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Auctions under Payoff Uncertainty: The Case with Heterogeneous Bidder Aversion to Downside Risk

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Auctions under Payoff Uncertainty: The Case with Heterogeneous Bidder-Aversion to Downside Risk*

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Abstract

This paper characterizes the optimal first-price auction (FPA) and second-price auction (SPA) for selling rights, contracts, or licenses that involve ensuing payoff uncertainty for the winning bidder. The distribution of the random payoff is common knowledge, except that bidders have private degrees of aversion to downside-risk. In this model, the optimal FPA entails a lower reserve price, a higher expected revenue, and higher expected utilities for at least some or all bidders than the optimal SPA does, which suggests that FPA dominates SPA in terms of both allocative and Pareto efficiency. Increasing risk or risk aversion generally leads to lower equilibrium bids.

Key words: auction, downside risk, risk aversion, payoff uncertainty, allocative efficiency, Pareto efficiency

JEL classification: D44

1 Introduction

A good number of auctions involve payoff uncertainty for the winning bidder. From antique, flowers, wine that are bought for resale, to licenses, rights, contracts for business, the winner's (monetary) payoff depends on the unknown prices and market conditions, among other factors, in the future. When every bidder is risk neutral and knows the *expected* value of the auctioned object to himself that is independent of other bidders' evaluations, such payoff uncertainty is immaterial and the situation can be analyzed as though the privately assessed payoff is certain. Indeed, risk-neutrality has been a crucial assumption in the development of auction theory, as it helps generate scores of valuable insights into important issues regarding, e.g. competitive bidding behavior, comparative revenue performance, and optimal auction design (e.g., Vickrey, 1961; Wilson, 1977; Myerson, 1981; Riley and Samuelson, 1981; Harris and Raviv, 1981; Milgrom and Weber, 1982; McAfee and McMillan, 1987). The same issues where bidders are risk averse and the winning bidder is subject to ensuing payoff uncertainty, however, have received far less attention.

Among the existing work that considers bidders' risk aversion, the focus is on the cases in which each bidder has a sure private value assigned to the auctioned object—consequently, the involved risk is related to the uncertainty of winning or losing in the auction only (e.g., Holt, 1980; Riley and Samuelson, 1981; Harris and Raviv, 1981; Matthews, 1983, 1987; Cox et al., 1982, 1988; Smith and Levin, 1996).[1] In these *sure-value* models, increasing bidders' degree of risk aversion leads to more aggressive bidding in the first-price auction (FPA), whereas bidding up to one's private value remains a dominant strategy in the second-price auction (SPA).[2] Thus, from the seller's point of view, the FPA is more

[1]See also Milgrom and Weber (1982, Section 8), Holt and Sherman (2000), and Goeree and Offerman (2003) for analyses about bidder risk aversion in common-value auctions; Waehrer (1998) and Eso and Futo (1999) about risk-averse seller's choice; and Eso (2005) about risk-averse bidders with correlated private values.

[2]In this article, the FPA refers to both the first-price sealed-bid auction and the Dutch (descending)

attractive than the SPA because it generates greater expected revenue when the buyers are risk averse rather than risk neutral. This comparative revenue implication can be directly derived from the famous revenue equivalence theorem (e.g., Myerson, 1981), which states that under certain conditions, the expected revenues from different standard auctions are the same when buyers are risk neutral and furthermore, share the common belief that everyone else is risk neutral.

The aim of the present study is to incorporate payoff uncertainty, or ensuing risk, into standard auction models and investigate the bidding behavior and optimal reserve prices with risk-averse bidders. The one well-known model that is related to this study is the "Case 3" preference considered in Maskin and Riley (1984). These authors present a fairly general model of private-value auction design with risk averse buyers.[3] In Maskin and Riley's Case 3 preference, the bidders are interpreted to have the *same* utility function but privately known distributions of the uncertain payoff of the object. These distributions are ranked in the sense of first-order stochastic dominance, which means that the most eager buyer is the one who has a payoff distribution that dominates all others'. Since Maskin and Riley are mainly concerned with the general properties of optimal mechanism that hold for a fairly large class of preferences, they do not single out this Case 3 and

auction, where the winner pays the price equal to his bid. The SPA refers to both the second-price sealed-bid auction (or Vickrey auction) and the English (ascending) auction, where the winner pays the price equal to the second-highest bid. A careful discussion of the conditions under which each pair of these auction forms entail the same equilibrium strategies can be found in Milgrom and Weber (1982).

[3] Payoff uncertainty is naturally present in the common value models where bidders receive only imperfect information about the value of the auctioned object. The issue of risk aversion in this area, however, has been mainly studied through experimental research focusing on explaining the "winner's curse," the tendency of the bidders to bid "too high" in first-price auctions. The issue turns out to be highly contentious (e.g., Harrison, 1989, 1992 and the references therein), part of the difficulty being acknowledged to be due to "the lack of theory about the behavior of common-value auctions with risk aversion" (Lind and Plott, 1991, p.344). Recent theoretical models include Holt and Sherman (2000) and Goeree and Offerman (2003b), where bidders have the same utility function and the distributions of signals are uniform and normal, respectively.

study its specific implications. Therefore, what we learn from Maskin and Riley (1984) are more about general conclusions, such that revenue-maximization implies a trade-off between incentives and risk sharing, with the less eager buyers bearing more risk than the more eager buyers (see also Matthews, 1983; Moore, 1984).

The main difference of our model from Maskin and Riley's is that we assume a common distribution of uncertain payoffs with heterogeneous degrees of bidder risk aversion. In situations where substantial risks are involved, it is conceivable that the bidding firms or the firm managers may exhibit considerable differences in their attitudes toward risk. For one thing, the degree of risk aversion can be shaped by the bidding firm's ideosyncratic conditions or by the firm manager's personal preferences, which are likely to be private information to the bidder.[4] The assumption of common payoff distribution, on the other hand, captures the essential feature of the mineral rights or common value models – that is, "to a first approximation, the values of these mineral rights to the various bidders can be regarded as equal" (Milgrom and Weber, 1982, p.1093). More precisely, though, bidders have common *expected value* of future payoffs. Owing to possible differences in risk attitudes in the present context, the bidders' *reservation values* may still differ.[5]

In order to make sense of interpersonal comparison that is an essential characteristic of auctions, the first thing one needs seems to be a cardinal measure of risk or utility. In light of empirical evidence that much of the risk-averse behavior can be explained by aversion to losses,[6] we focus on downside risk aversion[7] and examine its effects on

[4]When the object for sale is a business license and the bidders are firms competing in the same industry, the systematic risk of the future random payoffs of the license, on which investors require a risk premium, may be seen as associated with the average of the firms' private degrees of risk aversion.

[5]We leave the more general case in which the bidders have differing estimates of the common payoff distribution for future work. To our knowledge, auctions with heterogeneous risk averse bidders have only been studied for the sure-value cases (e.g., Cox et al., 1982, 1988) and the present model setup that also acknowledges ensuing payoff uncertainty is relatively new.

[6]See, e.g., Kahneman and Twersky (1979), Benartzi and Thaler (1995), Schmidt and Traub (2002), Shalev (2000), Rizzo and Zeckhauser (2004), and Kobberling and Wakker (2005).

[7]The mean-variance model (e.g., Markowitz, 1952) may be another modelling choice, but it does not

competitive bidding behavior, expected payments, optimal reserve prices, and bidders' expected utilities. Downside risk aversion has been incorporated in various studies, dating back to the classic of Domar and Musgrave (1944) on portfolio choice and taxation, and Bawa and Lindenberg's (1977) asset-pricing models. Closely related to the downside risk models are models of loss aversion as propounded in prospect theory (Kahneman and Tversky, 1979; Tversky and Kahneman, 1992). Both notions of downside risk and loss aversion refer to the similar observation that people are more sensitive to losses than to gains. The difference, in our opinion, lies in that the notion of loss aversion is nested in a set of more general behavioral assumptions, such as probability transformation (e.g., Prelec, 1998) and status-quo bias (e.g., Samuelson and Zeckhauser, 1988), which we do not consider. We focus instead on the mean-downside risk preferences with risk associated with "below-target payoffs" in the sense of Fishburn (1977). The reference level, or target, is the payment for the object when the bidder wins. As such, our approach remains congruent with expected utility theory, as demonstrated by earlier proponents of downside risk models (e.g., Bawa, 1975, 1976; and Fishburn, 1977).

The problem we consider involves n bidders who have private degrees of downside-risk aversion that are independently and identically distributed. The distribution of the future payoff of the auctioned object is common knowledge. Thus, the bidders' valuation of the object has two components: a pure common-expected value component v and a pure private-risk aversion component h (cf. Goeree and Offerman, 2002, 2003a). The mathematical formulation of the problem turns out to be isomorphic to a single dimension private value model thanks to a one-to-one relation between the private degree of risk aversion h and the private reservation value. In terms of economic implications, however, our model applies to a richer class of situations in which players face two dimensional uncertainties: the unknown future payoff as well as reservation values of the competitors.

The seller is assumed to be risk neutral, and we consider two possible cases of the seller objectives. In Case 1, the object must be sold and hence the seller does not have any

seem to be appealing, nor simplifying, for the analysis of auctions.

role in the model except the choice between FPA and SPA. This may be the case where the object is a contract or license about the development of certain projects that are of vital importance for the nation, such as those involved with military, defence, or national security systems. In Case 2, the revenue from the sale is also important, as in the sales of mineral or spectrum rights, so that the seller chooses a reserve price to maximize expected revenue under either the FPA or SPA policy.[8]

Our analysis yields several testable predictions. First, the more is the object's risk or the bidder's risk aversion, the less are the bids and hence the seller's expected revenue. This prediction is, indeed, quite intuitive given that risk or risk aversion reduces one's eagerness to pay. However, it is worth noting that this result is opposite to a wide-spread conclusion in the existing literature, as mentioned above, that risk aversion leads to more aggressive bidding in the FPA. This apparently paradoxical conclusion relies critically on the assumption that the auctioned object has a sure private value to the bidder, so that no one expects any monetary loss in standard auctions. A reason for there to be no "middle ground" between the existing and our conclusions is that in this study, bidders are assumed to dislike the downside risk only. Thus, in the absence of expected losses, the bidders would behave as though they were risk neutral. Second, the seller's expected revenue is always higher in the FPA than in the SPA. This prediction is not new, except that it extends the same finding in the existing literature to the case with ensuing risk of payoffs to the winner. Third, if the seller chooses the same reserve price for both FPA and SPA (Case 1), then all active bidders strictly prefer the SPA to the FPA.

