Terrorist Targeting, Information, and Secret Coalitions

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Abstract: We consider a game played by a state sponsor of terrorism, a terrorist group, and the target of terrorist attacks. The sponsoring state wishes to see as much damage inflicted on the target of attack as possible, but wishes to avoid retaliation. To do so, his relationship with the terrorist group must remain ambiguous. The target of attack, for his part, wishes to bring these attacks to an end as quickly as possible and will consider the option of retaliating against the sponsor to do so. There is a penalty, however, for retaliating against a state that is not supporting terrorist operations, and even if the victim is aware of this relationship, it is necessary to convince third parties that this relationship actually exists.

We approach the problem by introducing an "evidence" variable in a dynamic setting. We show that the interplay of different strategic and non-strategic effects boils down to three qualitatively different scenarios, determined by key parameters. Based on this result, two alternative instruments to retaliation are identified in order to resist terrorist activities. First, assuming that the target is able to change some parameters by monetary investments, the paper provides an economic analysis of how to invest optimally in order to make the sponsor lose incentives to support the terrorist group. Second, we propose changing the structure of the game. Here, the key insight is that the target country can make a unilateral statement as to his strategy. The sponsor cannot do so as he is in fact claiming that there is no cooperation with terrorist groups.

While our discussion, in this article, is motivated by an important problem in contemporary counterterrorism policy, it applies more generally to the study of secret coalitions.

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1. The problem

State-terrorist alliances have been a central feature of modern terrorism since the late 1960s. The motivations that underlie these alliances are, by now, well understood. For terrorists, state sponsorship is an important force multiplier, offering a source of material and financial aid, diplomatic cover, and often a comparatively secure environment from which to prepare and launch their operations. With such assistance, terrorist groups are frequently able to emerge as important regional and global actors in their own right. In the absence of such assistance, they are seldom more than a political nuisance. There are clear potential benefits for the sponsoring state, as well. State-terrorist alliances can be characterized as *secret coalitions*, where the specific nature, significance, and sometimes even the existence of the relationship between the sponsor and the group is unknown to the target of attack. Even where state sponsors are involved in supporting terrorist attacks they are often able to avoid being held accountable for the actions of their proxies[5]. This provides them with the opportunity to conduct a more (indirectly) aggressive and otherwise risky foreign policy than if they were not operating through an agent.

The problem that state-terrorist coalitions pose to the targets of terrorist attack is complicated by the fact that terrorist groups are hard to find and localize and, therefore, hard to respond to directly. This is due not only to any cover and protection they may gain by operating from the sovereign territory of another country, but also to their small size, informal organizational structures, and their own internal security measures. Because they are hard to see, they are hard to eliminate. This, in turn, makes them hard to deter. Strong states, in this way, are vulnerable to attack from objectively weaker terrorist groups because they are unable to bring their relative strength to bear against covert opponents. The force advantage enjoyed by one side, in such cases, is offset by the anonymity of the other. The net advantage in this contest generally resides with the terrorists. They may not be strong enough to strike decisively, but they are invisible enough to do so with a low expectation that they will be hit decisively in turn.

Because of the difficulty of responding to terrorist actions directly, many targets of attack are tempted to respond to these actions indirectly by threatening the use of force or other sanctions against "the source of the problem" -- the terrorists' state sponsor(s). The expected effectiveness of such a policy depends, among other things, on the assumption 1) that a sponsorship relationship actually exists and 2) that this relationship is essential to the terrorist group's continuing ability to pose a threat. To the degree that a terrorist group depends on continuing sponsor support to stay in the game, and the sponsoring state can be pressured into discontinuing that support, the group will be effectively neutralized as a future threat. On the other hand if no such relationship exists, or the relationship that exists is not central to the group's continuing ability to act, any program to indirectly attack the group by targeting its sponsor is destined to fail. The expected success of such a policy, in short, depends, on the confidence a retaliator has that the relationship between the sponsoring state and its proxy is actually what it seems.

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[5] One current example would be the relationship between Venezuela and the FARC guerrilla movement of Colombia, in which President Hugo Chávez, though he has nothing but praise for the guerrilla leadership, and allows them safe harbor in his country, can insist that he is in no way collaborating with them in their fight against the Colombian government.

This influences not only the expected *benefits* of a sponsor-focused counter terrorism policy, but also its expected *costs*. In general, there are two sets of costs associated with retaliating against a state sponsor of terrorism. The least important of these are the material costs of carrying out a program of sanctions. These costs are sensitive to the type and magnitude of the retaliatory action. The second and more important set of costs is political, both at home and abroad. These costs not only vary according to the nature and magnitude of the sanctions that are imposed; they are also highly sensitive to domestic and international perceptions concerning whether or not a causal relationship actually exists between terrorist attacks and the "sponsoring" state. The problem, in this case, is not simply one of identifying who a terrorist group's state sympathizers are -- this is generally well known -- but whether and to what degree these players can be shown to be involved in sponsoring terrorist operations and, therefore, the degree to which they can reasonably be held accountable for a group's activities.

The visibility of state-terrorist coalitions and, therefore, the payoff of retaliation are subject to change over time. One of the most important means of uncovering such a relationship is by evaluating the nature and pattern of the terrorist attacks themselves. While the victim of these attacks may, initially at least, have a very limited view of what is going on behind the scenes, it is clearly able to monitor the tempo, magnitude, and sophistication of its adversary's operations. In general, the more a group is able to do, and the longer and more frequently it is able to do it, the more likely it is to be receiving state assistance. Like a crime scene, furthermore, terrorist attacks generally leave behind trace evidence that can provide further insights into an attack's origins, its technical character, who carried it out, what forms of assistance were needed to conduct the operation, and what the likely sources of this assistance might be. What this means as a practical matter is that the longer a secret coalition exists, and the more important this relationship is to a terrorist group's capacity to act, the less secret the coalition is likely to become. As evidence of collusion grows, the net payoff of retaliation will increase, in turn.

This dynamic presents both the sponsor of terrorist attacks and the victim of these attacks with an interdependent decision in the face of incomplete information:
The victim must decide whether or not the expected benefits of retaliating against the terrorists' state sponsor outweigh the expected costs. Since his knowledge of the exact nature of the coalition will never be perfect, he must decide what level of evidence he will accept. If he decides to retaliate later rather than sooner in an effort to gain a more complete picture of this relationship and reduce his political costs, he risks experiencing more attacks than if he had acted earlier. On the other hand, if he acts sooner rather than later in an effort to avoid future attacks, he risks incurring a higher political cost for retaliating than if he had waited longer. The key assumption of our approach is that the cost of retaliation (political and economic) by the victim country decreases with the amount of evidence built up against the sponsor country. This evidence will be measured by a "suspicion factor" in a dynamic setting. The idea is to incorporate the fact that evidence against the sponsor goes up (down) each time a there is a terrorist attack while (not) being sponsored. This will be done with the use of Bayes' updating rule along with the fact that the probability of a successful terrorist attack increases when the terrorist groups is being sponsored.
In a similar way, the sponsor must decide whether or not the expected benefits of continuing to support terrorist attacks against the victim outweigh the expected costs. Besides the material costs of supporting the terrorist and costs of retaliation, the sponsor has repeated political costs to be suspected to help the terrorist group. While the sponsor may assume that, if this support

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continues, it will gradually be exposed, he cannot know the rate at which this will occur or its consequences. If he decides to sever or reduce his ties sooner rather than later in an effort to avoid retaliation, he will forego the benefits of working through his proxy for a longer period of time. On the other hand, if he acts later rather sooner in an effort to inflict as much damage as possible, he faces substantial political costs and risks being attacked in return.

While substantial work has been done on the problem of incomplete information in games (for a good overview see Aumann, Maschler and Stearns, 1995), this has dealt mainly with information regarding player utilities. Here we deal with a different problem, the role that information plays in the actions of and responses to a secret coalition where the existence and nature of the coalition is incompletely known to those who are not in the coalition but in the game. This problem is investigated by means of a game played by a terrorist organization, its victim, and its state sponsor.[6] While our discussion, in this article, is motivated by an important problem in contemporary counterterrorism policy, it applies more generally to the study of secret coalitions.

2. The model

We consider a game with three players: the terrorist (T), the Host (H) and the target or object of attack (V). Assuming that T has attacked V, it may be difficult for V to retaliate directly against T. T cannot, however, continue to operate at more than a low level of activity without support from H. Thus V has the option of retaliating, indirectly, against T by attacking or otherwise sanctioning H. The problem, as noted above, is that it is not always easy to determine whether and to what degree H is supporting T. Even when V feels quite certain that this is the case, there are often political reasons which make it impractical to act against H without clear evidence of collusion. In particular, there will be other parties (neutral states, international organizations, and even various domestic political constituencies) that will impose a cost on V (if V attacks H) unless they can be convinced that there is a link between H and T. In the absence of a "smoking gun" that definitively links H to T, however, it may still be possible to use mathematical analysis to infer a connection. This is based on the idea that if T is indeed receiving support from H, it will be able to operate at a higher level of activity and with a greater destructive effect than if it were operating on its own. Thus, for any given T, the higher the frequency and/or magnitude of T's attacks against V, the more likely it is that a supporting relationship actually exists, between T and H, and that neutral observers will accept its existence.

With this in mind, we distinguish between a **strong** T, who is in fact receiving aid from H, and a **weak** T, who is receiving no such support. We assume that, at the beginning of the game, T and H are not allied. At this time H must decide whether to form a coalition {H, T}, thus leading to a strong T, or to avoid such a coalition, leaving T to remain weak. Now T (knowing his type) can act (attack V) or not. For his part V, knowing T's action, but not T's type, can consider retaliation *after* he has been attacked. If in fact V retaliates, this retaliation can be

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[6] The present article focuses primarily on the bilateral relationship between the terrorist sponsor and the target of terrorist attack. In an earlier work, two of the authors examined the equally interesting relationship between the terrorist group and its sponsor. See, McCormick and Owen (2009). For an excellent and easily accessible overview of the state sponsorship of terrorism, see Byman (2005).

against T alone, against H alone, or against both T and H. We generally discard the idea of retaliation against H alone; we can reasonably assume that V may or may not attack H if he is prepared to attack T, but he will not attack H if he is not yet willing to attack T. H is always the more costly target. The question we investigate is whether or not he will retaliate against H and how this, in turn, influences H's calculations concerning the wisdom of continuing to support T.

We assume that, if there is no coalition, a weak T can attack at a level q. He will in fact do this, but cannot increase the average value of these attacks above q. A strong T, by contrast, with H's support, can raise the level to p. (This p is to be chosen jointly by T and H.) We assume $q \leq p \leq 1$. We assume a payoff to H of $A(p\text{-}q)$ per time period, where A is the payoff obtained from a unit-level attack. There is also a discount factor α. Payoffs or benefits of size z, obtained t time periods in the future, in this case, have a discounted present value of $ze^{-\alpha t}$. For a given level p, the total discounted value of attacks to H at time t is

$$\frac{A(p-q)}{\alpha}(1-e^{-\alpha t}) \, . \tag{1}$$

This term enters the payoff to V with a minus sign.

For V, there is a suspicion factor, μ (the "odds"), which corresponds to V's beliefs about the existence of valid evidence of an H-T coalition. In fact, V assigns subjective probability $\mu/(1+\mu)$ to this. Alternatively, this term can be thought of as the likelihood or odds that objective international and domestic actors will accept the existing evidence. (Note, however, that different actors will be more or less difficult to convince. Thus μ would be some average of the neutral observers' acceptance of the evidence.) At the beginning of the process, μ has a relatively low value $\mu(0) = c$.

