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Environmental Damage and Price Taking Behaviour by Firms and Consumers

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Environmental Damage and Price Taking Behaviour by Firms and Consumers *

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Abstract

Integrated assessment models lack a microeconomic foundation in modelling environmental damages to the economy. To overcome this, damage coefficients are incorporated in standard microeconomic models. Firms and consumers take both damages and prices as given. Demand, supply, profit and expenditure functions under damage coefficients are derived that allow easy implementation in applied economic models through appropriate price distortions related to such coefficients. For the consumer, Slutsky's equations are derived. The different speeds of equilibrium adjustment in economic and climate models is reconciled in the Recursive Equilibrium with Environmental Damages (REED). An exchange economy and Robinson Crusoe economy illustrate our approach.

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1 Introduction

The economic assessment of environmental policies combines the insights from several scientific disciplines into a single framework; a framework that requires the linking of several distinct models from each discipline involved. For example, to assess the economic impact of climate policies, one needs an integrated assessment consisting of an economic model, a climate model, and an ecological model. The economic model provides the climate model with the amounts of greenhouse gas emissions that result from the use of fossil fuels in the economy or from changes in land-use. In turn, the climate model adds these emissions to atmospheric concentrations and translates these concentrations into changes in climate variables such as mean global temperature, precipitation, and sun radiation. The values of these climate variables then serve as an input into the economic model in the form of environmental damages. Integrated Assessment Models link environmental and economic modules by interchanging updates of their variables in an iterative manner until convergence to a numerical solution is obtained. The economic module in an integrated assessment model can for example be based upon a computable general equilibrium model, see e.g. Manne et al. (1995) and Kemfert (2002).

The relation between changes in the values of particular climate variables and these economic damages is given by damage functions, which play a central role in integrated assessment. Economic damage often refers to the willingness to pay to avoid environmental degradation due to climate change, where climate change is regarded as a *public good*. Many economic assessment studies of climate impacts are based on the estimation of such damage functions. Messner et al. (2006) describe so-called best practices on the application of damage functions to climate-related flooding in the EU. Tol (2002a and 2002b) estimates climate-related damages as the percentage of current GDP that a society is willing to pay for the reduction of polluting activities in order to prevent environmental degradation.

Regarding climate change as a pure public good, a priori confines any economic analysis in either of the following ways: The public good can be seen as something worth preserving from environmental degradation, such as nature parks, the North Pole etc, and (the quality of) such good enters the utility function as a separate variable. Or, the public good is a non-marketed production factor that enters the production function as a separate variable, like precipitation and temperature patterns.

The impact of climate change is, however, more complicated than the simple degradation of a public good. Consider for instance the production side of the economy. Then, climate change causes two sources of potential damages. First, the production technology itself can be affected. For example, crops and fruit trees are sensitive to pests that flourish better in more humid weather conditions and this implies an increased proportion of crop spoils. Secondly, the productivity of certain inputs in a production process are degraded by the consequences of climate change. For example, sap flow requires moisture for transporting nutrients inside plants. Decreasing levels of precipitation affect such processes by decreasing nutrient uptake by plants and, therefore, directly affect the productivity of applied fertilizers. Or, an increase in outdoor temperatures requires a higher energy consumption to achieve a certain indoor temperature in offices, freezing chambers and homes. Not all effects of climate change are negative though, because an increase in temperature or precipitation may also stimulate plant growth or reduce the energy requirements for heating during cold seasons. Likewise, the consumer side of the economy is similarly affected. Conditions for, say, malaria become more favourable implying an increased need for health care and, simultaneously, diminished labour productivity involving a degradation of the consumer's initial endowments.

Since climate change may affect production factors, resources and endowments directly, it shifts productivity of technologies. In turn, shifts in productivity affect market prices, economic decisions, allocations and, ultimately, GDP. Hence, the need for a more fundamental approach in which the heterogeneity of environmental damages affect the economy at the level of individual agents. This paper proposes an innovative modelling approach in which economic damages related to climate change are modelled as so-called damage coefficients. Such coefficients allow for *heterogeneity* across all economic agents and all goods (including all private goods) in how climate change affects the use of goods by individual agents. We opt for modelling damage coefficients that allow easy implementation in applied economic models: Damage coefficients can be interpreted as percentages that reduce the effective use of goods applied in production or goods consumed. As we will show, our embedding can be easily implemented into, for example, existing computable general equilibrium models. Since individual economic agents are often relatively small compared to the environment and the economy, they are therefore best modelled as taking both environmental damages and prices as given.

The aim of this paper is to provide a general microeconomic foundation for producer and consumer behaviour under environmental damage represented by damage coefficients. For the individual producer, we derive the cost minimizing demand function (for inputs), the cost function, the profit maximizing supply function and the profit function. Our analysis establishes that all these functions are equivalent to the classical functions after applying appropriate price distortions. These price distortions are directly related to the damage coefficients and the extended functions can be easily implemented in practice. Comparative statics with respect to changes in prices and damages are provided, including an extension of Shephard's lemma. Also, for the consumer we derive both the Marshallian and Hicksian demand function, the indirect utility function and the expenditure function. Similar as for the producer, all these functions are related to the classical functions through price distortions that are related to the damage coefficients. The comparative statics include an extension of

Slutsky's equations.

Since damage coefficients affect productivity, utility preferences and endowments, the next step naturally involves incorporating consumers and producers into a model of an economy with environmental damage to study the complex and nonlinear interaction of economic decisions and environmental damage. Environmental damages on the one hand result in an adjusted cost of or expenditure on certain goods in production and consumption decisions, thereby often changing relative prices of these goods. Hence, the presence of environmental damages provides a substitution effect in the economy. On the other hand, environmental damages affect the economy's endowments and the firms' profitability, thereby resulting in an income effect.

We embed consumers and producers in a recursive model with an infinite time horizon. In this model, these agents regard their impact on greenhouse gases as infinitesimally small and they are both price and damage takers. Furthermore, today's initial environmental stock variables and today's economic activity imply tomorrow's environmental stocks and so on. Physical reality is represented by a climate model that can be represented by an impulse response function (IRF) as in e.g. Hooss et al. (2001). Also, tomorrow's environmental damages are related to tomorrow's environmental stocks through damage functions. Finally, economics and natural sciences typically deal with different adjustment processes towards equilibrium. In economic models, the equilibrium concept refers to price variables that are such that underlying markets clear immediately. In contrast, climate models use an equilibrium concept that refers to climate-related variables such as temperature, precipitation, and radiation that moves toward an equilibrium over long time horizons such as several decades or longer. The equilibrium concept used in climate models implies a certain underlying adjustment process, while the economic models do not define how an economy adjusts to a new equilibrium. We reconcile these differences in an appropriate concept of equilibrium that we call a Recursive Equilibrium with Environmental Damages, abbreviated as REED, and show its existence.

The time path of a REED allows to study the evolution of economic phenomena, such as equilibrium prices, equilibrium allocations and GDP, physical phenomena, such as emissions and environmental stock variables, and the interaction between economic activity, climate change and environmental damages. Comparing different periods within a time path of REED can be conducted by applying the standard equivalent variations approach with the first year as the base year. By applying REED to an exchange economy and a Robinson Crusoe economy, we illustrate that environmental damages affect economic agents and the economy in a nontrivial and nonlinear manner.

In Section 2, we extend the standard microeconomic theory of the producer and the consumer with the impact of environmental damages through the inclusion of damage coefficients into the production technology, the consumers utility function and the initial endowments. Section 3 and 4 provide the impact on producer respectively on consumer behavior in the extended microeconomic theory. Section 5 introduces REED and it is applied in Section 6 to a recursive exchange economy and a recursive Robinson Crusoe economy. Section 7 concludes.

2 The modelling of environmental damages

We consider a price-taking consumer and firm that are affected by environmental conditions imposed upon them by for example the surrounding climate. Climate refers to global variables such as global temperature, precipitation, or sun radiation levels. There are n commodities, indexed $k = 1, \ldots, n$, stacked into a vector $x \in \mathbb{R}^n_+$ that represents the consumption bundle or the vector $(y, -x) \in \mathbb{R}^n$, where $y \in \mathbb{R}_+$ and $x \in \mathbb{R}^{n-1}_+$, that represents a production plan. Production plans are such that the positive entry represents the amount of the output commodity and all non-positive entries the required amounts of inputs. All commodities are traded on markets, where $p_k > 0$ denotes the price of commodity k and $p \in \mathbb{R}^n_{++}$ is a price vector.

