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Defending Against Speculative Attacks[☆]

A Hybrid Model of Exchange Market Pressure and Crises

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Abstract

While virtually all currency crisis models recognise that the fate of a currency peg depends on how tenaciously policy makers defend it, they seldom model how this is done. We incorporate the mechanics of speculation and the interest rate defence against it in the model of [Morris and Shin](#) (American Economic Review 88, 1998). Our model captures that the interest rate defence reduces speculators' profits and thus postpones the crisis. It predicts that well before the fall of a currency interest rates are increased to offset the buildup of exchange market pressure, and this then unravels in a sharp depreciation. This pattern is at odds with predictions of standard models, but we show that it fits well with reality.

Key words: Exchange Market Pressure, Currency Crisis, Interest Rate Defence, Global Game.
JEL Codes: E58, F31, F33, G15.

1. Introduction

While virtually all modern currency crisis models recognise that the decision to abandon a currency peg depends on how tenaciously policy makers are willing to defend it, they seldom model in detail how this is done. Yet during the onset of a crisis much of the pressure on the exchange rate manifests itself through policy actions aimed at defending the currency peg, instead of through the ultimate decision whether to devalue or not. Policy makers undertake actions aimed at increasing the financing costs of speculators. In particular they raise the interest rate.

In this paper, we argue that a model that endogenously incorporates this interest rate defence captures decisive features of currency crises that are not captured by models that

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merely focus on the devaluation decision. We add it to the micro-founded global game currency crises model (Morris and Shin, 1998), which has been prominent in recent literature (see e.g. Corsetti et al. (2004), Goldstein (2005), Guimaraes and Morris (2007)). We also bring in the mechanics of speculation more explicitly. In our model, the policy maker is willing to increase the interest rate to offset the build up of pressure on exchange markets. This makes speculation riskier, and speculators postpone attacking the currency peg until the expected devaluation compensates for the risks they have to take. This idea is the main difference between our model and other approaches. It is the reason why our model outperforms traditional currency crisis models at explaining two salient features of how currency crises develop in reality. We illustrate this using empirical data from the EMS and East Asian crises.

Concretely, we focus on the following two empirically verifiable predictions regarding the onset, timing, and aftermath of the attack. First, our model implies that if an attack is successful, it will be followed by a substantial jump of the exchange rate. The resulting jump of the exchange rate is contrary to the predictions of the earliest, “first generation” models of currency crises, and, as we will argue in greater detail, essentially also contrary to the predictions of more basic global game currency crisis models, but it is in accordance with reality. Although a jump in the exchange rate is in principle consistent with “second generation”, multiple equilibria models, in these models a currency crisis is triggered by a sudden and exogenous shift in sentiment, which leaves the reason for the jump unexplained, in contrast to our model.

Second, our model implies that stress on exchange markets may be observed, through elevated interest rates, well before a crisis fully hits and the currency falls. Neither first generation models nor the basic global game model predict an extended period of stress preceding the fall of the currency, since these models do not explain well why speculators would postpone attacking it. Similarly, second generation models do not provide a reason why stress should be visible before the currency falls, since the collapse is triggered by an exogenous shift in sentiment.

Measuring stress on foreign exchange markets is exactly the focus of an empirical literature on exchange market pressure, originating with Girton and Roper (1977). To measure pressure, this literature uses a combination of data on exchange rates and data on policy actions that ward off stress. Our currency crisis model with endogenous policy responses brings these two faces of pressure together in a hybrid model and thus connects the theoretical currency crisis literature to the literature on exchange market pressure. Using this connection, we demonstrate the model’s implication of a build-up of pressure before the fall of the currency that culminates into a sharp depreciation, and we show that it fits well with reality.

Of course, there are a number of other papers that are concerned with improving the empirical performance of the traditional currency crisis models, and a subset of these are explicit about how policy makers may defend against speculative attacks. However, none of these explicitly

deals with the basic question of how the defence of the currency peg affects the profits that speculators hope to make from the attack, in other words, why such a defence works at all, and which empirical implications follow from this. While for instance [Flood and Jeanne \(2005\)](#) study the efficacy of an interest rate defence, they focus primarily on how the defence affects the sustainability of other government policies, in particular fiscal policy. [Drazen \(2000\)](#) considers a game theoretic signalling model in which the central bank increases the interest rate in order to signal the market of its *intention* of a dogged defence of the peg. [Broner \(2008\)](#) analyses a model in which a fraction of agents know the level reserves that the central bank commits to defending the currency peg, and focuses on the effects of private information.

Closer to our work on defence policies are some recent global game models of speculative attacks. In [Angeletos et al. \(2007\)](#) the central bank sets the interest rate just before speculators move. Speculators are unsure about the policy maker's eagerness to keep the peg, and view the interest rate defence as a signal of her tenacity. [Angeletos et al.](#) point out that the interest rate may transmit information about a policy maker's intentions in an intricate way, and that this may lead to multiple equilibria. Similarly, [Angeletos and Werning \(2006\)](#) and [Hellwig et al. \(2006\)](#) study global game models with multiple equilibria in which agents infer information through other channels, particularly prices and interest rates, rather than through information on fundamentals alone. The focus of these papers is on learning and inference in this non-trivial information structure. We share the view that studying such informational channels is an important line of research, but nevertheless believe that a number of crucial features of currency crises follow from more primitive aspects of the mechanics of attack and defence.

Our setup stays close to the original model of [Morris and Shin \(1998\)](#). In our model, all agents are aware of the strategic intentions of the policy maker, and there are no substantial asymmetries of information across agents. There are, however, two crucial differences with the model of [Morris and Shin](#), which are central to our results. First, in our model, actions of speculators are not obvious strategic complements. A larger amount of speculators attacking the peg may lead to a harsher defence by the policy maker, which means that the costs of speculation are higher. In this case, actions become strategic substitutes. Because [Morris and Shin](#) do not model the interest rate defence, they do not have to deal with this effect. Nevertheless, we show that our model has a unique equilibrium in threshold strategies, thus generalise their result to our setting.

Second, in contrast to the sequential timing structure adopted by [Morris and Shin](#)—in which speculators act first and the policy makers subsequently responds to their actions—in our model speculators and the policy maker act simultaneously. We believe this approach is more natural, since in reality, during crises, these agents react to each others' behaviour, without one particular party being the leader. In addition, this structure avoids the complica-

tions introduced by the informational channels in the global game models mentioned above. This means that we can fully focus on the cost-wise implications of raising the interest rate during an attack, in particular on how this affects the strategic decisions of speculators and the devaluation outcome.

The rest of the text is structured as follows. We motivate and develop our model in sections 2 through 4. In section 5, we prove that the model has a unique equilibrium. We compare the implications of our model with related empirical and theoretical literature in sections 6 and 7, and show how our model improves upon traditional currency crisis models. In section 8, we conclude. Proofs of lemmata and our main theorem appear in the appendix.

2. The Mechanics of Speculation

We develop a stylised approach to modelling a speculative attack on a currency peg, aiming to incorporate the most important features of such attacks. A good overview of the precise mechanics of speculative attacks can be found in an explanatory note by the International Monetary Fund (Folkers-Landau et al. 1997). Since these mechanics are the basis for the rest of our analysis, we briefly sketch them here.

2.1. Speculative Attacks in Practice

A speculative position against a weak currency is generally implemented by taking a short position in that currency in the forward market for foreign exchange (“forex”). Speculators enter into forward contracts with banks, selling the weak currency for a strong currency at some future date against a prearranged rate. Concretely, let f_t denote the “one-period-ahead” forward rate and s_{t+1} denote the exchange rate in the spot market at the time of maturity of the forward contract, both expressed logarithmically. The forward rate and spot rate are expressed as units of the domestic currency per unit of the foreign currency, and we will assume that the domestic currency is the weak currency. At the time of maturity of the forward contract, speculators conduct off-setting transactions on the spot market, by selling strong currency for weak currency. This way, they earn $s_{t+1} - f_t$ on each unit of the strong currency (neglecting transaction costs). Speculators make profits if $s_{t+1} > f_t$.

After signing the forward contracts, the banks involved face currency mismatches, because a forward transaction at time t maturing at $t + 1$ entails a long position in the weak currency. Due to standard risk balancing practises, the bank will immediately try to counter the mismatches that the forward transactions create by selling the weak currency spot already at time t . These spot sales lead to instantaneous pressure on the weak currency at time t . This is a crucial part of the mechanics of speculative attacks.

To support the weak currency, the domestic central bank has several instruments. Two instruments that are important in practice are active intervention on the forex markets, and

setting a high domestic interest rate. Raising interest rates may have three effects, which are standard but central to our approach. First, it increases the forward rate f_t speculators face, and thus deters speculation. To see how, note that the spot sale transforms a bank's currency mismatch into a maturity mismatch. Therefore, a forex swap with a third party is necessary to close off positions. The third party will compare the returns of holding the weak currency with revenue obtained by entering the swap. Thus, for the bank the cost of entering into the swap contract will reflect the third party's forgone interest. The income from the bank's forward and spot transactions, $f_t - s_t$, must compensate for this cost. Assume, for simplicity, that deposits in the strong currency earn no interest. Then the bank will set $f_t - s_t$ equal to the domestic interest rate. This reflects the covered interest rate parity (CIP) condition. If there is a large demand for forward contracts and no private third party is willing to carry out the swap, the bank obtains credit in the weak currency from the central bank against ceiling interest rates, and these costs are passed on to speculators, so that CIP also holds. In the rest of the paper we will abstract from the precise mechanics of the pricing of swaps, and assume the bank sets f_t using the CIP condition, fully in line with the ideas just sketched.

The second effect of the interest rate is on other agents that are active on the forex market whose desired positions in the weak currency are influenced by the interest rate. Quintessential examples of the kinds of agents that we have in mind are carry-traders. There may be others: forex traders and arbitrageurs of other banks, also perhaps of parties outside the banking system (traders on the goods market, hedge funds and large pension funds, etc.). These other agents may refuse to hold large positions in the weak currency at the prevailing fixed exchange rate in times of tension on the forex market, unless the currency risk is compensated by a sufficiently high domestic interest rate.

Lastly, an interest rate defence of the weak currency may also have adverse consequences. Increasing interest rates too much, or for a prolonged period of time, may have detrimental effects on economic activity and therefore increase the costs of maintaining the currency peg, weakening its credibility. The policy maker may, therefore, refuse to raise interest rates above a certain level. (A number of models explicitly address the economic costs of high interest rates, for instance [Flood and Jeanne \(2005\)](#) and [Lahiri and Végh \(2007\)](#).)

2.2. A Stylised Model of Speculative Trade

Currency trade during an episode of high exchange market pressure is a continuous interplay between different kinds of actors on financial markets, including central banks, where each action of some agent rapidly provokes reactions of the rest of the market. However, for the purpose of analysing speculative attacks, economists often have access to data of much lower frequency, e.g. weekly or monthly. In what follows, we will develop a model of speculative trade that is stylised enough to give clear implications for the exchange rate for such low frequencies, and preserves the intuitions sketched in the previous section (in particular that increasing the interest rate deters speculation).

In line with our discussion of the mechanics of speculative attacks in practice, we consider an economy populated by four kinds of agents:

- (I) A group of risk neutral speculators, speculating against the weak currency by entering into forward contracts with banks;
- (II) Commercial banks, that offer forward contracts to speculators and aim to reduce the resulting balance sheet mismatches through spot sales of the weak currency;
- (III) A policy maker that controls the central bank and manipulates the forex market using interest rates, in order to stabilize the exchange rate;¹
- (IV) A group of other traders, that will have to take on the role of counterpart to the spot market transactions induced by speculation, and are willing to do so if the domestic interest rate makes this sufficiently attractive.

The focus of the model is on how the mechanics of speculation and of a defence policy against it affect the decisions made by these agents in a given period, where the intended interpretation of a period is the time-unit of analysis for an empirical application (a month, in our own empirical analysis). This “intra-period” model can be used to analyse what happens over multiple periods by iterating it.

The group of other traders have a passive role in our model, and are discussed further below.² To disentangle the interplay of the decisions of the other agents, we assume that—in a given period—the decision of the policy maker occurs simultaneously with the decisions of the banks and speculators. We choose this approach for two reasons. First, this simultaneous set-up captures the continuous interaction between all types of agents in reality in a stylised way. It implies that neither the policy maker at the central bank, nor the commercial banks, nor the speculators are a “leading” party in the sequence of intra-day events; in contrast they *react to each others’* behaviour. Second, developing a more complex dynamical model of intra-day trade would not lead to the tractable results we are after.

