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Is There Really A Green Paradox?

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GROWTH, RENEWABLES AND THE OPTIMAL CARBON TAX*

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Abstract

Optimal climate policy is studied in a Ramsey growth model with exhaustible oil reserves, an infinitely elastic supply of renewables, stock-dependent oil extraction costs and convex climate damages. We concentrate on economies with an initial capital stock below that of the steady state of the carbon-free economy and the initial cost of oil (extraction cost plus scarcity rent and social cost of carbon) below that of renewables. There are then two regimes. If the oil stock is small, the social optimum path consists of an initial oil-only phase followed by a renewables-only phase. With a lower cost of renewables or a lower discount rate, more oil is left in situ and renewables are phased in more quickly. The optimal carbon tax rises along its development path during the oil-only phase, but the rise flattens off as less accessible reserves are explored and the social cost of carbon increases. Subsidizing renewables without an optimal carbon tax induces more oil to be left in situ and a quicker phasing in of renewables, but oil is depleted more rapidly initially. The net effect on global warming is ambiguous. The second regime occurs if the initial oil stock is large. The social optimum then consists of an initial oil-only phase followed with a final oil-renewables phase. This regime converges to the steady state of the carbon-free economy as well. The paper also gives a full characterization of two other regimes that occur if the initial capital stock is above its steady state or renewables have an initial cost advantage.

Keywords: Green Ramsey model, carbon tax, renewables, exhaustible resources, global warming, development, growth, intergenerational inequality aversion, second best, Green Paradox

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1. Introduction

Global warming and global economic development and how to reconcile the two are the most pressing issues facing our societies today. A substantial and possibly rising carbon tax is needed not only to curb demand for fossil fuel, but also to accelerate the switch from a carbon-based to a carbon-free economy. It does this, on the one hand, by encouraging the market to leave more oil in the crust of the earth, and, on the other hand, by making renewables more competitive and speeding up the moment of time that they are phased in. Subsidies for carbon-free renewables are, unless there are substantial learning-by-doing or other market failures, a poor substitute for a credible and lasting anticipated path of present and future carbon taxes. The optimal path for the carbon tax depends on the cost of renewables versus the cost of extracting fossil fuel, where the latter will increase as less accessible reserves have to be exploited. The optimal carbon tax should be set to the social external cost of carbon, which is the present value of all future marginal damages from global warming. But the social cost of carbon is highly endogenous as it depends on both the level of consumption and capital in the economy. More mature economies have a lower marginal utility of consumption and thus a higher social cost of carbon and a higher carbon tax.

Our main objective is to develop a Green Ramsey model, which can be used to analyze the intricate tradeoffs between economic development and fighting global warming with an emphasis on different regimes of energy use (i.e., only oil, only renewables or both). The transition and timing of the transition between these regimes is endogenous. Our model has exhaustible oil[1] reserves, an infinitely elastic supply of renewables, stock-dependent oil extraction costs and convex climate damages. If the initial social cost of oil consisting of the marginal extraction cost, the Hotelling rent and the social cost of carbon is less than that of renewables, the economy starts off using oil. For an economy starting with a capital stock below the steady state of the carbon-free economy, we then formally establish that there are two regimes. Regime I occurs if the initial stock of oil is small enough and has an initial oil-only phase followed by a final phase where only renewables are used. Regime II occurs if the initial stock of oil is large enough and has an initial oil-only phase followed by a final phase where renewables and oil are used simultaneously. In this regime consumption and capital overshoot the steady-state values of the carbon-free economy. Compared to regime I, oil use is higher in the early part of the initial oil-only phase and is curbed back substantially in the latter part. The oil-abundant regime II never phases out oil and phases in renewables at a much later date than the oil-scarce regime I. Furthermore, we establish that the "laissez-faire" outcome always has an initial oil-only phase followed by a carbon-free, renewables-only phase.

[1] We use for sake of brevity 'oil' rather than 'fossil fuel', so oil refers to natural gas, coal and the tar sands as well.

A third regime occurs if renewables have an initial cost advantage, which is the case if the cost of renewables is low, the initial oil stock is low and thus oil extraction costs are high, the discount rate is low and initial capital and consumption are high as then the social cost of carbon is high. Renewables are then used forever and oil is never phased in.

Finally, for economies with initial capital stocks above the carbon-free steady-state value, we establish three possibilities. Regime III of using renewables forever is enlarged for all initial oil stocks below the steady state. If the initial oil stock is above the steady state but not too much, regime IV occurs where the economy sets off using only renewables and then switches at some time to using renewables and oil alongside each other. If the initial oil stock is quite a bit above steady state, we get back to regime II with an initial oil-only phase followed by a phase with simultaneous use of oil and renewables. Until there is a breakthrough in renewables technology, regimes III and IV will not occur. So we focus mostly on the most relevant regimes I and II where oil has an initial cost advantage.

The innovation of our paper is not only the precise characterization of which of the various regimes takes place and whether oil and renewables are used on their own or used simultaneously in the production process, but also the endogenous determination of the optimal switch time between the different phases of each regime and the optimal amount of oil to be left unused in the crust of the earth.

A recent paper by Golosov et al. (2011) is in some respects close to ours. It looks at backstops in a Ramsey or Dasgupta-Heal-Solow-Stiglitz type model (Dasgupta and Heal, 1974; Solow, 1974; Stiglitz, 1974)), with stock-dependent extraction costs, albeit that they model global warming externalities as directly impacting aggregate production whereas we allow for them as losses to social welfare. For their analytical results they make five assumptions: (i) a logarithmic utility function; (ii) a Cobb-Douglas production function; (iii) a negative exponential function for multiplicative output damages; (iv) 100 per cent depreciation of manmade capital at the end of each discrete period; and (v) zero extraction costs for oil. These strong assumptions deliver a closed-form expression for the value function and a constant consumption-output ratio. The social cost of carbon is then proportional to output and high if the climate-sensitivity parameter is large and the social discount rate is small. The time path of their optimal carbon tax has an inverse U-shape, where the eventual decline of the carbon tax results from their assumption of natural decay of the atmospheric concentration of CO2 and that oil reserves are fully exhausted.

Our interests in characterizing the optimal climate policy are threefold: first, what are the determinants of the date that renewables kick in and the date that oil is phased out altogether; second, are renewables going to be used on their own or alongside oil and what is the optimal sequencing of oil and renewables; and third, how much oil is it socially optimal to leave in situ. We thus devote considerable attention to the intricate connections between the oil economy and the carbon-free economy. In the oil-scarce economy, the optimal carbon tax rises during the phase where oil is used until the moment that renewables replace oil. The rise in the optimal carbon tax flattens with time, since private agents hold back depletion by themselves as less accessible reserves have to be explored. Furthermore, the level of the optimal carbon tax depends on the stage of economic development of the economy. If the economy is at a low level of economic development, it has a low carbon tax rate which rises as the economy develops which echoes the prescriptions of those who argue in favor of a rising ramp for the carbon tax (e.g., Nordhaus, 2007). However, an economy starting with a higher capital stock, but still below the carbon-free steady state, has a higher carbon tax rate. If the initial level of the capital stock is high enough, the economy switches to oil-abundant regime with overshooting of consumption and capital. In that case, it is possible that the optimal carbon tax eventually falls.

The switch to renewables occurs more quickly in a socially optimum than in a market outcome which does not internalize global warming damages. The amount of oil reserves that is left in situ in the optimal outcome follows from the condition which says that the cost of extracting the last drop of oil plus the social cost of carbon (the present value of marginal global warming damages) must equal the cost of the backstop. We show that more oil reserves are left in situ if the social rate of discount is low, the climate challenge is acute, and the initial stock of oil reserves is high. The stock that is left in situ is higher if the economy is more developed and consumption is relatively close to its steady state. Conversely, if the economy is still underdeveloped and consumption low, less oil reserves will be left in situ.

We thus extend the classic Ramsey model of economic growth to allow for natural exhaustible resources, renewable backstops and global warming damages. Our analysis is related to the famous DHSS growth model with investment in manmade capital and natural exhaustible resources as factors of production, but differs in that we allow for renewable backstops and global warming damages. Our analysis is also related to earlier studies on pollution in the Ramsey model (e.g., van der Ploeg and Withagen, 1991), on capital accumulation, oil depletion and backstops (e.g., Tahvonen, 1997; Tsur and Zemel, 2003, 2005), on pollution and climate change in models with depletion of exhaustible resources but without investment and growth (e.g., Krautkraemer, 1985, 1998; Withagen, 1994), and on those studying regimes with simultaneous use of oil and renewables albeit in economies without capital (Hoel and Kverndokk, 1996).

Our analysis extends earlier results on the second-best effects of subsidizing clean backstops on oil exhaustion, speed and duration of phasing in of the backstop, and the effects on green and total welfare (Hoel, 2008; van der Ploeg and Withagen, 2010; Grafton, Kompas and Long, 2010; Gerlagh, 2011) to allow for saving, investment and capital accumulation. We thus offer a general-equilibrium analysis of the Green Paradox (Sinn, 2008ab), which says that subsidizing clean renewables (e.g., solar or wind energy) instead of implementing a carbon tax is counterproductive as owners of oil fields are encouraged to pump more quickly, global warming is accelerated and thus green welfare is reduced. However, if the social discount rate that is used is low (cf., the Stern Review, 2007) and global warming damages are acute, it is optimal to leave more oil reserves in the crust of the earth in which case cheaper renewables induce a bigger fraction of oil reserves to remain unexploited and global warming damages fall (van der Ploeg and Withagen, 2010). The Green Paradox then does not occur. Simulations of our Green Ramsey model suggest that this paradox is less likely to occur for a mature than a developing economy with relatively scarce capital, high marginal utility of consumption and low marginal global warming damages. This dilemma for the early stages of economic development reminds one of Bertolt Brecht's dictum: 'Erst kommt das Fressen: Dann kommt die Moral' (from *Die Dreigroschenoper)*.

Section 2 presents the Green Ramsey model and the optimality conditions for the social optimum. Section 3 gives an informal presentation of our main results regarding the four regimes that can occur in the socially optimal and "laissez-faire" outcomes and it also gives the two conditions necessary to pin down the optimal stock of oil to be left in situ and the time at which renewables are phased in. Section 4 gives formal proof of the regimes that can occur and details of the conditions that delineate the regimes. This makes use of the so called threshold economy, which is defined as the economy where after an initial oil-only phase the steady state of the carbon-free economy is exactly reached at the end of this oil-only phase and the economy stays there forever after. Section 5 formally establishes the two regimes of the "laissez-faire" economy and derives some properties of the level and time profile of the optimal carbon tax that replicates the socially optimal outcome. It also discusses second-best climate policy for the market economy if an optimal carbon tax is infeasible and why this is more likely to induce a Green Paradox if the economy is still developing. Section 6 present simulations for the social optimum and the "laissez-faire" outcome of the oil-scarce regime I of the Green Ramsey model. The level and time profile of the optimal carbon tax depends on the stage of economic development, the aversion to intergenerational inequality and the rate of time preference, and sheds light on the Green Paradox. Section 7 offers policy simulations for the oil-abundant regime which has a final phase where oil and renewables are used simultaneously. Section 8 concludes.

2. The Green Ramsey model

Let $O(t) \geq 0$ denotes oil use and $S(t) \geq 0$ the stock of remaining oil reserves at instant of time t. Then along a feasible program we have for all $t \geq 0$:

(1) $\dot{S}(t) = -O(t), \quad S(0) = S_0,$

where $S_0 > 0$ is the given initial stock of oil reserves. Hence, total oil depletion cannot exceed initial reserves: $\int_0^{\infty} O(t)dt \leq S_0$. We abstract from natural degradation of CO2 in the atmosphere, so the change in the stock of atmospheric carbon E is proportional to oil depletion for all $t \geq 0$:

(2) $\dot{E} = O, \quad E(0) = E_0,$

where $E_0 > 0$ is the initial stock of atmospheric carbon and we have normalized so that the CO2-emission ratio equals one. Hence, the stock of carbon in the atmosphere equals the initial stock plus the accumulated sum of past CO2 emissions: $E(t) = E_0 + \int_0^t O(t)dt = E_0 + S_0 - S(t), \forall t \geq 0.$

Manmade capital K and energy are inputs in the production process, which is described by the production function F. We suppose that energy from oil, O, and energy from renewables, R, are perfect substitutes.[2] We denote by $G(S)$ the cost needed to extract one unit of oil.

Assumption 1: $G'(S) < 0, \forall S > 0, \lim_{S \to 0} G(S) = \infty, \lim_{S \to \infty} G(S) = 0.$

We thus assume that the cost of extracting one unit of oil rises as fewer oil reserves are left, $G'(S) < 0$, and that oil extraction costs become infinitely large as oil reserves become fully exhausted. The latter ensures that oil reserves will never be fully exhausted.

The material balance equation of the economy and the investment dynamics are:

(3) $\dot{K} = F(K, O + R) - G(S)O - bR - C - \delta K, \quad K(0) = K_0,$

where C is consumption, δ the depreciation rate of manmade capital, and K_0 the initial capital stock.

Assumption 2: F has non-increasing returns to scale. It is strictly concave and increasing for positive inputs. $F(K, 0) = F(0, O + R) = 0$. $\lim_{O+R \to 0} F_{O+R}(K, O + R) = \infty, \lim_{O+R \to \infty} F_{O+R}(K, O + R) = 0, \forall K > 0$.[3]

$\lim_{K \to 0} F_K(K, O + R) = \infty, \lim_{K \to \infty} F_K(K, O + R) = 0, \forall O + R > 0.$

The production function thus satisfies the Inada conditions. Assumption 2, together with $\delta > 0$, implies that without the use of the backstop the economy cannot maintain a positive constant level of consumption (Dasgupta and Heal , 1974).