The fourth prediction of our analysis says that if the seller wishes to maximize expected revenue (Case 2), then the optimal reserve price must be *lower* in the FPA than in the SPA. This result derives from a property that the marginal expected revenue with

[8] We do not consider possible partnership or profit sharing between the seller and the license winner. Potential agency problems, although not modelled in the present context, can be invoked to justify excluding these considerations here. Otherwise, the auctioned object could become a "labor contract" in which the seller bears all residual risks and the winning bidder agrees to work for the seller on the basis of fixed wages.

respect to the screening level in the FPA is always higher than in the SPA. We discuss insights into this property in more detail in the main text. An important implication here is that the FPA is allocatively more efficient, for a lower reserve price reduces the probability of no-sale. The prediction also suggests that caution should be made in drawing conclusions about auction forms in non-standard contexts. It is well-known that the *benchmark* model (symmetric risk neutral bidders with independently distributed private values; see Milgrom, 2004) predicts that the optimal reserve price is the same for all standard auction forms (at least under the regularity condition). Perhaps for this reason, the reserve price is typically assumed to be the same in discussing or comparing between the FPA and SPA forms—even under bidder risk aversion. When the seller is also an active player who wishes to maximize revenue, however, it is necessary to check optimal choices of both the seller and the buyers before drawing any conclusion.

The final prediction is that, as long as the reserve price is effective in precluding some high risk-averse bidders from bidding, there exists always a portion, and possibly all, of the active bidders who prefer FPA to SPA. In other words, some bidders, namely bidders who are relatively more risk averse and hence less eager to bid, always share the same preference with the seller in favor of the FPA. The result that both the seller and bidders may jointly prefer the same auction policy, i.e., the FPA, is appealing, for then the FPA Pareto dominates the SPA. Put differently, in these cases a shift from the SPA to the FPA improves all players' expected utilities (cf. Smith and Levin, 1996).

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 investigates the bidding strategies under the FPA and the SPA. The reserve price is assumed to be given in the derivation of the strategies at this stage. In Section 4, then, we derive the seller's optimal reserve prices taking into account the strategic responses of the bidders. In Section 5, the two auction formats are compared in terms of the expected utilities of the bidders, as well as the expected revenues of the seller. Section 6 concludes the paper.

2 The Model

An indivisible object, e.g., a contract or a license, is to be sold to one of n (≥ 2) potential bidders through either a FPA or a SPA. The winner of the object will receive a random monetary payoff $\widetilde{v}$ in the future, e.g., through subsequent business activities. The probability distribution of $\widetilde{v}$ is publicly known to be Q, with density function Q' that is assumed to be continuous on its support $[0, \infty)$. The expected payoff of $\widetilde{v}$ is denoted by v ($\equiv E(\widetilde{v}) > 0$).[9]

Each bidder $i \in N = \{1, ..., n\}$ has a mean-risk preference, with risk associated with below-target payoffs. Specifically, we normalize each bidder's status-quo utility to be zero and assume that if bidder i wins the object and pays a price c, his utility equals

$$U(c, h_i) \equiv \int_0^{\infty} (\widetilde{v} - c) dQ(\widetilde{v}) - h_i \int_0^c \varphi(c - \widetilde{v}) dQ(\widetilde{v}) \tag{1}$$

where $h_i \in [0, \overline{H}]$ ($\overline{H} > 0$) is the bidder's *private* degree of downside-risk aversion.[10] The downside risk, denoted by L, is measured by (see Fishburn, 1977)[11]

$$L(c) \equiv \int_0^c \varphi(c - \widetilde{v}) dQ(\widetilde{v}) \tag{2}$$

where the paid price c (≥ 0) serves as the target, and $\varphi : \mathbb{R}^+ \to \mathbb{R}^+$ is the disutility function of losses that is assumed to be strictly increasing, (weakly) convex, twice differentiable, and normalized to take values $\varphi(0) = 0$ and $\varphi'(0) = 1$.

[9] The lower bound of $\widetilde{v}$ can be any finite real number without affecting the results; it is assumed to be zero for convenience, which is consistent with limited liability. We also abstract away any intertemporal discounting in this model.

[10] The lower bound of h_i is assumed to be zero in order to include the special case of risk neutrality. Since a rational player never likes to lose money, we rule out negative values of h_i.

[11] As Fishburn (1977, p. 118) argues: "The idea of a mean-risk dominance model [...] seems rather appealing since it recognizes the desire to come out well in the long run while avoiding potentially disastrous setbacks or embarrassing failures to perform up to standard in the short run." Another advantage of the mean-downside risk model is its relative simplicity, which turns out to be very useful for deriving sensible, yet tractible, results in this paper.

The h_i of the n bidders are assumed to be stochastically independent random variables. The cumulative distribution of h_i is F with density function f that is continuously differentiable and strictly positive on $(0, \overline{H})$. The whole situation is assumed to be common knowledge, except that h_i is known to bidder i only. It is common belief that each bidder will act to maximize expected utility defined in (1). For expositional convenience, we call h_i bidder i's *type*. The subscript i will be dropped hereafter for notational convenience, unless needed for clarity.

Since the model presented above is relatively new in the auction literature, we take a closer look at the properties of L and its relation to stochastic dominance. By assumption, L is a strictly increasing and convex function of payment c:

$$\begin{aligned} L'(c) &= \int_0^c \varphi'(c-\widetilde{v})dQ(\widetilde{v}) > 0, \\ L''(c) &= Q'(c) + \int_0^c \varphi''(c-\widetilde{v})dQ(\widetilde{v}) > 0 \end{aligned}$$

More precisely, the downside risk measure L defined in (2) depends on both the disutility function φ and the distribution function Q. Given any function φ, we say that L is the *downside risk associated with distribution* Q. In the present context, if another distribution $\widehat{Q}$ has the same mean as Q, and the downside risk $\widehat{L}$ associated with $\widehat{Q}$ satisfies $\widehat{L}(c) \geq L(c)$ for all $c \in [0, v]$, we say that $\widehat{Q}$ is (weakly) more risky than Q and hence is dominated by Q. Recall that the condition for Q to dominate $\widehat{Q}$ in the sense of second-order stochastic dominance (SSD) is

$$\int_0^c Q(\widetilde{v})d\widetilde{v} \leq \int_0^c \widehat{Q}(\widetilde{v})d\widetilde{v} \tag{3}$$

for all $c \geq 0$ with strict inequality for some c (see Rothchild and Stiglitz, 1970; Hardar and Russel, 1969, 1971). The SSD condition (3) has been shown to be a sufficient for *all* risk averse individuals to prefer Q to $\widehat{Q}$.[12] The connection between SSD and the more risky

[12] Hardar and Russel (1969, p.32) even argue that "any result within the framework of the theory of risk aversion can be established directly by means of SSD. Conversely, any case of preference under risk aversion must imply SSD; if the latter condition fails to hold, then the result must necessarily be due to a special assumption about the functional form of the utility function."

relation in the present context can be seen from

$$\int_0^c Q(\widetilde{v})d\widetilde{v} = Q(c)c - \int_0^c \widetilde{v}dQ = \int_0^c (c-\widetilde{v})dQ(\widetilde{v})$$

Therefore, if φ is a linear function and if $\widehat{L}(c) \geq L(c)$ for all $c \geq 0$, then Q also dominates $\widehat{Q}$ in the sense of SSD. Conversely, condition (3) implies that $\widehat{Q}$ is more risky than Q in that

$$\begin{aligned}
\widehat{L}(c) - L(c) &= \int_0^c \varphi(c-\widetilde{v})d\left(\widehat{Q} - Q\right) \\
&= \int_0^c \left(\widehat{Q}(\widetilde{v}) - Q(\widetilde{v})\right)\varphi'(c-\widetilde{v})d\widetilde{v} \\
&= \int_0^c \varphi'(c-\widetilde{v})d\left(\int_0^{\widetilde{v}}\left(\widehat{Q}(s) - Q(s)\right)ds\right) \\
&= \int_0^c \left(\widehat{Q}(s) - Q(s)\right)ds + \int_0^c \varphi''(c-\widetilde{v})\left(\int_0^{\widetilde{v}}\left(\widehat{Q}(s) - Q(s)\right)ds\right)d\widetilde{v} \geq 0
\end{aligned}$$

where the inequality follows from (3) and the assumption that $\varphi'(0) = 1$ and $\varphi'' \geq 0$. It is worth noting that, since the more risky relation in our context is defined more generally than SSD (e.g., by restricting $c \in [0, v]$), the ensuing comparative results in Proposition 2 hold for a larger class of situations.

3 Bidding strategies

We first derive the bidding strategies assuming that an arbitrary reserve price r is given. Any tie is assumed to be randomly resolved. Since in the present context a tie happens with zero probability, the case will be neglected henceforth. Since the lowest bidder, i.e., the one who bids the reserve price r, expects zero profit, the relation between reserve price and the *screening level* or *threshold type* H, i.e., the type who bids r, can be derived from

$$U(r(H), H) = 0 \text{ or } r(H) = v - HL(r(H)) \tag{4}$$

The last equation in (4) defines r as a function of $H \in [0, \overline{H}]$. Since U is continuously differentiable, so is r. Differentiating gives

$$r'(H) = -\frac{L(r)}{1 + HL'(r)} < 0 \tag{5}$$

which confirms that there is a monotonic negative relation between r and H. The threshold type H implies that bidders with $h > H$ will abstain from bidding. This derives from the fact that $U(r(H), h) < U(r(H), H) = 0$ for all $h > H$. We say that a bidder is *active* if his type $h \leq H$, i.e., it is optimal for him to participate in the auction. On the other hand, it is obvious that no rational bidder will bid higher than v. If all bidders were risk neutral ($\overline{H} = 0$), then the model would reduce to a pure common-value special case of, e.g., Wilson (1977), Milgrom and Weber (1982), and Pesendorfer and Swinkels (1997). Competition would then allow the seller to extract all expected profits from the buyers.

For a bidder of type h, the probability that all the other $n-1$ bidders have (weakly) higher types than h is

$$G(h) \equiv \prod_{j \neq i} \Pr(h_j \geq h) = (1 - F(h))^{n-1}$$

Note that G is a *decumulative* distribution and its derivative is *negative*:

$$g(h) \equiv G'(h) = -(n-1)\,(1 - F(h))^{n-2}\, f(h) < 0$$

3.1 Second-price equilibrium strategy

We start with the SPA. Arbitrarily given any bidder of type h, let y denote the lowest type among the other $n-1$ bidders, i.e., $y = \min\{h_j | j \neq i, j \in N\}$. For every active bidder of type $h \in [0, H]$, define function $a(h)$ implicitly by $a(h) = v - hL(a(h))$ and interpret a to be the bidder's *reservation value*. From (4), it is clear that $a(h) \equiv r(h)$ for all $h \in [0, H]$ (although the two functions have different interpretations).