In fact, the assumption that the speculators, the banks, and the policy maker act simultaneously leads to a key difference in the timing of events between our model and most second generation models of speculative attacks (e.g. [Obstfeld \(1996\)](#), and also [Morris and Shin \(1998\)](#)). These models have the following sequential structure. Speculators choose whether to attack the currency peg in the first stage of the game. The policy maker only acts at the second stage, at which it simply decides whether to abandon the peg or not. Since speculators have already committed to their decision at this point, the actions of the policy maker will

¹A defence strategy is often developed at the Ministry of Finance, and implemented by the central bank.

²These agents function to ensure that the spot market clears at *some* exchange and interest rate pair, and consequently to be able to determine a price for the currency when there is an excessive supply of it from banks. This is similar to the fundamental traders considered in [Morris and Shin \(2004\)](#).

regime in reality.

The second step that banks will need to take to close off positions is to enter into swap contracts. The costs of swaps are influenced by the policy maker's interest rate policy. The banks aim to pass on these costs to the speculators when they set the forward rate, so that a policy of setting a high interest rate reduces speculative activity. As noted, in our stylised setting banks do not immediately observe the actual costs associated with closing off positions, but form rational expectations about them, setting forward rates accordingly. This is because the policy maker sets the interest rate, r_t , at t_0 simultaneously with the decisions of banks and speculators.

Besides the direct effect of reducing speculative activity, a higher interest rate induces other traders to be the counterparty to spot market transactions. For simplicity, we assume that these other traders trade only at t_1 and that their demand for the weak currency is strictly increasing in r_t . The aim of the policy maker is to set the interest rate in such a way that the t_1 spot market clears at the fixed rate and that the central bank ends the period with a certain desired net position in the weak currency. We simply assume this position to be zero, though in more intricate models a target could be given in by other considerations. Thus, concretely, in our model at time t_1 the t_1 -traders will have to absorb any excess supply that the central bank has bought from the commercial banks, and the spot market rate prevailing at time t_1 will have to equal \bar{s} , for the peg to be defended in a successful and sustainable way. If the peg is not successfully defended, the currency devalues until the spot market clears.

To complete the description of the model, in the next sections we investigate the decision problems for the speculators, the banks, and the policy maker more closely. We first treat the decisions of the policy maker in full, derive her optimal interest rate policy, and then move on to the banks and the speculators and derive their optimal strategies. Subsequently, we characterise the equilibrium situation in which all agents make their decisions optimally.

3. The Policy Maker's Interest Rate Defence

The policy maker has a domestic interest rate target based on the desire to realise certain domestic policy objectives. Under the currency peg, this target must be subordinated to her objective of keeping the exchange rate fixed, so it is not realised as long as the peg remains in place. In what follows, assume u_t is the realisation of a fundamental that affects the policy maker's domestic target rate, $r_d(u_t)$ (say, unemployment). An increase in u_t is interpreted as a worsening of the fundamental. As the fundamental worsens, the policy maker would prefer a looser interest rate policy, so that the derivative $r'_d \leq 0$.

Suppose that setting $r_t = r_d(u_t)$ results in excess supply of weak currency on the spot market when the exchange rate is equal to \bar{s} . This has implications for the interest rate decision of the policy maker. To maintain the peg, the policy maker will have to increase the interest rate to achieve equilibrium on the end-of-period spot market. Let $r(u_t, \lambda_t, s_t)$ denote

the interest rate that clears the spot market against exchange rate s_t when the fundamental is u_t and speculative pressure is equal to λ_t . In particular, the interest rate $r(u_t, \lambda_t, \tilde{s})$ clears the spot market against the fixed exchange rate \tilde{s} . To simplify notation, we denote this particular interest rate by $r(u_t, \lambda_t)$, where the restriction to the peg \tilde{s} is implicitly understood.

We assume r is strictly increasing in λ_t —since speculative activity generates spot market sales by banks which have to be absorbed by other traders—and strictly decreasing in s_t . These two assumptions reflect the sensitivity of the demand of t_1 -traders to the interest rate and the exchange rate. Furthermore, we make the natural assumption that r is increasing in u_t —a worsening fundamental makes holding the currency less attractive, other things equal. Finally, we assume that r is continuously differentiable, and throughout the paper we maintain the following assumption on the derivatives of r :

$$\begin{aligned} \text{If both } r(u_t, \lambda_t) = r(\hat{u}_t, \hat{\lambda}_t) \text{ and } u_t < \hat{u}_t, \\ \text{then } r_{u_t}(u_t, \lambda_t) < r_{u_t}(\hat{u}_t, \hat{\lambda}_t) \text{ and } r_{\lambda_t}(u_t, \lambda_t) \leq r_{\lambda_t}(\hat{u}_t, \hat{\lambda}_t). \end{aligned} \quad (1)$$

In words, we compare the situation where the fundamental equals u_t with one where the fundamental is worse, equaling \hat{u}_t . If in both situations the interest rate has to be raised to the same level r_t in order to clear the spot market, then this market clearing interest rate is more sensitive to a further worsening of the situation, that is a further worsening of the fundamental or increase in speculative pressure under \hat{u}_t as it is under u_t . This assumption thus states that raising the interest rate to defend the peg becomes a less effective defence policy as the economic situation worsens, which we think is natural.

As argued in the previous section, increasing the interest rate may have detrimental effects on the economy. Assume that the *maximum* interest rate the policy maker is willing to use is $\bar{r}(u_t)$, where \bar{r} is a continuously differentiable and strictly decreasing function of u_t , with derivative \bar{r}' bounded away from 0. This amounts to the idea, crucial for all second generation models, that the policy maker is more averse to defending the peg under bad fundamentals than under good fundamentals. If $r(u_t, \lambda_t) > \bar{r}(u_t)$, then the interest rate that clears the spot market exceeds the maximum rate that the policy maker is willing to use to defend the currency peg, and thus the policy maker would prefer to abandon it. In this case, she prefers to set the interest rate to the domestic interest rate target $r_d(u_t) < \bar{r}(u_t)$ and the weak currency devalues. If $r(u_t, \lambda_t) \leq \bar{r}(u_t)$, the policy maker prefers to defend the peg and set $r_t = r(u_t, \lambda_t)$, so that indeed $s_t = \tilde{s}$. The policy maker's optimal decision can be characterised as follows.

Lemma 1. *There exists a continuously differentiable, strictly decreasing function $\lambda_t \mapsto u^*(\lambda_t)$, with compact range $[\ell, h]$, such that the policy maker prefers to devalue if and only if $u_t > u^*(\lambda_t)$.*

Figure 2 gives a graphical representation of this characterisation. If $u_t < \ell$, the policy maker

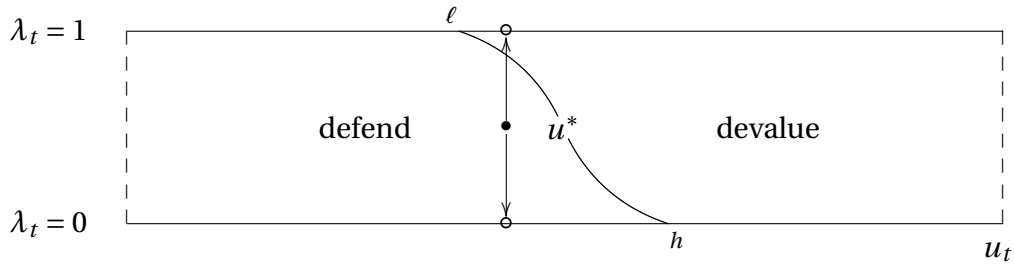


Figure 2: Characterising the decision to devalue or defend

does not abandon the peg for any value of $\lambda_t \in [0, 1]$. If $u_t > h$ the policy maker never defends the peg, whatever λ_t . If u_t is in the region between ℓ and h , then the optimal decision depends on the amount of speculative pressure λ_t . The weak currency is “ripe for attack”, and a currency crisis can be triggered if a sufficiently large number of speculators attack the weak currency. This tripartite classification is familiar from the literature on second generation currency crisis models (see [Jeanne \(1997\)](#) or [Morris and Shin \(1998\)](#)). The function u^* is also instrumental to characterise the policy maker’s optimal interest rate decision as a function of λ_t and u_t . This is given by:

$$r^*(u_t, \lambda_t) = \begin{cases} r(u_t, \lambda_t) & \text{if } u_t \leq u^*(\lambda_t); \\ r_d(u_t) & \text{if } u_t > u^*(\lambda_t). \end{cases} \quad (2)$$

4. The Decisions of Banks and Speculators

4.1. A Global Game Approach

What will speculators do in the region $[\ell, h]$, the region where the currency is ripe for attack? Following [Morris and Shin \(1998\)](#) and related literature, we will apply the global game technique of [Carlsson and van Damme \(1993\)](#) to resolve this question. The essential idea behind the global game technique is to follow a modelling approach that allows for a lack of common knowledge about the fundamental among agents. To this end, we assume that at the start of the period, the true value of the fundamental u_t is determined. At this moment, each bank and each speculator is endowed with some very precise private information about the true value of u_t , and this fact is common knowledge among agents. However, the actual value of u_t remains unknown to speculators and banks, so that there can be no common knowledge of the fundamental itself. Like in [Morris and Shin \(1998\)](#), the policy maker observes u_t perfectly.

Concretely, assume there is a large set of speculators, indexed on the real interval $[0, 1]$. At t_0 , each speculator is matched to a single bank—for notational simplicity we assume that speculator i is matched to bank i . At this stage, the speculator and the bank may enter into a forward contract. Observe that, in principle, any forward rate offered by bank i to speculator

i conveys some of the bank's information about u_t , since the forward rate is set based on the bank's information about u_t . Similarly, the decision of the speculator i to enter into the forward contract or not reveals some of her information on u_t to bank i . The final contract should reflect an informational equilibrium between both parties.³ We assume that, based on their ideas about the fundamental, both parties conclude that the true value of u_t is close to some $x_{it} \in \mathbf{R}$. This x_{it} will be called the *signal* of speculator i and bank i , and is uniformly distributed around u_t :

$$x_{it} \sim U(u_t - \epsilon, u_t + \epsilon), \text{ with } \epsilon \text{ fixed and } 2\epsilon < h - \ell, \quad (3)$$

so that the signal x_{it} noisily reflects the true fundamental. Signals are idiosyncratic and independently and identically distributed across all the bank-speculator pairs, and it is known to all agents that signals are distributed according to (3). From the perspective of speculator and bank i , the true fundamental u_t is in the set $X_{it} = [x_{it} - \epsilon, x_{it} + \epsilon]$, and each value is equally likely.

A *strategy* for a bank is a decision rule $\sigma_{it}^b : x_{it} \mapsto f_{it}$, where f_{it} is the forward rate offered when the bank's signal is equal to x_{it} . The forward rate that is offered may differ from bank to bank, which is actually a realistic feature of forex trade since in reality forex markets—including forward markets—are highly decentralised. A strategy for a speculator is a decision rule $\sigma_{it}^s : (x_{it}, f_{it}) \mapsto d_{it}$; where d_{it} is the short position of the speculator when her signal is equal to x_{it} and the forward rate offered is f_{it} . A *joint strategy profile* is a sequence σ_t that assigns a strategy to each speculator and each bank. Finally, a joint strategy profile is *symmetric* if $\sigma_{it}^s = \sigma_{jt}^s$ and $\sigma_{it}^b = \sigma_{jt}^b$ for all $i, j \in [0, 1]$.

4.2. The Bank's Decision Problem

Based on the signal x_{it} , bank i offers the forward rate $f_{it} = \sigma_{it}^b(x_{it})$ to speculator i . Bank i wishes to set this forward rate at a level that properly reflects the costs associated with signing a forward contract at time t_0 , as discussed in section 2. Its optimal strategy is to set:⁴

$$f_{it} = \tilde{s} + \mathbf{E}[r_t | x_{it}], \quad (4)$$

where $\mathbf{E}[r_t | x_{it}]$ is the interest rate it expects to prevail. As explained in section 2.2, the expectations operator appears here because of our “simultaneous action” global game approach. The condition in equation (4) is therefore the no-arbitrage condition for the forward market in our setting, since it entails that at the moment t_0 , when forward contracts are signed and

³Note that there is a theoretical justification for this approach. See the arguments, and the theorem on the coincidence of posteriors in this case, as put forward by [Aumann \(1976\)](#).

⁴Given that individual banks are small and have no influence on the behaviour of the policy maker, we can derive their optimal strategies while treating $\mathbf{E}[r_t | x_{it}]$ as being fixed. Like before, we assume that the foreign interest rate equals zero.

the exchange rate is fixed at \tilde{s} , no ex ante profitable, covered, forward market transactions can be made. When the policy maker is expected to increase the interest rate, the forward market tightens, that is, f_{it} increases.