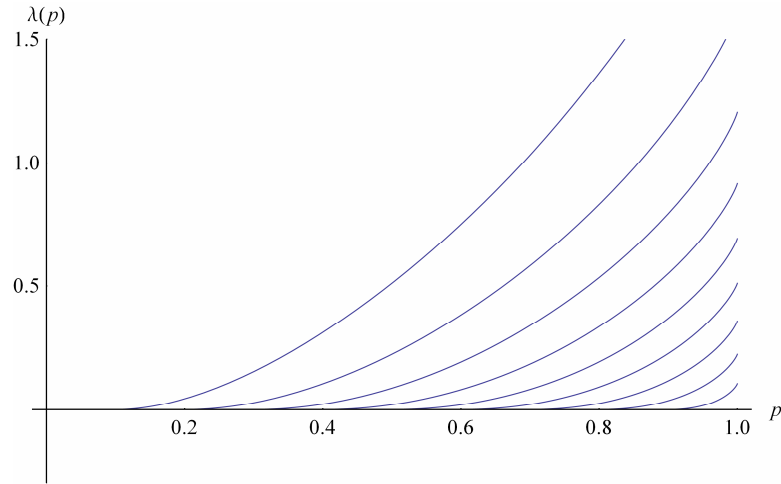As long as the coalition exists, as time progresses, μ increases exponentially, so $\mu(t) = ce^{\lambda t}$

$$\mu(t) = ce^{\lambda t} \tag{2}$$

where

$$\lambda = p \log(p/q) + (1-p)\log((1-p)/(1-q)) \tag{3}$$

[See subsection A1 of the Appendix for a justification of this λ.]

Figure 1 shows λ as a function of p, for different values of q.

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Figure 1

What would be the best *p*, from H's viewpoint? In general, we can assume that the attacks at this rate will continue until some critical level of μ has been reached. At that point, either V will attack H, or H will decide to discontinue assistance to T. Clearly, H would like to choose *p* in such a way as to maximize the damage done to V in that time.

To see how this work, let us assume that the attacks will continue until the level $\mu = ce = e\mu(0)$ is reached[7]. This will happen at time $\tau = 1/\lambda$. In that case, the total (undiscounted) payoff from these attacks will be $A(p-q)/\lambda$, as shown for *q*=0.3 in Figure 2.
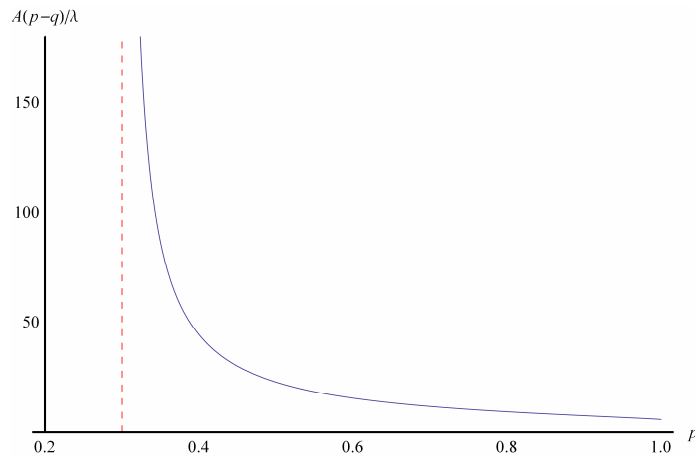
Figure 2

Note that, by choosing *p* slightly larger than 0.3 (the assumed value of *q*), H and T will do a lot of damage. However, this damage is spread out over a long period of time. (Think of 15 units of destruction per year, over the next 10,000 years. H and T, on the other hand, would clearly prefer to inflict 1000 units of destruction per year over the next 15 years.) The point, in this case,

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[7] There is no significance in choosing the critical μ in this way; we merely do this to illustrate the result. Note that *e* here is the base for natural logarithms, approximately equal to 2.718.

is that we have not discounted future payoffs. If we introduce our discount factor, we find that the discounted payoff is

$$A(p - q)(1 - e^{-\alpha\tau}) / \alpha \tag{4}$$

Figure 3, below shows this payoff, assuming a critical level $\mu = e\mu(0)$ is reached, and assuming $\alpha = 0.1$. As may be seen, this payoff reaches a maximum value at about $p = 0.5$. At higher values of p, the critical level of μ is reached very quickly and not enough damage is done. At lower levels, by contrast, the process lasts a long time, but the damage occurs slowly, and, because of discounting, its total present value is low.
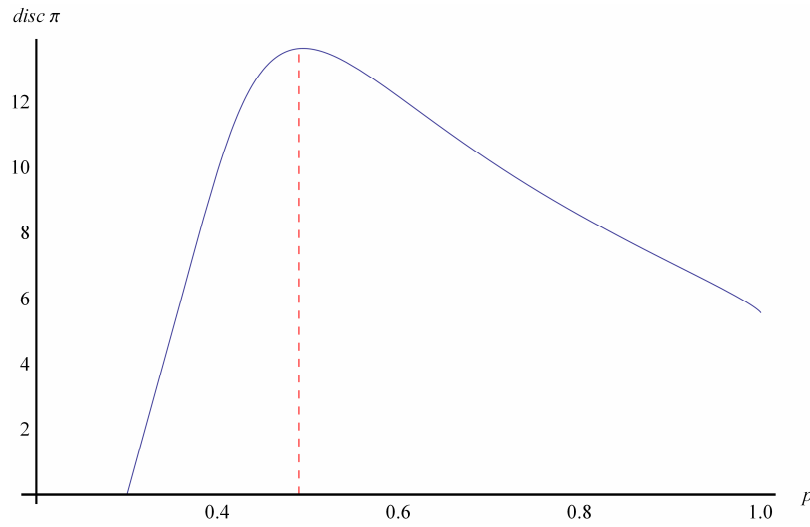
Figure 3

At some time, the process stops, either because V retaliates, or because H decides to break off his supporting relationship with T. Let us assume that V stays observant until time t and retaliates immediately afterwards.[8] H in turn, plans to sever his relationship with T at time s.

Suppose $t < s$. Then V stays observant until time t, when μ has reached a value $ce^{\lambda t}$. In that case there is probability $\mu/(1+\mu)$ of a positive benefit, B, to V and probability $1/(1+\mu)$ of a political cost, K. The expected, discounted value of this is

$$e^{-\alpha t} \frac{\mu B - K}{1 + \mu} \tag{5}$$

Apart from this, V has been suffering from T's attacks, which as seen before has a total discounted value

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[8] An alternative would be to interpret t as retaliation time. However, such an interpretation prevents determining H's best response as this would be $s<t$ as close as possible to t - unless we introduce ε-optimality (see Maschler 1966) implying H's best response to t is to stop supporting T at time $s=t-\varepsilon$. For notational convenience, we will stick to the interpretation of t as observant time.

$$-A(p-q)(1-e^{-\alpha t})/\alpha. \tag{6}$$

Suppose on the other hand $s \leq t$. Then the process ends because H breaks off at time s. V will never get to retaliate. There is then only the discounted payoff of the attacks given by $-A(p-q)(1-e^{-\alpha s})/\alpha$.
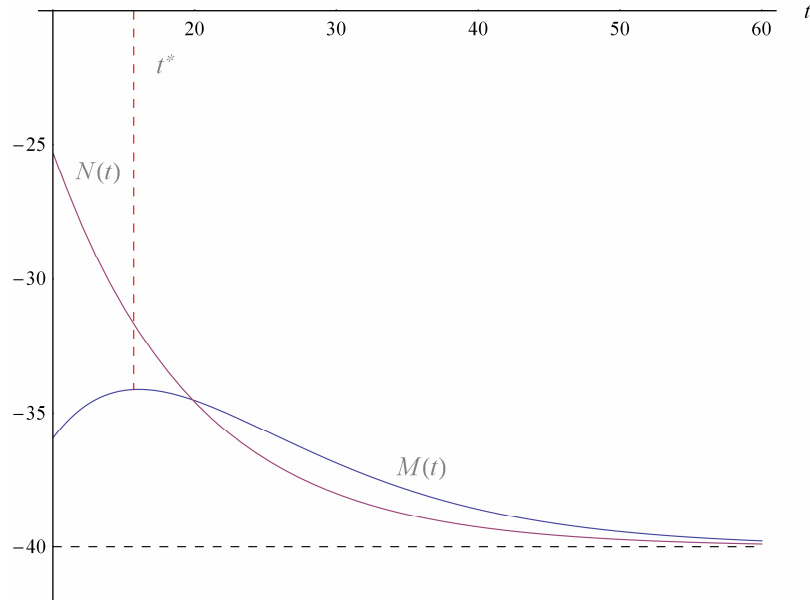
Thus we have the payoff to V

$$\Pi_V(s,t) = \begin{cases} M(t) & \text{if } t < s \\ N(s) & \text{if } t \geq s \end{cases} \tag{7}$$

where

$$M(t) = -\frac{A(p-q)}{\alpha}(1-e^{-\alpha t}) + e^{-\alpha t}\frac{ce^{\lambda t}B - K}{1+ce^{\lambda t}}$$

$$N(s) = -\frac{A(p-q)}{\alpha}(1-e^{-\alpha s}). \tag{8}$$

The strategy set of V is given by $S_V = \mathbb{R}_0^+ \cup \{\infty\}$. Note that $t = \infty$ means V never retaliates.

It will be of interest to visualize the function $M(t)$. Figure 4 gives a typical case, corresponding to $p-q = 0.2$, $A = 20$, $\alpha = 0.1$, $\lambda = 0.08$, $c = 0.4$, $B = 50$, and $K = 100$.
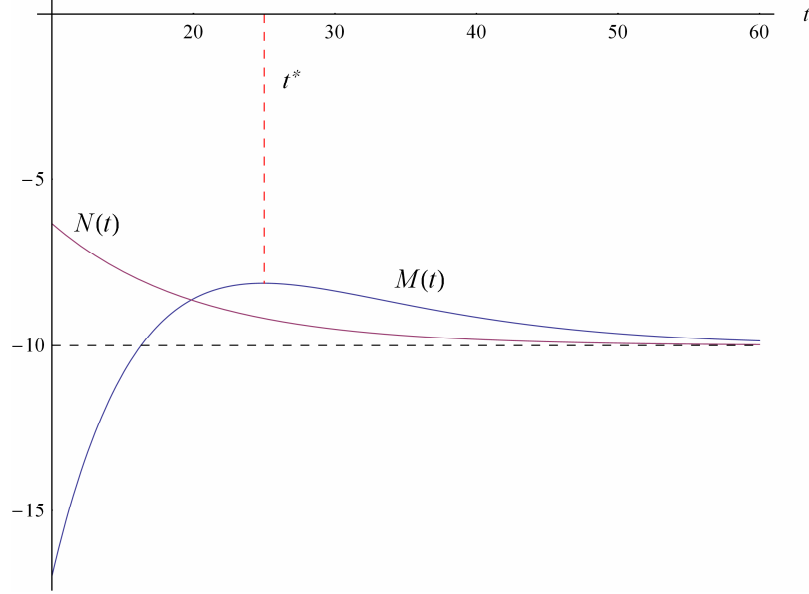

Figure 4

As may be seen, this increases to a maximum, equal to about -34.5, obtained at approximately $t = 19$, and then decreases monotonically to an asymptotic value -40. This is typical of the function, no matter what values are assigned to its several parameters, though in

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some cases, especially if c is relatively large, or if B is much larger than K, the maximizing point may correspond to a negative value of t.

