Large Swings in Currencies driven by Fundamentals

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Abstract

Exchange rate returns are fat-tailed distributed. We provide evidence that the apparent non-normality derives from the behavior of macroeconomic fundamentals. Economic and probabilistic arguments are offered for such a relationship. Empirical support is given by testing against normality and through investigating the tail shapes of the fundamentals' distributions. The currently available data sets on floating exchange rates permit a clearer picture than the relatively short spans with macroeconomic data available previously.

Keywords: Exchange rates, fundamentals, fat-tailed distributions.
JEL **Classification Codes:** E44, F31.

1 Introduction

International financial markets have experienced many crisis episodes. The EMS years of managed European exchange rates were littered with realignments. More recently Asia and Latin America have experienced severe currency crises, often spilling over from one country to the other. These large movements in exchange rates have been well researched and led to the hypothesis that exchange rate distributions exhibit non-normal fat tails, see e.g., Westerfield (1977), Boothe and Glassman (1987), Koedijk, Schafgans and de Vries (1990), Koedijk and Kool (1994), and in this journal Koedijk, Stork and de Vries (1992), and Susmel (2001).[1]

Since extreme movements occur infrequently and as exchange rates only started to float in 1973, this empirical research has almost exclusively relied on high frequency data. Now that the fat tail feature is widely recognized, it is of interest to uncover the driving forces behind this phenomenon. Standard exchange rate models such as the monetary-approach based model would suggest that this data feature must stem from the nature of the fundamentals' distributions. In this paper we provide empirical evidence for such a linkage and give further theoretical support for this relationship.

The low frequency nature of fundamental variables such as the money stock and real income has stalled such an empirical investigation. The fat tail feature shows

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[1] Moreover, at higher frequencies exchange rate returns exhibit volatility clustering, which adds to the fat tail nature of the returns.

primarily in the tail areas of the distribution. The fat tail property, therefore, requires relatively large data sets for reliable detection. The floating period now covers about 350 months (310 months for the euro participant countries) for which we collected data on standard fundamental variables. This permits a first investigation into the relationship between the distribution of the exchange rate and its macroeconomic fundamentals regarding the fat tail nature of these distributions. With some caution, since the series is still relatively short, the empirical results indicate that macroeconomic fundamentals are indeed heavy-tailed distributed as well.

The monetary model postulates an affine relation between the exchange rate returns and the relative changes in the fundamentals. It can be shown on the basis of Feller's (1971, VIII.8) convolution theorem for heavy-tailed random variables that the fat tail feature is preserved under addition. Therefore if the fundamentals' distributions exhibit fat tails and the linear monetary model applies, the fat tail feature is transferred to the exchange rate. Thus within the confines of the monetary model one may want to investigate the tail properties of the fundamentals. If the fundamentals do not exhibit fat tails, the entire burden falls on the residual, which would not help us much further in understanding the fat tail nature of the exchange rates. Whichever way the answer falls, it is of interest to know where the fat tail feature originates.

The next section provides evidence regarding the non-normality of the distributions of the exchange rate returns and the fundamentals' growth rates. Section 3 provides a summary on extreme value theory (EVT) and gives the argument for the transmission of fat tails from the fundamentals to the exchange rate returns. We combine theoretical economic and probabilistic arguments for the fat tail phenomenon. Section 4 and 5 discuss the estimation methods and empirical results, respectively. Conclusions are presented in Section 6.

2 Preliminary Evidence

This section provides some evidence regarding the non-normality of exchange rate returns and the changes in economic fundamentals. The variables are monthly observations on exchange rates, monetary aggregate (M2), industrial production, interest rates and the consumer price index from eight OECD countries, relative to US levels.[2] For all the series, the first differences are sufficient to induce stationarity. The fundamental series used throughout the paper are, therefore, the logarithmic rates of change of the domestic variables relative to the foreign (US) variables during the period of July 1973 to December 1998 for countries adopting the euro and the period of July 1973 to December 2002 for the other countries in the sample.

We start by providing some summary statistics for each variable in Table 5 to Table 8 in the appendix. In addition the Jarque-Bera (J-B) test is used to test whether the rate of change of the exchange rate and the rates of growth of the macroeconomic fundamentals are normally distributed. As most of the series exhibit volatility clusters, we applied Bollerslev's (1987) univariate GARCH model with Student-t distributed innovations to each series. The tables report the inverse of the degrees of freedom of the standardized residual series, i.e., after filtering out the ARMA-GARCH components. An asterisk indicates significance at the 10% level while two and three asterisks show significance at the 5% and 1% levels, respectively.[3]

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[2] The data set is described in Appendix A.

[3] The degrees of freedom of the standardized series is used to investigate the tail thickness. This gives a rough idea about the tail fatness. Below we also directly investigate the tail thickness and treat left and right tails separately.

In the tables, e, m, y, i, r, p represent the monthly rates of change of the European quoted exchange rate, money supply, real income, long-term interest rate, short-term interest rate and price level relative to the corresponding US variable, respectively.[4] The fundamental variables are selected on basis of the flexible-price and the sticky-price monetary-approach model variables. In the logarithmic form, the quasi-reduced-form specification of the models can be written as

$$e = \varphi_0 + \varphi_1 m + \varphi_2 y + \varphi_3 i + \varphi_4 p^e + u. \tag{1}$$

Monetary neutrality holds if $\varphi_1 = 1$, $\varphi_2 < 0$ and the sign of φ_3 depends on the version of the model, see Frankel (1979).

According to the summary statistics in Table 5 to Table 8, the average rates of change of most variables are smaller than 0.5% in absolute terms, except for the short-term interest rates of Canada, Germany and the UK. Standard deviations of the rates of change of e, m, y, i and p are less than 5% (except the French money supply). Standard deviations of inflation rates, in particular, are very low, when compared to other variables. On the other hand, standard deviations of changes in short-term interest rates range from 50% in Canada up to 150% in the UK. The spreads between the maximum and the minimum short-term interest rates are large, while these are small in the case of price levels.