4.3. The Speculator's Decision Problem

While it is impossible to enter into ex ante profitable *covered* forward market transactions, it is possible to enter into ex ante profitable *uncovered* forward market transactions, because the currency peg may be abandoned at time t_1 . Essentially, the policy maker may create a “window of opportunity” for speculators by guaranteeing the exchange rate \tilde{s} , but refusing to set the interest rate high enough to rule out gains in case she is forced to abandon this rate. This is precisely how speculators hope to make profits.

The speculator who is matched to bank i is offered the forward rate f_{it} . This forward rate represents the cost involved with speculation. The speculator may gain from speculation if the policy maker abandons the currency peg and the resulting devaluation is sufficiently large. If the prevailing peg is abandoned, it is not *a priori* clear what will be the exchange rate that can be expected to prevail at the date of maturity of the forward contracts ($t + 1$). We assume that, conditional on a collapse of the peg at time t_1 , the time t_0 expectation of the spot market rate that prevails in period $t + 1$, s_{t+1}^e , depends on the current values of λ_t and u_t :

$$s_{t+1}^e(u_t, \lambda_t) - \tilde{s} \begin{cases} = 0 & \text{if } u_t \leq u^*(\lambda_t); \\ \geq r_d(u_t) & \text{if } u_t > u^*(\lambda_t). \end{cases}$$

The expected amount of devaluation, $s_{t+1}^e - \tilde{s}$, is strictly increasing in both the size of the attack and a worsening of fundamentals. If $u_t \leq u^*(\lambda_t)$, so that the peg does not collapse, s_{t+1}^e is equal to \tilde{s} . Conditional on the collapse of the peg (that is $u_t > u^*(\lambda_t)$), we let $s_{t+1}^e(u_t, \lambda_t) - \tilde{s} \geq r_d(u_t)$. This means that speculators believe that speculation will be profitable when the peg indeed collapses, which appears consistent with how speculative attacks develop in reality.⁵

From the perspective of speculator i , expected profit from attacking the weak currency is:

$$\pi^e(x_{it}, f_{it}; \boldsymbol{\sigma}_t) := \mathbf{E}[s_{t+1}^e(u_t, \lambda_t) | x_{it}] - f_{it}. \quad (5)$$

Speculator i would like to attack if and only if $\pi^e(x_{it}, f_{it}; \boldsymbol{\sigma}_t) > 0$. Each speculator's wealth is equal to one unit of the weak currency. Since speculators are risk neutral, we may assume that $d_{it} = 1$ if speculator i attacks and $d_{it} = 0$ otherwise. Her *optimal* decision depends on the signal x_{it} and the joint strategy profile $\boldsymbol{\sigma}_t$, and the forward rate f_{it} .⁶ Expected profit is

⁵This assumption is implicitly made in virtually all global game currency crises papers. Technically, the assumption also guarantees that speculators find it optimal to attack when $x_{it} \geq h + \epsilon$, easing the application of the global game technique, and simplifies the proof of our main theorem below.

⁶Given that speculators are small and have no influence on the value of λ_t and f_{it} , we can determine their optimal strategy while treating these as being fixed.

decreasing in the forward rate, which is in turn influenced by the interest rate policy. Thus if the policy maker sets r_t sufficiently high, risk neutral speculators will postpone the attack until the expected devaluation compensates for the risks they take when going short. In equilibrium this risk will have a substantial impact on the behaviour of speculators.

5. Unique Equilibrium

5.1. Conditions for Equilibrium

A joint strategy profile σ_t describes the behaviour of all speculators and banks completely for any distribution of signals. If, moreover, the joint strategy profile σ_t is symmetric, the decisions of speculators and banks do not depend on which agent receives what signal, but only on the aggregate distribution. In this case we can write speculative pressure, λ_t , simply as a function of u_t and the joint strategy profile σ_t . Indeed we have:

$$\begin{aligned}\lambda_t(u_t; \sigma_t) &= \int_0^1 \sigma_{it}^s(x_{it}, f_{it}) di \\ &= \int_0^1 \sigma_{it}^s(x_{it}, \sigma_{it}^b(x_{it})) di = \int_0^1 \sigma_{jt}^s(x_{it}, \sigma_{kt}^b(x_{it})) di \quad \text{for all } j, k \in [0, 1] \quad (6) \\ &= \frac{1}{2\epsilon} \int_{u_t-\epsilon}^{u_t+\epsilon} \sigma_{jt}^s(x, \sigma_{kt}^b(x)) dx \quad \text{for all } j, k \in [0, 1].\end{aligned}$$

An *equilibrium* of the model is a joint strategy profile such that three optimality conditions, each following from the decision problems of agents, are satisfied simultaneously:

- (I) Each speculator attacks if and only if the expected profit from the attack is positive;
- (II) Banks set the forward rate in a way that rules out arbitrage opportunities from the bank's perspective;
- (III) The policy maker sets r_t so that she maintains end-of-period equilibrium on the spot market for the weak currency at \tilde{s} if she wishes to maintain the peg. Otherwise she sets $r_t = r_d(u_t)$.

We will now show that the model has a unique, symmetric equilibrium in which the choices of banks and speculators depend on the private information they receive about u_t . In fact, in equilibrium speculators use the threshold strategies that are familiar from the global games literature in general, in particular from [Morris and Shin \(1998\)](#). We start by defining threshold strategies. Let i be a speculator. A *threshold strategy* for speculator i is a strategy, characterised by a threshold value x , such that i attacks if and only if $x_{it} \geq x$. A *joint threshold strategy around x* is a (symmetric) joint strategy profile such that all speculators follow identical threshold strategies, characterised by the number x . Such profiles only specify the signals at which speculators attack, and not the strategies followed by banks. However, the following result shows that this is without loss of generality.

Lemma 2. *Any symmetric equilibrium is completely characterised by specifying the signals at which speculators attack.*

Moreover, the threshold strategies considered in most global game currency crisis models are also just strategies for speculators, so that this approach does not differ qualitatively from the one in the literature.

Nevertheless, our model still differs from the standard global game currency crisis models in that the actions of speculators are not obvious strategic complements. Of course, if the attack succeeds and brings about a devaluation, it pays off to take part in a large attack, because the resulting devaluation will be larger when more agents attack the peg. In this case, there are clear strategic complementarities. However, if the attack on the currency peg is unsuccessful, a larger amount of agents attacking the peg will only lead to a harsher defence by the policy maker, so that the costs of speculation are higher. Therefore, speculators would prefer to take part in a small rather than a large unsuccessful attack. In this case, actions are strategic substitutes. Because [Morris and Shin](#) do not explicitly model the defence, they do not have to deal with the reversed trade-off in case of an unsuccessful attack. As a consequence, we cannot directly apply their arguments to our model. However, under assumption (1) we have obtained the following, similar, result.

Theorem 3. *(i) There is a unique equilibrium in joint threshold strategies. (ii) There are no other equilibria, neither in symmetric nor in asymmetric joint strategy profiles.*

In other words, the weak currency is attacked if and only if the fundamental deteriorates beyond a certain threshold $\bar{u} \in \mathbf{R}$, which is precisely the kind of equilibrium derived by [Morris and Shin \(1998\)](#). In the “threshold equilibrium” two qualitatively different possible outcomes can obtain. For low values of u_t , speculators can expect a dogged defence of the currency peg. For high values of u_t , the defence will be lackluster. In any case, the forward rate will reflect the kind of defence fairly—it will be low when the defence is lackluster, and high when it is dogged—and the kind of defence only depends on the true fundamental u_t . A speculator’s signal thus reflects the likelihood of a dogged defence versus a lacklustre one.

Part (i) of theorem 3 is an immediate consequence of two lemmata we present next. The first lemma shows that when a joint threshold strategy is used, the optimal behaviour of any given individual speculator is also a threshold strategy.

Lemma 4. *Let σ_t be any joint strategy profile such that (i) speculators use a joint threshold strategy around x ; and (ii) both banks and the policy maker use their optimal responses to the joint threshold strategy. Then the optimal strategy for each individual speculator is a threshold strategy, and the associated threshold value, denoted $x^*(x)$, is a continuous function of x .*

A value x is called a fixpoint of x^* if $x^*(x) = x$. Lemma 4—which, in fact, does *not* depend on assumption (1)—entails that every fixpoint of x^* characterises an equilibrium in joint

threshold strategies. Since x^* is continuous, existence of an equilibrium follows from the well-known Brouwer fixpoint theorem. Our next lemma then completes the proof of the first part of theorem 3.

Lemma 5. *The function x^* has a unique fixpoint.*

The “partial” lack of complementarities in our model is similar, though not identical, to that in a model of [Goldstein and Pauzner \(2005\)](#). Theorem 3 generalises a similar result obtained by these authors to our setting; in particular, assumption (1) plays a similar role in our argument as the “single crossing condition” in that of these authors. The proof of part (ii) of the theorem chiefly involves algebraic manipulation and is fully delegated to the appendix.

5.2. The Locus of the Threshold Equilibrium

Theorem 3 states that there is a unique equilibrium, in joint threshold strategies. What can be said about the position of the speculators’ threshold inside the interval $[\ell, h]$ in figure 2? [Carlsson and van Damme \(1993\)](#) already observed that in a global game, an agent chooses the action that gives the highest expected payoff when she is completely unsure about the actions of others. With slight *abuse du langage*, this action may be called the “risk dominant action” (this follows existing terminology for two player games). In our setup, this works as follows. Consider a *marginal* speculator, that is, a speculator i who receives exactly the signal x_i^* that corresponds to the equilibrium threshold. For simplicity, assume ϵ is small.⁷ The decision problem faced by speculator i is crucial to understand the position of the speculators’ threshold, and makes clear why it is important to take the interest rate into account. Recall that from i ’s perspective, the true value of u_t is in the set $[x_i^* - \epsilon, x_i^* + \epsilon]$. In equilibrium, all speculators use the threshold strategy around x_i^* . The uniformity of signals implies that, on the interval $[x_i^* - \epsilon, x_i^* + \epsilon]$, λ_t increases linearly from the value 0 (at $x_i^* - \epsilon$) to 1 (at $x_i^* + \epsilon$). Even though i knows u_t up to ϵ , she faces much uncertainty about λ_t . Concretely, if she attacks she faces the risk that speculators fail to coordinate successfully, even though the currency is ripe for attack.

From speculator i ’s perspective, the expected devaluation of the weak currency is the average devaluation on the ϵ -interval around x_i^* . For small ϵ this is approximately:

$$\int_0^{\lambda^*} \tilde{s} d\lambda + \int_{\lambda^*}^1 s_{t+1}^e(x_i^*, \lambda) d\lambda - \tilde{s}, \quad (7)$$

where λ^* solves $u^*(\lambda^*) = x_i^*$. The forward rate f_{it} offered to speculator i reflects the average interest rate around x_i^* and is approximately equal to:

$$\tilde{s} + \int_0^1 r^*(x_i^*, \lambda) d\lambda \quad (8)$$

⁷The argument does not require that ϵ is small, but in this case equations (7) and (8) below take their simplest form.

Since all speculators attack around x_i^* , bank i expects the policy maker to increase interest rates sharply, to ward off speculative pressure. Thus the forward rate f_{it} will differ substantially from \tilde{s} . For a marginal speculator, the expected devaluation of the weak currency, given by equation (7), must be substantial, since it exactly compensates for the (substantial) cost of attacking it, given by equation (8). Since the expected devaluation is increasing in u_t , the speculators' threshold must be substantially to the right in the interval $[\ell, h]$ and, moreover, is increasing in the maximum interest rate $\bar{r}(\bar{u})$ that the policy maker is willing to use.

[Morris and Shin \(1998\)](#) model transaction costs, but not the effect of speculative activity on the forward rate. Transaction costs are usually taken to be small and fixed (as also emerges from the discussion by these authors in their Section III.B). In terms of our model, small and fixed transaction costs are tantamount to assuming a very low value of $\bar{r}(u_t)$, that does not vary with the amount of speculative pressure, or, in other words, to assuming that the currency peg is not actively defended. Therefore, the model of [Morris and Shin](#) and Shin predicts that the speculators' threshold should be rather close to ℓ in the interval $[\ell, h]$, in other words, that speculators will attack *en masse* when the expected devaluation is still modest, suggesting that the exchange rate will jump only slightly. In most of the subsequent global games literature on currency crises (including the recent literature that incorporates a role for interest rates, [Angeletos et al. \(2007\)](#) and [Hellwig et al. \(2006\)](#)), the size of devaluation is taken to be large and is fixed exogenously, which makes a further comparison of this point difficult.

