is one supplier and one user per location. Gravity-driven infrastructures dictate trade restrictions $K_{i,j} = \infty$ for all $i \leq j$ and and all other $K_{i,j} = 0$.

Further, we will relate this case to Laye and Laye (2008), who provide an existence and uniqueness result for multi-product oligopoly models as discussed in Section 3.2. They also provide a numerical implementation in which the equilibrium can be computed by means of a single convex program. However, these results require a specification with quadratic benefit functions. To apply their results, we assume that the quadratic benefit function for user i is given by $b_j(x_j) = \frac{1}{2}a_j x_j (2b_j - x_j)$, where $a_j, b_j > 0$, and the cost function $c_i(y_i; y_{-i}) = cy_i$, where $0 \leq c < a_j b_j$ for all j. We implement their numerical method in order to analyse the size of welfare losses due to market power. Welfare consists of the consumer surplus (CS), i.e., $u_j(x_j)$, and profits.

In this specific setting, the equilibrium with market power can be obtained from Proposition 2 in Laye and Laye (2008). The equilibrium solves:

$$\begin{aligned} &\max_x \sum\nolimits_{i=1}^{n} \sum\nolimits_{j=i}^{n} \tfrac{1}{2}a_j \left(\left[2b_j - x_j(-i)\right] x_{i,j} - x_{i,j}^2 \right) - \sum\nolimits_{i=1}^{n} \left(c \sum\nolimits_{k=i}^{n} x_{ik} \right) \\ &\text{s.t. } \sum\nolimits_{j=i}^{n} x_{i,j} \leq e_i, \qquad x_j(-i) = \sum\nolimits_{k=1,k\neq i}^{j} x_{kj}. \end{aligned} \tag{10}$$

For reasons of comparison, the competitive equilibrium solves:

$$\begin{aligned} &\max_x \sum\nolimits_{j=1}^{n} \tfrac{1}{2}a_j x_j (2b_j - x_j) - \sum\nolimits_{i=1}^{n} \left(c \sum\nolimits_{k=i}^{n} x_{ik} \right) \\ &\text{s.t. } \sum\nolimits_{j=i}^{n} x_{i,j} \leq e_i, \qquad x_j = \sum\nolimits_{k=1}^{j} x_{kj}. \end{aligned} \tag{11}$$

The differences between both programs boil down to a different treatment of the $x_{i,j}$'s in supplier i's objective function that reflect the different coefficients for $x_{i,j}$ in $b_j'(x_{i,j} + r_j) + b_j''(x_{i,j} + r_j)\, x_{i,j} = a_j b_j - 2a_j x_j$ and $b_j'(x_{i,j} + r_j) = a_j b_j - a_j x_j$ in the equilibrium with market power and the competitive equilibrium, respectively.

	Competitive				Market power			
i,j	1	2	3	Total	1	2	3	Total
y_i	1.00	1.00	1.00		1.00	1.00	0.84	
x_j	0.67	0.67	1.67		0.47	0.71	1.66	
p_j	2.67	2.67	2.67		3.05	2.58	2.68	
CS (j)	0.44	0.44	2.78	3.67	0.22	0.51	2.75	3.48
Profits (i)	1.67	1.67	1.67	5.00	1.83	1.63	1.42	4.89
Welfare				8.67				8.36

Table 2: The competitive equilibrium and the equilibrium with market power in case $e_1 = e_2 = e_3 = 1$.

We implemented (10) and (11) in MATLAB in order to numerically compute equilibrium deliveries from which we derived equilibrium prices, consumer surpluses and profits. Take, for instance the case with three locations such that $n = m = 3$. Assume parameter values $c = 1$, $a_1 = a_2 = a_3 = 2$, $b_1 = b_2 = 2$, $b_3 = 3$, and $e_1 = e_2 = e_3 = 1$. These values imply that resource capacity, costs, and benefit functions are equal across locations, except for the benefit function of user 3, whose marginal benefit of water use is higher. Equilibrium values of key variables for both equilibria are presented in Table 2.

Table 2 shows that in the competitive equilibrium resource extraction is efficient and prices are equal across locations. In the equilibrium with market power water resources are under-developed because supplier 3 limits his extraction. This is a response to the redistribution of supplies by suppliers 1 and 2, as discussed in Section 3.3. As a result, prices are higher for users 1 and 3 and lower for user 2. Interestingly, profits are higher with market power only for supplier 1 (by 10%), while those of suppliers 2 and 3 decrease (by 2.0% and 14.9% respectively). Market power increases the consumer surplus of user 2 by 13.7%, while consumer surplus of users 1 and 3 decreases (by 49.5% and 1.0% respectively).

Overall, market power decreases both the consumer surplus and suppliers' profits. Market power enables supplier 1 to increase its profits at the cost of both consumer surplus as well as the other suppliers' profits. Total welfare loss as a result of market power in our example equals 3% of the welfare in the competitive equilibrium.

Note that the sign and size of the effects of market power differs across locations. These effects depend among others on the scarcity of water. Using the above example, but limiting resources to $e_1 = e_2 = e_3 = 0.5$ we obtain an equilibrium that is capacity constrained at all locations in both equilibria. Equilibrium values of key variables for both equilibria are presented in Table 3.

	Competitive				Market power			
i, j	1	2	3	Total	1	2	3	Total
y_i	0.50	0.50	0.50		0.50	0.50	0.50	
x_j	0.17	0.17	1.17		0.17	0.25	1.08	
p_j	3.67	3.67	3.67		3.67	3.50	3.83	
CS (j)	0.03	0.03	1.36	1.42	0.03	0.06	1.17	1.27
Profits (i)	1.33	1.33	1.33	4.00	1.36	1.36	1.42	4.14
Welfare				5.42				5.40

Table 3: The competitive equilibrium and the equilibrium with market power in case $e_1 = e_2 = e_3 = 0.5$.

Table 3 shows that in this situation with limited resources, market power is beneficial to all suppliers and users, except for water user 3 whose consumer surplus decreases by 13.7%. All suppliers gain from market power (by 2.1 to 6.3%). Water user 1 is unaffected by market power while the consumer surplus of water user 2 increases by 115%. Due to market power, the strategic redistribution of supply by the suppliers causes a reduction in supply to user 3, who is the only user to see an increase in price. Total welfare loss as a result of market power in our example equals 0.2% of the welfare in the competitive equilibrium.

