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Pieter A. Gautier

VU University Amsterdam, Tinbergen Institute, and CEPR.

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Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

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Coordination frictions and the financial crisis*

Pieter A. Gautier[†]
Vrije Universiteit Amsterdam, Tinbergen Institute, CEPR

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Abstract

In this note I argue that the desirability of fiscal policy in response to the current crisis depends on whether one views the current crisis as a temporary deviation from a unique equilibrium or as a bad equilibrium out of multiple equilibria. The paper presents a simple Diamond (1982) type of model where firms must find an (investment) bank to finance their projects and the investment banks sell risky assets to get capital from investors. Due to coordination frictions, the economy can get stuck in an inefficient low-trade equilibrium. Finally, I briefly discuss some of the policies that have recently been put forward to stimulate the economy in the context of this model.

*I am grateful for comments received from Marloes Lammers and Arjen Siegmann.
[†]Department of Economics, Vrije Universiteit Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, <pgautier@feweb.vu.nl>.

1 Introduction

When entrepreneurs must make costs ex ante, i.e. write a business proposal, buy a patent etc. and the returns to their investment depend on the willingness of banks to provide credit, which on turn depends on the willingness of investors to buy risky assets, the economy can get stuck in an inefficient equilibrium. The fact that the total stock market losses are many times larger than the estimated 2.2 trillion potential deterioration in U.S.- originated credit assets held by banks and others is consistent with the view that there are multiple Pareto-rankable equilibria of the Diamond (1982) type.

In August 2007, the first signs came that housing market was overvalued in the US. Before this period, monetary policy in the US focussed on providing more liquidity and keeping interest rates low rather than at the quality and transparency of the bank's balance sheets. The low interest rates and the salary schemes in the banking sector stimulated the development of risky assets like the complex mortgage-backed securities. The interconnection of the banking system made the system more robust against small shocks but less robust to the risks of "tail" events, see Acemoglu (2009) and Hartmann, Straetmans and de Vries (2004). Since the house-price-and-related-asset bubble has bursted, there has been a flight to quality in the financial sector. Lucas (2008) describes this as: "Everyone wants to get into government-issued and government-insured assets, for reasons of both liquidity and safety". Caballero and Kurlat (2008) argue that while the US as a whole is regarded to be save (and this still leads to net capital inflows), all other forms of funding dried up. Similar arguments hold for savers, see Diamond and Dybvig (1983). Flight to quality is not specific for the current crisis but has been reported to take place in many cyclical downturns.[1] Reinhart and Rogoff (2008, 2009) give evidence that the current crisis shows a lot of similarities with past crises around the entire world.

In the model, I study the effect of this flight to quality on investment behavior and credit supply. I show that for certain configurations, flight-to quality can destroy an efficient trade equilibrium and that in that case, government intervention is required to prevent the economy from getting stuck in an inefficient no-trade equilibrium. I also show that because of macro-economic complementarities the social returns to investing in risky assets exceeds the private returns.

Besides Diamond (1982) the model is related to Silviera and Wright (2006) who consider a search model where entrepreneurs search for venture capital. In their model there is a more active role for the capital supplier in judging projects but they do not consider the coordination frictions which I focus on in this paper. Cooper and John (1988) give a nice overview of other sources of coordinations failures and sources of multiple equilibria in models with macroeconomic complementarities.

The paper is organized as follows. Section 2 presents the model and gives a numerical example and section 3 discusses some of the recent policy proposals in the light of this model.

[1] See e.g. Bernanke, Gertler and Gilchrist (1996) and Kashyap, Stein and Wilcox (1993).

2 The model

The purpose of this section is to illustrate how after a flight-to quality in the financial sector, the economy can get stuck in an inefficient equilibrium and that there is potential scope for government intervention. For this goal I choose to write down an extremely stripped down model where I leave many factors out that are important to understand the current crisis, like asymmetric information, deregulation, heterogeneity in projects etc. thereby sacrificing realism for simplicity. This enables me to isolate the effects of coordination frictions.