Note that the impact of the interest rate on the cost of speculation has an important policy implication for the effectiveness of other peg defences (e.g. sterilised intervention) compared to the interest rate defence. A defence of the peg that is not based on the interest rate instrument, but consists, say, of interventions in the spot market using foreign reserves, will be less effective, as it does not raise the cost of speculation. In the absence of interest rate hikes, the risks associated with speculation remain quite low.

This implication also distinguishes our approach from recent literature that emphasises the signalling function of a defence policy (e.g. [Drazen \(2000\)](#), and—to a considerable extent—[Angeletos et al. \(2007\)](#)). A precondition for an effective defence strategy based on signalling is that the signal is costly *for the policy maker*, so that it reveals her tenacity. Sterilised intervention in the spot market is costly for the policy maker because the central bank loses foreign reserves, so fulfils this precondition. However, it does not substantially increase the risk speculators face, and thus fails to exploit a more primitive aspect of the mechanics of attack and defence. Although signalling effects may play an additional role, our model shows that increasing the financing costs of speculators is the cornerstone of a successful defence.

5.3. A Dynamic Extension

Up to this point the discussion has focused on what we have called the intra-period model. We can also consider a dynamic interpretation of the model, where the intra-period model

is repeated over time as long as the currency peg is not abandoned. The fundamental u_t is driven by a process that makes moves between the periods. In each period, agents receive new information on the fundamental. If the dispersion of this information, given by ϵ , is sufficiently small, this implies that each agent's posterior on u_t is approximately uniform in each period, so that our intra-period results apply. A setup of this sort can be formalised straightforwardly in the style of the multi-period model in [Morris and Shin \(1999\)](#). These authors assume that the fundamental follows a persistent Markov process over time. This is plausible if u_t reflects the persistent variables such as unemployment or real currency overvaluation that are typically considered to be driving factors underlying currency crises. The persistence of u_t translates into a persistent path for the policy choice variable r_t over time in the multi-period model.

6. Pressure on the Forex Market

The model developed above is one of a pegged currency that comes under severe pressure. In this section, we deal with the question how this pressure can be observed in practice, and how to link this to the model. In the next section, we explore this connection to examine how some of the model's implications fit with reality.

In a floating exchange rate regime, exchange rate changes fully reflect tensions in the market. But in case of exchange rate rigidity, policy makers ward off exchange rate changes through policy measures, and the exchange rate change alone is no longer an appropriate measure of pressure. To nevertheless obtain an indicator of forex market tensions, [Girton and Roper \(1977\)](#) introduced the concept of exchange market pressure (EMP), which [Weymark \(1995\)](#) then further formalised. These authors defined EMP for the domestic currency as the relative depreciation required to remove excess supply of domestic currency on the forex market in the absence of policy actions to offset that excess supply. This concept of EMP is widely used in the literature.

Contrary to many of the currency crisis models in the literature, our model can be used to derive an expression for EMP. Indeed, the key to deriving EMP endogenously in the model is the expression we have for $r^*(u_t, \lambda_t)$, which we obtain because we explicitly model the interest rate defence. If the policy maker did not care about the currency peg, she would set $r_t = r_d(u_t)$, where $r_d(u_t)$ is the interest rate target based on domestic objectives.⁸ Recall that $r(u_t, \lambda_t, s_t)$ is the interest rate that clears the spot market against exchange rate s_t . It follows that if the policy maker (counter-factually) were to set $r_t = r_d(u_t)$ in the face of speculative pressure λ_t , the weak currency would depreciate *vis-à-vis* the strong currency from its fixed rate \bar{s} to the value s_t that solves $r(u_t, \lambda_t, s_t) = r_d(u_t)$. More generally, let $s(u_t, \lambda_t, r_d(u_t))$ be the uniquely⁹ induced function such that $r(u_t, \lambda_t, s(u_t, \lambda_t, r_d(u_t))) = r_d(u_t)$. Our model implies

⁸For sake of consistency with the theoretical model, we ignore other instruments that ward off pressure.

⁹Uniqueness requires the additional condition that the derivative of r_t w.r.t. s_t is bounded away from 0.

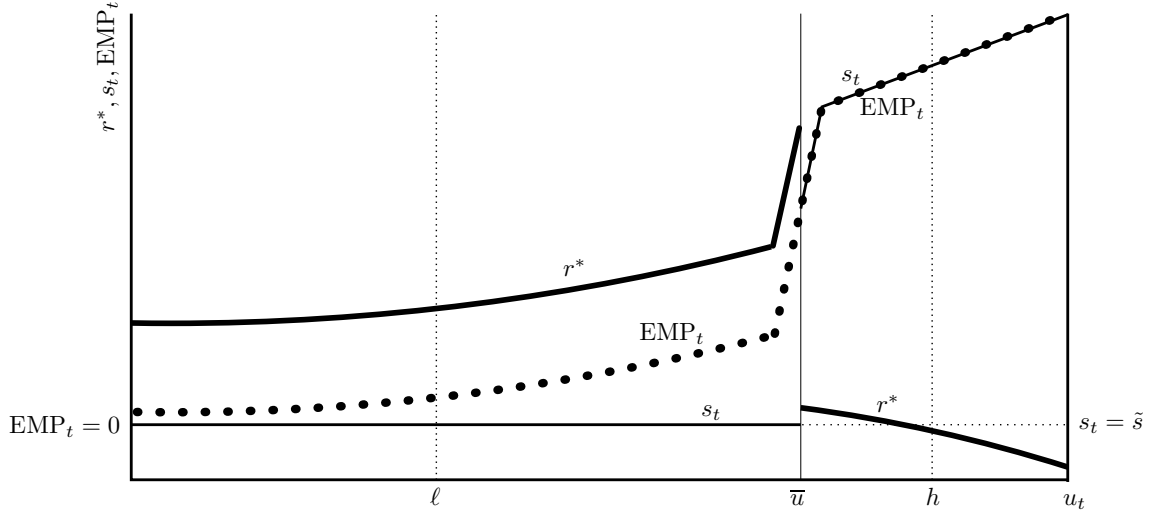


Figure 3: Exchange market pressure, the interest rate, and the exchange rate, as a function of u_t

that EMP is equal to:

$$\text{EMP}_t := s(u_t, \lambda_t, r_d(u_t)) - \tilde{s}. \quad (9)$$

Since the actual exchange rate equals $s_t = s(u_t, \lambda_t, r_t)$, EMP_t can be expressed as:

$$\begin{aligned} \text{EMP}_t &= s(u_t, \lambda_t, r_d(u_t)) - s(u_t, \lambda_t, r_t) + s(u_t, \lambda_t, r_t) - \tilde{s} \\ &=: w_t(r_t - r_d(u_t); u_t, \lambda_t) + \Delta s_t. \end{aligned} \quad (10)$$

The function w_t is a “weighting” function. The expression Δs_t is the change in the exchange rate from $t - 1$ to t . If the peg is abandoned, so that $r_t = r_d(u_t)$, then EMP_t is equal to the devaluation of the currency compared to the fixed value \tilde{s} , so to Δs_t . Therefore, the function w_t satisfies $w_t(0; u_t, \lambda_t) = 0$. Whenever the peg remains in place, EMP_t is a monotonic function of the wedge between the interest rate target based on domestic objectives and the interest rate set by the policy maker at time t , that is, of $r(u_t, \lambda_t) - r_d(u_t)$.¹⁰ Using some assumptions on the weight w_t , and a method to proxy for r_d , formula (10) can be used to express EMP_t as a function of the (observed) interest rate r_t and (observed) exchange rate s_t (and is more convenient than equation (9)). Note that EMP_t is not identical to speculative pressure λ_t . Speculative pressure is a partial determinant of EMP_t , since r_t is partly set in response to λ_t . Yet λ_t might have been kept low because the policy maker sets r_t sufficiently high, which still indicates the presence of EMP.

Figure 6 shows the relationship between the fundamental u_t , EMP_t , and the depreciation of the weak currency as predicted by the model. In the figure, the policy maker abandons the currency peg when u_t exceeds \bar{u} . There is exchange market pressure at values of u_t well below the critical value \bar{u} , revealed by the fact that the policy maker is forced to increase r_t in order

¹⁰In the EMP literature, this wedge is sometimes expressed using $r_t - r_{t-1}$. Yet using $r(u_t, \lambda_t) - r_d(u_t)$ is in line with the definition of EMP, as our model shows. [Klaassen and Jager \(2008\)](#) discuss this in a pure EMP setting.

to maintain equilibrium in the spot market while, at the same time, worsening fundamentals imply that r_d decreases. This pressure is reflected by the fact that r_t substantially exceeds $r_d(u_t)$. The policy maker's willingness to defend the peg keeps the speculators out, so that λ_t remains low. For values of u_t approaching \bar{u} , EMP_t increases sharply, because of vast speculative activity. At the point \bar{u} , speculators force the policy maker to abandon the currency peg; since she does no longer wish to set r_t to the interest rate $r(u_t, \lambda_t)$ that clears the spot market at \tilde{s} , the exchange rate jumps. Since the marginal speculator attacks only if expected profits compensate for a high forward rate, this jump must be substantial. Thus, for values of u_t to the right of the point \bar{u} , EMP_t reflects a sharp depreciation of the weak currency *vis-à-vis* the strong currency. If the peg is abandoned, the currency depreciates (or devalues) to its new equilibrium level. In the next period, EMP disappears from the market.

7. Relation to the Currency Crisis Literature

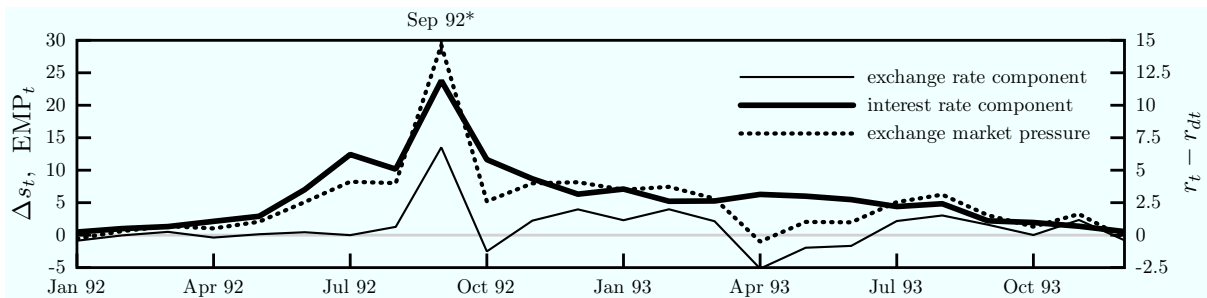
To substantiate that our model provides a good account of how currency crises develop in reality, this section compares its implications for EMP with some time series data from the empirical EMP literature, and reflects on how our model fares versus the traditional currency crisis models. The EMP time series in this section are taken from [Klaassen and Jager \(2008\)](#), who have developed an approach to derive EMP from observed data. They use observed interest rates for r_t^* and suggest methods to proxy $r_d(u_t)$.

The focus in this section is on the 1992–1993 crisis in the European Monetary System (“EMS”). The number of countries involved in the EMS crisis coupled to the absence of capital controls during the crisis and absence of other, further complicating issues (for instance the insolvency of governments, or a crisis in the banking sector), allow for the derivation of a number of clean and comparable time series, and make this group of countries a natural choice.¹¹ Moreover, the availability of good data for this group of countries enables us to use advanced methods for proxying r_d , deriving it from a Taylor rule (see [Taylor \(1993\)](#)). For each period we use the actual OECD forecasts that were made at that time for the expected inflation and output gaps in the Taylor rule. From forex data we compute Δs_t . Finally, to compute EMP_t in equation (10) we assume that the weighting function w_t is constant, derived as in [Eichengreen et al. \(1996\)](#), as this is the most common approach used in the EMP literature.