These two examples illustrate that market regulation will be complicated. The possibility that some water users might lose from such regulation requires compensation schemes. This is standard practice in General Equilibrium models but novel in a Cournot setting.

5 Extensions

The model introduced in Section 2 is a robust and flexible framework to address market power in water markets. For explanatory reasons, we restricted its generality in order to focus on the type of results that can be obtained without being distracted by too much notation. In this section, we will discuss how to generalize many of the restrictive assumptions. In order to focus the discussion, we distinguish between extensions to the river structure in Section 5.1 and extensions to the water market infrastructure in Section 5.2.

5.1 More general river structures

In this subsection we discuss an extension to more general river structures. This includes multiple sources and mouths of the river (i.e., a delta) as well as relaxation of our assumption of only one supplier per location.

Multiple sources and river branches imply that the water from a source can only reach downstream locations in its own branch of the river and not in other branches. Furthermore, once the water from this source reaches the main river flow it can only reach mainstream locations. Because all entries in the matrix R, as defined in Section 2, can be interpreted as $0,1$ indicators that indicate whether water of location i reaches location $\bar{\imath}$, we can easily modify the matrix R to model multiple sources and branches. For example, for a river in which location 1 and 2 are two distinct sources that are connected to the main stream at location 3, the water balances are $y_1 \leq e_1$, $y_2 \leq e_2$, $y_1 + y_2 + y_3 \leq e_1 + e_2 + e_3$, and as in (1) these can be modelled in matrix notation as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \leq \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}.$$

This example illustrates the general principle that a simple modification of the matrix R captures multiple sources and branches. Note that each river can be modelled like this, and that it is always possible to label locations in such a manner that the upper triangular matrix consists of zeros. The key observation is that all local water balances can be modelled as in (2).

Deltas have the property that the main stream of the river splits into branches that eventually reach its final destination, say the sea or ocean. Under the presumption that the main stream splits into fixed fractions, we must modify the $0,1$ indicator into the continuous interval $[0,1]$ to represent fractions. Furthermore, if the river splits into a number of branches, all the fractions should sum up to 1. For example, the river is the main stream at location 1 and then splits into two branches such that 33% reaches location 2 and 67% reaches location 3. The associated water balances are $y_1 \leq e_1$, $y_2 + \frac{1}{3}y_1 \leq e_2 + \frac{1}{3}e_1$, $y_3 + \frac{2}{3}y_1 \leq e_3 + \frac{2}{3}e_1$, and these can be modelled in matrix notation as

$$\begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \leq \begin{bmatrix} 1 & 0 & 0 \\ \frac{1}{3} & 1 & 0 \\ \frac{2}{3} & 0 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}.$$

If branches split further, we must multiply all the fractions between upstream location i and downstream location $\bar{\imath}$. In the above example, if the branch through location 3 would split into two branches such that 16.7% reaches location 4 and 83.3% reaches location 5, then $\frac{2}{3} \cdot \frac{1}{6} = \frac{1}{9}$ of location 1's unused resources end up in location 4 and $\frac{2}{3} \cdot \frac{5}{6} = \frac{5}{9}$ in location 5. So, $R_{4,1} = \frac{1}{9}$, $R_{5,1} = \frac{5}{9}$, $R_{4,4} = R_{5,5} = 1$ and all other entries on row 4 and 5 are 0. Once more, this example illustrates the general principle that a simple modification of the matrix R captures the idea of a river that breaks apart in its delta. The main point is that all local water balances can be modelled as in (2), and that it is always possible to label locations in such a manner that the upper triangular matrix consists of zeros.

To allow for more than one supplier per location requires a different approach, using the $\bar{r} \times n$ matrix L which assigns suppliers to locations. $L_{ri} = 1$ means supplier i belongs to location r, and otherwise $L_{i,j} = 0$. Without loss of generality, supplier i is either located upstream of supplier $i+1$ or both are located at the same location. Also, supplier 1 belongs to location 1 and supplier n to location $\bar{r}$. Then, L has the following structure:

$$L = \begin{bmatrix} 1,\ldots,1 & 0,\ldots,0 & \cdots & 0,\ldots,0 \\ 0,\ldots,0 & 1,\ldots,1 & \cdots & 0,\ldots,0 \\ \vdots & \vdots & \ddots & \vdots \\ 0,\ldots,0 & 0,\ldots,0 & \cdots & 1,\ldots,1 \end{bmatrix}.$$

Aggregate extraction at each location is given by the vector $L \cdot y$. For $n = 1, \ldots, \bar{r}$, we define location r's maximal inflow as $e_r + \ldots + e_1$, which is the r-th row of $R \cdot e$. By allowing multiple suppliers per location, water balance equation (2) changes to

$$R \cdot L \cdot y \leq R \cdot e.$$

For each of the three extensions discussed above, it is obvious that Propositions 2–4 also extend. The reason is that only the numerical values in the structure $R \cdot y \leq R \cdot e$ are adjusted, not the mathematical structure of the model. Therefore, the proof of existence does not require any modification and Proposition 2 still holds. Furthermore, Propositions 3–4 are stated without reference to this structure, and therefore both still hold. This generalises these results to realistic river structures with multiple sources, deltas, and relaxes the restriction of one supplier per location. We proceed by illustrating how allowing for more than one supplier per location can cause multiplicity of equilibria, based on the monopoly and duopoly examples of Sections 4.1 and 4.3.

Recall the example from Section 4.3 in which two suppliers can supply one water user on an infrastructure with an unlimited capacity, i.e., $n = 2$, $m = 1$ and $K_{1,1} = K_{2,1} = \infty$. The water user has benefit function $b_1(x_1) = x_1(2 - x_1)$ and a marginal willingness to pay $b_1'(x_1) = 2(1 - x_1)$. Contrary to any of the previous examples, we now assume one location that hosts two water suppliers and one water user. That is, we additionally take $\bar{r} = 1$ and $L = [1, 1]$. This means both suppliers share a common resource. Specifically, $e_1 \in (0, 1)$ is the common resource, and we assume extraction costs are given by $c_1(y_1) = c_2(y_2) = 0$. Then, the competitive equilibrium water demand is $x_1^* = e_1$. As in Section 4.3, we substitute y_i for $x_{i,1}$ and perform the analysis in y_1 and y_2.