2.1 Assumptions

Suppose firms find projects (or receive ideas) at rate μ per unit of time where time is continuous. Once the firm has a project, it can start it at a cost c. Think of c as the cost of writing a business proposal to the bank, getting a patent etc.. Before the project can be implemented and made profitable, the firm must find a bank with sufficient resources to finance the project. Normalize the total number of firms to 1 of which L have a project and $(1-L)$ have no project. Banks are very simple in this world. In total there are N banks, B of them have sufficient funds and $N-B$ is the number of banks that are looking for investors who want to buy a risky asset (the investors or investors). This goes as follows, the bank sells a risky asset in exchange for an amount of capital z. Assume that there are S investors who are willing to buy a risky asset. The current flight to quality can be interpreted as a reduction in S. The total number of contacts between banks and investors is given by $\lambda(N-B)S$, so investors who want to buy risky assets meet banks at rate $\lambda(N-B)$ and the banks meet the investors at rate λS. Each investor pays z' for the risky asset and in return she receives $z > z'$ if the bank finds a firm with a good project. So the asset is risky in the sense that if the bank finds no firm with a project, the investor makes a loss of $-z'$. So banks act as intermediaries between firms that need money and investors who buy risky assets. Roughly speaking, this is what investment banks did before the crisis. Assume that investors can hold at most one risky asset. A firm and a bank with sufficient resources who have contacted each other jointly implement the project and receive in total $2u+z$. I assume that this is shared, through symmetric Nash bargaining, i.e. both the firm and the bank receive u and the buyer of the risky asset receives her promised return z. Assume that firms can undertake only one project at a time. Further, assume that banks cannot do anything else with z' then finance the projects of firms and that there is no storage possible, i.e. banks can hold funds for only one investment project at a time. The idea is that banks need sufficient capital before they can lend to the firms. This simplification allows me to ignore the distribution of assets over banks. The total number of contacts between banks and firms is αLB per unit of time where L is the total number of firms with projects and B is the number of banks with funds (i.e. which sold a risky asset). Agents discount the future at rate r. $N, S, \alpha, \mu, \lambda, c, r, u, z'$ and z are all exogenous (although for the investors, there is a participation constraint that expected payoffs must be non-negative).[2] Finally, I only consider symmetric strategies and focus on the steady state where B, L are constant over time.

2.2 Characterization

Let V_{NP} and V_P be the values for firms of being in states NP (having no project and looking for one) and P (having a project and looking for a bank to finance it). Then,

$$rV_{NP} = \mu \max\left[V_P - V_{NP} - c, 0\right]. \tag{1}$$

At rate μ the firms find projects. When they find one, they gain the difference in value of state P and NP, and pay a cost c if they believe the project to be profitable. If they expect the investment to not be profitable, they do not invest and receive 0. The value for a firm of having an implementable project is,

$$rV_P = \alpha B\left(u + V_{NP} - V_P\right) \tag{2}$$

At rate αB an individual firm meets a bank with sufficient funds. In that case, the firm receives u and switches to state NP.

Denote the value of a bank without funds by V_{NF} and with funds by V_F. Then,

$$rV_{NF} = \lambda S \max[V_F - V_{NF}, 0]. \tag{3}$$

Banks meet investors who are willing to buy risky assets at rate λS and switch from state NF to state F if this occurs. Since banks cannot do anything else with z' than finance investments, there is no direct value of owning z'. The value of having funds for a bank is,

$$rV_F = \alpha L\left(u + V_{NF} - V_F\right). \tag{4}$$

Banks with funds meet firms at rate αL and they receive u if this occurs. The asset value for an investor who wants to buy a risky asset is:

$$rV_{NR} = \lambda(N - B)\left[V_R - V_{NR} - z'\right]. \tag{5}$$

At rate $\lambda(N - B)$ she meets a bank without funds, switches to state R and buys the risky asset for z' from the bank. When she owes a risky asset the flow value of her state is:

$$rV_R = \alpha L\left[z - (V_R - V_{NR})\right]. \tag{6}$$

At rate αL her bank meets a firm with a project and it receives z and switches to state NR.