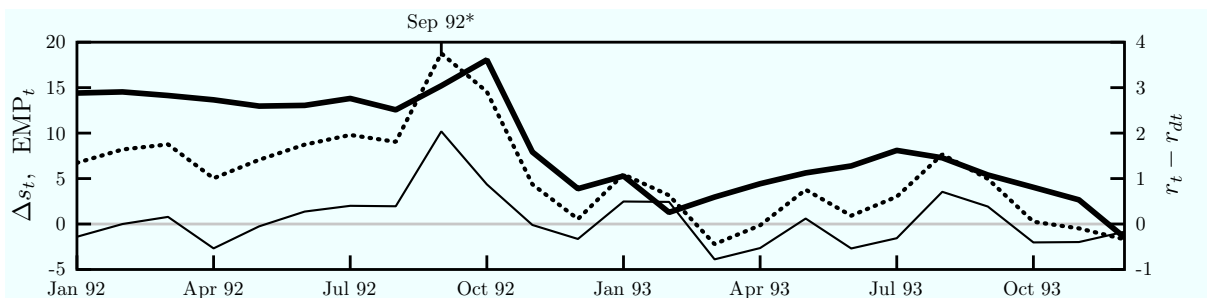
Figure 4 shows the EMP components and measure thus obtained, for the currencies of France, Italy and the United Kingdom—three countries that suffered heavily under speculative attacks—versus that of Germany, during the crisis.¹² To illustrate the robustness of our

¹¹E.g. [Goderis and Ioannidou \(2008\)](#) show empirically that high interest rates indeed defend currencies during crises, but also that weak private sector balance sheets, in particular a large stock of short term debt, can negate the efficacy of the defence. Such problems were absent during the EMS crisis.

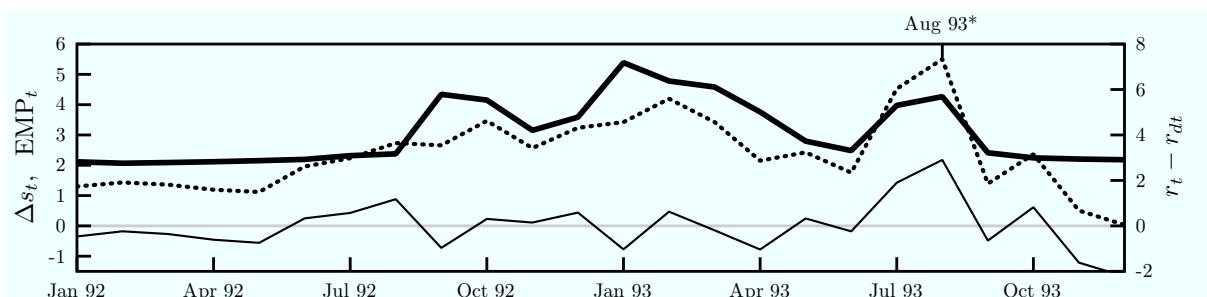
¹²The Italian and U.K. authorities were forced to leave the EMS in September 1992. The French authorities were forced to widen the EMS fluctuation band from 4.5% to 30% in August 1993.



(a) Italy: pressure on the lira



(b) United Kingdom: pressure on the pound



(c) France: pressure on the franc

* indicates the period of devaluation.

Figure 4: Exchange Market Pressure during the 1992–1993 EMS crisis

approach, figure 5 also shows EMP time series for Thailand, South-Korea, and Indonesia, countries that suffered during the 1997 East-Asian crisis. For these countries we do not have real-time expectations data, so that we have to proxy r_d with other means.¹³ Since EMP is expressed in terms of the counter-factual depreciation of the currency, it is measured on the same (left hand) scale as Δs_t .

While time series are not directly comparable to the static picture in figure 6, note that in (the dynamic extension of) our model, EMP_t will be a persistent process, provided that the stochastic process that drives u_t is persistent, so that its implications for EMP_t can be easily reinterpreted in the dynamic setting. Table 1 contains some economic indicators that may be considered as candidates for the fundamental u_t , all of which show persistence. We now evaluate three strands of traditional models of currency crises against the background of the

¹³The EMP measures in these figures are based on interest rate differentials with the United States; see Klaassen and Jager (2008) for details. Both the interest rate hikes and exchange rate increases were much bigger during the East-Asian crisis than during the EMS crisis. Therefore, using more complex methods to derive r_d would lead to economically insignificant differences with the EMP measures shown in figure 5.

| Italy | 1991 | | | | 1992 | | | |
|--------------------------------------|---------------|------|------|------|---------------|------|------|------|
| | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | *Q3 | Q4 |
| unemployment rate | 8.6 | 8.5 | 8.4 | 8.6 | 8.7 | 8.7 | 9.0 | 8.9 |
| current account (% of GDP) | -2.8 | -2.2 | -1.5 | -1.8 | -3.5 | -2.1 | -1.8 | -2.2 |
| real overvaluation (\dagger REER) | 100 | 99 | 99 | 100 | \dagger 100 | 100 | 101 | 90 |
| United Kingdom | 1991 | | | | 1992 | | | |
| | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | *Q3 | Q4 |
| unemployment rate | 7.8 | 8.5 | 8.9 | 9.1 | 9.3 | 9.7 | 9.8 | 10.2 |
| current account (% of GDP) | -3.0 | -1.7 | -2.3 | -0.2 | -2.2 | -2.5 | -2.8 | -0.8 |
| real overvaluation (\dagger REER) | 104 | 101 | 100 | 100 | \dagger 100 | 102 | 101 | 88 |
| France | 1992 | | | | 1993 | | | |
| | Q1 | Q2 | Q3 | Q4 | Q1 | Q2 | *Q3 | Q4 |
| unemployment rate | 9.5 | 9.6 | 9.9 | 10.1 | 10.5 | 10.9 | 11.3 | 11.6 |
| current account (% of GDP) | -0.5 | +0.6 | +0.2 | +0.8 | -0.3 | +0.5 | +1.5 | +1.1 |
| real overvaluation (\dagger REER) | \dagger 100 | 101 | 103 | 105 | 105 | 105 | 101 | 102 |

* indicates the period of devaluation. \dagger real effective exchange rate relative to 1992Q1. For unemployment and REER, period averages are taken.

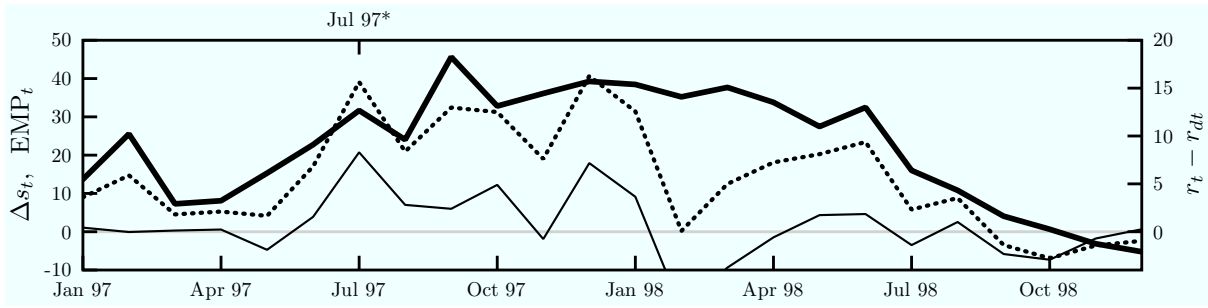
Data sources: IMF International Financial Statistics for current account and REER statistics, and OECD Main Economic Indicators for unemployment statistics.

Table 1: Economic Indicators

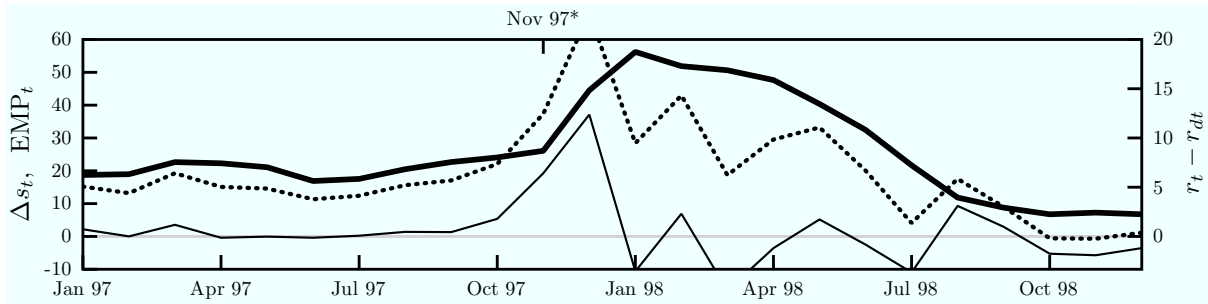
EMS time series.

Virtually every analysis of the fundamentals of the countries involved in the EMS crisis suggests that successful speculative attacks could have occurred much earlier. No major new developments are seen in important economic indicators such as the real exchange rate, the current account, and the unemployment rate (though all three countries experienced *modest* increases in unemployment)—as can be seen from table 1. Reflecting on the evolution of these fundamentals, [Obstfeld and Rogoff \(1995\)](#) observe: “The speculative attack on the British pound in September 1992 would certainly have succeeded had it occurred in August—so why did speculators wait?”; similar cases have been made for Italy and for France. This is at odds with first generation-style explanations (e.g. [Krugman \(1979\)](#), [Flood and Garber \(1984\)](#)) since first generation currency crisis models typically predict that a speculative attack will occur as soon as it is likely to succeed—predicting a negligible depreciation of the exchange rate. There are exceptions, of course: [Guimarães \(2006\)](#) presents a first generation model where the attack is possibly postponed, but this conclusion depends on the presence of frictions on asset markets. [Botman and Jager \(2002\)](#) consider a first generation speculative attack model with two vulnerable countries that cannot be distinguished *a priori*, so that it takes some time before speculators manage to coordinate. However, our model does not need such additional assumptions to explain the jump in the exchange rate.

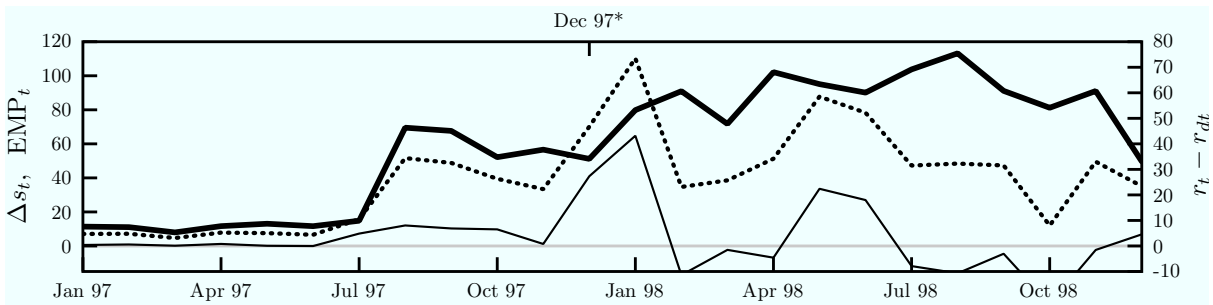
As emerges from the theoretical exposition in section 5, also global game models predict



(a) Thailand: pressure on the baht



(b) South Korea: pressure on the won



(c) Indonesia: pressure on the rupiah

* indicates the first period of substantial depreciation.

Figure 5: Exchange Market Pressure during the 1997 Asian crisis

that a currency will be attacked very quickly after it becomes vulnerable and that the jump in the exchange rate will be modest—at least as far as they do not take into account defence policies. In the basic global game speculative attack model, in which the only costs for speculators are transaction costs, the dominant action is to attack the weak currency, for almost all of the values of u_t in the region where it is “ripe for attack”. This is because the interest rate defence is not modelled, so that the expected profits of the speculators depend only the expected depreciation of the weak currency versus transaction costs, which will be small in comparison (a point also made by Chamley (2003)). In our model, the costs associated with attacking the currency can be substantial, so that a large depreciation is required to offset them; this induces speculators to postpone the attack.

Furthermore, the EMP measure suggests that the EMS crises did not come out of the blue, but were the culmination of periods of growing pressure. For instance, figure 4 shows that the September 1992 crisis of the lira did not come as a surprise, but was the climax of a period of gradually increasing interest rates to offset accumulating pressure on the lira.

For the currencies of France, the United Kingdom, and of the Asian countries in figure 5 similar patterns emerge. Such protracted periods of growing pressure on forex markets are not only at odds with first generation models and global game models, but also with the second generation style explanations that suggest that the crises were triggered by a sudden shift in sentiment on financial markets (e.g. Obstfeld (1996), Jeanne (1997), and Jeanne and Masson (2000)). In contrast, our model predicts that a crisis may be preceded by period of pressure on forex markets. Indeed, the analysis in figure 6 implies that while a full blown crisis only erupts if the fundamental u_t exceeds the critical threshold \bar{u} , EMP emerges already before this time. After all, the position of \bar{u} depends on the risk associated with speculation, and it may take a substantial period of time for the fundamental u_t to reach it, during which EMP gradually builds up.