We solve for the equilibrium with market power by solving each supplier's profit-maximization problem in order to obtain the best-response functions in terms of extraction:

$$R_1(y_2) = \min\left\{\tfrac{1}{2} - \tfrac{1}{2}y_2, e_1 - y_2\right\} \qquad \text{and} \qquad R_2(y_1) = \min\left\{\tfrac{1}{2} - \tfrac{1}{2}y_1, e_1 - y_1\right\},$$

where we defer details to the Appendix. The best-response function for each supplier is either identical to the best-response function in the standard Cournot model, or to the anticipated amount left by the competitor, which is different from the modified Cournot model with exogenous capacity constraints. The equilibrium extraction levels are

$$\begin{cases} y_1 \in [0, e_1], & y_2 = e_1 - y_1, \quad \text{if } e_1 \leq \frac{1}{2}, \\ y_1 \in [2e_1 - 1, 1 - 2e_1], & y_2 = e_1 - y_1, \quad \text{if } \frac{1}{2} < e_1 < \frac{2}{3}, \\ y_1 = \frac{1}{3}, & y_2 = \frac{1}{3}, \qquad\quad \text{if } e_1 \geq \frac{2}{3}. \end{cases}$$

The interval $[2e_1 - 1, 1 - e_1]$ for the middle case is a strict subinterval of $[0, e_1]$ that contains the symmetric equilibrium $y_1 = y_2 = \frac{1}{2}e_1$. As e_1 goes to $\frac{2}{3}$, both its upper and lower bound approach $\frac{1}{3}$. The equilibrium for the last case is the unconstrained equilibrium and it coincides with the traditional Cournot-duopoly equilibrium. We would obtain similar results if we had assumed positive marginal costs of extraction. A methodological implication is that multiple equilibria with market power may exists, and hence the uniqueness result discussed in Section 4.4, does not generally hold.

5.2 Conveyance losses, capacity constraints and operating costs

In this subsection we discuss an extension to the water supply infrastructure. This includes conveyance losses in run-off and supply as mentioned in Section 4.1, as well as relaxation of the assumption that capacity constraints $K_{i,j}$ are restricted to either 0 or ∞.

Conveyance losses may occur both in the run-off between locations as well as in supply to water users. Conveyance losses in the run-off between locations is modelled using the water balances as in (1). Specifically, we assume that of the water that is unused by supplier i, only the fraction $\rho_i \in [0, 1]$ runs off to the adjacent downstream supplier and fraction $1 - \rho_i$ evaporates, leaks, dissolves, or is simply lost. Using the same example with $n = 3$ as in (1), the water balance at location 1 dictates $y_1 \leq e_1$, with run-off $\rho_1(e_1 - y_1) \geq 0$. Consequently, the water balance at location 2 dictates $y_2 \leq e_2 + \rho_1 (e_1 - y_1)$, with run-off $\rho_2(\rho_1 (e_1 - y_1) + e_2 - y_2) \geq 0$. Similarly, the water balance at location 3 dictates $y_3 \leq e_3 + \rho_2 (\rho_1 (e_1 - y_1) + e_2 - y_2)$. Similar to (1), we can rewrite these water balances in matrix notation as

$$\begin{bmatrix} 1 & 0 & 0 \\ \rho_1 & 1 & 0 \\ \rho_1\rho_2 & \rho_2 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \leq \begin{bmatrix} 1 & 0 & 0 \\ \rho_1 & 1 & 0 \\ \rho_1\rho_2 & \rho_2 & 1 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix}. \tag{12}$$

In general, we only need to revise the matrix R to R' in order to preserve the structure $R' \cdot y \leq R' \cdot e$. The revised $n \times n$ matrix R' has elements $R'_{i,\hat{\imath}} = 0$ for $\hat{\imath} < i$, $R'_{i,i} = 1$, $R'_{i+1,i} = \rho_i$, $R'_{i+2,i} = \rho_{i+1} R'_{i+1,i} = \rho_{i+1}\rho_i$ up to $R'_{n,i} = \rho_{n-1} R'_{n-1,i}$. We mention that R' can also accommodate for multiple sources and deltas with conveyance losses, which we omit.

Conveyance losses in supply is modelled using a similar approach. Let $d_{i,j}$ indicate that if supplier i supplies $0 \leq x_{i,j} \leq K_{i,j}$ to water user j, then $d_{i,j}x_{i,j}$ arrives at this user. We put all $d_{i,j}x_{i,j}$ in the $n \times m$ matrix X^D. The i-th row sum of X^D consists of the total supply by supplier i to its water users. Similar, the j-th column sum of X^D, denoted as x_j^D, is the total water delivered to water user j, i.e., $x_j^D = d_{1,j}x_{1,j} + d_{2,j}x_{2,j} + \ldots + d_{n,j}x_{n,j} = \sum_{i=1}^n d_{i,j}x_{i,j}$.

Capacity constraints are obtained by relaxing the assumption that capacity constraints $K_{i,j}$ is restricted to either 0 or ∞. Specifically, $K_{i,j}$ dictates that supply is constrained even if $K_{i,j} \neq 0$ so that, given conveyance losses in supply, we have $d_{i,j}x_{i,j} \leq x_{i,j} \leq K_{i,j}$.

The model in Chakravorty et al. (2009) distinguishes between the costs of extraction and operating and maintenance (O&M) costs to run the delivery infrastructure. They study different institutional settings where operating the infrastructure is either in the hands of the single water supplier, or is regulated and charges marginal costs. The model in Section 2 can be easily extended by introducing O&M costs $c_{i,j}(x_{i,j})$ for the link between water supplier j and water user i. The different institutional settings can be modelled by attributing these costs to either the water supplier or the water user. In case suppliers legally control their links of the infrastructure, then the O&M costs should be attributed to water supplier i. By attributing these costs to the water users, we implicitly model that the infrastructure is regulated. Note that this modification requires $b_j(x_j) - \sum_{i=1}^n c_{i,j}(x_{i,j})$ as the benefit functions.