As for the function $N(s)$, this has a relatively straightforward behavior, decreasing monotonically from 0. Figure 4 shows this behavior for the same parametric values. It also approaches the value -40 asymptotically. As may be seen, both $M(t)$ and $N(s)$ approach the asymptotic value $-(p-q)A/\alpha$ as the time variable (t or s) goes to ∞.

Figure 5

In Figure 5, A has been changed from 20 to 5. All other parameters are the same as in Figure 4. In this case, M has a maximum of about -8.2, obtained at $t = 25$. Both M and N approach the asymptotic value of -10.

For a general set of parameters there are two critical values of the time variable. One of these is t^*, which satisfies $M'(t^*) = 0$, and which, if non-negative, maximizes the function M. See subsection A2 of the Appendix for an analytic derivation of this t^*. The other is $t^{\#}$, which corresponds to the value

$$\mu^{\#} = K / B. \tag{9}$$

Thus

$$t^{\#} = \frac{\log\left(K / Bc\right)}{\lambda} \tag{10}$$

It is easily seen that, for $t = t^{\#}$, the second term in the definition of M vanishes. What this means is that $t^{\#}$ is the break-even point for retaliation. Thus, $M(t^{\#}) = N(t^{\#})$. Similarly, we can see that, if $t < t^{\#}$, $M(t) < N(t)$, while for $t > t^{\#}$, $M(t) > N(t)$.

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For the strategy set of H, first note that he is not only able to determine s, the time at which H severs his relationship with T, but also p, the level with which a supported T is able to attack. Hence the strategy set of H is given by $\left(\mathbb{R}_0^+ \cup \{\infty\}\right) \times [q,1]$ which includes the case of non-cooperation with T by setting $p = q$.

Let β denote the time preference rate of H. Besides the total discounted profit value

$$\frac{A(p-q)}{\beta}(1-e^{-\beta t})$$

of attacks, H's the cooperation with T implies three kinds of costs:
(i) repeated *material* costs to support T,
(ii) repeated *political* costs to be suspected to help the terrorist group,
(iii) the costs of retaliation.

For (i) we assume a material costs $C_T(p-q)$ per time period, where C_T is the cost generated from a unit-level attack. For a given level p, the total discounted value of these costs at time s is

$$\frac{C_T(p-q)}{\beta}(1-e^{-\beta s}). \tag{11}$$

The costs of retaliation (iii) at time s are given by $C_R(p-q)e^{-\beta s}$.

The repeated political costs (ii) are more complicated. So long as there is some suspicion, H will suffer a loss proportional to the suspicion factor $\dfrac{\mu(s)}{\mu(s)+1}$ multiplied by C_P, the costs when the suspicion factor reaches it's maximum level 1. Because these are continuing costs, these have to be integrated. Moreover, they are being discounted, so we wind up with the integral

$$\int_0^s C_P \frac{\mu(v)}{\mu(v)+1}(p-q)e^{-\beta v}dv. \tag{12}$$

Thus we have the payoff to H

$$\Pi_H(t,(s,p)) = \begin{cases} F(s) & \text{if } s \leq t \\ G(t) & \text{if } t > s \end{cases} \tag{13}$$

where

$$F(s) = \frac{A-C_T}{\beta}(p-q)(1-e^{-\beta s}) - \int_0^s C_P \frac{\mu(v)}{\mu(v)+1}(p-q)e^{-\beta v}dv$$

$$G(t) = \frac{A-C_T}{\beta}(p-q)(1-e^{-\beta t}) - \int_0^t C_P \frac{\mu(v)}{\mu(v)+1}(p-q)e^{-\beta v}dv - C_R e^{-\beta t} \tag{14}$$

Note that for any given t, it is never a best response of H to choose $s > t$ since he can avoid retaliation costs by setting $s = t$.
For the following we assume that $A > C_T$ which means that the damage done to V is higher than the cost of support of the terrorist. Note that the first term of F and G has the same qualitative shape as $-N$ given by (8).

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The following figures correspond to q=0.3, A=20, C_T=5, C_P=15, β=0.1, c=0.4.
Figure 6 shows the political costs (12) which are monotonically increasing in s and p, except for $p=q$ or $s=0$ where it stays 0 for increasing s, p respectively.
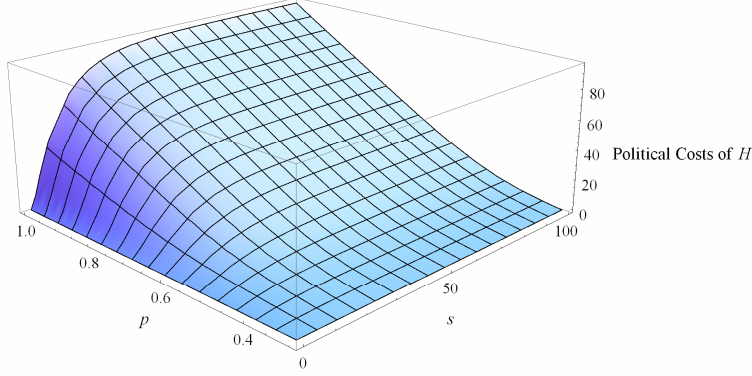
Figure 6

The payoff to H without retaliation given by F is illustrated in Figure 7. Here, F is interpreted as a function in p with given fixed levels of s=1,2,…,100. The red graph indicates $F(p)$ at the fixed level s=5. This figure shows that there is an inner maximum p^* for any fixed level of s. Since the suspicion factor is increasing over time for any $p > q$, political costs eventually become too painful for larger values of s. This provides an incentive for H to reduce the growth rate of the suspicion factor by choosing a lower value p^* when s increases.

Figure 7

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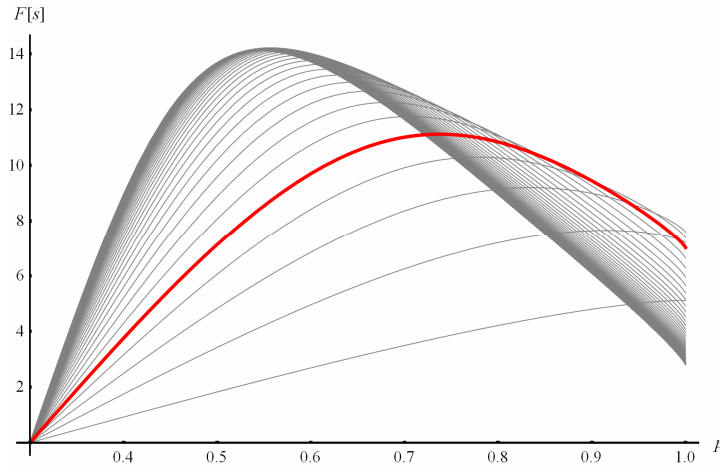
3. Game theoretic analysis – the normal form game

We start our analysis by assuming a fixed level of p in order to allow it to be chosen strategically by H at some later point. However, in case of dependence on p we will indicate it as an argument. In particular, note that $t^{\#}$ from (10) depends on p because λ does. This also holds for critical time t^* as shown in (44) of the Appendix.
Put

$$\Theta = \frac{(A - C_T)}{C_P}. \tag{15}$$

Proposition 1: *For any fixed level of p, the best response function of H is given by*

$$r_H(t) = \begin{cases} t & \text{for } \Theta \geq 1, \\ \min(s^*(p), t) & \text{for } \dfrac{c}{c+1} < \Theta < 1, \\ 0 & \text{otherwise,} \end{cases} \tag{16}$$

where $s^*(p)$ is the level at which the corresponding suspicion factor equals the ratio of the net profit of terror divided by the political costs

$$\frac{\mu(s^*(p))}{\mu(s^*(p)) + 1} = \Theta. \tag{17}$$

For a proof see subsection A4 of the Appendix.

The interpretation is that, for $c/(c+1) < \Theta < 1$, there is a point $s^*(p)$ in time where the political costs become so unbearable to H that he will stop even without the threat of retaliation. For $\Theta \leq c/(c+1)$, the material costs of supporting T as well as the political costs are too high in comparison to the benefit of damage to V, so that H has no incentive to choose any $s > 0$ regardless of the strategy choice of V.

Note that for $\Theta \geq 1$, this finding implies never ending support of T should V choose never to retaliate, $t = \infty$. However, for $t > t^{\#}(p)$, the expected, discounted value of retaliation to V given by (5) becomes positive. This rules out $s = t = \infty$ as Nash equilibrium strategies.

Corollary 1: *For all parameter values the time horizon of the game is finite. Hence the game will end eventually either by retaliation or by ending support of T.*

For the game theoretic analysis it turns out that the Nash equilibria are not unique. For example, there are infinitely many when A is large enough. For increasing A we have $\Theta \geq 1$ eventually and hence $r_H(t) = t$. On the other hand, V is not willing to retaliate unless either the break-even

point $t^{\#}(p)$ or his maximum payoff level of retaliation $M(t^{*}(p))$ is attained such that setting $t = s$ is a best response for early values of s. Hence for sufficiently large A there is a whole interval $[0, r]$ such that any (t, s) with $t = s \in [0, r]$ represents a Nash equilibrium.

The following Proposition shows that a whole set of equilibria can be ruled out by a simple refinement criterion.

Proposition 2: *The sets*

$$W_V = \left\{ t \mid t < \min\left\{ t^{*}(p), t^{\#}(p) \right\}, t > \max\left\{ t^{*}(p), t^{\#}(p) \right\} \right\}$$

$$W_H = \begin{cases} \left\{ s \mid s > s^{*}(p) \right\} & \text{for } \dfrac{c}{c+1} < \Theta < 1 \\[4mm] \left\{ s \mid s > 0 \right\} & \text{for } \Theta \leq \dfrac{c}{c+1} \end{cases} \qquad (18)$$

are weakly dominated strategies of H and V, respectively.

For a proof see subsection A5 of the Appendix.

Proposition 3: *Let p be given such that a strategy of H reduces to choosing $s \in \mathbb{R}_0^+ \cup \{\infty\}$. After elimination of weakly dominated strategies, the strategy tuple (t, s) with $t = \min(t^{*}(p), t^{\#}(p))$ and s given by*

$$\begin{aligned} s = t = \min(t^{*}(p), t^{\#}(p)) & \qquad \text{for } \Theta \geq 1, \\[2mm] s = \min(t^{*}(p), t^{\#}(p), s^{*}(p)) & \qquad \text{for } \dfrac{c}{c+1} < \Theta < 1, \\[2mm] s = 0 & \qquad \text{otherwise.} \end{aligned} \qquad (19)$$

is a Nash equilibrium.

For a proof see subsection A6 of the Appendix.

Again, note that the Nash equilibria of Proposition 3 are not the only one. In fact, depending on the parameter constellation there can be infinitely many as for example for $\Theta \geq 1$ and $t^{*}(p) < t^{\#}(p)$ (see Lemma 1 of subsection A6). However, due to the antagonistic character of the game we will focus on equilibria with $t(p) = \min(t^{*}(p), t^{\#}(p))$ as they also constitute the maximin solution of V at the same time.