There are two steady state conditions; one for the banks, and one for the firms. First, for the banks,

$$\lambda S\left(N - B\right) I_1 = \alpha BL, \tag{7}$$

[2] We can think of the size of agents who own or want to own a risky asset as M where S agents are looking for a bank to buy the risky asset from and $M - S$ agents own a risky asset. All relations below hold for any value of M, S.

where I_1 is an indicator variable which has value 1 if it is profitable for the investors to exchange a risky asset for cash with the bank and 0 otherwise. The inflow of banks with resources is equal to the number of banks without resources that meet and sell a risky asset in exchange for capital: $(\lambda S\,(N-B))$. The outflow of banks with capital is equal to the number of contacts between banks and firms with projects (αLB). The steady state condition for firms is,

$$\mu\,(1-L)\cdot I_2 = \alpha BL, \tag{8}$$

where I_2 is an indicator variable which has value 1 if firms who receive an idea/project pay the cost c and start to look for a bank and 0 otherwise. So the inflow of firms with feasible projects is the number of firms who receive a project and pay the setup cost $(\mu\,(1-L)\cdot I_2)$ and the outflow is equal to the number of firms with projects who meet a bank with sufficient funds (αBL).

Finally, for each value of S, (7) holds. The number of owners of risky assets increase when an investor buys a risky asset from the bank which occurs at rate $(\lambda S\,(N-B)\,I_1)$ while each time a bank meets a firm, one holder of a risky asset receives her payoff, looses her risky asset and starts to look for a new one. This happens at rate αBL. If the number of agents who either owns or who is willing to buy a risky asset is M then we have $(M-S)$ owners of risky assets and S investors looking for a risky asset if the participation constraint from Proposition 2 is fulfilled. Since (7) must hold for any value of S, M is irrelevant.

Depending on parameter values there are at most two equilibria.

Proposition 1. There always exists (for any $c, z > 0$) an equilibrium where firms do not start projects and no investors want to buy a risky asset $(B = L = 0)$.

Proof. If firms do not accept projects, $L = BL = 0$ and banks meet no firms with projects so the payoff of a risky asset is $-z$ and therefore investors do not participate, $S = 0$.

Besides the no trade equilibrium there is also an equilibrium where trade takes place.

Proposition 2. If $u > c\frac{r+\alpha B}{\alpha B}$ and $z > z'\frac{(r+\alpha L)}{\alpha L}$ there exists one equilibrium where $B, L, S > 0$, i.e. investors buy risky assets, firms work out their plans and banks finance the investment projects.

Proof. See the appendix

These conditions follow from $V_P - V_{NP} > c$,and $V_{NR} > 0$.

The equilibrium can be characterized as follows. First, we get from (8),

$$L = \frac{\mu}{(\mu+\alpha B)}. \tag{9}$$

Substituting this in (7) gives:

$$\lambda S\,(N-B)\,(\alpha B+\mu) - \alpha B\mu = 0,$$

which has one positive root for B.

$$B = \frac{-\left(\lambda S\left(\alpha N-\mu\right)-\alpha\mu\right)-\sqrt{\left(\left(\lambda S\left(\alpha N-\mu\right)-\alpha\mu\right)\right)^{2}+4\alpha\mu\lambda^{2}S^{2}N}}{-2\alpha\lambda S}. \tag{10}$$

For (9) and (10) to be an equilibrium, we must check whether the conditions in Proposition 2 are fulfilled.

2.3 Welfare

Steady state welfare is given by the weighted value of all states (where the weights are determined by the number of investors in each state).

$$r\Omega \equiv BrV_F + (N-B)rV_{NF} + LrV_P + (1-L)rV_{NP} + SrV_{NR} + (M-S)rV_R \tag{11}$$

Obviously, the no trade equilibrium with $B = L = S = 0$ generates lower welfare than the trade equilibrium. If S would be determined by free entry there would be too little entry because of the quadratic contact technology and the fact that part of the returns to the investments go to the banks and firms.