[2] Wind and solar energy are more likely to be complements than substitutes for oil. We assume perfect substitutes for analytical convenience; relaxing it would introduce regimes where the two types of fuel are used alongside each other. See Smulders and van der Werf (2008) and Michielsen (2011) for partial equilibrium studies which do allow for imperfect substitution between oil and the backstop in the partial equilibrium analysis of climate policy.

[3] For the derivative of the production function with respect to energy, we use F_O, F_R and F_{O+R} interchangeably.

Intertemporal social welfare W depends on utility of consumption and damage from accumulated CO2:

(4) $$W = \int_0^\infty e^{-\rho t} \left[U(C(t)) - D(E_0 + S_0 - S(t)) \right] dt,$$

where $\rho > 0$ is the constant social rate of discount. The instantaneous utility function, *U*, is concave and satisfies the Inada condition, which ensures positive consumption throughout. Global warming damages, *D*, and marginal damages increase in the stock of atmospheric carbon[4].

Assumption 3: $U'(C) > 0, U''(C) < 0, \forall C > 0.\ \lim_{C \to 0} U'(C) = \infty.\ \ D'(E) > 0, D''(E) > 0, \forall E > 0.$

The social planner maximizes (4) subject to (1)-(3) and the non-negativity constraints. The social cost of carbon is denoted by τ and corresponds to the present discounted value of marginal global warming damages (e.g., Nordhaus, 2011):

(5) $$\tau(t) \equiv \frac{\int_t^\infty e^{-\rho(s-t)} D'(E(s)) ds}{U'(C(t))}.$$

Defining the elasticity of intertemporal substitution $\sigma \equiv -U'(C) / CU''(C) > 0$, the social price of energy as $p = F_{O+R}(K, O+R)$, and the net rate of return on capital as $r \equiv F_K - \delta$, we obtain proposition 1.

Proposition 1: The social optimum satisfies (1)-(3) and the optimality conditions:

(6) $$p \le b,\ R \ge 0, \text{c.s.},$$

(7) $$\dot{p} = r\left[p - G(S) \right] - \frac{D'(E)}{U'(C)} \text{ if } O > 0,$$

(8) $$\dot{C} = \sigma(r - \rho)C,$$

(9) $$\dot{\tau} = r\tau - D'(E) / U'(C),$$

(10) $$\lim_{t \to \infty} \left[K(t) + \left(p(t) - G(S(t)) \right) S(t) - \tau(t)(E_0 + S_0) \right] U'(C(t)) e^{-\rho t} = 0.$$

Proof: The problem is to maximize (4) subject to (1)-(3). Letting μ_K be the marginal social value of manmade capital, μ_S the marginal social value of oil reserves and μ_E the marginal social cost of carbon

[4] Temperature is a concave function of accumulated CO2 emissions. So even if damages are a convex function of temperature, damages need not necessarily be a convex function of accumulated CO2 emissions. An alternative is to suppose that global warming damages production as in the RICE and DICE models of Nordhaus (2007) or in the Ramsey growth models of Golosov et al. (2011) and Gerlagh and Liski (2012).

in the atmosphere, the Hamiltonian function for this problem reads $H \equiv U(C) - D(E) +$ $\mu_K \left[F(K, O+R) - C - G(S)O - bR - \delta K \right] - \mu_S O - \mu_E O.$ We thus get the necessary optimality conditions:

(11a) $U'(C) = \mu_K, \quad \rho\mu_K - \dot{\mu}_K = \left[F_K(K, O+R) - \delta \right] \mu_K,$

(11b) $F_R(K, O+R) \le b$ and $R \ge 0$, c.s.,

(11c) $F_O(K, O+R) - G(S) - (\mu_S + \mu_E) / \mu_K \le 0$ and $O \ge 0$, c.s.,

(11d) $\rho\mu_S - \dot{\mu}_S = -G'(S)O\mu_K,$

(11e) $\rho\mu_E - \dot{\mu}_E = D'(E),$

(11f) $\lim_{t\to\infty} \left[\mu_K(t)K(t) + \mu_S(t)S(t) - \mu_E(t)E(t) \right] e^{-\rho t} = 0.$

If $R > 0$, $p = F_R = b$ which gives (6). If $O > 0$, (11c) gives $p = G(S) + (\mu_S + \mu_E) / \mu_K$ and thus (11c) together with (11a), (11d) and (11e) give (7). Equation (8) follows from (11a). From (11a) and (11e) we have $\tau = \mu_E / \mu_K$, and thus (9). Given that $U'(C) = \mu_K > 0$ (from assumption 3), the transversality condition (11f) can be written as (10). Q.E.D.

Equation (6) states that the social price of the renewable backstop, if in use, equals its marginal cost *b*. If oil is used in the production process, (7) indicates that the social price of oil follows a modified version of the Hotelling rule. The return on leaving a marginal barrel of oil in the earth (i.e., the capital gains on oil reserves) should thus equal the return from depleting a marginal barrel of oil, selling it and obtaining the social rate of return on capital *minus* the marginal cost of oil extraction *minus* the marginal global warming damages resulting from burning this barrel of oil (after converting these from utility to final goods units). Without global warming externalities and extraction costs, we get the familiar Hotelling rule which says that the capital gains on oil should equal the market rate of interest. Note that the Hotelling rent on oil $p - G(S)$ vanishes when the "laissez-faire" economy relying on only oil approaches the moment in time where the renewable backstop is introduced (cf., Heal, 1976). Equation (7) can also be written as $\frac{d\left[p - G(S)\right]}{dt} = r\left[p - G(S)\right] + G'(S)O - \frac{D'(E)}{U'(C)},$ which says that the Hotelling rent on oil increases at a lower rate than the social rate of return on capital for two reasons. First, extracting more oil pushes up extraction costs, which makes oil depletion more conservative. Second, extracting more oil raises the stock of atmospheric carbon and this pushes up marginal climate damages, which makes oil depletion also more conservative.

Equation (8) is the familiar Keynes-Ramsey rule for consumption growth. Growth in consumption is high, for a given σ, if the return on capital is high and consumers are relatively patient. The coefficients of relative intergenerational consumption inequality aversion and relative risk aversion equal $1/\sigma$. Consumption decreases in the social value of capital μ_K, since the marginal utility of consumption must equal the social value of capital, $U'(C) = \mu_K$.

If oil is in use, the social price of oil p must equal the total cost of oil which consists of three components. The first is the unit extraction cost. The second is the scarcity rent on oil, μ_S/μ_K, which is the discounted value of all future benefits in terms of lower extraction costs of having a higher oil stock. Indeed, it follows from (11d) that $\mu_S(t) = -\int_t^\infty e^{-\rho(s-t)} G'(S(s))O(s)\mu_K(s)ds$. The third component is the social cost of carbon defined in (5). Discounting in goods units, (9) gives an alternative expression for the social cost of carbon:

$$(5') \qquad \tau(t) = \int_t^\infty e^{-\int_t^s r(s')ds'} D'(E(s)) / U'(C(s))ds > 0.$$

The social price of oil must thus equal its marginal social cost: $p = G(S) + (\mu_S / \mu_K) + \tau$. Along an optimal program the transversality condition (10) has to be satisfied.

3. The socially optimal regimes of the Green Ramsey model

We first give a brief informal description of the regimes that can occur in the social optimum and the "laissez-faire economy before formally establishing our results in sections 4 and 5. Panel (a) of fig. 1 depicts the four socially optimal regimes that can occur in our Green Ramsey model, where each regime depends on a particular combination of the initial capital stock given on the horizontal axis and the initial stock of oil reserves given on the vertical axis.[5] The point (S^*, K^*) is where the four dividing curves meet. It corresponds to the *pivotal* economy which gives the initial conditions for which it is optimal to start using renewables and the economy starts from and remains forever at the steady state of the carbon-free economy. Before we discuss the pivotal economy, we must describe the carbon-free economy.

3.1. The carbon-free and the pivotal economy

By definition the carbon-free economy uses no oil: $O = 0$. Demand for renewables then follows from setting the marginal product of renewables to its cost, $F_R(K, R) = b$:

[5] Fig. 1 has been drawn for the parameter values used in sections 6 and 7, but its qualitative shape holds generally.

(12) $R = V(K,b),\quad \text{with } V_K = -F_{KV} / F_{VV} > 0 \text{ and } V_b = 1/F_{VV} < 0.$

Defining output net of fuel costs as $\tilde{F}(K,b) \equiv F(K,V(K,b)) - bV(K,b),$ where $\tilde{F}_K = F_K > 0$ and $\tilde{F}_b = -V(K,b) < 0,$ we write the material balance equation and the Keynes-Ramsey rule as:

(3R) $\dot{K} = \tilde{F}(K,b) - \delta K - C,$

(8R) $\dot{C} / C = \sigma\left[\tilde{F}_K(K,b) - \delta - \rho\right].$

Assumption 2 implies that there exists a unique interior steady state of (3R) and (7R) denoted by (K^*, C^*, R^*) and defined by $F_K(K^*, R^*) = \rho + \delta,\ F_R(K^*, R^*) = b$ and $C^* = F(K^*, R^*) - bR^* - \delta K^*.$ From any initial stock of capital, the system (3R) and (8R) converges towards the steady state. If there is an exogenous shock in, say, a drop in the resource cost b, this induces on impact an upward jump in consumption and subsequently consumption and capital increase along the saddlepath $C(t) = \Theta^R\left(K(t);b\right),$ where $\Theta^R_K\left(K;b\right) > 0, \Theta^R_b\left(K;b\right) < 0.$ The suffix R indicates the carbon-free economy. The carbon-free steady state is on the saddlepath, i.e., $C^* = \Theta^R(K^*;b).$

We define the 'pivotal' economy as the economy which starts at time zero from $K_0 = K^*$ and $S_0 = S^*$, where the pivotal stock of oil reserves follows from:

(13) $$G(S^*) + \frac{D'(E_0)}{\rho U'(\Theta^R(K^*;b))} = b.$$

This condition ensures that at time zero the economy is indifferent between using renewables and oil, because the marginal cost of extraction oil plus the social cost of carbon equals the cost of the backstop.

Assumption 4: $\dfrac{D'(E_0)}{\rho U'(\Theta^R(K^*;b))} < b.$

Given assumption 4, condition (12) yields a positive value of S^*.[6] The program $\{R(t) = 0,\ X(t) = X^*,$ $C(t) = C^*,\ S(t) = S^*$ and $K(t) = K^*, \forall t \geq 0\}$ satisfies the necessary conditions and is thus an optimal program. The pivotal economy thus corresponds to the initial stocks of oil reserves and capital stock so that it is optimal to remain forever in the carbon-free steady state.

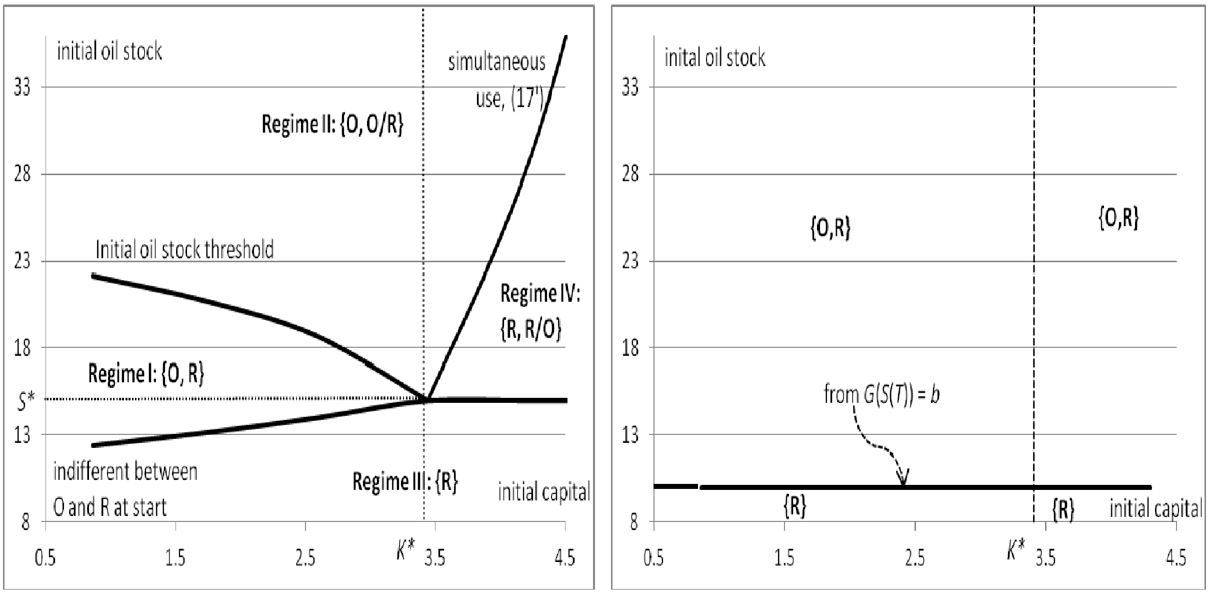
[6] If assumption 4 is not satisfied, it is then even with a large initial oil stock optimal to start with the backstop and never to use oil because the initial pollution level is too high compared to the cost of renewables to warrant any oil use. Note that this does not imply that for any initial capital stock the economy should always abstain from using oil. For a very small initial capital stock, the initial rate of consumption on the stable saddlepath is arbitrarily small and its marginal utility is arbitrarily close to zero. Hence, oil use is still an option but it will cease very quickly.