At the 5% significance level, the J-B test rejects the null hypothesis of normality for all variables, except for the Austrian money supply and the French real income. These variables have *p-values* of 0.0657 and 0.0567, respectively. For the money supply of Japan and the UK and the real incomes of France and Japan, the inverse degrees of freedom are not significantly different from zero, i.e., the innovations to the ARMA-GARCH processes appear to have a light-tailed distribution (even though the resulting stationary distribution for the output variable of the GARCH process will be heavy-tailed distributed). For the Austrian money supply and the French long-term interest rate one can reject the null hypothesis of thin tails at the 10% significance level. The rest of the variables have significantly fat-tailed distributed innovations. The picture which emerges is that not only exchange rate returns are heavy-tailed distributed, but also that the fundamental variables exhibit heavy tails. This provides a first confirmation of the linkage suggested by (1).

3   Theory

We first provide a short review of the probabilistic properties of fat-tailed distributed random variables and their scaling properties. Then we derive the implications for the monetary model.

3.1   Regular Variation and Tail Additivity

We adopt the following notion of heavy tails. A distribution function $F(x)$ is said to exhibit heavy tails if its tails vary regularly at infinity. The upper tail varies regularly at infinity with tail index α if [5]

$$\lim_{t \to \infty} \frac{1 - F(tx)}{1 - F(t)} = x^{-\alpha}, \ x > 0 \text{ and } \alpha > 0. \tag{2}$$

Regular variation implies that the distribution changes at a power rate. This contrasts with the normal distribution which has an exponential rate. The number of

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[4] We examine both long-run and short-run interest rates and use the price level as a substitute for the expected price level.

[5] For the lower tail, $\lim_{t \to \infty} F(-tx)/F(-t) = x^{-\alpha}$, $x > 0$ and $\alpha > 0$.

bounded moments is finite and equals the integer value of α.[6] One checks that the Student-t distribution satisfies (2).

Random variables with regularly varying distributions satisfy an important additivity property. Suppose a distribution has heavy tails, so that

$$P\{X > x\} = 1 - F(x) \sim Ax^{-\alpha}, \ as \ x \to \infty. \tag{3}$$

According to Feller's Convolution Theorem (1971, VIII.8), if X_1 and X_2 are iid with c.d.f. $F(x)$ which has regularly varying tails as in (3), then

$$P\{X_1 + X_2 > s\} \sim 2As^{-\alpha}, \ as \ s \to \infty. \tag{4}$$

If X and Y are iid and if X has a tail index of α and Y has a lighter tails (e.g., has a hyperbolic tail with a higher power than α or even has an exponential type tail), then

$$P\{X + Y > s\} \sim As^{-\alpha}. \tag{5}$$

In this case the convolution is dominated by the heavier tail.

This is a very powerful result. In the context of the monetary model (1), it predicts that if the distributions of the changes in the fundamentals variables exhibit heavy tails, the exchange rate return distribution has a heavy tail as well. Thus the tail shape of the exchange rate returns e is governed by the tail shape of the fundamentals' changes. In particular, (5) constrains the tail shapes of the fundamentals and the exchange rate in the following way

$$\alpha_e = \min(\alpha_m, \alpha_y, \alpha_i, \alpha_p, \alpha_u), \tag{6}$$

where α_x represents a tail index of a random variable x. This suggests a possible test of the validity of the monetary model.

The convolution result (4) assumes the random variables are iid. This may appear constraining. However, as long as the fundamental rates of change are linearly dependent, Feller's convolution theorem still holds, as the tail shape, i.e., α, is unaffected by linear dependence, only the scaling constant changes. Thus for example if the fundamental variables are all linearly related to the CAPM market factor, where the market factor and the idiosyncratic factors are all fat-tailed distributed, the exchange rate returns are also fat-tailed distributed. Hence, if the exchange rate model in equation (1) holds, then the exchange rate's extreme events should be governed by extreme events in the fundamentals or the error term. The fundamentals with the heaviest tail, should have a tail index similar as the tail index of exchange rates.

3.2 Tail Events and Macroeconomic Fundamentals

Even if we can relate the fat tail nature of the exchange rate distribution to the fundamentals' distribution in (1), one may still wonder how the latter variables end up having distributions with heavy tails. In this subsection, we provide an example of how a macroeconomic variable like the money stock can exhibit the fat tail feature. The idea is not to present a fully fledged theory, as this would be outside the scope of the paper, but to present a coherent argument for one of the macroeconomic variables. To this end consider the following stylized monetary macroeconomic model. The aggregate supply curve reads

$$Y_t = A_t(\Pi_t - E_{t-1}[\Pi_t]) + \varepsilon_t, \tag{7}$$

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[6]For instance, Pareto distributions satisfy the Power law and have a number of bounded moments equal to an integer of α. The Student-t distribution has moments equal to its degrees of freedom. The normal distribution has all moments bounded.

where Y_t is output, Π_t is inflation and $E_{t-1}[\Pi_t]$ is time t-1 expected inflation for time t, and ε_t is a noise term. In the short run, deviations from the long-run output are possible due to expectational errors. The elasticity of output with respect to inflation expectations' errors is A_t. Thus (7) is in a crude way the Lucas type supply curve. Aggregate demand depends on real interest rates, i.e., the nominal interest rate minus expected inflation $I_t - E_t[\Pi_{t+1}]$:

$$Y_t = -b(I_t - E_t[\Pi_{t+1}]) + \eta_t. \tag{8}$$

The reduced-form money market equation is based on the quantity equation

$$M_t = P_{t-1} + \Pi_t + Y_t - gI_t + \nu_t, \tag{9}$$

where M_t stands for the quantity of money. The three disturbances $(\varepsilon_t, \eta_t, \nu_t)$ are assumed to be mean zero iid noise.