Lemma 1 *In a SPA with reserve price r, (i) it is a (weakly) dominant strategy for a bidder with type h to bid $a(h)$ if $a(h) \geq r$ and to abstain from bidding if $a(h) < r$; (ii) the* *expected payment by the bidder of type $h \leq H$ is*

$$m^{II}(h, H) \equiv r(H)G(H) - \int_h^H a(y)g(y)dy \tag{6}$$

and (iii) the bidder's expected utility of type $h \leq H$ equals

$$U^{II}(h, H) \equiv \int_h^H G(y) \frac{[1 + hL'(a(y))]\, L(a(y))}{1 + yL'(a(y))} dy \tag{7}$$

Proof. The proof of (i) is standard, using the argument that bidding up to $U(a(h), h) = 0$ weakly dominates all other strategies. The details of this argument are analogous to Vickrey (1961), hence omitted. (ii) Given any bidder of type $h \leq H$, the term $r(H)G(H)$ equals his expected payment when $y \geq H$, in which case the bidder pays the reserve price. The expected payment when $y \in (h, H)$ is $\int_H^h a(y)dG(y) = -\int_h^H a(y)g(y)dy$. Thus, adding the two terms gives the total expected payment in (6). (iii) The bidder's expected utility also has two terms, $\int_H^{\overline{H}} (v - r - hL(r))\, dG(y)$ for $y \in [H, \overline{H}]$ and $\int_H^h [v - a(y) - hL(a(y))]\, dG(y)$ for $y \in (h, H)$. Adding yields

$$\begin{aligned} U^{II}(h, H) &= (v - r - hL(r))\, G(H) + \int_H^h [v - a(y) - hL(a(y))]\, dG(y) \\ &= vG(h) - (r + hL(r))\, G(H) - \int_H^h [a(y) + hL(a(y))]\, dG(y) \\ &= -\int_h^H G(y) \left[1 + hL'(a(y))\right] a'(y)dy \end{aligned}$$

where in the last equation we used integration by parts and the property that $v - a(h) - hL(a(h)) = 0$ for all $h \leq H$. Substituting $a'(y) = -L(a)/(1 + yL'(a))$ gives (7). ∎

Clearly, the more risk averse a bidder is (i.e., with a higher h), the less is his bid. In the limiting case where the bidder is risk neutral with $h = 0$, he bids the expected value v.

3.2 First-price equilibrium strategy

In the *benchmark* model where all bidders are risk neutral (e.g., Myerson, 1981), the reserve price maximizing the seller's expected revenue is the same for all standard auctions.

Although the bidding strategy in the FPA generally increases with the reserve price, this dependency does not cause any serious complication in the analysis of the benchmark models. When bidders are risk averse, however, such a dependency becomes endogenous in the derivation of the optimal reserve price, as will be seen in Section 4. Therefore, we define bidding strategies in the FPA as a two-dimensional function $b : (h, H) \in [0, \overline{H}]^2 \to b(h, H) \in [0, v]$.

Fixing any threshold type $H \leq \overline{H}$, suppose that each bidder uses a bidding strategy $b = b(h, H)$ that is a strictly decreasing and continuous function of h on $[0, H]$. The bidder's expected utility by bidding $b(z, H)$ for any $z \in [0, H]$, given his true type h, is then equal to

$$\overline{U}(z, h) \equiv G(z)U(b(z, H), h) = G(z)\left(v - b(z, H) - hL(b(z, H))\right)$$

Lemma 2 *In a FPA, (i) the unique symmetric equilibrium is characterized by bidding strategy*

$$b(h, H) = v - hL(b(h, H)) - \frac{1}{G(h)} \int_h^H G(y)L(b(y, H))dy \tag{8}$$

if $h \leq H$ and bidding nothing if $h > H$; (ii) the expected payment by the bidder of type $h \leq H$ is

$$m^I(h, H) \equiv b(h, H)G(h) \tag{9}$$

and (iii) the bidder's expected utility of type $h \leq H$ equals

$$U^I(h, H) \equiv \int_h^H G(y)L(b(y, H))dy \tag{10}$$

Proof. (i) For $h > H$, $U(r(H), h) < U(r(H), H) = 0$. Thus it is optimal for the bidder to bid nothing. For $h \leq H$, if $b = b(h, H)$ is a symmetric (Bayesian-Nash) equilibrium strategy, then by the envelope theorem (e.g., Milgrom and Segal, 2002)

$$\begin{aligned}
\frac{d\overline{U}(h,h)}{dh} &= \overline{U}_2(z,h)|_{z=h} = -G(h)L(b(h,H)) \\
&\Rightarrow \overline{U}(H,H) - \overline{U}(h,h) = -\int_h^H G(y)L(b(y,H))dy \\
&\Rightarrow \overline{U}(h,h) = G(h)\left(v - b(h,H) - hL(b(h,H))\right) = \int_h^H G(y)L(b(y,H))dy \\
&\Rightarrow b(h,H) = v - hL(b(h,H)) - \frac{1}{G(h)}\int_h^H G(y)L(b(y,H))dy
\end{aligned}$$

(ii) Straightforward by definition of expected payment. (iii) Rearranging terms in (8) gives directly

$$U^I(h,H) \equiv G(h)\left(v - b(h,H) - hL(b(h,H))\right) = \int_h^H G(y)L(b(y,H))dy \qquad (11)$$

■

As in the existing literature on the FPA with risk-averse bidders, the bidding strategy b here can only be defined implicitly. To obtain a (numerical) solution, it is often convenient to study the differential equation by differentiating b in (8) with respect to h, taking each value of H as fixed. This gives

$$b_1(h,H) = \frac{g(h)}{G(h)}\frac{(v-b-hL(b))}{(1+hL'(b))} = -\frac{(n-1)f(h)\,(v-b-hL(b))}{(1-F(h))\,(1+hL'(b))} \qquad (12)$$

Since in equilibrium the bidder must expect a positive expected utility, $v - b - hL(b) > 0$. Consequently, $b_1 < 0$ for all $h < H$. Also, the right side of (12) has a continuous partial derivative with respect to b for all $h < H$, which is uniformly bounded on the range of the bids $[0, v]$. Therefore, by the Theorem of Cauchy there exists a unique function b satisfying (12) and the initial condition $b(H,H) = r(H)$.

The results obtained so far are relatively straightforward; still, it is worth noting that the general conclusion $b_1 < 0$ for all $h < H$ is opposite to the one that is made in the existing literature (e.g., Holt, 1980; Riley and Samuelson, 1981; see also Krishna, 2002), which predicts that higher risk aversion leads to more aggressive bidding in the FPA. This existing prediction relies critically on the assumption that the auctioned object has a sure private value to the bidder. If, instead, the object carries ensuing risk of payoffs, we find that higher risk aversion leads to less aggressive bidding in the FPA.

Unlike in the SPA, a risk neutral bidder bids strictly lower than v. This can be seen by substituting 0 for h in (8) to get $b(0,H) = v - \int_0^H G(y)L(b(y,H))dy < v$. Intuitively, this result is due to the expectation that other bidders may be risk averse, so a risk neutral bidder has a strictly positive expected utility in equilibrium. To see how b changes with H, rewrite (8) as

$$[b(h,H) + hL(b(h,H)) - v]\, G(h) = -\int_h^H G(y)L(b(y,H))dy$$

Differentiating with respect to H then yields

$$\begin{aligned} \left[1 + hL'(b)\right] G(h) b_2(h,H) &= -G(H)L(r) - \int_h^H G(y)L'(b(y,H))b_2(y,H)dy \qquad (13) \\ \text{or } b_2(h,H) &= -\frac{G(H)L(r)}{[1 + hL'(b)]\, G(h)} - \frac{\int_h^H G(y)L'(b(y,H))b_2(y,H)dy}{[1 + hL'(b)]\, G(h)} \end{aligned}$$

It is easily seen that for $h = H$,

$$b_2(H,H) = \frac{-L(r)}{1 + HL'(r)} = r'(H) < 0 \qquad (14)$$

The next lemma establishes that b is a strictly decreasing function of H in general, for all $h \leq H$. This property will be useful for the analysis in the next two sections.

Lemma 3 *For all h and H such that $0 \leq h \leq H \leq \overline{H}$, $b_2(h,H) \leq 0$, with strict inequality for at least some h and H.*

Proof. It can be shown that b_{12} exists (e.g., Lemma 4). Since $b_2(H,H) < 0$ as shown in (14), $b_2(h,H) < 0$ at least for some h and H by continuity. Suppose there were some values of h and H such that $b_2(h,H) > 0$. Then, by continuity of b_2 again, there must exist some $\widehat{h} \in (h,H)$ such that $b_2(y,H) > 0$ for all $y \in (h,\widehat{h})$, and $b_2(\widehat{h},H) = 0$ (since

$b_2(H,H) < 0$). But this is impossible since, from (13), we would then have

$$\begin{aligned}
0 &< \left[1 + hL'(b)\right] G(h) b_2(h,H) \\
&= -G(H)L(r) - \int_h^H G(y)L'(b(y,H))b_2(y,H)dy \\
&= -G(H)L(r) - \int_h^{\widehat{h}} G(y)L'(b(y,H))b_2(y,H)dy \\
&\quad - \int_{\widehat{h}}^H G(y)L'(b(y,H))b_2(y,H)dy \\
&< -G(H)L(r) - \int_{\widehat{h}}^H G(y)L'(b(y,H))b_2(y,H)dy \\
&= \left[1 + \widehat{h}L'(b)\right] G(\widehat{h}) b_2(\widehat{h},H) = 0
\end{aligned}$$

■

What Lemma 3 says is that the equilibrium bids are pushed higher as the reserve price increases. The main reason is that the bidder whose type equals the threshold type H has to bid up to the reserve price that equals his reservation value, whereas he would bid below this payment if the reserve price were lower. Thus, increasing the reserve price results in a "ratcheting effect," forcing every active bidder to bid higher.

3.3 Effects of increasing risk aversion and the screening level

The general effects of increasing risk aversion h and increasing the screening level H can be derived straightforwardly by differentiating the expected payments and utilities.

Proposition 1 *For all h and H such that $0 < h < H \leq \overline{H}$, and for $A \in \{I, II\}$, $m_1^A(h,H) < 0$, $m_2^A(h,H) < 0$, $U_1^A(h,H) < 0$, and $U_2^A(h,H) > 0$.*

Proof. Using the subscript to denote the partial derivative with respect to the corresponding argument, we get from (9) that

$$m_1^{II}(h,H) = a(h)g(h) < 0 \tag{15}$$

$$m_2^{II}(h,H) = -\frac{G(H)L(r(H))}{1 + HL'(r(H))} = G(H)r'(H) < 0 \tag{16}$$

Differentiating (7) gives

$$\begin{aligned} U_1^{II}(h,H) &= -G(h)L(r(h)) - \int_h^H G(y)dL(a(y)) \\ &= -G(H)L(r(H)) + \int_h^H L(a(y))g(y)dy < 0 \qquad (17) \\ U_2^{II}(h,H) &= -G(H)\left[1 + hL'(r)\right] r'(H) > 0 \qquad (18) \end{aligned}$$

Similarly, differentiating (9) and (10) yields

$$m_1^I(h,H) = b_1(h,H)G(h) + b(h,H)g(h) < 0 \qquad (19)$$

$$m_2^I(h,H) = b_2(h,H)G(h) < 0 \qquad (20)$$

$$\begin{aligned} U_1^I(h,H) &= -G(h)L(b(h,H)) < 0 \qquad (21) \\ U_2^I(h,H) &= G(H)L(r(H)) + \int_h^H G(y)L'(b(y,H))b_2(y,H)dy \\ &= -\left[1 + hL'(b)\right] G(h)b_2(h,H) > 0 \qquad (22) \end{aligned}$$

■

Thus, according to Proposition 1, in both FPA and SPA the bidders with lower types h (or lower degrees of downside-risk aversion) have higher expected payments and higher expected utilities; whereas in general, increasing reserve price r (hence lowering H) hurts all bidders. A subtile difference between FPA and SPA, however, can be discerned by comparing (16) with (20). Namely, that the marginal change of expected payment with respect to H is the same for all types $h < H$ in SPA, but it varies with h in FPA. This difference, as will be seen, leads to different optimal reserve prices in the two auction forms.