Given the market-clearing prices expressed as a continuous function of local supply, the extraction decision by supplier $i = 1, \ldots, n$, in the presence of conveyance losses, O&M costs attributed to water suppliers, and capacity constraints is given by

$$\begin{array}{ll} \max_{y_i, x_{i,1}, \ldots, x_{i,m} \geq 0} & \sum_{j=1}^m \left[b_j'\left(d_{1,j}x_{1,j} + \ldots + d_{i,j}x_{i,j} + \ldots + d_{n,j}x_{n,j}\right) d_{i,j}x_{i,j} - c_{i,j}(x_{i,j}) \right] - c_i(y_i, y_{-i}), \\ \text{s.t.} & R_i' \cdot y \leq R_i' \cdot e, \qquad (\beta_i) \\ & x_{i,1} + \ldots + x_{i,m} = y_i, \\ & x_{i,j} \leq K_{i,j}, \qquad (\gamma_{i,j}) \end{array}$$

Similar to Section 5.1, Propositions 2–4 can be adjusted in a straightforward manner. As will be clear from the proof of Proposition 2, the arguments accommodate for the change from R to R' and general $K_{i,j} \in [0, \infty)$. Concavity of the profit functions is guaranteed if the function $c_{i,j}(x_{i,j})$ is differentiable, increasing and convex, and (5) is relaxed to

$$b_j''' (d_{i,j}x_{i,j} + r_j) < \frac{2 \left| b_j'' (d_{i,j}x_{i,j} + r_j) \right|}{d_{i,j}x_{i,j}}.$$

In Proposition 3, a binding constraint $x_{i,j} = K_{i,j} > 0$ requires the minor modification

$$b_j' (d_{i,j}K_{i,j} + r_j) + b_j'' (d_{i,j}K_{i,j} + r_j) d_{i,j}K_{i,j} \geq c_i' \left(d_{i,j}K_{i,j} + \sum\nolimits_{\hat{\jmath} \neq j} d_{i,\hat{\jmath}}x_{i,\hat{\jmath}}; y_{-i} \right) + c_{i,j}' (K_{i,j}),$$

due to each link's individual O&M costs. Capacity restricts profits if and only if the inequality is strict. Then, Proposition 4 requires two minor modifications. First, the attractiveness of serving a market also depends on the individual O&M costs, and these need to be subtracted from the marginal revenue. So, whenever the optimal supply $x_{i,j} < K_{i,j}$ and $x_{i,\hat{\jmath}} < K_{i,\hat{\jmath}}$ by supplier i to both water users j and $\hat{\jmath}$ is positive, it also holds that

$$\begin{aligned} & b_j' (d_{i,j}x_{i,j} + r_j) + b_j'' (d_{i,j}x_{i,j} + r_j) d_{i,j}x_{i,j} - c_{i,j} (K_{i,j}) \\ = \; & b_{\hat{\jmath}}' (d_{i,\hat{\jmath}}x_{i,\hat{\jmath}} + r_{\hat{\jmath}}) + b_{\hat{\jmath}}'' (d_{i,\hat{\jmath}}x_{i,\hat{\jmath}} + r_{\hat{\jmath}}) d_{i,\hat{\jmath}}x_{i,\hat{\jmath}} - c_{i,\hat{\jmath}} (K_{i,\hat{\jmath}}) \\ \geq \; & c_i' (y_i, y_{-i}). \end{aligned}$$

In case $x_{i,j} = K_{i,j}$ and $x_{i,\hat{\jmath}} < K_{i,\hat{\jmath}}$, than the equality becomes a strict inequality, because supplier i would like to increase his supply to water user j but is physically constrained to do so. To summarize, our main results generalise to realistic water market infrastructures with conveyance losses, O&M costs and finite capacity.

6 Concluding remarks

We analyse water markets with market power as multi-market Cournot competition in which an infrastructure constrains access to differentiated local markets and a river structure with interdependencies constrains resource use. Our analysis shows that an equilibrium under market power exists under mild concavity conditions. Binding resources or capacity constraints impose some physical commitment not to serve too many markets, and directly lead to a gap between the relative mark-up on markets served and the traditional expressions for the Lerner indices. Compared to the competitive equilibrium, market power results in the strategic substitution of water supply over the suppliers' markets. The competitive equilibrium is not necessarily efficient (it is typically not if cost functions depend on inflow from upstream) and market power may increase such inefficiencies. All four cases in Section 4 indicate under-development of water resources, but the interdependency of extractions costs creates a counter effect to the traditional contraction effect of oligopolies on extraction, which creates an ambiguity with respect to under-development. Nevertheless, market power should not be seen as an instrument in mitigating over-exploitation of water. Standard intuition is valid in those cases where market power is bad for all consumers, which justifies the regulation of water markets. Our numerical example in Section 4.4 illustrates, however, that market power may not unequivocally harm water users and benefit suppliers. Depending on their location and parameter values, some water users may benefit, while suppliers may be harmed by the presence of market power. So, Pareto improving regulation for water users may involve financial compensation schemes for those users that would be harmed by regulation.

Our paper develops a robust multi-market Cournot model that is applicable in many different circumstances. This facilitates wide applicability of our model, especially because software is available to numerically solve such models. The Cournot model is motivated by the observation in Holland (2006) that there might be an incentive to reduce deliveries. Alternatively, one might model market power as price competition. Price competition would be equivalent in the monopoly and local monopolies cases in Section 4.1 and 4.2, similar as in the standard monopoly model. In the duopoly model of Section 4.3 with constant marginal costs, classic Bertrand competition would seem to lead to the competitive equilibrium. One can then ask whether Bertrand competition underestimates market power, or that Cournot competition overestimates the effects of market power. Empirical research has to settle this issue, but it requires economic models that can deal with market power and the specific structures of water markets. For that reason, we regard our analysis of Cournot competition as a necessary first step towards a formal treatment of market power in water markets.

Two issues have not received attention yet but should be considered when putting our model to work. First, an implicit assumption is that water users are financially unconstrained. This means that either their budget exceeds the purchases they made, or perfect capital markets exist in order to provide loans that are paid back by some of the monetary benefit $b_j(x_j)$.