Frequently model estimates and new data lead to parameter revisions, see Sack (2000). We capture the model uncertainty via the Brainard (1967) effect and assume the coefficient for the short-run Phillips effect A_t is an iid random variable. Suppose A_t has a beta distribution

$$P\{A \leq x\} = x^\alpha, \ \alpha > 2. \tag{10}$$

The support of this distribution is $[0, 1]$. The fact that zero is in the support reflects the possibility that the short-run supply curve can be vertical, i.e., coincide with the long run curve.

Suppose the goal of monetary policy is to stabilize the level of inflation around a target π^*. This reflects the European Central Bank's single price stability objective, since the ECB does not have real income stabilization or employment as its prime objectives. Specifically, assume that the objective resembles the ECB's main task reads

$$\min_{I_t} E[(\Pi_t - \pi^*)^2]. \tag{11}$$

Note that this implies setting $E_t[\Pi_{t+1}] = E_{t-1}[\Pi_t] = \pi^*$. Substituting out Y_t from the first two equations (7) and (8) gives

$$\Pi_t = \frac{(b + A_t)\pi^* - bI_t + \eta_t - \varepsilon_t}{A_t}.$$

As $\alpha > 2$ in (10) $E[1/A_t]$ is bounded (see (12) below), thus we can take expectations

$$E_t[\Pi_t] = b(\pi^* - I_t)E[1/A_t] + \pi^*.$$

Equate $E_t[\Pi_t] = \pi^*$. Hence, given its objective function (11), it is optimal for the central bank to set

$$I_t = \pi^*.$$

This implies for the money equation (9) that

$$M_t = P_{t-1} + \Pi_t + Y_t - g\pi^* + \nu_t.$$

Use the first two equations (7) and (8) to substitute out Y_t and Π_t, to get

$$M_t = P_{t-1} + (1 - g)\pi^* + (1 + \frac{1}{A_t})\eta_t - \frac{\varepsilon_t}{A_t} + \nu_t.$$

Solving for the two other endogenous variables we find

$$\Pi_t = \pi^* + \frac{\eta_t - \varepsilon_t}{A_t}$$

and

$$Y_t = \eta_t.$$

Now Π_t and M_t are heavy-tailed distributed since $(\eta_t - \varepsilon_t)/A_t$ is heavy-tailed distributed by the fact that the random Phillips effect coefficient appears in the denominator. This follows from the beta distribution (10) assumption regarding A_t. The distribution of the inverse is

$$P\{\frac{1}{A} \le x\} = 1 - P\{A \le \frac{1}{x}\} = 1 - \frac{1}{x^\alpha}, \tag{12}$$

with support $x\epsilon[1, \infty)$. Thus the inverse is a heavy-tailed Pareto distribution (conditional on the distribution of η_t and ε_t) and has moments up to α. Therefore the unconditional distributions of Π_t and M_t are also heavy-tailed, due to the random Phillips curve coefficient.

4 Estimation

In this section, we explain the tail estimation procedures. We provide a test statistic which permits a comparison of the tail features of the variables on both sides of the exchange rate equation (1).

4.1 Tail Estimators

To estimate the tail index α from (2), there are several estimators, e.g., the log-moment based Hill (1975) estimator, the Dekkers-Einmahl-de Haan (1989) (DEdH) estimator and the quasi maximum likelihood estimator, see Smith (1987). Based on the evidence from the normality tests presented in Table 5 to Table 8, it is assumed that the data are heavy tailed. Since we start from the presumption that the underlying distribution is heavy tailed, the Hill estimator is more efficient than the alternative estimators.

Define the ascending order statistics from the sample of size n, as $X_{(1)} \le X_{(2)} \le \dots \le X_{(n)}$. The nonparametric Hill (1975) estimator reads

$$\widehat{\gamma}(m) = \frac{1}{m} \sum_{i=1}^{m} [\log X_{(n+1-i)}/X_{(n-m)}], \tag{13}$$

where $\widehat{\gamma}$ denotes the inverse tail index $\widehat{1/\alpha}$ and $X_{(n-m)}$ is a threshold. Thus, there are m observations above the threshold. For $m(n) \to \infty$, while $m(n)/n \to 0$, $\widehat{1/\alpha}$ is asymptotically normally distributed with zero mean and variance $1/\alpha^2$.

The use of extreme value theory for economics and finance resides in the estimation of extreme probability-quantile $(\widehat{p}_q, q)$ combinations, where $\widehat{p}_q = 1 - F(q)$ and q is at the border or outside the sample. For example, the banking industry uses this to provide stress test estimates. De Haan, Jansen, Koedijk and de Vries (1994) developed the following estimator:

$$\widehat{p}_q = \frac{m}{n}(\frac{X_{(n-m)}}{q})^{\widehat{\alpha}}. \tag{14}$$

For the reverse problem of estimating the quantile at a certain low probability level, one simply inverts (14). One shows that the statistical properties of $\widehat{p}_q$ are driven by the statistical properties of the tail index estimator, since this statistic appears in (14) as a power.

