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A note on passepartout problems*

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Abstract:

This note provides a methodological contribution to the allocation of joint revenues obtained from passepartouts. In a passepartout system a group of service providers offers a passepartout that allows its owners the use of specified services for an unlimited number of times during a fixed period of time. The corresponding allocation problem is then how to share the total joint revenues of the passepartout system adequately among the service providers. Arguments are provided to model a passepartout problem within the framework of bankruptcy and context-specific properties are considered in order to select an appropriate allocation rule.

Keywords: Passepartout problem, bankruptcy problem, allocation rule.

JEL Classification Number: D70

1 Introduction

In a passepartout system a group of service providers decides to emit a passepartout to costumers that allows an unlimited use of specific services during a limited period of time at a fixed price. Passepartout systems are quite common. For example, in The Netherlands, the OV-jaarkaart allows unlimited use of public transport (train, bus, metro and tram) during one year and the Museumkaart allows unlimited visit to more than 400 museums during one year. In Copenhagen, the Card Plus allows its owners both to visit a large number of museums (and amusement parks) and to use public transport during a limited amount of time. Note that

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these cases typically do not lead to surplus-sharing problems since they are concerned with semi-public facilities and are heavily subsidized for aims of promotion or social well-being. In particular, without a steep rise of the total number of customers, the amount of money that is obtained by means of the passepartout system will be less than the total amount of money that service providers would have obtained without this system.

As an example of a passepartout system concerning private facilities, consider the “3 Parken Pas” that allows to its owners an unlimited number of visits during one year to three amusement parks in The Netherlands: Walibi World, Dolfinarium, and Avonturenpark Hellendoorn. In case of private facilities, the passepartout system may not cover all the services provided by the members of the system, but only some specific ones. In this case, the additional revenues obtained by the regular services that are not covered by the passepartout will be enough to compensate the “losses” generated by the passepartout. In particular, the 3 Parken Pas only covers the entrance to the parks and extra services as parking fees, the rental of lockets, and the purchase of food, drinks, and souvenirs have to be paid separately. Although we could not get the exact data on the 3 Parken Pas due to privacy rules, we were informed that basically each amusement park keeps the full price of all passepartouts sold at their own gates. Thus no explicit reallocation of the revenues takes place. They very well realize that this is not a “fair” way to share the joint revenues generated by the 3 Parken Pas. However, the extra profit and exposure revenues due to additional consumption and marketing considerations prove to be enough incentive to maintain the cooperation.

A passepartout problem consists of how to share the joint revenues obtained by the sales of passepartouts. Generally speaking, the total revenue obtained from a passepartout system will be less than the total revenue that the service providers would have obtained if they had charged regular prices for the services provided to passepartout holders. On the basis of this feature, we propose a bankruptcy model (for a survey on this topic we refer to Thomson (2003)) to solve passepartout problems. In the corresponding bankruptcy problem the estate will represent the total revenues obtained from the sales of the passepartouts and the vector of claims will represent the total amounts that each service provider would have obtained if its customers with passepartout had paid regular prices instead. Note that whereas the estate is the result of cooperation in the system, the claims are in principle non-cooperative: the providers take as reference point the amount of money that they individually would have obtained in the new

setting on the basis of their individual regular prices.

Various bankruptcy rules have been proposed and characterized in the literature. In order to select an adequate rule for the passepartout problem, we consider context-specific properties that a rule should satisfy. Since the vector of claims reflects the opportunity cost in terms of number of users and regular price per users, we argue that claim truncation is an undesirable property for a passepartout rule. As a result, well-known rules as the constrained equal awards rule, the Talmud rule, and the adjusted proportional rule are not good candidates to become passepartout rules. We further argue that every service provider should be initially entitled to that part of the estate that the others do not claim: the property of minimal rights first. Other properties that we propose are respect of positive claims, equal treatment of equals, claim monotonicity, order preservation, and two more technical ones: self-duality for problems with zero minimal rights and composition down for problems with zero minimal rights. The proportional rule is a well-known and highly accepted rule which does not satisfy claim truncation, but unfortunately does not satisfy minimal rights first, although it does satisfy the above list of further properties. The violation of minimal rights first can be circumvented by assigning first the minimal rights to the service providers and then allocating the remaining estate proportional to the adjusted claims. We will call this rule the proportional rule with minimal rights first (called conceded proportional rule in Moreno-Tertero (2006)), and promote this rule as an excellent candidate for a passepartout rule.