To fix ideas, consider a simple numerical example.

Example 1 *Assume $n = 2$, $F(h) = h$ on $[0,1]$, $Q(\widetilde{v}) = \widetilde{v}$ on $[0,1]$ (so that $v = 0.5$), and $\varphi(x) = x$. Thus $G(h) = 1 - h$ and*

$$\begin{aligned} L(c) &= \int_0^c (c - \widetilde{v})d\widetilde{v} = \frac{c^2}{2} \\ a(h) &= v - h\frac{a^2(h)}{2} = \frac{1}{h}\left(\sqrt{2hv+1} - 1\right) \end{aligned}$$

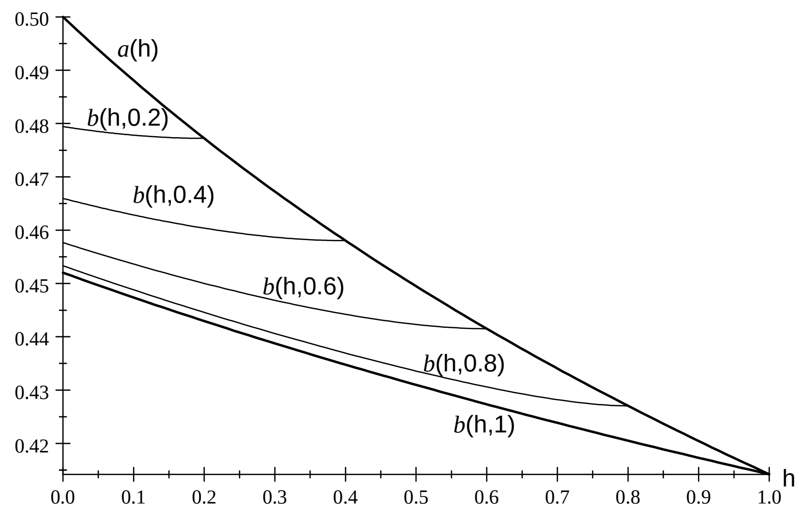

Figure 1: The bidding strategies are inversely related to degree of risk aversion h. The second-price bidding strategy is unaffected by the reserve price or the threshold type H, whereas the first-price bidding strategy increases as H decreases.

The first-price bidding function can be solved numerically from the differential equation with initial condition $b(H) = r(H) = \frac{1}{H}\left(\sqrt{H+1}-1\right)$, *treating* H *as a constant in each of the computations:*

$$b' = -\frac{\left(v - b - hb^2/2\right)}{(1-h)(1+hb)}$$

As shown in Figure 1, a is invariant with the reserve price whereas b is.

3.4 Effects of increasing risk

We conclude this section by examining how risk affects the bidding strategies and expected payments. Intuitively, when the payoff becomes more risky (keeping the mean as constant), the object becomes less attractive to all risk averse bidders and hence lower equilibrium bids are to be expected. This intuition is indeed correct, as shown in the next proposition.

Proposition 2 *Let two payoff distributions Q and $\widehat{Q}$ be given that have the common mean v. Assume that $L(c) \leq \widehat{L}(c)$ for all $c \in [0, v]$, i.e., that Q dominates $\widehat{Q}$ in the sense of having less downside risk. Then, under both FPA and SPA, $\widehat{Q}$ entails lower equilibrium bids (hence, expected revenue) than Q.*

Proof. Let the "hat" symbol designate the association with $\widehat{Q}$ of the relevant functions. We want to show that for all $h \in (0, H)$ and all $H \leq \overline{H}$, (i) $a(h) \geq \widehat{a}(h)$ and (ii) $b(h, H) \geq \widehat{b}(h, H)$.

(i) The bidding function $a(h)$ and $\widehat{a}(h)$ are given by

$$a(h) = v - hL(a(h)), \;\; \widehat{a}(h) = v - h\widehat{L}(\widehat{a}(h))$$

Subtracting gives

$$a(h) - \widehat{a}(h) = -h\left[L(a(h)) - \widehat{L}(\widehat{a}(h))\right] \geq -h\left[\widehat{L}(a(h)) - \widehat{L}(\widehat{a}(h))\right] \tag{23}$$

where the last inequality uses the assumption that $\widehat{Q}$ is more risky than Q. We prove the claim by contradiction. If $a(h) < \widehat{a}(h)$ for some h, then $\widehat{L}(a(h)) < \widehat{L}(\widehat{a}(h))$. But then the term on the right side of (23) is strictly positive, indicating that $a(h) > \widehat{a}(h)$. (ii) From (11), we have

$$\begin{aligned} U^I(h, H) &= [v - b(h, H) - hL(b(h, H))]\, G(h) \\ &= \int_h^H G(y)L(b(y, H))dy \\ \widehat{U}^I(h, H) &= \left[v - \widehat{b}(h, H) - h\widehat{L}(\widehat{b}(h, H))\right] G(h) \\ &= \int_h^H G(y)\widehat{L}(\widehat{b}(y, H))dy \end{aligned}$$

We prove a stronger claim that $L(b(h, H)) \geq \widehat{L}(\widehat{b}(h, H))$ for all h, which implies $b(h, H) \geq \widehat{b}(h, H)$. This is because if $L(b(h, H)) < \widehat{L}(\widehat{b}(h, H))$ for some h, then by continuity, there exists some $z \in (h, H]$ such that $L(b(y,H)) < \widehat{L}(\widehat{b}(y,H))$ for all $y \in (h, z)$. Moreover, if $z < H$ then $L(b(z,H)) = \widehat{L}(\widehat{b}(z,H))$, or else $z = H$. In any case, $U^I(z,H) \leq \widehat{U}^I(z,H)$ because for $z = H$, $U^I(H,H) = \widehat{U}^I(H,H) = 0$, and for $z < H$, $L(b(z,H)) = \widehat{L}(\widehat{b}(z,H))$ implies $b(z,H) \geq \widehat{b}(z,H)$, which in turn implies $U^I(z,H) \leq \widehat{U}^I(z,H)$. A contradiction then follows:

$$
\begin{aligned}
0 &< U^I(h,H) - \widehat{U}^I(h,H) = \int_h^H G(y)\left(L(b(y,H)) - \widehat{L}(\widehat{b}(y,H))\right) dy \\
&= \int_h^z G(y)\left(L(b(y,H)) - \widehat{L}(\widehat{b}(y,H))\right) dy \\
&\quad + \int_z^H G(y)\left(L(b(y,H)) - \widehat{L}(\widehat{b}(y,H))\right) dy \\
&= \int_h^z G(y)\left(L(b(y,H)) - \widehat{L}(\widehat{b}(y,H))\right) dy + U^I(z,H) - \widehat{U}^I(z,H) < 0
\end{aligned}
$$

■

Somewhat surprisingly, the proof of Proposition 2 is more involved than one might expect – especially with the FPA strategies. The reason is that the characterization of bidding function b is implicit, which prevents a direct comparison of the two functions, b and $\widehat{b}$. Note that Proposition 2 also predicts that increasing risk generally reduces the seller's expected revenue under either the FPA or the SPA. This is true even if we allow the seller to choose different reserve prices for Q and $\widehat{Q}$, for then H can be interpreted in Proposition 2 as the optimal threshold type that the seller sets for the case with $\widehat{Q}$.

4 The seller's problem

We assume that the seller is risk neutral and consider two cases of seller objectives. In Case 1, the object must be sold and therefore the threshold type is $\overline{H}$. In Case 2, the seller imposes a reserve price to maximize expected revenue. In this second case, the seller must run the risk of not selling the object and be able to credibly commit to it, as we assume. In general, the two cases can be combined together by assuming that if the seller fails to sell the object, there will be a deadweight (social) loss so that the *reserve value* to the seller is v_0 ($< v$). Then, Case 1 can be associated with a significantly low (or negative) value of v_0, and Case 2 with a v_0 that is moderately lower than v.

In order to ensure that the seller's problem of maximizing expected revenue is well defined, we invoke the following assumption.

Assumption 1 *For all* $h \in [0, \overline{H}]$, $1 + \left(\frac{F(h)}{f(h)}\right)' \geq \frac{L'(v)}{(1+hL'(v))} \frac{F(h)}{f(h)}$.

Assumption 1 implies that the seller's objective function in the SPA is quasi-concave in H. This property can then be used for the comparative analysis about optimal reserve prices between the two auction forms. The somewhat peculiar condition in Assumption 1 is probably due to the fresh model in the present study, which has not been formulated before. This condition is actually "tight" if the derived results are to hold for all possible seller valuations $v_0 \leq v$, as will be seen in the proof of Proposition 3. The condition does not imply, nor is implied by, the more familiar conditions such as the regularity or the logconcavity conditions (e.g., Krishna, 2002). However, many familiar distributions can be verified to satisfy Assumption 1, including uniform, exponential, and more generally the gamma distributions with properly delineated parameters. Note that Assumption 1 is about the primitives only.

4.1 Seller's problem in the SPA

For arbitrary threshold type $H \leq \overline{H}$, the seller's expected utility can be written as

$$\begin{aligned} V^{II}(H) &\equiv (1 - F(H))^n v_0 + n \int_0^H m^{II}(h, H) dF(h) \qquad (24) \\ &= (1 - F(H))^n v_0 + n \int_0^H \left(r(H)G(H) - \int_h^H a(y)g(y)dy \right) dF(h) \qquad (25) \end{aligned}$$

The first term on the right side of (24) is related to the no-sale event, which happens with probability $(1 - F(H))^n$ when all bidders have types $h > H$. The last term in (24) is the total expected payment from the n bidders; the possibility of no bids (higher than the reserve price) is taken into account in the integration, which runs from 0 to H (rather than $\overline{H}$).