4.1.1 Small-sample Bias Correction

An essential step in the computation of any tail index estimator is the selection of the number of upper order statistics m, in other words the selection of the threshold, $X_{(n-m)}$ in (13). The statistical properties of the Hill estimator crucially depend on the selection of the threshold. Too few observations enlarge the variance of the estimator, while too many observations reduce the variance at the expense of biasedness (by including observations from the central range, the first order approximation (3) becomes marred by second order terms). There are several automated procedures available to deal with this trade-off problem, but these bootstrap based procedures only work in large samples such as are available in the high frequency domain.[7]

Our study relies on rather coarse macroeconomic data, there are only around 350 monthly observations from the period 1973-2002, and we are mainly interested in comparing tail index values. Therefore, we prefer an estimator which is feasible in moderate samples and which kees the bias small relative to the variance (this favors the null hypothesis of tail equality when true). For this reason wewill use a modified version of the Hill estimator proposed by Huisman, Koedijk, Kool, and Palm (2001). The procedure is feasible in moderately sized samples, and tries to keep the bias to a minimum. This estimator exploits the fact that the asymptotic bias in many cases eventually changes monotonically with respect to m. Suppose $\widehat{\gamma}(m)$ from (13) is evaluated for a number of different m-values. A linear projection of $\widehat{\gamma}(m)$ against m then yields

$$\widehat{\gamma}(m) = \rho_0 + \rho_1 m + \varepsilon(m), \ \ m = 1, ..., \ \kappa. \tag{15}$$

Since theoretically the bias tends to zero as m/n tends to zero, the idea is to select ρ_0 as the estimate for γ. Note that the procedure still requires the selection of the nuisance parameter κ, which is chosen such that $\widehat{\gamma}(m)$ is approximately linear in m.

Huisman et al. (2001) propose a weighted-least squared (WLS) technique with a $(\kappa \times \kappa)$ weighting matrix, that has $\left\{\sqrt{1}, ..., \sqrt{\kappa}\right\}$ as diagonal elements and zero elsewhere, to correct for heteroskedasticity. Therefore, a modified version of the Hill estimator $\widehat{\gamma}^m$ is a weighted average of a set of conventional Hill estimates $\widehat{\gamma}$, i.e.,

$$\widehat{\gamma}^m(\kappa) = \sum_{m=1}^{\kappa} \varpi(m)\widehat{\gamma}(m), \tag{16}$$

where the weights $\varpi(m)$ are a function of m. Due to the overlapping data problem, one needs to take into account the autocorrelation in the error term $\varepsilon(m)$. In combination with the weighted least squares regressions, we apply the Newey-West statistics to correct for possible serial correlation.

4.2 Statistical Tests

To investigate amount of tail fatness and the equality of different tails, we employ two statistical procedures. The first procedure is used to determine the amount of tail fatness. The second is a test used to investigate the association between extreme events of the exchange rate returns and changes in the economic fundamentals, which is the main topic of the paper.

The J-B tests and the degrees of freedom in Table 5 to Table 8 already provided evidence of the non-normality due to tail fatness. The tail index estimates provide a measure of the amount of (unconditional) tail fatness for the different variables. Recall that the tail index signifies the number of bounded moments. Thus the lower is α, the thicker is the tail. Since the tail estimator is asymptotically normally

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[7]It can be shown that the Hill estimator's rate of convergence is best under the mean squared error criterion, see Hall and Welsh (1984).

distributed, we report standard error bands of the point estimates. In case the upper boundary of the bands is small, the number of bounded moments is likely to be small as well. To estimate the asymptotic standard error $\sigma\left(\widehat{\gamma}_x\right)$ we use the bootstrap technique with replacement sampling. The number of bootstrap replications is equal to 1000 and the standard error $\sigma\left(\widehat{\gamma}_x\right)$ is derived from the empirical distribution of the 1000 gamma estimates.

The second statistical procedure tests whether or not the tail index of the exchange rate corresponds to the (minimum) tail index of the fundamental variables from the monetary model. If the heaviest fundamental tail is similar to the tail of exchange rate returns, the monetary exchange rate model captures and explains at least the largest swings in the exchange rates on basis of the largest irregular movements of the fundamentals; recall equation (6). We first test $H_0 : \gamma_e \leq \gamma_f$ against $H_1 : \gamma_e > \gamma_f$, where γ_e is the inverse tail index of the exchange rate returns, while γ_f is the inverse of the lowest tail index of the fundamental variables. If the null hypothesis of this one-sided test is rejected, the exchange rate return exhibits a thicker tail than the fundamentals, and the fundamental variable is not a potential contributor to the extreme movement of the exchange rate. In such a case, the burden is placed on the error term, but which does not yield an economic explanation for the observed tail fatness of the exchange rate returns.

The variables for which we cannot reject the initial null hypothesis are subjected to a second one-sided test. This is done to identify possible fundamentals with a fatter tail than the exchange rate return. If fundamentals have a fatter tail than the exchange rate, one would reject the null hypothesis of $H_0 : \gamma_e \geq \gamma_f$ for the alternative hypothesis of $H_1 : \gamma_e < \gamma_f$. If both null hypotheses cannot be rejected, we may conclude that the tail behavior of the exchange rate returns accords well with the tail behavior of the fundamentals. This would validate the monetary model, at least for the larger movements in the variables. To test the hypotheses on tail equivalences, we use the following t-test

$$T_{[e,f]} = \frac{\widehat{\gamma}_e - \widehat{\gamma}_f}{\sigma_{\widehat{\gamma}_e - \widehat{\gamma}_f}},$$

where the asymptotic standard error is estimated by bootstrapping. The same bootstrap procedure is employed as was described above.

In the tables below results are presented on a per country basis, with separate rows for the depreciation and the appreciation (of the domestic currency) sides. Factors that raise the demand for the domestic currency relative to the demand for US dollars cause an appreciation of the domestic currency, and vice versa. Therefore, an appreciation of the domestic currency, is compared to decreases in log money supply and log prices, and increases in log real income and interest rates. For the flexible-price model, the set of fundamentals contains money supply, real income and interest rate. The sluggish-price model consists of these three variables and price level.

5   Empirical Results

This section is devoted to the empirical investigation of the extreme behavior of exchange rates and macroeconomic variables, and their extreme linkages. The first subsection presents the estimates of inverse tail indices of the exchange rate returns and the fundamentals' growth rates. The second subsection investigates the validity of the monetary-approach exchange rate models with respect to tail events, by testing for tail equivalence.