8. Conclusion

During currency crises, policy makers undertake actions aimed at increasing the financing costs of speculators, such as raising the interest rate. Any approach that abstracts from these actions, and solely focuses on the policy makers' decision whether or not to devalue under speculative pressure, gives an incomplete picture. Much of the pressure on a currency manifests itself through policy actions aimed at defending it, instead of through its eventual depreciation. The literature on exchange market pressure deals with measuring stress on forex markets using a combination of data on exchange rates and data on policy actions that ward off pressure on the exchange rate.

Inspired by this literature, and basing ourselves on a discussion of the mechanics of speculation and the mechanics of a defence against such speculative activity, we have extended the well-known global game speculative attack model of Morris and Shin (1998) by incorporating these mechanics into the model. In our extended model, the policy maker's interest rate defence is endogenous. Therefore, it is a hybrid model of exchange market pressure, rather than just a model of the devaluation decision. We proved that the model has a unique equilibrium, which is similar to the threshold equilibrium in the model of Morris and Shin.

The focus on the interest rate defence leads to a number of predictions that are broadly consistent with how currency crises develop in practice. First, the model is consistent with the persistent increase in pressure on forex markets that often precedes currency crises. Second, the model suggests that a successful speculative attack will lead to a substantial jump of the exchange rate following the attack, and shows this is due to the substantial risks involved with speculation. Finally, the model may be used to clarify the timing of the attack. All of these points touch upon weak spots of traditional currency crises models. The model in this paper provides an explanation of these points based on quite primitive features of the interest rate defence during currency crises.

Appendix: Proofs

Proof of Lemma 1. Define $\mathcal{B}(u_t, \lambda_t) := \bar{r}(u_t) - r(u_t, \lambda_t)$. Since r_{λ_t} is positive, \mathcal{B} is decreasing in λ_t . Since r_{u_t} is positive, and \bar{r}' is negative and bounded away from 0, there exist largest ℓ and least h such that $\mathcal{B}(\ell, \lambda_t) \geq 0$ and $\mathcal{B}(h, \lambda_t) \leq 0$ for all $\lambda_t \in [0, 1]$. Under these conditions, the implicit function theorem implies that there is a unique, continuously differentiable, strictly decreasing function u^* with domain $[\ell, h]$ such that $\mathcal{B}(u^*(\lambda_t), \lambda_t) = 0$. ■

Proof of Lemma 2. Suppose σ_t^* is a symmetric equilibrium. Let $d_t^*(x) := \sigma_{it}^{s*}(x, \sigma_{it}^{b*}(x))$. Then $d_t^*(x) = 1$ if and only if a speculator attacks when her signal is x . We will show σ_t^* —that is, each σ_{it}^{s*} and each σ_{it}^{b*} —is fully determined by d_t^* . First, substitution in equation (6) gives:

$$\lambda_t(u_t; \sigma_t^*) = \frac{1}{2\epsilon} \int_{u_t - \epsilon}^{u_t + \epsilon} d_t^*(x) dx.$$

Using equation (2), we can now express the policy maker's optimal interest rate decision as a function of u_t . Then by condition (III), that is, the spot market clears at \tilde{s} whenever the policy maker prefers to defend the currency peg, in this equilibrium the policy maker must set $r_t = r^*(u_t, \lambda_t(u_t; \sigma_t^*))$.

Next, we find the expectation value of the interest rate set by the policy maker from the perspective of bank i , which receives signal x_{it} . Since the policy maker's decision is given by $r^*(u_t, \lambda_t(u_t; \sigma_t^*))$, this expectation value is:

$$\mathbf{E}[r_t | x_{it}] = \frac{1}{2\epsilon} \int_{X_{it}} r^*(v, \lambda_t(v; \sigma_t^*)) dv = \frac{1}{2\epsilon} \int_{X_{it}} r^*\left(v, \frac{1}{2\epsilon} \int_{v-\epsilon}^{v+\epsilon} d_t^*(x) dx\right) dv, \quad (11)$$

a function solely of the signals at which speculators attack and the bank's own signal x_{it} . By condition (II), bank i will set $f_{it} = \tilde{s} + \mathbf{E}[r_t | x_{it}]$ as given by equation (11).

Finally, by condition (I), speculator i attacks when expected profit from attacking, given by equation (5), is positive. Condition (I) now becomes:

$$\frac{1}{2\epsilon} \int_{X_{it}} s_{t+1}^e \left(v, \frac{1}{2\epsilon} \int_{v-\epsilon}^{v+\epsilon} d_t^*(x) dx \right) dv - f_{it} > 0, \quad (12)$$

and, for a given forward rate, a speculator's decision depends solely on the signals at which speculators attack and the speculator's own signal x_{it} . In equilibrium, a speculator attacks if and only if inequality (12) holds, so that $\sigma_{it}^{s*}(x_{it}, f_{it})$ is fully determined by d_t^* for all f_{it} . ■

Proof of Lemma 4. For arbitrary $x \in \mathbf{R}$, let $\sigma_t(x)$ denote the joint threshold strategy around x . For fixed $\sigma_t(x)$, write speculative λ_t as a continuous and piecewise differentiable function of u_t :

$$\lambda_t(u_t; \sigma_t(x)) = \begin{cases} 0 & \text{if } u_t \in (-\infty, x - \epsilon) \\ \frac{u_t - (x - \epsilon)}{2\epsilon} & \text{if } u_t \in [x - \epsilon, x + \epsilon] \\ 1 & \text{if } u_t \in (x + \epsilon, +\infty) \end{cases} \quad (13)$$

By lemma 1, the inverse of $u^*(\lambda_t)$, denoted $\lambda^*(u_t)$, is continuously differentiable and strictly decreasing on the compact convex set $[\ell, h]$. Now, $\lambda^*(\ell) = 1 \geq \lambda_t(\ell; \sigma_t(x))$ and $\lambda^*(h) = 0 \leq \lambda_t(h; \sigma_t(x))$, so

that by the intermediate value theorem, $\lambda^*(u_t)$ and $\lambda_t(u_t; \boldsymbol{\sigma}_t(x))$ have a (unique) intersection point, which we denote by $a^*(x)$. By the implicit function theorem a^* is a continuous function of x near the point $a^*(x)$. If u_t is to the left of the point $a^*(x)$, speculative pressure falls short of bringing down the peg; to the right the peg collapses.

Using equations (2) and (6) and the optimality of the policy maker's interest rate decision, for fixed $\boldsymbol{\sigma}_t(x)$, $r^*(u_t, \lambda_t(u_t; \boldsymbol{\sigma}_t(x)))$ is a function of u_t , which is strictly increasing for $u_t \leq a^*(x)$, and decreasing and equal to $r_d(u_t)$ for $u_t > a^*(x)$. Now turn to bank i receiving signal x_{it} . From the bank's perspective, the true value of u_t must be in the interval $X_{it} = [x_{it} - \epsilon, x_{it} + \epsilon]$. By optimality against $\boldsymbol{\sigma}_t(x)$, the forward rate offered by the bank is:

$$f_{it}(x_{it}; \boldsymbol{\sigma}_t(x)) = \frac{1}{2\epsilon} \int_{X_{it}} r^*(v, \lambda_t(v; \boldsymbol{\sigma}_t(x))) dv + \bar{s}, \quad (14)$$

which is continuous in x_{it} and in x (since r^* is bounded).

Finally, turn to the speculator receiving signal x_{it} . From the speculator's perspective, the true value of u_t must be in the interval $X_{it} = [x_{it} - \epsilon, x_{it} + \epsilon]$, and the expected amount of depreciation is:

$$\Delta s(x_{it}; \boldsymbol{\sigma}_t(x)) = \frac{1}{2\epsilon} \int_{X_{it} \cap \{v \in \mathbf{R} \mid v > a^*(x)\}} s_{t+1}^e(v, \lambda_t(v; \boldsymbol{\sigma}_t(x))) - \bar{s} dv. \quad (15)$$

This expression is continuous and strictly increasing in x_{it} , since the integrand term is bounded and strictly increasing in u_t , and since the support $X_{it} \cap \{v \in \mathbf{R} \mid v > a^*(x)\}$ expands in x_{it} , and it is continuous in x . $\pi^e(x_{it}; \boldsymbol{\sigma}_t(x)) = \Delta s(x_{it}; \boldsymbol{\sigma}_t(x)) - (f_{it}(x_{it}; \boldsymbol{\sigma}_t(x)) - \bar{s})$ and may be rewritten as:

$$\frac{1}{2\epsilon} \left[\int_{X_{it} \cap \{v \in \mathbf{R} \mid v > a^*(x)\}} s_{t+1}^e(v, \lambda_t(v; \boldsymbol{\sigma}_t(x))) - \bar{s} - r_d(v) dv - \int_{X_{it} \cap \{v \in \mathbf{R} \mid v \leq a^*(x)\}} r^*(v, \lambda_t(v; \boldsymbol{\sigma}_t(x))) dv \right].$$

This shows $\pi^e(x_{it}; \boldsymbol{\sigma}_t(x))$ is continuous in x_{it} and x , negative when $x_{it} \leq a^*(x) - \epsilon$, strictly increasing when $x_{it} \geq a^*(x) - \epsilon$, and strictly positive when $x_{it} \geq a^*(x) + \epsilon$. There is a unique $x^* \in [\ell - \epsilon, h + \epsilon]$ (which depends on $\boldsymbol{\sigma}(x)$), such that π^e is strictly positive if and only if $x_{it} > x^*$. The optimal strategy for speculator i is the threshold strategy around $x^*(x)$.

By continuity we have that $\pi^e(x^*(x), f_{it}(x^*(x), \boldsymbol{\sigma}_t(x)); \boldsymbol{\sigma}_t(x)) = 0$. This implies that $a^*(x)$ lies in the interior of $[x^*(x) - \epsilon, x^*(x) + \epsilon]$. In this case π^e is strictly increasing in x_{it} near $x^*(x)$, so that the implicit function theorem for locally one-one functions (e.g. [Kumagai \(1980\)](#)) implies that $x^*(x)$ is continuous at x . The threshold x was chosen arbitrarily, so $x^*(x)$ is continuous at every point, therefore continuous. ■

Proof of Lemma 5. The function x^* is continuous and bounded by $\ell - \epsilon$ and $h + \epsilon$. Therefore the set of fixpoints of x^* is closed and bounded (thus compact) and, by the Brouwer fixpoint theorem, non-empty. So x^* has a least fixpoint \underline{x} and a largest fixpoint \bar{x} . For $x \in \mathbf{R}$, when agents use the joint threshold strategy $\boldsymbol{\sigma}_t(x)$, λ_t increases linearly from 0 to 1 on the set $[x - \epsilon, x + \epsilon]$. Let i be the speculator receiving the signal \bar{x} when agents use the joint threshold strategy $\boldsymbol{\sigma}_t(\bar{x})$. Since $\boldsymbol{\sigma}_t(\bar{x})$ is an equilibrium and π^e is continuous in x_{it} , speculator i must be indifferent:

$$\pi^e(\bar{x}; \boldsymbol{\sigma}_t(\bar{x})) = \Delta s(\bar{x}; \boldsymbol{\sigma}_t(\bar{x})) - (f_{it}(\bar{x}; \boldsymbol{\sigma}_t(\bar{x})) - \bar{s}) = 0, \quad (16)$$

where $\Delta s(x_{it}; \boldsymbol{\sigma}_t(x))$ is defined as in equation (15), and f_{it} is given by:

$$f_{it}(\bar{x}; \boldsymbol{\sigma}_t(\bar{x})) = \frac{1}{2\epsilon} \left[\int_0^{a^*(\bar{x}) - (\bar{x} - \epsilon)} r(v + (\bar{x} - \epsilon), \frac{v}{2\epsilon}) dv + \int_{a^*(\bar{x})}^{\bar{x} + \epsilon} r_d(v) dv \right] + \bar{s}.$$

Suppose $\underline{x} \neq \bar{x}$ and let j be the agent that receives the signal \underline{x} when the joint threshold around \underline{x} (i.e. $\boldsymbol{\sigma}_t(\underline{x})$) is used. This agent must also be indifferent, so that equation (16) holds with i replaced by j and \bar{x} replaced with \underline{x} . Since $\lambda^*(u_t)$ is a strictly decreasing function, we have $a^*(\underline{x}) - \underline{x} > a^*(\bar{x}) - \bar{x}$. This means that the support of the integral in equation (15) is narrower under the signal \underline{x} than under \bar{x} . Letting $\delta := (a^*(\underline{x}) - \underline{x}) - (a^*(\bar{x}) - \bar{x})$, write $f_{jt}(\underline{x}; \boldsymbol{\sigma}_t(\underline{x}))$ as:

$$f_{jt}(\underline{x}; \boldsymbol{\sigma}_t(\underline{x})) = \frac{1}{2\epsilon} \left[\int_0^\delta r((\underline{x} - \epsilon) + v, \frac{v}{2\epsilon}) dv + \int_0^{a^*(\bar{x}) - (\bar{x} - \epsilon)} r((\underline{x} - \epsilon) + v + \delta, \frac{v + \delta}{2\epsilon}) dv + \int_{a^*(\underline{x})}^{\underline{x} + \epsilon} r_d(v) dv \right] + \bar{s}.$$