The seller's problem is

$$\max_H V^{II}(H) \text{ subject to } 0 \leq H \leq \overline{H} \tag{26}$$

To facilitate notation, we define a function $\Phi : H \in [0, \overline{H}] \to \mathbb{R}$ by

$$\Phi(H) \equiv r(H) - \frac{L(r(H))}{1 + HL'(r(H))} \frac{F(H)}{f(H)} \tag{27}$$

Proposition 3 *Assume that $v_0 < v$ and that Assumption 1 holds. Then the optimal reserve price r^{II} under the SPA is given by $r^{II} = r(\overline{H})$ if $\Phi(\overline{H}) \geq v_0$, and $r^{II} = r(H)$ with H satisfying $\Phi(H) = v_0$ if otherwise – that is,*

$$r(H) = v_0 + \frac{L(r(H))}{1 + HL'(r(H))} \frac{F(H)}{f(H)} \tag{28}$$

Proof. The derivative of V^{II} with respect to H is given by, noting that $a(H) = r(H)$,

$$\begin{aligned} \frac{dV^{II}}{dH} &= -nG(H)f(H)v_0 + n\left[m^{II}(H,H)f(H) + \int_0^H m_2^{II}(h,H)dF(h)\right] \\ &= -nG(H)f(H)v_0 + n\left[r(H)f(H) + r'(H)F(H)\right]G(H) \qquad \text{(see (16))} \\ &= nG(H)f(H)\left(r(H) - v_0 - \frac{L(r)}{1+HL'(r)}\frac{F(H)}{f(H)}\right) \\ &= nG(H)f(H)\left(\Phi(H) - v_0\right) \end{aligned} \tag{29}$$

The first-order condition for maximizing (26) is thus

$$nf(H)G(H)\left(\Phi(H) - v_0\right) - \lambda = 0$$

where λ is the Lagrangian multiplier associated with constraint $H \leq \overline{H}$. The constraint $H \geq 0$ is not binding because, under the assumption that $v_0 < v$, the term in (29) is strictly positive for $H = 0$. Consequently, the candidates H for an optimal solution are given by $\Phi(H) = v_0$ with $\lambda = 0$, or $H = \overline{H}$ with $\lambda \geq 0$. Whenever an interior solution H satisfying $\Phi(H) = v_0$ exists, note that

$$\frac{d^2V^{II}}{dH^2}|_{\Phi(H)=v_0} = n\Phi'(H)f(H)G(H)$$

Thus, if we can show that $\Phi'(H) < 0$ whenever $\Phi(H) = v_0$ for $H < \overline{H}$, then H will be a unique global optimal solution to (26). To proceed, write

$$\Phi(h) = r(h) + r'(h)\frac{F(h)}{f(h)} \tag{30}$$

Differentiating yields

$$\Phi'(h) = r'(h)\left(1 + \left(\frac{F(h)}{f(h)}\right)'\right) + r''(h)\left(\frac{F(h)}{f(h)}\right) \tag{31}$$

where

$$\begin{aligned}
r''(h) &= -\frac{L'(r)r'\left[1 + hL'(r)\right] - L(r)[L'(r) + hL''r']}{\left[1 + hL'(r)\right]^2} \\
&= -\frac{-L'(r)L(r) - L(r)[L'(r) + hL''r']}{\left[1 + hL'(r)\right]^2} \\
&= \frac{2L'(r)L(r) + L(r)hL''r'}{\left[1 + hL'(r)\right]^2} \\
&= -\frac{2L'(r)r'}{[1 + hL'(r)]} - \frac{hL''r'^2}{[1 + hL'(r)]} \\
&= -\frac{L'(r)r'}{[1 + hL'(r)]}\left(2 + \frac{hL''r'}{L'}\right) = \frac{L'(r)r'}{L(r)}\left(2 + \frac{hL''r'}{L'}\right)r' \qquad (32)
\end{aligned}$$

Substituting the last term in (32) for r'' in (31) gives

$$\Phi'(h) = r'(h)\left[\left(1 + \left(\frac{F(h)}{f(h)}\right)'\right) + \frac{L'(r)}{L(r)}\left(2 + \frac{hL''r'}{L'}\right)\left(\frac{F(h)}{f(h)}\right)r'\right] \tag{33}$$

Alternatively, (30) and $\Phi(h) = v_0$ implies

$$r'(h) = \frac{f(h)}{F(h)}(v_0 - r) \quad (= -\frac{L(r)}{(1 + hL'(r))}) \tag{34}$$

Therefore, substituting (34) for r' in (33), and recalling that $r' < 0$, showing $\Phi'(h) < 0$ is equivalent to showing that for all v_0 such that $\Phi(h) = v_0$,

$$\left(1 + \left(\frac{F(h)}{f(h)}\right)'\right) > \frac{L'(r)}{L(r)}\left(2 - \frac{hL''}{L'}\frac{f(h)}{F(h)}(r - v_0)\right)(r - v_0) \tag{35}$$

The right side of (35) is a concave function of v_0. Its maximum is attained when h satisfies the first order condition, yielding

$$\frac{hL''}{L'}\frac{f(h)}{F(h)}(r - v_0) = 1$$

Consequently, (35) holds for all $v_0 < v$ if and only if (which suggests that Assumption 1 is "tight")

$$1 + \left(\frac{F(h)}{f(h)}\right)' > \frac{L'(r)}{L(r)}(r - v_0) = \frac{L'(r)}{(1 + hL'(r))}\frac{F(h)}{f(h)} \tag{36}$$

where the last equality comes from (34). Finally, noting that $r \le v$ and that the term on the right side of (34) is a strictly increasing function of r, we conclude that Assumption 1 implies that H satisfying $\Phi(H) = v_0$, or equivalently, (28), attains a global maximum for (26). ■

Although the optimal reserve price is characterized only implicitly in (28), it is clear that this reserve price is always higher than the seller's reserve value v_0. This result is analogous to the one obtained in the risk-neutral benchmark model (e.g., Krishna, 2002), except that the derivation, especially the process of identifying a tight sufficient condition for global maximization, is much more complicated here. We shall denote by H^{II} the threshold type that solves problem (26). We say that the reserve price is *effective* if $H^{II} < \overline{H}$, in which case the first-order condition $\Phi(H^{II}) = v_0$, together with the second-order condition $\Phi'(H^{II}) < 0$, defines H^{II} implicitly as a *decreasing* function of v_0. Figure 2 shows the relations between H, v_0, and $r(H)$.

Example 2 *Assume the case in Example 1. We have*

$$\Phi(H) = \frac{1}{H}\left(\sqrt{H+1} - 1\right) - \frac{\frac{1}{H}\left(\sqrt{H+1} - 1\right)^2/2}{\sqrt{H+1}} = \frac{1}{2\sqrt{H+1}}$$

Therefore

$$\Phi(H^{II}) = v_0 \Rightarrow H^{II}(v_0) = \frac{1}{4v_0^2} - 1$$

The upper bound of H is $\overline{H} = 1$ in this example, so the seller will choose an effective reserve price only if

$$\frac{1}{4v_0^2} - 1 \le 1 \textit{ or } v_0 \ge \frac{1}{4}\sqrt{2} = 0.353\,55$$

The optimal reserve price can then be solved as a function of $v_0 \in (0.35355, 0.5)$, i.e.,

$$\widehat{r}^{II}(v_0) \equiv r\left(H^{II}(v_0)\right) = \frac{1}{H^{II}(v_0)}\left(\sqrt{H^{II}(v_0) + 1} - 1\right) = \frac{2}{\sqrt{\frac{1}{v_0^2}} + 2}$$

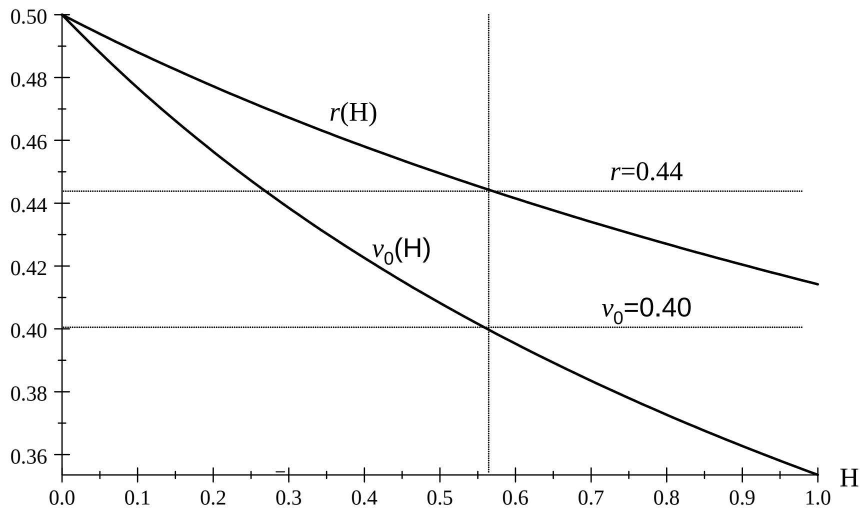

Figure 2: The seller's reserve value v_0 and the optimal threshold type H are negatively related via a one-to-one relation. For example, if $v_0 = 0.4$, then $H = 0.5625$ and the reserve price $r(0.5625) = 0.44$.

For instance, for $v_0 = 0.4$*, the threshold type is* $H = 0.5625$ *and the reserve price is* $r(H) = 0.4444$.

4.2 Seller's problem in the FPA

Unlike in the SPA, the bidder's strategy in the FPA depends on both his own type h and the threshold type $H \leq \overline{H}$, which is $b(h, H)$ for $h \in [0, H]$ as derived in (8). Before deriving the optimal reserve price in the FPA, we present a lemma that will be useful later.

Lemma 4 *For all $h \in (0, H) \subset [0, \overline{H}]$, $m^I_{12}(h, H) < 0$.*

Proof. Recall from (12) that

$$b_1(h, H) = \frac{g}{G} \times \frac{(v - b - hL(b))}{(1 + hL'(b))} < 0$$

Differentiating $b_1(h, H)$ with respect to H yields

$$b_{12}(h, H) = -\frac{g}{G} \times \left(1 + \frac{(v - b - hL(b))\, hL''}{(1 + hL'(b))^2}\right) b_2(h, H) < 0$$

where $b_2(h, H) < 0$ by Lemma 3. It follows that

$$\begin{aligned} m^I_{12}(h, H) &= b_{12}(h, H)G(h) + b_2(h, H)g(h) \\ &= g(h)b_2(h, H) - g\left(1 + \frac{(v - b - hL(b))\, hL''}{(1 + hL'(b))^2}\right) b_2(h, H) \\ &= -g(h)\left(\frac{(v - b - hL(b))\, hL''}{(1 + hL'(b))^2}\right) b_2(h, H) \end{aligned}$$

By inspecting the signs, we obtain

$$m^I_{12}(h, H) \begin{cases} < 0 \text{ if } h \in (0, H) \\ = 0 \text{ if } h \in \{0, H\} \end{cases}$$

■

Note that $m^I_2(h, H)$ is the marginal expected payment in the FPA with respected to the threshold type H. It can be verified that in the risk-neutral benchmark model (e.g., Myerson, 1981), this term depends only on the threshold type and not on each bidder's private type. Under risk aversion, the SPA preserves this property as can be seen from (16) or $m^{II}_{12}(h, H) = 0$. Lemma 4 shows, however, that this is no longer the case when bidders are risk averse in the FPA. Specifically, the marginal expected payment with respect to H is found to be a decreasing function in each bidder's private type h. The sign of $m^I_2(h, H)$ being negative (see (20)) suggests that increasing reserve price leads to higher expected payment by each active bidders. The sign of $m_{12}^I(h,H)$ being negative for all $h < H$ suggests, however, that the effect of increasing the reserve price (or decreasing the threshold type) weakens as h becomes lower. This observation helps explain the finding in the next proposition that it is optimal to set a lower reserve price in the FPA than in the SPA.