3.2. Characterization of the four regimes of the socially optimal outcome

Before describing the four optimal regimes it should be stressed that fig. 1(a) should not be read as a phase diagram. On the axes we have the initial oil and capital stocks and the initial CO2 stock is given. The latter stock changes along all optimal paths where oil extraction takes place. Hence, if we take an initial point in the region that is indicated by regime I we can say that it is optimal to have first only oil and then only renewables, but it is not true in general that the development of the economy, as given by the development of oil, capital and CO2 can be depicted as a path in this region.

Figure 1: Characterization of regimes of the Green Ramsey model

(a) Social optimum (proposition 2) **(b) "Laissez-faire" (proposition 3)**



Panel (a) of fig. 1 indicates that, starting from the pivotal economy, a smaller initial capital stock (while keeping the oil stock fixed) implies that it is attractive to start using oil and switch to using renewables after some point of time before the carbon-free steady-state level of capital is reached (the oil-scarce regime I). If a high enough initial oil stock is available and capital is below its steady state, the economy crosses the threshold and overshoots the carbon-free steady state. It is then optimal to start with oil and phase in renewables alongside oil after some time (the oil-abundant regime III). In the later phases of this optimal program it is optimal to approach the steady-state capital stock from above with oil and renewables used forever alongside each other and with oil use vanishing eventually.

Starting from the pivotal economy, a lower initial oil stock but keeping the initial capital stock at its steady state level, implies that oil is never used (regime III). The reason is that with a lower initial oil stock it is more and thus too expensive to extract oil. For an initial capital stock higher than the carbon-free steady state stock and an initial oil stock higher than S^* (but not too much higher), oil will be used at some future date, but only after a period of time where only renewables are employed. Moreover, oil is then used alongside with renewables (regime IV). For a high enough initial oil stocks we are back in regime II again where it is optimal to have oil use only from the outset, whereas after some time a final phase with simultaneous use begins.

3.3. Optimal amount of oil to be left in situ and timing of phasing in of renewables

Here we discuss how to determine the optimal amount of oil to be left in the crust of the earth and the time that renewables are phased in regimes I, II and IV. During a phase where oil and renewables are used alongside each other, the economy can be described by two differential equations in K and C and the stable manifold of this economy can be written as $C(t) = \Theta^{RO}\left(K(t);b\right)$ (see appendix 2.1, part II (ii)). Denoting the time that renewables are phased in by T, we can solve for T and $S(T)$ from the following arbitrage conditions must hold at that moment of time for regime I and regime II (see lemma 2 below):

(14a) $$p(T) = b \quad \text{and} \quad G\left(S(T)\right) + \frac{D'\left(E_0 + S_0 - S(T)\right)}{\rho U'\left(\Theta^R\left(K(T);b\right)\right)} = b,$$

(14b) $$p(T) = b \quad \text{and} \quad G\left(S(T)\right) + \frac{D'\left(E_0 + S_0 - S(T)\right)}{\left(\tilde{F}_K(K(T);b) - \delta\right)U'\left(\Theta^R\left(K(T);b\right)\right)} = b.$$

These conditions paste the two systems of differential equations corresponding to the two phases occurring in these regimes (see appendix 2). We thus have that the price of energy must be continuous at the moment that renewables are phased in, i.e., $p(T) = b,$ so that there are no unexploited opportunities for improving social welfare. The paths of capital, consumption and the stock of oil reserves must also be continuous and thus should not jump at the switch time T. Furthermore, the cost of extracting the last drop of oil plus the resulting social cost of carbon must equal the cost of renewables.

4. Formal characterization of the socially optimal regimes

Proposition 2 below gives formal proof of our characterization of the optimal sequencing and phases of the socially optimal outcome depicted in fig. 1(a) and which conditions delineate the different regimes.

4.1. Delineating regimes I and III and regimes III and IV

Lemma 1: Suppose the initial oil stock and initial capital stock satisfy

(15) $$b < G(S_0) + \frac{D'(E_0)}{\rho U'(\Theta^R(K_0;b))} \quad \text{and} \quad K_0 < K^*$$

or

$$(16)\qquad b < G(S_0) + \frac{D'(E_0)}{\rho U'(\Theta^R(K^*;b))} \text{ and } K_0 > K^*,$$

then it is optimal to start the optimal program using only renewables. Afterwards, it is never optimal to phase in oil.

Proof: Take $O(t) = 0, \mu_E(t) = D'(E_0)/\rho$ and $C(t) = \Theta^R\left(K(t);b\right), \forall t \geq 0.$ With this choice we have $G(S(t)) + \frac{\mu_S(t)+\mu_E(t)}{\mu_K(t)} = G(S_0) + \frac{D'(E_0)}{\rho U'(\Theta^R(K(t);b))}, \forall t \geq 0.$ If (15) holds, then $b < G(S(t)) + \frac{\mu_S(t)+\mu_E(t)}{\mu_K(t)}, \forall t \geq 0,$ because $C(t) = \Theta^R\left(K(t);b\right) > \Theta^R\left(K_0;b\right), \forall t \geq 0$ and consumption is increasing. If (16) holds then the same result is obtained because $C(t) = \Theta^R\left(K(t);b\right) > \Theta^R\left(K^*;b\right), \forall t \geq 0.$ The proposed program satisfies all the necessary conditions and is thus the unique optimum. Q.E.D.

It now follows that, if in condition (15) the first part holds with equality, it yields the upward-sloping locus arriving from the left at the pivotal economy in fig. 1(a) and delineates regimes I and III. If the economy starts off satisfying (15), it will never phase in oil (regime III). If the economy satisfies (16) or equivalently $\{S_0 < S^*, K_0 > K^*\}$, it will never phase in oil either (regime III). However, if $\{S_0 > S^*, K_0 > K^*\}$, it could be the case that after some time it is optimal to make the transition to simultaneous use (regime IV). We will turn to this case now. The horizontal segment starting towards the right from the pivotal economy in fig. 1(a) delineates regimes III and IV.

4.2. Delineating regimes II and IV

Lemma 2: Simultaneous use of oil and renewables can only occur if $K > K^*$ and requires that:

$$(17)\qquad p = b \text{ and } G(S) + \frac{D'(E)}{\left[F_K(K,V(K;b)) - \delta\right]U'(C)} = b.$$

Proof: Along any interval of time with simultaneous use we have from (11b) and (11c) that $b = G(S(t)) + \frac{\mu_S(t)+\mu_E(t)}{\mu_K(t)}$. Using (11d) and (11e), and (11b) again, this implies that along the interval:

$$(18)\qquad 0 = \rho\mu_E(t) - D'(E(t)) + \rho\mu_S(t) + \left[F_K(K(t),V(K(t);b)) - \delta - \rho\right]\left[b - G(S(t))\right],$$

because total energy use if $R > 0$ is given by $O(t) + R(t) = V(K(t);b)$. We have $\rho\mu_E(t) - D'(E(t)) \geq 0$, $\forall t \geq 0$. Indeed if μ_E would ever fall, it will decrease without bound because there is no decay of CO2. That cannot be optimal. So, simultaneous use only occurs if $F_K(K(t), V(K(t);b)) < \rho + \delta$ which demands $K(t) > K^*$. We now use $b = G(S(t)) + \dfrac{\mu_S(t) + \mu_E(t)}{\mu_K(t)}$, $\mu_K = U'(C)$, and $(\mu_S + \mu_E)/\mu_K = F_{O+R}(K, O+R) - G(S)$ from (11c) into (18) to obtain (17). Q.E.D.

We can use (17) at time zero to characterize the locus of initial oil and capital stocks, given the initial CO2 stock, for which it is optimal to have simultaneous use throughout:

$$(17') \qquad p(0) = b \quad \text{and} \quad G(S_0) + \frac{D'(E_0)}{\left[F_K(K_0, V(K_0;b)) - \delta\right] U'(\Theta^{RO}(K_0;b))} = b.$$

Clearly, this locus starts in the pivotal economy, defined in section 3.1. It slopes upwards from (K^*, S^*) for initial values slightly above the carbon-free steady-state stocks (K^*, S^*) because ultimately the economy converges to these values.[7] In fact, the locus separating regimes II and IV is increasing globally.[8] Furthermore, capital stocks on the curve are larger than K^* but smaller than some finite upper bound.[9] Oil stocks on the curve are larger than S^*. Since $\Theta_K^{RO} > \Theta_K^R > 0$ (see appendix 2.1, part II (ii)) and $F_{KK} < 0$, we find that the locus (17′) separating regimes II and IV is steeper than the locus (15) separating regimes I and III in panel (a) of fig. 1.

Lemma 3: For any $K_0 > K^*$ and $S_0 > S^*$ for which $G(S_0) + \dfrac{D'(E_0)}{\left[F_K(K_0, V(K_0;b)) - \delta\right] U'(\Theta^{RO}(K_0;b))} > b$, it is optimal to start with an initial phase where only renewables are used and a final phase where oil and renewables are used alongside each other forever (regime IV).

Proof: This is clear from (17′) and from the fact that the separating locus is increasing.

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[7] Suppose the economy is at $K_0 = K^*$ and $S_0 = S^*$. If we then increase K_0, the economy uses only renewables. If we keep $K_0 = K^*$ and increase S_0, the economy uses only oil. Hence, simultaneous use in the neighborhood of (K^*, S^*) can only occur to the north east of (K^*, S^*). Note that at (K^*, S^*), $\Theta_K^{RO} > 0$ (appendix 2.1, part II (ii)), which confirms that the locus must slope upwards immediately to the right of the pivotal economy.

[8] If this were not the case there would exists an initial stock $K_0 > K^*$ and initial oil stocks $S_{03} > S_{02} > S_{01} > S^*$ such that with $S_{02} > S_0 > S_{01}$ it is optimal to use only oil initially, with $S_0 = S_{02}$ it is optimal to start with simultaneous use of oil and renewables, and with $S_{03} > S_0 > S_{02}$ it is optimal to start with only renewables. But this contradicts the fact that with increasing abundance of oil, ceteris paribus, it becomes more attractive to use oil only.

[9] Since $D'(E_0)/\left[F_K(K_0, V(K_0;b)) - \delta\right] U'(\Theta^{RO}(K_0;b))$ goes to infinity if the initial capital stock goes to infinity, there is an upper bound on the initial capital stock for which it is optimal to start with simultaneous use.

For initial stocks to the north-west of the locus of simultaneous use in fig. 1(a) (and with $K_0 > K^*$), the economy is relatively oil abundant, and there will be an initial interval of time with an oil-only regime. Along this phase E increases. If we then reconsider the equation $b = G(\hat{S}_0) + D'(E_0) / \rho U'(C^*)$ at some instant of time larger than zero, then $\hat{S}_0$ is higher. Hence, the entire curve shifts upward over time. Therefore it is hit at some instant of time and simultaneous use is warranted. This reasoning also makes clear that once the economy is a regime with simultaneous use, it will stay there. A lower backstop cost enhances the use of the backstop. The right-hand side of the inequality in lemma 3 decreases. The left-hand side increases, because upon a downward jump of b steady-state consumption in the carbon-free economy jumps upwards, and initial consumption also increases, thereby decreasing marginal utility. A higher initial stock of atmospheric carbon also enhances use of renewables. A lower initial capital stock will decrease initial consumption on the stable branch, making the inequality in lemma 3 less likely to hold. So, a lower initial capital stock enhances the use of oil. The intuition is that with low wealth the marginal utility of consumption is much more important than climate damage.

4.4. Delineating regimes I and II: the threshold economy

We will now show that the locus of initial oil and capital stocks such that the 'threshold' economy prevails corresponds to the downward-sloping locus labelled the 'initial stock threshold' in fig. 1(a). This locus delineates the oil-scarce regime I from the oil-abundant regime II. Let the initial capital stock $K_0 < K^*$ be given. The threshold economy is defined as the economy that is endowed with a particular initial oil stock that we denote by S_0^* (that depends on K_0). The endowments ((K_0, S_0^*) are such that it is optimal to start using only oil until a critical time T^* when the economy exactly reaches the steady-state of the carbon-free economy (K^*, C^*) and stays there forever after: $(K(t), O(t), R(t), C(t), S(t)) = (K^*, 0, R^*, C^*, S(T^*))\ \forall t \geq T^*$.[10] At time T^*, the economy must be indifferent between oil and renewables:

(18) $$G(S(T^*)) + \frac{D'(E_0 + S_0^* - S(T^*))}{\rho U'(C^*)} = b.$$

We show that for any given $K_0 \leq K^*$, there is at most one unique S_0 for which the path described here is optimal. If $K_0 = K^*$ and $S_0 = S^*$, so that $b = G(S_0) + D'(E_0) / \rho U'(C^*)$, then $T^* = 0$ and we have the pivotal economy. For a smaller S_0, and still $K_0 = K^*$, oil will never be used. For a larger S_0, and still

[10] If the initial oil extraction cost plus initial social cost of carbon exceeds the cost of renewables, i.e., $G(S_0) + D'(E_0) / \rho U'\left(\Theta^R(K_0, b)\right) > b$, then it never pays to use oil and thus no value of S_0^* exists if S_0 is too small.