Since r_d is decreasing in u_t , and since s_{t+1}^e is strictly increasing in u_t , we find:

$$\int_{a^*(\underline{x})}^{\underline{x} + \epsilon} s_{t+1}^e(v, \lambda_t(v; \boldsymbol{\sigma}(\underline{x}))) - r_d(v) dv - \int_0^\delta r((\underline{x} - \epsilon) + v, \frac{v}{2\epsilon}) dv < \int_{a^*(\bar{x})}^{\bar{x} + \epsilon} s_{t+1}^e(v, \lambda_t(v; \boldsymbol{\sigma}(\bar{x}))) - r_d(v) dv. \quad (17)$$

(note here that the length of the supports of the two left hand side integrals combined are equal the support of the right hand side integral; moreover, the integrand of the right hand side integral is non-negative). Moreover, we will show that assumption (1) implies that:

$$\int_0^{a^*(\bar{x}) - (\bar{x} - \epsilon)} r((\underline{x} - \epsilon) + v + \delta, \frac{v + \delta}{2\epsilon}) dv > \int_0^{a^*(\bar{x}) - (\bar{x} - \epsilon)} r((\bar{x} - \epsilon) + v, \frac{v}{2\epsilon}) dv. \quad (18)$$

Equations (17) and (18) combined entail $\Delta s(\underline{x}) - (f_{jt} - \bar{s}) < \Delta s(\bar{x}) - (f_{it} - \bar{s})$. This means that speculators i and j cannot be both indifferent—and thus we conclude that $\underline{x} \neq \bar{x}$ leads to a contradiction.

To see inequation (18), note that for the upper limits of these integrals we have:

$$\begin{aligned} r((\underline{x} - \epsilon) + a^*(\bar{x}) - (\bar{x} - \epsilon) + \delta, \frac{a^*(\bar{x}) - (\bar{x} - \epsilon) + \delta}{2\epsilon}) &= \bar{r}(a^*(\underline{x})) \\ &> \bar{r}(a^*(\bar{x})) = r(a^*(\bar{x}) - (\bar{x} - \epsilon) + (\bar{x} - \epsilon), \frac{a^*(\bar{x}) - (\bar{x} - \epsilon)}{2\epsilon}) \quad (\text{since } \bar{r} \text{ is strictly decreasing}), \end{aligned}$$

showing that at the upper limit of the support of the integral in the left hand side of (18) exceeds that of the right hand side. If, for some v , the integrands are equal, then by assumption (1),

$$\begin{aligned} \frac{dr}{dv} r((\underline{x} - \epsilon) + v + \delta, \frac{v + \delta}{2\epsilon}) &= r_{u_t}((\underline{x} - \epsilon) + v + \delta, \frac{v + \delta}{2\epsilon}) + \frac{1}{2\epsilon} r_{\lambda_t}((\underline{x} - \epsilon) + v + \delta, \frac{v + \delta}{2\epsilon}) \\ &> \frac{dr}{dv} r(v + (\bar{x} - \epsilon), \frac{v}{2\epsilon}) = r_{u_t}(v + (\bar{x} - \epsilon), \frac{v}{2\epsilon}) + \frac{1}{2\epsilon} r_{\lambda_t}(v + (\bar{x} - \epsilon), \frac{v}{2\epsilon}) \end{aligned}$$

(the inequality follows from $(\underline{x} - \epsilon) + v + \delta < v + (\bar{x} - \epsilon)$). Thus, using the fundamental theorem of calculus and assumption (1) we see that, in equation (18), when moving from the upper to the lower limits of the integrals, the left hand side integrand remains above that of the right hand side, so that the inequality in indeed holds. ■

*Proof of Theorem 3(ii).*¹⁴ Let σ_t be any equilibrium and let $\tilde{\lambda}(u_t)$ be the fraction of speculators that attack when the true fundamental equals u_t . If σ_t is not a symmetric strategy, then $\tilde{\lambda}(u_t)$ is a random variable, since this fraction depends on which agents receive what signals (as well as on the true fundamental u_t). Using the fact that in equilibrium banks and the policy maker use their optimal responses to substitute for f_{it} , equation (5) becomes:

$$\pi^e(x_{it}, f_{it}; \sigma_t) = \frac{1}{2\epsilon} \int_{X_{it}} \mathbf{E}[s_{t+1}^e(v, \tilde{\lambda}(v))] dv - \bar{s} - \frac{1}{2\epsilon} \int_{X_{it}} \mathbf{E}[r^*(v; \tilde{\lambda}(v))] dv, \quad (19)$$

which is a continuous function of x_{it} . Since σ_t is an equilibrium, speculator i speculates against the weak currency if (19) is strictly positive, refrains from speculation if (19) is strictly negative, and is indifferent when (19) vanishes. Let x_{it}^b be the largest signal such that speculators do not strictly prefer to refrain from attacking: $x_{it}^b := \sup\{x_{it} \in \mathbf{R} \mid \pi^e(x_{it}, f_{it}; \sigma_t) \leq 0\}$. By continuity $\pi^e(x_{it}^b, f_{it}; \sigma_t) = 0$. The derivative of $\pi^e(x_{it}, f_{it}; \sigma_t)$ w.r.t. x_{it} at x_{it}^b exists and is strictly positive. (It is given by: $s_{t+1}^e(x_{it}^b + \epsilon, 1) - r_d(x_{it}^b + \epsilon) - \mathbf{E}[s_{t+1}^e(x_{it}^b - \epsilon, \tilde{\lambda}(x_{it}^b - \epsilon))] + \mathbf{E}[r^*(x_{it}^b - \epsilon, \tilde{\lambda}(x_{it}^b - \epsilon))] > 0$). Assume σ_t is not a joint threshold strategy. There must be speculators that attack when they receive signals smaller than x_{it}^b , and (by the continuity of π^e) in particular there is a *largest* signal x_{it}^a , satisfying $x_{it}^a < x_{it}^b$ (since the derivative of π^e w.r.t. x_{it} is strictly positive at x_{it}^b), such that $\pi^e(x_{it}^a, f_{it}; \sigma_t) = 0$. The sets X_{it}^a and X_{it}^b may or may not be disjoint. Let $C = X_{it}^a \cap X_{it}^b$, and $U_a = X_{it}^a - C$, $U_b = X_{it}^b - C$. We also denote $U_b =: [u_1, u_2]$. Now, $\pi^e(x_{it}^b, f_{it}; \sigma_t) = \pi^e(x_{it}^a, f_{it}; \sigma_t) = 0$ implies the following equality:

$$\int_{U_a} \mathbf{E}[s_{t+1}^e(v, \tilde{\lambda}(v))] - \mathbf{E}[r^*(v; \tilde{\lambda}(v))] - \bar{s} dv = \int_{U_b} \mathbf{E}[s_{t+1}^e(v, \tilde{\lambda}(v))] - \mathbf{E}[r^*(v; \tilde{\lambda}(v))] - \bar{s} dv. \quad (20)$$

Even if σ_t is not a symmetric equilibrium, the terms inside the expectation operators on the right hand side of equation (20) are not random. In fact, the right hand side is equal to:

$$\int_{u_1}^{u_2} s_{t+1}^e(v, \lambda_t(v, \sigma_t(x_{it}^b))) - r^*(v; \lambda_t(v, \sigma_t(x_{it}^b))) - \bar{s} dv, \quad (21)$$

with $\lambda_t(v, \sigma_t(x_{it}^b))$ defined as in equation (13). This is because $u_1 \geq x_{it}^a + \epsilon$, so for any $u_t \in (u_1, u_2)$, a speculator that receives the signal $u_t - \epsilon$ does not attack (we have $x_{it}^a < u_t - \epsilon < x_{it}^b$); while a speculator that receives the signal $u_t + \epsilon$ attacks (since $x_{it}^b < u_t + \epsilon$). Hence on the interval $[u_1, u_2]$, λ_t increases linearly *as if* speculators are using the joint threshold strategy around x_{it}^b . We will denote the value of expression (21) by V_b . Now rewrite the left hand side of equation (20) as:

$$V_a := \mathbf{E} \left[\int_{U_a} s_{t+1}^e(v, \tilde{\lambda}(v)) - r^*(v; \tilde{\lambda}(v)) - \bar{s} dv \right].$$

Denote the term inside the remaining expectation operator by $g(\tilde{\lambda}(v))$. We claim:

¹⁴The proof of this part of the theorem adapts the argument given in part C of the proof of theorem 1 in Goldstein and Pauzner (2005) to our model. The main difference is in the crucial last step of the proof (claim 1 below).

Claim 1. For *any* possible distribution of signals on the true fundamental among agents:

$$g(\tilde{\lambda}(u_t)) := \int_{U_a} s_{t+1}^e(v, \tilde{\lambda}(v) - r^*(v; \tilde{\lambda}(v)) - \tilde{s}) dv < V_b. \quad (22)$$

It follows straightforwardly that $V_a < V_b$, yet this contradicts the equality in (20). The absurdity implies that σ_t must be a joint threshold strategy after all, which proves the theorem.

Proof of claim 1. For any distribution of signals among agents and any $\eta \in [-\epsilon, \epsilon]$ let $\mathbf{i}(\eta)$ be the speculator and bank that receive the signal $u_t + \eta$ when the true fundamental is u_t . Keeping the relative distribution of signals among agents (the function \mathbf{i}) fixed, $\tilde{\lambda}(u_t)$ can be written as

$$\tilde{\lambda}(u_t) = \frac{1}{2\epsilon} \int_{-\epsilon}^{\epsilon} \sigma_{\mathbf{i}(\eta)}^s(\eta + u_t, \sigma_{\mathbf{i}(\eta)}^b(\eta + u_t)) d\eta, \quad (23)$$

thus can be regarded as a deterministic and continuous function of u_t . Moreover, for any $u_t \in C$, $\tilde{\lambda}(u_t) \leq \lambda_t(u_1; \sigma_t(x_{it}^b))$. To see this, note that $C = [x_{it}^b - \epsilon, x_{it}^a + \epsilon]$ and that $x_{it}^a + \epsilon = u_1$; in the evaluation of expression (23), when moving from the lower end of C to its upper end, we “replace” speculators receiving signals below x_{it}^a —who may or may not attack—with speculators with signals above x_{it}^b who always choose to attack. Thus, fixing \mathbf{i} , $\tilde{\lambda}$ is monotonic on C .

First we will prove that $V_b \geq 0$. If $C = \emptyset$ then this follows from the fact that $\pi^e(x_{it}^b, f_{it}; \sigma_t) = 0$. Otherwise, suppose that $C \neq \emptyset$ and $V_b < 0$. This means that there are at least some $u_t \in [u_1, u_2]$ at which the policy maker does not devalue. Since $\lambda_t(u_t; \sigma_t(x_{it}^b))$ increases on the interval $[u_1, u_2]$, we have $\lambda_t(u_1; \sigma_t(x_{it}^b)) \leq \lambda_t(u_t; \sigma_t(x_{it}^b))$, so the policy maker also does not devalue when the true state is equal to u_1 . Since for all $u_t \in C$, and any distribution of signals $\tilde{\lambda}(u_t) \leq \lambda_t(u_1; \sigma_t(x_{it}^b))$, this implies she does not devalue at any $u_t \in C$ under any distribution of signals \mathbf{i} . So:

$$\int_C s_{t+1}^e(v, \tilde{\lambda}(v) - r^*(v; \tilde{\lambda}(v)) - \tilde{s}) dv < 0. \quad (24)$$

If $V_b < 0$, then inequality (24) holds for any distribution of signals; this contradicts that $\pi^e(x_{it}^b, f_{it}; \sigma_t)$, which is the sum of V_b and the expectation value of expression (24) is equal to 0.