The consequences of environmental damages on the economic behaviour of the consumer or the producer are modelled through the introduction of damage coefficients $d_k > 0$ into their preference relations or technologies. These damage coefficients affect the expenditure on, respectively the cost of obtaining the individual commodities in these relations. A damage coefficient d_k can take a value in $(0, 1]$ to represent a deteriorating effect or it can take a value $d_k \in (1, \infty)$ in case the effect is beneficial. In case $d_k \in (0, 1]$, it means that the consumer's actual satisfaction of consuming x_k units of good k, or the productivity of x_k units of good k in the producer's technologies is reduced to $d_k x_k$, where $d_k = 1$ for all commodities corresponds to standard microeconomic theory. Otherwise, $d_k x_k > x_k$ can be seen as an improvement in the consumer's satisfaction of good k or the productivity of good k in the technology. For the analysis, it is convenient to introduce a diagonal matrix D with d_k as its diagonal elements, which has a dimension of $n \times n$ for the consumer and of $(n-1) \times (n-1)$ for the producer. Similarly, we also introduce $d_0 > 0$ to indicate the overall impact on the output of production or the overall well-being of the consumer. All impact coefficients are treated as additional parameters in the consumer's and producer's optimization programs that are, similar to prices, taken as given by the economic agents.

Formally, the producer has a technology that produces commodity $y \in \mathbb{R}_+$ units of the single output good using the other $n-1$ commodities as its inputs, i.e., $-x \in \mathbb{R}^{n-1}_-$. In describing the microeconomic behaviour of the producer, it is convenient to use the vector $x \in \mathbb{R}^{n-1}_+$ to refer to the amounts of inputs and to represent the technology by the production function $d_0 f(Dx)$, which is consistent with the production set $Y(D, d_0) = \{(y, -x) \in \mathbb{R}^n \mid y \leq d_0 f(Dx)\}$. We assume that $f : \mathbb{R}_+^{n-1} \to \mathbb{R}$ is continuous, strictly concave, and homogeneous, i.e., $f(0) = 0$. The producer's profit maximisation problem is given by

$$\max_{y \geq 0; x \geq 0} \quad qy - p^\top x, \text{ s.t. } y \leq d_0 f(Dx), \tag{1}$$

where $q > 0$ denotes the price per unit of output. The inequality in (1) is due to free disposal.

The consumer's preferences over the commodities in the economy are described by a utility function $u(Dx)$, where $u : \mathbb{R}_+^n \to \mathbb{R}$ is continuous, strictly quasi-concave, and monotonically increasing.[1] The consumer is endowed with initial endowments $\omega \in \mathbb{R}_+^n \backslash \{0\}$ that are also subject to environmental damage. The damage coefficient that works on endowment k is denoted as $\phi_k > 0$. As before, $\phi_k \in (0, 1]$ indicates a deterioration of the consumer's endowment in commodity k, while $\phi_k > 1$ indicates an improvement in the consumer's endowment in commodity k following an environmental impact. For convenience, we introduce an $(n \times n)$-dimensional diagonal matrix Φ with ϕ_k as its diagonal elements. The consumer's utility maximisation program is then given by

$$\max_{x \geq 0} \quad u(Dx), \text{ s.t. } p^\top x \leq p^\top \Phi \omega. \tag{2}$$

3 The impact of environmental damage on the firm

We analyse how damage coefficients affect the producer's cost and profit maximisation problems, in particular the derived supply and demand functions. An economy with damages d_0 and D is compared to a benchmark economy where there are no damages − hence $d_0 = 1$ and $D = I$.

3.1 Cost minimisation

Let $C(p, y; d_0, D)$ define the producer's cost function under market prices p, output level y, and damages given by d_0 and D. Let $h(p, y; d_0, D)$ define the input demand functions of each good under these conditions. Then, by definition

$$\begin{aligned} C(p, y; d_0, D) &= \min_{x \geq 0} \; p^\top x, \text{ s.t. } d_0 f(Dx) \geq y, & (3) \\ h(p, y; d_0, D) &= \arg\min_{x \geq 0} \; p^\top x, \text{ s.t. } d_0 f(Dx) \geq y. & (4) \end{aligned}$$

Note that $C(p, y; 1, I)$ and $h(p, y; 1, I)$ are the standard cost function, respectively, input demand function, which we denote in their usual form, i.e. without the explicit addition of

[1] Mathematically, we could allow for $d_0 u(Dx)$, but since $d_0 > 0$ implies a linear affine transformation of the utility scale, we can set d_0 equal to 1.

the damage parameters as arguments. Thereby, we can refer back to the original cost function and input demand function. The following result relates $C(p,y;d_0,D)$ and $h(p,y;d_0,D)$ to the standard cost and demand functions.

Proposition 1 *The producer's cost function*

$$C(p,y;d_0,D) = C\left(D^{-1}p, d_0^{-1}y\right)$$

is differentiable in $\hat{p} = D^{-1}p$ *and its demand for inputs*

$$h(p,y;d_0,D) = D^{-1}h\left(D^{-1}p, d_0^{-1}y\right).$$

Proof. Under the assumption of free disposal, there exists a unique cost minimising vector. The differentiability of the cost function in $\hat{p} = D^{-1}p$ then follows with the Duality Theorem, see e.g., Mas-Colell et al. (1995). A change of variables $z = Dx$ yields

$$\begin{aligned} C(p,y;d_0,D) &= \min_{x\geq 0}\ p^\top x,\ \text{s.t. } d_0 f(Dx) \geq y, \\ &= \min_{z\geq 0}\ p^\top D^{-1}z,\ \text{s.t. } f(z) \geq d_0^{-1}y, \\ &= C\left(D^{-1}p, d_0^{-1}y\right), \end{aligned}$$

where $z^* = h\left(D^{-1}p, d_0^{-1}y\right)$ is the minimising demand. Hence, $x^* = D^{-1}h\left(D^{-1}p, d_0^{-1}y\right)$. ■

The inclusion of environmental damages into the production cost of the economy influences the commodity prices. Prices should be denominated with environmental damage, hence $D^{-1}p$, under the existence of environmental damages given by d_0 and D. In case all environmental effects are deteriorating, i.e., $d_0, d_k < 1$, the environmental cost of production $C\left(D^{-1}p, d_0^{-1}y\right) > C(p,y)$, hence these cost increase. The difference $C\left(D^{-1}p, d_0^{-1}y\right) - C(p,y)$ quantifies the environmental impact on the production costs under given prices p and activity levels y.

We can compare the impact of environmental damages on the production side of the economy with imposing an ad-valorem tax rate equal to $1/d_k$ on the use of input good k in the production of the output good. The imposition of such taxes distorts the economic decisions of the production sector, and so does environmental damage. The overall damage d_0 acts as a lump-sum tax on total production and does not induce price distortions, but does discourage production.

Some economic models include transport costs in the price of a good in a form comparable to an iceberg that looses volume during its trip from one point to another. Taking a similar interpretation with respect to the impact of environmental damages on the economy's production side, there exists a difference between a 'first-on-board' (fob) price p_k and the 'cost-insurance-freight' (cif) price equal to $p_k\left(1-1/d_k\right)$.

The following result quantifies marginal price effects $\frac{\partial C(p,y;d_0,D)}{\partial p_k}$ and marginal environmental effects $\frac{\partial C(p,y;d_0,D)}{\partial d_k}$ and should be seen as an extension of Shephard's Lemma.

