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International Trade with Firm Heterogeneity in Factor Shares

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The short and long–run impact of globalization if firms differ in factor input ratios*

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Abstract

Empirical evidence has shown that exporters are more capital intensive than non–exporters. Based on this evidence, I construct a two–factor general equilibrium model with firm heterogeneity in factors intensities, monopolistic competition, scale economies and international trade. This setting can explain several empirical regularities on international trade, factor market competition, factor relocations and factor returns: (*i*) exporters are more capital intensive than non–exporters, regardless of a country's relative factor endowments; (*ii*) finite supply of capital limits a country's export activities; (*iii*) trade liberalization increases the relative return to capital; (*iv*) new profit opportunities in export markets change the distribution of firms towards the more capital intensive ones. Finally, I extend the setting to endogenous capital accumulation and show that trade liberalization induces economic growth and, in the long–run, benefits all factors in real terms.

JEL classification: F12; L11; O41; D33

Keywords: international trade; economic growth; firm heterogeneity in factor input ratios; factor market competition; income distribution

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1 Introduction

Bernard, Jensen et al. (2007) show for narrowly defined US industries that the capital employment per worker is, on average, 12% higher for exporters, compared to non–exporters. Using Chilean data, Alvarez and Lopez (2005) show that exporters are, on average, 60% more capital intensive than non–exporters. This evidence suggests that firm heterogeneity in factor input ratios is crucial to explain firm selection into export markets.[1]

This paper analyzes how firm heterogeneity in factor input ratios contributes to the 'new' trade theory, in which international trade is due to product differentiation and scale economies. I extend the Krugman (1980) setup by considering (*i*) two factors of production, (*ii*) CES production functions and (*iii*) firm heterogeneity in the factor share parameters of the production function. In equilibrium firms produce with different factor input ratios. This setting rationalizes several empirical findings on international trade, factor market competition, factor relocations and factor returns.

First, this setting can explain why exporters are more capital intensive than non–exporters, regardless of a country's relative factor endowments. I show that exporters are more capital intensive than non–exporters if the effective price of capital is smaller than the one of labor and if exporting is costly. The relationship between effective factor prices and a country's relative factor endowments depends on the firm distribution. If the firm distribution is sufficiently skewed towards the more labor intensive firms, the effective price of capital is smaller than the one of labor, also if the country's labor endowment exceeds its capital endowment. Thus, this setting can rationalize the empirical finding that exporters are more capital intensive than non–exporters, also in less developed countries like Chile or Mexico.

Second, there is quite some anecdotal evidence stating that in export–oriented economies like China or Germany, the limited supplies of those factors, which are used intensively by exporters, limit a sector's export activities (e.g., McKinsey, 2009, for Germany or World Bank, 2012, for China). In the present setting a finite supply of capital, which is used intensively by exporters, limits a country's export activities. The reason is that trade liberalization intensifies relative competition for capital. This makes it more costly for the capital intensive exporters to serve the foreign market and, thus, limits their relative frequency in the firm distribution. Notice that in a setting with firm homogeneity in factor intensities (e.g., Bernard et al., 2007), the share of exporting firms in a sector's firm distribution depends only on the export cost parameters and is not related to a country's relative factor endowments.

[1] If the notion of capital is extended to include human capital, the same picture emerges for Denmark (Munch and Skaksen, 2008), Mexico (Harrison and Hanson, 1999) and Portugal (Martins and Opromolla, 2011). Klein et al. (2010) and Leonardi (2007) document the general prevalence of firm heterogeneity in factor input ratios.

Third, trade liberalization is correlated with a country's income distribution and, on an aggregate level, with labor's share in national income. While the empirical evidence on the correlation between trade liberalization and income distribution is ambiguous (Milanovic, 2005) the correlation between trade liberalization and labor's share in national income is typically found to be negative (IMF, 2007). This paper argues that the time dimension might explain the ambiguity concerning the correlation between globalization and income distribution. I show that, if exporters are more capital intensive than non–exporters, trade liberalization increases the relative price of capital in the short–run. The resulting incentives for investments into capital increase a country's capital stock in the long–run which, in turn, brings the relative price of capital down again. Concerning labor's share in national income, the impact of globalization does not differ between the short and the long–run in this paper. The reason is that labor's share not only depends on factor prices, but also on factor endowments. The negative short–run impact of globalization on labor's share results from the increase in the relative price of capital. In the long–run, it is the increase in the country's capital stock that impacts labor's share negatively.

Fourth, Bernard and Jensen (1997) report for the US that trade liberalization relocates resources towards the more capital intensive exporters. If the notion of capital is extended to include human capital, the empirical literature which is reviewed in Goldberg and Pavcnik (2007) supports this finding: trade liberalization is positively correlated with demand for skilled labor, a resource shift towards the more skill intensive firms and an increase in the sector–wide skill intensity. Importantly, Goldberg and Pavcnik (2007) emphasize that these correlations hold also for developing countries. This is in line with this paper's result that exporters can be the more capital intensive firms, regardless of a country's relative factor endowments. I show that, if a country's effective factor prices are such that only the more capital intensive firms export, trade liberalization increases relative capital demand. In the short–run with a fixed capital stock, the resulting adjustment of relative factor prices shifts the firm distribution either towards the non–exporters or towards the exporters, depending on the magnitude of export costs. In the long–run with an endogenous capital stock, the firm distribution shifts towards the exporters, regardless of the magnitude of export costs.

Finally, a large number of empirical studies has documented the positive impact of trade liberalization on economic growth (see, e.g., the literature which is reviewed in Singh, 2010). This paper shows that the trade–induced relocation of production factors towards the more capital intensive exporters induces capital accumulation by households. In addition, this paper shows that, due to the growth impact of trade liberalization, both capital and labor gain in real terms in the long–run after trade liberalization. Empirical evidence on the short–run versus long–run welfare impact of trade liberalization is scarce. This is mainly due to the ambiguity in separating the long–run from the short–run. Still, Porto (2007) finds evidence suggesting that, after households had the time to adjust their production and consumption patterns, the welfare impact of trade liberalization is positive for all households, regardless of whether they are producers of consumption goods or workers.

The analysis in this paper starts with the short–run, which is characterized by fixed factor supplies. I show that firms with different factor input ratios produce with different levels of marginal costs. Thus, when a country opens up to costly trade, exporters and non–exporters use factors in different intensities. I show for which combinations of (*i*) the firm distribution and (*ii*) a country's relative factor endowments, exporters are more capital intensive than non–exporters. I will highlight that, if the firm distribution is sufficiently skewed towards the more labor intensive ones, exporters are more capital intensive than non–exporters, even when a country's relative capital endowment is smaller than unity. I will first focus on symmetric countries, which are such that exporters are more capital intensive than non–exporters. Thus, trade liberalization increases the relative price of capital and decreases labor's share in national income. Furthermore, it is a priori ambiguous into which direction trade liberalization shifts the firm distribution. On the one hand, increased profit opportunities abroad ceteris paribus shift the firm distribution towards capital–intensive exporters. On the other hand, increased competition for capital ceteris paribus shifts the firm distribution towards labor intensive non–exporters. Thus, the sector's export volume is restricted by a limited supply of capital.

Afterwards, I extend the analysis to the long–run and assume that the countries' capital stocks are determined endogenously in terms of the Ramsey growth model. I highlight that, also with endogenous capital stocks, exporters can be more capital intensive than non–exporters, regardless of a country's relative factor endowments. I derive the following long–run impact of globalization for the two symmetric countries: (*i*) the industry–wide capital intensity in production increases due to factor relocations towards the more capital intensive firms; (*ii*) capital accumulation by households increases; (*iii*) labor's share in national income decreases, while the real returns to all factors increase.

In an extension I consider asymmetric countries. In the home country, exporters are still the more capital intensive firms. In the foreign country, in contrast, exporters are now the more labor intensive firms. I show that making the foreign country different does not change the qualitative impact of trade liberalization for the home country. The reason is that the qualitative impact of trade liberalization depends on the capital intensity of exporters relative to the one of non–exporters. Since this paper considers trade due to product differentiation—not Heckscher–Ohlin trade—, the capital intensity of exporters relative to the one of non–exporters depends only on how domestic factor prices compare to each other. As long as the effective price of capital is smaller than the one of labor, home country's exporters are more capital intensive than home country's non–exporters, regardless of the characteristics of the foreign country. For the foreign country, the short–run impact of trade liberalization is the mirror image, as compared to the initial scenario: the relative price of capital decreases. The long–run impact of trade liberalization is ambiguous for the foreign country. While a resource shift towards the more labor intensive exporters reduces relative capital demand, the decrease in the relative price of capital increases relative capital demand. Thus, the foreign country's capital stock and welfare can increase or decrease in the long–run after trade liberalization. I also show that the trading equilibrium is not unique when countries are asymmetric.

Finally, I parameterize the model with values for the US from the literature in order to evaluate the model's quantitative performance. The simulated results match several empirical findings for the US concerning the dissimilarities between exporters and non–exporters. Thus, the simulation supports the modeling of the production side as it is proposed in this paper.

One of the key elements of the present setup is the market entry procedure. Following Melitz (2003), firms randomly draw their technology parameter after they have paid a sunk market entry fee. However, the technology parameter is not a total factor productivity (TFP) parameter, but a factor share parameter of a CES production function in the present setup. The random factor share parameter can reflect the firms' initial uncertainty about, e.g., the workers' skills. Depending on the draw of the factor share parameter, profit maximizing firms choose different factor input ratios.

This paper is related to the literature in the following ways.

First, this paper is related to the literature on international trade with firm heterogeneity (e.g., Melitz, 2003). This literature focusses on a random TFP parameter as the source of heterogeneity and explains, amongst others, the positive correlation between trade liberalization and industry–wide productivity.[2] Bernard, Redding and Schott (2007) also extend the Melitz (2003)–setup by two factors of production and, additionally, two monopolistically competitive sectors with different capital–labor ratios in production. Within sectors firms are heterogeneous in TFP, but homogeneous in capital–labor ratios. The model by Bernard, Redding and Schott (2007) provides important insights into the inter– and intra–industry factor relocations due to trade liberalization. By construction, though, it does not analyze how heterogeneity in capital–

[2]More recent contributions extend the Melitz (2003) setting into different directions. For example, Van Long et al. (2011) and Vannoorenberghe (2008) also allow firms to engage in costly R&D, while Unel (2010) extends the original setting by two factors of production and a second monopolistcally competitive sector; in both sectors firms are heterogeneous in TFP. Jørgensen and Schröder (2008), in contrast, assume that firms are heterogeneous in their productivity for producing fixed costs.

labor input ratios interacts with globalization. The reason is that within sectors, a firm's export status depends on its TFP and not on its factor shares in production. The setup by Crozet and Trionfetti (2010) comes closest to mine as they also model firm heterogeneity in factor input ratios. However, their focus is different. Crozet and Trionfetti (2010) analyze how a firm's factor input ratio and a country's relative factor endowments jointly determine the firm's market share. Also related is the work by Lu (2010), who considers a Heckscher–Ohlin setting, in which firms within sectors are heterogeneous in TFP. She shows, theoretically and empirically with Chinese data, that it depends on (*i*) a sector's factor intensities and (*ii*) a country's relative factor abundance whether exporters are more or less productive than non–exporters. Her study's focus is different from mine as she looks at the relationship between a firm's TFP and its export status, controlling for factor intensities. In my setting, TFP is identical across firms and I analyze how factor intensities and export status are related.

Second, this paper is related to the literature on globalization and economic growth. One strand of this literature focuses on 'new' trade models and ascribes the positive growth impact of trade liberalization either to international R&D spillovers (e.g., Dinopoulos and Unel, 2011), more innovation due to increased competition (e.g., Baldwin and Robert–Nicoud, 2008) or to increased productivity due to exploitation of economies of scale (e.g., Ades and Glaeser, 1999). Another strand of this literature combines a Heckscher–Ohlin trade setting with a neoclassical growth setting and shows that such a setting leads to a continuum of steady states (Chen, 1992) or to non–convergence between countries if countries start from different initial factor endowments (Bajona and Kehoe, 2006). While I also consider neoclassical growth, I consider trade in differentiated varieties instead of Heckscher–Ohlin trade. If countries are symmetric, this leads to a unique open economy steady state and, if countries are asymmetric, to a finite number of open economy steady states. The channel through which globalization impacts economic growth is also different in my paper. If a country is such that exporters are more capital intensive than non–exporters, the factor relocations to exporters can explain a positive temporary growth effect of trade liberalization.

Third, this paper is related to the literature on the gains from trade, which has experienced a resurgence due to recent work by Arkolakis et al. (2012). These authors show that the gains from trade are solely determined by aggregate trade flows and the elasticity of imports with respect to variable trade costs, regardless of whether the Krugman (1980) model is extended by firm heterogeneity in TFP or not. However, Arkolakis et al. (2010) focus on trade models with only labor or firm homogeneity in factor input ratios. Thus, the setting by Arkolakis et al. (2012) is silent about the distributional impact of globalization. In the present setup trade changes the income distribution, but benefits all factors of production if exporters are more capital intensive than non–exporters and, as a consequence, trade induces growth.

The structure of the paper is as follows. Section 2 describes the setup of the model. Section 3 analyzes trade liberalization in the short–run, while section 4 analyzes trade liberalization in the long–run with endogenous capital stocks. Sections 3 and 4 consider symmetric countries. Section 5 extends the model to asymmetric countries. Section 6 parameterizes the long–run setup with values for the US from the literature and compares the simulated dissimilarities between exporters and non–exporters with the empirical findings. Section 7 concludes.

2 The model

2.1 Overview

This paper analyzes trade between home country H and foreign country F.

Households in each country are characterized by Dixit–Stiglitz preferences (Dixit and Stiglitz, 1977) and consume a continuum of imperfectly substitutable varieties of a differentiated good Q. Thus, firm behavior can be described by large–group monopolistic competition, i.e. each firm regards the prices of all other varieties and factor returns as given. The production side of each country consists of this single monopolistically competitive sector.

Firms produce with capital K and labor L. After a firm has paid a sunk market entry fee, it randomly draws the factor share parameters of its CES production function from an exogenous probability distribution. Different factor share parameters imply that profit maximizing firms choose different factor input ratios. Thus, if factor prices differ, firms produce with different marginal costs. Production also leads to fixed costs. Due to fixed costs a firm only starts with production if the draw of the factor share parameters allows it to produce with sufficiently low marginal costs.

To keep the model simple, I assume that labor and capital are perfectly mobile between firms within a country, but perfectly immobile between countries. Furthermore, I assume symmetry across countries in sections 3 and 4. Only in section 5 I allow for asymmetric countries.

2.2 Production

A single firm produces its unique variety of good Q with the following CES function:

$$q(\phi) = \left[\phi^{1-\alpha} (kA_K)^\alpha + (1-\phi)^{1-\alpha} (lA_L)^\alpha\right]^{1/\alpha}, \qquad \phi_L \leq \phi \leq 1, \qquad 0 < \alpha < 1. \quad (1)$$

k and l denote the input of capital and labor, $\phi_L \geq 0$ denotes the lower bound for the capital share parameter ϕ and A_K and A_L are productivity parameters, which are identical across firms. In order to simplify the exposition, I normalize $A_K = A_L = 1$ and k and l will stand for the effective capital and labor input in the following.

Later, when I explain the market entry procedure, I will follow Crozet and Trionfetti (2010) and assume that a firm's production function results from a random draw of ϕ after market entry. Thus, firms differ with respect to ϕ. Regardless of its ϕ, a firm can choose any factor input ratio. However, I assume that firms minimize production costs for a given ϕ. Thus, a firm's marginal costs are given by:

$$c(\phi) \;=\; \left[\phi\, r^{1-\sigma} + (1-\phi)\, w^{1-\sigma}\right]^{1/(1-\sigma)}, \qquad \sigma \;=\; \frac{1}{1-\alpha} > 1. \tag{2}$$

σ stands for the elasticity of substitution between k and l and r and w denote the returns per unit capital and labor. If $r \neq w$, the capital share parameter ϕ influences marginal costs.

I will explain in section 2.5 that a firm only starts with production after market entry if it has drawn a ϕ from the sub–interval $[\underline{\phi}, \overline{\phi}]$ of the interval $[\phi_L, 1]$. The boundaries $\underline{\phi}$ and $\overline{\phi}$ are endogenous and their general equilibrium values are determined in sections 3 and 4.

Production requires a fixed cost which takes the following form: $F(\phi) = c(\widetilde{\phi})f$. This structure of fixed costs is common in two–factor trade models (e.g., Markusen and Venables, 2000) and implies that firms have to pay for the fixed input requirement f in terms of the variety which is produced with the capital share parameter $\widetilde{\phi}$. I will define $\widetilde{\phi}$ in the next subsection.[3]

2.3 Demand

Households aggregate varieties $q(\phi)$ to give the aggregate consumption good Q:

$$Q \;=\; \left[\int_{\underline{\phi}}^{\overline{\phi}} q(\phi)^{(\xi-1)/\xi}\, \mu(\phi)\, N d\phi\right]^{\xi/(\xi-1)}, \qquad \xi \;>\; 1. \tag{3}$$

ξ stands for the elasticity of substitution between the varieties and N for the mass of heterogeneous firms, which are distributed on $[\underline{\phi}, \overline{\phi}]$ according to the density $\mu(\phi)$.

In order to simplify the algebra, without affecting the results in a qualitative sense, I impose assumption 1 for the remainder of the analysis:

Assumption 1 $\sigma = \xi$. *In the following the parameter σ will denote the elasticity of substitution between capital and labor in production* and *the elasticity of substitution between varieties in consumption.*

Assumption 1 simplifies several proofs since the term $c(\phi)^{1-\xi}$ becomes linear in ϕ if $\xi = \sigma$.[4]

[3] Alternatively, I could assume that firms have to pay for f in terms of labor, i.e. $F = wf$, in terms of capital, i.e. $F = rf$ or in terms of their own variety, i.e. $F = c(\phi)f$. The results are robust to these alternative specifications of F.

[4] The results of this paper will depend on (*i*) how ϕ influences $c(\phi)$ and (*ii*) how ϕ and factor prices influence the per unit factor demands by firms. These relationships are not influenced by assumption 1 in a qualitative sense. The proofs for the case of $\sigma \neq \xi$ are tedious, but are available from the author.

The price index P, which is dual to the CES function in equation 3, is given by:

$$P \quad = \quad \left[\int_{\underline{\phi}}^{\overline{\phi}} p\,(\phi)^{1-\sigma} \;\; \mu(\phi)\, N d\phi \right]^{1/(1-\sigma)} . \tag{4}$$

Applying Shephard's Lemma to P, demand for a single variety can be derived as:

$$q\,(\phi) \quad = \quad Y\, P^{\sigma-1}\, p(\phi)^{-\sigma}. \tag{5}$$

Y denotes total factor income, i.e. $Y = r\,K + w\,L = PQ$, with K and L denoting the country's endowments with capital and labor. Profit maximizing firms apply the following pricing rule: $p(\phi) = \frac{\sigma}{\sigma-1}\, c(\phi)$. Finally, the average revenue over all firms on the interval $[\underline{\phi}, \overline{\phi}]$ results as follows:

$$\int_{\underline{\phi}}^{\overline{\phi}} p(\phi)\, q(\phi)\, \mu(\phi) d\phi \quad = \quad Y\, P^{\sigma-1} \int_{\underline{\phi}}^{\overline{\phi}} [\phi r^{1-\sigma} + (1-\phi) w^{1-\sigma}]\, \mu(\phi) d\phi \quad = \quad p(\widetilde{\phi})\, q(\widetilde{\phi}), \tag{6}$$

where $\mu(\phi)$ denotes the conditional probability density for ϕ on the interval $[\underline{\phi}, \overline{\phi}]$. I call $\widetilde{\phi}$ the "average capital share parameter" and it is defined as follows:

$$\widetilde{\phi} \quad = \quad \int_{\underline{\phi}}^{\overline{\phi}} \phi\, \mu(\phi) d\phi. \tag{7}$$

2.4 Profits, the capital share parameter ϕ and factor returns r and w

Labor is chosen as numéraire and only relative returns to capital $\frac{r}{w} \equiv r$ matter in the following. Later, when I derive the general equilibrium, I will show that both $r > 1$ or $r < 1$ are possible, depending on parameters of the model. Depending on how r relates to 1, ϕ has a positive or a negative influence on a firm's profits $\pi(\phi)$. This is shown by equation 8:

$$\pi\,(\phi) \quad = \quad \frac{p(\phi)\, q(\phi)}{\sigma} - c(\widetilde{\phi})\, f \;\; = \;\; Y\, P^{\sigma-1}\, \frac{\phi\, \left[r^{1-\sigma} - 1\right] + 1}{\sigma^{\sigma}\, (\sigma-1)^{1-\sigma}} - c(\widetilde{\phi})\, f. \tag{8}$$

Y, P and $c(\widetilde{\phi})$ depend on $\widetilde{\phi}$ and r, but they are exogenous for a single firm due to large–group monopolistic competition. If $r < 1$, profits increase with ϕ. If $r > 1$ profits decrease with ϕ.[5]

Finally, if f is sufficiently large, there is always a range of ϕ for which profits are negative.