$K_0 = K^*$, the economy should overshoot the carbon-free steady state and $T^* > 0$. Now assume that $K_0 < K^*$. Given the initial capital and oil stocks, the terminal conditions $C(T^*) = C^*$ and $p(T^*) = b$, the switch time for the threshold economy, T^*, and oil demand $O = V(K;p)$ from $F_O(K,O) = p$, we can solve the two-point-boundary-value-problem for the oil-only economy:

(1O) $\dot{S} = -V(K;p), \quad S(0) = S_0,$

(4O) $\dot{K} = \tilde{F}(K;p) + [p - G(S)]V(K;p) - C - \delta K, \quad K(0) = K_0,$

(7O) $\dot{p} = \left[\tilde{F}_K(K;p) - \delta\right]\left[p - G(S)\right] - \frac{D'(E_0 + S_0 - S)}{U'(C)},$ and

(8O) $\dot{C} = \sigma\left[\tilde{F}_K(K;b) - \delta - \rho\right]C.$

This gives us $K(T^*)$ and $S(T^*)$. We then solve for S_0^* and T^* so that $K(T^*) = K^*$ and (19) are satisfied. We restrict attention to those values of K_0 for which a solution satisfying $S_0^* \geq 0$ and $T_0^* \geq 0$ exists. The resulting threshold value for the initial oil stock and the associated switch time and initial consumption and social price of oil can be written as follows:

(19) $S_0^* = S_0^*(K_0;b), \quad T^* = T^*(K_0;b), \quad C_0^* = C_0^*(K_0;b), \quad p_0^* = p_0^*(K_0;b) \leq b.$

The threshold value of S_0^* depends negatively on the initial stock of capital as a lower K_0 requires a higher S_0^* and is associated with a higher switch time T^*.[11] A lower K_0 also depresses initial consumption $C(0) = C_0^*$ and the initial social price of oil $p(0) = p_0^*$ in the threshold economy. The locus $S_0 = S_0^*(K_0;b)$ corresponds to a downward-sloping locus in fig. 1(a) delineating regimes I and II. For a lower (higher) initial oil stock or a lower (higher) initial capital stock, the economy switches from the threshold economy to regime I (regime II).

4.5. Summing up

We have already shown that once we are in regime III with only use of the renewables, we will stay there, and capital and consumption monotonically increase or decrease to the carbon-free steady state. The construction of the locus dividing regimes II and IV makes clear that once we are in a regime with

[11] With more capital, the economy uses more oil, produces more and reaches the carbon-free steady state more quickly. However, the economy will also consume more and save less, so that capital will grow less quickly and this tends to postpone the date at which K^* is reached. Computations with the parameter values used in sections 6 and 7 suggest that the former effect dominates the latter effect.

simultaneous use of oil and renewables, we continue with simultaneous use indefinitely. Indeed, a deviation to the right of the locus brings us in a regime with exclusive of renewables and a decreasing capital stock. A deviation to the left leads to exclusive oil use and an increasing capital stock, steering the economy back to simultaneous use immediately. Then it also follows that starting to the east of the locus, there will never be only oil use. If it is optimal to start with oil only, then at some point in time it is optimal to switch to either simultaneous use or to renewables only. The latter occurs in regime I. It is clear that once renewables have taken over in this regime, oil will never be used again. The former occurs for high initial oil stocks. The economy will overshoot the steady state in the oil-only phase. It is impossible that renewables will ever be phased in because if that would be the case, oil would never be used again. Finally, appendix 1 shows that capital and consumption increase monotonically as long as they are smaller than K^*. For regime III this is obvious, but it may be less obvious for the initial phases of regimes I and II.

We can now formally establish the regimes depicted in fig. 1(a).

Proposition 2: The following regimes exist for the social optimum of the Green Ramsey model:

I. For intermediate initial oil stocks satisfying $G(S_0)+\dfrac{D'(E_0)}{\rho U'(\Theta^R(K_0;b))}<b$ and $S_0<S_0^*(K_0;b)$ and $K_0<K^*$, there is an initial oil-only phase and a final renewables-only phase, and capital and consumption rise monotonically throughout the two phases;

II. For high enough initial oil stocks satisfying $S_0>S_0^*(K_0;b)$ and $K_0<K^*$ or if $G(S_0)+\dfrac{D'(E_0)}{\left[F_K(K_0,V(K_0;b))-\delta\right]U'(\Theta^{RO}(K_0;b))}<b$ and $K_0>K^*$, there is an initial oil-only phase and a final oil-renewables phase. In the first case, capital and consumption overshoot their steady-state values;

III. For low enough initial oil stocks satisfying $K_0<K^*$ and $G(S_0)+\dfrac{D'(E_0)}{\rho U'(\Theta^R(K_0;b))}>b$ or satisfying $K_0>K^*$ and $S_0<S^*$, only renewables are used from the outset and forever thereafter, and consumption and capital rise monotonically in the first case and decline in the second case;

IV. For intermediate initial oil stocks satisfying $G(S_0)+\dfrac{D'(E_0)}{\left[F_K(K_0,V(K_0;b))-\delta\right]U'(\Theta^{RO}(K_0;b))}>b$, $S_0>S^*$, and $K_0>K^*$, there is an initial only-renewables phase and a final renewables-oil phase.

Appendix 2 gives details of how the solution trajectories of regimes I and II can be computed.

5. Climate policy, the market and the Green Paradox

5.1. Realizing the socially optimal outcome in the market economy

The "laissez-faire" market economy does not internalize global warming externalities ($D \equiv 0$).

Proposition 3: The "laissez-faire" economy never has simultaneous use of oil and renewables. If oil is abundant initially, $S_0 > G^{-1}(b)$, there is first an oil-only phase followed at time T by a final phase of using only renewables. The amount of oil left in situ follows from $b = G(S(T))$, i.e., $S(T) = G^{-1}(b)$, and is less than in the socially optimal outcome. If oil is scarce initially, $S_0 < G^{-1}(b)$, it is optimal to start with renewables and keep using them forever. In both cases, the economy converges asymptotically to the carbon-free steady state.

Proof: From (17) simultaneous use requires $G(S) = b$ if $D = 0$ which gives a constant stock of oil which is inconsistent with positive oil use. Instead of (14), we now have $p(T) = G\left(S(T)\right) = b.$ Comparing this with (14) shows that now less oil is left in situ than in the social optimum.The rest is obvious Q.E.D.

The two regimes of the "laissez-faire" outcome are depicted in fig. 2(b); note that the critical value $G^{-1}(b) < S^*$ as the "laissez-faire" outcome does not internalize global warming externalities and the social optimum does. If the initial oil stock is high and it is cheap to deplete oil, the economy starts with using only oil. As oil reserves get depleted and oil extraction costs rise, the economy switches to using renewables. If the initial oil stock of oil is low enough, oil is never used.

Since there are no other distortions apart from the climate externality and lump-sum taxes/subsidies are available, the optimal carbon tax must equal the social cost of carbon (5) which was defined as the present discounted value of all future marginal global warming damages.

Proposition 4: The government can reproduce the socially optimal outcome by levying a carbon tax τ equal to the social cost of carbon (5) or (5′). The optimal carbon tax rises over time if and only if

$$r(t) > D'(E(t))e^{-\rho t} / \int_t^\infty D'(E(s))e^{-\rho s} ds.$$

Proof: Compare first-order conditions of the "laissez-faire" outcomes with those of proposition 1. The numerator in (5′) increases over time, so the optimal carbon can only decline with time if consumption declines over time and the denominator in (5′) rises with time. Q.E.D.

It is important that the carbon tax is also maintained after the era only-oil era has finished and only renewables are used. If this were not the case, the economy would switch back to using oil.

If the economy starts off with a low capital stock, consumption is low, the interest rate is high and the marginal value of consumption is high so that the social cost of carbon and the optimal carbon tax are low. Hence, as the economy develops and consumption and capital rise, the social cost of carbon and optimal carbon tax rise. Once renewables kick in, the optimal carbon tax stays constant at the level that prevails at the end of the oil-only phase. Hence, the magnitude of the optimal carbon tax depends on the state of economic development. If the economy has high enough degree of economic development, it may be optimal for the carbon tax to start off high and then diminish with time. Indeed, proposition 6 indicates that, if consumption and capital overshoot their steady-state values, the interest rate is relatively small and the marginal utility of consumption is small, there is a real possibility that the optimal carbon tax falls with time, especially if the optimal carbon tax is low and the atmospheric stock of CO2 is high.

5.2. Second-best outcome if a carbon tax is infeasible

What happens if for political or other reasons it is infeasible to levy a carbon tax?[12] The Green Paradox states that subsidizing the backstop fuel with the aim of curbing oil demand and carbon emissions is counterproductive as it encourages private well owners to pump up their oil more rapidly, thereby aggravating global warming damages. This paradox has been studied before in partial equilibrium models without capital accumulation (Sinn, 2008ab; Hoel, 2008; Gerlagh, 2011; Grafton et al., 2010), but it relies on oil reserves being fully exhausted. If oil extraction costs rise rapidly as reserves diminish, the market leaves less oil in the crust of the earth than the social optimum. In that case, the Green Paradox need not occur as reducing the cost of renewables then leaves more oil in situ (van der Ploeg and Withagen, 2010).

We extend earlier analysis of the Green Paradox to a general equilibrium framework with economic growth. The Green Paradox highlights the second-best effects of introducing a constant backstop subsidy υ financed by a lump-sum tax, to phase out oil more quickly and mitigate global warming if a carbon tax is ruled out, $\tau = 0$. We are interested in the effects of a renewables subsidy on consumption, accumulation of capital, growth and economic development. There is no case for a subsidy (or tax) on renewables once the extraction cost of oil is larger than the production cost of renewables. We suppose that the government can commit and keep the renewables subsidy in place once oil is no longer used.

The economy without subsidy on renewables leaves some oil reserves unexploited, but less than in the social optimum. A subsidy has the effect of leaving more oil unexploited[13], because $b - \upsilon = G(S(T))$.

[12] In practice, politicians do charge a price for carbon (e.g., the European Union Emission Trading Scheme) but lower than the social optimum. For simplicity, we assume that the government levies no carbon tax at all.
[13] If full exhaustion of oil reserves is feasible (i.e., if oil extraction costs as oil reserves vanish tend to a small enough finite number rather than infinity), a backstop subsidy only leads to more rapid pumping of oil, faster exhaustion of oil reserves and thus to a Green Paradox (van der Ploeg and Withagen, 2010).

which reduces the amount of carbon in the atmosphere and brings the economy closer to the first best. As we will see in the simulations of section 6 and 7 and is known from the literature on the Green Paradox, initially more oil is extracted. Hence, the total effect on green welfare (the negative of climate damages) is ambiguous. With a subsidy the steady-state stock of capital is higher (from $\tilde{F}_K(K^*; b-\upsilon) = \delta + \rho$). The steady-state rate of consumption is lower now, because the subsidy is financed by lump-sum taxes which curbs consumption. The effect on total welfare depends on the state of the economy. With a low level of development (low initial capital stock), the subsidy may be counterproductive. The loss of consumption implies a large loss in utility, whereas the possible gain in terms of reduced damages may be negligible compared to the utility loss. The optimal second-best renewables subsidy will thus be lower if the economy is in the early stages of development. As we have seen already, the optimal first-best carbon tax is then lower as well.

6. Policy simulations: oil-scarce regime (undershooting)

To start we offer some policy simulations for the case where the initial conditions are such that the social optimum has an initial period where only oil is used and a final period where only renewables are used (i.e., regime I of proposition 2 which an intermediate level of initial stock oil stock). For a given initial oil stock, this occurs for a sufficiently low initial capital stock. We simulate both the social optimum and the decentralized market outcome (described in sections 4 and 5). We set $S_0 = 20$ and $E_0 = 24$.[14] We use as a discount rate, $\rho = 0.014$. We use the CES utility function $U(C) = C^{1-1/\sigma} / (1-1/\sigma)$ with a ballpark estimate of the elasticity of intertemporal substitution equal to $\sigma = 0.5$ [15] and explore the sensitivity with respect to σ to gain insight into the effect of intergenerational inequality aversion on global warming and economic growth. We use a Cobb Douglas production function $F(K, O+R)) = K^{\alpha}(O+R)^{\beta}$. The shares of capital and of oil/gas in GDP have been set at $\alpha = 0.2$ and $\beta = 0.1$. The average lifetime of manmade capital has been set at twenty years, so $\delta = 0.05$. The initial stock of capital is set at half the

[14] In 2000 there were oil and gas reserves in the crust of the earth corresponding to 469 and 1,121 Giga tons of carbon, respectively, whereas there had been emitted 224 plus 111 Giga tons of carbon into the atmosphere resulting from burning, respectively, oil and gas (Edenhofer and Kalkuhl, 2009). Normalizing so that $S_0 = 20$, we set $E_0 = 24$ (rather than 335x20/1,590 = 4.2) to allow for the substantial CO2 concentration that was already in the atmosphere for non-anthropogenic reasons.