Second, water markets may apply other trading mechanisms than the quantity competition used in this paper. Important alternative mechanism are water banks, (double) auctions, water leasebacks, and bilateral bargaining. Such mechanisms may coexist. In the Australian Goulburn-Murray Irrigation District, for example, a small informal bargaining market coexists with a formal auction-based water exchange. This informal market is mainly used by neighbouring farms who prefer private transactions of small amounts of water (Bjornlund 2003). For this type of informal trade, a bargaining model may be more suitable, such as Saleth et al. (1991), who demonstrate that strategic bargaining in thin water markets leads to inefficiencies and therefore welfare loss.

Appendix: Mathematical proofs and other derivations

Proof of Proposition 2

The proof consists of verifying whether the sufficient conditions for existence of a social equilibrium in Debreu (1952) hold in our water market. These conditions concern the sets of feasible actions and the objective functions.

For supplier i, the set of feasible extractions $[0, e_i + R_{i-1} \cdot (e - y)] \subseteq [0, R_i \cdot e]$ is a non-empty, compact and convex set whose bounds are continuous in y_{-i}. Due to linearity of the constraints, the set $\left\{y \in \mathbb{R}_+^n | R_i \cdot y \leq R_i \cdot e \text{ for all } i = 1, \ldots, n\right\}$ is a non-empty, compact and convex polyhedron in the Cartesian product $[0, R_1 \cdot e] \times \ldots \times [0, R_n \cdot e]$.

We will apply the existence theorem on the domain of feasible deliveries in $\mathbb{R}_+^{n \times m}$. Given the matrix of deliveries $X \in \mathbb{R}_+^{n \times m}$, X_i and X_{-i} denote the i-th row of X, respectively, all rows of X except row i. We also put all capacity constraints into the $n \times m$ matrix K, and write $X \leq K$ whenever $x_{i,j} \leq K_{i,j}$ for all $i = 1, \ldots, n$ and $j = 1, \ldots, m$. After substitution and due to linearity of the constraints, we obtain that the set

$$P \equiv \left\{ X \in \mathbb{R}_{+}^{n \times m} | \sum\nolimits_{\hat{\imath}=1}^{i} \left(x_{\hat{\imath},1} + \ldots + x_{\hat{\imath},m}\right) \leq R_i \cdot e \text{ for all } i = 1, \ldots, n, \text{ and } X \leq K \right\}$$

is a non-empty, compact and convex polyhedron in the Cartesian product $[0, R_1 \cdot e]^n \times \ldots \times [0, R_n \cdot e]^n$. The set P satisfies the conditions on the domain in Debreu (1952).

The profit function of supplier $i = 1, \ldots, n$ can be written as

$$\sum_{j=1}^{m} b_j' \left(x_{1,j} + \ldots + x_{i,j} + \ldots + x_{n,j}\right) x_{i,j} - c_i \left(x_i, X_{-i}\right),$$

and this function is well-defined on the domain $P \subset \mathbb{R}_{+}^{n \times m}$. Furthermore, it is continuous in X and strictly concave in supplier i's own decision variables $(x_{i,1}, \ldots, x_{i,m})$ by assumption. Therefore, supplier i's profit function satisfies the conditions on the objective function in Debreu (1952).

To summarise, all conditions in the existence theorem for a social equilibrium in Debreu (1952) hold, and hence, such an equilibrium exists. □

Proof of Proposition 3

Given X_{-i}, $x_i = \sum_{j=1}^{m} x_{i,j}$, $x_j = \sum_{i=1}^{n} x_{i,j}$, $r_j = x_j - x_{i,j}$, $\hat{r}_i = x_i - x_{i,j}$ and y_{-i} are also determined. Costly extraction implies $x_i \leq y_i$ is binding, and we substitute x_i for y_i. Then, supplier i's profit-maximization program (4) can be rewritten as

$$\begin{aligned} \max_{x_{i,1},\ldots,x_{i,m} \geq 0} \quad & \sum\nolimits_{j=1}^{m} b_j' \left(x_{i,j} + r_j\right) x_{i,j} - c_i \left(x_i, y_{-i}\right), \\ \text{s.t.} \quad & x_i \leq a_i, \qquad (\beta_i) \\ & x_{i,j} \leq K_{i,j}, \qquad (\gamma_{i,j}) \end{aligned} \tag{13}$$

where $a_i = R_i \cdot e - R_{i-1} \cdot y$. The FOCs for supplier i are

$$\begin{aligned} x_{i,j}: \quad & b_j' \left(x_{i,j} + r_j\right) + b_j'' \left(x_{i,j} + r_j\right) x_{i,j} - c_i' \left(x_i, y_{-i}\right) - \beta_i - \gamma_{i,j} \leq 0 \quad \perp \quad x_{i,j} \geq 0, \\ \beta_i: \quad & x_{i,j} + \hat{r}_i - a_i \leq 0 \quad \perp \quad \beta_i \geq 0, \\ \gamma_{i,j}: \quad & x_{i,j} - K_{i,j} \leq 0 \quad \perp \quad \gamma_{i,j} \geq 0. \end{aligned}$$

To characterise $x_{i,j}$, we must distinguish the following cases. First, we distinguish the main cases $K_{i,j} = \infty$ and $K_{i,j} = 0$. Then, we distinguish the first sub-level of cases $a_i > \hat{r}_i$ and

$a_i = \hat{r}_i$. At the deepest level, we distinguish the sub-cases $x_{i,j} = 0$, $x_{i,j} = a_i - \hat{r}_i$, and $x_{i,j} < a_i - \hat{r}_i$, provided such sub-cases exist.

$K_{i,j} = \infty$ Then, $\gamma_{i,j} = 0$ must hold.