Passepartout problems were first studied in Ginsburgh and Zang (2001, 2003) for the specific case of museum passes. These two papers model passepartout problems as cooperative games where the value of a coalition is the total amount of revenues from the passepartouts of those costumers that only use their passepartouts to make use of the service providers in the coalition. Subsequently, the Shapley value of this game is proposed as a solution to the allocation problem at hand. Obviously, one of the drawbacks of this type of model is the high degree of information needed to derive this game: a historical record of the exact use of each single passepartout is needed. Moreover, this approach lacks a context specific motivation for selecting a general solution as the Shapley value. The novelty of our current approach is twofold. First, the modeling of passepartout problems within the framework of bankruptcy problems. This is done with a relatively low need regarding informational data on costumers behavior (only the number of times that a service provider is used by passepartout holders), while explicitly

taking into account differences in both regular prices and asymmetries in the users' attitudes. Secondly, the normative context-specific analysis of passepartout problems in order to provide an adequate rule for passepartout problems.

2 A bankruptcy approach to passepartout problems

A *passepartout problem* is denoted by (N, π, p, μ, m) where N is the (finite) set of service providers that are involved in the passepartout system, π is the price of the passepartout, $p = (p_i)_{i \in N} \in \mathbb{R}^N$ is the vector of regular prices of the various services, and (μ, m) is a realization of the passepartout system, with μ the number of passepartouts sold and $m = (m_i)_{i \in N} \in \mathbb{R}^N$ indicating for each service provider $i \in N$ the number of times^{1,2} that its services were employed by passepartout owners. Since it is reasonable to assume that $\mu\pi \leq \sum_{i \in N} m_i p_i$ (i.e. customers want to “profit” from their passepartouts), one can define an associated bankruptcy problem (N, E, c) as follows. The estate E equals the total revenues that are obtained from the sale of the passepartouts³, i.e. $E = \mu\pi$, and the vector of claims $c \in \mathbb{R}^N$ represents the opportunity costs for each provider $i \in N$, i.e. the amount $c_i = m_i p_i$ that the provider would have obtained if its services had been charged on the basis of its regular price.

There exist various types of allocation rules for bankruptcy problems in the literature (cf. Thomson (2003)). Formally, a bankruptcy rule R assigns to each bankruptcy problem (N, E, c) a vector of payoffs $R(N, E, c) \in \mathbb{R}^N$. Following Thomson (2003), such a rule is always supposed to satisfy *efficiency*, ($\sum_{i \in N} R_i(N, E, c) = E$), *non-negativity* ($R_i(N, E, c) \geq 0$ for every $i \in N$), and *claim boundedness* ($R_i(N, E, c) \leq c_i$ for every $i \in N$). In order to decide which rule is more adequate for passepartout problems, we analyze passepartout problems in a normative way.

A provider's claim indicates its opportunity cost in terms of the number of users and the regular price per user. The claims of the service providers give an indication of how important they are to the system: the higher a claim of a given service provider, the higher the number

¹If a particular customer with a passepartout card uses the same service twice, this counts as two services.

²Note that (contrary to the model of Ginsburgh and Zang (2003)) we do not need the exact behavior of each passepartout holder, but only the number of times that a service provider is employed by passepartout holders.

³Of course, if there are costs to providing the passepartout facility, this can be taken into account as well, for instance in the definition of the estate E , which is simply the “bag of money” that is left to be divided among the service providers.

of costumers that use its services, indicating that the service provider is attracting customers to the passepartout system. Ignoring any part of this claim is not economically motivated and may punish, in particular those providers that attract clients to buy a passepartout. Due to this, we argue, contrary to standard considerations, that a bankruptcy rule suitable for the passepartout problem should *not* satisfy claim truncation. A bankruptcy rule R satisfies *claim truncation* if the amount that the rule assigns to each provider does not change if the claims are previously truncated by ignoring the part of a claim that exceeds the total estate. Formally, R satisfies claim truncation if for every bankruptcy problem (N, E, c)

$$R(N, E, c) = R(N, E, c')$$

with $c'_i = \min\{c_i, E\}$ for every $i \in N$. Note that since claim truncation seems an undesirable property in the passepartout setting, no game theoretic rule (see Thomson (2003) and Curiel et al. (1987)) is a good candidate to become an adequate rule for passepartout problems. Therefore, well studied bankruptcy rules as the constrained equal awards rule, the Talmud rule, and the adjusted proportional rule are not good candidates to become passepartout rules.