Proposition 4: *The maximin strategy of V is given by $\min(t^{*}(p), t^{\#}(p))$. The strategy combination $(t, s) = (\min(t^{*}(p), t^{\#}(p)), t^{\#}(p))$ represents a Nash equilibrium of the zero sum game where the payoff of H is given by $\Pi_H = -\Pi_V$.*

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For a proof see subsection A7 of the Appendix.

We now turn to the question of H's strategic choice of p.

Proposition 5: *For any given strategy tuple* (t, s) *there exists an optimal level of support* p^* *given by*

$$p^*(t, s) = \begin{cases} \in (q, 1) & \text{for} \quad \Theta > \dfrac{c}{c+1} \\ q & \text{otherwise.} \end{cases} \tag{20}$$

For a proof see subsection A8 of the Appendix.

Note that this implies that when Θ is too low, H will not cooperate with T at all by setting $p = q$. The first row of (20), however, is confirmed by Figure 7.

In combination with (16), the best response of H given a strategy t of V is given by

$$r_H(t) = \begin{cases} (t, p^*) & \text{for } \Theta \geq 1, \\ (\min(s^*(p^*), t), p^*) & \text{for } \dfrac{c}{c+1} < \Theta < 1, \\ (0, q) & \text{otherwise.} \end{cases} \tag{21}$$

In summary, in this section we identified the following Nash equilibrium

$$t_N = \min(t^*(p^*), t^\#(p^*))$$

$$s_N = \begin{cases} t_N & \text{for } \Theta \geq 1, \\ \min(t^*(p), t^\#(p), s^*(p)) & \text{for } \dfrac{c}{c+1} < \Theta < 1, \\ s = 0 & \text{otherwise.} \end{cases} \tag{22}$$

$$p_N = p^*(t_N, s_N).$$

The following figure illustrates p_N as a function of A, Θ respectively. Moreover, the figure shows three contours which divide the A, p- plane into different areas of ranking of $t^*, t^\#$ and s^*, also as functions of A.

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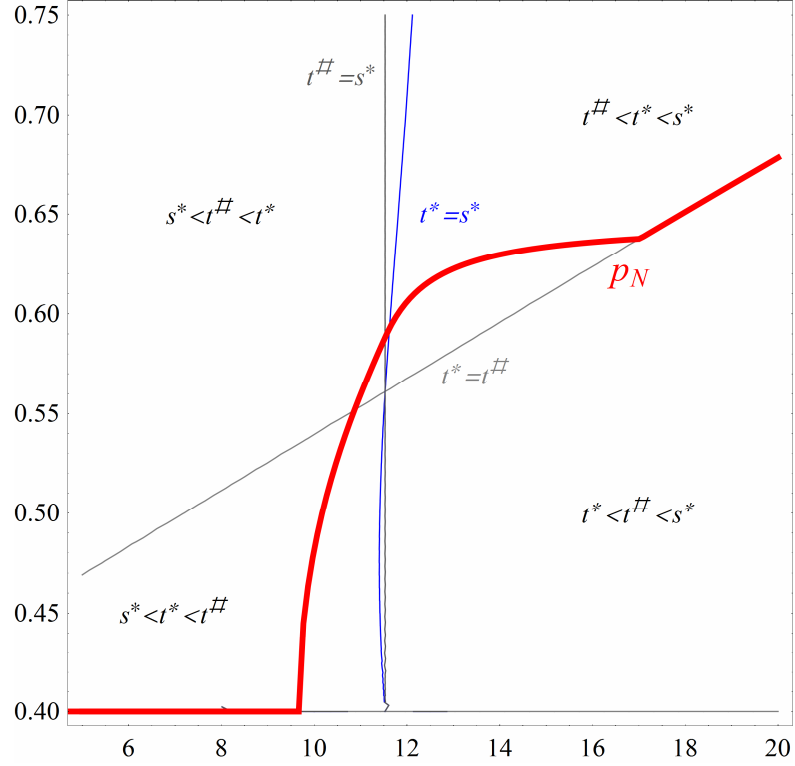
Figure 8: Nash equilibrium strategies for different values of A
$q = 0.4, C_T = 8, C_P = 5, B = 50, K = 120, \ \alpha = \beta = 0.1, \ c = 0.5.$

Observe that for values of A such that $\Theta \leq c/(c+1)$ there is no incentive for H to raise the level p above q. However, for values of $A \geq C_T + C_P \ c/(c+1) \approx 9.6667$ the equilibrium value p_N increases dramatically. Note that for values of A between 9.6667 and 11.3, the graph of p_N passes the two areas where $s^* < t^* < t^\#$ and $s^* < t^\# < t^*$, respectively. In these parts the corresponding equilibrium time s_N for H according to (22) equals s^* such that we don't observe any qualitative change when passing the line defined by $t^* = t^\#$. Here, in both areas the game ends at s^*. When p_N passes through the line $t^\# = s^*$, however, the minimum of s^*, t^* and $t^\#$ is $t^\#$ is such that it becomes the equilibrium choice for both H and V. From $A=17$ on, H chooses p_N such that he avoids ending up in the area where t^* is the strict minimum. This is to avoid a "too early" retaliation which is characteristic for this area: although V's expected payoff from retaliation is still negative he is not willing to postpone it any further.

4. Interpretation

The critical time

$$t^\# = \frac{\log[K/cB]}{\lambda} \tag{23}$$

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(the break-even point) is the moment for V at which the evidence of H's collusion with T is large enough that the expected payoff from retaliation, $(\mu B\text{-}K)/(\mu+1)$, first becomes positive. It is logical for V to retaliate no later than $t^{\#}$. Of course it may actually be in V's interest to retaliate at a prior time, even though he will experience a negative expected payoff. This happens if the periodic cost of continuing attacks, $(p\text{-}q)A$, is sufficiently large in relation to B and K. In such case, V might retaliate rather than tolerate T's attacks on a continuing basis.[9] Here, $t^* < t^{\#}$.

As shown in subsection A2 of the Appendix, the other critical time t^* is determined by the existence of a positive root μ^* of the quadratic equation

$$-\left[A(p-q)+\alpha B\right]\mu^2 + \left[-2A(p-q)-(\alpha-\lambda)B+(\lambda+\alpha)K\right]\mu + \left[\alpha K - A(p-q)\right] = 0. \qquad (24)$$

To be more specific, t^* is given by

$$t^* = \begin{cases} \dfrac{\log(\mu^*/c)}{\lambda} & \text{if } \mu^* \geq c, \\[2mm] 0 & \text{otherwise} \end{cases} \qquad (25)$$

Note that if $A(p-q)$ keeps increasing, the quadratic term becomes dominant eventually and the root μ^* shifts to zero. Hence for a sufficiently large term $A(p-q)$ the critical time t^* is equal to zero. This implies immediate retaliation should V observe any activity in the very beginning of the game.

From subsection A3 of the Appendix it follows that the condition for $t^{\#} < t^*$ reduces to the inequality

$$(p-q)A < \lambda BK/(K+B). \qquad (26)$$

This agrees with our previous intuition that, for large A, V is more likely to act before the break-even point, $\mu^{\#}$, is reached. As for dependence on p, this is more problematic as an increase in p will also cause an increase in λ, and it is not clear how it will affect (26). Stronger attacks mean, on the one hand, that V will want to retaliate sooner, but, on the other, that evidence of H's collusion is increasing rapidly, and so V may be more willing to wait for the break-even moment.

From (2) and (17) we get

$$s^* = \frac{1}{\lambda}\log\left[\frac{1}{c}\frac{\Theta}{1-\Theta}\right] = \frac{1}{\lambda}\log\left[\frac{1}{c}\frac{A-C_T}{C_P-(A-C_T)}\right], \qquad (27)$$

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[9] Something like this apparently happened in the Colombian government's March 1 2008 attack on a FARC (Revolutionary Armed Forces of Colombia) base inside Ecuadorian territory, which caused significant tension between Colombia's president Alvaro Uribe and his Ecuadorian counterpart Rafael Correa. Correa appears to have believed that Colombia would hesitate to attack in the absence of strong evidence of Ecuador's collaboration with the guerrillas. Uribe however seems to have felt that some political cost was preferable to the FARC's continuing use of bases across the border in Ecuador. As it happened, computers captured during the raid belonging to the local rebel leader, Raúl Reyes, had sufficient evidence of Ecuador's support of the FARC to mitigate these costs. Correa nevertheless did his best to maximize this cost, traveling throughout both Europe and Latin America to present his case to other heads of state. As of this writing (July 2009), diplomatic relations between the two countries have not been reestablished.

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which is defined for $c/(c+1) < \Theta < 1$. Note that s^* increases with Θ (with increasing A respectively). In particular, we get that s^* tends to infinity with Θ approaching 1.

From (16) follows that for low values of Θ, the political costs C_P weigh so heavily in relation to the damage factor $(A - C_T)$ that H has an interest in cutting support of T at an early time $s^* < t$, or doesn't have an interest to support T in the first place. For increasing Θ, we have eventually $s^* \geq t$ so that H would like to continue to support T if there wasn't the threat of retaliation. Here, H has no interest in disengaging prior to t, as the only point of disengaging is to avoid a retaliatory attack.

5. An alternative to retaliation

Proposition 5 shows how to prevent any terrorist activity, independent of what H expects V to choose as a strategy -- the level of $\Theta = (A - C_T)/C_P$ has to be smaller or equal to the initial suspicion factor $c/(c+1)$. This section discusses the options of the victim country in order to have an impact on Θ.

There are several ways the destructive power obtained from a unit-level attack A can be curbed by V. As an example, consider an increasing level of damage and safety precautions such as investing in better methods of detecting explosives or supervisions of endangered people and areas. This, however, implies considerable monetary investments which have to be traded off against other actions affecting Θ. Analogously, C_T, the material cost to H generated by a unit-level attack, will respond to a range of sanctions such as supervision of suspects and international monetary flow as well as espionage. In fact, any strategy that hinders exchange of communication, money or material within the secret coalition causes C_T to increase. Political costs can be interpreted to what degree the suspicion at time t will be accepted as evidence for a coalition. The parameter C_P will possibly be determined by the voters and media in both countries H and V. In addition, it will be influenced by other interested parties such as third countries, the UN or other multi-country bodies. Therefore, the term C_P will respond to propaganda, as well as international pressure such as international controls, trade barriers and sanctions.

In what follows we will assume that A, C_T and C_P can be influenced by V by some monetary investment. When the expense budget for V is limited, however, the question is now how to invest the money on lowering Θ in an optimal way.

Let x,y and z be the investment in A, C_T, C_P such that they can be read as functions $A(x), C_T(y), C_P(z)$ with $\Theta(x, y, z) = (A(x) - C_T(y))/C_P(z)$. We will not assume any particular shape of these functions, however, we will assume that they are twice continuously differentiable as well as

$$A'(x) < 0$$
$$C_T'(y) > 0 \qquad (28)$$
$$C_P'(z) > 0$$

and

$$A''(x) > 0$$
$$C_T''(y) < 0 \qquad (29)$$
$$C_P''(z) < 0.$$

Assume further that without any investment

$$\Theta(0,0,0) > \frac{c}{(c+1)} \qquad (30)$$

and

$$\lim_{x,y,z \to \infty} \Theta(x, y, z) < \frac{c}{(c+1)}. \qquad (31)$$

Proposition 6: *The solution to the optimization problem*

$$\min x + y + z$$
$$\text{s.t. } \frac{A(x) - C_T(y)}{C_P(z)} \leq \frac{c}{c+1} \qquad (32)$$

is given by

$$A'(x) = -C_T'(y) = -\frac{c}{c+1} C_P'(z),$$
$$\Theta = \frac{A(x) - C_T(y)}{C_P(z)} = \frac{c}{c+1}. \qquad (33)$$

For a proof see subsection A9 in the Appendix.