2.5 Market entry and supply decision to the domestic market

An infinitely dividable number of potential entrants into the market exists. By the time of market entry, firms do not know their capital share parameter ϕ yet, i.e. firms are identical by the time of market entry. The entry procedure can be divided into three steps.

[5] Notice that, without the normalization $A_K = A_L = 1$, the relationship between profits and ϕ would depend on how r relates to $\frac{A_K}{A_L}$.

First, market entry leads to a sunk input requirement f_E, which is also produced with the average technology parameter $\widetilde{\phi}$. Thus, the sunk market entry costs are given by $c(\widetilde{\phi})\, f_E$.

Second, after entry firms draw the factor share parameter ϕ from the interval $[\phi_L, 1]$ according to a probability distribution with density $g(\phi)$.[6] The lower bound ϕ_L of the support is equal or larger than zero. The variable costs of a firm are then given by $c\,(\phi)$. The ex–ante uncertainty about the capital share parameter ϕ reflects a firm's ex–ante uncertainty about which factor input ratio maximizes profits. Uncertainty about the profit–maximizing factor input ratio, in turn, can reflect uncertainty about the workers' abilities or about how consumers perceive a firm's variety. Each firm keeps the ϕ it has drawn for the rest of its life.

Third, after the draw of ϕ, firms decide whether to start with production or not. Since fixed production costs $c(\widetilde{\phi})\, f$ exist and since r can be larger or smaller than 1 in general equilibrium, two cases have to be distinguished:

(*i*) if $r < 1$ in general equilibrium, only firms with a ϕ from the interval $[\phi^*, 1]$ start with production after entry; ϕ^* is equal or larger than ϕ_L. Thus, $\underline{\phi} = \phi^*$ and $\overline{\phi} = 1$. Firms with $\phi < \phi^*$ immediately exit.

(*ii*) if $r > 1$ in general equilibrium, only firms with a ϕ from the interval $[\phi_L, \phi^*]$ start with production after entry; ϕ^* is equal or smaller than 1. Thus, $\underline{\phi} = \phi_L$ and $\overline{\phi} = \phi^*$. Firms with $\phi > \phi^*$ immediately exit.

Furthermore, a firm's profits are zero if $\phi = \phi^*$. This leads to a zero cutoff profit condition:

$$\pi\,(\phi^*) \;=\; \frac{q\,(\phi^*)\,p\,(\phi^*)}{\sigma} - c(\widetilde{\phi})\, f \;=\; 0. \tag{9}$$

Considering $p\,(\phi) = \frac{\sigma}{\sigma-1}\, c(\phi)$ and $\frac{q(\phi^*)}{q(\widetilde{\phi})} = \left[\frac{c(\phi^*)}{c(\widetilde{\phi})}\right]^{-\sigma}$, equation 9 can be transformed:

$$q(\widetilde{\phi}) \;=\; (\sigma - 1)\left[\frac{c(\widetilde{\phi})}{c(\phi^*)}\right]^{1-\sigma} f. \tag{10}$$

Finally, in each period a firm may be hit by a negative shock with probability θ, $0 < \theta < 1$. The shock forces the firm to exit the market. Due to the negative shock a constant amount of sunk entry costs arises in each period of the steady state.

3 General equilibrium — symmetric countries, short–run

The short–run is characterized by constant factor endowments K and L. To avoid cluttering up the exposition, I will include country indices only in section 5 when I study trade between asymmetric countries.

[6] Crozet and Trionfetti (2010) also consider random factor share parameters. Notice that, without sunk market entry costs, firms could repeatedly enter the market and draw their factor share parameter until they have received the most preferred one.

3.1 Closed economy

The general equilibrium for either country is characterized by the following 4 conditions:

(*i*) production equals demand for each variety at price $p(\phi) = \frac{\sigma}{\sigma-1}\, c(\phi)$ (equation 5);

(*ii*) the zero cutoff profit condition (equation 10);

(*iii*) a free entry condition;

(*iv*) the factor market equilibrium conditions.

These 4 conditions can be solved for the following 4 variables in the closed economy general equilibrium for either country: $\widetilde{\phi}$, $q(\widetilde{\phi})$, r and N.

3.1.1 Free entry condition (*FEC*)

Firms enter the market only if the sum of expected discounted lifetime profits at least covers the sunk market entry costs.

Section 2.4 has shown that the relationship between a firm's capital share parameter ϕ and its profits depends on whether $r < 1$ or $r > 1$ in the country's general equilibrium. Therefore, the free entry condition (*FEC*) is defined piecewise:

$$c(\widetilde{\phi})\, f_E \;=\; \begin{cases} [1 - G(\phi^*)] \sum_{t=t'}^{\infty} E\left[\pi(\phi)\middle|\phi \geq \phi^*\right] \left(\frac{1-\theta}{1+\rho}\right)^t & \text{if } \; r < 1 \\ G(\phi^*) \sum_{t=t'}^{\infty} E\left[\pi(\phi)\middle|\phi \leq \phi^*\right] \left(\frac{1-\theta}{1+\rho}\right)^t & \text{if } \; r > 1. \end{cases} \tag{11}$$

The left hand side of equation 11 denotes the sunk market entry costs, while the right hand side denotes expected discounted lifetime profits. Period t' denotes the period in which a firm enters the market and G denotes the cumulative density for ϕ on the interval $[\phi_L, 1]$. The term $(1-\theta)^t$ accounts for the risk of death in each period and the term $(1+\rho)^{-t}$ discounts future profits to current period values.

If $r < 1$ in general equilibrium, firms are active after entry only if their ϕ is from the interval $[\phi^*, 1]$. Thus, $1 - G(\phi^*)$ denotes the probability for a successful entry and $E[\pi(\phi)\big|\phi \geq \phi^*]$ denotes the expected profits, given that the firm is active.

If, in contrast, $r > 1$ in general equilibrium, firms are active after entry only if their ϕ is from the interval $[\phi_L, \phi^*]$. Thus, $G(\phi^*)$ denotes the probability for a successful entry in this case and $E[\pi(\phi)\big|\phi \leq \phi^*]$ denotes the expected profits, given that the firm is active.

Firms ex–ante expect that a successful market entry will bring them the average profits over all active firms. Therefore (see also equation 6):

$$E[\pi(\phi)] \;=\; \frac{p(\widetilde{\phi})\, q(\widetilde{\phi})}{\sigma} - c(\widetilde{\phi})\, f. \tag{12}$$

Using the formula for an infinite geometric series, the *FEC* can be simplified:

$$f_E \frac{\rho+\theta}{1+\rho} \quad = \quad \Theta(\phi^*) \left[\frac{q(\widetilde{\phi})}{\sigma-1} - f\right], \tag{13}$$

with $\Theta(\phi^*) = 1 - G(\phi^*)$ if $r < 1$ and $\Theta(\phi^*) = G(\phi^*)$ if $r > 1$. Notice that the definition of $\widetilde{\phi}$ (equation 7) differs between the cases of $r < 1$ and $r > 1$: if $r < 1$, then $\widetilde{\phi} = \int_{\phi^*}^{1} \phi \frac{g(\phi)}{1-G(\phi^*)} d\phi$, while $\widetilde{\phi} = \int_{\phi_L}^{\phi^*} \phi \frac{g(\phi)}{G(\phi^*)} d\phi$ if $r > 1$.

$q(\widetilde{\phi})$ in equilibrium is a function of the threshold capital share parameter ϕ^* and the average capital share parameter $\widetilde{\phi}$ (see equation 10). Since $\widetilde{\phi}$ is a function of ϕ^* and the respective boundary, equation 13 can be solved for $\widetilde{\phi}$ or ϕ^* as a function of relative factor returns r. The resulting curves (*FEC*–curve and ϕ^*–curve) are illustrated by figure 1.[7]

[Figure 1 about here]

The shape of the *FEC*–curve is intuitive. For example, take the part for $r < 1$. If r increases, the relative cost advantage of capital intensive firms decreases, i.e. more labor intensive firms enter the market successfully and $\widetilde{\phi}$ decreases. If r is close to 1, firms with each possible capital share parameter ϕ can afford to produce, implying that the *FEC*–curve becomes horizontal. The same logic applies to the part for $r > 1$.

3.1.2 Factor market equilibrium conditions

Applying Shephard's Lemma to the marginal cost function (equation 2) leads to the following factor market equilibrium conditions:[8]

$$(1-\widetilde{\phi})\, c(\widetilde{\phi})^{\sigma} \left[q(\widetilde{\phi})\, N + \widetilde{f}\, N\right] \quad = \quad L \tag{14}$$

$$\widetilde{\phi}\, r^{-\sigma}\, c(\widetilde{\phi})^{\sigma} \left[q(\widetilde{\phi})\, N + \widetilde{f}\, N\right] \quad = \quad K, \tag{15}$$

where $\widetilde{f}\, N$ denotes total fixed input requirements in general equilibrium and $\widetilde{f} = f + \frac{\theta f_E}{\Theta(\phi^*)}$.[9]

Dividing equations 14 and 15 by each other results in:

$$\frac{1-\widetilde{\phi}}{\widetilde{\phi}}\, r^{\sigma} \quad = \quad \frac{L}{K}. \tag{16}$$

[7] In appendix A I show analytically that both curves have a negative slope, except for the horizontal part around $r = 1$. If r is close to 1, firms with each possible ϕ can afford to produce, implying that the *FEC*–curve has a kink and becomes horizontal. The *FEC*–curve is drawn for $\phi_L = 0$ and a right–skewed distribution of ϕ.

[8] See appendix B for the derivation of equations 14 and 15.

[9] f_E is divided by $\Theta(\phi^*)$ in order to account for the fact that only the share $\Theta(\phi^*)$ of the entering firms actually becomes active. Thus, if N firms are active in general equilibrium, $\frac{N}{\Theta(\phi^*)}$ firms had entered. Since unsuccessful entry also leads to entry costs, f_E is divided by $\Theta(\phi^*)$. Furthermore, f_E is multiplied by θ since the share θ of active firms is replaced by new firms in each period of the steady state.

Equation 16 can be solved for $\widetilde{\phi}$ as a function of relative capital returns r. Figure 1 illustrates the resulting curve ($\frac{L}{K}$–curve), which is upward sloping: if $\widetilde{\phi}$ increases, all firms ceteris paribus produce more capital intensively and r increases for an equilibrium on factor markets. Furthermore, the $\frac{L}{K}$–curve shifts rightward with an increase in $\frac{L}{K}$ since this leads to an increase in r. The *steady state*–curve in figure 1 should be ignored for the time being.

3.1.3 Closed economy general equilibrium

The intersection point between the *FEC*–curve and the $\frac{L}{K}$–curve determines the general equilibrium values for $\widetilde{\phi}$ and r. The *FEC*–curve has a non–positive slope, while the $\frac{L}{K}$–curve is monotonously upward sloping from $\widetilde{\phi} = 0$ at $r \to 0$ to $\widetilde{\phi} = 1$ at $r \to \infty$. Thus, I can formulate proposition 1:

Proposition 1 *A unique and stable autarkic general equilibrium exists.*

The position of the *FEC*–curve depends on the distribution of ϕ on $[\phi_L, 1]$. If relatively more firms have their ϕ close to ϕ_L (close to 1), the *FEC*–curve shifts downward (upward). The position of the $\frac{L}{K}$–curve depends on the country's relative factor endowments. The smaller is $\frac{L}{K}$, the further to the left is the $\frac{L}{K}$–curve. This yields lemma 1:

Lemma 1 *The relative return to capital is smaller than* 1 *in general equilibrium if:* (*i*) *the distribution of* ϕ *is sufficiently skewed to the right,* (*ii*) *the country's relative labor endowment* $\frac{L}{K}$ *is sufficiently small. If neither of these conditions holds,* $r > 1$ *in general equilibrium.*

Notice that, if ϕ were distributed symmetrically around $\widetilde{\phi} = 0.5$, $r < 1$ ($r > 1$) could result in general equilibrium only if $\frac{L}{K} < 1$ ($\frac{L}{K} > 1$). However, if ϕ does not follow such a symmetric distribution, the *FEC*–curve runs below or above the point $(1; 0.5)$, implying that $r < 1$ ($r > 1$) can also result in general equilibrium if $\frac{L}{K} > 1$ ($\frac{L}{K} < 1$).[10]

Thus, with heterogeneity in factor shares, it is a priori ambiguous whether a higher or a lower value of ϕ implies smaller marginal costs $c(\phi)$. If $r < 1$, a higher value of ϕ implies a smaller level of $c(\phi)$. If $r > 1$, a lower value of ϕ implies a smaller level of $c(\phi)$. Thus, if trade is sufficiently costly so that only firms with sufficiently small marginal costs can afford to export, it is a priori ambiguous whether exporting firms produce more or less capital intensively than non–exporting firms.

This illustrates an important difference between the current setup and the literature with firm heterogeneity in TFP (e.g., Melitz, 2003). With firm heterogeneity in TFP, the cost advantage

[10]Notice for figure 1 that $r < 1$ could result even if the country's relative labor endowment became larger, which would shift the $\frac{L}{K}$–curve rightward. If the distribution of ϕ were even more right–skewed, so that the *FEC*–curve were closer to the ϕ^*–curve, the *FEC*–curve and the $\frac{L}{K}$–curve would still intersect at a point with $r < 1$.

of a single firm relative to its competitors is fixed by the ratio of TFP parameters. In the current setup, in contrast, the cost advantage of any single firm relative to its competitors depends on (i) the firms' capital share parameters ϕ and (ii) on the relative return to capital. The latter is determined endogenously and will adjust with trade liberalization.

3.2 Open economy

I assume that import tariffs are initially prohibitively high and that trade liberalization is reflected by a reduction of these tariffs to zero. Following the existing literature, I impose two assumptions: (i) entering the foreign market leads to sunk costs; (ii) firms make their export decision after the draw of ϕ. For simplicity, iceberg transport costs are zero.

The open economy general equilibrium is characterized by the following 5 conditions for each of the two symmetric countries:

(i) production equals demand for each variety at price $p(\phi) = \frac{\sigma}{\sigma-1}\, c(\phi)$; notice that 'demand' is 'worldwide demand' if a firm exports;

(ii) the zero cutoff profit condition for the domestic market (equation 10);

(iii) a zero cutoff profit condition for the foreign market;

(iv) an open economy free entry condition;

(v) the factor market equilibrium conditions.

These conditions—5 per country—can be solved for the following 5 variables for either country in the open economy general equilibrium: $\widetilde{\phi}$, $q(\widetilde{\phi})$, r, N and the mass of exporting firms N_X.

3.2.1 Supply decision to the foreign market

Foreign demand for a domestic variety is given by:

$$q_X(\phi) \quad = \quad Y\, P^{\sigma-1}\, p(\phi)^{-\sigma}. \tag{17}$$

The subscript X denotes exports. Neither Y nor P have a country index due to symmetry across countries in this section. Since iceberg transport costs are zero, aggregate sales of an exporting firm ceteris paribus double with trade liberalization:

$$q(\phi) + q_X(\phi) \quad = \quad 2\, Y\, P^{\sigma-1}\, p(\phi)^{-\sigma}. \tag{18}$$

Entering the foreign market leads to a sunk input requirement which is also produced with the average factor share parameters $\widetilde{\phi}$ and $1-\widetilde{\phi}$. The per period equivalent of the sunk entry costs into the foreign market is given by $c(\widetilde{\phi})f_X$.[11]

As long as $r \neq 1$ in autarky, not all firms necessarily export after trade liberalization due to export costs. Therefore, an additional threshold capital share parameter ϕ_X^* exists. At ϕ_X^*, a firm's profits from exporting, which are denoted by $\pi_X(\phi)$, are zero:

$$\pi_X\left(\phi_X^*\right) \quad = \quad \frac{q_X(\phi_X^*)\,p(\phi_X^*)}{\sigma} - c(\widetilde{\phi})\,f_X \quad = \quad 0. \tag{19}$$

Equations 9 and 19 can be jointly solved for ϕ_X^*:

$$\phi_X^* \quad = \quad \frac{f_X}{f}\phi^* + \frac{\frac{f_X}{f}-1}{r^{1-\sigma}-1}. \tag{20}$$

In order to get partitioning of firms with respect to export status, I assume $f_X > f$. Equation 20 implies that the trading equilibrium is characterized by the following partitioning of firms:

Lemma 2 *If $r < 1$, then $\phi_X^* > \phi^*$, i.e. exporters are more capital intensive than non–exporters. If $r > 1$, then $\phi_X^* < \phi^*$, i.e. exporters are more labor intensive than non–exporters.*

Since it is the empirically more relevant case that exporters are capital intensive relative to non–exporters, I limit myself in sections 3 and 4 to analyzing trade liberalization for countries which are characterized by $r < 1$. This case has been illustrated by figure 1. Notice that, as shown by lemma 1, $r < 1$ is not limited to countries with $\frac{L}{K} < 1$.

Equation 20 also implies proposition 2:

Proposition 2 *If countries are symmetric, more equal factor returns ceteris paribus decrease the share of exporting firms in the firm distribution.*

Proof. See appendix C. ■

The intuition is as follows: more equal factor returns (i.e. r is closer to 1, but still smaller than 1) imply a smaller cost advantage of capital intensive firms relative to labor intensive firms. Thus, more equal factor returns make it less likely that even the firm with the largest ϕ can afford to export.

3.2.2 Free entry condition in the open economy

The free entry condition for the closed economy (equation 13) is extended by the ex–ante expected export profits. The free entry condition for the open economy is given by:

$$f_E\,\frac{\rho+\theta}{1+\rho} \quad = \quad \Theta(\phi^*)\left[\frac{q(\widetilde{\phi})}{\sigma-1} - f\right] + \frac{\Theta(\phi_X^*)}{c(\widetilde{\phi})}\left[\frac{q_X(\widetilde{\phi}_X)p(\widetilde{\phi}_X)}{\sigma} - c(\widetilde{\phi})\,f_X\right]. \tag{21}$$

[11] If $c(\widetilde{\phi})f_{Ex}$ denotes the one–time sunk entry costs into the foreign market, $c(\widetilde{\phi})\,f_{Ex}\,\frac{\rho+\theta}{1+\rho}$ denotes the per–period equivalent of the sunk entry costs. In order to simplify notation, I define $f_X \equiv f_{Ex}\,\frac{\rho+\theta}{1+\rho}$.

Notice that $\Theta(\phi^*) = 1 - G(\phi^*)$ and $\Theta(\phi_X^*) = 1 - G(\phi_X^*)$ since I focus here on the case of $r < 1$. The term $\Theta(\phi_X^*)$ stands for the probability that an entrant will be an exporting firm. $\widetilde{\phi}_X$ denotes the average capital share parameter over the exporting firms, while the term $\frac{q_X(\widetilde{\phi}_X)\,p(\widetilde{\phi}_X)}{\sigma} - c(\widetilde{\phi})\,f_X \equiv \pi_X(\widetilde{\phi}_X)$ stands for the average export profits over the exporting firms. $\pi_X(\widetilde{\phi}_X) \geq 0$ since firms export due to profit opportunities abroad. Thus, the expected profits from market entry increase with trade liberalization. Equation 21 leads to lemma 3:

Lemma 3 *For given relative factor returns r, trade liberalization increases the threshold capital share parameter ϕ^*, i.e. the FEC–curve shifts upward.*

Proof. See appendix D. ■

Lemma 3 does not give any information about the equilibrium $\widetilde{\phi}$ in the open economy. Whether the equilibrium $\widetilde{\phi}$ increases or decreases with trade liberalization also depends on how the $\frac{L}{K}$–curve shifts with trade liberalization.

The economic mechanisms that lead to lemma 3 are as follows. The additional export profits induce additional market entry, which implies that each incumbent firm sells less, i.e. $q(\phi)$ decreases.[12] Therefore, for a given r, the zero cutoff profit condition (equation 9) fails to hold at the initial threshold level ϕ^*. Thus, only firms with a larger capital share parameter ϕ start with production after market entry. Figure 2 illustrates the upward shift of the *FEC*–curve due to trade liberalization (from "*FEC–curve (aut)*" to "*FEC–curve (ft)*").