If $a_i > \hat{r}_i$, then $x_{i,j} \in [0, a_i - \hat{r}_i]$ can be equal to the lower bound, can be interior, or can be equal to the upper bound. In the first sub-case, $x_{i,j} = 0 < a_i - \hat{r}_i$ implies $\beta_i = 0$ and $b'_j(r_j) \leq c'_i(\hat{r}_i, y_{-i})$. In the second sub-case, $\beta_i = b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j} - c'_i(x_i, y_{-i}) \geq 0$ with equality and $b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j} = c'_i(x_{i,j} + \hat{r}_i, y_{-i})$ if $0 < x_{i,j} < a_i - r_j$. In the remaining sub-case $0 < x_{i,j} = a_i - \hat{r}_i$, $\beta_i \geq 0$ implies $b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j} \geq c'_i(x_i, y_{-i})$.

If $a_i = \hat{r}_i$, then $x_{i,j} = 0$ but with the understanding that either $x_{i,j} \geq 0$ is binding and $\beta_i = 0$ whenever $b'_j(r_j) < c'_i(\hat{r}_i, y_{-i})$, or $x_{i,j} \leq a_i - r_j = 0$ is binding and $\beta_i > 0$ whenever $b'_j(r_j) > c'_i(\hat{r}_i, y_{-i})$. Equality $b'_j(r_j) = c'_i(\hat{r}_i, y_{-i})$ implies $x_{i,j} = 0$ is optimal under classical optimization without the constraints $x_{i,j} \geq 0$ and $x_{i,j} \leq 0$. In the latter case, $\beta_i = 0$.

$K_{i,j} = 0$ Then, independent of $a_i \geq \hat{r}_i$, $x_{i,j} = 0$ must hold with the understanding that either $x_{i,j} \geq 0$ is binding whenever $b'_j(r_j) < c'_i(\hat{r}_i, y_{-i})$, or $x_{i,j} \leq \min\{K_{i,j}, a_i - \hat{r}_i\} = 0$ is binding whenever $b'_j(r_j) > c'_i(\hat{r}_i, y_{-i})$. Equality $b'_j(r_j) = c'_i(\hat{r}_i, y_{-i})$ implies $x_{i,j} = 0$ is optimal under classical optimization without the constraints $x_{i,j} \geq 0$ and $x_{i,j} \leq 0$. Note that $a_i > \hat{r}_i$ additionally implies that $\beta_i = 0$. □

Proof of Proposition 4

First, consider $K_{i,j} = K_{i,\hat{\jmath}} = \infty$ and positive supply $x_{i,j}$ and $x_{i,\hat{\jmath}}$ to both water users j and $\hat{\jmath}$. Independent of constrained or unconstrained water resources $x_i \leq a_i$, it follows from the first order conditions of the proof of Proposition 3 that

$$\beta_i = b'_j(x_{i,j} + r_j) + b''_j(x_{i,j} + r_j)\, x_{i,j} - c'_i(x_i, y_{-i}) = b'_{\hat{\jmath}}(x_{i,\hat{\jmath}} + r_{\hat{\jmath}}) + b''_{\hat{\jmath}}(x_{i,\hat{\jmath}} + r_{\hat{\jmath}})\, x_{i,\hat{\jmath}} - c'_i(x_i, y_{-i}).$$

Combined with $\beta_i \geq 0$ we must have

$$b_j'(x_{i,j}+r_j)+b_j''(x_{i,j}+r_j)\,x_{i,j} = b_{\hat{\jmath}}'(x_{i,\hat{\jmath}}+r_{\hat{\jmath}})+b_{\hat{\jmath}}''(x_{i,\hat{\jmath}}+r_{\hat{\jmath}})\,x_{i,\hat{\jmath}} \geq c_i'(x_i, y_{-i}),$$

with equality if supplier i is unconstrained. Next, consider $K_{i,j} = K_{i,\hat{\jmath}} = \infty$ and $x_{i,j} > 0$ and $x_{i,\hat{\jmath}} = 0$. Then, additionally

$$b_j'(x_{i,j}+r_j)+b_j''(x_{i,j}+r_j)\,x_{i,j} \geq b_{\hat{\jmath}}'(x_{i,\hat{\jmath}}+r_{\hat{\jmath}})+b_{\hat{\jmath}}''(x_{i,\hat{\jmath}}+r_{\hat{\jmath}})\,x_{i,\hat{\jmath}} = b_{\hat{\jmath}}'(r_{\hat{\jmath}}),$$

because $<$ would contradict that $(x_{i,1},\ldots,x_{i,n})$ maximizes supplier i's profit. □

Derivations of the example in Section 4.1

Consider the monopoly. The derivative of the monopolist's profit function is proportional to $(1-2x_1)(e_1-x_1)-\frac{1}{2}c_1 = 2x_1^2 - x_1(1+2e_1)+e_1-\frac{1}{2}c_1$. The derivative is negative at $x_1 = e_1$, and it is positive at $x_1 = 0$ if and only if $e_1 > \frac{1}{2}c_1$. So, for $e_1 \leq \frac{1}{2}c_1$, we have $x_1 = 0$, and for $e_1 > \frac{1}{2}c_1$ we have an interior solution $x_1 \in (0, e_1)$. We obtain (6) from solving the first-order condition, because $\frac{1}{2}\left(\frac{1}{2}+e_1+\sqrt{\left(e_1-\frac{1}{2}\right)^2+c_1}\right) > \frac{1}{2}\left(\frac{1}{2}+e_1+\sqrt{\left(e_1-\frac{1}{2}\right)^2}\right) = e_1$ is infeasible, and $-\sqrt{\left(e_1-\frac{1}{2}\right)^2+c_1} < -\sqrt{\left(\frac{1}{2}-e_1\right)^2} = -\left(\frac{1}{2}-e_1\right)$ implies the other root is feasible, i.e., less than e_1.