We further argue that a passepartout rule should satisfy minimal rights first. The *minimal right*, $r_i(N, E, c)$, of provider $i \in N$ is the amount that remains if all other providers' claims have been fully satisfied (or 0 if the estate is not high enough to satisfy these claims), i.e. for every bankruptcy problem (N, E, c)

$$r_i(N, E, c) = \max\{0, E - \sum_{j \in N \setminus \{i\}} c_j\}$$

for every $i \in N$. Roughly speaking, the minimal right of provider i is the part of the estate that i is entitled to since nobody else claims it. A bankruptcy rule R satisfies *minimal rights first* if applying the rule directly or first paying each provider its minimal right and then applying the rule to the remaining problem given by the updated estate and updated claims, gives the same result. Formally, R satisfies minimal rights first if for every bankruptcy problem (N, E, c)

$$R(N, E, c) = r(N, E, c) + R(N, E - \sum_{i \in N} r_i(N, E, c), c - r(N, E, c)).$$

Since a passepartout system is based on cooperation among service providers, every service provider that is required to indeed carry out a service for passepartout holders should receive a positive share of the revenues generated by the passepartout. Therefore, we argue that a passepartout rule should satisfy *respect of positive claims*. Formally, a bankruptcy rule

R satisfies respect of positive claims if for every bankruptcy problem (N, E, c)

$$R_i(N, E, c) > 0$$

for every $i \in N$ with $c_i > 0$.

Another important principle of fairness is non-discrimination between service providers, i.e. two service providers with the same claim should get the same amount from the revenues generated by the passepartout system. Therefore, we argue that a passepartout rule should satisfy *equal treatment of equals*. Formally, a bankruptcy rule R satisfies equal treatment of equals if for every bankruptcy problem (N, E, c)

$$R_i(N, E, c) = R_j(N, E, c)$$

for every $i, j \in N$ with $c_i = c_j$.

A further natural requirement on a passepartout rule is that no service provider should suffer from an increase of passepartout holders using its services. Therefore we argue that a passepartout rule should satisfy *claim monotonicity*. Formally, a bankruptcy rule R satisfies claim monotonicity if for every bankruptcy problem (N, E, c)

$$R_i(N, E, (\tilde{c}_i, (c_j)_{j \in N \setminus \{i\}})) \geq R_i(N, E, c)$$

for every $i \in N$ and every $\tilde{c}_i \geq c_i$.

Furthermore, the fact that the claim of a service provider indicates the importance of the service provider to the system requires that a service provider should never get less than another one with a lower claim. On the other hand, in order to protect service providers with a low claim, a service provider with a high claim should lose at least as much as another one with a lower claim. Therefore, we argue that a passepartout rule should satisfy *order preservation*. Formally, a bankruptcy rule R satisfies order preservation if for every bankruptcy problem (N, E, c)

$$R_i(N, E, c) \geq R_j(N, E, c) \text{ and } c_i - R_i(N, E, c) \geq c_j - R_j(N, E, c)$$

for every $i, j \in N$ with $c_i \geq c_j$.

The following two properties are of a more technical nature and are concerned with computability issues. A bankruptcy rule R satisfies *self-duality for problems with zero minimal rights* if given a bankruptcy problem with zero minimal rights, applying the rule directly or indirectly via first giving the full claims to the providers and then subtracting the surplus according to the rule does not make a difference. Formally, a bankruptcy rule R satisfies order

preservation if for every bankruptcy problem (N, E, c) with $r(N, E, c) = 0$

$$R(N, E, c) = c - R(N, \sum_{i \in N} c_i - E, c).$$

A bankruptcy rule R satisfies *composition down for problems with zero minimal rights* if given a bankruptcy problem with zero minimal rights, if the estate is lower than initially recorded, then every claimant gets the same in the new situation as if the rule is applied to the new estate using as claims what the claimants had been initially allocated. Formally, if for every bankruptcy problem (N, E, c) with $r(N, E, c) = 0$ and every $E' < E$

$$R(N, E', c) = R(N, E', R(N, E, c)).$$

The *proportional rule* is a well-known and widely accepted rule for bankruptcy problems. For a bankruptcy problem (N, E, c) , the proportional rule assigns

$$\text{PROP}(N, E, c) = \lambda c$$

with $\lambda = \frac{E}{\sum_{j \in N} c_j}$.

It is well-known that the proportional rule is the only rule satisfying both self-duality and composition down (see Thomson (2003)). Unfortunately, the proportional rule does not satisfy minimal rights first. A way to circumvent the violation of minimal rights first is by first allocating the minimal rights and then sharing the remaining estate proportionally to the adjusted claims in a second stage. This new rule is called the *proportional rule with minimal rights*⁴. Formally, for a bankruptcy problem (N, E, c) , the proportional rule with minimal rights assigns

$$\text{MRPROP}(N, E, c) = r(N, E, c) + \text{PROP}\left(N, E - \sum_{i \in N} r_i(N, E, c), c - r(N, E, c)\right).$$

Next, we provide two examples that illustrate the computation of the proportional rule with minimal rights and that points out how the proportional rule with minimal rights takes into account possible asymmetries among the service providers.