Notice that trade liberalization does not affect the intersection point of the *FEC*–curve with the vertical $r = 1$–line. The reason is that no firm can afford to export if $f_X > f$ and $r \rightarrow 1$.[13]

[Figure 2 about here]

3.2.3 Factor market equilibrium conditions

Adding the additional factor demands by the exporting firms to a country's closed economy factor market equilibrium conditions leads to:[14]

$$(1-\widetilde{\phi})\,c(\widetilde{\phi})^{\sigma}\left[q(\widetilde{\phi})\,\frac{1-\widetilde{\phi}+s_X(1-\widetilde{\phi}_X)}{1-\widetilde{\phi}}+\widetilde{f}+s_X f_X\right]N \;=\; L \qquad (22)$$

$$\widetilde{\phi} r^{-\sigma} c(\widetilde{\phi})^{\sigma}\left[q(\widetilde{\phi})\,\frac{\widetilde{\phi}+s_X\widetilde{\phi}_X}{\widetilde{\phi}}+\widetilde{f}+s_X f_X\right]N \;=\; K. \qquad (23)$$

[12] Notice that P (equation 4) depends negatively on the mass of firms N. Thus, $q(\phi) = YP^{\sigma-1}p(\phi)^{-\sigma}$ decreases if N increases.

[13] Equation 20 shows that $\phi_X^* > 1$ if $f_X > f$ and $r \rightarrow 1$. However, ϕ is restricted by the interval $[\phi_L, 1]$.

[14] See appendix E for the derivation of equations 22 and 23.

$s_X \equiv \frac{\Theta(\phi_X^*)}{\Theta(\phi^*)}$ stands for the share of exporters in the firm distribution, i.e. $s_X N$ denotes the mass of exporters in the country. Dividing equations 22 and 23 by each other leads to:

$$\frac{1-\widetilde{\phi}}{\widetilde{\phi}} r^{\sigma} \Lambda \;=\; \frac{L}{K}, \qquad \text{with} \quad \Lambda = \frac{q(\widetilde{\phi}) + \widetilde{f} + q(\widetilde{\phi})\, s_X \, \frac{1-\widetilde{\phi}_X}{1-\widetilde{\phi}} + s_X\, f_X}{q(\widetilde{\phi}) + \widetilde{f} + q(\widetilde{\phi})\, s_X \, \frac{\widetilde{\phi}_X}{\widetilde{\phi}} + s_X\, f_X}. \tag{24}$$

Compared to the autarkic equilibrium (equation 16), the term Λ adds to the left hand side.

Furthermore, $\Lambda < 1$ since $1 > s_X > 0$ and $1 > \phi_X^* > \phi^*$ in the trading equilibrium. $\Lambda < 1$ implies that trade liberalization increases relative capital demand, i.e. the left hand side of equation 24, which denotes relative labor demand, decreases. This leads to lemma 4:

Lemma 4 *Trade liberalization increases the relative return to capital r for a given $\widetilde{\phi}$. The $\frac{L}{K}$–curve shifts to the right.*

Proof. Since $\Lambda < 1$, relative labor demand decreases with trade liberalization. This is compensated by an increase in r, which has two impacts on relative labor demand. First, it increases each firm's relative labor input, which is given by $\frac{1-\phi}{\phi r^{-\sigma}}$. Second, it increases ϕ_X^* (see equation 20) and, thus, decreases the share of exporting firms s_X. The latter brings the country closer to the autarkic equilibrium and, thus, increases relative labor demand. ■

Figure 2 illustrate the shift of the $\frac{L}{K}$–curve due to trade liberalization (from "*L/K–curve (aut)*" to "*L/K–curve (ft, short–run)*").

Notice that the $\frac{L}{K}$–curve after trade liberalization converges to the $\frac{L}{K}$–curve before trade liberalization if r is sufficiently close to 1. The reason is, again, that no firm exports if $f_X > f$ and if r is close to 1. Thus, trade liberalization does not affect the intersection point of the $\frac{L}{K}$–curve with the vertical $r = 1$–line.

In summary, neither the *FEC*–curve nor the $\frac{L}{K}$–curve change their intersection point with the vertical $r = 1$–line due to trade liberalization. Since the *FEC*–curve has a non–positive slope, while the $\frac{L}{K}$–curve is monotonously upward sloping, I can formulate lemma 5:

Lemma 5 *If $r < 1$ in the autarkic equilibrium, $r < 1$ also in the trading equilibrium.*

3.2.4 Impact of trade liberalization — short–run

Considering the trade induced shifts of the *FEC*–curve and the $\frac{L}{K}$–curve, as illustrated by figure 2, leads to proposition 3:

Proposition 3 *If $r < 1$ in the autarkic equilibrium, r increases in the short–run after trade liberalization.*

Proposition 3 implies that labor's share in national income, which is given by $\frac{L}{L+r\,K}$, decreases with trade liberalization. In particular, proposition 3 establishes a Stolper–Samuelson theorem for the case of trade between identical countries: the relative price of a factor increases with trade liberalization if it is smaller than unity in autarky.

Furthermore, proposition 2 has shown that an increase of r ceteris paribus increases ϕ_X^*, which decreases the share of exporters a country's firm distribution. Thus, a finite supply of that factor, which is intensively used by exporters, limits a country's export activities.

The consequences of trade liberalization for the average capital share parameter $\widetilde{\phi}$ and, thus, for the firm distribution, are a priori ambiguous. On the one hand, trade liberalization shifts the *FEC*–curve upward, which ceteris paribus increases $\widetilde{\phi}$. On the other hand, it shifts the $\frac{L}{K}$–curve to the right, which ceteris paribus decreases $\widetilde{\phi}$. This leads to proposition 4:

Proposition 4 *In the short–run after trade liberalization, the average capital share parameter $\widetilde{\phi}$ increases if the sunk export costs are sufficiently small, i.e. if f_X is sufficiently close to f. Otherwise, $\widetilde{\phi}$ decreases in the short–run after trade liberalization.*

Proof. See appendix F. ■

The intuition is as follows. The adjustment of $\widetilde{\phi}$ to trade liberalization follows from two counteracting forces. First, trade liberalization increases market entry due to additional expected export profits. The resulting decrease in sales per firm only allows firms with a larger ϕ to survive, i.e. the *FEC*–curve shifts upward. Second, trade liberalization increases relative demand for capital, which is intensively used by exporters. The resulting increase in r decreases the cost advantage of exporters and, thus, ceteris paribus shifts the firm distribution towards non–exporters, which are the more labor intensive firms. The increased factor market competition is reflected by the rightward shift of the $\frac{L}{K}$–curve. Notice that the second effect is absent in a setup with firm heterogeneity in TFP. Thus, in such a setup, the threshold technology parameter always improves with trade liberalization.

4 General equilibrium — symmetric countries, long–run

In this section, households can accumulate capital and optimize intertemporally in terms of the Ramsey growth model. Thus, the countries' capital stocks are endogenous and adjust to changes in relative capital demand due to trade liberalization.

Section 4.1 derives relative factor returns in the steady state. Afterwards, these relative factor returns are substituted into the equilibrium conditions from section 3.

4.1 Relative factor returns in the steady state

Households use the variety $q(\widetilde{\phi})$ for investment purposes and choose their consumption and investment level each period such that lifetime utility V is maximized.[15]

ρ denotes the time discount rate and u the instantaneous utility function. Including the time index t, lifetime utility of a country's representative household is given by:

$$V = \sum_{t=0}^{\infty} \frac{u(Q_t)}{(1+\rho)^t}, \tag{25}$$

where Q_t is the aggregate consumption good as defined by equation 3. Each country's labor endowment is assumed to be constant over time. Investments therefore only compensate for depreciation of capital. If δ denotes the capital depreciation rate, investments into a country's capital stock in any period t of the steady state are given by:

$$I_t = K_{t+1} - (1-\delta)K_t = \delta K. \tag{26}$$

I_t denotes the amount of variety $q(\widetilde{\phi})$ which is invested in period t, K_t and K_{t+1} denote a country's capital stocks in t and $t+1$. $I_t = \delta K_t$ since K is constant in the steady state.

Households own the production factors and lend them out to firms for production. Given that households behave perfectly competitively, the steady state of a Ramsey growth setup is characterized by several necessary first order conditions, which determine r in the steady state as a function of the parameters ρ, δ, σ and the endogenous average capital share parameter $\widetilde{\phi}$ (see also Baxter, 1992). This is summarized by lemma 6.

Lemma 6 *The relative return to capital in the steady state is given by:*

$$r = \left[\frac{(1-\widetilde{\phi})(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma} - \widetilde{\phi}(\rho+\delta)^{1-\sigma}}\right]^{1/(1-\sigma)}. \tag{27}$$

Proof. See appendix G.[16] ■

[15] Total demand for variety $q(\widetilde{\phi})$ will be given by $Y_{cons}P^{\sigma-1}p(\widetilde{\phi})^{-\sigma} + I$ in the following. I denotes demand for variety $q(\widetilde{\phi})$ for investment purposes and Y_{cons} denotes that part of factor income which is used for consumption purposes. Since the distribution of ϕ on the interval $[\phi_L, 1]$ is exogenous, the model remains analytically solvable, even if any other variety $q(\phi)$ with $\phi \neq \widetilde{\phi}$ were used for investments.

[16] The more familiar expression for r in the steady state would result if variety $q(\widetilde{\phi})$ were taken as numéraire: $\frac{r}{p(\widetilde{\phi})} = \rho + \delta$. Equation 27 shows that r is defined for all possible values of σ only if $\widetilde{\phi} < \left(\frac{1-1/\sigma}{\rho+\delta}\right)^{1-\sigma}$. However, $\widetilde{\phi} \to \left(\frac{1-1/\sigma}{\rho+\delta}\right)^{1-\sigma}$ cannot result in general equilibrium. The argument is as follows. Assume for the moment that $\widetilde{\phi} \to \left(\frac{1-1/\sigma}{\rho+\delta}\right)^{1-\sigma}$. This implies $r \to 0$ and $\phi^* = \widetilde{\phi} = 1$ since no firm employs labor if $r \to 0$. However, if $\phi^* = \widetilde{\phi}$, all firms are identical, implying that the zero cutoff profit condition holds for the average firm. Thus, the *FEC* does not hold and less firms will enter the market. Less entry increases sales for each firm, and, thus, reduces $\widetilde{\phi}$ so that $\widetilde{\phi}$ becomes strictly smaller than $\left(\frac{1-1/\sigma}{\rho+\delta}\right)^{1-\sigma}$.

The time index t has been removed from equation 27 since it denotes a relationship in the steady state. Equation 27 shows that the parameters σ, ρ and δ determine whether $r > 1$ or $r < 1$ in the steady state. This leads to lemma 7:

Lemma 7 *If* $1 < \frac{1-1/\sigma}{\rho+\delta}$*, then* $r < 1$ *in the steady state. Conversely, if* $1 > \frac{1-1/\sigma}{\rho+\delta}$*, then* $r > 1$ *in the steady state.*

Lemma 7 shows that it does not depend on relative factor endowments or the firm distribution whether $r < 1$ or $r > 1$ in the steady state.[17] Equation 27 implies lemma 8:

Lemma 8 *If* $r < 1$*, an increase in* $\widetilde{\phi}$ *decreases the steady state value of* r*. Conversely, if* $r > 1$*, an increase in* $\widetilde{\phi}$ *increases the steady state value of* r*.*

Proof. See appendix H. ■

Finally, lemma 7 and equation 20 imply lemma 9:

Lemma 9 $\phi_X^* > \phi^*$ *in the open economy equilibrium, i.e. only the more capital intensive firms export, if (i)* $f_X > f$ *and (ii)* $1 < \frac{1-1/\sigma}{\rho+\delta}$*. Otherwise,* $\phi_X^* < \phi^*$ *in the open economy equilibrium, i.e. only the more labor intensive firms export.*

4.2 Closed economy steady state

Since the steady state value of r is determined by ρ, δ, σ and $\widetilde{\phi}$, I can use the zero cutoff profit condition, the free entry condition and the factor market equilibrium conditions to solve successively for $q(\widetilde{\phi})$, $\widetilde{\phi}$ and K in the closed economy steady state.

First, substituting r from equation 27 into the zero cutoff profit condition for the supply to the domestic market (equation 10) leads to:

$$q(\widetilde{\phi}) = \frac{(\sigma-1)(1-\widetilde{\phi})\left(1-1/\sigma\right)^{1-\sigma} f}{(\phi^*-\widetilde{\phi})(\rho+\delta)^{1-\sigma}+(1-\phi^*)\left(1-1/\sigma\right)^{1-\sigma}}. \tag{28}$$

Second, substituting $q(\widetilde{\phi})$ (equation 28) into the closed economy *FEC* (equation 13) leads to:

$$\frac{f_E}{f}\frac{\rho+\theta}{1+\rho} = \Theta(\phi^*)\,\frac{1-\frac{(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}}}{\frac{(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}}+\frac{1-\phi^*}{\phi^*-\widetilde{\phi}}}, \tag{29}$$

with $\Theta(\phi^*) = 1 - G(\phi^*)$ if $1 < \frac{1-1/\sigma}{\rho+\delta}$ and $\Theta(\phi^*) = G(\phi^*)$ if $1 > \frac{1-1/\sigma}{\rho+\delta}$. Equation 29 leads to proposition 5:

[17] Notice that, with yearly capital depreciation and time discount rates of around 0.1 and 0.05, respectively, the condition for $r > 1$ would require unreasonably low values for σ. However, if I do not impose the normalization $A_K = A_L = 1$, the relationship between ϕ and a firm's profits would, instead, depend on how r relates to $\frac{A_K}{A_L}$: profits increase (decrease) with ϕ if $r < \frac{A_K}{A_L}$ $\left(r > \frac{A_K}{A_L}\right)$. Furthermore, $r < \frac{A_K}{A_L}$ if $\frac{1-1/\sigma}{\rho+\delta} > A_K$ and $r > \frac{A_K}{A_L}$ if $\frac{1-1/\sigma}{\rho+\delta} < A_K$. Thus, if A_K is sufficiently larger than 1, a negative relationship between profits and the capital share parameter ϕ could also result for reasonable values for ρ, δ and σ.

Proposition 5 *If ϕ is distributed on $[\phi_L, 1]$ according to a Pareto–distribution, the average capital share parameter $\widetilde{\phi}$ in the closed economy steady state is uniquely determined by σ (elasticity of substitution), ρ (time discount rate), δ (capital depreciation rate), θ (death probability) and the fixed costs parameters f and f_E.*

Proof. See appendix I. ■

Third, the additional factor demands for producing variety $q(\widetilde{\phi})$ for investment purposes add to the factor market equilibrium conditions (equations 14 and 15). Thus, the factor market equilibrium conditions in the autarkic steady state are given by:

$$(1-\widetilde{\phi})\, c(\widetilde{\phi})^{\sigma} \left\{ N \left[q(\widetilde{\phi}) + \widetilde{f} \right] + \delta K \right\} = L \tag{30}$$

$$\widetilde{\phi}\, r^{-\sigma}\, c(\widetilde{\phi})^{\sigma} \left\{ N \left[q(\widetilde{\phi}) + \widetilde{f} \right] + \delta K \right\} = K. \tag{31}$$

Solving equations 30 and 31 for K in the autarkic steady state leads to:

$$K = \frac{\widetilde{\phi}}{1-\widetilde{\phi}} r^{-\sigma} L = \frac{\widetilde{\phi}\,(1-\widetilde{\phi})^{1/(\sigma-1)}(\rho+\delta)^{-\sigma} L}{\left[(1-1/\sigma)^{1-\sigma} - \widetilde{\phi}\,(\rho+\delta)^{1-\sigma}\right]^{\sigma/(\sigma-1)}}. \tag{32}$$

Equation 32 leads to proposition 6:

Proposition 6 *For any given value of r, the relative capital stock $\frac{K}{L}$ can take any value from the interval $[0, \infty)$, depending on the general equilibrium value for $\widetilde{\phi}$.*

Proof. See appendix J. ■

Importantly, proposition 6 implies that, in general equilibrium, r can be smaller or larger than 1, regardless of whether $\frac{K}{L} > 1$ or $\frac{K}{L} < 1$. The reason is that only the parameters σ, ρ and δ determine how the steady state level of r relates to 1 (lemma 7). However, the steady state level of K adjusts according to relative capital demand, which also depends on $\widetilde{\phi}$. For example, both $r < 1$ and $\frac{K}{L} < 1$ can result in general equilibrium if the distribution of ϕ on $[\phi_L, 1]$ is sufficiently skewed to the right so that $\widetilde{\phi}$ is sufficiently small (see also lemma 1).

Figure 1 illustrates the autarkic steady state. The *steady state*–curve results from equation 27 and is drawn for the case of $1 < \frac{1-1/\sigma}{\rho+\delta}$. The intersection point between the *FEC*–curve and the *steady state*–curve determines the steady state values for r and $\widetilde{\phi}$. In the long–run, the capital stock K adjusts so that the $\frac{L}{K}$–curve goes through the intersection point between the *FEC*–curve and the *steady state*–curve.

4.3 Open economy steady state

Again, since $\phi_X^* > \phi^*$ is the empirically more relevant case, I will analyze here trade liberalization for symmetric countries for which conditions (i) and (ii) of lemma 9 hold.

The steady state value of r (equation 27) is now substituted into the open economy equilibrium conditions, which can be solved successively for $q(\widetilde{\phi})$, $\widetilde{\phi}$, ϕ^*_X and K in the open economy steady state.

First, the zero cutoff profit condition for exports (equation 19) is transformed to:

$$q_X(\widetilde{\phi}_X) \;=\; (\sigma-1)\,\frac{c(\widetilde{\phi})}{c(\phi^*_X)}\,\frac{c(\phi^*_X)^\sigma}{c(\widetilde{\phi}_X)^\sigma}\,f_X. \tag{33}$$

Equation 33 shows that $q_X(\widetilde{\phi}_X)$ in the open economy steady state is determined by the parameters σ, ρ, δ, f, f_X and the steady state value for ϕ^*.[18]

Second, substituting $q(\widetilde{\phi})$ (equation 28) and $q_X(\widetilde{\phi}_X)$ (equation 33) into the open economy free entry condition (equation 21) and simplification leads to:

$$\frac{f_E}{f}\,\frac{\rho+\theta}{1+\rho} \;=\; \Theta(\phi^*)\frac{1-\frac{(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}}}{\frac{(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}}+\frac{1-\phi^*}{\phi^*-\widetilde{\phi}}}+\Theta(\phi^*_X)\frac{\left[1-\frac{(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}}\right]\frac{f_X}{f}}{\frac{\phi^*_X-\widetilde{\phi}}{\phi^*_X-\widetilde{\phi}_X}\frac{(\rho+\delta)^{1-\sigma}}{(1-1/\sigma)^{1-\sigma}}+\frac{1-\phi^*_X}{\phi^*_X-\widetilde{\phi}_X}}. \tag{34}$$

Comparing equation 34 with equation 29 shows that trade liberalization adds the ex–ante expected export profits, the second term, to the right hand side of the free entry condition. Equation 34 leads to proposition 12:

Proposition 7 *If ϕ is distributed on $[\phi_L, 1]$ according to a Pareto–distribution, the average capital share parameter $\widetilde{\phi}$ in the open economy steady state is uniquely determined by σ (elasticity of substitution), ρ (time discount rate), δ (capital depreciation rate), θ (death probability) and the fixed costs parameters f, f_E and f_X.*

Proof. The proof is along the same lines as the one for proposition 5: given a Pareto–distribution for ϕ on $[\phi_L, 1]$, equation 34 holds for a unique level of ϕ^*. ■

Third, the additional factor demands for producing variety $q(\widetilde{\phi})$ for investment purposes add to the factor market equilibrium conditions (equations 22 and 23). The factor market equilibrium conditions in the open economy steady state are given by:

$$(1-\widetilde{\phi})\,c(\widetilde{\phi})^\sigma\left[q(\widetilde{\phi})\,N\frac{1-\widetilde{\phi}+s_X(1-\widetilde{\phi}_X)}{1-\widetilde{\phi}}+\widetilde{f}N+f_Xs_XN+\delta K\right] \;=\; L \tag{35}$$

$$\widetilde{\phi}\,\frac{c(\widetilde{\phi})^\sigma}{r^\sigma}\left[q(\widetilde{\phi})\,N\frac{\widetilde{\phi}+s_X\widetilde{\phi}_X}{\widetilde{\phi}}+\widetilde{f}\,N+f_Xs_XN+\delta K\right] \;=\; K. \tag{36}$$

Solving equations 35 and 36 for K leads to:

$$K \;=\; \frac{\widetilde{\phi}r^{-\sigma}\,L\,\Omega}{1-\widetilde{\phi}}, \quad \text{with} \;\; \Omega \;=\; \frac{\widetilde{f}\left(1+\frac{s_Xf_X}{f}\right)+q(\widetilde{\phi})\left(1+\frac{s_X\widetilde{\phi}_X}{\widetilde{\phi}}\right)}{\widetilde{f}\left(1+\frac{s_Xf_X}{f}\right)+q(\widetilde{\phi})\left(1+s_X\frac{1-\widetilde{\phi}_X}{1-\widetilde{\phi}}\right)+q(\widetilde{\phi})\frac{s_Xc(\widetilde{\phi})^\sigma}{\delta^{-1}r^\sigma}\frac{\widetilde{\phi}_X-\widetilde{\phi}}{1-\widetilde{\phi}}}. \tag{37}$$

[18] See appendix K for the proof.