2.4 A simple numerical example

This section illustrates that (i) for certain configurations, a flight to quality eliminates the efficient trade equilibrium, and (ii) that even if the trade equilibrium is not eliminated, social welfare increases more than the total expected payoffs that firms of risky assets receive when S goes from 0.5 to 1 (keeping $M-S$ constant and equal to 1). Details are in Appendix 3.

First, let

$$\alpha = 1, \lambda = 1, \mu = 1, S = 1, N = 0.5, r = 0.05.$$

Then B and L can be calculated from (9) and (10) which gives: $B = 0.28$ and $L = 0.78$. Then, using Proposition 2, a trade equilibrium exists if $u > 1.27c$ and $z > 1.18z'$. Next, we can think of the flight to quality as a reduction of S, suppose to 0.5. Then, $B = 0.19$ and $L = 0.84$. This equilibrium exists for $u > 1.27c$ and $z > 1.18z'$. So if $u \in < 1.18c, 1.27c >$, and the economy was in the trade equilibrium with $S = 1$, the flight to quality destroys the equilibrium and the no-trade equilibrium is the unique equilibrium.

A less dark scenario is that $u > 1.27c$ and $z > 1.18z'$, suppose $u = z = 2$ and $c = z' = 1$. Then, the flight to quality does not eliminate the trade equilibrium (although it could still push the economy in the no-trade equilibrium). However, in that case, the total increase in welfare when S would go back to 1 exceeds the total payoffs that go to the firms of the risky assets. $r\Omega(1) - r\Omega(0.5) = 0.08 > rV_{NR}(1) - 0.5 \cdot rV_{NR}(0.5) = 0.04$. This is due to the macroeconomic complementarities, see Cooper and John (1988).

In this very simple setting there are only two equilibria but it is easy to imagine that if firms are heterogeneous in terms of the profitability of their projects or if the investors who buy the risky asset pay a fixed entry cost (i.e. the cost of studying the profitability of projects) there can be more than two Pareto rankable equilibria. I also assumed that the investors bear all the risks but the model can be adjusted such that the banks bear some risks, i.e. must pay back z' after some time irrespective of whether they found a firm with a business plan or not. Banks will then only invest if they expect that there are enough firms with projects and similarly, firms are only willing to develop their projects if they expect that there are enough banks with funds. Finally, it is straightforward to make the risky asset more risky by letting some projects fail at rate δ.

3 Discussion

During the credit crisis, banks engaged in deleveraging, which made it more difficult for firms to find banks that were willing to supply loans which as an extreme case can be thought of as a reduction in α (it becomes harder to find a bank with sufficient capital). As mentioned before, the flight to quality can be interpreted as a reduction in S. Because of the macro-economic complementarities, the flight to quality affected the beliefs of the firms which affected the beliefs of the banks and vice versa. In normal times, the FED would buy treasury bills in the federal-funds market to reduce the Federal funds rate but when this rate is essentially zero, the difference between treasury bills and cash vanishes. Lucas (2008) argues that the FED could still satisfy the demand for quality by using reserves to buy other securities. In terms of the model above, if the FED buys securities, it increases αB. Without enough risk free securities, the desire to hold quality securities just reduces the prices of other assets.

Is this is enough? Cochrane (2009) argues that sooner or later, firms and banks will realize that the 2% returns to treasury bills is less attractive than the 9% returns for corporate bonds. This makes sense if there is a unique market equilibrium and if we are temporarily in a disequilibrium However, in the presence of coordination frictions, getting out of a low activity equilibrium requires a lot of coordination.