Example 2.1. Suppose that we have two service providers, 1 and 2, with a passepartout system. The regular prices of the services for provider 1 and 2 are 10 and 5 Euro, respectively. The

⁴This rule is called the conceded proportional rule in Moreno-Ternero (2006).

price of the passepartout is 12 Euro. Assume that each person who buys a passepartout uses each service once and that the total number of passepartouts sold equals 100. For the associated bankruptcy problem (N, E, c) we find that $E = 1200$ and $c = (1000, 500)$. In this case, the vector of minimal rights is $r(N, E, c) = (700, 200)$ and the allocation proposed by the proportional rule with minimal rights is given by $\text{MRPROP}(N, E, c) = (700, 200) + \text{PROP}(N, 300, (300, 300)) = (850, 350)$. This outcome clearly takes into account the asymmetry in regular prices between the two services. \diamond

Example 2.2. Suppose again that we have two service providers, 1 and 2, with a passepartout system. The regular price of the services for both providers is 10 Euro while the price of the passepartout is 15 Euro. Assume that each person who buys a passepartout uses service 1 once and service 2 twice and that the total number of passepartouts sold equals 100. For the associated bankruptcy problem (N, E, c) it holds that $E = 1500$ and $c = (1000, 2000)$. In consequence, the vector of minimal rights is $r(N, E, c) = (0, 500)$ and the allocation proposed by the proportional rule with minimal rights is given by $\text{MRPROP}(N, E, c) = (0, 500) + \text{PROP}(N, 1000, (1000, 1500)) = (400, 1100)$. This outcome reflects the fact that there is a clear difference in the utilization attitude of passepartout holders although the providers are symmetric on the basis of regular prices. \diamond

The following theorem is a direct consequence of the characterization of the proportional rule by means of self-duality and composition down (see Thomson (2003)).

Theorem 2.3. The proportional rule with minimal rights is the unique rule satisfying minimal rights first, self-duality for problems with zero minimal rights, and composition down for problems with zero minimal rights.

Moreover, it is readily verified that

Proposition 2.4. The proportional rule with minimal rights satisfies respect of positive claims, equal treatment of equals, claim monotonicity and order preservation. Besides, it does not satisfy claim truncation.

3 Passepartout games á la Ginsburgh and Zang

A game theoretical approach to passepartout problems was proposed by Ginsburgh and Zang (2003) for the special case in which the service providers are museums. Given a passepartout problem (N, π, p, μ, m) , they denote by M the set of customers that buy a passepartout card (note that $|M| = m$) and for every $j \in M$, they denote by $K_j \subset N$ the set of service providers that customer j uses (at least once). Then, a corresponding passepartout game (N, v) is defined by

$$v(S) = \pi \cdot |\{j \in M \mid K_j \subset S\}|$$

for every $S \subset N$. The Shapley value of this game is proposed as a way to share the joint revenues $m\pi$ from the passepartout system. Ginsburgh and Zang (2001) admit that their approach may not be suitable when the regular prices of the service providers are different. Quoting Ginsburgh and Zang (2001), p 372:

“[S]ince the nature of the service provided and its pricing are different, it seems reasonable . . . to use a variant of the scheme offered below where the allocations are weighted by the individual admission prices”

As argued in the introduction, there are two other drawbacks: the lack of a methodological study of the passepartout problem to select the Shapley value of the corresponding game as a passepartout rule, and the high degree of informational data needed to obtain the coalitional values of the game.

We now reconsider our two earlier examples to illustrate serious drawbacks of Ginsburgh and Zang’s proposal, not only concerning difference in prices but also concerning asymmetries in actual users’ attitude.

Example 3.1. Reconsider the problem introduced in Example 2.1. Using Ginsburgh and Zang’s solution, each provider would obtain 600 Euro. In this proposal service provider 2 obtains more than it would have obtained if all passepartout services had been paid according to its regular price (500 Euro). \diamond

Example 3.2. Reconsider the problem introduced in Example 2.2. Using Ginsburgh and Zang’s solution, each service provider would obtain 750 Euro from this museum pass system. Therefore, in this case both service providers obtain the same amount, although service providers 1

and 2 would have earned 1000 and 2000 Euro respectively if all services had been carried out on the basis of regular prices. \diamond

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