<u>Table 1: The modified Hill estimates</u>

$$\widehat{\gamma}^m(\kappa) = \sum_{m=1}^{\kappa} \varpi(m)\widehat{\gamma}(m)$$

		e	m	y	i	r	p
Austria	γ_{dep}	0.1946	0.0812	0.2635	0.2722	0.3701	0.1241
	Std. error	0.0728	0.0409	0.0713	0.0597	0.0821	0.0886
	γ_{app}	0.1424	0.2120	0.0714	0.3242	0.3613	0.1283
	Std. error	0.0587	0.0543	0.0501	0.0943	0.1189	0.0523
Canada	γ_{dep}	0.2197	0.2535	0.1341	0.3104	0.3314	0.3228
	Std. error	0.0728	0.0712	0.0749	0.0673	0.0742	0.0760
	γ_{app}	0.0943	0.2767	0.3389	0.2420	0.3840	0.1817
	Std. error	0.0625	0.0806	0.0991	0.0691	0.0993	0.0730
France	γ_{dep}	0.1543	0.3355	0.1512	0.2494	0.4313	0.1414
	Std. error	0.0776	0.1121	0.0727	0.0582	0.0860	0.0803
	γ_{app}	0.1637	0.3737	0.1448	0.2565	0.3619	0.0828
	Std. error	0.0673	0.1816	0.0680	0.0717	0.1296	0.0800
Germany	γ_{dep}	0.2041	0.0813	0.2777	0.2130	0.6998	0.3270
	Std. error	0.0756	0.0727	0.0818	0.0707	0.1239	0.1414
	γ_{app}	0.1820	0.2161	0.2057	0.1216	0.5476	0.2406
	Std. error	0.0689	0.0968	0.1039	0.1004	0.1280	0.0677

This table gives the estimated inverse tail indices of monthly exchange rate returns and monthly changes in the economic fundamentals inducing depreciation (γ_{dep}) and appreciation (γ_{app}) of the domestic currency. The asymptotic standard error is presented below each estimate.

5.1 Tails of Economic Variables

Table 1 and 2 provide the estimates of the inverse tail index, $\widehat{\gamma} = 1/\widehat{\alpha}$, for the exchange rate returns and the rates of change of the domestic fundamentals relative to the rates of change of the US fundamentals. For each country, the top row, denoted γ_{dep}, shows the tail estimate for the right tail of the exchange rate return distribution and the estimates for the tails of the economic fundamentals which are associated with a domestic currency depreciation. Those include the tail estimates for the right tails (positive side) of changes in money supply and price level, and the estimates for the left tails (negative side) of changes in real income, long-term interest rates and short-term interest rates.

The bottom row γ_{app} represents the appreciation side of the domestic currency and movements of the economic fundamentals that induce the appreciation of the domestic currency. On top of each column, e, m, y, i, r, p denote the rates of changes of the exchange rate, money supply, real income, long-term interest rate, short-term interest rate and price level, respectively. The asymptotic standard errors are presented beneath each estimated inverse tail index.

Excluding France, the tail associated with a depreciation (positive) of the exchange rate return is heavier than the tail associated with the appreciation side. This means that the probability of a large domestic currency depreciation relative to the US dollar is higher than the probability of a large currency appreciation. For Canada, Germany, the Netherlands and the UK, estimated tail indices of the positive rates of return of holding a US dollar are below 5, as $\widehat{\gamma}^m > 0.20$. This implies that a maximum bounded moment of the right tail of these currencies is equal to 4, i.e. the kurtosis just exists, and even less than 4 in the case of the Dutch

Table 2: The modified Hill estimates

$$\widehat{\gamma}^m(\kappa) = \sum_{m=1}^{\kappa} \varpi(m)\widehat{\gamma}(m)$$

		e	m	y	i	r	p
Italy	γ_{dep}	0.1215	0.2544	0.2346	0.2106	0.3347	0.2082
	Std. error	0.0821	0.1240	0.0837	0.0786	0.0853	0.0541
	γ_{app}	0.0800	0.2460	0.2829	0.2543	0.1793	0.3161
	Std. error	0.0844	0.0919	0.0853	0.0809	0.1268	0.1281
Japan	γ_{dep}	0.1623	0.1841	0.1021	0.1309	0.4627	0.3980
	Std. error	0.0722	0.0447	0.0516	0.0556	0.1009	0.1125
	γ_{app}	0.0749	0.0199	0.1077	0.2407	0.4714	0.2217
	Std. error	0.0843	0.0625	0.0551	0.0814	0.1176	0.0435
Netherlands	γ_{dep}	0.2531	0.2588	0.3086	0.2990	0.4424	0.0264
	Std. error	0.0675	0.1256	0.0791	0.0583	0.1257	0.0937
	γ_{app}	0.1588	0.1699	0.2371	0.1994	0.4936	0.2536
	Std. error	0.0704	0.0484	0.0640	0.0858	0.1345	0.0590
UK	γ_{dep}	0.2344	0.0851	0.2199	0.2060	0.5524	0.2735
	Std. error	0.0699	0.0697	0.0848	0.0742	0.1040	0.0799
	γ_{app}	0.1323	0.2300	0.2206	0.2136	0.3177	0.3730
	Std. error	0.0758	0.0945	0.0868	0.0824	0.0854	0.0749

This table gives the estimated inverse tail indices of monthly exchange rate returns and monthly changes in the economic fundamentals inducing depreciation (γ_{dep}) and appreciation (γ_{app}) of the domestic currency. The asymptotic standard error is presented below each estimate.

guilder.