Proposition 2 *For each input good k,*

$$\begin{aligned} d_k \cdot \left(\frac{\partial C\left(p,y;d_0,D\right)}{\partial p_k}\right) &= h_k\left(D^{-1}p, d_0^{-1}y\right) > 0, \\ d_k \cdot \left(\frac{\partial C\left(p,y;d_0,D\right)}{\partial d_k}\right) &= -p_k \cdot \left(\frac{h_k\left(D^{-1}p, d_0^{-1}y\right)}{d_k}\right) < 0. \end{aligned}$$

Proof. By Proposition 1 and Shephard's Lemma, the marginal impact of a change in p_k is

$$\begin{aligned} d_k\left(\frac{\partial C\left(p,y;d_0,D\right)}{\partial p_k}\right) &= d_k\left(\frac{\partial C\left(D^{-1}p, d_0^{-1}y\right)}{\partial p_k}\right) \\ &= d_k\left(\left.\frac{\partial C\left(\hat{p}, d_0^{-1}y\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right)\cdot\left(\frac{\partial d_k^{-1}p_k}{\partial p_k}\right) \\ &= h_k\left(D^{-1}p, d_0^{-1}y\right). \end{aligned}$$

Similarly,

$$\begin{aligned} d_k\left(\frac{\partial C\left(p,y;d_0,D\right)}{\partial d_k}\right) &= d_k\left(\left.\frac{\partial C\left(\hat{p}, d_0^{-1}y\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right)\cdot\left(\frac{\partial d_k^{-1}p_k}{\partial d_k}\right) \\ &= -\left(d_k^{-1}p_k\right)\cdot d_k^{-1}h_k\left(D^{-1}p, d_0^{-1}y\right), \end{aligned}$$

which provides the stated result. This completes the proof. ■

Corollary 3 *Let $\eta_k\left(\hat{p},\hat{y}\right)$ be the price elasticity of the cost function with respect to input k at $\left(\hat{p},\hat{y}\right)$. Then, for each input good k,*

$$\frac{\partial C\left(p,y;d_0,D\right)}{\partial p_k} = -\left(\frac{\eta_k\left(D^{-1}p, d_0^{-1}y\right)}{d_k}\right)\cdot\left(\frac{C\left(D^{-1}p, d_0^{-1}y\right)}{d_k}\right).$$

Proof. From the proof of Proposition 2, it follows that

$$\begin{aligned} \frac{\partial C\left(p,y;d_0,D\right)}{\partial p_k} &= -d_k^{-2}\cdot p_k\left(\left.\frac{\partial C\left(\hat{p}, d_0^{-1}y\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right) \\ &= -d_k^{-2}\cdot\eta_k\left(D^{-1}p, d_0^{-1}y\right)\cdot C\left(D^{-1}p, d_0^{-1}y\right). \end{aligned}$$

■

Proposition 1 implies that, in the presence of environmental damages, we should denominate prices and output levels in units of damage, i.e. $\hat{p} = D^{-1}p$ and $\hat{y} = d_0^{-1}y$. This has its consequences for applying Shephard's Lemma to obtain the cost minimising amounts of each input good where, according to Proposition 2, Shephard's Lemma results in the cost minimizing amount of input per damage unit.

Note that the derivative of the cost function with respect to the price of input k implies the standard Shephard's Lemma for $d_0 = 1$ and $D = I$, which follows from the fact that the right-hand side is then equal to $h_k(p, y; d_0, D)$. According to Proposition 2, an improvement in damages, i.e. an increase in the coefficients d_0 or d_k, results in decreasing production costs. Notice, however, that an overall damage improvement on output d_k results in a cost savings proportional to current expenditure on input k.

The next result relates the derivatives of the cost minimising demand functions to the second derivatives of the standard cost function.

Corollary 4 *Let $C(\hat{p}, \hat{y})$ be twice differentiable at $(\hat{p}, \hat{y})$ and $\hat{p} \gg 0$. Then, for inputs k and l,*

$$
\begin{aligned}
d_l \left(\frac{\partial h_k (p, y; d_0, D)}{\partial p_l} \right) &= C_{kl} \left(D^{-1}p, d_0^{-1}y \right), \\
d_l \left(\frac{\partial h_k (p, y; d_0, D)}{\partial d_l} \right) &= -\frac{C_{kl} \left(D^{-1}p, d_0^{-1}y \right)}{d_l},
\end{aligned}
$$

where $C_{kl}(\hat{p}, \hat{y})$ denotes $\frac{\partial C(\hat{p},\hat{y})}{\partial \hat{p}_k \partial \hat{p}_l}$.

Proof. For inputs k, the propositions 1 and 2 imply that

$$
\begin{aligned}
d_l \left(\frac{\partial h_k (p, y; d_0, D)}{\partial p_l} \right) &= d_l d_k \left(\frac{\partial^2 C (p, y; d_0, D)}{\partial p_l \partial p_k} \right) \\
&= d_l d_k \left(\left. \frac{\partial C \left(\hat{p}, d_0^{-1}y \right)}{\partial \hat{p}_k \partial \hat{p}_l} \right|_{\hat{p}=D^{-1}p} \right) \cdot \left(\frac{\partial d_k^{-1} p_k}{\partial p_k} \right) \cdot \left(\frac{\partial d_l^{-1} p_l}{\partial p_l} \right) \\
&= -\frac{C_{kl} \left(D^{-1}p, d_0^{-1}y \right)}{d_l}.
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
d_l \cdot \left(\frac{\partial h_k\left(p, y; d_0, D\right)}{\partial d_l}\right) &= d_l d_k \left(\frac{\partial^2 C\left(p, y; d_0, D\right)}{\partial d_l \partial p_k}\right) \\
&= d_l d_k \left(\left.\frac{\partial C\left(\hat{p}, d_0^{-1} y\right)}{\partial \hat{p}_k \partial \hat{p}_l}\right|_{\hat{p}=D^{-1}p}\right) \cdot \left(\frac{\partial d_k^{-1} p_k}{\partial p_k}\right) \cdot \left(\frac{\partial d_l^{-1} p_l}{\partial d_l}\right) \\
&= -d_l^{-1} C_{kl}\left(D^{-1}p, d_0^{-1} y\right).
\end{aligned}
$$

■

According to Corollary 4, an improvement in damages, i.e. an increase in the coefficients d_k, results in decreasing production costs. Notice however that an overall damage improvement on output d_0 results in a decreased demand for each input good k. Hence, such improvements lead to efficiency gains. An improvement in input good l's effectiveness due to decreased damage impacts d_k, increases the use of input good l into the production process since it has become more efficient compared to the other inputs. An improvement in input good k's effectiveness decreases the use of each other input good k for similar reasons.

3.2 Profit maximisation

The profit maximisation problem associated with the producer's cost function defined in Proposition 1, can be expressed as

$$
\Pi\left(q, p; d_0, D\right) = \max_{y \geq 0}\ qy - C\left(D^{-1}p, d_0^{-1} y\right), \tag{5}
$$

and the supply function defined by

$$
s\left(q, p; d_0, D\right) = \arg\max_{y \geq 0}\ qy - C\left(D^{-1}p, d_0^{-1} y\right). \tag{6}
$$

We denote the standard profit function $\Pi\left(q, p; 1, I\right)$ and supply function $s\left(q, p; 1, I\right)$ are the standard profit, respectively, supply function, which we denote in their usual form. Substitution of the producer's supply function into the cost minimising demand for inputs $h\left(p, y; d_0, D\right)$ as defined in (4), determines the profit maximising demand for inputs. Under the assumption of differentiability of the cost function, supply function (6) is the solution to the first-order-condition to the optimisation problem in (5),

$$
q - C_y\left(p, y; d_0, D\right) \leq 0 \qquad \perp \qquad y \geq 0, \tag{7}
$$

where $C_y\left(p, y; d_0, D\right) = \frac{\partial C(p, \hat{y})}{\partial \hat{y}}$. From this condition we obtain

Proposition 5 *The producer's profit function*

$$\Pi\left(q,p;d_0,D\right)=\Pi\left(d_0q,D^{-1}p\right)$$

is differentiable in q and p and its supply of output

$$s\left(q,p;d_0,D\right)=d_0 s\left(d_0q,D^{-1}p\right).$$

Proof. The assumption of free disposal implies the existence of a unique profit maximising quantity. Differentiability of the profit function in prices is then guaranteed by the Duality Theorem, see e.g. Mas-Colell et al. (1995). A change of variables $z=d_0^{-1}y$ and Proposition 1 yields

$$\begin{aligned}\Pi\left(q,p;d_0,D\right) &= \max_{y\geq 0}\ qy-C\left(p,y;d_0,D\right),\\ &= \max_{z\geq 0}\ \left(d_0q\right)z-C\left(D^{-1}p,z\right),\\ &= \Pi\left(d_0q,D^{-1}p\right),\end{aligned}$$

where $z^{*}=s\left(d_0q,D^{-1}p\right)$ is the profit maximising supply. Hence, $y^{*}=d_0s\left(d_0q,D^{-1}p\right)$. ■