Next, we consider the competitive equilibrium. The derivative of the objective is proportional to $(1-x_1)(e_1-x_1)-\frac{1}{2}c_1 = x_1^2-(e_1+1)\,x_1+e_1-\frac{1}{2}c_1$. Note that the derivative is negative at $x_1 = e_1$, and it is positive at $x_1 = 0$ if and only if $e_1 > \frac{1}{2}c_1$. So, for $e_1 \leq \frac{1}{2}c_1$, we have $x_1^* = 0$, and for $e_1 > \frac{1}{2}c_1$ we have an interior solution $x_1^* \in (0, e_1)$. We obtain (7) from solving the first-order condition, because $\frac{1}{2}\left(1+e_1+\sqrt{(e_1-1)^2+2c_1}\right) > \frac{1}{2}\left(1+e_1+\sqrt{(e_1-1)^2}\right) = e_1$ is infeasible, and $-\sqrt{(e_1-1)^2+2c_1} < -\sqrt{(1-e_1)^2} = -(1-e_1)$ implies the other root is feasible. □

Derivations of the two examples in Section 4.3

Supplier 1's best-response function is found by solving

$$\begin{aligned} R_1(y_2) &= \arg \max_{y_1 \in [0,e_1]} y_1 p_1(y_1 + y_2) - 0 \cdot y_1 \\ &= \arg \max_{y_1, \lambda_1 \geq 0} 2y_1(1 - y_1 - y_2) - \lambda_1(y_1 - e_1) \\ &= \min\left\{\max\left\{0, \tfrac{1}{2} - \tfrac{1}{2}y_2\right\}, e_1\right\}. \end{aligned}$$

Similarly, supplier 2's best-response function is given by

$$\begin{aligned} R_2(y_1) &= \arg \max_{y_2 \in [0,e_1+e_2-y_1]} y_2 p_1(y_1 + y_2) - 0 \cdot y_2 \\ &= \arg \max_{y_2, \lambda_2 \geq 0} 2y_2(1 - y_1 - y_2) - \lambda_2(y_1 + y_2 - e_1 - e_2) \\ &= \min\left\{\max\{0, \tfrac{1}{2} - \tfrac{1}{2}y_1\}, e_1 + e_2 - y_1\right\}. \end{aligned}$$

In the main text, we write $\frac{1}{2} - \frac{1}{2}y_1$ instead of $\max\{0, \frac{1}{2} - \frac{1}{2}y_1\}$ for convenience. We rewrite $\frac{1}{2} - \frac{1}{2}y_2 \leq e_1$ and $\frac{1}{2} - \frac{1}{2}y_1 \leq e_1 + e_2 - y_1$ as $y_2 \geq 1 - 2e_1$ and $y_1 \leq 2e_1 + 2e_2 - 1$. Clearly, we need to distinguish four distinct cases.

D. $y_2 \geq 1 - 2e_1$ and $y_1 \leq 2e_1 + 2e_2 - 1$. So, $y_1 = \frac{1}{2} - \frac{1}{2}y_2$ and $y_2 = \frac{1}{2} - \frac{1}{2}y_1$. Then, $y_1 = y_2 = \frac{1}{3} > 0$. This case's conditions impose $e_1 \geq \frac{1}{3}$ and $e_1 + e_2 \geq \frac{2}{3}$.

C. $y_2 \geq 1 - 2e_1$ and $y_1 > 2e_1 + 2e_2 - 1$. So, $y_1 = \frac{1}{2} - \frac{1}{2}y_2$ and $y_2 = e_1 + e_2 - y_1$, because $\frac{1}{2} - \frac{1}{2}y_2 \leq e_1$. Taking non-negativity of y_1 and y_2 for granted, we obtain $y_1 = 1 - e_1 - e_2$ and $y_2 = 2e_1 + 2e_2 - 1$. This case's conditions further impose $2e_1 + e_2 \geq 1$ and $e_1 + e_2 < \frac{2}{3}$. Feasibility $0 \leq y_1 \leq e_1$ and $0 \leq y_2 \leq e_1 + e_2 - y_1$ hold. Total extraction is $y_1 + y_2 = e_1 + e_2$.

B. $y_2 < 1 - 2e_1$ and $y_1 \leq 2e_1 + 2e_2 - 1$. So, $y_1 = e_1$ and $y_2 = \frac{1}{2} - \frac{1}{2}y_1$. Taking feasibility of y_1 and y_2 for granted, we obtain $y_1 = e_1$ (feasible) and $y_2 = \frac{1}{2} - \frac{1}{2}e_1$. This case's conditions impose $e_1 < \frac{1}{3}$ and $e_1 + 2e_2 \geq 1$, and these conditions guarantee feasibility $0 \leq y_1 \leq e_1$ and $0 \leq y_2 \leq e_1 + e_2 - y_1$.

A. $y_2 < 1 - 2e_1$ and $y_1 > 2e_1 + 2e_2 - 1$. So, $y_1 = e_1$ and $y_2 = \max\{e_2, e_1 + e_2 - y_1\} = e_2$. Then, $y_1 = e_1$ and $y_2 = e_2$ satisfy feasibility. This case's conditions impose $2e_1 + e_2 < 1$ and $e_1 + 2e_2 < 1$.

Summarizing the conditions of all four cases, we have four important lines: $e_1 = \frac{1}{3}$, $e_1+e_2 = 1$, $2e_1 + e_2 = 1$ and $e_1 + 2e_2 = 1$. In the (e_1, e_2)-space, all these lines intersect at $e_1 = e_2 = \frac{1}{3}$, as Figure 2 in the main text illustrates.

In the second example, we consider $c_1(y_1) = -c_1 \ln(e_1 - y_1)$ and $c_2(y_2, y_1) = -c_2 \ln(e_1 + e_2 - y_1 - y_2)$. Because $y_1 < e_1$ in the optimum, supplier 1's optimal response solves

$$\max_{y_1 \in [0,e_1)} y_1 (2 - y_1 - y_2) + c_1 \ln(e_1 - y_1).$$

Similar, $y_2 < e_1 + e_2 - y_1$ in the optimum, supplier 2's optimal response solves

$$\max_{y_2 \in [0,e_1+e_2-y_1)} y_2 (2 - y_1 - y_2) + c_2 \ln(e_1 + e_2 - y_1 - y_2).$$

Combining and rewriting the both first-order conditions for an interior solution, implies the following non-linear system:

$$\begin{aligned} 0 &= (2 - 4y_1 - 2y_2)(e_1 - y_1) - c_1, \\ 0 &= (2 - 2y_1 - 4y_2)(e_1 + e_2 - y_1 - y_2) - c_2. \end{aligned}$$

Obviously, $y_1 < e_1$ and $y_1 + y_2 < e_1 + e_2$, because otherwise the right-hand side of one of the conditions would be negative. Expanding the expressions yields

$$\begin{aligned} 0 &= 4y_1^2 - y_1(2 + 4e_1 - 2y_2) + e_1(2 - 2y_2) - c_1, \\ 0 &= 4y_2^2 - y_2(2 + 4e_1 + 4e_2 - 6y_1) + (e_1 + e_2 - y_1)(2 - 2y_1) - c_2. \end{aligned}$$

Note that the first line's right-hand side is positive at $y_1 = 0$ if and only if $c_1 < 2e_1(1 - y_2)$. Similar, the second line's right-hand side $c_1 < 2(e_1 + e_2 - y_1)(1 - y_1)$. Note that both conditions are more restrictive than $c_1 < 2e_1$ and $c_1 < 2(e_1 + e_2 - y_1)$, respectively. So, only under these conditions do we have an interior solution.