Caballero and Kurlat (2008) argue that in order to stop the extreme risk aversion, the government should purchase some of the bank's securities by an auction. This will signal to firms who play a waiting strategy (for prices to fall further) that prices will stop falling and eliminate the gains from speculative waiting. Finally, in the model, the social value of investing in risky assets is larger than the private value because of macroeconomic complementarities. This suggests that we should, as Alesina and Zingales (2009) argue, subsidize risk taking, for example by decreasing or eliminating the capital gain and dividend taxes.

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Appendix

A Proof of Proposition 2

We must derive under which condition,

$$V_P - V_{NP} = \frac{\alpha B u + \mu c}{(r + \alpha B + \mu)} > c$$

or,

$$u > c\frac{r+\alpha B}{\alpha B}$$

The participation constraint for the investors who buy risky assets is:

$$rV_{NR} = \lambda(N-B)\left[V_R - V_{NR} - z'\right] > 0.$$

(6)-(5) gives

$$(V_R - V_{NR}) = \frac{\alpha L z + \lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))}.$$

Plugging this in the participation constraint gives:

$$\begin{aligned} rV_{NR} &= \lambda(N-B)\left[\frac{\alpha L z + \lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))} - z'\right] > 0. \\ z &> \frac{z'\left(r+\alpha L\right)}{\alpha L}. \end{aligned}$$

In the trade equilibrium, (2)-(1) yields:

$$(V_P - V_{NP}) = \frac{\alpha B u + \mu c}{(r+\alpha B+\mu)}$$

substituting this back in (1) and (2)-(5) yields:

$$\begin{aligned} rV_{NP} &= \mu\left(\frac{\alpha B u + \mu c}{(r+\alpha B+\mu)} - c\right) \\ rV_P &= \alpha B\left(u - \frac{\alpha B u + \mu c}{(r+\alpha B+\mu)}\right) \end{aligned}$$

Similarly (4)-(3) yields:

$$(V_F - V_{NF}) = \frac{\alpha L u}{r+\alpha L+\lambda S}$$

substituting this back in (3) and (4) gives,

$$\begin{aligned} rV_{NF} &= \lambda S\left(\frac{\alpha L u}{r+\alpha L+\lambda S}\right) \\ rV_F &= \alpha L\left(u - \frac{\alpha L u}{r+\alpha L+\lambda S}\right) \\ rV_F &= \alpha L u\left(\frac{r+\lambda S}{r+\alpha L+\lambda S}\right). \end{aligned}$$

Plugging (8) in (7) yields:

$$\begin{aligned} \lambda S\left(N-B\right)\left(\alpha B+\mu\right) - \alpha B\mu &= 0 \\ -\alpha\lambda S B^2 + \left(\lambda S\left(\alpha N-\mu\right) - \alpha\mu\right)B + \lambda\mu S N &= 0 \end{aligned}$$

There is one positive solution for B.

$$B = \frac{-\left(\lambda S\left(\alpha N-\mu\right)-\alpha\mu\right) - \sqrt{\left(\left(\lambda S\left(\alpha N-\mu\right)-\alpha\mu\right)\right)^2 + 4\alpha\mu\lambda^2 S^2 N}}{-2\alpha\lambda S}$$

B Numerical example (not to be included in the paper)

Since, the inflow of investors does not depend on S (only on the number of contacts between banks with funds and firms with projects), $M-S$ can take any value and I decide to set it equal to 1. So we can think of the flight to quality either as a drop in M or a drop in S (as I do here).

$$\alpha = 1, \lambda = 1, \mu = 1, S = 1, M - S = 1, N = 0.5, r = 0.05.$$

For those values, $B = 0.28078$ and $L = 0.780\,77$. If the number of investors that is willing to take risks (S) drops to 0.5, we get $B = 0.18614$ and $L = 0.84307$