For the rest of the proof of the claim, we keep the distribution of signals \mathbf{i} fixed. We “pair” each true state u_t in the set X_{it}^a with a state in the set X_{it}^b , by setting $\vec{u}_t = x_{it}^a + x_{it}^b - u_t$. That is, \vec{u}_t is the “mirror image” of u_t , mirrored in the midpoint between x_{it}^a and x_{it}^b . For each $u_t \in [u_1, u_2]$, we then have $\tilde{\lambda}(\vec{u}_t) \leq \tilde{\lambda}(u_t)$. To see this, note $C = [\vec{u}_1, u_1]$, and so $\tilde{\lambda}(\vec{u}_1) \leq \tilde{\lambda}(u_1)$. As we move up from u_1 to u_2 on the interval $[u_1, u_2]$ and evaluate expression (23), at first we have $u_t - \epsilon \leq x_{it}^a$, which means we “replace” speculators who may or may not attack at the signal $u_t - \epsilon$ with others who attack for sure at the signal $u_t + \epsilon$; at the same time, moving down from \vec{u}_1 to \vec{u}_2 we replace speculators who attack for sure at $\vec{u}_t + \epsilon \geq x_{it}^b$ with others who may or may not attack at $\vec{u}_t - \epsilon$. So $\tilde{\lambda}(u_t)$ increases while $\tilde{\lambda}(\vec{u}_t)$ decreases. Next, we have $u_t - \epsilon > x_{it}^a$, and we replace speculators who do not attack at $u_t - \epsilon$ with others who do for sure at $u_t + \epsilon$, while we replace speculators who do not attack at $\vec{u}_t + \epsilon$ with those who may or may not at $\vec{u}_t - \epsilon$. So $\tilde{\lambda}(u_t)$ increases faster as u_t moves up from u_1 to u_2 than $\tilde{\lambda}(\vec{u}_t)$ as it moves down from \vec{u}_1 to \vec{u}_2 .

If the policy maker devalues when the true fundamental is equal to u_1 , then inequation (22) holds

and the proof is complete. This is because $\lambda_t(u_t)$ is increasing on $[u_1, u_2]$, so that in this case the policy maker devalues for all $u_t \in [u_1, u_2]$. Now, if the policy maker does not devalue at some $\vec{u}_t \in [\vec{u}_2, \vec{u}_1]$, then $s_{t+1}^e(\vec{u}_t, \tilde{\lambda}(\vec{u}_t)) - r^*(\vec{u}_t, \tilde{\lambda}(\vec{u}_t)) < \bar{s} < s_{t+1}^e(u_t, \lambda_t(u_t; \sigma_t(x_{it}^b))) - r_d(u_t)$; if, in contrast, the policy maker devalues then $\tilde{\lambda}(\vec{u}_t) \leq \lambda_t(u_t; \sigma_t(x_{it}^b))$ implies $s_{t+1}^e(\vec{u}_t, \tilde{\lambda}(\vec{u}_t)) - r_d(\vec{u}_t) < s_{t+1}^e(u_t, \lambda_t(u_t; \sigma_t(x_{it}^b))) - r_d(u_t)$. This shows that $g(\tilde{\lambda}(u_t)) < V_b$ must hold.

Finally we deal with the case where the policy maker does not devalue when the true fundamental is u_1 . Suppose, contrary to our claim, that $g(\tilde{\lambda}(u_t)) \geq V_b \geq 0$. There must be $u_t \in U_a = [\vec{u}_2, \vec{u}_1]$ such that the policy maker devalues (otherwise we would find $g(\tilde{\lambda}(u_t)) < 0$). But, $\tilde{\lambda}(\vec{u}_1) \leq \lambda_t(u_1; \sigma_t(x_{it}^b))$, and moreover the policy maker does not devalue when the fundamental equals u_1 . By continuity $v^* = \max\{u_t \in U_a \mid r_t(u_t; \tilde{\lambda}(u_t)) = \bar{r}(u_t)\}$, the largest signal $v^* \in U_a$ such that $\bar{r}(v^*) = r_t(v^*; \tilde{\lambda}(v^*))$, is well-defined. Let $\Delta\lambda = \tilde{\lambda}(v^*) - \tilde{\lambda}(\vec{u}_1)$ (note $\Delta\lambda > 0$), and let $z = v^* + 2\epsilon\Delta\lambda - \epsilon$. We will be interested a modification $\hat{\sigma}_t$ of the joint strategy profile σ_t :

$$\text{for all speculators } i \in [0, 1], \text{ and all } f_{it}, \hat{\sigma}_{it}^s(x_{it}, f_{it}) = \begin{cases} 1 & \text{if } x_{it} \leq z \\ 0 & \text{if } z < x_{it} \leq \vec{u}_1 - \epsilon \\ \sigma_{it}^s(x_{it}, \sigma_{it}^b(x_{it})) & \text{otherwise.} \end{cases}$$

Following equation (23), let $\hat{\lambda}(u_t) = \frac{1}{2\epsilon} \int_{-c}^c \hat{\sigma}_{i(\eta)t}^s(\eta + u_t, f) d\eta$ (note that, by construction, $\hat{\lambda}(u_t)$ is independent of the choice of f). Assume that when speculators use $\hat{\sigma}_t$ the policy maker sets r_t to $r_d(u_t)$ if $u_t < v^*$, and that she sets $r_t = r(u_t, \hat{\lambda}(u_t))$ if $u_t \geq v^*$. (Of course, $\hat{\sigma}_t$ is not an equilibrium, but this is not important for our argument.) In the joint strategy profile $\hat{\sigma}_t$, let banks set f_{it} optimally with respect to the policy maker's interest decision.

The interval $[\vec{u}_2, \vec{u}_1]$ can now be decomposed in two parts. If $u_t \in [z + \epsilon, \vec{u}_1]$, $\hat{\lambda}(u_t)$ is, by construction, constant and equal to $\tilde{\lambda}(\vec{u}_1)$. This is because, as we move through the region $[\vec{u}_2, \vec{u}_1]$, under the profile $\hat{\sigma}_t$ speculators with signals $u_t - \epsilon$ do not attack the currency (we have $z < u_t - \epsilon < \vec{u}_1 - \epsilon$), and speculators with signals $u_t + \epsilon$ also do not attack the currency (we have $x_{it}^a \leq u_t + \epsilon \leq x_{it}^b$), so that $\hat{\lambda}(u_t)$ is constant. Moreover, compared to the profile σ_t , we replace speculators who attack by speculators who don't, so that for $u_t \in [z + \epsilon, \vec{u}_1]$, $\hat{\lambda}(u_t) \leq \tilde{\lambda}(u_t)$.

On the interval $[\vec{u}_2, z + \epsilon]$, $\hat{\lambda}(u_t)$ decreases with derivative $-1/(2\epsilon)$. This is because when $u_t \in [\vec{u}_2, z + \epsilon]$, under the profile $\hat{\sigma}_t$ speculators with signals $u_t - \epsilon$ attack the currency (we have $u_t - \epsilon < z$) and agents with signals $u_t + \epsilon$ do not attack the currency (we have $x_{it}^a \leq u_t + \epsilon < x_{it}^b$).

Finally, note that the profile $\hat{\sigma}_t$ is constructed such that $\hat{\lambda}(v^*) = \tilde{\lambda}(v^*)$. (This is because $\hat{\lambda}(v^*) = \hat{\lambda}(z + \epsilon) + \frac{z + \epsilon - v^*}{2\epsilon} = \hat{\lambda}(z + \epsilon) + \Delta\lambda = \hat{\lambda}(\vec{u}_1) + \Delta\lambda = \tilde{\lambda}(\vec{u}_1) + \Delta\lambda = \tilde{\lambda}(v^*)$). Now, letting $D \subseteq U_a$ be the subset of fundamentals of U_a where the policy maker devalues under the joint strategy profile σ_t and the distribution of signals \mathbf{i} , we have:

$$\begin{aligned} g(\tilde{\lambda}(v)) &= \int_{[\vec{u}_2, v^*] \cap D} s_{t+1}^e(v, \tilde{\lambda}(v)) - r_d(v) - \bar{s} dv - \int_{[\vec{u}_2, v^*] - D} r(v; \tilde{\lambda}(v)) dv - \int_{v^*}^{\vec{u}_1} r(v; \tilde{\lambda}(v)) dv \\ &\leq \int_{[\vec{u}_2, v^*] \cap D} s_{t+1}^e(v, \hat{\lambda}(v)) - r_d(v) - \bar{s} dv - \int_{[\vec{u}_2, v^*] - D} r(v; \tilde{\lambda}(v)) dv - \int_{v^*}^{\vec{u}_1} r(v; \hat{\lambda}(v)) dv \end{aligned}$$

(the above inequality holds since $\hat{\lambda}$ decreases at the fastest rate on $[\vec{u}_2, z + \epsilon)$ and is constant on $[z + \epsilon, \vec{u}_1]$, so we have $\hat{\lambda}(u_t) \geq \tilde{\lambda}(u_t)$ for all $u_t \in [\vec{u}_2, v^*]$, and $\hat{\lambda}(u_t) \leq \tilde{\lambda}(u_t)$ for all $u_t \in [v^*, \vec{u}_1]$)

$$\begin{aligned} &\leq \int_{[\vec{u}_2, v^*] \cap D} s_{t+1}^e(v^* + (v^* - v), \hat{\lambda}(v)) - r_d(v^* + (v^* - v)) - \bar{s} \, dv \\ &\quad + \int_{[\vec{u}_2, v^*] - D} s_{t+1}^e(v^* + (v^* - v), \hat{\lambda}(v)) - r_d(v^* + (v^* - v)) - \bar{s} \, dv - \int_{v^*}^{\vec{u}_1} r(v; \hat{\lambda}(v)) \, dv \end{aligned}$$

(the inequality holds since $s_{t+1}^e(u_t, \lambda_t)$ is strictly increasing in u_t and λ_t , $r_d(u_t)$ is non-increasing in u_t , and $s_{t+1}^e(u_t, \lambda_t) - r_d(u_t) - \bar{s} \geq 0$ when $\lambda_t \geq \lambda^*(u_t)$)

$$\leq \int_{\vec{u}_2}^{v^*} s_{t+1}^e(v^* + (v^* - v), \tilde{\lambda}(\vec{v})) - r_d(v^* + (v^* - v)) - \bar{s} \, dv - \int_{v^*}^{\vec{u}_1} r(v; \hat{\lambda}(v)) \, dv$$

(combining the first two integral terms and using $\hat{\lambda}(v) \leq \tilde{\lambda}(\vec{v}) = \lambda_t(v, \sigma_t(x_{it}^b))$)

$$\leq \int_{\vec{v}^*}^{u_2} s_{t+1}^e(v, \tilde{\lambda}(v)) - r_d(v) - \bar{s} \, dv - \int_{v^*}^{\vec{u}_1} r(v; \hat{\lambda}(v)) \, dv \quad (25)$$

(using “mirroring”, and because $s_{t+1}^e(u_t, \lambda_t)$ is increasing in u_t , r_d is non-increasing in u_t , and $v^* + (v^* - \vec{v}) \leq v$ for $v \in [u_1, u_2]$). Let $v^{**} \in U_b$ be the (unique) point where $r^*(v^{**}, \lambda_t(v^{**}, \sigma(x_{it}^b))) = \bar{r}(v^{**})$.

Rewrite expression (25) as:

$$\int_{\vec{v}^*}^{u_2} s_{t+1}^e(v, \tilde{\lambda}(v)) - r_d(v) - \bar{s} \, dv - \int_{v^*}^{v^* + (v^{**} - u_1)} r(v; \hat{\lambda}(v)) \, dv + \int_{v^* + (v^{**} - u_1)}^{\vec{u}_1} -r(v; \hat{\lambda}(v)) \, dv.$$

The final integral term in this expression is negative. On the other hand V_b equals:

$$\int_{\vec{v}^*}^{u_2} s_{t+1}^e(v, \tilde{\lambda}(v)) - r_d(v) - \bar{s} \, dv - \int_{u_1}^{v^{**}} r(v; \tilde{\lambda}(v)) \, dv + \int_{v^{**}}^{\vec{v}^*} s_{t+1}^e(v, \tilde{\lambda}(v)) - r_d(v) - \bar{s} \, dv,$$

and the final integral term is positive. To complete the proof of the claim it suffices to show that:

$$\int_{v^*}^{v^* + (v^{**} - u_1)} r(v; \hat{\lambda}(v)) \, dv \geq \int_{u_1}^{v^{**}} r(v; \tilde{\lambda}(v)) \, dv.$$

But $r(v^*, \hat{\lambda}(v^*)) = r(v^*, \tilde{\lambda}(v)) = \bar{r}(v^*) > \bar{r}(v^{**}) = r(v^{**}, \hat{\lambda}(v^{**}))$. The supports of these integrals are of equal length. Using the fact that the support of the left hand integral is entirely below the support of the right hand integral, assumption (1) implies that, when moving from the lower limit of the left hand integral to its upper limit, the integrand remains above that of the right hand side integrand when moving from the upper limit of the right hand side integral to the lower limit (the argument is similar to that in the proof of lemma 5). The claim follows. ■

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