4.4 The impact of trade liberalization — long–run

As in the short–run, trade liberalization ceteris paribus has a positive impact on $\widetilde{\phi}$ since new profit opportunities abroad increase market entry. In the long–run, the negative impact on $\widetilde{\phi}$ due to increased factor market competition is eliminated. The reason is that the increase in relative capital demand induces households to investment more. Thus, in the long–run, the relative capital endowment $\frac{K}{L}$ reacts to increased competition for capital. The average capital share parameter $\widetilde{\phi}$ accordingly increases with trade liberalization in the long–run. This leads to proposition 8:

Proposition 8 *If $r < 1$ in the autarkic equilibrium and if ϕ is distributed on $[\phi_L, 1]$ according to a Pareto–distribution, the average capital share parameter $\widetilde{\phi}$ increases in the long–run after trade liberalization.*

Proof. See appendix L. ■

The increase in $\widetilde{\phi}$ in the long–run after trade liberalization implies the following further adjustments to trade liberalization, as summarized by proposition 9:

Proposition 9 *Trade liberalization has the following long–run impact on either country:* (*i*) *the capital stock K increases;* (*ii*) *the relative return to capital r decreases;* (*iii*) *the average firm produces more capital intensively;* (*iv*) *the price $p(\widetilde{\phi})$ of the average variety decreases;* (*v*) *labor's share in national income $\frac{L}{L+r\,K}$ decreases;* (*vi*) *the price index P decreases;* (*vii*) *the real returns to capital and labor increase.*

Proof. See appendix M. ■

The intuition is as follows. Since r increases in the short–run after trade liberalization, households increase investments, which increases K as the country approaches the new steady state. A higher K implies a lower steady state level of r. The average firm produces more capital intensively in the new steady state since $\widetilde{\phi}$ has increased and r has decreased. $p(\widetilde{\phi})$ decreases due to the decrease of r and the increase of $\widetilde{\phi}$. I can show formally that net effect of trade liberalization on labor's share in national income is positive, even though r decreases, while K increases. The decrease of P results from the decrease of $p(\widetilde{\phi})$ and the availability of additional varieties. Finally, the real returns to both factors increase since P decreases.

Figure 2 illustrates a country's adjustment from the autarkic equilibrium to the short–run trading equilibrium and, finally, to the long–run trading equilibrium. The adjustment from the short–run to the long–run trading equilibrium results from the increase in K, which shifts the $\frac{L}{K}$–curve leftward (from "*L/K–curve (ft, short–run)*" to "*L/K–curve (ft, long–run)*").

5 Asymmetric countries

In this section I analyze trade liberalization between asymmetric countries and, thus, include country indices $i, j = H, F$. Country H is still characterized by $r_H < 1$, while country F is characterized by $r_F > 1$ in this section. Countries H and F differ only with respect to those parameters which determine whether $r < 1$ or $r > 1$, but are still symmetric in all other respects.

Importantly, country H's trading pattern, i.e. whether country H's exporters are more or less capital intensive than its non–exporters, depends on how r_H compares to unity. The relationship between r_H and r_F, in contrast, does not play any role for country H's trading pattern. The reason is that countries engage in 'new' trade in this setting, not in Heckscher–Ohlin trade. This implies that foreign households do not demand a domestic variety because it is cheaper, but because it is different from the foreign variety. Thus, a domestic firm exports as long as its marginal costs are sufficiently small so that variable export profits cover fixed export costs. The relationship between marginal costs and factor intensities depends on whether $r_H < 1$ or $r_H > 1$ (see subsection 2.4). Thus, country H's trading pattern is not affected by dropping the assumption of symmetry across countries.

As a consequence, I will focus in this section only on (i) the impact of trade liberalization on country F and (ii) how country F's characteristics impact country H's threshold capital share parameter $\phi^*_{H,X}$.

To describe the equilibria in this section, I will mostly use the previously derived equilibrium conditions, but complement them with country indices $i, j = H, F$. Country F's autarkic equilibrium with $r_F > 1$ is illustrated by figure 3, and I will start the formal analysis directly with the open economy equilibrium for country F.

[Figure 3 about here]

5.1 Open economy equilibrium — short–run analysis

Country i's threshold capital share parameter $\phi^*_{i,X}$, which separates exporters from non–exporters, results from jointly solving equations 9 and 19 for $\phi^*_{i,X}$:

$$\phi^*_{i,X} = \frac{Y_i P_i^{\sigma-1}}{Y_j P_j^{\sigma-1}} \frac{f_X}{f} \phi^*_i + \frac{\frac{Y_i P_i^{\sigma-1}}{Y_j P_j^{\sigma-1}} \frac{f_X}{f} - 1}{r_i^{1-\sigma} - 1}, \quad i, j = H, F, i \neq j. \tag{38}$$

Lemma 10 describes the condition that must be imposed on $\frac{f_X}{f}$, so that only part of country i's domestically active firms export:

Lemma 10 *If* $\frac{f_X}{f} > \frac{Y_j P_j^{\sigma-1}}{Y_i P_i^{\sigma-1}}$, *only part of country i's domestically active firms exports.*

Proof. See appendix N. ■

I will assume here that f_X is sufficiently large so that $\frac{f_X}{f} > \frac{Y_j P_j^{\sigma-1}}{Y_i P_i^{\sigma-1}}$ always holds and, as a consequence, $\phi^*_{H,X} > \phi^*_H$ and $\phi^*_{F,X} < \phi^*_F$. $\phi^*_{F,X} < \phi^*_F$ implies that the exporting firms are more labor intensive than the non–exporting firms in country F.

While the relationship between r_i and unity alone determines whether $\phi^*_{i,X} \in [\phi^*_i, 1]$ or $\phi^*_{i,X} \in [\phi_L, \phi^*_i]$, the trading partner's characteristics still impact the magnitude of $\phi^*_{i,X}$ within either of these two intervals. This leads to proposition 10:

Proposition 10 *For given ϕ^*_i and $\frac{f_X}{f}$, the magnitude of $\phi^*_{i,X}$ depends on the economic size of country i relative to country j. If country F becomes larger (smaller) relative to country H, both $\phi^*_{H,X}$ and $\phi^*_{F,X}$ decrease (increase).*

Proof. See appendix O. ■

The intuition for proposition 10 is as follows. If the relative size of country F increases, its export volume ceteris paribus increases since the absolute number of country F exporters becomes larger. To keep trade balanced, both $\phi^*_{H,X}$ and $\phi^*_{F,X}$ decrease so that the share of exporters in country H's (F's) firm distribution increases (decreases).

Equation 38 also shows that relative factor prices r_H and r_F impact $\phi^*_{H,X}$ and $\phi^*_{F,X}$, both directly and indirectly via their impact on $Y_H P_H^{\sigma-1}$ and $Y_F P_F^{\sigma-1}$. Thus, the relationship between $\phi^*_{i,X}$ and r_H and r_F is not linear. Still, $r_i \to 1$ implies that $s_{H,X} \to 0$ and $s_{F,X} \to 0$.[19] The intuition is that, if $r_i \to 1$, even those country i firms with the most favorable ϕ cannot afford to export. Thus, $s_{i,X} = 0$ if $r_i \to 1$. Balanced trade implies that also $s_{j,X} = 0$ if $s_{i,X} = 0$.

Finally, equation 38 allows for the conclusion that, if f_X is chosen appropriately, the trading equilibrium with asymmetric countries mimics the one with symmetric countries. The reason is that country j influences country i only via its impact on $\phi^*_{i,X}$.[20] Once $\phi^*_{i,X}$ is determined, country i's trading equilibrium does not depend on whether countries are symmetric or not. If f_X is chosen appropriately, $\phi^*_{i,X}$ in the asymmetric trading equilibrium (equation 38) equals $\phi^*_{i,X}$ in the symmetric trading equilibrium (equation 20).

To derive the short–run impact of trade liberalization on country F, I first consider how trade liberalization shifts country F's *FEC*–curve. Comparing equation 13 with equation 21 and considering that $r_F > 1$, which implies $\Theta(\phi^*_F) = G(\phi^*_F)$, $\Theta(\phi^*_{F,X}) = G(\phi^*_{F,X})$, $\widetilde{\phi}_F = \int_{\phi_L}^{\phi^*_F} \phi \frac{g(\phi)}{G(\phi^*_F)} d\phi$ and $\widetilde{\phi}_{F,X} = \int_{\phi_L}^{\phi^*_{F,X}} \phi \frac{g(\phi)}{G(\phi^*_{F,X})} d\phi$, leads to lemma 11:

[19] See appendix O for the proof.

[20] Notice that, after including a country index i, equation 19 can be solved for the equilibrium sales of country i's average exporter to country j: $q_X(\widetilde{\phi}_{i,X}) = (\sigma-1)\frac{c(\widetilde{\phi}_i)}{c(\phi^*_{i,X})}\frac{c(\phi^*_{i,X})^\sigma}{c(\widetilde{\phi}_{i,X})^\sigma} f_X$. Thus, in equilibrium, country j influences $q_X(\widetilde{\phi}_{i,X})$ only via its impact on $\phi^*_{i,X}$.

Lemma 11 *For given relative factor returns r_F, trade liberalization decreases the threshold capital share parameter ϕ_F^*, i.e. country F's FEC–curve shifts downward.*

Proof. See appendix P. ■

Figure 4 illustrates the shift of country F's *FEC*–curve (from "*FEC–curve (aut)*" to "*FEC–curve (ft)*"). Notice that country F's *FEC*–curve does not change its intersection point with the vertical $r = 1$–line since no firm exports if $r_F \rightarrow 1$.

Comparing equation 24 with equation 16 reveals that trade liberalization increases relative labor demand in country F. This is because $r_F > 1$, which implies $\widetilde{\phi}_F > \widetilde{\phi}_{F,X} > \phi_L$ and, thus, $\Lambda_F > 1$. The resulting shift of country F's $\frac{L}{K}$–curve is explained by lemma 12:

Lemma 12 *Trade liberalization decreases the relative return to capital r_F for a given $\widetilde{\phi}_F$. Country F's $\frac{L}{K}$–curve shifts to the left.*

Proof. The proof of lemma 4 has shown that an increase in r decreases relative capital demand. This implies that a decrease in r decreases relative labor demand. ■

Figure 4 illustrate the shift of country F's $\frac{L}{K}$–curve (from "*L/K–curve (aut)*" to "*L/K–curve (ft, short–run)*"). Notice that, again, the $\frac{L}{K}$–curve after trade liberalization converges to the $\frac{L}{K}$–curve before trade liberalization if r_F is sufficiently close to 1.

Thus, neither country F's *FEC*–curve nor its $\frac{L}{K}$–curve change their intersection point with the vertical $r = 1$–line due to trade liberalization. Furthermore, since the *FEC*–curve has a non–positive slope for all possible ϕ, while the $\frac{L}{K}$–curve is monotonously upward sloping, I can formulate lemma 13:

Lemma 13 *If $r_F > 1$ in the autarkic equilibrium, $r_F > 1$ also in the trading equilibrium.*

Thus, if $r_H < 1$ and $r_F > 1$ in the countries' autarkic equilibria, the same holds also in the trading equilibrium. Considering how country F's *FEC*–curve and $\frac{L}{K}$–curve shift due to trade liberalization leads to proposition 11:

Proposition 11 *Trade liberalization has the following short–run impact on country F:*

(i) r_F decreases in the short–run after trade liberalization;

(ii) $\widetilde{\phi}_F$ decreases in the short–run after trade liberalization if f_X is sufficiently close to $\frac{Y_H P_H^{\sigma-1}}{Y_F P_F^{\sigma-1}} f$. Otherwise, $\widetilde{\phi}_F$ increases in the short–run after trade liberalization.

Proof. See appendix Q. ■

5.2 Open economy equilibrium — long–run analysis

The steady state value of r_i (equation 27) is now substituted into the open economy equilibrium conditions, which can be jointly solved for $q(\widetilde{\phi}_i)$, $\widetilde{\phi}_i$, $\phi^*_{i,X}$ and K_i, $i = H, F$, in the open economy steady state. The capital depreciation rate δ_i and the time discount rate ρ_i get a country index i in order to allow for $r_H < 1$ and, at the same time, $r_F > 1$.

First, equation 33 implies that $q_X(\widetilde{\phi}_{i,X})$ in the open economy steady state is a function of the parameters σ, ρ_i, ρ_j, δ_i, δ_j, f, f_X and the steady state values for ϕ^*_i and ϕ^*_j.[21]

Second, considering country i's free entry condition in the steady state with asymmetric countries leads to proposition 12:

Proposition 12 *If ϕ is distributed on $[\phi_L, 1]$ according to a Pareto–distribution, a finite number of open economy steady states exists. The average capital share parameters $\widetilde{\phi}_H$ and $\widetilde{\phi}_F$ in either steady state are determined by: σ (elasticity of substitution), ρ_H and ρ_F (time discount rates), δ_H and δ_F (capital depreciation rates), θ (death probability) and the fixed costs parameters f, f_E, f_X.*

Proof. See appendix S. ■

Proposition 12 implies that the quantitative impact of trade liberalization between asymmetric countries is not necessarily unique, since the magnitude of the upward (downward) shift of country H's (F's) *FEC*–curve is not necessarily unique. The reason is that ϕ^*_j impacts $\phi^*_{i,X}$ (equation 38), while, at the same time, $\phi^*_{i,X}$ impacts ϕ^*_j via its influence on ϕ^*_i. Still, it is unambiguous that trade liberalization shifts country H's (F's) *FEC*–curve upward (downward) (see lemma 3 and lemma 11). Thus, the qualitative impact of trade liberalization is unique, and I can formulate proposition 13:

Proposition 13 *If $r_F > 1$ in the autarkic equilibrium and if ϕ is distributed on $[\phi_L, 1]$ according to a Pareto–distribution, the average capital share parameter $\widetilde{\phi}_F$ and the relative return to capital r_F decrease in the long–run after trade liberalization.*

Proof. Trade liberalization shifts country F's *FEC*–curve downward (lemma 11), while country F's *steady state*–curve has a positive slope (see figure 4 and lemma 8). Thus, the new intersection point of the two curves implies lower levels of $\widetilde{\phi}_F$ and r_F. ■

Third, equation 37 reveals that the long–run impact of trade liberalization on country F's capital stock K_F is ambiguous since both $\widetilde{\phi}_F$ and r_F decrease in the long–run. While the decrease in $\widetilde{\phi}_F$ ceteris paribus decreases K_F, the decrease in r_F ceteris paribus increases K_F. Thus, the long–run impact of trade liberalization on K_F can be positive or negative.[22]

[21] See appendix R for the proof.
[22] See appendix T for the proof.

Finally, the long–run impact of trade liberalization on country F's welfare is also ambiguous. While trade liberalization ceteris paribus exerts a positive variety and price effect, a decrease in K_F ceteris paribus decreases country F's welfare. I show in appendix T that the net effect on country F's welfare can be negative if K_F decreases with trade liberalization.

Figure 4 illustrates country F's adjustment from autarky to the short and long–run trading equilibrium. The adjustment from the short to the long–run trading equilibrium results from the adjustment in K_F, which shifts the $\frac{L}{K}$–curve to the intersection point between the open economy *FEC*–curve and the *steady state*–curve. The *steady state*–curve is drawn with a larger slope than the $\frac{L}{K}$–curve. In this case K_F increases during the long–run adjustment to the trading equilibrium. If the *steady state*–curve were drawn with a smaller slope than the $\frac{L}{K}$–curve, K_F could also decrease during the long–run adjustment to the trading equilibrium. Appendix T shows that both, an increase and a decrease in K_F are possible.

[Figure 4 about here]

6 A numerical exercise

The analytical results are able to qualitatively replicate many of the empirical relationships between international trade, factor market competition, factor relocations and factor returns which are described in the introduction. How does the model perform quantitatively? To evaluate this, I take a simulation approach, calibrate the model parameters with recent empirical estimates for the US and compare the simulated outcomes with those for the US.

I limit the analysis to the open economy steady state. The focus on the steady state seems to be reasonable for the purposes of this paper since the establishment of the North American Free Trade Agreement in 1994 can be considered as the last major trade shock for the US. Indeed, the UNCTAD comtrade database shows that the US trade intensity (exports plus imports, divided by GNI) has stabilized at around 0.2 since then.

I aggregate the US trading partners to a single foreign country. In addition, I assume that the US and the single foreign country are symmetric and characterized by $r < 1$. The reasons for these two assumptions are the following. First, the UNCTAD comtrade database reveals that US trade (exports plus imports) with other OECD countries has been around two thirds of total US trade in the last decade. Thus, the largest part of US trade has been with countries that have relative factor endowments comparable to those of the US. Second, I have argued in section 5 that a country's trading pattern does not depend on the trading partner's relative factor prices, but only on how domestic relative factor prices compare to unity. Thus, the assumption of symmetry across countries simplifies the simulation analysis, but is not necessary to obtain the results which are presented below.

The parametrization of the baseline model is as follows. First, I assume that ϕ follows a Pareto distribution over the interval $[0.025, 1]$. Smaller supports, e.g., $[0.05, 0.95]$, lead to similar outcomes which are not reported here. Most other papers on international trade with firm heterogeneity also assume a Pareto distribution for the technology parameter (e.g., Bernard et al. 2007). While the technology parameter in those papers is a TFP parameter, it is the capital share parameter of a CES production function here. I will perform the baseline simulation with a shape parameter k equal to 5.

Second, most studies on US manufacturing sectors estimate an elasticity of substitution between input factors in production of $\sigma = 2$ (e.g., Acemoglu, 2002, and the literature reviewed therein). Concerning the elasticity of substitution between varieties in consumption (ξ), Broda and Weinstein (2006) report for US imports within three–digit SITC categories an average value of around 4. Thus, I will choose $\sigma = 2$ and $\xi = 4$ for the baseline simulation.[23]

Third, most of the literature uses quarterly discount factors of 0.01 and quarterly capital depreciation rates of 0.025 for the US and other developed economies (Tsoukalas, 2011). Furthermore, time–to–build has been estimated to be between 4 and 8 quarters for most industries (Koeva, 2000). Thus, I assume that a period t in the present setup corresponds to one year, and I get a yearly discount factor of $\rho = 0.05$ and a yearly depreciation rate of $\delta = 0.105$. These values are in line with those used in related simulation studies (e.g., Backus et al., 1992; Jonsson and Klein, 2006) and imply that $r < 1$ in the steady state (lemma 7).

Fourth, empirical estimates for the firm turnover rate θ and the fixed costs parameters f_E and f are, if available at all, industry–specific. Furthermore, the free entry condition (equation 34) shows that θ and the ratio $\frac{f_E}{f}$ jointly influence the equilibrium capital share parameter $\widetilde{\phi}$. I find that the simulation leads to outcomes which fit well actual US data if: $\theta = 0.02$, $\frac{f}{f_E} = 4$.

Fifth, since L only scales the economy to a larger or smaller size, I normalize it to unity.

In order to evaluate the model, I will determine the open economy steady state and choose a value of $\frac{f_X}{f} = 1.1167$ for the ratio of fixed export costs relative to fixed production costs. This leads to a share of exporters in the firm distribution of $s_X = 0.15$. This is the average magnitude across three–digit NAICS US manufacturing sectors (Bernard et al., 2007).