The seller's expected utility in the FPA equals, for any $H \leq \overline{H}$,

$$\begin{aligned} V^I(H) &\equiv (1-F(H))^n v_0 + n\int_0^H m^I(h,H)dF && (37)\\ &= (1-F(H))^n v_0 + n\int_0^H b(h,H)G(h)f(h)dh && (38)\end{aligned}$$

The seller's problem is thus

$$\max_H V^I(H) \text{ subject to } 0 \leq H \leq \overline{H} \tag{39}$$

Similar to the previous analysis, define $\Psi : H \in [0,\overline{H}] \to \mathbb{R}$ by

$$\Psi(H) \equiv r(H) + \frac{1}{G(H)f(H)}\int_0^H b_2(h,H)G(h)dF(h) \tag{40}$$

Proposition 4 *Assume that $v_0 < v$ and that Assumption 1 holds. Then (i) the seller's expected utility is strictly higher in the FPA than in the SPA; (ii) the optimal threshold type in the FPA, denoted by H^I, is strictly higher than H^{II} whenever $H^{II} < \overline{H}$; and (iii) if $H^I < \overline{H}$, then $\Psi(H^I) = v_0$ or equivalently, the interior optimal threshold type necessarily satisfies*

$$r(H) = v_0 - \frac{1}{G(H)f(H)}\int_0^H b_2(h,H)G(h)dF(h) \tag{41}$$

Proof. See Figure 3. The derivative of V^I with respect to H equals

$$\begin{aligned} \frac{dV^I}{dH} &= -nG(H)f(H)v_0 + n\left(r(H)G(H)f(H) + \int_0^H b_2(h,H)G(h)dF(h)\right)\\ &= nG(H)f(H)\left(\Psi(H) - v_0\right) && (42)\end{aligned}$$

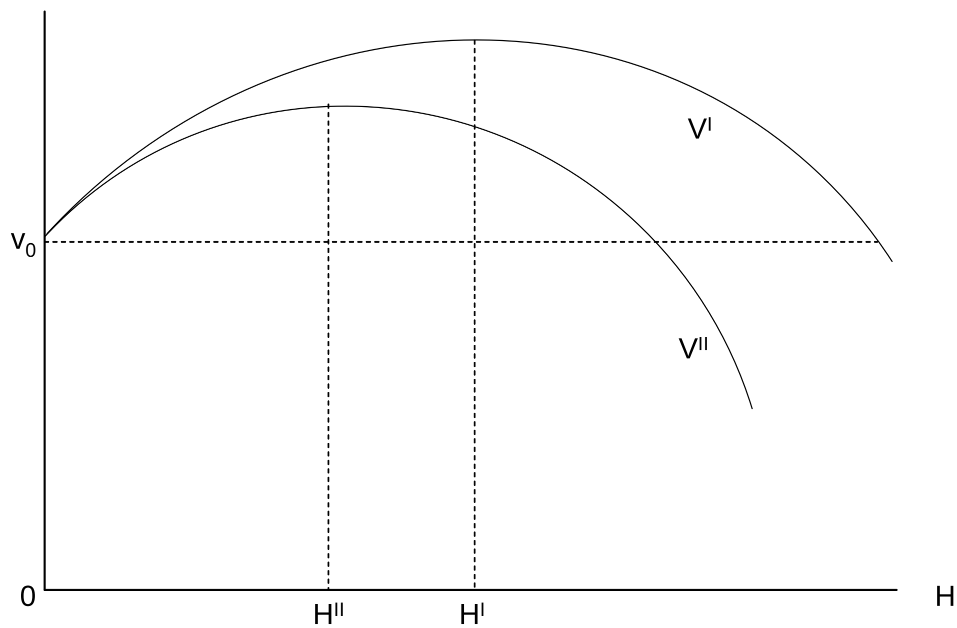

Figure 3: The slope of the seller's expected utility in the FPA, or V^I, is always higher than that in the SPA, or V^{II}. Therefore the optimal threshold type H^I is greater than H^{II}, indicating that the reserve price in the FPA is lower than in the SPA.

Applying integration by parts, and using (14), we observe that

$$
\begin{aligned}
\Psi(H) &\equiv r(H) + \frac{1}{G(H)f(H)} \int_0^H b_2(h,H)G(h)dF(h) \\
&= r(H) + r'(H)\frac{F(H)}{f(H)} - \frac{1}{G(H)f(H)} \int_0^H F(h)\frac{\partial}{\partial h}(b_2(h,H)G(h))dh \\
&= \Phi(H) - \frac{1}{G(H)f(H)} \int_0^H F(h)m_{12}^I(h,H)dh
\end{aligned}
\tag{43}
$$

with $\Phi(\cdot)$ defined in (27). It follows from Lemma 4 that $\Psi(H) > \Phi(H)$. Comparing (42) with (29), we obtain $dV^I/dH > dV^{II}/dH$ for all $H \in \left(0, \overline{H}\right)$.

(i) Since $V^I(0) = V^{II}(0) = v_0$, $\Psi(H) > \Phi(H)$ implies that $V^I(H) > V^{II}(H)$ for all $H \in (0, \overline{H}]$, and hence the conclusion. (ii) Again from $\Psi(H) > \Phi(H)$, $\Psi(H^{II}) > \Phi(H^{II}) = v_0$. We know from Proposition 3 that $\Phi(H) > v_0$ for all $H < H^{II}$, so it is also true for $\Psi(H)$. It follows that $H^I > H^{II}$ as long as $H^{II} < \overline{H}$. (iii) Assuming an interior solution, the first-order condition for the maximization problem in (39) implies $\Psi(H) = v_0$. Thus the optimal reserve price necessarily satisfies (41) unless $H^I = H^{II} = \overline{H}$. ■

The fact that $m_{12}^I(h,H) < 0$ drives the results in Proposition 4, as can be seen in (43). Since in the SPA the marginal effect of increasing H (or reducing the reserve price) is the same for all types $h < H$, whereas in the FPA this effect diminishes in intensity as type h becomes smaller (or the bidder more eager to pay), it is an intuitive conclusion that the seller prefers to set a lower reserve price in the FPA.

Note that we have not attempted to find a sufficient condition in the proof of Proposition 4 for H^I. It seems unavoidable that any sufficient condition for the general case will have to involve the bidding function in it (cf. Maskin and Riley, 1984, condition (45)). Fortunately, establishing the optimal screening level H^{II} for SPA in Proposition 3 together with the observation that $m_{12}^I(h,H) < 0$ in Lemma 4, we are able to characterize H^I in Proposition 4 without invoking a sufficient condition; the differentiability of the objective function V^I and the fact that H^I is chosen from a closed interval $[0, \overline{H}]$ turns out to be sufficient for the results.

A corollary concerning the expected revenue for the seller follows.

Corollary 1 *Assume $v_0 < v$. Then the seller's expected revenue is strictly higher in the FPA than in the SPA.*

Proof. By Proposition 4, $H^I \geq H^{II}$ and hence $(1 - F(H^I))^n v_0 \leq (1 - F(H^{II}))^n v_0$. Since $V^I(H^I) > V^{II}(H^{II})$, we must have $n \int_0^{H^I} m^I(h, H^I) dF > n \int_0^{H^{II}} m^{II}(h, H^{II}) dF$ for all $n \geq 1$ and $v_0 < v$. ■

Since the reserve price r is higher than the seller's reserve value v_0, it entails potential allocative inefficiencies in case the object is not sold. In view that the seller chooses a lower reserve price in the FPA than in the SPA, the former entails a lower probability of no-sale. In this sense, the FPA is allocatively more efficient than the SPA.

5 Bidders' preferences for the auction forms

We now turn to the bidders' preferences for the auction forms (cf. Matthews, 1987; Smith and Levin, 1996). In the existing literature, the seller's preference for the FPA has been established for the case in which bidders have sure private values concerning the auctioned object. In these situations, Matthews (1987) shows that a buyer need not prefer the SPA even though the seller prefers FPA. The reason is that the bidder's payment is a riskier random variable in the SPA than it is in the FPA.

Revenue (or utility) comparison between the FPA and SPA is usually conducted under the assumption that the reserve prices for the two auctions are the same (e.g., Maskin and Riley, 1984; and Matthews, 1987). This assumption is valid only in our Case 1, however, where the reserve price is ineffective and the threshold type is $\overline{H}$ under both auction forms. As soon as the seller finds it optimal to set an effective reserve price in any of the auctions, the matter becomes more involved.

We first show some comparative marginal effects on bidders' expected payments and utilities as type h changes.

Lemma 5 *Suppose $0 < H^{II} < \overline{H}$. Then (i) $m_1^I(h, H^I) < m_1^{II}(h, H^{II})$ for all $h \in [0, H^{II}]$, and (ii) $U_1^I(h, H^I) > U_1^{II}(h, H^{II})$ for all $h \in [0, H^I]$.*

Proof. Since $L'' > 0$, the following inequality holds for all $a > b$:

$$L(a) - L(b) > (a - b)\, L'(b) \tag{44}$$

(i) Assume $h \in [0, H^{II}]$. From the proof of Proposition 1), $m_1^I(h, H^I) < m_1^{II}(h, H^{II})$ is the same as

$$b_1(h, H^I)G(h) + b(h, H^I)g(h) < a(h)g(h)$$

Substituting (12) for $b_1(h, H^I)$ and recalling that $g(h) < 0$, we need to show that

$$\frac{(v - b - hL(b))}{(1 + hL'(b))} > a - b \tag{45}$$

For $a = a(h)$, however, $v - b - hL(b) = a - b + hL(a) - hL(b)$. Substituting into (45) and cancelling terms, we find that (45) reduces to (44). Hence the conclusion.

(ii) Assume first $h \in [0, H^{II}]$. The expected utilities $U^I(h, H^I)$ and $U^{II}(h, H^I)$ are given in Lemma 2 and Lemma 1, respectively. Twice differentiating with respect to h yields

$$\begin{aligned} U_{11}^I(h, H^I) &= -\frac{\partial}{\partial h}\left[G(h)L(b(h, H^I)\right] \qquad (46)\\ &= -L'(b)\frac{(v - b - hL(b))}{(1 + hL'(b))}g - L(b)g \\ U_{11}^{II}(h, H^{II}) &= -L(a(h))g(h) \qquad (47) \end{aligned}$$

We show that $U_{11}^I(h, H^I) < U_{11}^{II}(h, H^{II})$, which is equivalent to

$$\begin{aligned} L'(b)\frac{(v - b - hL(b))}{(1 + hL'(b))} - L(b) &< L(a) \\ \text{or } L'(b)\,(a - b + h(L(a) - L(b))) &< (L(a) - L(b))\left(1 + hL'(b)\right) \end{aligned}$$

for $a = a(h)$. Cancelling the term $(L(a) - L(b))hL'(b)$ on both sides reduces the above inequality to (44), thus

$$U_{11}^I(h, H^I) < U_{11}^{II}(h, H^{II})$$

Integrating between h and H^{II} yields

$$
\begin{aligned}
U_1^I(H^{II}, H^I) - U_1^I(h, H^I) &< U_1^{II}(H^{II}, H^{II}) - U_1^{II}(h, H^{II}) \\
\text{or } U_1^I(h, H^I) - U_1^{II}(h, H^{II}) &> U_1^I(H^{II}, H^I) - U_1^{II}(H^{II}, H^{II}) \\
&= G(H^{II}) \left[L(r(H^{II})) - L(b(H^{II}, H^I))\right] \\
&> 0 \quad (\text{since } b(H^{II}, H^I) < r(H^{II}))
\end{aligned}
$$

Finally, for $h \in (H^{II}, H^I]$, bidders do not bid in the SPA; hence from Proposition 1, $U_1^I(h, H^I) > U_1^{II}(h, H^{II}) = 0$. ■

In words, Lemma 5 says that the marginal expected payment with respect to h is lower, and the marginal expected utility is higher, in the FPA than in the SPA. These inequalities do not arise in the risk-neutral benchmark model because of the revenue equivalence theorem (Myerson, 1981, p.65). From Lemma 5, an immediate corollary is that if the seller chooses the same reserve price in both auctions, then all bidders have higher expected payments and lower expected utilities in the FPA than in the SPA. This is shown in the next proposition.