Around half of the economic fundamental series have sizable fat tails with the estimated tail indices, and thus the number of existing moments, less than 4 ($\widehat{\gamma}^m > 0.25$). For more than half of the fundamental series, the upper bound of the 95% confidence interval of the estimated tail index is in single digits. For example, with 95% confidence for both tails of the Canadian money supply the maximum number of bounded moments is less than or equal to 8, while for the left and right tails of the Dutch real income the maximum number of bounded moments appears to be below 6 (left tail). Note that the point estimates for the tail index of these series are close to 3, which implies that the fourth moment is unbounded.

Furthermore, the upper bound of the 95% confidence interval of the estimated number of existing moments is less than or equal to 5 for about twenty percent of the economic fundamental series. These heavy-tailed fundamental series mainly consist of interest rates and inflation rates. As in Section 2, the short-term interest rate, in particular, has the thickest tail in most cases and in some cases only the mean exists, i.e., $\widehat{\gamma}^m > 0.5$. In all countries, with 95% confidence the upper bound of the maximum number of moments for the negative tail of the short-term interest rate is less than or equal to 5. In the case of Germany for example, the estimated tail index (and the 95%-interval upper bound) for the short-term interest rate's left and right tail are 1.4290 (2.1869) and 1.8262 (3.2926), respectively.

The evidence thus indicates the tail fatness of the logarithmic macroeconomic variables. The stylized fact that exchange rate returns are fat-tailed distributed is now accompanied by the not so well known fact that macroeconomic fundamentals are also heavy tailed and in some cases appear to have even fatter tails than the exchange rate.

5.2   Relations between the Tails

In this subsection, we examine the relationship between the extreme exchange rate returns and the largest growth rates of the fundamentals. From Feller's convolution theorem we have that if the exchange rate is an additive function of macroeconomic fundamentals, the tails of the exchange rate return distribution should be governed by the heaviest tails of the fundamentals rates of change as exposed in (1). Given that the tail index estimator is asymptotically normally distributed, one can test the validity of the monetary models in the tail areas by testing whether the heaviest fundamental tail in the exchange rate models has the same tail index as the exchange rate returns.

Table 3 and 4 present differentials between the estimated inverse tail indices of the exchange rate returns and the rates of change of the economic fundamentals. In the tables, we report tests of $H_0 : \gamma_e - \gamma_f \leq 0$ against $H_1 : \gamma_e - \gamma_f > 0$; *p-values* are reported in the rows below the differentials $\gamma_e - \gamma_f$. Again, for each country the rows denoted by *Dep.* and *App.* respectively, give the depreciation side values and the appreciation side values.

Table 3: Test results based on the Feller tail additivity theorem

$$\widehat{\gamma}^m(\kappa) = \sum_{m=1}^{\kappa} \varpi(m)\widehat{\gamma}(m)$$

		$\gamma_e - \gamma_m$	$\gamma_e - \gamma_y$	$\gamma_e - \gamma_i$	$\gamma_e - \gamma_r$	$\gamma_e - \gamma_p$
Austria	Dep.	0.1116	**-0.0689**	**-0.0807**	-0.1755	**0.0705**
	p-value	0.0975	0.7452	0.8026	0.9405	0.2722
	App.	**-0.0696**	**0.0711**	-0.1818	-0.2189	**0.0141**
	p-value	0.8033	0.1823	0.9470	0.9464	0.4280
Canada	Dep.	**-0.0338**	0.0856	**-0.0907**	**-0.1117**	**-0.1031**
	p-value	0.6379	0.2012	0.8175	0.8591	0.8299
	App.	-0.1824	-0.2447	-0.1477	-0.2898	**-0.0875**
	p-value	0.9619	0.9811	0.9410	0.9907	0.8126
France	Dep.	-0.1812	**0.0031**	**-0.0952**	-0.2770	**0.0128**
	p-value	0.9096	0.4887	0.8325	0.9922	0.4519
	App.	**-0.2100**	**0.0189**	**-0.0927**	-0.1982	**0.0809**
	p-value	0.8599	0.4189	0.8165	0.9116	0.2253
Germany	Dep.	**0.1228**	**-0.0737**	**-0.0089**	-0.4957	**-0.1229**
	p-value	0.1209	0.7524	0.5354	0.9990	0.7761
	App.	**-0.0342**	**-0.0237**	**0.0604**	-0.3656	**-0.0586**
	p-value	0.6164	0.5731	0.3132	0.9923	0.7162

This table tests for differences between the (inverse of) the tail index of the exchange rate return and the rates of change of the economic fundamentals, namely $\widehat{\gamma}_e - \widehat{\gamma}_f$. We test $H_0 : \gamma_e \leq \gamma_f$ against $H_1 : \gamma_e > \gamma_f$, and where $\gamma_f = \gamma_{m,y,i,r,p}$. The *p-value* from the t-statistic for each pair is presented below each estimate. Values in bold indicate the cases for which we cannot reject equality of the tail indices between the exchange rate return and the growth rates of the fundamentals.

Table 4: Test results based on the Feller tail additivity theorem

$$\widehat{\gamma}^{m}(\kappa) = \sum_{m=1}^{\kappa} \varpi(m)\widehat{\gamma}(m)$$

		$\gamma_e - \gamma_m$	$\gamma_e - \gamma_y$	$\gamma_e - \gamma_i$	$\gamma_e - \gamma_r$	$\gamma_e - \gamma_p$
Italy	Dep.	**-0.1329**	**-0.1131**	**-0.0891**	-0.2132	**-0.0867**
	p-value	0.8145	0.8352	0.7672	0.9612	0.8063
	App.	-0.1660	-0.2029	-0.1743	**-0.0993**	-0.2361
	p-value	0.9257	0.9518	0.9341	0.7414	0.9297
Japan	Dep.	**-0.0219**	**0.0602**	**0.0314**	-0.3004	-0.2358
	p-value	0.5990	0.2478	0.3683	0.9918	0.9605
	App.	**0.0550**	**-0.0329**	-0.1658	-0.3966	-0.1468
	p-value	0.2968	0.6302	0.9330	0.9969	0.9331
Netherlands	Dep.	**-0.0057**	**-0.0555**	**-0.0459**	-0.1893	0.2266
	p-value	0.5166	0.7122	0.7014	0.9155	0.0233
	App.	**-0.0111**	**-0.0783**	**-0.0406**	-0.3348	**-0.0949**
	p-value	0.5553	0.7850	0.6423	0.9878	0.8474
UK	Dep.	0.1493	**0.0145**	**0.0284**	-0.3180	**-0.0390**
	p-value	0.0655	0.4438	0.3911	0.9933	0.6444
	App.	**-0.0976**	**-0.0883**	**-0.0812**	-0.1854	-0.2407
	p-value	0.7894	0.7828	0.7652	0.9412	0.9846