In case all environmental effects are deteriorating, i.e. $d_0,d_k<1$, it can be shown that the profits of the production sector decrease, i.e., $\Pi\left(d_0q,D^{-1}p\right)<\Pi\left(q,p\right)$, due to a higher output price and lower input prices in the non damage case. Hence, Π is an increasing function of environmental damage. The difference $\Pi\left(d_0q,D^{-1}p\right)-\Pi\left(q,p\right)$ quantifies the environmental impact on the production sector under given prices. This environmental impact decomposes into the cost impact of Section 3.1 and an environmental impact on the producer's revenues:

$$\Pi\left(q,p\right)-\Pi\left(d_0q,D^{-1}p\right)=\left(1-d_0\right)qy-\left[C\left(d_0^{-1}q,D^{-1}p\right)-C\left(q,p\right)\right]. \tag{8}$$

The following result quantifies marginal price effects and marginal environmental effects and should be seen as an extension of Hotelling's Lemma.

Proposition 6 *Differentiability of* $\Pi\left(\hat{q},\hat{p};d_0,D\right)$ *implies*

$$\frac{\partial\Pi\left(q,p;d_0,D\right)}{\partial q}=d_0s\left(d_0q,D^{-1}p\right)>0,$$

and for each input k,

$$d_k\left(\frac{\partial\Pi\left(q,p;d_0,D\right)}{\partial d_k}\right)=p_k\left(\frac{h_k\left(D^{-1}p,d_0^{-1}y\right)}{d_k}\right)<0.$$

Proof. Note that $\Pi(\hat{q},\hat{p};d_0,D)>0$ is due to $s(d_0q,D^{-1}p)>0$, which implies equality in (7). By Proposition 5 and Hotelling's Lemma, the marginal impact of a change in output price q is

$$\begin{aligned}
\frac{\partial \Pi(q,p;d_0,D)}{\partial q} &= \frac{\partial \Pi(d_0q,D^{-1}p)}{\partial q} \\
&= \left(\left.\frac{\partial \Pi(\hat{q},D^{-1}p)}{\partial \hat{q}}\right|_{\hat{q}=d_0q}\right)\cdot\left(\frac{\partial d_0q}{\partial q}\right) \\
&= \left(d_k^{-2}p_k\right)\left[q-\left(\left.\frac{\partial C(D^{-1}p,\hat{y})}{\partial \hat{y}}\right|_{\hat{y}=d_0^{-1}y}\right)\cdot d_0^{-1}\right]\left(\left.\frac{\partial s(d_0q,\hat{p})}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right) \\
&\quad +d_0s\left(d_0q,D^{-1}p\right) \\
&= d_0s\left(d_0q,D^{-1}p\right)>0,
\end{aligned}$$

where we have used equality in (7). Similarly, for inputs k,

$$\begin{aligned}
d_k\left(\frac{\partial \Pi(q,p;d_0,D)}{\partial d_k}\right) &= d_k\left(\left.\frac{\partial \Pi(d_0q,\hat{p})}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right)\cdot\left(\frac{\partial d_k^{-1}p_k}{\partial d_k}\right) \\
&= \left(d_k^{-1}p_k\right)\left[\left(\left.\frac{\partial C(D^{-1}p,\hat{y})}{\partial \hat{y}}\right|_{\hat{y}=d_0^{-1}y}\right)d_0^{-1}-q\right]\cdot\left(\left.\frac{\partial s\left(\hat{p},d_0^{-1}y\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right) \\
&\quad -\left(\frac{\partial C\left(D^{-1}p,d_0^{-1}y\right)}{\partial d_k}\right)d_k \\
&= \left(d_k^{-1}p_k\right)\left[\left(\frac{\partial C}{\partial \hat{y}}\right)d_0^{-1}-q\right]\left(\frac{\partial s}{\partial \hat{p}_k}\right)+p_kd_k^{-1}h_k\left(D^{-1}p,d_0^{-1}y\right).
\end{aligned}$$

The latter in combination with (7) completes the proof. ■

4 Environmental damage impact on the consumer

We include damage coefficients into the consumer's expenditure minimisation problem and the utility maximisation problem, especially into the derived demand functions. Although the analysis of the firm's cost minimisation problem directly applies to the consumer's expenditure problem with C representing the expenditure function and h the associated Hicksian demand function, we study each optimisation problem in a separate subsection. The reason is that we invoke the Hicksian demand functions in modifying Slutsky's equations. An economy with damages d and D is compared to a benchmark economy where no damages are assumed, assuming market prices p and income levels m.

4.1 Expenditure minimisation

The Hicksian demand function is the expenditure minimising consumption bundle. Formally, the expenditure function and Hicksian demand function, including damage impacts, are defined as

$$\begin{aligned} E(p,u;D) &= \min_{x\geq 0} p^\top x, \quad \text{s.t.} \quad u(Dx)\geq u, \\ H(p,u;D) &= \arg\min_{x\geq 0} p^\top x, \quad \text{s.t.} \quad u(Dx)\geq u. \end{aligned}$$

The functions E and H for the consumer have the same properties as the producer's cost function C and demand for inputs h, and therefore we can derive the same results for E and H as we did in Section 3.

Proposition 7 *$E(p,u;D) = E(D^{-1}p,u)$ and $H(p,u;D) = D^{-1}H(D^{-1}p,u)$.*

Proof. Similar to Proposition 1 ■

4.2 Utility maximisation

The utility maximisation problem is given in (2). Let $m \equiv m(p,\omega;\Phi) = p^\top \Phi\omega$ denote the consumer's market income, and $v(p,m;D)$ denote the consumer's indirect utility function under market prices p, market income m, and damages given by Φ and D. Let $z(p,m;D)$ define the demand functions of each good under these conditions. Then, by definition

$$\begin{aligned} v(p,m;D) &= \max_{x\geq 0} u(Dx), \quad \text{s.t.} \quad p^\top x\leq m, \\ z(p,m;D) &= \arg\max_{x\geq 0} u(Dx), \quad \text{s.t.} \quad p^\top x\leq m. \end{aligned}$$

As before, we simplify notation by taking $v(p,m) \equiv v(p,m;I)$ and $z(p,m) \equiv z(p,m;I)$ as the standard indirect utility function, respectively, demand function.

Proposition 8 *The consumer's indirect utility function*

$$v(p,m;D) = v\left(D^{-1}p,m\right)$$

and its demand function

$$D\cdot z(p,m;D) = z\left(D^{-1}p,m\right).$$

Proof. The change of variables $z = Dx$ yields

$$\begin{aligned} v(p, m; D) &= \max_{x \geq 0} u(Dx), \quad \text{s.t.} \quad p^\top x \leq m, \\ &= \max_{z \geq 0} u(z), \quad \text{s.t.} \quad \left(D^{-1}p\right)^\top z \leq m, \\ &= v\left(D^{-1}p, m\right), \end{aligned}$$

where $z^* = z(D^{-1}p, m)$ is the maximising demand. Hence, $D \cdot x^* = z(D^{-1}p, m)$. ■

Corollary 9 *If u is a Cobb-Douglas utility function, then $z(p, m; D) = z(p, m)$ and $v(p, m; D) = v(p, m)$.*

In case all environmental effects are deteriorating, $d_k < 1$, the indirect utility $v(D^{-1}p, m) < v(p, m)$, hence indirect utility decreases. The difference $v(D^{-1}p, m) - v(p, m) < 0$ quantifies the environmental impact on consumer utility under given prices p and income m. The effect of changes in D are different from those in Φ. Changes in D are price distorting, whereas changes in Φ act as a lump sum income effect on market income $m(p, \omega; \Phi)$. To study these effects in terms of income and substitution effects, we extend Slutsky's equations to express changes in both types of damage coefficients.