Proposition 5 *If the threshold type H $(\leq \overline{H})$ is the same in both FPA and SPA, then $m^I(h, H) > m^{II}(h, H)$ and $U^I(h, H) < U^{II}(h, H)$ for all $h \in (0, H)$.*

Proof. Since $m^I(H, H) = m^{II}(H, H) = r(H)$ and $U^I(H, H) = U^{II}(H, H) = 0$, using Lemma 5 and integrating both sides of the inequalities $m_1^I(y, H) < m_1^{II}(y, H)$ and $U_1^I(y, H^I) > U_1^{II}(y, H^{II})$ for y between h and H yields the results. ■

The unambiguous result in Proposition 5, once again, is due to the assumption that only downside risks matter for the bidders. Thus, the kind of indeterminacy with general concave utility functions, as shown in Matthews (1987), concerning bidder preferences over auction forms does not arise in the present context.

Now we turn to the Case 2 of seller preference in which the seller chooses reserve prices optimally. As the next two propositions show, the FPA will then be preferred by both the seller and at least some of the bidders with relatively higher degrees of risk aversion. Figure 4 illustrates the results graphically.

Proposition 6 *Suppose $H^{II} < \overline{H}$. Then either (i) $m^I(h, H^I) < m^{II}(h, H^{II})$ for all $h \in [0, H^{II}]$ or (ii) there exists a unique $H_1 \in [0, H^{II})$ such that $m^I(H_1, H^I) = m^{II}(H_1, H^{II})$, $m^I(h, H^I) < m^{II}(h, H^{II})$ for all $h \in (H_1, H^{II}]$, and $m^I(h, H) > m^{II}(h, H)$ for all $h \in [0, H_1)$ if $H_1 > 0$.*

Proof. See Figure 4. A type-H^{II} bidder has an expected payment equal to $r(H^{II})$ in the SPA, and an expected payment $b(H^{II}, H^I)G(H^{II})$ in the FPA. Since $b_2(h, H) \leq 0$, $b_2(H^I, H^I) < 0$, and $H^{II} < H^I$, we have

$$\begin{aligned} b(H^{II}, H^I) &< b(H^{II}, H^{II}) = r(H^{II}) \text{ and} \\ m^I(H^{II}, H^I) &= b(H^{II}, H^I)G(H^{II}) < r(H^{II})G(H^{II}) < r(H^{II}) = m^{II}(h, H^{II}) \end{aligned}$$

By continuity, there exists $H_1 < H^{II}$ such that $m^I(h, H^I) < m^{II}(h, H^{II})$ for all $h \in (H_1, H^{II}]$. If $H_1 = 0$, then we are done. If $H_1 > 0$, then $m^I(H_1, H^I) = m^{II}(H_1, H^{II})$. The rest is to show that H_1 is unique. This follows from Lemma 5 that $m_1^I(h, H^I) < m_1^{II}(h, H^{II})$. Integrating both sides of the inequality between h and H_1 yields $m^I(h, H) > m^{II}(h, H)$ for all $h \in [0, H_1)$. ■

In words, either all bidders, or a proportion of the more risk averse bidders (with types $h \in (H_1, H^{II}]$), expect to pay strictly less in the FPA than in the SPA. Note that Proposition 6 concerns the *interim* expected payments by the bidders. Therefore it does not counteract the prediction in Proposition 3 that the *ex ante* expected payments are higher in the FPA in general.

A similar interim result holds as with the bidders' expected utilities as shown in the next proposition. It says that as long as the reserve price is effective, then there is a cut-off point H_0 such that bidders with $h > H_0$ prefer the FPA to the SPA.

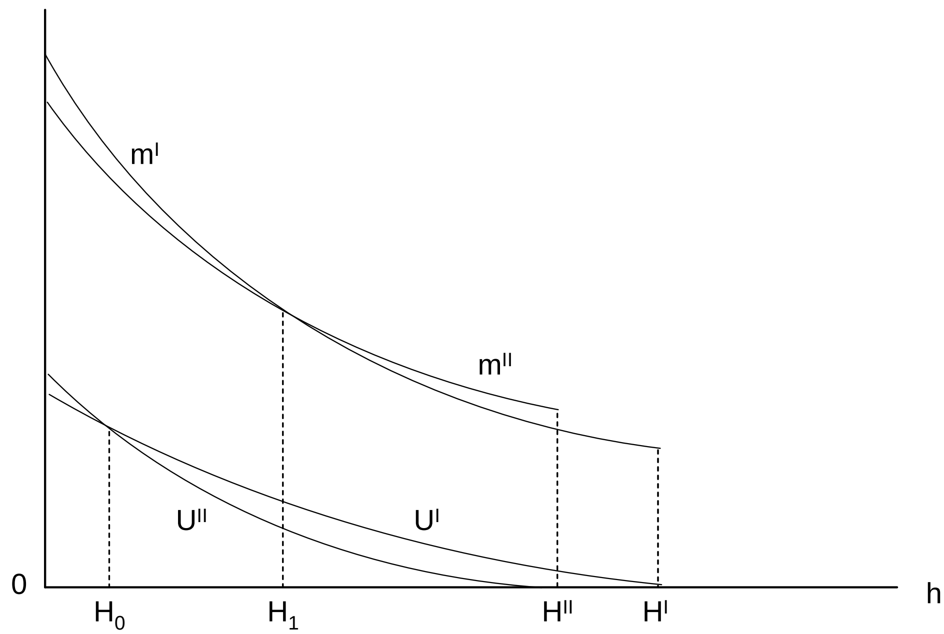

Figure 4: There is a cut-off type $H_1 \geq 0$ such that the expected payment m^I in the FPA is higher for type $h < H_1$ and lower for $h > H_1$ than m^{II} in the SPA. Likewise, provided $H_1 > 0$, there is a cut-off type $H_0 < H_1$ such that the expected utility U^I in the FPA is lower for type $h < H_1$ and higher for $h > H_1$ than U^{II} in the SPA. If $H_0 = H_1 = 0$, then all bidders prefer the FPA to the SPA.

Proposition 7 *Suppose $H^{II} < \overline{H}$. Then there exists a unique $H_0 \in [0, H_1)$, with H_1 given in Proposition 6, such that $U^I(h, H^I) > U^{II}(h, H^{II})$ for all $h \in (H_0, H^I]$ and, if $H_0 > 0$, $U^I(h, H) \le U^{II}(h, H)$ for all $h \in [0, H_0]$ with strict inequality for $h \in [0, H_0)$.*

Proof. See Figure 4. First notice that $U^I(h, H^I) > U^{II}(h, H^{II}) = 0$ for all $h \in [H^{II}, H^I]$ since these types do not bid in the SPA. By continuity, then, there exists $H_0 < H^{II}$ such that $U^I(h, H^I) > U^{II}(h, H^{II})$ for all $h \in (H_0, H^I]$. If $H_0 = 0$, then we are done. Otherwise, define H_0 by $U^I(H_0, H^I) = U^{II}(H_0, H^{II})$. From Lemma 5, integrating both sides of $U_1^I(h, H^I) > U_1^{II}(h, H^{II})$ from h to H_0 yields $U^I(h, H^I) < U^{II}(h, H^{II})$ and from H_0 to higher h yields $U^I(h, H^I) > U^{II}(h, H^{II})$. Thus H_0 is unique. What remains is to show that $H_0 < H_1$ whenever $H_1 > 0$. From Lemmas 1 and 2,

$$\begin{aligned}
U^I(H_1, H^I) &= vG(H_1) - m^I(H_1, H^I) - H_1 L(b(H_1, H^I))G(H_1) \\
U^{II}(H_1, H^{II}) &= vG(H_1) - m^{II}(H_1, H^{II}) \\
&\quad -H_1 \left(L(r(H^{II}))G(H^{II}) - \int_{H_1}^{H^{II}} L(a(y))g(y)dy \right)
\end{aligned}$$

Since $m^I(H_1, H^I) = m^{II}(H_1, H^{II})$, $U^I(H_1, H^I) > U^{II}(H_1, H^{II})$ if and only if

$$L(b(H_1, H^I))G(H_1) < L(r(H^{II}))G(H^{II}) - \int_{H_1}^{H^{II}} L(a(y))g(y)dy \tag{48}$$

From the proof of Lemma 5, e.g., (46) and (47), $\frac{\partial}{\partial h}\left[G(h)L(b(h, H^I)\right] > L(a(h))g(h)$. Integrating both sides of this inequality between H_1 and H^{II} verifies that the inequality in (48) does hold true. It follows then, from the preceding analysis, that $H_0 < H_1$. ■

According to Propositions 6 and 7, the type set $[0, \overline{H}]$ can be partitioned into a number of intervals (assuming $H_0 > 0$), as shown in Figure 4, in which the relative magnitudes of the expected payments, m^I vs. m^{II}, and of the bidders' expected utilities, U^I vs. U^{II}, are ranked unambiguously.

The next example shows a case where $H_0 = H_1 = 0$, that is, all bidders prefer the FPA to the SPA. In these situations, the FPA dominates the SPA in terms of Pareto efficiency as well as allocative efficiency.

Example 3 *Continue to assume $n = 2$ and $F(h) = h$ on $[0,1]$, so that $G(h) = 1 - h$, $g(h) = -1$; and $\varphi(x) = x$, as in the previous examples. But we change the payoff distribution to $Q(\widetilde{v}) = \widetilde{v}/100$ on $[0,100]$ (so that $v = 50$) in order to enlarge the numbers and make the differences more discernible. The same previous examples generate the same qualitative results but the differences are to small. Working out the numbers, we have*

$$\begin{aligned} L(c) &= \int_0^c \widetilde{v}/100 d\widetilde{v} = \frac{1}{800}(c+100)^2 \\ a(h) &= \frac{1}{h}\left(100\sqrt{h+1} - 100\right) \\ b' &= -\frac{100}{(hb+100)(h-1)}\left(\frac{1}{200}hb^2 + b - 50\right) \end{aligned}$$

The following table summarizes the numerical results.