First look at the constraints in Proposition 2 for a trade equilibrium for each of the values of B. First, the firms want to trade for the various value of B if:

$$\begin{aligned}
\frac{1}{B+0.05}\left(Bu - 0.05c - Bc\right) &> 0 \\
S &= 1 : u > c\frac{(1+20\cdot 0.28078)}{20\cdot 0.28078} = 1.178\,1c \\
S &= 0.5 : u > c\frac{(1+20\cdot 0.18614)}{20\cdot 0.18614} = 1.268\,6c
\end{aligned}$$

and investors want to buy risky assets if:

$$\begin{aligned}
\frac{\alpha L z + \lambda(N-B)z'}{(r+\alpha L + \lambda(N-B))} &> z' \\
S &= 1 : z > \frac{z'\left(r + \alpha 0.186\,14\right)}{\alpha 0.186\,14} = 1.1921z' \\
S &= 0.5 : z > \frac{z'\left(r + \alpha 0.84307\right)}{\alpha 0.84307} = 1.177\,9z'
\end{aligned}$$

To calculate welfare and the social value of an investor who is willing to buy a risky asset, we use the explicit solutions of the asset equations. Suppose that u and z are sufficiently high that both equilibria exist, i.e.

$$u = 2, c = 1, z = 2, z' = 1, B = 0.28078, L = 0.78077, S = 1$$

For the flows this implies:

$$\lambda S\left(N - B\right) = 0.22 = \alpha B L = \mu\left(1 - L\right).$$

The asset values for $S = 1, B = 0.28078, L = 0.780\,77$ are given first and then the asset values for $S = 0.5, B = 0.186\,14$ and $L = 0.843\,07$ are calculated.

$$\begin{aligned}
rV_{NF} &= \lambda S\left(\frac{\alpha L u}{r + \alpha L + \lambda S}\right) = 0.58671 \\
rV_{F} &= \alpha L u\left(\frac{r + \lambda S}{r + \alpha L + \lambda S}\right) = 0.64538
\end{aligned}$$

$$rV_{NP} = \mu\left(\frac{\alpha Bu+\mu c}{(r+\alpha B+\mu)}-c\right) = 0.17342$$

$$rV_P = \alpha B\left(u-\frac{\alpha Bu+\mu c}{(r+\alpha B+\mu)}\right) = 0.23209$$

$$rV_{NR} = \lambda(N-B)\left(\frac{\alpha Lz+\lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))}-z'\right) = 0.12024$$

$$rV_R = \alpha L\left(z-\frac{\alpha Lz+\lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))}\right)-\frac{\alpha Lz+\lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))} = 0.19767$$

For $S = 0.5, B = 0.186\,14$ and $L = 0.843\,07$:

$$rV_{NF} = \lambda S\left(\frac{\alpha Lu}{r+\alpha L+\lambda S}\right) = 0.66151$$

$$rV_F = \alpha Lu\left(\frac{r+\lambda S}{r+\alpha L+\lambda S}\right) = 0.72767$$

$$rV_{NP} = \mu\left(\frac{\alpha Bu+\mu c}{(r+\alpha B+\mu)}-c\right) = 0.11013$$

$$rV_P = \alpha B\left(u-\frac{\alpha Bu+\mu c}{(r+\alpha B+\mu)}\right) = 0.165\,64$$

$$rV_{NR} = \lambda(N-B)\left(\frac{\alpha Lz+\lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))}-z'\right) = 0.16644$$

$$rV_R = \alpha L\left(z-\frac{\alpha Lz+\lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))}\right)-\frac{\alpha Lz+\lambda(N-B)z'}{(r+\alpha L+\lambda(N-B))} = 0.24296$$

$$r\Omega \equiv BrV_F+(N-B)rV_{NF}+LrV_P+(1-L)rV_{NP}+SrV_{NR}+1\cdot rV_R$$

$$r\Omega(1) \equiv B\cdot 0.64538+(N-B)0.58671+L\cdot 0.23209+(1-L)0.17342+S\cdot 0.12024+1\cdot 0.19767 = 0.88378$$

$$r\Omega_2(0.5) \equiv B\cdot 0.66571+(N-B)0.60519+L\cdot 0.165\,64+(1-L)0.11013+S\cdot 0.16644+1\cdot 0.24296 = 0.796\,97$$