To our knowledge, there isn't yet any literature or data that would support any specific functional form of $A(x), C_T(y), C_P(z)$. However, since our goal is to lend precision to a concept rather than quantitative policy prescriptions, we opt for discussing a simple and transparent example.

Example: *Due to the invisibility of the secret connection and hence the absence of direct influence, C_T might respond more inert to actions than A. In economic terms, C_T might be less elastic with respect to investments.*

$$\left| \frac{y}{C_T(y)} C_T'(y) \right| < \left| \frac{x}{A(x)} A'(x) \right|. \qquad (34)$$

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One could argue, on the other hand, that the unpredictability of terrorist attacks requires investments in A to operate on a large scale. In contrast, propaganda, as well as international pressure can be established very targeted such that $C_P(z)$ is probably the most elastic quantity. As an example we put

$$A(x) = (x + a)^{-2/3}$$
$$C_T(y) = y^{1/2} + c_T$$
$$C_P(z) = z^{99/100} + c_P,$$

(35)

such that for increasing x,y and z the elasticities approach -2/3, 1/2 and 99/100.
From the first order conditions (33) we get for the optimal solution

$$y^* = \frac{9}{16}(x^* + a)^{10/3}$$

$$z^* = \left(\frac{297c}{200(c+1)}\right)^{100} (x^* + a)^{500/3}$$

(36)

$$z^* = \left(\frac{99c}{50(c+1)}\right)^{100} (y^* + a)^{50}$$

Note that the weak curvature of $C_P(z)$ implies that the marginal returns to investment decrease very slowly in comparison to $A(x)$ and $C_T(y)$. This is reflected by the high exponents in the second and third row of (36).

6. Game theoretic analysis – the Stackelberg approach

Maschler (1966) introduced so called inspector's games in which one player has promised to perform a certain duty and the other is allowed to inspect and verify occasionally that the duty has indeed been performed. The key insight is that there is a clear asymmetry in the possibility of communication between the inspection role and the (possible) violation role. The inspector can announce his strategy in advance, but the potential violator cannot, as he is in fact claiming that he would never violate the agreement. We follow Maschler in pointing out that this asymmetry in communication possibilities also applies to the present model. V can announce his strategy that his observant time stops at a given time, *t*. In that case, and assuming that retaliation always hurts H, it should follow that H's best choice is to stop helping T at time *s = t*. Again, the argument is that V can make a unilateral statement as to his strategy, but H cannot do so as he denies any cooperation with T.[10] In game theoretic terms, we now assume an extensive form of the game where V, the Stackelberg leader, is in the position to move first.

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[10] For example, after some rocket launchers which Sweden had sold to the Venezuelan government appeared in FARC hands , Chavez's justice minister "told state television that the case of the rocket launchers appears 'a cheap film of the U.S. government'." (NYTimes, 27 July 2009)

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Additionally, we assume that V has the option of committing himself, prior to playing the game. For example, we assume that V would not risk his reputation by not fulfilling his commitments in a way which is easy to discover. To make sure that H believes him, V may sign a resolution or any other legal statement that rules out the possibility of an empty threat.

According to Proposition 1 and 5, the best response $(s(t), p(t))$ of H and therefore the strategy of him as Stackelberg follower is given by

$$(s(t), p(t)) = \begin{cases} (t, p^*), & p^* \in (q,1) & \text{for } \Theta \geq 1, \\ (\min(s^*(p^*),t), p^*), & p^* \in (q,1) & \text{for } \dfrac{c}{c+1} < \Theta < 1, \\ (0, q) & & \text{otherwise.} \end{cases} \quad (37)$$

where s^* is the level at which the corresponding suspicion factor equals the ratio of the net profit of terror divided by the political costs $\Theta = \mu(s^*)/(\mu(s^*)+1)$. Note that whatever the parameter values for Θ in (37), we will have $s(0) = 0$.

Corollary 2: *For any parameter specification of A, C_T, C_P, the Stackelberg leadership enables V to avoid any cooperation of H with T by announcing $t = 0$.*

7. Summary and Discussion

The theoretical contribution of this paper is to deal with the role that information plays in the actions of and responses to secret coalitions. This is different to the problem of incomplete information games where the lack of information regards player's utilities. With secret coalitions, the existence and nature of the coalition is incompletely known to those who are not in the coalition. We motivated this problem by an application in counterterrorism policy; however, wish to point out that it applies more generally to the study of secret coalitions.

Secret coalitions provide clear potential benefits when it comes to state-terrorist alliances. For the terrorist groups, it provides an important force multiplier allowing them to inflict much more damage than without such assistance. For the sponsoring state, the clear benefit is that by operating through an agent it can avoid being held accountable for the actions of their proxies. The anonymity of the secret coalitions offsets any force advantage of the otherwise strong target state for two reasons. First, the invisibility of the terrorist groups makes them hard to eliminate. Second, retaliation against the sponsor state is complicated by the fact that enough evidence of cooperation with the terrorist is needed to make it a valuable strategy.

In the absence of a "smoking gun", however, it is still possible to use mathematical analysis to infer a connection. In the present paper, the key tool to approach the game theoretic analysis of secret coalitions is the introduction of a "suspicion factor" variable in a dynamic setting which measures evidence. This is based on the idea that if the terrorist group is indeed receiving support from the sponsor, it will be able to operate at a higher level of activity and with a greater destructive effect than if it were operating on its own. Therefore evidence goes up (down) each

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time a terrorist group attacks while (not) being sponsored. Based on the suspicion factor, the insight is that the cost of retaliation by the victim country decreases with the amount of evidence built against the sponsor country.

The analysis has shown a number of interesting effects when it comes to different levels of terrorist attacks. First, a high level of attacks implies that the critical level of evidence causing retaliation is reached very quickly and not enough damage is done. At lower levels, however, the process lasts a long time but the present value is low due to time discounting. A second aspect is that stronger attacks and an increasing suspicion factor not only enforces the threat of retaliation but also increases the sponsor's political costs to be suspected to help the terrorist group. Finally, an increasing level of terrorist attacks doesn't automatically lead to an earlier retaliation. Besides damage, a higher level also leads to a faster accumulation of evidence such that the victim may be more willing to wait for more evidence.

We found that there is a key term which determines three possible outcomes, depending on parameter values. The sponsor (1) breaks off his terrorist support at some point in time in order to avoid a retaliatory attack (2) breaks it off after some time even in absence of the threat of retaliation due to increasing political costs (3) has no incentive in terrorist support in the first place. The key term determining this categorization is determined by the damage obtained from a unit-level attack as well as material and the sponsor's political costs of supporting the terrorist group.

The finding of the third category of possible outcomes leads to an important conclusion. There is an alternative to retaliation as a counter-terror strategy if the victim is in the position to influence the magnitude of the key term --shifting it into the third category removes any incentive for terrorist support. Under the assumption that the victim is able to influence the corresponding parameters by monetary investments, the economic analysis shows how to invest optimally. These necessary investments could outweigh the expected benefits and costs of entering the game with potential retaliation.

A third instrument to resist terrorist activities was shown to be changing the structure of the game in order to put the target country in a position to announce his strategy. Here, the key insight is that the target country can make a unilateral statement as to his strategy, but the sponsor cannot do so as he is in fact claiming that there is no cooperation with terrorist groups.

An obvious question is whether it might be more profitable for the secret coalition to vary the level of attacks over time, perhaps decreasing as the suspicion factor approaches its critical value. While such a procedure seems reasonable, a continuously varying level of attack seems rather difficult to implement, especially so as communications between the two partners must of necessity be secret. This would suggest an approach allowing the level to vary a few times, at discrete intervals which would turn it into a sequential game solvable by backward induction.

The work of the present paper is clearly exploratory inasmuch as functional forms and parameters are not derived from empirical data. The basic structure of the model solution, however, is quite robust with respect to parameter values, which is helpful since estimating parameter values is difficult. With the present paper, our goal is provide a concept rather than computing specific quantitative policy prescriptions. We know of no data or literature that

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would support any specific functional forms, such that we adopt for simple forms and focus on qualitative behavior of the solutions.

Appendix

A1. Derivation of λ

In general, we can expect (i) that μ, the suspicion factor, will increase so long as the attacks (by T) are at a rate p, greater than q, which is what T could do all by itself, and (ii) that the rate of this increase will be greater, the greater p is. There are of course many possibilities. We choose a particular functional increase which can be thought of as *continuous Bayesian updating*.

Let p and q represent frequencies of attack: in each time period, a weak T will attack with probability q, whereas a strong T attacks with probability p. Let π_n be the probability (after n periods) that T is in fact strong. Assume that there is an attack in period $n+1$. Then, by Bayes' rule, the $(n+1)$- period probability, π_{n+1}, is given by

$\pi_{n+1} = p\pi_n / [p\pi_n + q(1-\pi_n)]$
From this, we obtain
$1-\pi_{n+1} = q(1-\pi_n) / [p\pi_n + q(1-\pi_n)]$

and so,

$$\frac{\pi_{n+1}}{(1-\pi_{n+1})} = \frac{\pi_n}{(1-\pi_n)}\frac{p}{q} \tag{38}$$

Similarly we have

$$\frac{\pi_{n+1}}{(1-\pi_{n+1})} = \frac{\pi_n}{(1-\pi_n)}\frac{(1-p)}{(1-q)} \tag{39}$$

if there is no attack in that period.

Letting

$$\mu_n = \frac{\pi_n}{(1-\pi_n)} \tag{40}$$

we see that μ is multiplied by the factor p/q in every time period with an attack, and by $(1-p)/(1-q)$ in every time period with no attack. In terms of logarithms, we see that $\log(\mu)$ increases by $\log(p/q)$ or by $\log[(1-p)/(1-q)]$ respectively, in the two cases. Since, with a strong T, the attacks occur with frequency (probability) p, we see that, in expectation, $\log(\mu)$ will increase by

$$\lambda = p\log\frac{p}{q} + (1-p)\log\frac{1-p}{1-q} \tag{41}$$

per time period.

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A2. Calculation of the critical times

As mentioned above, one of the critical times is the time that maximizes $M(t)$. We start from equation (8)

$$M(t) = -\frac{A(p-q)}{\alpha}(1-e^{-\alpha t}) + e^{-\alpha t}\frac{ce^{\lambda t}B-K}{1+ce^{\lambda t}}$$

Differentiating with respect to t, we obtain an expression with a common factor $e^{-\alpha t}$. Dividing by this, we find

$$e^{\alpha t}M'(t) = -A(p-q) + \frac{c(\lambda-\alpha)Be^{\lambda t}+c^2(\lambda-\alpha)Be^{2\lambda t}+\alpha K+\alpha cKe^{\lambda t}-c^2\lambda Be^{2\lambda t}+c\lambda Ke^{\lambda t}}{(ce^{\lambda t}+1)^2}$$

Thus after multiplying $M'(t)$ by the two factors $e^{\alpha t}$ and $(ce^{\lambda t}+1)^2$, which are always positive, and substituting $\mu = ce^{\lambda t}$, we obtain the expression

$$-A(p-q)(\mu+1)^2 + (\lambda-\alpha)B\mu + (\lambda-\alpha)B\mu^2 + \alpha K - \lambda B\mu^2 + (\lambda+\alpha)K\mu$$

or

$$-[A(p-q)+\alpha B]\mu^2 + [-2A(p-q)-(\alpha-\lambda)B+(\lambda+\alpha)K]\mu + [\alpha K - A(p-q)] \quad (42)$$

which is equivalent to M' in the sense that *they always have the same sign*. We note that, in this last expression, the coefficient of the quadratic term is always negative; thus M decreases for large values of t. This will mean that M has a maximum value over the interval from 0 to ∞.