This table tests for differences between the (inverse of) the tail index of the exchange rate return and the rates of change of the economic fundamentals, namely $\widehat{\gamma}_e - \widehat{\gamma}_f$. We test $H_0 : \gamma_e \leq \gamma_f$ against $H_1 : \gamma_e > \gamma_f$, and where $\gamma_f = \gamma_{m,y,i,r,p}$. The *p-value* from the t-statistic for each pair is presented below each estimate. Values in bold indicate the cases for which we cannot reject equality of the tail indices between the exchange rate return and the growth rates of the fundamentals.

At the 10% significance level, there are few fundamental variables for which we can reject the null hypothesis in favor of the conclusion that their rates of change have lighter tails than the exchange rate return. The fundamental variables which do seem to have lighter tails are the positive rates of change of the Austrian money supply and the Dutch and UK inflation rates.

The other macroeconomic fundamentals appear to have equal or fatter tails than the exchange rate returns. For these variables, we run the second t-test $H_0 : \gamma_e - \gamma_f \geq 0$ against $H_1 : \gamma_e - \gamma_f < 0$, in order to examine whether the fundamentals have equal or significantly fatter tails. The *p-values* for this t-test are the complements of the p-values reported for the previous test. Thus the reader can easily deduce the p-values for the present test by taking one minus the p-values reported in Table 3 and 4. At the 10% significance level, variables for which we cannot reject the null hypothesis of equal tail indices are shown in bold in Table 3 - 4.

All the fundamental variables display similar tail thickness, except for the short term interest rate. Since most of the short-term interest rates appear to have thicker tails than the exchange rates, these would be incompatible with any version of the monetary model. If we use the long-term interest rates, however, the monetary model appears to do a good job if judged by the shape of the tails. Recall that the monetary model is often regarded as a relationship between the fundamentals that should hold in the longer term (for examle this is how the monetary model is used in the overshooting literature). Using the long-term interest rates, the flexible-price model is consistent with the frequency of extreme depreciations of the domestic currency for all countries, except in case of France; but only the domestic currency appreciation in France, Germany, the Netherlands and the UK do not violate the monetary model predictions. The sluggish-price model is also consistent with the extreme events in the foreign exchange market just like the flexible-price model, except for the depreciation of the Japanese yen and the appreciation of the British pound.

Evidence thus suggests the validity of the monetary-approach exchange rate models with respect to tail events. As can be theoretically anticiapted on basis of the Feller's convolution theorem, the extreme events of the exchange rate returns are associated with the largest changes in the economic fundamentals. In every country, large movements in real income and long-term interest rates are associated with extreme movements of exchange rate returns. Except for the domestic currency depreciation in Austria and the UK, dramatic changes in money supply are related to large swings in currency prices. Large changes in price levels can also explain extreme exchange rate returns, except for the depreciation of the Japanese yen and the Dutch guilder and the appreciation of the British pound. But the extreme movements in short-term interest rates do not seem to be related to the extreme changes in the exchange rate, except for the depreciation of the Canadian dollar.

6 Conclusion

Exchange rate returns are fat-tailed distributed. This is indicative of unconditional risk due to for instance currency crises, exchange rate realignments and volatility clusters. We provide evidence that the apparent non-normality derives from the behavior of macroeconomic fundamentals. Economic and probabilistic arguments are offered for this relationship. We find that both exchange rate returns and macroeconomic fundamentals' growth rates are heavy-tailed distributed. The monetary models of the exchange rate imply a linear relationship between the exchange rate and the fundamentals. The convolution theorem for heavy-tailed random variables holds that the sum has an hyperbolic tail shape equal to the thickest tailed random variable in the sum.

When we tested for this restriction, it turned out that especially the depreciation tail of the exchange rate compares well with the thickest tails of the fundamentals. Based on our criteria, in most cases the real income growth rates and the long-term interest rate changes are linked to the exchange rate returns. In a number of cases, monetary policy variables, like the money growth rate and the inflation rate, also play a role in explaining large swings in currency prices. The reason as to why short term rates are generally appear to be fatter than the the long term rates and why the latter better correspond to the tail shapes of the exchange rate returns is an interesting question for future work. Moreover, we would like to examine further the causes of heavy-tailed economic fundamentals. Further empirical work, especially for non-OECD countries would also be welcome once sufficiently long time series for the fundamentals are available.

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7 Appendix A: Data Source

The data are monthly observations on the exchange rates, money supply (M2), industrial production, long-term interest rate, short-term interest rate and consumer price index, starting from July 1973 to December 1998 for countries that adopted the euro and from July 1973 to December 2002 for the other countries in the sample. There are nine OECD countries involved. These are Austria, Canada, France, Germany, Italy, Japan, the Netherlands, the United Kingdom (UK) and the United States (US). The exchange rate is quoted on a per US dollar basis. The fundamentals' variables are relative to the US variables.