[15] Some argue that the elasticity of intertemporal substitution σ is very low with an implied coefficient of relative risk aversion of about 10 (e.g., Mehra and Prescott, 1985; Campbell and Mankiw, 1989; Obstfeld, 1994); others argue that σ is one or greater than one with a much smaller and more realistic implied coefficient of relative risk aversion (e.g., Hansen and Singleton, 1982). $\sigma = 0.5$ implies a coefficient of relative risk aversion 2 which seems a little high. Rather than breaking the link between risk aversion and intertemporal substitution to allow for an elasticity of intertemporal substitution in the range 0.05 to 1 and a coefficient of relative risk aversion in the range 0.4 to 1.4 (Epstein and Zin, 1991), we will explore the sensitivity of our results with respect to different values of σ.

value that prevails in the steady state of the carbon-free economy, i.e., $K_0 = K^* / 2$. We set $b = 0.5$[16] and the initial cost of extracting one unit of oil at $G(S_0) = 0.2$. We suppose that unit extraction costs become infinitely large as more and more oil is extracted and capture this with the specification $G(S) = \gamma S_0 / S$ with $\gamma = 0.2$. This implies that in the market outcome where global warming externalities are not internalized, half of the initial stock of oil is left in situ at the time of the switch to the renewable backstop, $S(T) = S_0 / 2 = 10$. If global warming externalities are internalized, a bigger stock is left in situ. The cost of extracting the last drop of oil thus exactly equals the cost of renewables in the market outcome, but will be less than in the socially optimal outcome. For global warming damages we use the specification $D(E) = \kappa E^2 / 2$ with $\kappa = 0.00012$.[17] With these parameters, we calculate $S_0^* = 20.8$ and $T^* = 24.3$. Since we start with $S_0 = 20 < S_0^*$, proposition 4 indicates that it is optimal to start with an initial oil-only phase and end with a final renewables-only phase.

6.1. The optimal carbon tax needed to attain the first-best outcome in the market economy

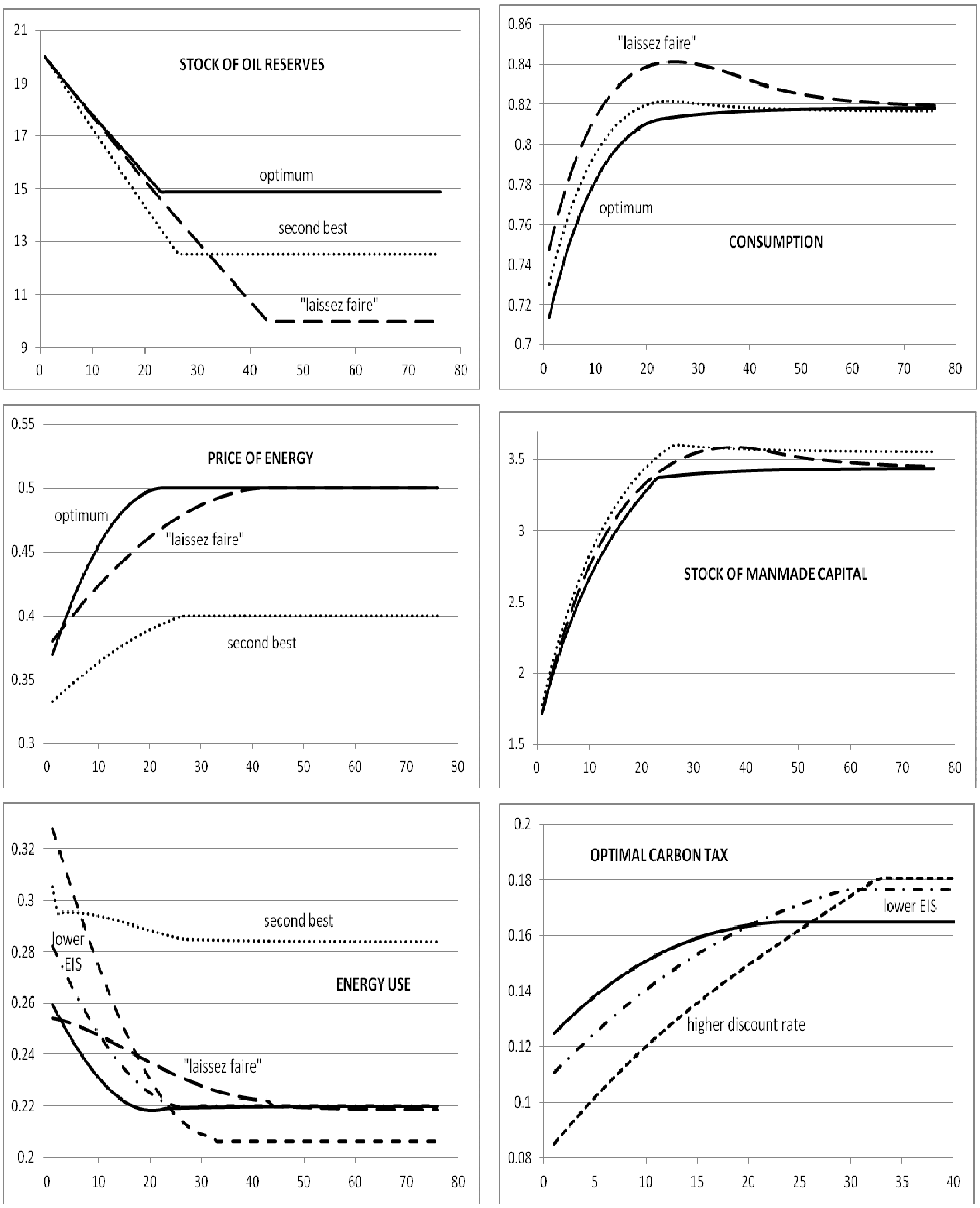
Fig. 2 plots the time paths of the key variables for these two outcomes (solid lines represent the optimum, long-dashes the market and also gives the time path of the optimal carbon tax that ensures that the market properly internalizes the global warming externality. Both manmade capital and consumption rise over the entire optimal path, confirming the theory. In this benchmark simulation there is no overshooting of capital in the optimum, but the market does overshoot. Under "laissez-faire", manmade capital and consumption decline during the carbon-free phase and even before. Optimally internalizing global warming externalities implies that renewables get phased in more quickly than under the "laissez-faire" outcome, namely at time 22.0 rather than 42.3[18], and that 14.9 rather than 10.0 units of oil are left in situ at the time of the switch to the carbon-free economy. Switching more quickly to the carbon-free economy and leaving more oil in situ is an effective way to curb CO2 emissions and global warming. Consumption and manmade capital are smaller for the optimal than for the "laissez-faire" outcome, since oil use is cut

[16] Solar and wind energy are more expensive than fossil fuel, especially if one looks beyond marginal production costs once capacity is installed and considers the costs needed to increase capacity, deal with intermittence and repair offshore wind mills. Wind energy can be at least three times as expensive as 'grey' electricity (Wikipedia). As far as the electricity industry is concerned, costs of renewables have fallen substantially: solar energy is currently 50% more expensive than conventional electricity; wind energy has the same cost and is (apart from the problem of intermittence) competitive; and biomass, CCS coal/gas and advanced natural gas combined cycle have mark-ups of 10%, 60% and 20%, respectively (Paltsev et al., 2009). These mark-ups are measured from a very low base and may not be so impressive when they account for a much larger market share. Hence, we use a 100% mark-up.

[17] Peer-reviewed estimates of the social cost of carbon for 2005 have an average of \$43 per ton of carbon and a standard deviation of \$83 dollar per ton of carbon, and these estimates are likely to increase by 2 to 4 percent per year (Yohe et al., 2007). A ballpark estimate of the social cost of carbon is \$30 dollar per ton (Nordhaus, 2007).

[18] The switch occurs earlier than in the threshold economy, $T = 22.0 < T^* = 24.3$, as oil is scarcer.

Figure 2: Simulation trajectories for oil-scarce economy



Key: solid lines – optimum (benchmark): $T = 22.0,\ S(T) = 14.9$;
long dashes – "laissez-faire": $T = 42.3,\ S(T) = 10.0$;
dots – no carbon tax and backstop subsidy (second best): $T = 25.3,\ S(T) = 12.5$;
dots and dashes – optimum with higher inequality aversion: $T = 28.9,\ S(T) = 13.1$;
short dashes – optimum with higher rate of time preference: $T = 31.8,\ S(T) = 12.0$.

back more quickly to limit global warming. We also observe a steeper time path for the social price of oil in the socially optimal outcome due to the rising time profile of the optimal carbon tax from 0.24 to 0.31. This contrasts with the inverse U-shape for the time profile of the optimal carbon tax found in Golosov et al. (2011), where the eventual decline of the carbon tax results from their assumption of natural decay of atmospheric CO2.

The Green Solow model put forward by Brock and Taylor (2010) has CO2 emissions as an inevitable by-product of production and abstracts from renewables. Social welfare is maximized by choosing a constant savings rate and a constant share of abatement in output. They find an Environmental Kuznets Curve: emissions initially increase and later decrease with economic development. Within our Green Ramsey framework CO2 emissions per unit of output are initially high, since initially the marginal utility of consumption is large compared to the marginal damages of accumulated CO2. During the development growth path, the rapid accumulation of manmade capital compensates for the falling use of oil resulting from the rising price of oil and growth tapering off. Consequently, CO2 emissions are initially high and then fall rapidly over time. Once the economy has switched to a clean backstop, CO2 emissions are reduced to zero and the accumulated pollution in the atmosphere is stabilized. So, we also get an Environmental Kuznets Curve in a different framework with a benevolent policy maker.

6.2. Effects of intergenerational inequality aversion and time preference

If the elasticity of intergenerational inequality aversion ($1/\sigma$) is increased from 2 in the benchmark to 4 (corresponding to halving the elasticity of intertemporal substitution ($EIS = \sigma$), we find that the time to phase out oil and switch to renewables in the social optimum is postponed from instant 22.0 to 28.9, and that, as a result of more aggressive oil use, the stock of oil that is left in situ at the end of the oil phase is decreased from 14.9 to 13.1. Both these factors tend to increase CO2 emissions and global warming, as may be expected if intergenerational inequality aversion is higher and thus more priority is given to current, relatively poor generations who have to shoulder most of the burden of combating climate change rather than to distant, relatively rich generations. This way the economy develops faster initially at the expense of global warming, albeit that the steady-state levels of the capital stock and consumption are not affected by more intergenerational inequality aversion. Fig. 2 indicates that, with a lower *EIS*, the optimal carbon tax rate for this case (dots and dashes) is lower in the initial part of the oil-only phase but higher in the latter part of this phase. Furthermore, as the oil-only phase lasts longer with a lower EIS. With a higher intergenerational equality aversion, current generations are better off in terms of consumption than future generations.

One might argue that the private sector employs a higher rate of time preference than the government, say a rate of time preference of 0.03 rather than 0.014 for the "laissez-faire" economy. The time of the switch towards renewables is then reduced by a tiny amount from 42.34 to 42.30 whilst the stock of oil that remains in situ remains 10.0. However, as the economy is impatient and consumes more upfront and thus invests less, it ends up in the long run with much less manmade capital (2.58 rather than 3.57) and somewhat lower consumption (0.80 rather than 0.83).[19] If the government also employs the higher rate of discount of 0.03, it initially pursues a less aggressive climate change policy resulting in much more oil use. In the latter part of the oil-only phase climate policy becomes less aggressive and oil use is below than the case where the government employs the prudent discount rate. Still, renewables are phased in more quickly than under "laissez-faire" at instant 31.8, but a lot more slowly than if the government employs a precautionary discount rate of 0.014 (at time instant 22.0). Furthermore, oil left in situ, 12.0, is less than with a prudent discount rate of 0.014, but more than in the "laissez-faire" outcome. Fig. 3 indicates that the optimal carbon tax for the case of a low discount rate (solid line) is for the most part lower than that for a high discount rate (short dashes) but in the final part of the oil-only phase is higher.

6.3. Second-best outcome: subsidizing renewables does not lead to the Green Paradox

Fig. 2 also plots the time paths of the key macroeconomic and resource variables under the renewables subsidy (dotted lines) amounting to $\upsilon = 0.1$ and financed with lump-sum taxes. The date of switching from oil to renewables becomes 25.3, later than in the socially optimal outcome and earlier than in the market without the subsidy. The amount of oil left in situ increases from 10 in the "laissez-faire" economy to 12.5, which is less than in the socially optimal outcome. The time path for consumption is higher than in the social optimum but lower than in the "laissez-faire" market economy. As a result of renewables subsidy, the price of energy is much lower both during the oil-only and the carbon-free phase in this simulation. Initially private agents are encouraged to use much more oil in production than even in the "laissez-faire" outcome, because the loss of higher extraction now on future extraction cost is lower due to the fact that renewables will be phased in earlier and more oil is left in situ. This is what underpins the inexorable logic of the Green Paradox: despite renewables being phased in more quickly and more oil being left in situ, private agents pump up oil much more vigorously. For our numerical example the present value of global warming damages is reduced from 2.19 in the "laissez-faire" market outcome to 1.98 (more than in the social optimum, 1.73). Hence, despite that more oil is pumped up initially, global warming damages need not increase under the backstop subsidy as renewables are phased in more quickly

[19] The reason that C^* changes only a little compared with K^* is that the share of capital is much smaller than the combined share of all the non-energy factors (capital and labor) in value added.

and more oil is left in situ. As the renewables subsidy distorts private decisions, private welfare falls from -67.8 in the "laissez-faire" to -69.0 in the market outcome with the subsidy. The renewables subsidy thus boosts green welfare, but curbs social welfare from -70.0 to -70.9. Clearly, such a subsidy also performs worse than the outcome with an optimal carbon tax.

7. Policy simulations: oil-abundant regime (overshooting)

We now offer policy simulations for regime II of proposition 2 in which the oil-only phase is followed by a final phase where oil and renewables are used simultaneously. In contrast, in the final phase of the "laissez-faire" outcome only renewables are used. We use the same parameters as in section 6 except that we set the initial oil stock equal to $S_0 = 25 > S_0^* = 20.8$ instead of 20. The policy simulations of this oil-abundant regime are depicted in fig. 3.