The constant term in (42) is usually positive, and so M is increasing at $t = 0$. Thus there will (usually) be one positive root and one negative root of the corresponding quadratic equation

$$-[A(p-q)+\alpha B]\mu^2 + [-2A(p-q)-(\alpha-\lambda)B+(\lambda+\alpha)K]\mu + [\alpha K - A(p-q)] = 0. \quad (43)$$

Since μ is an exponential, the negative root is extraneous; the positive root, $\mu^*(p)$, is the one desired. Then

$$t^*(p) = \frac{\log(\mu^*(p)/c)}{\lambda(p)} \quad (44)$$

is the maximizing value. Note however that if $\mu^* < c$, then $t^* < 0$. In this case $t = 0$ is the maximizing value.

It is of course possible that the constant term in expression (42), $[\alpha K - A(p-q)]$, might also be negative. This would require that K be very small, or A very large, or some such thing, but we will not rule it out. In such case there might be no positive roots of the equation, and, hence, no maximizing value of t. The maximizing point would then also be at $t = 0$. There may also be two positive roots of the equation; the larger root would then be a local maximum, and it would be necessary to compare it with $t = 0$ to determine the global maximum. We omit further details.

A3. Relation between t^* and $t^{\#}$

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For notational convenience we omit functional dependence on p.

As mentioned in section 2, $t^{\#}$, the point where $M(t) = N(t)$, is given by $t^{\#} = \log(K/cB) / \lambda$. To see conditions for t^* to be smaller than $t^{\#}$ (or vice-versa), we note that, if $t^{\#} < t^*$, then $M'(t^{\#}) > 0$.

Starting from the expression

$-A(p\text{-}q)(\mu+1)^2 + (\lambda\text{-}\alpha)B\mu + (\lambda\text{-}\alpha)B\mu^2 + \alpha K - \lambda B\mu^2 + (\lambda+\alpha) K\mu$

above, we note that, if $\mu B = K$, this expression reduces to

$-A(p\text{-}q)(K/B +1)^2 + \lambda K + \lambda K^2/B$

and this has the same sign as

$$-A(p-q)+\frac{\lambda K+\lambda K^2/B}{(K/B+1)^2}$$

or equivalently

$$-A(p-q)+\frac{\lambda BK}{K+B} \tag{45}$$

As mentioned above, the condition for $t^{\#} < t^*$ is that $M'(t^{\#})$, which has the same sign as (45), be positive; this reduces to the inequality $(p\text{-}q)A < \lambda BK/(K+B)$.

A4. Proof of Proposition 1

For H, it is never a best response to choose $s>t$ since retaliation can be avoided by setting $s = t$. We therefore focus on the payoff given by $F(s)$ in (14).

However, can there ever be an incentive to choose $s<t$? To answer this we have to consider benefit and cost of H. From (14) we get that the net benefit is given by

$$B(s)=\frac{A-C_T}{\alpha}(p-q)(1-e^{-\alpha s}) ,$$

whereas the costs are given by

$$C(s)=\int_0^s C_P \frac{\mu(v)}{\mu(v)+1}(p-q)e^{-\alpha v}dv .$$

For the derivatives we get

$$B'(s)=(A-C_T)(p-q)e^{-\alpha s}$$

$$C'(s)=C_P \frac{\mu(s)}{\mu(s)+1}(p-q)e^{-\alpha s}.$$

Setting them equal yields the FOC

$$\Theta=\frac{\mu(s^*)}{\mu(s^*)+1} \tag{46}$$

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which is equivalent to $s^* = \dfrac{1}{\lambda} \ln \dfrac{\Theta}{c(1-\Theta)}$.

The second order condition $B''(s) - C''(s) < 0$, using the FOC (46), simplifies to

$$-C_P \frac{\mu'(s)}{(\mu(s)+1)^2} < 0 \qquad (47)$$

which is always fulfilled since $\mu'(s) > 0$ for all $s \geq 0$.

Note that the suspicion factor $\dfrac{\mu(s)}{\mu(s)+1}$ is a strictly increasing function with $\dfrac{\mu(0)}{\mu(0)+1} = \dfrac{c}{c+1}$ and bounded by 1. This implies that a necessary condition for the FOC to be fulfilled is

$$\frac{c}{c+1} \leq \Theta < 1, \qquad (48)$$

where $s^* = 0$ for $\Theta = \dfrac{c}{c+1}$.

When $\Theta \geq 1$ we have $B'(s) > C'(s)$ for all s such that the optimal response is $r_H(t) = t$.

When $\Theta < \dfrac{c}{c+1}$ we have $B'(s) < C'(s)$ for all s such that $r_H(t) = 0$.

q.e.d.

A5. Proof of Proposition 2

Case I: $t^* \leq t^\#$. Take any fixed $t' < t^*$. Figure 9 illustrates the payoff function $\Pi_V(t^*, s)$ and $\Pi_V(t', s)$ as a function in s.
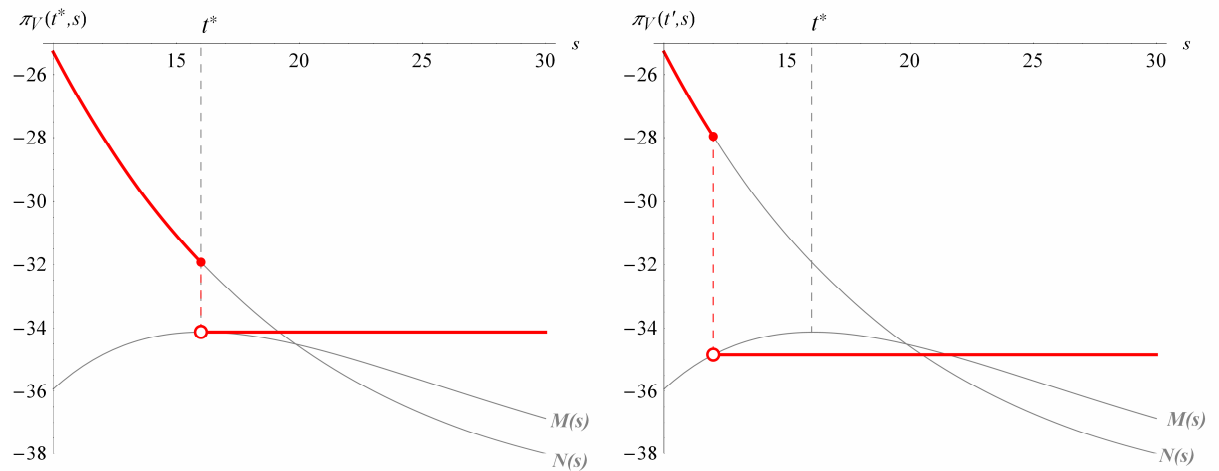

Figure 9

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Recall that the game stops with retaliation such that the payoff function stays at a fixed level for values s higher than observant time t. As we can see from Figure A1, it holds that for $s \leq t'$ the payoffs are given by $\Pi_V(t^*,s) = \Pi_V(t',s) = N(s)$. For $s > t'$, however, it holds that $\Pi_V(t^*,s) > \Pi_V(t',s) = M(t')$. We conclude that t^* weakly dominates t'.

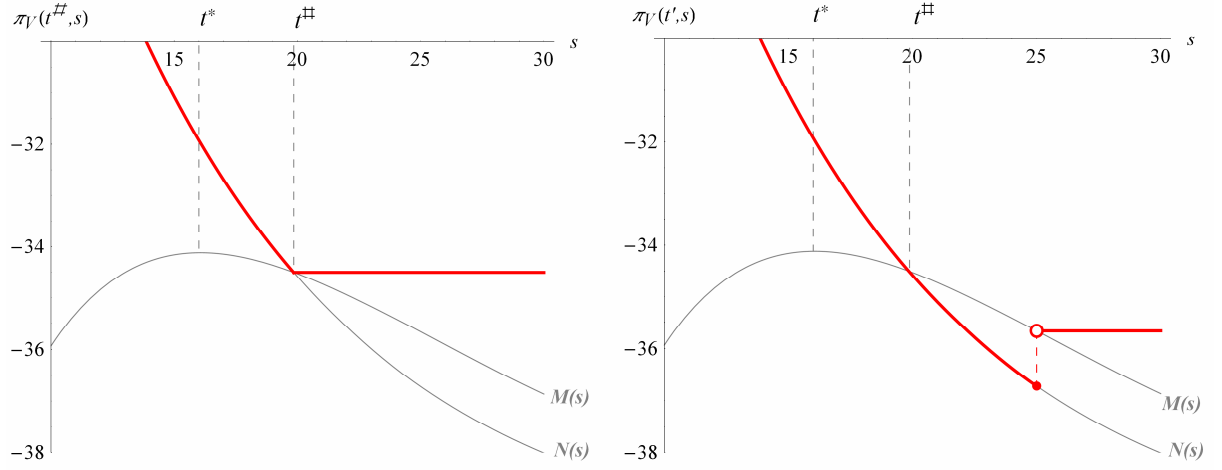Analogously, it follows that any $t' > t^\#$ is weakly dominated by $t^\#$. This is illustrated in Figure 10.