The Hicksian demand functions of Section 4.1 allows us to extend Slutsky's equations and characterize the effects of damage coefficients.

Proposition 10 *Let $E(\hat{p}, \hat{u})$ be differentiable at $(\hat{p}, \hat{u})$ and $\hat{p} \gg 0$. Then, for $k, l = 1, \ldots, n$,*

$$\begin{aligned} \frac{\partial z_l(D^{-1}p, \bar{m})}{\partial \hat{p}_k} &= \frac{\partial D^{-1} H_l(D^{-1}p, \bar{u})}{\partial \hat{p}_k} - \frac{\partial z_l(D^{-1}p, \bar{m})}{\partial m}\left[\bar{x}_k - \phi_k \omega_k\right], \\ \frac{\partial z_l(p, m; D)}{\partial d_k} &= \left(\frac{\partial z_l(D^{-1}p, \bar{m})}{\partial \hat{p}_k}\right)\left(-d_k^2 p_k\right), \\ \frac{\partial z_l(p, m; D)}{\partial \phi_k} &= \left(\frac{\partial z_l(D^{-1}p, m)}{\partial m}\right) p_k \omega_k, \end{aligned}$$

where $\bar{m} = p^\top \Phi \omega$, $\bar{u} = v(p, \bar{m}; D)$ and $\bar{x}_k = H_k(D^{-1}p, \bar{u})$.

Proof. Define the minimal compensation function $T : \mathbb{R}_+^n \times \mathbb{R}_+ \to \mathbb{R}$ to obtain the utility level $\bar{u} = v(p, \bar{m}; D)$ at prices p' as

$$T\left(D^{-1}p', \bar{u}\right) = E\left(D^{-1}p', \bar{u}\right) - \left(D^{-1}p'\right)^\top \cdot D\Phi\omega.$$

Obviously, the function T has the property that $T\left(D^{-1}p', \bar{u}\right) = 0$. Given the function T, prices p, and market income $\bar{m} = p^{\top}\Phi\omega$, it follows directly from the definition of the minimal compensation function T that

$$E\left(D^{-1}p, \bar{u}\right) = \bar{m} + T\left(D^{-1}p', \bar{u}\right). \tag{9}$$

By Shephard's lemma, the function T has the following partial derivative,

$$\left.\frac{\partial T\left(\hat{p}, \bar{u}\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p} = \left(\left.\frac{\partial E\left(\hat{p}, \bar{u}\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right) - d_k\phi_k\omega x_k = d_k^{-1}\bar{x}_k - d_k\phi_k\omega_k.$$

Then, by the standard properties of the primal and dual consumer problem and Proposition 8, we have that $H\left(p, \bar{u}; D\right) = z\left(p, E\left(p, \bar{u}; D\right); D\right)$. Substitution of (9) yields

$$H\left(p, \bar{u}; D\right) = z\left(p, \bar{m} + T\left(D^{-1}p', \bar{u}\right); D\right) \Longrightarrow D^{-1}H\left(D^{-1}p, \bar{u}\right) = z\left(D^{-1}p, \bar{m} + T\left(D^{-}p', \bar{u}\right)\right),$$

where $\bar{m}$ and $\bar{u}$ are constants. Differentiating both sides of the last equality for commodity l with respect to p_k yields

$$\begin{aligned}\left.\frac{\partial D^{-1}H_l\left(D^{-1}p, \bar{u}\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p} &= \left(\left.\frac{\partial z_l\left(D^{-1}p, \bar{m}\right)}{\partial \hat{p}_k}\right|_{\hat{p}=D^{-1}p}\right) + \left(\frac{\partial z_l\left(D^{-1}p, \hat{m}\right)}{\partial \hat{m}}\right) \cdot \left(\frac{\partial T\left(\hat{p}, \bar{u}\right)}{\partial \hat{p}_k}\right) \\ &= \left(\frac{\partial z_l\left(D^{-1}p, \bar{m}\right)}{\partial \hat{p}_k}\right) + \left(\frac{\partial z_l\left(D^{-1}p, \bar{m}\right)}{\partial m}\right) \cdot \left[\bar{x}_k - \phi_k\omega_k\right],\end{aligned}$$

after discarding of the common term d_k^{-1}. So, we obtain the stated expression.

Next, it follows that

$$\frac{\partial z_l\left(D^{-1}p, \bar{m}\right)}{\partial d_k} = \left(\frac{\partial z_l\left(\hat{p}, \bar{m}\right)}{\partial \hat{p}_k}\right) \cdot \left(\left.\frac{\partial \hat{p}_k}{\partial d_k}\right|_{\hat{p}=D^{-1}p}\right) = \left(\frac{\partial z_l\left(D^{-1}p, \bar{m}\right)}{\partial \hat{p}_k}\right) \cdot \left(-d_k^2 p_k\right).$$

and that

$$\frac{\partial z_l\left(D^{-1}p, \bar{m}\right)}{\partial \phi_k} = \left(\left.\frac{\partial z_l\left(D^{-1}p, \hat{m}\right)}{\partial \hat{m}}\right|_{\hat{m}=\bar{m}}\right) \cdot \left(\frac{\partial p^{\top}\Phi\omega}{\partial \phi_k}\right),$$

which yields the other stated results. ■

Modified Slutsky's equations are derived from differentiating $h_k\left(p, \bar{u}\right) = d_k z_k\left(D^{-1}p, E\left(D^{-1}p, d_0^{-1}\bar{u}\right)\right)$ with respect to price p_l. This results into

$$\left(\frac{d_k}{d_l}\right)\left(\frac{\partial z_k}{\partial p_l}\right) + \left(\frac{d_k}{d_l}\right)\left(\frac{\partial z_k}{\partial \omega}\right)\left[\left(\frac{\partial E}{\partial p_l}\right) + \left(\frac{\partial E}{\partial u}\right)\left(\frac{\phi_l}{d_0}\right)\omega_l\right].$$

The first part of the latter formula refers to the substitution effect of a price change and includes the damages on the consumption of good l with respect to the damage on the consumption of good k. The second part of the formula refers to the income effect of a price change in good l, and it contains the damage done on income obtained from selling good l.

5 Recursive Equilibrium with Environmental Damages

The economy consists of consumers $i \in \{1, \ldots, I\}$ and producers $j \in \{1, \ldots, J\}$ whose behaviour under the presence of environmental damage has been described in the previous sections. Consumers and producers are assumed to be small relative to the economy. Therefore, they take both prices and environmental damages as given, but current economic activity adds to the cumulative environmental stock variables. The damage coefficients d_0, D, and Φ are specific to the consumers and producers, which we express with the addition of an appropriate super index. Similarly, we denote consumer i's consumption bundle as $x^i \in \mathbb{R}^n_+$ and producer j's production plan as $y^j \in \mathbb{R}^n$. Firm j produces commodity k^j, meaning that firm j's production plan $y^j \in \mathbb{R}^n$ has elements $y^j_{k^j} \geq 0$, $y^j_k \leq 0$, $k \neq k^j$, which we write more conveniently as $y^j = (y^j_{k^j}, y^j_{-k^j})$ in order to distinguish between output and inputs. We extend firm j's $(n-1) \times (n-1)$ matrix of damage coefficients related to inputs by inserting a row and column at the k^j-th position and define the (k^j, k^j)-th element of the extended matrix as $\left(d^j_0\right)^{-1}$. This notation is consistent with $\left(d^j_0\right)^{-1} y_{k^j} \leq f^j\left(D^j y^j_{-k^j}\right)$ in the production set $Y(D^j, d^j_0)$ as defined in Section 2. We suppress d^i_0 in our notation and simply refer to D^i whenever we actually mean (d^i_0, D^i). An allocation in the economy consists of all human activities related to consumption $x = \left(x^1, \ldots, x^i, \ldots, x^I\right)^\top$ and production plans $y = \left(y^1, \ldots, y^j, \ldots, y^J\right)^\top$, which we write as (x, y).