In contrast with the case of oil scarcity, it is now optimal to extract more oil initially than in the market economy. The aim is not to increase consumption but to build up capital more rapidly than in the market economy. The social price of energy in the social optimum starts below that under "laissez-faire" and then during the latter part of the oil-only phase is above it. As a result, oil use is only lower than the "laissez-faire" oil use in the latter part of the only-oil phase. As soon as renewables take over under "laissez-faire", the market price has caught up with the social price of oil again. Still, renewables fall ever so slightly from then on as capital falls during this final phase under "laissez-faire". We also observe that *clean* renewables are phased in much later under "laissez-faire" (i.e., at time 61.4 instead of 34.0), albeit that the social optimum never phases out oil completely and only gradually ramps up the use of renewables. Output overshoots in both outcomes. The swinging time profile of output in the social optimum reflects, on the one hand, the rapid growth of the capital stock during the early parts of the initial oil-only phase, and, on the other hand, the substantial curbing of oil use during the oil-only phase. The reason for the kink in net investments and output is that at the transition, capital, consumption and energy use are continuous, but oil and renewables are not.[20] Oil use jumps down at the moment renewables are phased in.

The social optimum achieves an improvement in green welfare at the expense of private welfare by substantially curbing oil use and carbon emissions and thus flattening output growth during the latter part of the oil-only phase, despite the increase in oil use during the early part of this phase. Consumption under "laissez-faire" is higher throughout, especially during the latter part of the oil-only phase. The social optimum also leaves much more oil in situ and thus puts less carbon in the atmosphere.

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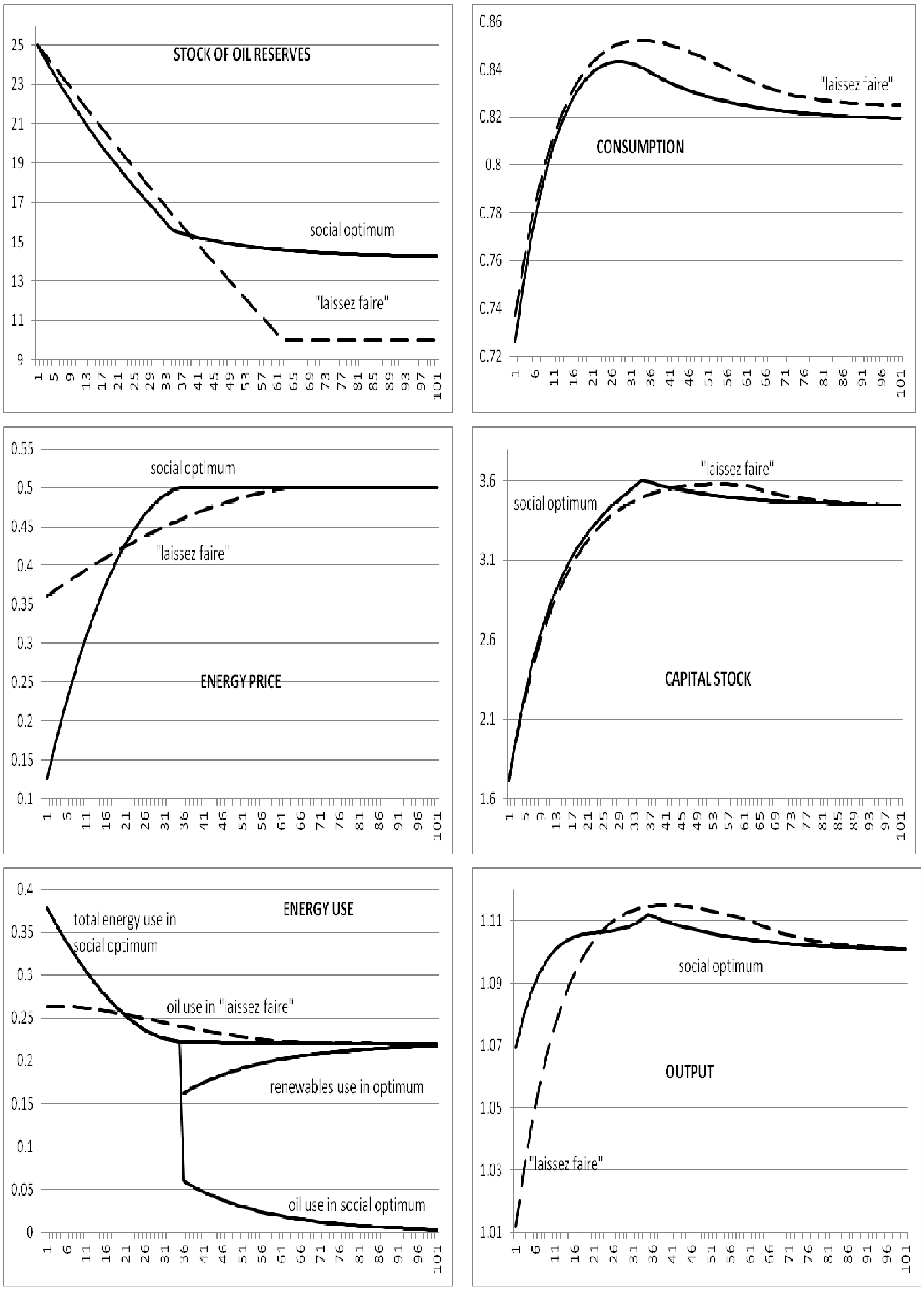
[20] $bR + G(S)O = b(O+R) + [G(S) - b]O$ is discontinuous at the transition because $[G(S) - b] = (\mu_S + \mu_E)/\mu_K \neq 0$.

Figure 3: Simulation trajectories for oil-abundant economy

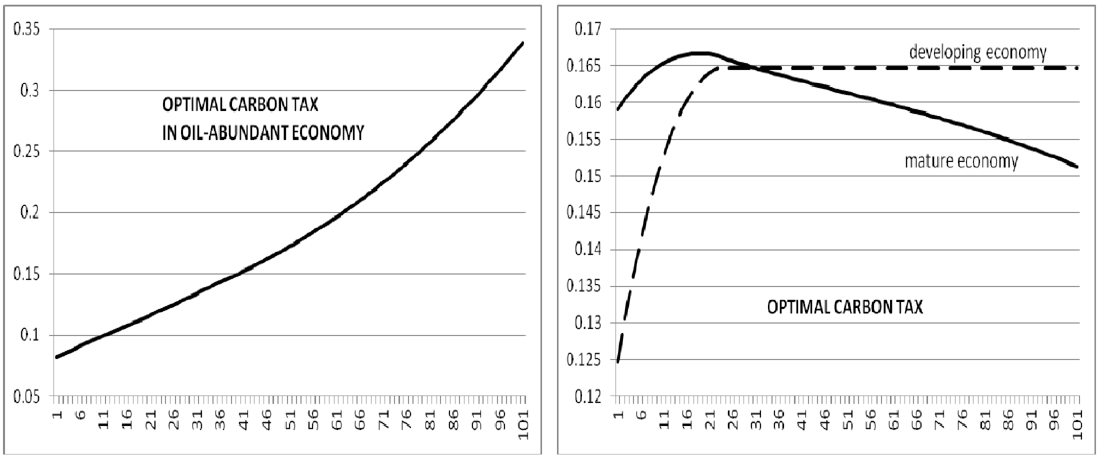


Note: Social optimum $T = 34.0$, $S(T) = 15.6$; $S(\infty) = 14.2$; "Laissez-faire" $T = 61.4$, $S(T) = S(\infty) = 10$.

The optimal carbon tax is depicted in fig. 4(a). It continues to rise after renewables have been phased in, since during this final phase capital and consumption fall so that the interest rate and marginal utility of consumption rise. This combined with the increase in marginal climate damages as more carbon is emitted into the atmosphere leads to an upward time profile of the optimal carbon tax in the final phase. During this final phase use of oil in the production process is gradually winded down whilst that of renewables is gradually ramped up.

Figure 4: The optimal carbon tax in the oil-abundant and the mature economy

(a) $K_0 = 0.5K^*, S_0 = 25 > S_0^* = 20.8$ (b) $K_0 = 0.75K^*, S_0 = 20 > S_0^* = 18.7$



If we would start with a larger initial capital stock, say $K_0 = 0.75K^*$ and keep $S_0 = 20$, the threshold for the initial oil stock will be lower, i.e., $S_0^* = 18.7 < 20.8$, which moves the economy also from an oil-scarce to an oil-abundant regime (see proposition 2) and thus oil is never phased out. At instant of time 20.1 renewables are phased in alongside oil (see appendix 3). Fig. 4(b) indicates that the time profile of the optimal carbon tax in the mature economy has an inverted U-shape. It starts off higher but ends up lower than in the developing economy discussed in section 6. The social cost of carbon for the mature economy is initially higher due to the lower interest rate and the lower marginal utility of consumption. In spite of the fact that there is no decay of the CO2 stock, the carbon tax eventually decreases.

8. Conclusion

We have analyzed optimal climate policy in a Ramsey growth model with exhaustible oil reserves, an infinitely elastic supply of renewables, stock-dependent oil extraction costs and convex climate damages. In a market economy there are only two regimes: either the initial stock of oil reserves is so low that there is an initial oil-only phase followed by a final renewables-only phase or else renewables are used on their own from the outset. There is never a phase with simultaneous use of oil and renewables. We have also characterized the four regimes that can occur in the socially optimal outcome. The two most likely regimes occur if the economy has an initial cost advantage of oil and an initial capital stock below the carbon-free steady state.

Regime I occurs for intermediate values of the initial stock of oil. It is then optimal to have an initial oil-only phase followed by a renewables-only, carbon-free phase. Our simulations suggest that in this regime a lower cost of renewables, a lower discount rate or a lower degree of intergenerational inequality aversion command a higher long-run carbon tax for the market economy, which ensures that more oil is left in situ and renewables are phased in more quickly. The optimal carbon tax rises as the economy moves along its development path during the oil-only phase. The carbon tax rises over time for two reasons. First, as oil reserves diminish and the stock of atmospheric carbon rises, the marginal cost of global warming rises. Second, as consumption increases, the marginal utility of consumption falls. The rise in the carbon tax flattens off as less accessible reserves have to be explored, because oil well owners deplete more conservatively then which is good for the environment. If a carbon tax is infeasible and renewables are subsidized, renewables are phased in more quickly and more oil is left in situ. However, oil is also pumped up more vigorously initially (a manifestation of the Green Paradox), so the effect on global warming is ambiguous. The second-best policy of subsidizing the renewables in regime I may have adverse effects: it leads to faster pumping of oil initially, but less oil is used in total as more oil is left in situ. The renewables subsidy goes at the cost of consumption, which for low capital stocks is already low. If the economy is not very developed, the optimal carbon tax rises as the economy moves along its development path during the oil-only phase, but the rise in the carbon tax flattens off as less accessible reserves have to be explored and the marginal cost of global warming increases as the amount of carbon in the atmosphere and the marginal utility of consumption falls.

Regime II occurs if the initial oil stock is large enough. The social optimum then has an initial oil-only phase followed with a final phase where oil and renewables are used alongside each other; in the "laissez-faire" outcome oil is phased out and renewables phased in much later. The energy price starts below that in "laissez-faire", but during the latter part of the oil-only phase rises above it. As a result, oil use is thus initially higher than "laissez-faire" and is only lower in the latter part of the only-oil phase. In the final phase use of renewables is gradually ramped up. The social optimum boosts green welfare at the expense of private welfare by substantially curbing oil use and carbon emissions and thus flattening output growth during the latter part of the oil-only phase, despite the increase in oil use during the early part of this phase. Consumption under "laissez-faire" is higher throughout, especially during the latter part of the oil-only phase. The social optimum also leaves more oil in situ and thus less carbon in the atmosphere. In spite of the fact that we suppose that there is no decay of CO2 the optimal carbon tax may decline in regime II but not in regime I.

The oil-abundant regime II also applies to a *mature* economy which is an economy with an initial capital stock that exceeds the carbon-free steady state and a high enough initial oil stock and thus low enough initial oil extraction cost to warrant starting with oil only. Although the economy ends up in the same steady state, the mature economy of regime II keeps on using oil alongside renewables forever whilst the *developing* economy of regime I switches from using only oil to only renewables, albeit that the mature economy phases renewables in more quickly. The mature economy leaves less oil in situ and has a higher carbon tax than the developing economy for the oil-only phase, since it faces a higher social cost of carbon. In a developing economy the optimal carbon tax is gradually ramped up until the moment renewables take over from oil. In the mature economy, despite there being no decay of the CO2 stock, the carbon tax may decrease if the level of development is high enough.

For regime II to occur in the mature economy it is necessary that the initial oil stock is high enough and thus oil extraction costs and marginal climate damages low enough for it to remain attractive to start with oil. However, if the initial stock of oil is low enough, it is attractive to start with renewables rather than oil. If in addition the initial stock of oil is below a threshold value, regime III prevails where renewables are used forever (also if the initial capital stock is below steady state). For intermediate values of the initial oil stock with the initial capital stock still above its steady state, regime IV prevails with an initial renewables-only phase followed by a final oil-renewables phase. Until there is a breakthrough in renewables technology, these latter two regimes seem unlikely to occur.