Figure 10

Case II: $t^* > t^\#$. Analogously, Figure 11and 12 illustrate that any $t' < t^\#$ is (weakly) dominated by $t^\#$, whereas any $t' > t^*$ is dominated by t^*.
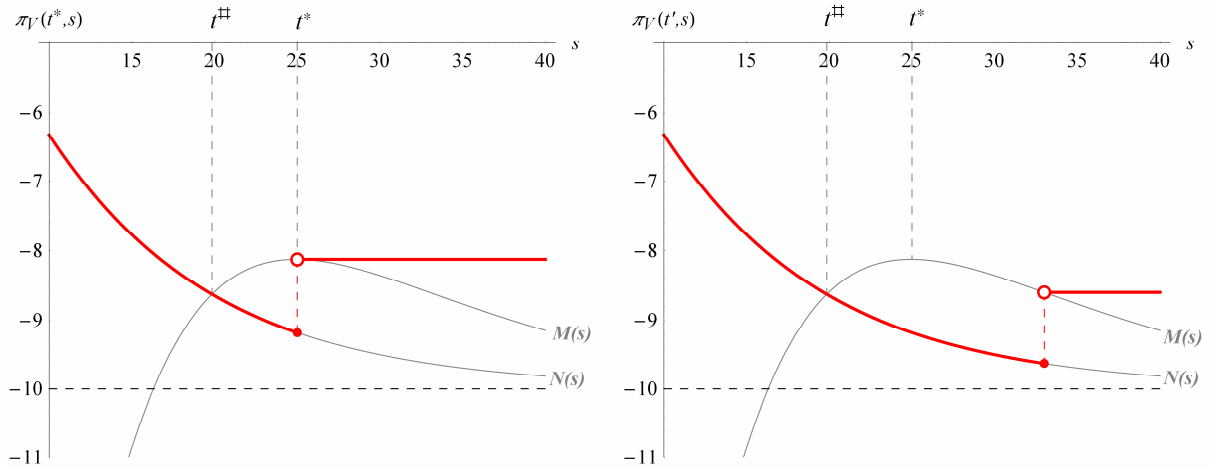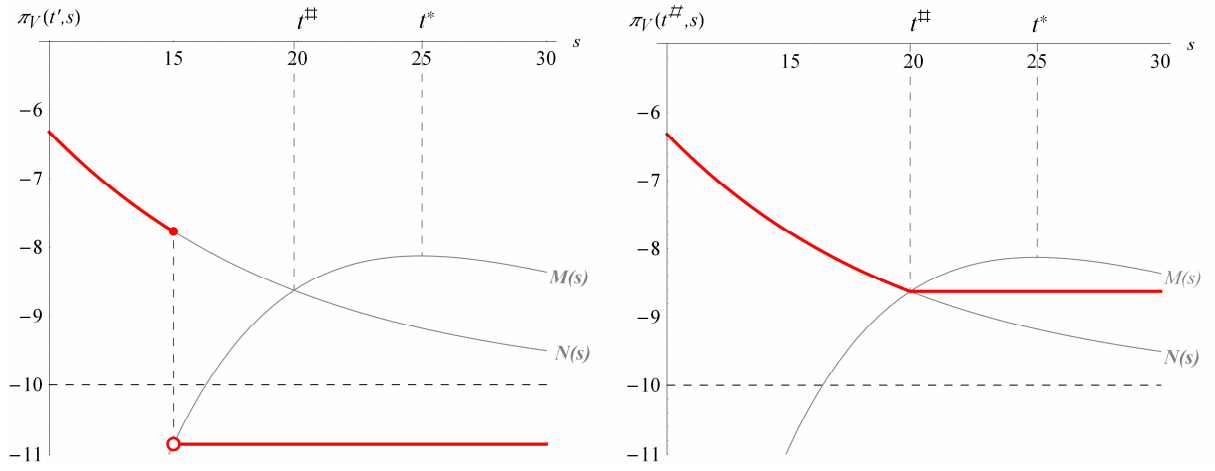

Figure 11

Figure 12

For $c/(c+1) < \Theta < 1$, there exists a maximum s^* in $F(s)$, the payoff of H without retaliation. Choose an $s' > 0$ with $s' > s^*$. Figure 13 illustrates $\Pi_H(t, s^*)$ and $\Pi_H(t, s')$ as a function in t.
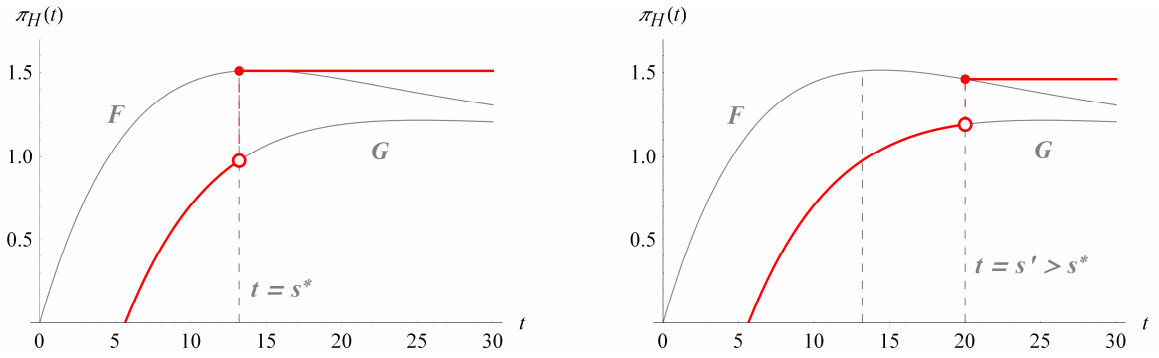

Figure 13

As can be seen, s^* weakly dominates s'. For $t < s^*$, both provide the same payoff $G(t)$ since retaliation takes place for early values of t. At $t = s^*$, however, retaliation is avoided such that $\Pi_H(t, s^*)$ jumps to the level $F(s^*)$ and stays there -- since this is the maximum level that can be achieved, it holds that $\Pi_H(t, s^*) > \Pi_H(t, s')$ for $t \geq s^*$.

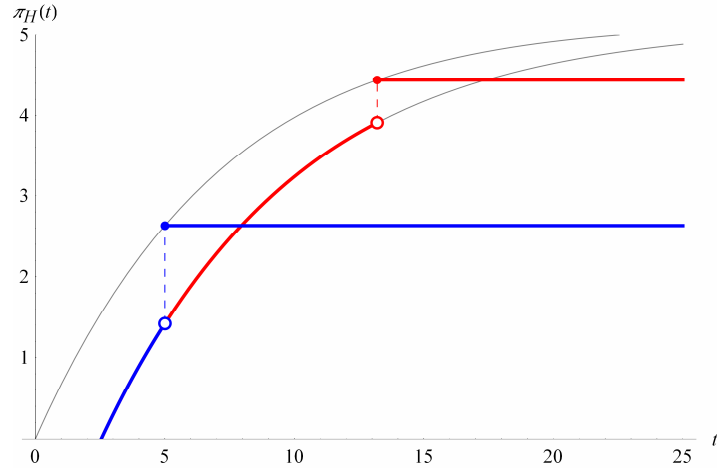Note that this argument does not apply when $\Theta \geq 1$ as is illustrated in Figure 14.

Figure 148

With any two strategies $s_1 > 0$ and $s_2 > 0$ there is an interval for t where s_1 provides a higher value than s_2 and the other way around.

For $\Theta < c/(c+1)$, the payoff of H is negative unless $s = 0$ which provides zero payoff. Hence $s = 0$ weakly dominates any other strategy $s > 0$ since it provides the same payoff of zero for $t = 0$ and a higher payoff (non-negative) for $t > 0$.

q.e.d

## A6. Proof of Proposition 3

We will first determine the reaction function of V.

Case I: $t^* \leq t^\#$

Let $\tilde{t}$ be defined by $N(\tilde{t}) = M(t^*)$ such that $N(s) < M(t^*)$ for $s > \tilde{t}$. Note that $\tilde{t} > t^*$.

This implies that retaliation at $t = t^*$ is the best answer to $s > \tilde{t}$. For $s < \tilde{t}$, we get $N(s) > M(s)$ such that non-retaliation provides the highest payoff to V. In summary, we get for the best reaction of V

$$r_V(s) = \begin{cases} \text{any } t \text{ with } t \geq s & \text{for } s \leq \tilde{t} \quad \text{(no retaliation)} \\ t^* & \text{for } s > \tilde{t} \quad \text{(retaliation)} \end{cases} \tag{49}$$

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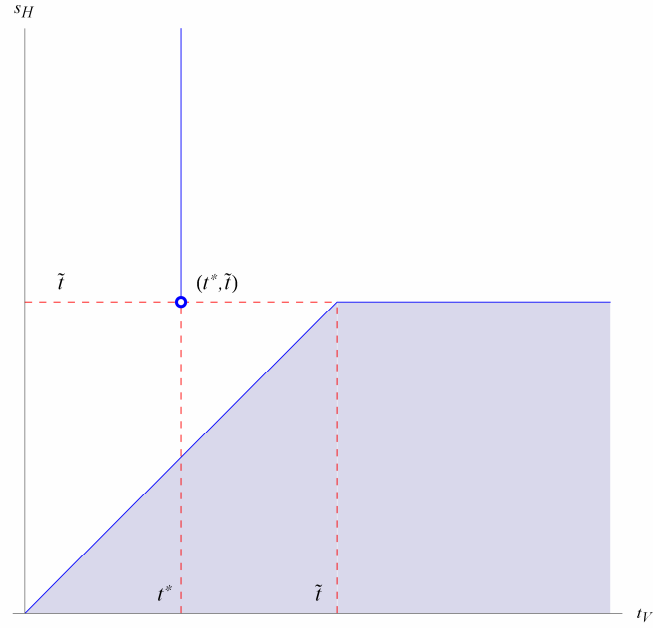
Figure 159: Sketch of Case I.

Case II follows analogously.

Case II: $t^* > t^\#$

$$r_V(s) = \begin{cases} \text{any } t \text{ with } t \geq s & \text{for } s \leq t^\# & \text{(no retaliation)} \\ \text{undetermined} & \text{for } t^\# < s \leq t^* & \text{(retaliation)} \\ t^* & \text{for } s > t^* & \text{(retaliation)} \end{cases} \tag{50}$$

Note that the case $t^\# < s \leq t^*$ is undetermined since after $t^\#$ the expected payoff of retaliation is positive and $M(t)$ increasing in time. This implies that the best retaliation time would be just before s, say $t = s - \varepsilon$, with ε as small as possible.

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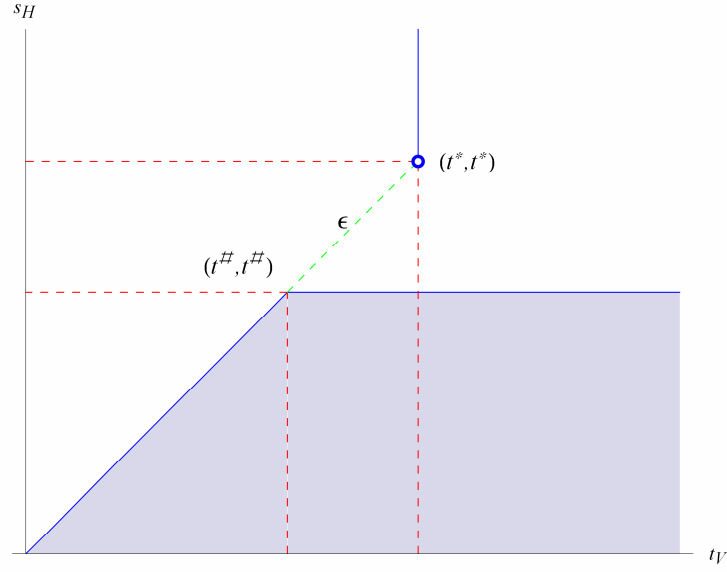
Figure 16: Sketch of Case II

We now turn back to the proof of Proposition 3.

(i) $\Theta \geq 1$

From (16) we get that the best answer of H is given by $s = t$.

(a) For $t^* \leq t^\#$, recall that $t^* < \tilde{t}$. For the first row of (49) we get that a best answer of V is given by $t = s$ such that $t = s = t^*$ is in equilibrium.

(b) For $t^* > t^\#$, the first row of (50) provides a mutual best answer for $t = s = t^\#$.

(ii) $\Theta \leq \dfrac{c}{c+1}$

From (16) we get that H chooses not to cooperate with T, $s = 0$. Here, any $t \geq 0$ is a best answer of V, in particular $t = \min(t^*, t^\#)$.

(iii) $\dfrac{c}{c+1} < \Theta < 1$

Case I: $s^* \leq \min(t^*, t^\#)$

For $t = \min(t^*, t^\#)$ follows from (16) that H's best answer is given by $s = s^*$. In turn, from the first rows of (49) and (50) we see that a best answer is given by any t with $t \geq s$, in particular $t = \min(t^*, t^\#)$.

Case II: $t^* \leq \min(s^*, t^\#)$

We need to show that $(t, s) = (t^*, t^*)$ is a Nash equilibrium.