The first row of table 1 displays the empirical estimates for three–digit NAICS US manufacturing sectors in 2002 (Bernard et al., 2007). The second row displays the results of the baseline simulation. The remaining rows report the findings of the sensitivity analyses, in which I consider alternative values for the parameters f_X, ξ, f and k. I start with changing f_X, so

[23]Notice that, since $\sigma \neq \xi$, the average capital share parameter $\widetilde{\phi}$ results as the solution to the following equation: $c(\widetilde{\phi})^{1-\xi} = \int_{\underline{\phi}}^{\overline{\phi}} \left(\phi r^{1-\sigma} + 1 - \phi\right)^{\frac{1-\xi}{1-\sigma}} \mu(\phi) d\phi$ (see also equation 6).

that s_X becomes 0.1 in one case and 0.2 in the other. Both these values for s_X are empirically relevant (Bernard et al., 2007). Afterwards, I perform the simulation with $\xi = 3$ and $\xi = 5$. Both values for ξ are within the empirically relevant range (Broda and Weinstein, 2006). Finally, I try alternative values for f and k without direct reference to the empirical literature.

The variable $\frac{\widetilde{(k/l)}_X}{\widetilde{(k/l)}_N}$ stands for the capital–labor input ratio of the average exporter, relative to that of the average non–exporter. The variable $\frac{[q(\widetilde{\phi}_X)+q_X(\widetilde{\phi}_X)]p(\widetilde{\phi}_X)}{q(\widetilde{\phi}_N)p(\widetilde{\phi}_N)}$ denotes the value of total shipments of exporters, relative to the value of total shipments of non–exporters. $\frac{\widetilde{l}_X}{\widetilde{l}_N}$ denotes labor employment of the average exporter, relative to that of the average non–exporter. Finally, the variable $\frac{exports}{total\ shipments}$ measures the sector's export intensity, i.e. the value of the sector's exports, divided by the value of the sector's total shipments.

[Table 1 about here]

Table 1 illustrates that the baseline simulation replicates all four variables fairly well. The sensitivity analyses show that those variables, which measure the dissimilarities between exporters and non–exporters (columns 2–4) are fairly robust to alternative parameter values. The sector's export intensity, in contrast, is rather sensitive, especially to alternative values for ξ and f. The underlying reason is that the variable s_X is rather sensitive and ranges from $s_X = 0.0327$ if $\frac{f}{f_E} = 3.6$ up to $s_X = 0.4112$ if $\xi = 5$. However, we can observe a comparable range for s_X also in the data. While Bernard et al. (2007) report an average magnitude of $s_X = 0.15$, it ranges from $s_X = 0.02$ in some sectors up to $s_X = 0.4$ in others.

The sensitivity analyses also highlight the channels that are at work in this model. First, the elasticity parameter ξ reflects the degree of competition between firms, which produce different varieties. If ξ becomes larger, varieties become more substitutable and competition on goods markets increases. Thus, when ξ becomes larger, ϕ^* increases, implying that the dissimilarities between the average exporter and non–exporter become smaller and the sector's export intensity increases. Second, the fixed costs ratio $\frac{f}{f_E}$ impacts the threshold capital share parameter ϕ^*. If $\frac{f}{f_E}$ increases, ϕ^* becomes larger. A larger ϕ^* implies, again, that the dissimilarities between the average exporter and non–exporter become smaller and the sector's export intensity increases. Finally, a change in k alters the skewness of the Pareto–distribution for ϕ. If k increases, the Pareto–distribution becomes more skewed to the right, ϕ_X^* ceteris paribus falls and the average exporter and non–exporter become more alike. On the other hand, if the Pareto–distribution becomes more skewed to the right, s_X ceteris paribus decreases. In the specific numerical setup, the sector's export intensity falls with the increase in k.

7 Conclusions

Empirical research suggests that firm heterogeneity in factor input ratios is crucial in order to understand firm selection into export markets. Therefore, this paper extends the existing trade literature by a new dimension of firm heterogeneity. Firms in this paper produce with capital and labor and profit maximizing firms choose different capital–labor input ratios.

This paper has analyzed the short and long–run impact of globalization and has also highlighted the role of factor market competition for the adjustment of the industry to trade liberalization. The analytical results in this paper are able to account for several empirical findings on the firm–level and industry–level adjustments to trade liberalization.

Furthermore, this paper has highlighted under which conditions exporters are more capital intensive than non–exporters. If this is the case, trade liberalization has the following short–run impact on the economy: the returns to capital increase and labor's share in national income decreases, while the firm distribution may change in favor of more labor intensive firms or in favor of more capital intensive firms. Considering the long–run, this paper has shown that the increase in the returns to capital induce households to invest more. The corresponding increase in the countries' capital endowments has decreased the long–run returns to capital, has increased the real returns to all factors of production and has shifted the firm distribution in favor of more capital intensive firms.

Thus, this paper has also established a new mechanism in order to explain the positive growth impact of trade liberalization. Furthermore, even though trade liberalization changes a country's income distribution, this paper has shown that, in the long–run, all factors benefit. Future research could try to empirically validate the specific channels, which have lead to the short and long–run impact of trade liberalization.

Appendix

A *FEC*–curve

Considering equation 10, the free entry condition (equation 13) can be transformed to:

$$\frac{f_E}{f}\frac{\rho+\theta}{1+\rho} \quad = \quad \underbrace{\Theta(\phi^*)\left[\frac{\widetilde{\phi}\,r^{1-\sigma}+1-\widetilde{\phi}}{\phi^*\,r^{1-\sigma}+1-\phi^*}-1\right]}_{\equiv\,\Xi(\phi^*,r)}. \tag{39}$$

$\Theta(\phi^*)$ denotes the probability for a successful market entry. Total differentiation leads to:

$$0 \quad = \quad \Theta(\phi^*)\frac{(\widetilde{\phi}\,r^{1-\sigma}+1-\widetilde{\phi})\,\left(1-r^{1-\sigma}\right)}{\left(\phi^*\,r^{1-\sigma}+1-\phi^*\right)^2}\,d\phi^*+\Theta(\phi^*)\frac{(\widetilde{\phi}-\phi^*)(1-\sigma)r^{-\sigma}}{\left(\phi^*\,r^{1-\sigma}+1-\phi^*\right)^2}\,dr. \tag{40}$$

Thus, I get: $\frac{d\phi^*}{dr}=\frac{(\widetilde{\phi}-\phi^*)(\sigma-1)r^{-\sigma}}{(\widetilde{\phi}\,r^{1-\sigma}+1-\widetilde{\phi})\,(1-r^{1-\sigma})}$. If $r<1$, then $\Theta(\phi^*)=1-G(\phi^*)$ and $\widetilde{\phi}-\phi^*>0$. If $r>1$, then $\Theta(\phi^*)=G(\phi^*)$ and $\widetilde{\phi}-\phi^*<0$.

From this follows that $\frac{d\phi^*}{dr}<0$, both with $r<1$ and $r>1$. Thus, the ϕ^*–curve has a negative slope. The only exception is the part in which r is close to unity since all firms produce if $r\rightarrow1$, regardless of their ϕ. Thus, if r approaches unity from below (above), ϕ^* reaches its lower (upper) bound, the *FEC*–curve has a kink and becomes horizontal. Furthermore, if $r\rightarrow\infty$, then $\phi^*\rightarrow0$ since no firm produces with capital if its price goes to infinity. Since $\frac{\partial\widetilde{\phi}}{\partial\phi^*}>0$ due to the Leibniz rule, $\widetilde{\phi}$ reacts to r the same way as ϕ^* does.

B Closed economy factor market equilibrium conditions

Applying Shephard's Lemma to the marginal cost function (equation 2) and considering demand $q(\phi)$ for a single variety (equation 5) leads to the factor market equilibrium conditions:

$$\frac{Y\,N}{P^{1-\sigma}}\int_{\underline{\phi}}^{\overline{\phi}}(1-\phi)\left[\frac{c\,(\phi)}{p\,(\phi)}\right]^{\sigma}\mu(\phi)d\phi+c(\widetilde{\phi})^{\sigma}\,(1-\widetilde{\phi})\,\widetilde{f}\,N \quad = \quad L \tag{41}$$

$$\frac{Y\,N}{P^{1-\sigma}\,r^{\sigma}}\int_{\underline{\phi}}^{\overline{\phi}}\phi\left[\frac{c\,(\phi)}{p\,(\phi)}\right]^{\sigma}\mu(\phi)d\phi+\frac{c(\widetilde{\phi})^{\sigma}}{r^{\sigma}}\,\widetilde{\phi}\,\widetilde{f}\,N \quad = \quad K, \tag{42}$$

with $\mu(\phi)=\frac{g(\phi)}{G(\overline{\phi})-G(\underline{\phi})}$. The boundaries $\underline{\phi}$ and $\overline{\phi}$ of the integrals depend on whether $r<1$ or $r>1$ in general equilibrium. Considering $\left[\frac{c(\phi)}{p(\phi)}\right]^{\sigma}=\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}=\left[\frac{c(\widetilde{\phi})}{p(\widetilde{\phi})}\right]^{\sigma}$ and $q(\widetilde{\phi})=Y\,P^{\sigma-1}\,p(\widetilde{\phi})^{-\sigma}$, equations 41 and 42 can be transformed to equations 14 and 15.

C Proof of proposition 2

The partial derivative of ϕ^*_X with respect to r is given by: $\frac{\partial\phi^*_X}{\partial r}=\frac{(f_X/f-1)\,(\sigma-1)\,r^{-\sigma}}{\left(r^{1-\sigma}-1\right)^2}>0$. Notice that, if $r<1$, an increase in r implies more equal factor prices.

D Proof of lemma 3

In the trading equilibrium the *FEC*–curve results from the following free entry condition:

$$\frac{f_E}{f}\,\frac{\rho+\theta}{1+\rho} = \underbrace{[1-G(\phi^*)]\left\{\left[\frac{c(\widetilde{\phi})}{c(\phi^*)}\right]^{1-\sigma}-1\right\}}_{\equiv\Xi(\phi^*,r)} + \underbrace{[1-G(\phi_X^*)]\left\{\left[\frac{c(\widetilde{\phi}_X)}{c(\phi_X^*)}\right]^{1-\sigma}-1\right\}}_{\equiv\Xi(\phi_X^*,r)}\frac{f_X}{f}. \quad (43)$$

The term $\Xi(\phi_X^*,r)f_X > 0$ denotes the ex–ante expected export profits. Since $\frac{\partial\Xi(\phi^*,r)}{\partial\phi^*} = \frac{[1-G(\phi^*)]c(\widetilde{\phi})^{1-\sigma}}{(1-r^{1-\sigma})^{-1}c(\phi^*)^{2-2\sigma}} < 0$, $\frac{\partial\Xi\left(\phi_X^*,r\right)}{\partial\phi_X^*} = \frac{[1-G(\phi_X^*)]c(\widetilde{\phi}_X)^{1-\sigma}}{(1-r^{1-\sigma})^{-1}c\left(\phi_X^*\right)^{2-2\sigma}} < 0$ and $\frac{\partial\phi_X^*}{\partial\phi^*} > 0$, it follows that ϕ^* increases if the term $\Xi(\phi_X^*,r)\frac{f_X}{f}$ adds to the free entry condition. An increase in ϕ^*, which decreases $\Xi(\phi^*,r)$ and $\Xi(\phi_X^*,r)$, guarantees that the free entry condition holds again after trade liberalization.

E Open economy factor market equilibrium conditions

Adding the factor demands by the exporting firms to equations 41 and 42 and considering $\left[\frac{c(\phi)}{p(\phi)}\right]^{\sigma} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma}$ leads to:

$$\frac{Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}}{P^{1-\sigma}}\left[\int_{\phi^*}^{1}(1-\phi)\mu(\phi)d\phi+\int_{\phi_X^*}^{1}(1-\phi)\mu_X(\phi)d\phi\right]+c(\widetilde{\phi})^{\sigma}(1-\widetilde{\phi})\left(\widetilde{f}+s_Xf_X\right)=\frac{L}{N} \quad (44)$$

$$\frac{Y\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}}{P^{1-\sigma}r^{\sigma}}\left[\int_{\phi^*}^{1}\phi\mu(\phi)d\phi+\int_{\phi_X^*}^{1}\phi\mu_X(\phi)d\phi\right]+\frac{c(\widetilde{\phi})^{\sigma}}{r^{\sigma}}\widetilde{\phi}\left(\widetilde{f}+s_Xf_X\right)=\frac{K}{N}, \quad (45)$$

with $\mu_X(\phi) \equiv \frac{g(\phi)}{1-G(\phi_X^*)}$ and $s_X \equiv \frac{1-G\left(\phi_X^*\right)}{1-G(\phi^*)}$. Considering $\left[\frac{c(\widetilde{\phi})}{p(\widetilde{\phi})}\right]^{\sigma} = \left(\frac{\sigma-1}{\sigma}\right)^{\sigma} = \left[\frac{c(\widetilde{\phi}_X)}{p(\widetilde{\phi}_X)}\right]^{\sigma}$ and $q(\widetilde{\phi}) = Y\,P^{\sigma-1}\,p(\widetilde{\phi})^{-\sigma}$, equations 44 and 45 can be simplified to equations 22 and 23.

F Proof of proposition 4

I analyze how the shifts of the *FEC*–curve and the $\frac{L}{K}$–curve due to trade liberalization depend on f_X. In the trading equilibrium the *FEC*–curve is given by equation 43.

I proceed in four steps. First, I analyze how the term $\Xi(\phi_X^*,r)$ reacts to f_X. Since $\phi_X^* = \phi^*\frac{f_X}{f}+\frac{f_X/f-1}{r^{1-\sigma}-1}$ in general equilibrium (equation 20), I can calculate the following partial derivative: $\frac{\partial\Xi(\phi_X^*,r)}{\partial f_X} = \frac{-\left[1-G(\phi_X^*)\right]}{f}\frac{c(\widetilde{\phi}_X)^{1-\sigma}c(\phi^*)^{1-\sigma}}{c(\phi_X^*)^{2(1-\sigma)}}$. Thus, $\frac{\partial\Xi(\phi_X^*,r)}{\partial f_X} < 0$, implying that a smaller f_X leads to a larger upward–shift of the *FEC*–curve with trade liberalization.

Second, I analyze how the *FEC*–curve shifts if trade is liberalized with $f_X = f$. If $f_X = f$, then $s_X = 1$ and $\widetilde{\phi}_X = \widetilde{\phi}$ and the open economy free entry condition becomes:

$$\frac{f_E}{f}\,\frac{\rho+\theta}{1+\rho} = 2\underbrace{[1-G(\phi^*)]\left\{\left[\frac{c(\widetilde{\phi})}{c(\phi^*)}\right]^{1-\sigma}-1\right\}}_{\equiv\Xi(\phi^*,r)}. \quad (46)$$

Relative to autarky the factor 2 has added to the right hand side of equation 46. Thus, relative to autarky, ϕ^* increases so that the term $\Xi(\phi^*, r)$ decreases for a given r.

Third, I analyze how the *FEC*–curve shifts if trade is liberalized with a "high" level of f_X, which I denote by $\overline{f}_X$ and define as follows: if $f_X = \overline{f}_X$, then $s_X = 0$ and $\phi_X^* = \widetilde{\phi}_X = 1$. If f_X falls marginally below $\overline{f}_X$, s_X becomes positive. To determine the resulting shift of the *FEC*–curve, I totally differentiate equation 43 with respect to f_X and ϕ^*, consider afterwards that $1 - G(\phi_X^*) = 0$ in the initial equilibrium with $f_X = \overline{f}_X$, and rearrange to:

$$\left\{\frac{g(\phi^*)\Xi(\phi^*, r)}{1 - G(\phi^*)} - [1 - G(\phi^*)]\frac{\partial \Xi(\phi^*, r)}{\partial \phi^*}\right\}\frac{d\phi^*}{df_X} = -g(\phi_X^*)\frac{\partial \phi_X^*}{\partial f_X}\left\{\left[\frac{c(\widetilde{\phi}_X)}{c(\phi_X^*)}\right]^{1-\sigma} - 1\right\}\overline{f}_X. \quad (47)$$

Since $\left[\frac{c(\widetilde{\phi}_X)}{c(\phi_X^*)}\right]^{1-\sigma} = 1$ if $\phi_X^* = \widetilde{\phi}_X = 1$, equation 47 can be simplified to $\frac{d\phi^*}{df_X} = 0$. Thus, the *FEC*–curve does not shift if f_X falls marginally below $\overline{f}_X$ and the autarkic *FEC*–curve ($s_X = 0$) coincides with the free trade *FEC*–curve if s_X is strictly positive, but "small".

Fourth, I consider the term Λ. If $f_X > f$, such that $0 < s_X < 1$ and $\widetilde{\phi} < \widetilde{\phi}_X < 1$, the $\frac{L}{K}$–curve shifts to the right with trade liberalization. If $f_X = f$ and, thus, $s_X = 1$ and $\widetilde{\phi} = \widetilde{\phi}_X$, the term Λ equals 1 and the $\frac{L}{K}$–curve does not shift with trade liberalization.

In summary, if f_X is sufficiently close to f, the upward–shift of the *FEC*–curve is non–marginal, while the rightward–shift of the $\frac{L}{K}$–curve is marginal. Thus, $\widetilde{\phi}$ increases with trade liberalization. However, if f_X is "large" so that s_X is "small", the upward shift of the *FEC*–curve is marginal, while the rightward shift of the $\frac{L}{K}$–curve is non–marginal. Thus, $\widetilde{\phi}$ decreases with trade liberalization in this case.

G Proof of lemma 6

Extending the setup of Baxter (1992) by monopolistic competition between firms, the steady state of a Ramsey growth model is characterized by four necessary first order conditions:

$$r_t + (1 - \delta)\, p(\widetilde{\phi}_t) = p_t^K \quad (48)$$

$$r_t = p(\widetilde{\phi}_t)\left[\widetilde{\phi}_t^{1-\alpha} + (1 - \widetilde{\phi}_t)^{1-\alpha}\,(l_t/k_t)^{\alpha}\right]^{(1-\alpha)/\alpha}\widetilde{\phi}_t^{1-\alpha} \quad (49)$$

$$w_t = p(\widetilde{\phi}_t)\left[\widetilde{\phi}_t^{1-\alpha}\,(k_t/l_t)^{\alpha} + (1 - \widetilde{\phi}_t)^{1-\alpha}\right]^{(1-\alpha)/\alpha}(1 - \widetilde{\phi}_t)^{1-\alpha} \quad (50)$$

$$p_{t+1}^K = (1 + \rho)\, p(\widetilde{\phi}_t), \quad (51)$$

where p_t^K is the price per unit capital in period t, r_t the capital rental rate in t and w_t the wage rate in t. $p(\widetilde{\phi}_t)$ is the price of the average variety, which is used for investments.

Equation 48 is a zero profit condition for the households' capital lending behavior. Households realize zero profits from lending capital out to firms if p_t^K equals r_t, plus what is left from the unit of capital in $t+1$; since one unit of $q(\widetilde{\phi})$ in t leads to one unit of capital in $t+1$, the remaining $1-\delta$ units of capital in $t+1$ are evaluated with $p(\widetilde{\phi}_t)$. Equations 49 and 50 imply that, in the steady state, factor prices are equal to the value of the marginal product for each factor. Equation 51 denotes the Euler equation.

The time index is removed now for a steady state analysis. Equations 51 and 49 can be substituted into equation 48, which is then rearranged to:

$$\frac{l}{k} = \left\{ \frac{\left[\frac{(\rho+\delta)(\sigma-1)}{\sigma\,\widetilde{\phi}^{1-\alpha}}\right]^{\alpha/(1-\alpha)} - \widetilde{\phi}^{1-\alpha}}{(1-\widetilde{\phi})^{1-\alpha}} \right\}^{1/\alpha}. \tag{52}$$

Substituting equation 52 into equation 50 leads to:

$$\frac{w}{p(\widetilde{\phi})} = \left\{ \frac{(1-\widetilde{\phi})\left[(\rho+\delta)(\sigma-1)\right]^{\alpha/(1-\alpha)}}{\left[(\rho+\delta)(\sigma-1)\right]^{\alpha/(1-\alpha)} - \widetilde{\phi}\,\sigma^{\alpha/(1-\alpha)}} \right\}^{(1-\alpha)/\alpha}. \tag{53}$$

Combining equations 48 and 51 gives:

$$\frac{r}{p(\widetilde{\phi})} = (\rho+\delta)\,\frac{\sigma-1}{\sigma}. \tag{54}$$

Dividing equations 54 and 53 by each other and considering $\sigma = \frac{1}{1-\alpha}$ leads to equation 27.