We have used a stylized model to highlight the importance of distinguishing different regimes and of endogenously determining the time that the economy switches from oil to renewables and the optimal amount of oil to be left in situ. In practice, there may be an upward-sloping supply schedule of renewables (e.g., Sinn, 2008ab) which will introduce regimes where more and more renewables are phased in alongside oil (van der Ploeg and Withagen, 2010). There may be technical progress in renewables (e.g., Bovenberg and Smulders, 1996; Popp, 2002; Bosetti, et al., 2009; Acemoglu et al., 2012; van der Meijden and Smulders, 2012; Daubanes et al., 2012) leading to a gradual decline in the price of renewables, thus bringing forward the date of the switch from fossil fuel to renewables and kick-starting green innovation. Technical progress and population growth will affect the optimal carbon tax. Imperfect substitution between energy and other production factors is weak in the short run, but due to directed energy-saving technical change strong in the long run (Hassler et al., 2011). Imperfect substitution between the various sources of energy also plays a role (Smulders and van der Werf, 2008; Michielsen, 2011; Pelli, 2012). Natural decay of the atmospheric stock of CO2 makes the optimal carbon tax eventually fall over time (cf, Golosov et al., 2011). If coal instead of renewables is the relevant backstop, the optimal strategy is to have a more conservative oil depletion strategy and delay the time one has to switch to using coal (van der Ploeg and Withagen, 2011). The robustness of our results if global warming damages affect production multiplicatively (rather than utility additively) should be addressed with great scrutiny; more generally, the elasticity of substitution between damages and economic output (or consumption) might have an important effect on both the time profile of the optimal carbon tax. Finally, China and India are growing rapidly and have little appetite for an aggressive climate policy whilst the more mature OECD economies have more inclination to fight global warming. OPEC has monopoly power and no immediate interest in climate policy. A multi-country model will shed more light on the different tradeoffs between climate and development facing different parts of the world (e.g., Fischer et al., 2012; Michielsen, 2012; Jaakkola, 2012; Wirl, 2012).

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Appendix 1:

In this appendix we show that capital and consumption are monotonically increasing if $K < K^*$.

Define total energy use $V \equiv O + R$. Recall that (K^*, V^*) is defined by $F_V(K^*, V^*) = b$ and $F_K(K^*, V^*) = \rho + \delta$. It follows from concavity of F that $[\rho + \delta - F_K(K,V)](K - K^*) + [b - F_V(K,V)](V - V^*) \geq 0$. We have $K < K^*$ by assumption, and $b \geq F_V(K,V)$. Suppose $F_K(K,V) - \rho - \delta \leq 0$. Then $V \geq V^*$. But with $K < K^*$ and $V \geq V^*$ we have $F_K(K,V) > F_K(K^*, V^*) = \rho + \delta$, which is a contradiction. It follows that *C* is rising. Regarding the proof that *K* is rising, we do not have to consider the case of simultaneous use. The statement is true if we are in a renewables-only phase, because once such a phase starts, it will last forever and the economy approaches its steady state with K monotonically increasing. So, we assume that along some interval of time with only fossil fuel use, K is decreasing and establish a contradiction. Since the economy will eventually approach the steady-state capital stock, the decrease will not be permanent. Hence, there exist instants of time $t_1 < t_2$ with $K(t_1) = K(t_2) = K < K^*$ and $S(t_1) > S(t_2)$ such that $\dot{K}(t_1) = F(K, O(t_1)) - G(S(t_1))O(t_1) - \delta K - C(t_1) < 0$ and either

Case 1: $\dot{K}(t_2) = F(K, O(t_2)) - G(S(t_2))O(t_2) - \delta K - C(t_2) > 0$ or

Case 2: $\dot{K}(t_2) = F(K, R(t_2)) - bR(t_2) - \delta K - C(t_2) > 0$.

Consumption is increasing over time, so that $C(t_1) < C(t_2)$. In case 1 we therefore have $F(K, O(t_1)) - G(S(t_1))O(t_1) < F(K, O(t_2)) - G(S(t_2))O(t_2)$. Moreover, we have $F_O(K, O(t_i)) = G(S(t_i)) + \mu(t_i) / \mu_K(t_i), i = 1,2$. From the fact that we have the same capital stock at both instants of time but higher social cost of oil in t_2 we have $\mu(t_2) / \mu_K(t_2) > \mu(t_1) / \mu_K(t_1)$. It then follows that $F_O(K, O(t_1)) < F_O(K, O(t_2))$ so that $O(t_2) < O(t_1)$. Reduce $O(t_1)$ to $O(t_2)$. Then we have $F(K, O(t_2)) - G(S(t_1))O(t_2) = F(K, O(t_2)) - G(S(t_2))O(t_2) + (G(S(t_2)) - G(S(t_1))O(t_2)$ $> F(K, O(t_1)) - G(S(t_1))O(t_1)$. So, by decreasing $O(t_1)$ we get more net production and less pollution. Welfare can be improved and we were not on an optimal path. In case 2 we have $F(K, R(t_2)) - bR(t_2) > F(K, O(t_1)) - G(S(t_1)O(t_1)$. Indeed, we now have $F_R(K, R(t_2)) = b > G(S(t_1))$ and $F_O(K, O(t_1)) < b$ from the necessary conditions ($b \geq G(S(t_1)) + \mu(t_1) / \mu_K(t_1)$). Hence, $R(t_2) < O(t_1)$ and the same contradiction is reached. Q.E.D.

Appendix 2: Details of solving for the optimal time paths of regimes I and II

A2.1. Optimal pasting conditions, switch time and stock of oil to be left in situ

I. Initial oil and capital stocks small: the oil-only, renewables-only regime

The optimal program of regime I of proposition 3 consists of two phases.

(i) *Oil-only phase* ($0 \leq t \leq T$):

The initial oil-only phase is described by equations (1), (3O), (6O) and (7O). Given the switch time T, initial values for S_0 and K_0 and terminal values $C(T) = \Theta^R\left(K(T); b\right)$, where $\Theta^R(.)$ is the stable manifold of the renewables-only phase, and $p(T) = b$ at some of time T, we can solve the resulting two-point-boundary-value problem for the time paths of the initial oil-only phase. Hence, this gives $K(T)$, which is used as initial condition in the subsequent phase renewables-only phase, and $S(T)$, both as functions of the unknown switch time T.

(ii) *Renewables-only phase* ($t \geq T$):

The carbon-free economy is described by equations (1), (3R) and (7R).

(iii) Pasting the oil-only and renewables-only phases

To paste the two phases of the optimal program, we use three pasting conditions: (a) $p(T)=b$; (b) *K*, *C* and *S* must not jump at time *T*; (c) $G(S(T))+\dfrac{D'(E_0+S_0-S(T))}{\rho U'(\Theta^R(K(T);b))}=b$ which gives

(A1) $\quad 0<S(T)=\Upsilon(\overset{-}{b},\overset{+}{K(T)},\overset{+}{E_0+S_0},\overset{-}{\rho})<S_0.$

The optimal pasting of the two regimes is completed by solving for the optimal switch time *T* from the condition that (A1) must equal *S*(*T*) at the end of the oil-only phase

The stock of oil left in situ at time *T* is endogenous as it depends on the capital stock that the carbon-free economy starts off with. However, in two cases *S*(*T*) can be found directly. If utility is linear and the elasticity of intertemporal substitution is infinite, i.e., $U'(C)=\varphi$ is a constant and $\sigma\to\infty$, condition (c) indicates that *S*(*T*) is independent of *C*(*T*) or *K*(*T*).

II: Initial oil and capital stocks large: the oil-only, oil-renewables regime

If the initial stocks of oil and manmade capital are sufficiently high (i.e., $S_0>S_0^*(K_0;b,\rho)$), we get regime II with the following two phases of the optimal program:

(i) *Oil-only phase* ($0\le t\le T$):

The oil-only phase follows from (1), (3O), (6O) and (7O) and can be solved given *T*, the initial conditions $K(0)=K_0$ and $S(0)=S_0$ and the terminal conditions $p(T)=b$ and $C(T)=\Theta^{OR}(K(T),b,\rho)$, where $\Theta^{OR}(.)$ is the stable manifold of the oil-renewables phase defined below. This yields *K*(*T*) and *S*(*T*) as functions of the unknown switch time *T*.

(ii) *Oil- renewables phase* ($t\ge T$):

With simultaneous use of oil and renewables, we have $p=F_R(K,V)=b$ and thus $O+R=V(K,b)$. The dynamics of the stock of manmade capital and consumption are now given by (7R) and

(3RO) $\quad \dot{K}=\tilde{F}(K,b)+[b-G(S)]O-\delta K-C,\quad K(T)$ given.

The indifference condition (17) gives the stock of oil in situ:

(A2) $\quad S=S(b,C,K,E_0+S_0).$

The difference with (A1) is that the social rate of interest rather than the rate of time preference is used to calculate the present value of marginal global warming damages. Differentiating (A2) and using (3RO) and (7R), we obtain the oil-depletion dynamics:

(A2′) $$O=-\dot{S}=\frac{-S_C\sigma C\left[\tilde{F}_K(K;b)-\delta-\rho\right]-S_K\left[\tilde{F}(K;b)-\delta K-C\right]}{1+S_K\left[b-G(S)\right]}.$$

Equations (3RO) and (7R) with *S* given by (A2) and *O* given by (A2′) can be solved as a two-dimensional, two-point-boundary value problem for a given switch time *T* and *K*(*T*). The saddlepath corresponding to the stable manifold of this system is denoted by $C(t)=\Theta^{RO}(K(t);b)$. It is not difficult to establish that the slope of this saddlepath evaluated at the carbon-free steady state is positive and equal to $\Theta_K^{RO}=0.5\sqrt{r^{*2}-4r^{*}\sigma F_{KK}C^{*}}+0.5r^{*}>0,$ where $r^{*}\equiv\rho-\sigma F_{KK}C^{*}S_C\left[b-G(S)\right]>\rho>0.$[21] Note that as $r^{*}>\rho,$ we have $\Theta_K^{RO}>\Theta_K^{R}>0.$

(iii) Pasting the oil-only and oil-renewables phases

The solution of the final oil-renewables phase also gives consumption at the beginning of that phase, $C(T)=\Theta^{RO}\left(K(T),b,\rho\right),$ where $\Theta^{OR}(.)$ is the stable manifold of the system. This serves as terminal condition for the oil-only phase. The switch time T is chosen such that the $S(T)$ at the end of the oil-only phase matches the oil stock at the beginning of the oil-renewables phase, so from (A2) we require $S(T)=S\left(b,\Theta^{RO}\left(K(T),b,\rho\right),K(T),E_0+S_0\right).$ Similarly, we require that capital at the end of the oil-only phase must equal capital at the beginning of the oil-renewables phase. The economy converges to the steady state of the carbon-free economy, $(K^{*},C^{*}),$ and $S^{*}=\Upsilon(b,C^{*},E_0+S_0,\rho)$. Since energy prices and energy use must be continuous at time T, renewables use starts with a positive amount and thus oil use must fall at time T by a corresponding amount. Oil and renewables use are thus not continuous at time T.

A2.2. Stable manifold of the renewables-only and the renewables-oil economies

The stable manifold of the carbon-free (renewables-only) economy can be found from eliminating time and solving the resulting first-order differential equation where the steady-state values of K and C pin down the solution:

$$\frac{dC}{dK}=\frac{\sigma C\left[\tilde{F}_K(K,b)-\delta-\rho\right]}{\tilde{F}(K,b)-\delta K-C}\quad\Rightarrow\quad C=\Theta^{R}(K;b)\ \text{ with }\ C^{*}=\Theta^{R}(K^{*};b).$$

The pasting conditions for pasting the oil-only and the carbon-free phase of regime I are given by $C(T)=\Theta^{R}\left(K(T);b\right)$ and $S(T)$ from (A1). Notice that for simulation of the carbon-free economy only a one-dimensional equation needs to be integrated forwards once this stable manifold is substituted into (3R), i.e., $\tilde{F}(K,b)-\delta K-C(K),$ where $K(T)$ comes from the oil-only economy.

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[21] Denoting deviations from the carbon-free steady state by Δ, the saddlepath can be written as $\Delta C=\theta\Delta K$ with the guess $\Theta^{RO}=\theta$. We get $\Delta\dot{C}=\sigma F_{KK}C^{*}\Delta K=\theta(\alpha_1\Delta K-\alpha_2\Delta C),$ where the linearization around steady state gives $\alpha_1\equiv\left\{\rho-\sigma F_{KK}C^{*}S_C[b-G(S^{*})]\right\}\alpha_2>0$ and $\alpha_2\equiv\left\{1+[b-G(S^{*})]S_K\right\}^{-1}>0.$ The method of undetermined coefficients requires solving the quadratic $\alpha_2\theta^2-\alpha_1\theta+\sigma F_{KK}C^{*}=0,$ which yields the solution $\theta=\left(0.5\alpha_1\pm0.5\sqrt{\alpha_1^{2}-4\alpha_2\sigma F_{KK}C^{*}}\right)/\alpha_2$. Picking the positive value of θ corresponds to the saddlepath associated with the stable manifold and gives $\Theta_K^{RO}=0.5\sqrt{r^{*2}-4r^{*}\sigma F_{KK}C^{*}}+0.5r^{*}>0.$

A2.3. Solving for the boundary conditions

To connect the oil-only and the oil-renewables phase in regime I, we use $\Theta^R(.)$ to obtain the right pasting condition. The TPBVP for the oil-only phase can be solved with fourth-order Runge-Kutta integration or with a spectral decomposition algorithm for the linearized model. The Runga-Kutta algorithm is nested within a Newton-Raphson method to solve for *T* and *S*(*T*) from $S(T) = \Upsilon(b, \Theta^R(K(T); b), E_0 + S_0, \rho)$ and *S*(*T*) = *b*. To solve for regime II, we use a spectral decomposition algorithm to solve for the system (3OR) and (7O) with *S* given by (A2) and *O* given by (A2′) and nest this within a Newton-Raphson algorithm to solve for *T* and *S*(*T*) from $S(b, C(T+), K(T+), E_0 + S_0) = S(T-)$.