Let $t \in \mathbb{N}$ denote discrete time and the vector $\varepsilon_t \in \mathbb{R}^m_+$, $m \in \mathbb{N}$, a vector containing m cumulative stock variables representing the initial stock at time t of the environmental variables. Environmental damage is assumed to be associated with these environmental stock variables and its dynamics are governed by the physical world and the current period's human activities consisting of consumption and production plans (x_t, y_t). The physical world is represented by a climate model that can be reduced to an impulse response function (IRF) $L : \mathbb{R}^{In}_+ \times \mathbb{R}^{Jn}_+ \times \mathbb{R}^m_+ \to \mathbb{R}^m_+$ such that next period's stock ε_{t+1} is given by

$$\varepsilon_{t+1} = L\left(x_t, y_t, \varepsilon_t\right).$$

We assume that L is continuous and monotone in the environmental variables ε_t and (x_t, y_t). Furthermore, $L\left(x_t, y_t, 0\right) \geq 0$. Hence, L maps the current stock ε_t and current human activities x_t and y_t into the next period's environmental stock ε_{t+1}. A possible interpretation of ε_t is the value of climate variables such as global temperature, precipitation, or sun radiation. The value of these variables is dependent on the amount of greenhouse gases in the atmosphere. Such concentrations depend on the global economy's emissions associated with its consumption and production of fossil fuel energy goods. We refer to e.g. Hooss et al. (2001) with respect to the estimation of impulse response functions on the outcomes of more elaborate climate models.

The relation between the environmental stocks $\varepsilon_t = \varepsilon$ and the environmental damages to consumer i are given by the damage functions $D^i = F^i(\varepsilon)$ and $\Phi^i = \bar{F}^i(\varepsilon)$.[2] Similarly, $D^j = F^j(\varepsilon)$ denotes the environmental damage function to producer j. We assume that all damage functions F^i, $\bar{F}^i$, and F^j are continuous and map from $\mathbb{R}^m_+$ to $[0, d^{\max}]^n$ for some $d^{\max} > 1$.[3]

The equilibrium concept that we propose is a recursive equilibrium in which the economy is in equilibrium in every period given the current damage coefficients. All economic agents maximise their own objective function given prevailing market prices and given the environmental damages related to the environmental stock variable.

Definition 11 *The sequence of allocations, prices, and environmental stocks,* $\{(x_t, y_t, p_t, \varepsilon_t)\}_{t\in\mathbb{N}}$ *is a* Recursive Equilibrium with Environmental Damages (REED)*, if for each period* $t \in \mathbb{N}$*,*

1) y^j_t maximises producer j's profit function $p_t y^j_t$ at prices p_t, damages D^j_t, and his technology $Y^j(D^j_t)$.

2) x^i_t maximises consumer i's utility function u^i at prices p_t, damages D^i and Φ^i, and market income $p_t \phi^i_t \omega^i$.

3) markets are in equilibrium,

$$\sum_i x^i_t - \sum_j y^j_t - \sum_i \phi^i_t \omega^i_t \leq 0.$$

4) environmental damages are related to the environmental stock ε_t through damage functions,

$$\begin{aligned} D^j_t &= F^j(\varepsilon_t), \\ D^i_t &= F^i(\varepsilon_t), \\ \Phi^i_t &= \bar{F}^i(\varepsilon_t). \end{aligned}$$

5) the environmental stock evolves according to an impulse response function L:

$$\varepsilon_{t+1} = L(x_t, y_t, \epsilon_t).$$

Computable general equilibrium models apply recursive dynamic models which generate a sequence of static equilibria following an update of relevant stock variables, endowments,

[2] These functions are vector mappings from $\mathbb{R}^m_+$ to $\mathbb{R}^n_+$, but for notational considerations, we relate these functions directly to the diagonal matrices D^i and Φ^i.

[3] We write a uniform upper bound for notational convenience. In applications, we would allow for differentiated bounds $0 \leq d^i_k \leq \bar{d}^i_k$ and $0 \leq d^j_k \leq \bar{d}^j_k$ and $0 \leq \phi^i_k \leq \bar{\phi}^i_k$.

and productivity parameters over time. In REED, the static general equilibrium in a certain period has an impact on the environmental variables and the damage parameters in the next period. This update therefore results in a different static general equilibrium in the next period. The following result establishes the existence of the REED equilibrium.

Proposition 12 *There exists a* REED $\{(x_t, y_t, p_t, \varepsilon_t)\}_{t\in\mathbb{N}}$ *given any initial environmental stock* ε_0*.*

Proof. For each period $t \in \mathbb{N}$ and given any damage coefficients $D_t^i, \bar{D}_t^i, D_t^j \in [0, d^{\max}]^n$, conditions 1 to 3 of Definition 11 define a standard general equilibrium with these damage coefficients as parameters. Since the production technologies $Y^j(D^j)$ and the utility functions u^i satisfy the standard assumptions for existence, the existence of an (x_t, y_t, p_t) in period t satisfying conditions 1 to 3 of Definition 11, follows straightforward with Kakutani's fixed point theorem. Since L is continuous in x_t, y_t, and ε_t, and the functions F^i, $\bar{F}^i$, and F^j are continuous functions that map to $[0, d^{\max}]^n$, the evolution of equilibria over time is also well-defined. ■

Notice that a REED does not require single peaked-ness of the impulse response functions for long time horizons. This allows for the existence of multiple steady states, like in Brock and de Zeeuw (2002). In the case of convergence, the initial conditions are decisive which of the stable steady states is reached. The issue whether or not a steady state is reached, is of less importance since changes in the economic environment, i.e., unmodeled changes in production technologies and consumer preferences, occur more rapidly over time than changes in ecological processes.

When applying this equilibrium concept in environmental policy analysis, we set all damages equal to unity in a benchmark equilibrium. The approach to add environmental damage into the general equilibrium model through the use of damage coefficients then allows us to assess the cost of environmental damage on the economy. This approach is taken in Kemfert and Kremers (2008), where they introduce damage coefficients into a simple production function of the German apple orchard sector in order to assess the cost of climate change to this area. Environmental damage or environmental cost are given by the willingness-to-pay of the consumers when comparing REED's first period's equilibrium with unit damage coefficients with a REED's later-period equilibrium where damages are included leading to non-unit damage coefficients. Kemfert and Kremers (2008) only consider a specific production sector, hence the environmental damages are given by the change in the sector's profits.

Recursive dynamic models are calibrated in such a way that they reproduce certain scenarios. The definition of REED allows for defining a scenario in the environmental stock

ε_t over time t, for example in climate change, a scenario referring to the development of global mean temperature over time t. Alternatively, it can also be applied to calibrate the model such as to reproduce the IPCC emission scenarios. Using the damage functions, this translates into changes in productivity and endowments over time t.

The recursive dynamic model can be regarded as a special case of a temporary equilibrium, see Ginsburgh and Keyzer (1997) for a survey. In the case of REED, expectations about future environmental stock variables and future environmental damages would supplement the more traditional expectations about future prices. Without going into details, there are several ways to obtain REED from such modified temporary equilibria. Assuming myopic behavior by all the agents is the simplest of such models. Alternatively, the model can accommodate intertemporal optimisation by agents if individual decisions have an incremental impact on future environmental stocks. Furthermore, these agents have naive expectations with respect to the future evolution of stock variables, i.e. for all $\tau > t$ the agents' expectations about ε_τ equal ε_t. Alternatively, we may assume inter-temporal optimisation by the agents and their awareness of climate change but naive expectations about future damages, i.e. for all $\tau > t$, all agents hold expectations $D^i_\tau = D^i_t$, $\Phi^i_\tau = \Phi^i_t$ and $D^j_\tau = D^j_t$. Similarly, we may assume inter-temporal optimisation by the agents and their awareness of climate change but naive expectations about future price developments due to climate change, i.e. for all $\tau > t$, all agents hold expectations $p^*_\tau = p^*_t$.

6 Applications

In order to illustrate the proposed approach of damage coefficients, we derive the REED for some simplified computable general equilibrium models with environmental damage.