H Proof of lemma 8

Lemma 8 follows from the following partial derivative: $\frac{\partial r}{\partial \widetilde{\phi}} = \frac{(\rho+\delta)^{1-\sigma}\left[(1-1/\sigma)^{1-\sigma}-(\rho+\delta)^{1-\sigma}\right]}{(\sigma-1)r^{-\sigma}\left[(1-1/\sigma)^{1-\sigma}-\widetilde{\phi}\,(\rho+\delta)^{1-\sigma}\right]^2}$. The sign of $\frac{\partial r}{\partial \widetilde{\phi}}$ depends on the sign of the squared bracket in the numerator, i.e. it depends on whether $r<1$ or $r>1$ in the steady state.

I Proof of proposition 5

First, consider the case of $1 < \frac{1-1/\sigma}{\rho+\delta}$, i.e. the *steady state*–curve is in the range of $r<1$. The *FEC*–curve and the *steady state*–curve have a negative slope in this case. Still, I can show that the *FEC*–curve and the *steady state*–curve intersect only once, implying that the autarkic equilibrium is unique. To prove this, I show that the right hand side of equation 29, which I will denote by $\Psi(\phi^*)$, depends monotonously negatively on ϕ^*:

$$\frac{\partial\Psi(\phi^*)}{\partial\phi^*} = \frac{\left[\frac{(1-1/\sigma)^{1-\sigma}}{(\rho+\delta)^{1-\sigma}} - 1\right]\left\{[1-G(\phi^*)]\,\frac{(1-1/\sigma)^{1-\sigma}}{(\rho+\delta)^{1-\sigma}}(1-\widetilde{\phi}) - g\,(\phi^*)\,(\phi^*-\widetilde{\phi})^2\right\}}{\left[(\phi^*-\widetilde{\phi})(\rho+\delta)^{1-\sigma} + (1-\phi^*)(1-1/\sigma)^{1-\sigma}\right]^2(\rho+\delta)^{\sigma-1}}. \tag{55}$$

Since I focus on the case of $\frac{(1-1/\sigma)^{1-\sigma}}{(\rho+\delta)^{1-\sigma}} < 1$, the partial derivative $\frac{\partial\Psi(\phi^*)}{\partial\phi^*}$ is negative if:

$$\frac{(1-1/\sigma)^{1-\sigma}}{(\rho+\delta)^{1-\sigma}} > \frac{g\,(\phi^*)\,(\phi^*-\widetilde{\phi})^2}{[1-G(\phi^*)]\,(1-\widetilde{\phi})}. \tag{56}$$

I have argued in footnote 16 that, in general equilibrium, $\left(1-\frac{1}{\sigma}\right)^{1-\sigma} - \widetilde{\phi}(\rho+\delta)^{1-\sigma} > 0$. Thus, condition 56 definitely holds if:

$$\widetilde{\phi} \;>\; \frac{g(\phi^*)\,(\phi^*-\widetilde{\phi})^2}{[1-G(\phi^*)]\,(1-\widetilde{\phi})}. \tag{57}$$

Assuming a Pareto distribution with finite mean and variance for ϕ on $[\phi_L, 1]$, I get:

$$\frac{g(\phi^*)}{1-G(\phi^*)} = \frac{k\,(\phi^*)^{-1}}{1-(\phi^*)^k} \quad \text{and} \quad \widetilde{\phi} = \frac{k}{k-1}\,\phi^*\,\frac{1-(\phi^*)^{k-1}}{1-(\phi^*)^k}, \quad \text{with } k > 2.$$

Thus, condition 57 can be transformed to:

$$\left[1-(\phi^*)^k\right]\left[1-(\phi^*)^{k-1}\right] \;\geq\; \frac{\left\{(k-1)\left[1-(\phi^*)^k\right]-k\left[1-(\phi^*)^{k-1}\right]\right\}^2}{(k-1)\left[1-(\phi^*)^k\right]-k\phi^*\left[1-(\phi^*)^{k-1}\right]}. \tag{58}$$

Notice that the denominator on the right hand side of condition 58 is positive since:

$$1>\widetilde{\phi} \iff 1>\frac{k\phi^*}{k-1}\,\frac{1-(\phi^*)^{k-1}}{1-(\phi^*)^k} \implies (k-1)[1-(\phi^*)^k] > k\phi^*[1-(\phi^*)^{k-1}].$$

Therefore, and since $1>\phi^*>0$, condition 58 definitely holds if:

$$\begin{aligned} \left[1-(\phi^*)^k\right]\left[1-(\phi^*)^{k-1}\right] \;&\geq\; \left\{(k-1)\left[1-(\phi^*)^k\right]-k\left[1-(\phi^*)^{k-1}\right]\right\} \\ \iff \underbrace{2-(\phi^*)^{k-1}(1+k)-2(\phi^*)^k+k(\phi^*)^k+(\phi^*)^{2k-1}}_{\equiv\Delta} \;&\geq\; 0. \end{aligned} \tag{59}$$

Evaluating Δ at $\phi^*=0$ and $\phi^*=1$ leads to: $\Delta\Big|_{\phi^*=0} = 2$ and $\Delta\Big|_{\phi^*=1} = 0$. Furthermore:

$$\frac{\partial\Delta}{\partial\phi^*} \;=\; (\phi^*)^{k-2}\underbrace{\left(1-k^2-2k\phi^*+k^2\phi^*+(2k-1)(\phi^*)^k\right)}_{\equiv\Gamma}. \tag{60}$$

Evaluating Γ at $\phi^*=0$ and $\phi^*=1$ leads to: $\Gamma\Big|_{\phi^*=0} = 1-k^2 < 0$ and $\Gamma\Big|_{\phi^*=1} = 0$. Finally:

$$\frac{\partial\Gamma}{\partial\phi^*} \;=\; -2k+k^2+k(2k-1)(\phi^*)^{k-1} \;>\; 0 \quad \text{since } k > 2.$$

Since Γ is negative for all $\phi^* \in (0,1]$, it follows that Δ is positive for all $\phi^* \in (0,1]$. Thus, condition 59 holds, implying that $\frac{\partial\Psi(\phi^*)}{\partial\phi^*} < 0$, i.e. the right hand side of equation 29 depends monotonously negatively on ϕ^* if $1 < \frac{1-1/\sigma}{\rho+\delta}$.

Second, consider the case of $\frac{1-1/\sigma}{\rho+\delta} < 1$, i.e. the *steady state*–curve is in the range of $r > 1$. The *FEC*–curve has a negative slope, while the *steady state*–curve has a positive slope in this case. Thus, both curves intersect only once, implying that the steady state levels of $\widetilde{\phi}$ and r are uniquely determined as well if $r > 1$.

J Proof of proposition 6

First, consider the case of $r < 1$. Equation 32 then shows that: (i) $\frac{K}{L} = 0$ if $\widetilde{\phi} = 0$; (ii) $\frac{K}{L} \to \infty$ if $\widetilde{\phi} \to \left(\frac{1-1/\sigma}{\rho+\delta}\right)^{1-\sigma}$ since $r = 0$ in this case. Furthermore, $\frac{\partial(K/L)}{\partial\widetilde{\phi}} = \frac{r^{-\sigma}}{(1-\widetilde{\phi})^2} - \sigma\frac{\partial r}{\partial\widetilde{\phi}} > 0$ since $\frac{\partial r}{\partial\widetilde{\phi}} < 0$. Thus, depending on $\widetilde{\phi}$, $\frac{K}{L}$ can reach any value from $[0, \infty)$.

Second, consider the case of $r > 1$. Equation 32 then shows that $\frac{K}{L} = 0$ if $\widetilde{\phi} = 0$. Furthermore, equation 32 shows that $\frac{K}{L} \lesseqgtr 1 \iff \widetilde{\phi}^{(\sigma-1)/\sigma}(1-\widetilde{\phi})^{1/\sigma} + \widetilde{\phi} \lesseqgtr \frac{(1-1/\sigma)^{1-\sigma}}{(\rho+\delta)^{1-\sigma}}$. While $\frac{\partial(K/L)}{\partial\widetilde{\phi}}$ can be positive or negative if $\frac{1-1/\sigma}{\rho+\delta} < 1$, $\frac{K}{L}$ approaches infinity if $\frac{1-1/\sigma}{\rho+\delta} \to 1$ and $\widetilde{\phi} \to 1$.

K $q_X(\widetilde{\phi}_X)$ in the open economy steady state

$\frac{c(\widetilde{\phi})}{c(\phi_X^*)}$ and $\frac{c(\phi_X^*)}{c(\widetilde{\phi}_X)}$ are functions of r, $\widetilde{\phi}$, $\widetilde{\phi}_X$ and ϕ_X^*. Furthermore, equation 20 shows that ϕ_X^* is a function of ϕ^*, r, f, f_X, while $\widetilde{\phi}$ and $\widetilde{\phi}_X$ are functions of ϕ^* and ϕ_X^* and the probability distribution of ϕ on the interval $[\phi_L, 1]$. Finally, r is a function of ρ, σ, δ and $\widetilde{\phi}$ (see equation 27). Therefore, $q_X(\widetilde{\phi}_X)$ is a function of ρ, σ, δ, f, f_X and ϕ^*.

L Proof of proposition 8

Proposition 8 follows from the proof of proposition 5, which has shown that the term $\Psi(\phi^*)$ depends negatively on ϕ^*. It can be shown along the same lines that the term $\Psi(\phi_X^*)$ depends negatively on ϕ_X^* Thus, if $\Psi(\phi_X^*)$ adds to the right hand side of the free entry condition, ϕ^* has to increase, so that both $\Psi(\phi^*)$ and $\Psi(\phi_X^*)$ decrease (remember that $\frac{\partial\phi_X^*}{\partial\phi^*} > 0$).

M Proof of proposition 9

Result (i). The capital stock in the autarkic steady state (equation 32) is smaller than the capital stock in the free trade steady state (equation 37) since $\widetilde{\phi}$ increases with trade liberalization and since $\Omega \geq 1$ if $s_X \geq 0$:

$$\Omega \;\geq\; 1 \quad\iff\quad \delta\widetilde{\phi} \leq \frac{r^\sigma}{c(\widetilde{\phi})^\sigma} \quad\iff\quad (\delta\,\widetilde{\phi})^{1/\sigma}\left(1-\frac{1}{\sigma}\right) \leq \rho+\delta. \tag{61}$$

Condition 61 holds in the steady state since $\widetilde{\phi} < \left(\frac{1-1/\sigma}{\rho+\delta}\right)^{1-\sigma}$ (see footnote 16), $1 \geq \delta \geq 0$, and since $\sigma > 1$.

Result (ii). r decreases since $\widetilde{\phi}$ increases (see lemma 8).

Result (iii). The average firm produces more capital intensively since $\widetilde{\phi}$ increases and r decreases.

Result (iv). Substituting the steady state value for r into $c(\widetilde{\phi})$ and calculating the partial derivative with respect to $\widetilde{\phi}$ leads to: $\frac{\partial c(\widetilde{\phi})}{\partial\widetilde{\phi}} = \frac{c(\widetilde{\phi})^\sigma}{1-\sigma}\frac{(1-1/\sigma)^{1-\sigma}\left[(\rho+\delta)^{1-\sigma}-(1-1/\sigma)^{1-\sigma}\right]}{\left[(1-1/\sigma)^{1-\sigma}-\widetilde{\phi}(\rho+\delta)^{1-\sigma}\right]^2} < 0$.

Result (v). Multiplying the capital stocks in the autarkic and the free trade steady state (equations 32 and 37) by r and including superscripts aut and ft to denote autarkic and free trade variables leads to: $r^{aut}K^{aut} = \frac{\widetilde{\phi}^{aut}L}{1-\widetilde{\phi}^{aut}}(r^{aut})^{1-\sigma}$ and $r^{ft}K^{ft} = \frac{\widetilde{\phi}^{ft}L\,\Omega}{1-\widetilde{\phi}^{ft}}(r^{ft})^{1-\sigma}$. $r^{ft}K^{ft} > r^{aut}K^{aut}$ since $\widetilde{\phi}$ increases with trade liberalization, $\Omega > 1$ and $(r^{ft})^{1-\sigma} > (r^{aut})^{1-\sigma}$. The increase of rK decreases labor's share in national income.

Result (vi). Substituting the term for $q(\widetilde{\phi})$ (equation 5) into equation 10 and dividing both sides by $c(\widetilde{\phi})^{-\sigma}$ leads to:

$$\frac{Y_{cons}}{P^{1-\sigma}}\left(\frac{\sigma}{\sigma-1}\right)^{-\sigma} = (\sigma-1)\frac{c(\widetilde{\phi})}{c(\phi^*)^{1-\sigma}}\,f, \tag{62}$$

where $Y_{cons} = Y - p(\widetilde{\phi})\delta K$ denotes that part of income, which is used for consumption in the steady state. The right hand side of equation 62 decreases with trade liberalization since $c(\widetilde{\phi})$ and $c(\phi^*)$ decrease. Thus, the left hand side of equation 62 has to decrease as well with trade liberalization and I can conclude:

$$\frac{Y^{aut}_{cons}}{p(\widetilde{\phi}^{aut})^{1-\sigma}}\frac{1}{N^{aut}} > \frac{Y^{ft}_{cons}}{p(\widetilde{\phi}^{ft})^{1-\sigma}}\frac{1}{N^{ft}\left[1+s_X\frac{p(\widetilde{\phi}^{ft}_X)^{1-\sigma}}{p(\widetilde{\phi}^{ft})^{1-\sigma}}\right]}. \tag{63}$$

Furthermore, the steady state values for K, r and $p(\widetilde{\phi})$ imply

$$\underbrace{\frac{(1-1/\sigma)^{1-\sigma}+\widetilde{\phi}^{aut}(\rho+\delta)^{-\sigma}\delta}{1-\widetilde{\phi}^{aut}}}_{=Y^{aut}_{cons}/p\left(\widetilde{\phi}^{aut}\right)^{1-\sigma}} < \underbrace{\frac{(1-1/\sigma)^{1-\sigma}+\widetilde{\phi}^{ft}(\rho+\delta)^{-\sigma}\left[(\Omega-1)\rho+\delta\right]}{1-\widetilde{\phi}^{ft}}}_{=Y^{ft}_{cons}/p\left(\widetilde{\phi}^{ft}\right)^{1-\sigma}} \tag{64}$$

due to $\Omega \geq 1$ and $\widetilde{\phi}^{ft} > \widetilde{\phi}^{aut}$. Thus, inequalities 63 and 64 imply: $N^{aut} < N^{ft}\left[1+s_X\frac{p(\widetilde{\phi}^{ft}_X)^{1-\sigma}}{p(\widetilde{\phi}^{ft})^{1-\sigma}}\right]$. Since $p(\widetilde{\phi}^{aut}) > p(\widetilde{\phi}^{ft})$ and since the price indices are given by $P^{aut} = \left[p(\widetilde{\phi}^{aut})^{1-\sigma}N^{aut}\right]^{1/(1-\sigma)}$ and $P^{ft} = \left\{p(\widetilde{\phi}^{ft})^{1-\sigma}N^{ft}\left[1+s_X\frac{p(\widetilde{\phi}^{ft}_X)^{1-\sigma}}{p(\widetilde{\phi}^{ft})^{1-\sigma}}\right]\right\}^{1/(1-\sigma)}$, I immediately get $P^{aut} > P^{ft}$.

Result (vii). First, the real returns to labor are given by $\frac{1}{P}$. Due to result (vi), $\frac{1}{P}$ increases with trade liberalization. Second, the real returns to capital in the autarkic and the free trade steady state, respectively, are given by:

$$\frac{r^{aut}}{P^{aut}} = \frac{r^{aut}}{p(\widetilde{\phi}^{aut})\,(N^{aut})^{1/(1-\sigma)}} \quad \text{and} \quad \frac{r^{ft}}{P^{ft}} = \frac{r^{ft}}{p(\widetilde{\phi}^{ft})\left\{N^{ft}\left[1+s_X\frac{p(\widetilde{\phi}^{ft}_X)^{1-\sigma}}{p(\widetilde{\phi}^{ft})^{1-\sigma}}\right]\right\}^{1/(1-\sigma)}}.$$

Due to $\frac{r^{aut}}{p(\widetilde{\phi}^{aut})} = \frac{r^{ft}}{p(\widetilde{\phi}^{ft})} = \rho+\delta$, it follows that $\frac{r}{P}$ increases with trade liberalization since $N^{aut} < N^{ft}\left[1+s_X\frac{p(\widetilde{\phi}^{ft}_X)^{1-\sigma}}{p(\widetilde{\phi}^{ft})^{1-\sigma}}\right]$, as I have argued with the proof for result (vi).

N Proof of lemma 10

All firms from the interval $[\phi_H^*, 1]$ are active in country H's general equilibrium, while all firms from the interval $[\phi_L, \phi_F^*]$ are active in country F's general equilibrium. Thus, if $\phi_{H,X}^* > \phi_H^*$ ($\phi_{F,X}^* < \phi_F^*$) only part of country H (F) firms, which are active domestically, export as well.

Assume now that $\frac{f_X}{f} > \frac{Y_j P_j^{\sigma-1}}{Y_i P_i^{\sigma-1}}$, $i, j = H, F$. If $r_H < 1$, equation 38 implies that $\phi_{H,X}^* > \phi_H^*$ since (i) $\frac{f_X}{f}\frac{Y_H P_H^{\sigma-1}}{Y_F P_F^{\sigma-1}} > 1$ and (ii) $r_H^{1-\sigma} - 1 > 0$. If $r_F > 1$, equation 38 implies that $\phi_{F,X}^* < \phi_F^*$ as long as $\left(\frac{Y_F P_F^{\sigma-1}}{Y_H P_H^{\sigma-1}}\frac{f_X}{f} - 1\right)\left[\phi_F^*(r_F^{1-\sigma} - 1) + 1\right] > 0$. This last condition holds since (i) $\frac{Y_F P_F^{\sigma-1}}{Y_H P_H^{\sigma-1}}\frac{f_X}{f} > 1$ and (ii) $r_F^{1-\sigma} - 1 > -1$, i.e. $\phi_F^*(r_F^{1-\sigma} - 1) + 1 > 0$.

O Proof of proposition 10

I will make use of the trade balance equation (TBE) in this proof. The TBE holds since the demand functions (equation 5) imply that, in each country, the value of aggregate production equals the value of aggregate consumption. The TBE is given by:

$$\int_{\phi_{H,X}^*}^{1} \frac{Y_F}{P_F^{1-\sigma}} p(\phi)^{1-\sigma} N_H s_{H,X} \mu_{H,X}(\phi) d\phi = \int_{\phi_L}^{\phi_{F,X}^*} \frac{Y_H}{P_H^{1-\sigma}} p(\phi)^{1-\sigma} N_F s_{F,X} \mu_{F,X}(\phi) d\phi, \quad (65)$$

with $s_{F,X} = \frac{G(\phi_{F,X}^*)}{G(\phi_F^*)}$ and $\mu_{F,X}(\phi) = \frac{g(\phi)}{G(\phi_{F,X}^*)}$. Equation 65 can be transformed to $\left[\frac{c(\widetilde{\phi}_{H,X})}{c(\widetilde{\phi}_{F,X})}\right]^{1-\sigma} \frac{N_H s_{H,X}}{N_F s_{F,X}} = \frac{Y_H P_H^{\sigma-1}}{Y_F P_F^{\sigma-1}}$. Thus, I can rewrite equation 38:

$$\phi_{i,X}^* = \Upsilon_i \frac{f_X}{f}\phi_i^* + \frac{\Upsilon_i \frac{f_X}{f} - 1}{r_i^{1-\sigma} - 1}, \text{ with } \Upsilon_i = \frac{c(\widetilde{\phi}_{i,X})^{1-\sigma} N_i s_{i,X}}{c(\widetilde{\phi}_{j,X})^{1-\sigma} N_j s_{j,X}}, \quad i, j = H, F, \quad i \neq j. \quad (66)$$

Furthermore, to prove proposition 10, I will determine how the absolute size of country F impacts N_F. Considering country F's zero cutoff profit conditions for domestic supply and for exports (equations 10 and 33) reveals that both $q(\widetilde{\phi}_F)$ and $q_X(\widetilde{\phi}_{F,X})$ do not change if country F experiences a proportional increase in K_F and L_F, which keeps r_F constant. This implies that a proportional increase in K_F and L_F increases N_F.