A2.4. Spectral decomposition algorithm for the linearized model

The carbon-free phase starts at time *T* and is given by (3R) and (7R) starting with the initial condition *K*(*T*). *C*(*T*) must be on the saddlepath of the carbon-free economy, which we linearize as follows:

$$C(T) = \Theta^R(K(T)) \cong C^*(b) + \theta\big(K(T) - K^*(b)\big),\ \Theta^R{}' = \theta \equiv \frac{1}{2}\rho + \frac{1}{2}\sqrt{(\rho+\delta)^2 + 4\sigma\left(\frac{1-\alpha-\beta}{1-\beta}\right)(\rho+\delta)\frac{C^*}{K^*}} > 0.$$

We linearize around (S^*, K^*, C^*, b) with *S** from $b = G(S^*) + \dfrac{D'(E_0 + S_0 - S^*)}{\rho U'(C^*)}$. Defining $R^* = V(K^*, b)$, we get the state-space system $\dot{\underset{\sim}{x}} = \mathrm{A}\underset{\sim}{x} + \underset{\sim}{a}$, where $\underset{\sim}{x} \equiv (S - S^*, K - K^*, C - C^*, p - b)'$ and $\underset{\sim}{a} \equiv (-R^*, K^{*\alpha} R^{*\beta} - G(S^*) - \delta K^* - C^*, 0, 0)'$. Spectral decomposition gives $\mathrm{A} = \mathrm{M}\Lambda\mathrm{M}^{-1} = \mathrm{N}^{-1}\Lambda\mathrm{N}$ where the diagonal matrix Λ contains the eigenvalues in descending order and the matrix M contains the eigenvectors. Defining the canonical variables $\underset{\sim}{y} = \mathrm{N}\underset{\sim}{x}$ yields $\dot{\underset{\sim}{y}} = \Lambda\underset{\sim}{y} + \underset{\sim}{n}$, $\underset{\sim}{n} \equiv \mathrm{N}\underset{\sim}{a}$. The system has two eigenvalues with positive real part, collected in the vector $\underset{\sim}{\lambda}_u$, and two with negative real part, collected in $\underset{\sim}{\lambda}_s$, and thus satisfies the saddlepoint property. We thus have $\Lambda = \mathrm{diag}(\lambda_{u1}\lambda_{u2}, \lambda_{s1}, \lambda_{s2})$, so we get:

(A3)
$$y_{ui}(t) = e^{\lambda_{ui}(t-T)}\left[y_{ui}(T) + \overline{n}_{ui}\right] - \overline{n}_{ui},\quad \overline{n}_{ui} \equiv n_{ui} / \lambda_{ui},\quad i = 1, 2,$$
$$y_{si}(t) = e^{\lambda_{si}t}\left[y_{si}(0) + \overline{n}_{si}\right] - \overline{n}_{si},\quad \overline{n}_{si} \equiv n_{si} / \lambda_{si},\quad i = 1, 2,\quad \forall t \in [0, T].$$

Decomposing so that $\mathrm{M} = \begin{pmatrix} \mathrm{M}_{su} & \mathrm{M}_{ss} \\ \mathrm{M}_{uu} & \mathrm{M}_{us} \end{pmatrix}$ and $\underset{\sim}{x} = (\underset{\sim}{x}_s{}', \underset{\sim}{x}_u{}')'$, we write the initial conditions as:

(A4)
$$\underset{\sim}{x}_s(0) = \mathrm{M}_{ss}\underset{\sim}{y}_s(0) + \mathrm{M}_{su}\begin{pmatrix} e^{-\lambda_{u1}T} & 0 \\ 0 & e^{-\lambda_{u2}T} \end{pmatrix}\left[\underset{\sim}{y}_u(T) + \underset{\sim}{\overline{n}}_u\right] - \underset{\sim}{\overline{n}}_u = \underset{\sim}{x}_{so} \equiv (S_0 - S^*, K_0 - K^*)'.$$

The terminal conditions $C(T) = \Theta(K(T))$ given above and *p*(*T*) = *b* become:

(A5)
$$\mathrm{E}\left(\mathrm{M}_{ss}\left\{\begin{pmatrix} e^{\lambda_{s1}T} & 0 \\ 0 & e^{\lambda_{s2}T}\end{pmatrix}\left[\underset{\sim}{y}_s(0)+\underset{\sim}{\overline{n}}_s\right]-\underset{\sim}{\overline{n}}_s\right\}+\mathrm{M}_{su}\underset{\sim}{y}_u(T)\right)+\mathrm{M}_{us}\left\{\begin{pmatrix} e^{\lambda_{s1}T} & 0 \\ 0 & e^{\lambda_{s2}T}\end{pmatrix}\left[\underset{\sim}{y}_s(0)+\underset{\sim}{\overline{n}}_s\right]-\underset{\sim}{\overline{n}}_s\right\}$$
$$+\mathrm{M}_{uu}\underset{\sim}{y}_u(T)=\underset{\sim}{0} \text{ from } \mathrm{E}\underset{\sim}{x}_s(T)+\underset{\sim}{x}_u(T)=\underset{\sim}{0}, \quad \mathrm{E}=\begin{pmatrix} 0 & -\theta \\ 0 & 0\end{pmatrix}.$$

The initial and terminal conditions (A4) and (A5) can be solved as follows:

$$\begin{pmatrix}\underset{\sim}{y}_s(0) \\ \underset{\sim}{y}_u(T)\end{pmatrix}=\begin{pmatrix} \mathrm{M}_{ss} & \mathrm{M}_{su}\begin{pmatrix} e^{-\lambda_{u1}T} & 0 \\ 0 & e^{-\lambda_{u2}T}\end{pmatrix} \\ (\mathrm{M}_{us}+\mathrm{EM}_{ss})\begin{pmatrix} e^{\lambda_{s1}T} & 0 \\ 0 & e^{\lambda_{s2}T}\end{pmatrix} & \mathrm{M}_{uu}+\mathrm{EM}_{su}\begin{pmatrix} e^{-\lambda_{u1}T} & 0 \\ 0 & e^{-\lambda_{u2}T}\end{pmatrix}\end{pmatrix}^{-1}\begin{pmatrix} \underset{\sim}{x}_{s0}+\mathrm{M}_{su}\begin{pmatrix} 1-e^{-\lambda_{u1}T} & 0 \\ 0 & 1-e^{-\lambda_{u2}T}\end{pmatrix}\underset{\sim}{n}_u \\ \left(\mathrm{M}_{us}+\mathrm{EM}_{ss}\right)\begin{pmatrix} 1-e^{\lambda_{s1}T} & 0 \\ 0 & 1-e^{\lambda_{s2}T}\end{pmatrix}\underset{\sim}{n}_s\end{pmatrix}.$$

Given this we can calculate $\underset{\sim}{y}$ from (A3) and thus finally obtain $\underset{\sim}{x}=\mathrm{M}\underset{\sim}{y}$ for $\forall t\in[0,T]$. The resulting solution trajectories satisfy the necessary initial and terminal conditions, (A4) and (A5). To obtain the switch time, we use (A1) and solve for time *T* from $b=G(x_{s1}(T)+S^*)+\dfrac{D'(E_0+S_0-x_{s1}(T)-S^*)}{\rho U'(x_{u1}(T)+C^*)}$. This procedure implies that *S*(*T*) depends on *C*(*T*). Given *K*(*T*) obtained from the fossil-fuel economy, the carbon-free economy can be found from multiple shooting or directly from linearization:

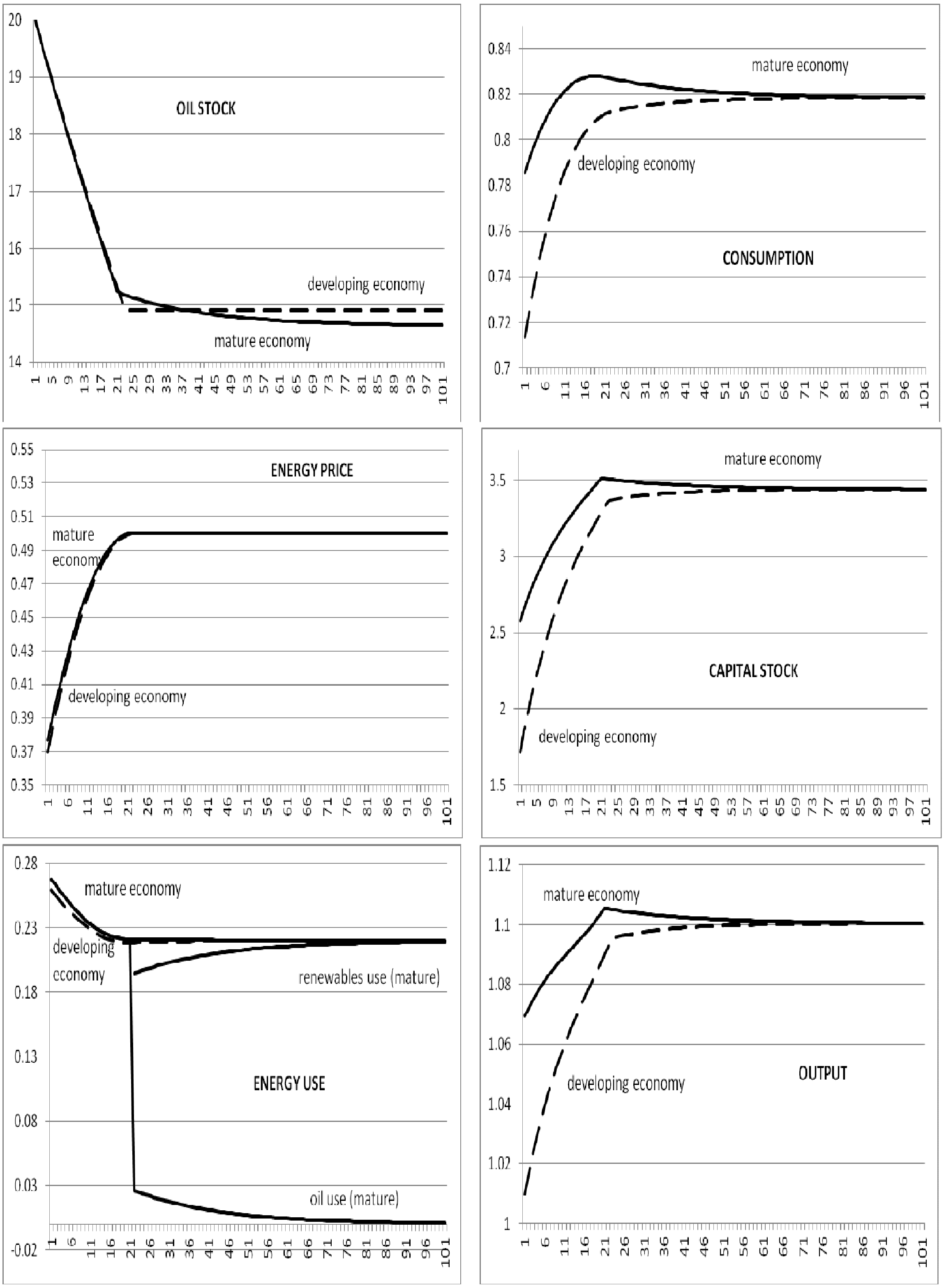
$$\dot{K}(t)=K^*+(\rho-\theta)\left[K(t)-K^*\right] \;\Rightarrow\; K(t)=K^*+e^{(\rho-\theta)(t-T)}\left[K(T)-K^*\right] \text{ and}$$
$$C(t)=C^*+\theta e^{(\rho-\theta)(t-T)}\left[K(T)-K^*\right], \quad \forall t\geq T, \text{ where } \theta>\rho.$$

We have also tried a fourth-order Runge-Kutta algorithm to solve (1), (3O), (6O) and (7O) given *K*(0) = K_0, *S*(0) = S_0 and guesses for *C*(0) and *p*(0); and Gauss-Newton iterations to adjust *C*(0), *p*(0) and *T* to satisfy $p(T)=b, b-G(S_T)=\dfrac{D'(E_0+S_0-S_T)}{\rho U'(C(N))}$, and $C(T)=\Theta(K(T))$. This was numerically sensitive, hence we report the results from our linearized model.

Appendix 3: Overshooting in a mature economy

If we start off with S_0 = 20 and $K_0=0.75K^*$, we get $S_0^*=18.7<20.8$ and $T^*=16.4<24.3$. The paths for the social optimum starting with K_0 = 0.5 *K** and starting with K_0 = 0.75 *K** are shown in fig. A1. They are qualitatively very similar to the paths of fig. 4 for an oil-abundant economy. A mature economy starts with a higher rate of consumption, but ends up with the same capital stock and rate of consumption in steady state. The rate of consumption overshoots first and sometime later capital overshoots its steady-state value; this does not occur in a developing, oil-scarce economy. In the mature economy $T=20.1>T^*=16.4$, but in the developing economy $T=22>T^*=24.3$. The mature economy leaves less oil in the crust of the earth than the developing economy, which results from oil use throughout the only-oil phase being less than in the developing economy and no oil being use alongside renewables in the final phase. The more negative impact on the climate in a developing economy is less as, despite advancing more rapidly along its development path, oil use is somewhat less in the oil-only phase and zero in the final renewables-only phase. This is why the mature economy has higher carbon tax than the developing economy for the oil-only phase.

Figure A1: Comparing social optimum for a mature economy with a developing economy



Note: Mature economy $T = 20.1$, $S(T) = 15.2$; developing economy $T = 22$, $S(T) = 14.9$.