	$H_0 = H_1 = 0$	$H^{II} = 0.5625$	$H^I = 0.5645$
m^I	45.883	19.438	19.348
m^{II}	45.887	19.444	0
U^I	4.117	0.0086	0
U^{II}	4.113	0	
$v_0 = 0.4,\ U_0^I = 44.115,\ U_0^{II} = 44.086$			

We see in the above table that at $H = 0$, the risk-neutral bidder's expected payment is lower, and thus the expected utility higher, in the FPA than in the SPA. This implies from Propositions 6 and 7 that $H_0 = H_1 = 0$, hence all bidders prefer the FPA to the SPA. In the bottom row of the table, the values also confirm that the seller prefers the FPA to the SPA. The difference, however, is surprisingly small in this example.

6 Conclusion

This paper is a first attempt to incorporate both payoff uncertainty and bidders' private degrees of risk aversion into standard auction models. In line with the empirical evidence that risk-averse behavior can be largely explained by aversion to losses, we have focused on the case in which the bidders have the mean-downside risk preferences.

An important conclusion is that the reserve price that maximizes the seller's expected revenue is strictly lower in the FPA than in the SPA—provided it is optimal to preclude some more risk averse bidders from bidding. Several testable predictions with clear policy implications are then derived, showing superiority of the FPA over the SPA in the present setup. For the seller, the FPA dominates the SPA in generating higher expected revenue–with or without an effective reserve price. For the bidders, given that the reserve price is optimally chosen and effective, at least part of the bidders–namely the more risk-averse types–also derive higher expected utilities in the FPA than in the SPA. It is possible that all bidders prefer the FPA as well, in which case the FPA Pareto dominates the SPA. Finally, from the society's point of view, since the FPA entails a lower probability of no-sale, it dominates the SPA in terms of allocative efficiency.

Pragmatically, however, the SPA has a salient advantage that bidders need less information and hence are less prone to errors (e.g., Vickrey, 1961). Since in private values models the equilibrium is characterized by dominant strategies in the SPA (i.e., bidding up to one's reservation value), a bidder only needs to know his own type and need not estimate the other bidders' types.[13] Combining Vickrey's insight with the new observations from our analysis, we conclude that the SPA is likely to be more appropriate in circumstances where the auctioned object entails little or no payoff uncertainty to the winner, or where assessing the other bidders' types in addition to one's own is difficult. On the other hand, the FPA may be more appropriate where the bidders have a similar estimation of the expected value of the object, or where large payoff uncertainty remains after winning.

The above conclusions seem to be consistent with general practices. For example, in the United States, federal oil, gas, or mineral rights (conceivably, with high payoff uncertainty) have been sold exclusively through first-price sealed-bid auctions, whereas timber rights, fine art, or non-durable consumer goods (conceivably, with low payoff uncertainty)

[13] In our model, though, if the optimal reserve price is to be chosen in a SPA, the seller still needs a good assessment of both the downside-risk function L and the ratio (F/f) of the distribution of bidders' types.

have traditionally been sold through oral, or second-price, auctions.

A promising line of future research is to extend the present model to the more general case in which the bidders have differing estimates of the common payoff distribution, as well as differing degrees of risk aversion. Since a lower degree of risk aversion contributes to a higher bid (given the same payoff estimate), the winning bidder in this general case need not have the highest payoff estimate. An unverified conjecture is that incorporating heterogeneous risk aversion may alleviate some "winner's curse" as observed in empirical and experimental studies based on traditional common value models (see Crawford and Iriberri, 2007; and the references therein).

7 References

Bawa, Vijay S. 1975. "Optimal Rules for Ordering Uncertain prospects." *Journal of Financial Economics*, 2: 95-121.

Bawa, Vijay S.1976. "Admissible Portfolios For All Individuals." *The Journal of Finance*, 31: 1169-1183.

Bawa, Vijay S. and Eric B. Lindenberg. 1977. "Capital Market Equilibrium in a Mean-lower Partial Moment Framework." *Journal of Financial Economics*, 5(2): 189-200.

Benartzi, Shlomo, and Richard H. Thaler. 1995. "Myopic Loss Aversion and the Equity Premium Puzzle." *Quarterly Journal of Economics*, 110: 73–92.

Cox, James C., Vernon L. Smith, and James M. Walker. 1982. "Auction Market Theory of Heterogeneous Bidders." *Economics Letters*, 9: 319–325.

Cox, James C., Vernon L. Smith, and James M. Walker. 1988. "Theory and Individual Behavior of First-price Auctions." *Journal of Risk and Uncertainty*, 1(1): 61–99.

Crawford, Vincent P., and Nagore Iriberri. 2007. "Level-k Auctions: Can A Nonequilibrium Model of Strategic Thinking Explain The Winner's Curse and Overbidding in

Private-Value Auctions?" *Econometrica*, 75(6): 1721-1770.

Domar, Evsey D., and Richard A. Musgrave. 1944. "Proportional Income Taxation and Risk-Taking." *The Quarterly Journal of Economics*, 58(3): 388-422.

Eso, Peter. 2005. "An Optimal Auction with Correlated Values and Risk Aversion." *Journal of Economic Theory*, 125(1): 78-89.

Eso, Peter, and Gabor Futo. 1999. "Auction Design with a Risk Averse Seller." *Economics Letters*, 65(1): 71–74

Fishburn, Peter C. 1977. "Mean-Risk Analysis with Risk Associated with Below-Target Returns." *The American Economic Review*, 67(2): 116-126.

Goeree, Jacob K., Charles A. Holt, and Thomas Palfrey. 2002. "Quantal Response Equilibrium and Overbidding in Private-Value Auctions." *Journal of Economic Theory*, 104(1): 247-272.

Goeree, Jacob K., and Theo Offerman. 2002. "Efficiency in Auctions with Private and Common Values: An Experimental Study." *American Economic Review*, 92(3): 625-643.

Goeree, Jacob K., and Theo Offerman. 2003a. "Competition Bidding in Auctions With Private and Common Values." *Economic Journal*, 113: 598-614.

Goeree, Jacob K., and Theo Offerman. 2003b. "Winner's Curse without Overbidding." *European Economic Review*, 47(4): 625-644.

Hadar, J., and W. Russell. 1969. "Rules for Ordering Uncertain Prospects," *American Economic Review* 59, 25-34.

Harris, Milton, and Artur Raviv.1981. "Allocation Mechanisms and the Design of Auctions." *Econometrica*, 49(6): 1477-1499.

Harrison, Glenn W. 1989. "Theory and misbehavior of first-price auctions." *The American Economic Review*, 79(4): 749–762.

Harrison, Glenn W. 1992. "Theory and Misbehavior of First-Price Auctions: Reply." *The American Economic Review*, 82(5): 1426-1443.

Holt, Charles A. 1980. "Competitive Bidding for Contracts under Alternative Auction Procedures." *The Journal of Political Economy*, 88(3): 433-445.

Holt, Charles A., and Roger Sherman. 2000. "Risk Aversion and the Winner's Curse." Working paper.

Kahneman, Daniel, and Amos Tversky. 1979. "Prospect Theory: An Analysis of Decision under Risk." *Econometrica*, 47: 263-291.

Köbberling, Veronika, and Peter P.Wakker. 2005. "An Index of Loss Aversion." *Journal of Economic Theory*, 122: 119 – 131.

Krishna, Vijay. 2002. *Auction Theory.* San Diego: Academic Press.

Lind, Barry and Charles R. Plott. 1991. "The Winner's Curse: Experiments with Buyers and with Sellers." *The American Economic Review*, 81(1): 335–346.

McAfee, R. Preston, and John McMillan. 1987. "Auctions and Bidding." *Journal of Economic Literature*, 25(2): 699–738.

Markowitz, H. M. 1952. "Portfolio Selection." *Journal of Finance* 7, 77-91.

Maskin, Eric, and John Riley. 1984. "Optimal Auctions with Risk Averse Buyers." *Econometrica*, 52(6): 1473-1518.

Matthews, Steven.1987. "Comparing Auctions for Risk Averse Buyers: A Buyer's Point of View." *Econometrica*, 55(3): 633-646.

Matthews, Steven.1983. "Selling to Risk Averse Buyers with Unobservable Tastes." *Journal of Economic Theory*, 30: 370-400.

Milgrom, Paul, and Robert J. Weber. 1982. "A Theory of Auctions and Competitive Bidding." *Econometrica*, 50(5): 1089-1122.

Milgrom, Paul. R., and I. Segal 2002. "Envelope Theorems for Arbitrary Choice Sets." *Econometrica*, 70: 583-601.

Milgrom, Paul. 2004. *Putting Auction Theory to Work*. Cambridge: Cambridge University Press.

Moore, John. 1984. "Global Incentive Constraints in Auction Design." *Econometrica*, 52(6): 1523-1535.

Myerson, Roger B. 1981. "Optimal Auction Design." *Mathematics of Operations Research*, 6(1): 58-73.

Pesendorfer, Wolfgang, and Jeroen Swinkels. 1997. "The Loser's Curse and Information Aggregation in Common Value Auctions." *Econometrica*, LXV 1, 1247-1282.

Prelec, D. 1998. "The probability weighting function." *Econometrica*, 66(3), 497–527.

Rizzo, John A., and Richard J. Zeckhauser. 2004. "Reference Incomes, Loss Aversion, and Physician Behavior", *Review of Economics and Statistics*, 85: 909–922.

Riley, John G., and William F. Samuelson. 1981. "Optimal Auctions." T*he American Economic Review*, 71(3): 381-392.

Rothchild, M. and J. Stiglitz. 1970. "Increasing Risk: I. A Definition," *Journal of Economic Theory* 2, 225-243. ('Addendum', 1972, *Journal of Economic Theory* 5, p. 306).

Samuelson, William F., and Richard J. Zeckhauser. 1988. "Status Quo Bias in Decision Making." *Journal of Risk and Uncertainty*, 1: 7-59.

Schmidt, Ulrich, and Stefan Traub. 2002. An Experimental Test of Loss Aversion, *J. Risk Uncertainty,* 25: 233–249.

Shalev, Jonathan. 2000. "Loss Aversion Equilibrium." *International Journal of Game Theory*, 29: 269–287.

Smith, James L., and Dan Levin.1996. "Ranking Auctions with Risk Averse Bidders." *Journal of Economic Theory*, 68: 549-561.

Tversky, Amos, and Daniel Kahneman. 1992. "Advances in Prospect Theory: Cumulative Representation of Uncertainty," *Journal of Risk and Uncertainty*, 5: 297-323.

Vickrey, William.1961."Counterspeculation, Auctions, and Competitive Sealed Tenders." *The Journal of Finance*, 16(1): 8-37.

Waehrer, Keith, Ronald M. Harstad and Michael H. Rothkopf. 1998. "Auction Form Preferences of Risk-Averse Bid Takers." The RAND Journal of Economics, 29(1): 179-192.

Wilson, Robert. 1977. "A Bidding Model of Perfect Competition." *The Review of Economic Studies*, 44(3): 511-518.