6.1 The cost of climate change in a pure exchange economy

Consider a pure exchange economy without production with two consumers indexed with $i = 1, 2$, and two goods indexed with $k = 1, 2$. The market prices p_i of each good i are collected into a vector $p \in \mathbb{R}^2_+$. An allocation x consists of a pair (x^1, x^2) where $x^i \in \mathbb{R}^2_+$ denotes the consumption bundle of consumer i. The environmental damage coefficients with respect to consumption are included in a (2×2)-dimensional diagonal non-negative matrix D^i with the k-th diagonal element d^i_k. The consumption of $x^i \in \mathbb{R}^2_+$ provides consumer i with a utility $u^i(Dx^i) = \sqrt{(d^i_1 x^i_1) \cdot (d^i_2 x^i_2)}$. Consumer i only holds a single unit of commodity $k = i$ as his endowment, i.e. $\omega^1 = (1, 0)$ and $\omega^2 = (0, 1)$. So, total endowments in the presence of environmental damage are $\Phi^1\omega^1 + \Phi^2\omega^2 = \left(\phi^1_1, \phi^2_2\right)$. Market income for consumer i equals $m^i = p^\top \omega^i = p_i \phi^i_i$. Solving utility maximisation problem (2) with this Cobb-

Douglas specification of the utility function and the initial endowments provide the following expression for the demand for good k by consumer i, $z_k^i(p;\Phi^i) = \frac{p_i \phi_i^i}{2p_k}$. Notice that the demand for each good is independent of any damage impact on the consumer's utility due to the property of constant budget shares.

We first solve for a static general equilibrium in the economy, given any damage vector (ϕ_1^1, ϕ_2^2), by computing prices p_1^* and p_2^* such that

$$\frac{\phi_1^1 p_1^*}{2p_1^*} + \frac{\phi_2^2 p_2^*}{2p_1^*} = \phi_1^1, \tag{10}$$

and assuming that good 2 is the numeraire good, hence $p_2^* = 1$. Solving this equation results into the equilibrium price $p_1^* = \phi_2^2/\phi_1^1$ for good 1. Then, the equilibrium demand for good k equals $z_k^i(p^*;\Phi^i, D^i) = \frac{1}{2}\phi_k^k$ and indirect utility of consumer i, $v^i(p^*;\Phi^i, D^i) = \frac{1}{2}\sqrt{(\phi_1^1 d_1^i)(\phi_2^2 d_2^i)}$. Finally, consumer i's equilibrium market income $m^{*i} = \phi_2^2$.

Let us consider the economic costs of climate change as environmental damage, where we can think of changes in productivity, via the d^i damage coefficients, or in resources or endowments, via the ϕ^i damage coefficients. We define the first-period of REED as the one-period benchmark equilibrium with unit damage coefficients and compare it with an alternative, climate-change included, REED equilibrium with non-unit damage coefficients in some later period. We can then compute the cost of climate change on the economy by considering the Equivalent Variation (EV). This is a measure of the income compensation that should be given to consumer i in order for him to be as well of in the climate equilibrium as in the benchmark equilibrium. This implies that we should determine EV^i such that

$$v^i\left(p^B, m_B^i + \mathrm{EV}^i; I, I\right) = v^i\left(p^C, m_C^i; \Phi^i, D^i\right),$$

where p^B (m_B^i) denotes the price vector (i's market income) in the benchmark equilibrium and p^C (m_C^i) denotes the price vector (i's market income) in the climate equilibrium. This interpretation of EV^i to compute the cost of environmental damage on the welfare of each consumer in the economy coincides with the environmental cost or the willingness-to-pay concepts in cost-benefit analyses in Fankhauser et al. (1997).

As in e.g. Mas-Colell et al. (1995), we can compute the cost of climate change EV^i to each consumer i in terms of the expenditure function, $-$ $E^i(p,u) = 2u\sqrt{p_1 p_2}$ for the applied Cobb-Douglas functional form $-$, as

$$\mathrm{EV}^i = E^i\left(p^B, v^i\left(p^C, m_C^i; \Phi^i, D^i\right)\right) - m_B^i,$$

Hence, we obtain $\mathrm{EV}^i = \sqrt{(\phi_1^1 d_1^i)(\phi_2^2 d_2^i)} - 1$. Note that consumer i bears costs $|\mathrm{EV}^i|$ of climate change whenever $\mathrm{EV}^i < 0$, which is equivalent to $\phi_1^1 d_1^i \phi_2^2 d_2^i \leq 1$. So, each damage coefficient has a linear effect on EV^i, but simultaneous changes in damage coefficients have a nonlinear effect. The condition implies that consumer i suffers costs of climate change if the negative effects of climate change, i.e., damage coefficients less than 1, outweigh the positive effects of climate change. For the whole economy, the total costs are equal to

$$\mathrm{EV} = \mathrm{EV}^1 + \mathrm{EV}^2 = \sqrt{\phi_1^1 \phi_2^2}\left(\sqrt{d_1^1 d_2^1} + \sqrt{d_1^2 d_2^2}\right) - 2.$$

Note the possibility that, say, consumer 1 benefits from climate change while the whole economy suffers from climate change.

The values of the damage coefficients, Φ^i and Δ^i, are determined from the values of certain economic variables, using a damage function. In climate-change policy research, greenhouse gas emissions are associated with the demand for fossil fuels in the production households. Similarly, we can associate a coefficient $\gamma_k > 0$ to the demand x_k^i of good k by each consumer i to denote associated greenhouse gas emissions. This environmental degradation can then be described by the impulse response function L on the environmental stock variable ε, which may represent the atmospheric stock of greenhouse gas concentrations,

$$L(x, \varepsilon) = \delta\varepsilon + \sum_{k=1,2} \gamma_k \left(x_k^1 + x_k^2\right),$$

where $\delta \in (0, 1)$ denotes natural decay. In the current equilibrium, p^*, the environmental stock variable becomes $\delta\varepsilon + \gamma_1\phi_1^1 + \gamma_2\phi_2^2$. This results into a steady-state environment equilibrium stock $\varepsilon^* = \left(\frac{1}{1-\delta}\right)\left[\gamma_1\phi_1^1 + \gamma_2\phi_2^2\right]$. For the ease of presentation, assume there are only damage effects associated with the level ε on consumer 2's income parameter ϕ_2^2, i.e. $d_k^i = 1$ and $\phi_1^1 = 1$. We then postulate the following damage function $\bar{F}_2^2$ relating the value ϕ_2^2 of the income damage coefficient to the environmental stock variable ϵ as

$$\phi_2^2 = \bar{F}_2^2(\varepsilon) = e^{-\alpha(\ln\varepsilon - \beta)}, \text{ where } \alpha, \beta > 0.$$

Notice that $\bar{F}_2^2(0) = e^{\alpha\beta} > 1$, $\bar{F}_2^2(\beta) = 1$, and $\lim_{\varepsilon\to\infty} \bar{F}_2^2(\varepsilon) = 0$. This damage function $\bar{F}_2^2$ implicitly incorporates the complete relationship between atmospheric concentrations of greenhouse gases and climate related damages that is usually modelled within the far more elaborate climate models. Here, according to such models, the economy's emissions are added to the concentrations in the atmosphere, $-$ air, land, or oceans $-$, while a second sub-model translates these concentrations into changes in climate variables such as mean global temperature, radiation, and precipitation. The link between these variables' values and climate related damages to productivity and resources is also taken up into our damage functions. Notice that other types of damage functions, such as Tol (2002a) and Tol (2002b), only relate changes in these climate variables and cost as a percentage of GDP.

In the steady-state atmospheric greenhouse gas concentrations ε^*, climate change damages to income adds up to $\phi_2^{2*} = Ae^{\alpha\beta}$ where $A = \frac{1-\delta}{1-\gamma_2}$. These damages cause the equilibrium price level to be $p_1^* = Ae^{\alpha\beta}$, $p_2^* = 1$, at which consumer 1 obtains a utility $v^1 = \frac{1}{2}\sqrt{Ae^{\alpha\beta}}$ from consuming $z^{1*} = \left(\frac{1}{2}, \frac{1}{2}Ae^{\alpha\beta}\right)$, and consumer 2 obtains a utility $v^2 = v^1$ from consuming $z^{2*} = z^{1*}$. Each consumer's steady-state cost of climate change then equals $\sqrt{Ae^{\alpha\beta}} - 1 < 0$.