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Note that $r_H(t^*) = \min(s^*, t^*) = t^*$. Suppose, next, that H disengages at t^*. From the first row of (49) we get that a best answer is given by any t with $t \geq s$, in particular $t = s = t^*$.

Case III: $t^\# \leq \min(s^*, t^*)$

Here, we need to show that $(t, s) = (t^\#, t^\#)$ is a Nash equilibrium. For H we have $r_H(t^\#) = \min(s^*, t^\#) = t^\#$. Suppose, next, that H disengages at $t^\#$. From the first row of (50) we get that a best answer is given by any t with $t \geq s$, in particular $t = s = t^\#$.
q.e.d.

Lemma 1: *For* $\Theta \geq 1$ *and* $t^* < t^\#$, *all strategy tuples* (t, s) *with* $s = t \in [t^*, \tilde{t}]$ *are in equilibrium.*

Proof: From (16) we get that the best answer of H is given by $s = t$. From the first row of (49) we get that any t with $t = s$ is a best answer for $s \leq \tilde{t}$.

q.e.d

A7. Proof of Proposition 4

Suppose $\Pi_H = -\Pi_V$.

Case I: $t^* < t^\#$
We will show that the strategies $t = t^*$, $s = t^\#$ are mutual best answers, with payoff $\Pi_V(t^*, t^\#) = M(t^*)$.
Suppose that V chooses to retaliate at $t = t^*$. If H chooses $s \geq t^*$, then

$$\Pi_V(t^*, s) = -\Pi_H(t^*, s) = M(t^*).$$

If, on the other hand, H chooses $s < t^*$, then

$$\Pi_V(t^*, s) = N(s).$$

Since N is monotone decreasing, we have $N(s) > N(t^*)$, and, since $t^* < t^\#$, $N(t^*) > M(t^*)$.

Thus, for any choice of s, $\Pi_V(t^*, s) \geq M(t^*)$.

Suppose, next, that H chooses to disengage at $s = t^\#$. Then, if V chooses $t \leq t^\#$,

$$\Pi_V(t, t^\#) = M(t) \leq M(t^*).$$

If, on the other hand, V chooses $t > t^\#$,

$$\Pi_V(t, t^\#) = N(t^\#) = M(t^\#) \leq M(t^*)$$

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And so, for all t, $\Pi_V (t, t^{\#}) \le M(t^*)$.

Thus we see that, in Case I, both V and H can guarantee the value $M(t^*)$ of Π_V. This case is illustrated in Figure 4.

Case II: $t^{\#} \le t^*$
We will show that the strategies $t = s = t^{\#}$ are mutual best answers, with payoff $\Pi_V(t^*, t^{\#}) = M(t^*)$.

Suppose V chooses to retaliate at time $t^{\#}$. Then, if H chooses $s \ge t^{\#}$, we will have

$$\Pi_V (t^{\#}, s) = M(t^{\#})$$

If, on the other hand, H disengages at $s < t^{\#}$, then

$$\Pi_V (t^{\#}, s) = N(s).$$

Since N is monotone decreasing, and $s < t^{\#}$, it follows that $N(s) > N(t^{\#}) = M(t^{\#})$. Thus, for every value of s, we have $\Pi_V (t^{\#}, s) \ge M(t^{\#})$.

Suppose, next, that H decides to disengage at time $t^{\#}$ while V plans to retaliate at some time t. Then, if $t \le t^{\#}$, we have
$$\Pi_V (t, t^{\#}) = M(t)$$

Since M increases monotonically for $t < t^*$, and in this case $t \le t^{\#} < t^*$, it follows that $M(t) \le M(t^{\#})$.

If, on the other hand, $t > t^{\#}$, then

$$\Pi_V (t, t^{\#}) = N(t^{\#}) = M(t^{\#}).$$

Thus, for all t, we have $\Pi_V (t, t^{\#}) \le M(t^{\#})$.

We see, then, that, in Case II, both V and H can guarantee the value $M(t^{\#})$ of Π_V.

q.e.d

A8. Proof of Proposition 5

Case I: $\Theta > c/(c+1)$
Assume $s > 0$ and $c > 0$ are fixed, while q is fixed and $q < 1$. From (14) we get
$$\frac{\partial F(s, p)}{\partial p} = \varphi_1 - \varphi_2 - \varphi_3, \tag{51}$$

with

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$$\varphi_1 = \frac{(C_T - A)(1 - e^{-\beta s})}{\beta}$$

$$\varphi_2 = \int_0^s C_P \frac{\mu}{\mu + 1} e^{-\beta v} dv \tag{52}$$

$$\varphi_3 = \int_0^s C_P \frac{\mu'(p - q)}{(\mu + 1)^2} e^{-\beta v} dv,$$

where ' denotes differentiation with respect to p. Now φ_1 is fixed, while clearly $\varphi_2 > 0$. As for φ_3, note that since $\mu = ce^{\lambda v}$ we will have $\mu' = \lambda' v c e^{\lambda v} = \lambda' v \mu$. Thus every term in the integrand is positive so long as $v > 0$. Therefore, we have

$$\varphi_3 \geq \int_{s/2}^s C_P \frac{\mu'(p - q)e^{-\beta v}}{(\mu + 1)^2} dv = \int_{s/2}^s C_P \frac{\lambda' v \mu(p - q)e^{-\beta v}}{(\mu + 1)^2} dv. \tag{53}$$

Note that, as p increases towards 1, λ increases to the limit $\log(1/q)$, whereas λ' increases to ∞. This means that μ is bounded between c and $(1/q)^s$, and $(\mu + 1)^2$ in the denominator is also bounded, finite above and away from zero below. Also, we note that, for p close to 1, we have $p - q > (1 - q)/2$. Finally, $v \geq s/2$. Thus we can put a lower bound, $H > 0$, on the term $C_P v \mu (p - q)/(\mu + 1)^2$. We conclude that

$$\varphi_3 \geq H\lambda' \int_{s/2}^s e^{-\beta v} dv = H\lambda' \left(e^{-\beta s/2} - e^{-\beta s} \right). \tag{54}$$

Clearly, this goes to ∞ with λ'. We conclude that $\frac{\partial F}{\partial p} = \varphi_1 - \varphi_2 - \varphi_3$ goes to $-\infty$ as p goes to 1.

Case II: $\Theta \leq c/(c+1)$

We wish to prove that, for $s > 0$, $p = q$ is optimal. Clearly, $F(s, q) = 0$.

Suppose, on the other hand, that $p > q$. Let us now divide by $C_P(p - q)$ which is positive, so that signs are unchanged:

$$\frac{F(s, q)}{C_P(p - q)} = \frac{\Theta(1 - e^{-\beta s})}{\beta} - \int_0^s \frac{\mu e^{-\beta v}}{\mu + 1} dv. \tag{55}$$

We know that $\mu(0) = c$, and that μ is increasing in v. Thus, for all $v \geq 0$, we have

$$\frac{\mu}{\mu + 1} \geq \frac{c}{c + 1} \geq \Theta$$

And it follows that the integral here is

$$\int_0^s \frac{\mu e^{-\beta v}}{\mu + 1} dv \geq \int_0^s \Theta e^{-\beta v} dv \geq \frac{\Theta(1 - e^{-\beta v})}{\beta}. \tag{56}$$

Thus $F(s, p) \leq 0$. It follows that $p = q$ is optimal.

q.e.d.

A9. Proof of Proposition 6

For the proof of this Proposition the following Lemma will be helpful.

Lemma 2: *The solution to the optimization problem*

$$\min x + y + z$$
$$\text{s.t. } f_1(x) + f_2(y) + f_3(z) \leq 0 \qquad (57)$$

with

$$f_1'(x) < 0, f_2'(y) < 0, f_3'(z) < 0,$$
$$f_1''(x) > 0, f_2''(y) > 0, f_3''(z) > 0 \qquad (58)$$

is given by

$$f_1'(x) = f_2'(y) = f_3'(x)$$
$$f_1(x) + f_2(y) + f_3(z) = 0. \qquad (59)$$

Proof of Lemma 2:

From (58) follows that the subject constraint (57) can be rewritten as

$$f_1(x) + f_2(y) + f_3(z) = 0. \qquad (60)$$

Applying the method of Lagrange leads to

$$1 + \eta f_1'(x) = 0$$
$$1 + \eta f_2'(y) = 0$$
$$1 + \eta f_3'(x) = 0$$
$$f_1(x) + f_2(y) + f_3(z) = 0. \qquad (61)$$

where η is the Lagrange multiplier. System (61) simplifies to the FOCs

$$f_1'(x) = f_2'(y) = f_3'(x)$$
$$f_1(x) + f_2(y) + f_3(z) = 0. \qquad (62)$$

It remains to be proven that a solution to (62) is a minimum solution to (57).

The Hessian matrix of the Lagrange function reads

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$$H = \begin{pmatrix} 0 & -f_1'(x) & -f_2'(y) & -f_3'(z) \\ -f_1'(x) & -\eta f_1''(x) & 0 & 0 \\ -f_2'(y) & 0 & -\eta f_2''(y) & 0 \\ -f_3'(z) & 0 & 0 & -\eta f_3''(z) \end{pmatrix}. \tag{63}$$

A sufficient condition for the solution to (62) to be a minimum is if the determinant of the submatrix

$$H_3 = \begin{pmatrix} 0 & -f_1'(x) & -f_2'(y) \\ -f_1'(x) & -\eta f_1''(x) & 0 \\ -f_2'(y) & 0 & -\eta f_2''(y) \end{pmatrix} \tag{64}$$

as well as the determinant of H is negative. For (64) we get

$$\det(H_3) = -\det \begin{pmatrix} 0 & f_1'(x) & f_2'(y) \\ f_1'(x) & \eta f_1''(x) & 0 \\ f_2'(y) & 0 & \eta f_2''(y) \end{pmatrix} \tag{65}$$

$$= -\eta \left\{ [f_1'(x)]^2 f_2''(y) + [f_2'(y)]^2 f_1''(x) \right\}.$$

Note that from (58) and (61) follows that $\eta > 0$. From (65) follows that $\det(H_3) < 0$ is equivalent to

$$[f_1'(x)]^2 f_2''(y) + [f_2'(y)]^2 f_1''(x) > 0$$

which is always fulfilled due to (58).

For the determinant of H we get
$$\det(H) = \det(-H)$$
$$= -\eta^2 \left\{ [f_1'(x)]^2 f_2''(y) f_3''(z) + [f_2'(y)]^2 f_1''(x) f_3''(z) + [f_3'(z)]^2 f_1''(x) f_2''(y) \right\} \tag{66}$$
which is always negative due to (58).

q.e.d.

Proof of Proposition 6:

The side constraint of (32) can be rewritten as

$$A(x) - C_T(y) - \frac{c}{c+1} C_P(z) \le 0, \tag{67}$$

such that setting

$$f_1(x) = A(x), \ f_2(y) = -C_T(y), \ f_3(z) = -\frac{c}{c+1} C_P(z) \tag{68}$$

allows us to apply Lemma 2. With (68) the FOCs (62) read

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$$A'(x) = -C_T{}'(y) = -\frac{c}{c+1}C_P{}'(z),$$

$$\Theta = \frac{A(x) - C_T(y)}{C_P(z)} = \frac{c}{c+1}. \tag{69}$$

From (58) follows that the solution to (69) is a minimum if

$$A''(x) > 0$$
$$C_T{}''(y) < 0 \tag{70}$$
$$C_P{}''(z) < 0.$$

<div align="right">q.e.d.</div>

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