To evaluate how the increase in N_F impacts $\phi_{F,X}^*$ and $\phi_{H,X}^*$, I differentiate equation 66 totally for both countries and write the resulting system of equations in matrix notation:

$$\begin{pmatrix} \frac{\partial \Upsilon_H}{\partial \phi_{H,X}^*}\frac{f_X}{f}\frac{c(\phi_H^*)^{1-\sigma}}{r_H^{1-\sigma}-1} - 1 & \frac{\partial \Upsilon_H}{\partial \phi_{F,X}^*}\frac{f_X}{f}\frac{c(\phi_H^*)^{1-\sigma}}{r_H^{1-\sigma}-1} \\ \frac{\partial \Upsilon_F}{\partial \phi_{H,X}^*}\frac{f_X}{f}\frac{c(\phi_F^*)^{1-\sigma}}{r_F^{1-\sigma}-1} & \frac{\partial \Upsilon_F}{\partial \phi_{F,X}^*}\frac{f_X}{f}\frac{c(\phi_F^*)^{1-\sigma}}{r_F^{1-\sigma}-1} - 1 \end{pmatrix} \begin{pmatrix} d\phi_{H,X}^* \\ d\phi_{F,X}^* \end{pmatrix} = \begin{pmatrix} \frac{-c(\phi_H^*)^{1-\sigma}}{r_H^{1-\sigma}-1}\frac{\partial \Upsilon_H}{\partial N_F}\frac{f_X}{f} dN_F \\ \frac{-c(\phi_F^*)^{1-\sigma}}{r_F^{1-\sigma}-1}\frac{\partial \Upsilon_F}{\partial N_F}\frac{f_X}{f} dN_F \end{pmatrix}, \quad (67)$$

with $\frac{\partial \Upsilon_i}{\partial \phi_{i,X}^*} = \left[\frac{c(\phi_{i,X}^*)}{c(\widetilde{\phi}_{j,X})}\right]^{1-\sigma} \frac{N_i}{N_j s_{j,X}}\frac{\partial s_{i,X}}{\partial \phi_{i,X}^*}$, $\frac{\partial \Upsilon_i}{\partial \phi_{j,X}^*} = -\left[\frac{c(\widetilde{\phi}_{i,X}) c(\phi_{j,X}^*)}{c(\widetilde{\phi}_{j,X})^2}\right]^{1-\sigma} \frac{N_i s_{i,X}}{N_j s_{j,X}^2}\frac{\partial s_{j,X}}{\partial \phi_{j,X}^*}$, $\frac{\partial \Upsilon_H}{\partial N_F} =$

$-\left[\frac{c(\widetilde{\phi}_{H,X})}{c(\widetilde{\phi}_{F,X})}\right]^{1-\sigma}\frac{N_H s_{H,X}}{N_F^2 s_{F,X}}$ and $\frac{\partial \Upsilon_F}{\partial N_F}=\left[\frac{c(\widetilde{\phi}_{F,X})}{c(\widetilde{\phi}_{H,X})}\right]^{1-\sigma}\frac{s_{F,X}}{N_H s_{H,X}}$. Solving equation 67 for $\frac{d\phi^*_{i,X}}{dN_F}$ yields:

$$\frac{d\phi^*_{H,X}}{dN_F}=\frac{\frac{\partial \Upsilon_i}{\partial N_F}\frac{c(\phi^*_i)^{1-\sigma}}{r_i^{1-\sigma}-1}}{\left[\frac{c(\phi^*_{H,X})}{c(\widetilde{\phi}_{F,X})}\right]^{1-\sigma}\frac{N_H\frac{g(\phi^*_{H,X})}{1-G(\phi^*_H)}}{N_F s_{F,X}}\frac{c(\phi^*_H)^{1-\sigma}}{r_H^{1-\sigma}-1}-\left[\frac{c(\phi^*_{F,X})}{c(\widetilde{\phi}_{H,X})}\right]^{1-\sigma}\frac{N_F\frac{g(\phi^*_{F,X})}{G(\phi^*_F)}}{N_H s_{H,X}}\frac{c(\phi^*_F)^{1-\sigma}}{r_F^{1-\sigma}-1}+\frac{f}{f_X}}. \tag{68}$$

Equation 68 shows that $\frac{d\phi^*_{i,X}}{dN_F}<0$, $i=H,F$. This follows from $\frac{\partial \Upsilon_H}{\partial N_F}<0$, $\frac{\partial \Upsilon_F}{\partial N_F}>0$, $r_H^{1-\sigma}-1>0$ and $r_F^{1-\sigma}-1<0$, implying that the numerator on the right hand side of equation 68 is negative, while the denominator is positive.

Finally, equation 66 also shows the following: (i) $\phi^*_{H,X}\to 1$ if $r_H\to 1$ or $s_{F,X}\to 0$; (ii) $\phi^*_{F,X}\to 0$ if $r_F\to 1$ or $s_{H,X}\to 0$. Since $s_{H,X}\to 0$ if $\phi^*_{H,X}\to 1$ and $s_{F,X}\to 0$ if $\phi^*_{F,X}\to 0$, it follows that trade ceases if either or both relative factor prices r_H and r_F approach 1.

P Proof of lemma 11

In the trading equilibrium country F's *FEC*–curve results from equation 69:

$$\frac{f_E}{f}\frac{\rho_F+\theta}{1+\rho_F}=\underbrace{G(\phi^*_F)\left\{\left[\frac{c(\widetilde{\phi}_F)}{c(\phi^*_F)}\right]^{1-\sigma}-1\right\}}_{\equiv\Xi_F(\phi^*_F,r_F)}+\underbrace{G(\phi^*_{F,X})\left\{\left[\frac{c(\widetilde{\phi}_{F,X})}{c(\phi^*_{F,X})}\right]^{1-\sigma}-1\right\}}_{\equiv\Xi_F(\phi^*_{F,X},r_F)}\frac{f_X}{f}. \tag{69}$$

The term $\Xi_F(\phi^*_{F,X},r_F)\frac{f_X}{f}>0$ adds to country F's free entry condition with trade liberalization. Since $\frac{\partial \Xi_F(\phi^*_F,r_F)}{\partial \phi^*_F}=\frac{-G(\phi^*_F)c(\widetilde{\phi}_F)^{1-\sigma}}{(r_F^{1-\sigma}-1)^{-1}c(\phi^*_F)^{2-2\sigma}}>0$, $\frac{\partial \Xi_F(\phi^*_{F,X},r_F)}{\partial \phi^*_{F,X}}=\frac{-G(\phi^*_{F,X})c(\widetilde{\phi}_{F,X})^{1-\sigma}}{(r_F^{1-\sigma}-1)^{-1}c(\phi^*_{F,X})^{2-2\sigma}}>0$ and $\frac{\partial \phi^*_{F,X}}{\partial \phi^*_F}>0$, it follows that ϕ^*_F decreases with trade liberalization. The resulting decrease in $\Xi_F(\phi^*_F,r_F)$ and $\Xi_F(\phi^*_{F,X},r_F)$ guarantees that country F's free entry condition holds again after trade liberalization.

Q Proof of proposition 11

The decrease in r_F in the short–run after trade liberalization follows from the shifts of country F's *FEC*– and $\frac{L}{K}$–curves, as described by lemma 11 and lemma 12, respectively.

The proof of part (ii) is along the same lines as the one for proposition 4:

First, country F's $\frac{L}{K}$–curve does not shift with trade liberalization if $f_X=\frac{Y_H P_H^{\sigma-1}}{Y_F P_F^{\sigma-1}}f$, which implies that each domestically active firm also exports. However, country F's *FEC*–curve shifts downward since $\Xi_F(\phi^*_{F,X},r_F)>0$ in this case. Thus, if f_X is sufficiently close to $\frac{Y_H P_H^{\sigma-1}}{Y_F P_F^{\sigma-1}}f$, $\widetilde{\phi}_F$ decreases in the short–run after trade liberalization.

Second, country F's *FEC*–curve does not shift with trade liberalization if f_X is sufficiently large so that $\phi^*_{F,X}\to\widetilde{\phi}_{F,X}$, which implies that only the most labor intensive firms export. Still, country F's $\frac{L}{K}$–curve shifts to the left since trade liberalization increases relative labor demand in this case. Thus, if f_X is sufficiently large so that $\phi^*_{F,X}$ is sufficiently close to $\widetilde{\phi}_{F,X}$, $\widetilde{\phi}_F$ increases in the short–run after trade liberalization.

R $q_X(\widetilde{\phi}_{i,X})$ in the open economy steady state

First, including a country index i, $c(\widetilde{\phi}_i)$, $c(\phi^*_{i,X})$ and $c(\widetilde{\phi}_{i,X})$ on the right hand side of equation 33 are functions of σ, r_i, ϕ^*_i and $\phi^*_{i,X}$. Thus, $q_X(\widetilde{\phi}_{i,X})$ is a function of σ, r_i, ϕ^*_i, $\phi^*_{i,X}$ and f_X. Second, equation 38 shows that $\phi^*_{i,X}$ is a function of ϕ^*_i, r_i, $\frac{f_X}{f}$ and the ratio $\frac{Y_iP_i^{\sigma-1}}{Y_jP_j^{\sigma-1}}$. In general equilibrium the terms $Y_HP_H^{\sigma-1}$ and $Y_FP_F^{\sigma-1}$ result as functions of σ, r_i, ϕ^*_i and f. This follows from the respective zero cutoff profit condition for the supply to the domestic market (see equation 10). Considering that $q(\widetilde{\phi}_i) = Y_iP_i^{\sigma-1}p(\widetilde{\phi}_i)^{-\sigma}$, equation 10 can be transformed to $Y_iP_i^{\sigma-1} = \frac{(\sigma-1)^{1-\sigma}}{\sigma^{-\sigma}}\frac{c(\widetilde{\phi}_i)}{c(\phi^*_i)^{1-\sigma}}f$. Thus, in general equilibrium, $\phi^*_{i,X}$ is a function of σ, r_H, r_F, f_X, f, ϕ^*_H and ϕ^*_F. Third, r_i in the steady state is a function σ, ϕ^*_i, ρ_i and δ_i.

Thus, $q_X(\widetilde{\phi}_{i,X})$ in the steady state is a function of σ, ρ_H, ρ_F, δ_H, δ_F, f_X, f and the steady state values of ϕ^*_H and ϕ^*_F.

S Proof of proposition 12

To prove proposition 12, I will first analyze how the general equilibrium value of $\phi^*_{i,X}$ (equation 38) is impacted by ϕ^*_j. Considering that, in general equilibrium, $Y_iP_i^{\sigma-1} = \frac{(\sigma-1)^{1-\sigma}}{\sigma^{-\sigma}}\frac{c(\widetilde{\phi}_i)f}{c(\phi^*_i)^{1-\sigma}}$, which follows from combining equations 5 and 10, equation 38 can be rewritten:

$$\phi^*_{i,X} \;=\; \frac{c(\widetilde{\phi}_i)}{c(\widetilde{\phi}_j)c(\phi^*_j)^{\sigma-1}}\,\frac{f_X/f}{r_i^{1-\sigma}-1} - \frac{1}{r_i^{1-\sigma}-1}.$$

The partial derivative of $\phi^*_{i,X}$ with respect to ϕ^*_j results as follows:

$$\frac{\partial\phi^*_{i,X}}{\partial\phi^*_j} \;=\; \Pi\left[\frac{\partial\widetilde{\phi}_j}{\partial\phi^*_j}\left(\frac{r_j^{1-\sigma}-1}{\sigma-1} - \frac{\widetilde{\phi}_j}{r_j^{\sigma}}\frac{\partial r_j}{\partial\widetilde{\phi}_j}\right) + \frac{c(\phi^*_j)^{\sigma-1}}{c(\widetilde{\phi}_j)^{\sigma-1}}\left(r_j^{1-\sigma} - 1 - \phi^*_j\frac{\sigma-1}{r_j^{\sigma}}\frac{\partial r_j}{\partial\widetilde{\phi}_j}\frac{\partial\widetilde{\phi}_j}{\partial\phi^*_j}\right)\right], \quad (70)$$

with $\Pi = \frac{c(\widetilde{\phi}_i)c(\phi^*_j)^{1-\sigma}f_X/f}{c(\widetilde{\phi}_j)^{2-\sigma}(r_i^{1-\sigma}-1)}$. Equation 70 shows that $\frac{\partial\phi^*_{H,X}}{\partial\phi^*_F} < 0$ and $\frac{\partial\phi^*_{F,X}}{\partial\phi^*_H} < 0$, and this follows from: (i) $r_H^{1-\sigma} - 1 > 0$, (ii) $r_F^{1-\sigma} - 1 < 0$, (iii) $\frac{\partial r_H}{\partial\widetilde{\phi}_H} < 0$ and (iv) $\frac{\partial r_H}{\partial\widetilde{\phi}_H} > 0$. Thus, I can write $\phi^*_{i,X}$ as a function of ϕ^*_i and ϕ^*_j, $i, j = H, F$:

$$\phi^*_{i,X} \;=\; \phi^*_{i,X}\left(\phi^*_j, \phi^*_i\right), \quad \text{with} \quad \frac{\partial\phi^*_{i,X}}{\partial\phi^*_i} > 0 \quad\text{and}\quad \frac{\partial\phi^*_{i,X}}{\partial\phi^*_j} < 0. \quad (71)$$

Furthermore, appendix D has shown that the term $\Xi_H(\phi^*_{H,X}, r_H)$ depends negatively on $\phi^*_{H,X}$, while appendix P has shown that the term $\Xi_F(\phi^*_{F,X}, r_F)$ depends positively on $\phi^*_{F,X}$. Notice that $\Xi_H(\phi^*_{H,X}, r_H)$ and $\Xi_F(\phi^*_{F,X}, r_F)$ add to the respective country's free entry condition due to trade liberalization. Thus, I can write ϕ_i^* as a function of $\phi_{i,X}^*$, $i = H, F$:

$$\phi_i^* \;=\; \phi_i^*\Big(\phi_{i,X}^*\Big), \quad \text{with} \;\; \frac{\partial \phi_i^*}{\partial \phi_{i,X}^*} \;<\; 0. \tag{72}$$

Substituting equation 71 into equation 72 leads to:

$$\phi_i^* \;=\; \phi_i^*\Big(\overbrace{\phi_{i,X}^*}^{-}\Big(\overbrace{\phi_j^*}^{-}, \overbrace{\phi_i^*}^{+}\Big)\Big), \quad i, j \;=\; H,\, F, \quad i \neq j, \tag{73}$$

where the sign above a variable denotes the sign of the corresponding partial derivative. Totally differentiating equation 73 and rearranging leads to:

$$d\phi_i^*\Big(1 - \overbrace{\frac{\partial \phi_i^*}{\partial \phi_{i,X}^*}}^{-}\overbrace{\frac{\partial \phi_{i,X}^*}{\partial \phi_i^*}}^{+}\Big) \;=\; \overbrace{\frac{\partial \phi_i^*}{\partial \phi_{i,X}^*}}^{-}\overbrace{\frac{\partial \phi_{i,X}^*}{\partial \phi_j^*}}^{-} d\phi_j^*. \tag{74}$$

Thus, $\frac{d\phi_H^*}{d\phi_F^*} > 0$ and $\frac{d\phi_F^*}{d\phi_H^*} > 0$, and I can draw a $\phi_H^*(\phi_F^*)$–curve and a $\phi_F^*(\phi_H^*)$–curve in a diagram with ϕ_H^* on the vertical axis and ϕ_F^* on the horizontal axis. Both curves have a positive slope. Still, the slopes of the two curves differ since the partial derivatives in equation 74 depend on, amongst others, relative domestic factor prices. In addition, the axis intercepts of the two curves differ as well since $\phi_i^* > 0$ if $\phi_j^* = 0$. Thus, while these curves do not coincide, they intersect at least once.

T The long–run impact of trade liberalization on country F

First, I focus on country F's capital stock. Including a country index F and substituting the steady state term for r (equation 27) into equation 37 leads to $K_F = \frac{\widetilde{\phi}_F(1-\widetilde{\phi}_F)^{\frac{1}{\sigma-1}}(\rho_F+\delta_F)^{-\sigma}L_F\Omega_F}{\left[(1-1/\sigma)^{1-\sigma}-\widetilde{\phi}_F(\rho_F+\delta_F)^{1-\sigma}\right]^{\frac{\sigma}{\sigma-1}}}$.

For the remainder of this appendix I assume that trade is liberalized with a "small" magnitude of f_X, so that $s_{F,X} = 1$ and $\widetilde{\phi}_{F,X} = \widetilde{\phi}_F$. Thus, $\Omega_F = 1$ in the trading and the autarkic equilibrium and I can evaluate the impact of this trade liberalization exercise on K_F by determining how the term $\frac{\widetilde{\phi}_F r_F^{-\sigma}}{1-\widetilde{\phi}_F}$ reacts to the decrease of $\widetilde{\phi}_F$ due to trade liberalization:

$$\frac{\partial}{\partial \widetilde{\phi}_F}\left(\frac{\widetilde{\phi}_F r_F^{-\sigma}}{1-\widetilde{\phi}_F}\right) = \frac{(1-\widetilde{\phi}_F)\sigma\,(1-1/\sigma)^{1-\sigma} + \widetilde{\phi}(\rho_F+\delta_F)^{1-\sigma} - (1-1/\sigma)^{1-\sigma}}{(\sigma-1)(\rho_F+\delta_F)^{\sigma}(1-\widetilde{\phi}_F)^{\frac{\sigma-2}{\sigma-1}}\left[(1-1/\sigma)^{1-\sigma} - \widetilde{\phi}_F(\rho_F+\delta_F)^{1-\sigma}\right]^{\frac{2\sigma-1}{\sigma-1}}}. \tag{75}$$

Thus, $\frac{\partial}{\partial \widetilde{\phi}_F}\left(\frac{\widetilde{\phi}_F r_F^{-\sigma}}{1-\widetilde{\phi}_F}\right) > (<)\ 0$ if $\widetilde{\phi}_F < (>) \frac{(\sigma-1)(1-1/\sigma)^{1-\sigma}}{\sigma(1-1/\sigma)^{1-\sigma}-(\rho_F+\delta_F)^{1-\sigma}} \equiv \nu$, where ν denotes a critical magnitude for $\widetilde{\phi}_F$. $0 < \nu < 1$ since $\sigma > 1$ and $\left(1-\frac{1}{\sigma}\right)^{1-\sigma} > (\rho_F+\delta_F)^{1-\sigma}$ (see lemma 7). Thus, depending on its initial level, the decrease in $\widetilde{\phi}_F$ impacts K_F negatively or positively.

Second, I focus on country F's welfare. In order to do so, I need three sub–results. The first one refers to the domestic supply of the average firm, which is pinned down by equation 10 in the autarkic and trading equilibrium: $q(\widetilde{\phi}_F) = (\sigma - 1)\left[\frac{c(\widetilde{\phi}_F)}{c(\phi_F^*)}\right]^{1-\sigma} f$. Substituting the steady state term for r_F (equation 27) into $c(\widetilde{\phi}_F)$ and $c(\phi_F^*)$ and rewriting leads to:

$$\frac{q(\widetilde{\phi}_F)}{(\sigma-1)\,(1-1/\sigma)^{1-\sigma} f} = \frac{1-\widetilde{\phi}_F}{(\phi_F^* - \widetilde{\phi}_F)(\rho_F+\delta_F)^{1-\sigma} + (1-\phi_F^*)\,(1-1/\sigma)^{1-\sigma}} \equiv \chi. \quad (76)$$

The adjustment of $q(\widetilde{\phi}_F)$ with trade liberalization depends on the sign of $\frac{\partial \chi}{\partial \phi_F^*}$:

$$\frac{\partial \chi}{\partial \phi_F^*} = \frac{B\left[(1-1/\sigma)^{1-\sigma} - (\rho_F+\delta_F)^{1-\sigma}\right]}{\left[(\phi_F^* - \widetilde{\phi}_F)(\rho_F+\delta_F)^{1-\sigma} + (1-\phi_F^*)\,(1-1/\sigma)^{1-\sigma}\right]^2}, \quad (77)$$

with $B \equiv -\frac{\partial \widetilde{\phi}_F}{\partial \phi_F^*}(1-\phi_F^*) + 1 - \widetilde{\phi}_F$ and $\frac{\partial \widetilde{\phi}_F}{\partial \phi_F^*} = \frac{g(\phi_F^*)}{G(\phi_F^*)}(\phi_F^* - \widetilde{\phi}_F)$. Thus, the sign of $\frac{\partial \chi}{\partial \phi_F^*}$ equals the sign of B. Assuming a Pareto distribution with finite mean and variance for ϕ on an interval $[\phi_L, 1]$ yields:

$$B = 1 - \frac{(\phi_F^*)^{k-1} - \phi_L^{k-1}}{(\phi_F^*)^k - \phi_L^k}\frac{k\phi_F^*\phi_L}{k-1} - \frac{k\phi_L^k(1-\phi_F^*)}{(\phi_F^*)^k - \phi_L^k} + \frac{\phi_L^{k+1}k^2}{k-1}\frac{(\phi_F^*)^{k-1} - \phi_L^{k-1}}{\left[(\phi_F^*)^k - \phi_L^k\right]^2}(1-\phi_F^*), \quad k > 2.$$

Since I cannot determine the sign of B for all possible values for k and ϕ_L, I will consider here a numerical example. Assuming $k = 3$ and $\phi_L = 0.25$ results in:

$$B = \frac{320(\phi_F^*)^4 + 160(\phi_F^*)^3 + 96(\phi_F^*)^2 - 8\phi_F^* - 1}{2\left[16(\phi_F^*)^2 + 4\phi_F^* + 1\right]^2}. \quad (78)$$

Equation 78 shows that $B > 0$ if $\phi_F^* = 0.25$ and $\frac{\partial B}{\partial \phi_F^*} > 0$ if $\phi_F^* \geq 0.25$. Thus, $\frac{\partial \chi}{\partial \phi_F^*} > 0$, implying that $q(\widetilde{\phi}_F)$ decreases with trade liberalization. The absolute magnitude of the decrease in $q(\widetilde{\phi}_F)$ depends on, amongst others, the magnitude of the term $\left(1-\frac{1}{\sigma}\right)^{1-\sigma} - (\rho_F+\delta_F)^{1-\sigma}$ (see equation 77). Importantly, if σ, ρ_F and δ_F are chosen such that $\frac{\partial \chi}{\partial \phi_F^*} \longrightarrow 0$, then also $\frac{\partial q(\widetilde{\phi}_F)}{\partial \phi_F^*} \longrightarrow 0$.