6.2 The cost of "clean technology" in a "Robinson Crusoe economy"

Consider an economy with one consumer, one producer, and two goods, known in the literature as a "Robinson Crusoe economy". Good 1 is the consumption good and good 2 is the production factor, say capital or land. We suppress superscripts indicating consumer $i = 1$ and firm $j = 1$. An allocation (x, y) consists of a consumption bundle $x = (x_1, x_2)$ and the production plan $y = (y_1, y_2)$.

From consuming $x \in \mathbb{R}_+^2$, the consumer obtains a utility $u(Dx) = d_1 x_1$. The consumer only holds the production factor as initial endowments, hence his initial endowment is $(0, \phi_2\omega_2)$, assuming the existence of environmental damage on his endowments with a factor ϕ_2.

Since market income $m = p_2\phi_2\omega_2 + \Pi$, Π denoting the firm's profits which are here assumed to accrue to the consumer, the consumer's demand function $z(p, m; D) = \left(\frac{m}{p_1}, 0\right)$. The firm produces the consumer good from the production factor using a decreasing returns to scale production function $y_1 = 2d_0\sqrt{d_2 x_2}$. Profit maximisation leads to $y(p; D) = \left(2d_0^2 d_2\left(\frac{p_1}{p_2}\right), -d_0^2 d_2\left(\frac{p_1}{p_2}\right)^2\right)$ and $\Pi(p; D) = py(p, D) = d_0^2 d_2\left(\frac{p_1^2}{p_2}\right) > 0$. Substitution of the firm's profit into the consumer's market income yields $m = p_2\phi_2\omega_2 + d_0^2 d_2\left(\frac{p_1^2}{p_2}\right)$ and into the consumer's demand function for good 1 gives $z_1(p; \Phi, D) = \left(\frac{p_2\phi_2\omega_2}{p_1}\right) + d_0^2 d_2\left(\frac{p_1}{p_2}\right)$.

We first solve for the equilibrium in the economy by computing prices p_1^* of the consumption good and p_2^* of the production factor such that the market for good 1 clears

$$\left(\frac{p_2^*}{p_1^*}\right)\phi_2\omega_2 = d_0^2 d_2\left(\frac{p_1^*}{p_2^*}\right),$$

and the production factor is the numeraire, hence $p_2^* = 1$. We obtain $p_1^* = \frac{1}{d_0}\sqrt{\frac{\phi_2\omega_2}{d_2}}$. The utility maximising bundle for the consumer equals $z(p^*; \Phi, D) = \left(2d_0\sqrt{d_2\phi_2\omega_2}, 0\right)$ with utility $v(p^*; \Phi, D) = 2d_0 d_1\sqrt{d_2\phi_2\omega_2}$, and the profit maximising bundle $y(p^*; D) = (z_1(p^*; \Phi, D), -\phi_2\omega_2)$ with profit $\Pi(p^*; D) = \phi_2\omega_2$. Finally, $m^* = 2\phi_2\omega_2$.

Like in the previous example, we consider the climate as the environment and take ε again to be the atmospheric stock of greenhouse gases. The cost of climate change in this economy can be computed as the equivalent variation EV for the consumer such that

$$\mathrm{EV} = E\left(p^B, v\left(p^C, m^C; \Phi, D\right)\right) - m^B.$$

Hence, we can compute the consumer's willingness-to-pay for clean technology in terms of the expenditure function, $-$ $E(p,u) = up_1$ for the applied utility functional form $-$, as

$$\mathrm{EV} = 2\left(d_0 d_1 \sqrt{d_2 \phi_2} - 1\right)\omega_2.$$

Note that the nonlinear condition that determines whether the consumer suffers from climate change, i.e., $\mathrm{EV} < 0$, is given by $d_0 d_1 \sqrt{d_2 \phi_2} < 1$. This shows that, ceteris paribus, some damage coefficients have a linear effect on EV, while others have a nonlinear effect. Definitely, simultaneous changes in damage coefficients have a nonlinear effect. Note that the condition can be decomposed into the effect $d_0\sqrt{d_2}$ on the production side and $d_1\sqrt{\phi_2}$ for the consumption side of the economy. For specific parameter values, it is possible that the impact to one side of the economy is favourable, but such positive effects can still be offset by a larger negative impact on the other side.

We associate environmental pollution in the form of greenhouse gas emissions, with the input of good 2 into the production process using a parameter $\gamma > 0$. Such pollution results into a degradation on the environmental stock ε through the impulse response function L given by

$$L(y, \varepsilon) = \delta\varepsilon - \gamma y_2,$$

where $\delta \in (0,1)$ denotes natural decay. In the current equilibrium (p^*, y^*, z^*), this environmental stock becomes $\delta\varepsilon + \gamma\phi_2\omega_2$, providing a stead-state solution $\varepsilon^* = \left(\frac{\gamma\phi_2}{1-\delta}\right)\omega_2$.

For the ease of presentation, we now only consider environmental damage through the damage parameter d_0, making $d_1 = d_2 = \phi_2 = 1$. We then postulate the following damage function F relating damages d_0 with the environmental stock parameter ε through

$$d_0 = F(\varepsilon) = \alpha \ln \varepsilon + \beta, \ \alpha, \beta > 0.$$

Hence, increases in d_0 are caused by increases in the environmental stock. In the steady-state $d_0^* = A + \alpha \ln \omega_2$, with $A = \beta + \alpha \ln\left(\frac{\gamma}{1-\delta}\right)$. This damage implies equilibrium prices $p_1^* = \frac{\sqrt{\omega_2}}{A+\alpha\ln\omega_2}$, $p_2^* = 1$, equilibrium production $y^* = \left(2(A + \alpha\ln\omega_2)\sqrt{\omega_2}, -\omega_2\right)$ and equilibrium consumption vector $z^* = (y_1^*, 0)$. The consumer obtains a utility from z^* of $u^* = 2(A + \alpha\ln\omega_2)\sqrt{\omega_2}$ while the producer uses the production bundle y^* to obtain profits $\Pi^* = \omega_2$. The cost of climate change is then given by the willingness-to-pay of the consumer $2(A + \alpha\ln\omega_2 - 1)\omega_2$.

Let us suppose that the current 'dirty' production technology, hence with a parameter $\gamma_d > 0$ instead of γ, is replaced with a cleaner technology, $-$ "dirty" referring to a technology emitting relatively more emissions than a comparable "cleaner" technology $-$, hence with a parameter $0 < \gamma_c < \gamma_d$. Associated with these parameters is a parameter A_c respectively A_d such that $A_c < A_d$. The environmental benefit of replacing dirty with clean technology results into a societal's cost saving of $2(A_d - A_c)\omega_2 > 0$.

7 Concluding Remarks

The impact assessment of implementing environmental policies on the world and local economies is often based on the integration of models from various scientific disciplines related to the subject, which is currently the case in assessing various climate policies on emission abatement following the Kyoto Protocol. In most cases, the economic computable general equilibrium model is integrated with climate models and ecological models. This paper deals with the methodology of the economists' perspective on such integration: How to model environmental damages that change productivity of inputs and endowments. We resolve this issue by introducing so-called damage coefficients.

The consequences of damage coefficients on the standard microeconomic model applied in many economic climate studies is elaborated in our study and the extensions of consumer and producer behaviour with damage coefficients are derived. Damage coefficients form the link within the economic model to the environment modelled using so-called damage functions. In order to study this dynamic interaction, we enrich a popular recursive dynamic equilibrium concept, often used in policy-oriented research as a convenient way to model dynamics over time, with the inclusion of these damage functions and with the development of the environmental stock over time. The resulting equilibrium definition is what we call a recursive equilibrium with environmental damage, shortly REED. We show the existence of a REED under the usual economic assumptions. We also demonstrate the applicability of the concept in two popular applications in climate policy impact studies, namely on the determination of the cost of environmental damage to an economy and on the cost of introducing cleaner technology. The impact of damage coefficients on the economy is nonlinear.

An interesting angle for future research is to extend the concept of REED, similar as the extension of temporary equilibrium in e.g. Radner (1972), to a distribution of states and associated probabilities at some future date, say ten, fifty or one-hundred years from today. This idea extends the applicability of REED to the analysis of different climate scenarios with associated probability assessments, which would be relevant to study the economic impact of the rise in sea level.

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