The first-order conditions do not allow an analytical solution. To see this, supplier 1's first-order condition is linear in y_2, and by $y_1 \in (0, e_1)$, implies

$$y_2 = \frac{\left(4y_1^2 - (2 + 4e_1) y_1 + 2e_1 - c_1\right)}{2(e_1 - y_1)}$$

Substitution into supplier 2's first-order condition yields

$$\begin{aligned} 0 \quad = \quad & -6y_1^4 + (6e_1 - 6e_2 + 8) y_1^3 + \left(2e_2 - 14e_1 + 5c_1 + c_2 + 6e_1^2 + 12e_1e_2 - 2\right) y_1^2 \\ & + \left(4e_1 - 3c_1 + 4e_1^2 - 6e_1^3 - 6e_2e_1^2 - 4e_1e_2 - 3e_1c_1 - 2e_1c_2 + 2e_2c_1\right) y_1 \\ & + 2e_1^3 - 2e_1^2 + 2e_2e_1^2 - 2c_1e_1^2 + c_2e_1^2 - c_1^2 + 3e_1c_1 - 2e_1e_2c_1. \end{aligned}$$

This polynomial of degree four does not allow an analytical solution and can only be solved numerically. In principle, there can be four roots. A unique equilibrium would imply that all except one will be infeasible.

As a numerical example, we take $e_1 = e_2 = 1$ and consider all four combinations $c_1, c_2 \in \{\frac{1}{4}, \frac{1}{2}\}$. The numerical method is applied to the quadratic system of first-order conditions and is implemented in MATLAB. We report all numerical solutions (y_1, y_2) for the system of first-order conditions in the following table. Clearly, two roots are infeasible, because one of the extractions is negative. Another root is infeasible because its aggregate extraction is above $e_1 = e_2 = 2$. For these numerical values, we obtain a unique equilibrium.

c_1, c_2	Equilibrium (Root 1)	Root 2	Root 3	Root 4
$c_1 = c_2 = \frac{1}{4}$	$(0.251, 0.330)$	$(1.060, 1.002)$	$(-1.090, 3.12)$	$(1.113, -0.119)$
$c_1 = \frac{1}{4}, c_2 = \frac{1}{2}$	$(0.276, 0.276)$	$(1.058, 1.058)$	$(-1.118, 3.177)$	$(1.118, -0.177)$

This completes the derivations. □

Derivation of the example in Section 5.1

Supplier i's profit-maximization problem is given by

$$y_i = \arg\max_{y_i \geq 0} 2y_i \left(1 - y_i - y_{-i}\right), \quad \text{s.t. } y_i \leq e_1 - y_{-i}.$$

From the Lagrange function $2y_i (1 - y_i - y_{-i}) - \lambda_i (y_i + y_{-i} - e_1)$, the Kuhn-Tucker first-order conditions can be derived. There are two cases, either $y_1 + y_2 < e_1$ or $y_1 + y_2 = e_1$.

$y_1 + y_2 < e_1$. Then, $\lambda_1 = \lambda_2 = 0$ and we obtain the best-response functions $y_1 = \frac{1}{2} - \frac{1}{2}y_2$ and $y_2 = \frac{1}{2} - \frac{1}{2}y_1$. The equilibrium extractions are $y_1 = y_2 = \frac{1}{3} > 0$. Finally, $y_1 + y_2 < e_1$ requires $e_1 > \frac{2}{3}$.

$y_1 + y_2 = e_1$. For supplier 1, the best-response functions is $y_1 = e_1 - y_2$ and the shadow prices are $\lambda_1 = 2 - 4y_1 - 2y_2$. Note that $\lambda_1 \geq 0$ is equivalent to $y_1 \leq \frac{1}{2} - \frac{1}{2}y_2$, where the left-hand side is the best-response function from the first case. By symmetry, $y_2 = e_2 - y_1$ and $\lambda_2 = 2 - 2y_1 - 4y_2 \geq 0$ if and only if $y_2 \leq \frac{1}{2} - \frac{1}{2}y_1$. Because the best-response functions form a dependent linear system, these only impose $y_1 + y_2 = e_1$.

The equilibrium values of y_1 and y_2 are restricted by the non-negativity of the shadow prices. Substitution of $y_1 = e_1 - y_2$ into the condition for $\lambda_1 \geq 0$ yields $y_2 \geq 2e_1 - 1$, and similar substitution of $y_2 = e_1 - y_1$ yields $y_1 \leq 1 - e_1$. By symmetry, substitution into $\lambda_2 \geq 0$ yields $y_1 \geq 2e_1 - 1$ and $y_2 \leq 1 - e_1$. So, $y_1 \in [2e_1 - 1, 1 - e_1]$ and $y_2 = e_1 - y_1$. The interval is non-empty if and only if $e_1 \leq \frac{2}{3}$. The interval is a strict subset of $[0, e_1]$ if and only if $e_1 > \frac{1}{2}$. So, for $e_1 \in (\frac{1}{2}, \frac{2}{3}]$ we must have that all $y_1 \in [2e_1 - 1, 1 - e_1]$ are equilibria, and for $e_1 \in [0, \frac{1}{2})$ we have that all $y_1 \in [0, e_1]$ are equilibria.

For completeness, we note that qualitatively the same result holds for $c_i(y_i) = cy_i$, where $c \in (0, 1)$, after we substitute $1 - c$ for 1 in the Lagrangian function.

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