The second sub–result refers to the term $\frac{Y_F}{p(\widetilde{\phi}_F)}$. Considering the steady state values for r_F and K_F yields $\frac{Y_F}{p(\widetilde{\phi}_F)} = \frac{\left[(1-1/\sigma)^{1-\sigma} - \widetilde{\phi}_F(\rho_F+\delta_F)^{1-\sigma}\right]^{\frac{\sigma}{1-\sigma}}}{(1-1/\sigma)^{\sigma-1}(1-\widetilde{\phi}_F)^{\frac{1}{1-\sigma}} L_F^{-1}}$, and I can determine the following partial derivative:

$$\frac{\partial}{\partial \widetilde{\phi}_F}\left[\frac{Y_F}{p(\widetilde{\phi}_F)}\right] = \frac{(1-\sigma)(\rho_F+\delta_F)^{1-\sigma}\widetilde{\phi}_F + \sigma(\rho_F+\delta_F)^{1-\sigma} - (1-1/\sigma)^{1-\sigma}}{\sigma^{1-\sigma}(\sigma-1)^{\sigma}\left[(1-1/\sigma)^{1-\sigma} - \widetilde{\phi}_F(\rho_F+\delta_F)^{1-\sigma}\right]^{\frac{2\sigma-1}{\sigma-1}}(1-\widetilde{\phi}_F)^{\frac{\sigma-2}{\sigma-1}}} L_F.$$

Thus, $\frac{\partial}{\partial \widetilde{\phi}_F}\left[\frac{Y_F}{p(\widetilde{\phi}_F)}\right] > (<)\ 0$ if $\widetilde{\phi}_F < (>) \frac{(1-1/\sigma)^{1-\sigma} - \sigma(\rho_F+\delta_F)^{1-\sigma}}{(1-\sigma)(\rho_F+\delta_F)^{1-\sigma}} \equiv \kappa$. Again, κ denotes a critical magnitude for $\widetilde{\phi}_F$. $\kappa > 0$ if $\frac{(1-1/\sigma)^{1-\sigma}}{(\rho_F+\delta_F)^{1-\sigma}} < \sigma$ and $\kappa < 1$ since $\frac{(1-1/\sigma)^{1-\sigma}}{(\rho_F+\delta_F)^{1-\sigma}} > 1$. Thus, again, depending on its initial level, the decrease in $\widetilde{\phi}_F$ impacts $\frac{Y_F}{p(\widetilde{\phi}_F)}$ negatively or positively.

The third sub–result refers to domestic demand for the average firm's variety:

$$q(\widetilde{\phi}_F) = \frac{Y_{cons,F}}{p(\widetilde{\phi}_F)N_F\left[1 + \frac{s_{H,X}\,p(\widetilde{\phi}_{H,X})^{1-\sigma}N_H}{p(\widetilde{\phi}_F)^{1-\sigma}N_F}\right]}. \quad (79)$$

Notice that $s_{H,X} = 0$ in autarky and $s_{H,X} > 0$ after trade liberalization. Equation 79 allows for the following conclusion. If σ, ρ_F and δ_F are chosen such that $\frac{\partial q(\widetilde{\phi}_F)}{\partial \phi^*_F} \longrightarrow 0$, the terms $\frac{Y_{cons,F}}{p(\widetilde{\phi}_F)}$ and $N_F \left[1 + \frac{s_{H,X} p(\widetilde{\phi}_{H,X})^{1-\sigma} N_H}{p(\widetilde{\phi}_F)^{1-\sigma} N_F}\right]$ have to adjust into the same direction with trade liberalization.

Finally, real income results as:

$$\frac{Y_{cons,F}}{P_F} = \frac{Y_{cons,F}}{p(\widetilde{\phi}_F)} \left\{ N_F \left[1 + \frac{s_{H,X} p(\widetilde{\phi}_{H,X})^{1-\sigma} N_H}{p(\widetilde{\phi}_F)^{1-\sigma} N_F}\right]\right\}^{\frac{1}{\sigma-1}}. \tag{80}$$

Thus, if $\frac{\partial q(\widetilde{\phi}_F)}{\partial \phi^*_F} \longrightarrow 0$ and if the autarkic $\widetilde{\phi}_F$ is such that $\frac{Y_{cons,F}}{p(\widetilde{\phi}_F)}$ decreases (increases) with trade liberalization, the term $N_F \left[1 + \frac{s_{H,X} p(\widetilde{\phi}_{H,X})^{1-\sigma} N_H}{p(\widetilde{\phi}_F)^{1-\sigma} N_F}\right]$ decreases (increases) as well. Country F's welfare decreases (increases) with trade liberalization in this scenario.

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Figure 1: Autarkic equilibrium with $r < 1$

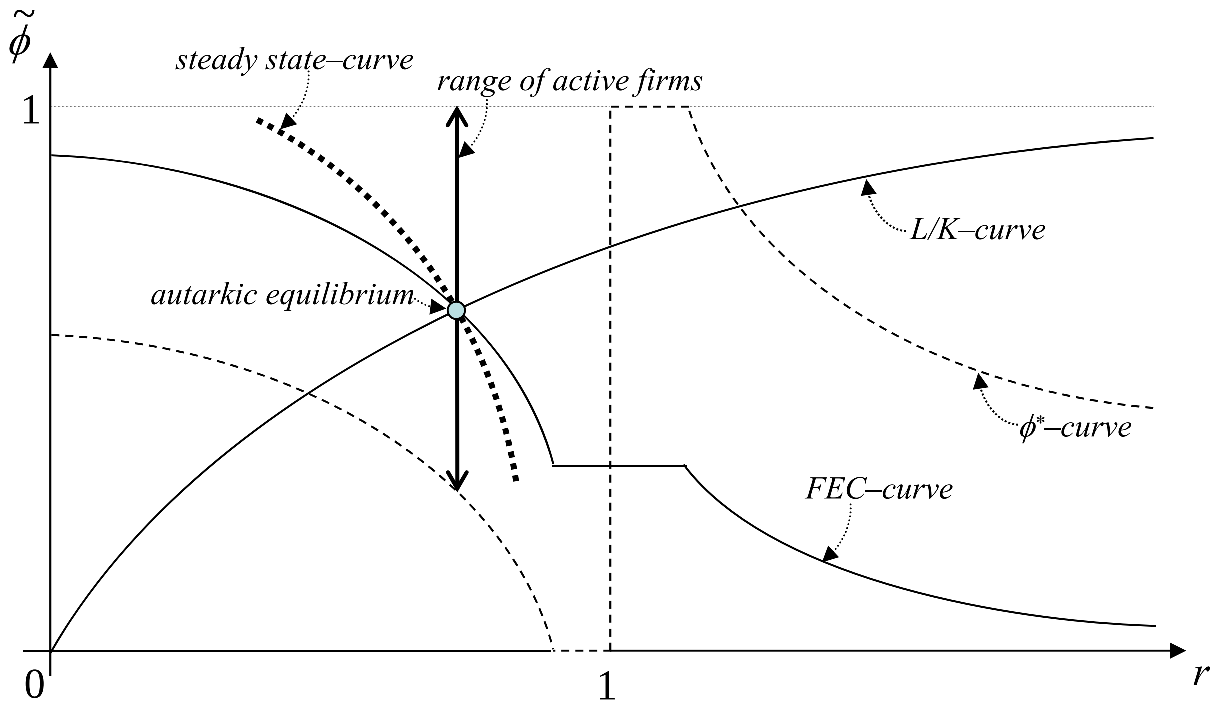

Figure 2: Open economy equilibrium with $r < 1$

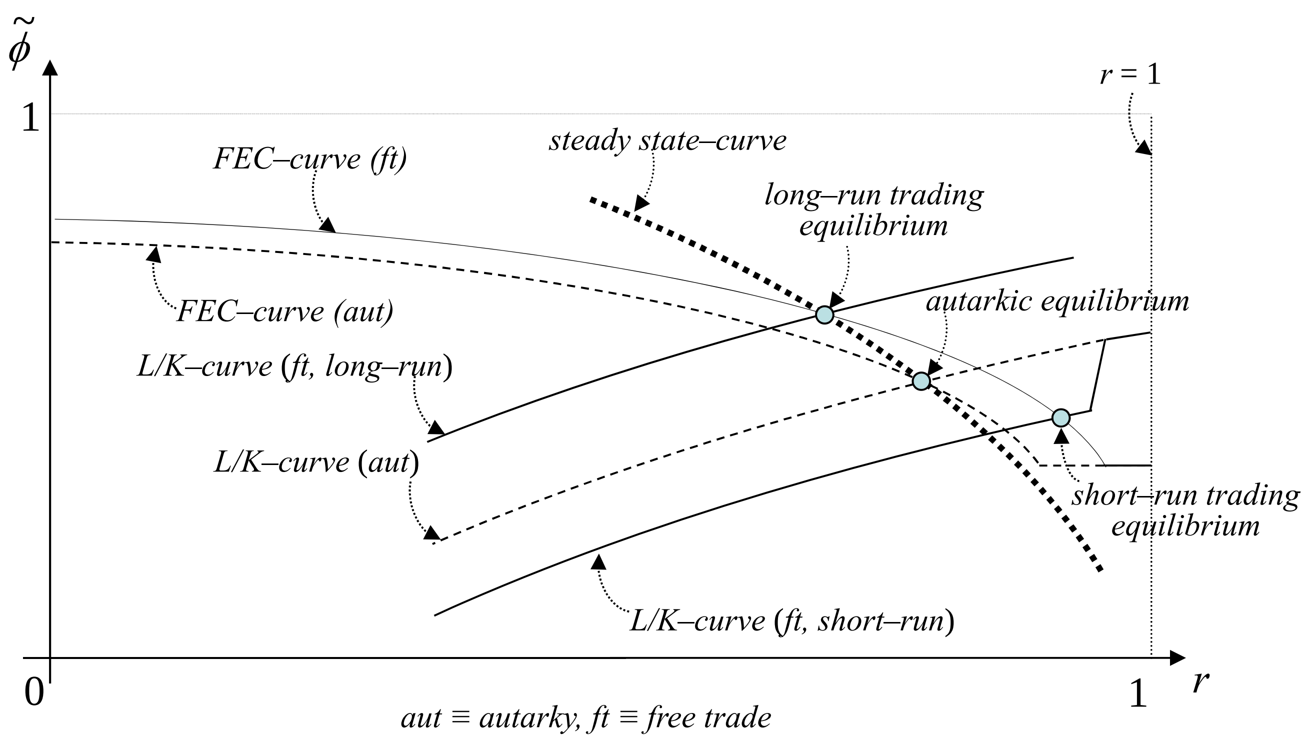

Figure 3: Autarkic equilibrium with $r > 1$

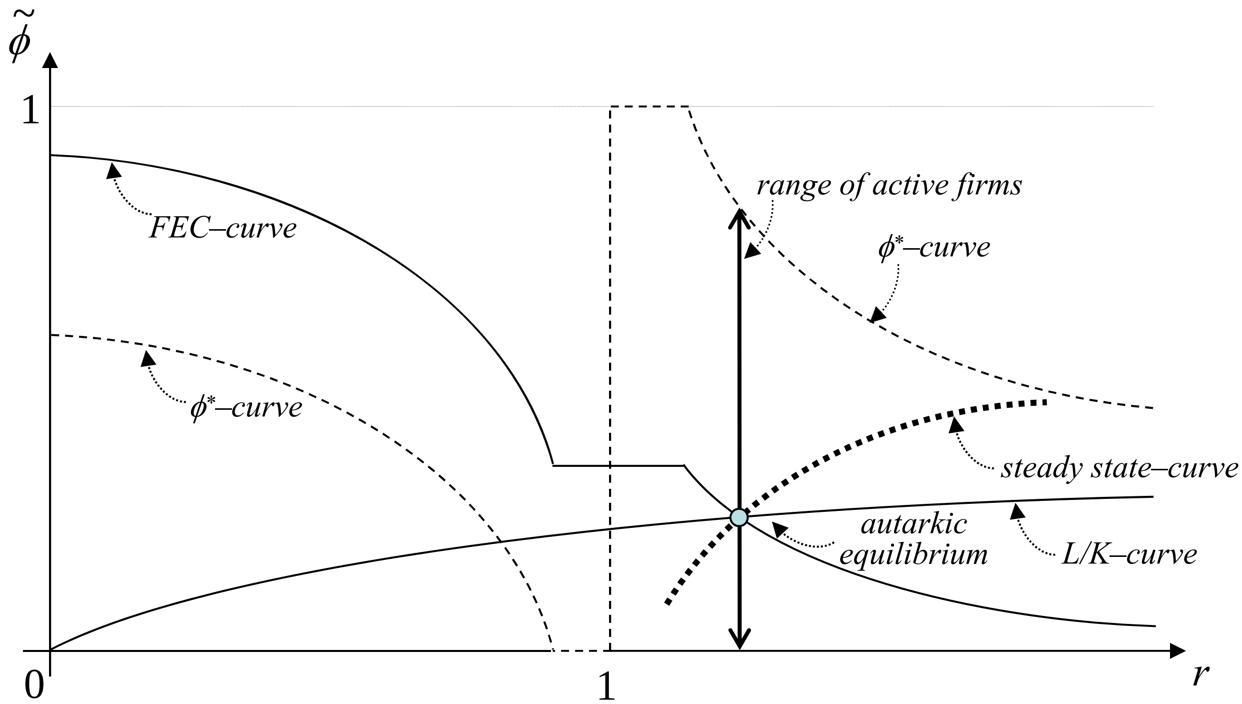

Figure 4: Open economy equilibrium with $r > 1$

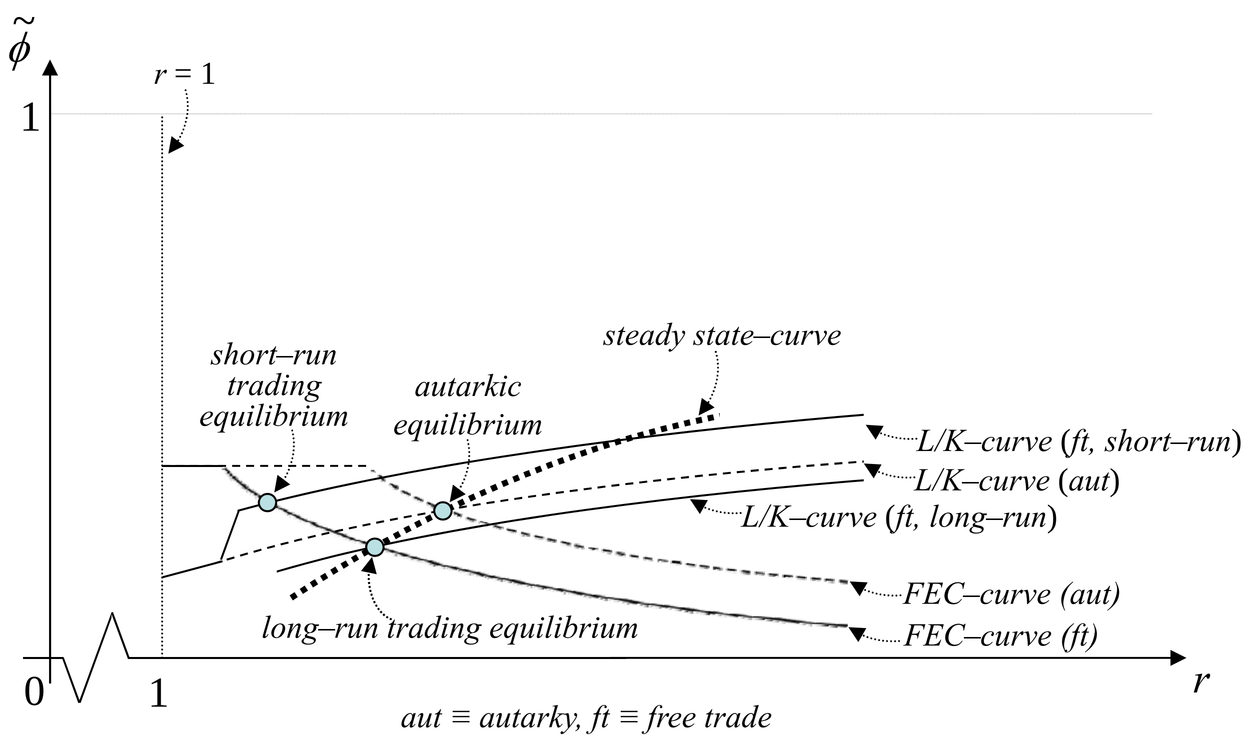

Table 1: Simulation results versus empirical evidence

	$\frac{\left(\widetilde{k/l}\right)_X}{\left(\widetilde{k/l}\right)_N}$	$\frac{[q(\widetilde{\phi}_X)+q_X(\widetilde{\phi}_X)]p(\widetilde{\phi}_X)}{q(\widetilde{\phi}_N)p(\widetilde{\phi}_N)}$	$\frac{\widetilde{l}_X}{\widetilde{l}_N}$	$\frac{exports}{total\ shipments}$
US in 2002 (source: Bernard et al., 2007)	1.32	2.48	2.19	0.14
Baseline simulation:	1.6456	2.3538	2.1742	0.1468
Sensitivity analysis 1: $\frac{f_X}{f}$				
$\frac{f_X}{f} = 1.1472 \quad (s_X = 0.1):$	1.7616	2.4204	2.2049	0.1059
$\frac{f_X}{f} = 1.0967 \quad (s_X = 0.2):$	1.5720	2.3121	2.1547	0.1831
Sensitivity analysis 2: ξ				
$\xi = 3:$	1.9685	2.3087	2.0774	0.0553
$\xi = 5:$	1.5767	3.2742	2.6843	0.5407
Sensitivity analysis 3: $\frac{f}{f_E}$				
$\frac{f}{f_E} = 3.6:$	2.1630	2.6388	2.3018	0.0410
$\frac{f}{f_E} = 4.4:$	1.3114	2.1730	2.0879	0.4389
Sensitivity analysis 4: k				
$k = 4:$	1.6684	2.4978	2.2422	0.2687
$k = 6:$	1.6273	2.3004	2.1487	0.0804

Notes: The empirical values refer to the average over three–digit NAICS US manufacturing sectors. The simulation results refer to the open economy steady state; s_X stands for the share of exporting firms in the industry's firm distribution; k is the shape parameter of the Pareto distribution for ϕ on the interval $[0.025, 1]$; the simulation of the baseline economy is based on the following numerical values: $\frac{f_X}{f} = 1.1167$ (this leads to $s_X = 0.15$), $\xi = 4$, $\frac{f}{f_E} = 4$, $k = 5$, $\sigma = 2$, $\rho = 0.05$, $\delta = 0.105$ and $\theta = 0.02$. In the sensitivity analyses the numerical value of a single parameter is changed, while the other parameters are kept at the numerical values of the baseline simulation.