The data source is the IMF International Financial Statistics (IFS). The US dollar exchange rates from IFS are coded AE. The monetary aggregate M2 is a sum of IFS codes 34 and 35. Seasonally adjusted industrial production, code 66, is used as a proxy for real income. In case of Austria, to make the series compatible with others, we use X-11 to correct for seasonality. For the long-term interest rate, the long-term government bond yield, coded 61, is employed. For the short-term interest rate and inflation rate, we use the treasury bill rate (code 60 C) and the consumer price index (code 64), respectively.

8 Appendix B: Summary Statistics

Table 5: Descriptive statistics for Austria and Canada

Country	Variable	Mean	Standard Deviation	Max	Min	J-B test p-value	Inverse d.f.	Number of Observations
Austria	e	-0.0015	0.0328	0.1213	-0.1001	0.0038	0.1282**	306
	m	0.0003	0.0281	0.0642	-0.0967	0.0657	0.0713*	306
	y	0.0005	0.0337	0.0935	-0.1335	0.0273	0.1157**	306
	i	-0.0006	0.0304	0.1558	-0.1150	0.0000	0.1414***	306
	r	0.0009	0.8533	7.0300	-3.4000	0.0000	0.3527***	306
	p	-0.0011	0.0048	0.0186	-0.0125	0.0023	0.1905***	306
Canada	e	0.0013	0.0135	0.0626	-0.0337	0.0000	0.1993***	353
	m	0.0030	0.0248	0.0965	-0.1218	0.0000	0.2043***	353
	y	0.0001	0.0123	0.0740	-0.0472	0.0000	0.1852***	353
	i	0.0002	0.0262	0.1025	-0.0950	0.0000	0.0792**	353
	r	0.0110	0.5859	3.2900	-2.2700	0.0000	0.2754***	353
	p	0.0002	0.0035	0.0199	-0.0111	0.0000	0.1702***	353

In this table, all statistics are based on monthly rates of change of exchange rates, money supply (M2), industrial production, interest rates and consumer price index. Exchange rates are local currency prices per one US dollar. The fundamental series are the rates of change of the domestic variables relative to the foreign (US) variables.

Table 6: Descriptive statistics for France and Germany

Country	Variable	Mean	Standard Deviation	Max	Min	J-B test p-value	Inverse d.f.	Number of Observations
France	e	0.0010	0.0322	0.1164	-0.0918	0.0003	0.1495**	306
	m	0.0001	0.0524	0.2229	-0.6865	0.0000	0.4976***	306
	y	-0.0011	0.0161	0.0510	-0.0520	0.0567	0.0572	306
	i	-0.0005	0.0326	0.1250	-0.0958	0.0000	0.0708*	306
	r	-0.0035	0.8094	6.3200	-3.5100	0.0000	0.3013***	306
	p	0.0006	0.0033	0.0133	-0.0112	0.0000	0.2542***	306
Germany	e	-0.0012	0.0330	0.1217	-0.1100	0.0030	0.1214**	306
	m	0.0018	0.0296	0.1044	-0.0757	0.0227	0.2437***	306
	y	-0.0012	0.0188	0.1108	-0.0980	0.0000	0.1678***	306
	i	-0.0011	0.0270	0.1133	-0.1008	0.0000	0.1756***	306
	r	-0.0129	1.0299	7.3800	-7.4600	0.0000	0.3889***	306
	p	-0.0017	0.0039	0.0133	-0.0225	0.0000	0.2305***	306

In this table, all statistics are based on monthly rates of change of exchange rates, money supply (M2), industrial production, interest rates and consumer price index. Exchange rates are local currency prices per one US dollar. The fundamental series are the rates of change of the domestic variables relative to the foreign (US) variables.

Table 7: Descriptive statistics for Italy and Japan

Country	Variable	Mean	Standard Deviation	Max	Min	J-B test p-value	Inverse d.f.	Number of Observations
Italy	e	0.0034	0.0309	0.1343	-0.0778	0.0000	0.2292***	306
	m	0.0044	0.0332	0.1219	-0.1115	0.0000	0.4899***	306
	y	-0.0009	0.0285	0.1399	-0.1351	0.0000	0.2069***	306
	i	-0.0003	0.0431	0.1913	-0.1642	0.0000	0.1929***	306
	r	0.0033	0.9866	6.5500	-3.2300	0.0000	0.3281***	306
	p	0.0034	0.0050	0.0228	-0.0104	0.0000	0.2256***	306
Japan	e	-0.0022	0.0334	0.1153	-0.1501	0.0000	0.2125***	353
	m	0.0020	0.0440	0.1325	-0.1027	0.3937	0.0056	353
	y	-0.0006	0.0149	0.0397	-0.0426	0.4303	0.0297	353
	i	-0.0008	0.0304	0.1317	-0.0892	0.0000	0.1455***	353
	r	0.0017	0.7639	6.9800	-4.5800	0.0000	0.3570***	353
	p	-0.0014	0.0065	0.0322	-0.0199	0.0000	0.2200***	353

In this table, all statistics are based on monthly rates of change of exchange rates, money supply (M2), industrial production, interest rates and consumer price index. Exchange rates are local currency prices per one US dollar. The fundamental series are the rates of change of the domestic variables relative to the foreign (US) variables.

Table 8: Descriptive statistics for the Netherlands and the UK

Country	Variable	Mean	Standard Deviation	Max	Min	J-B test p-value	Inverse d.f.	Number of Observations
Netherlands	e	-0.0011	0.0331	0.1221	-0.1172	0.0008	0.1323**	306
	m	0.0009	0.0308	0.1386	-0.0683	0.0000	0.3965***	306
	y	-0.0008	0.0278	0.0817	-0.1192	0.0000	0.1622***	306
	i	-0.0003	0.0269	0.1275	-0.0767	0.0000	0.1634***	306
	r	0.0070	1.4381	9.7300	-5.6000	0.0000	0.3485***	306
	p	-0.0013	0.0046	0.0164	-0.0215	0.0000	0.2005***	306
United Kingdom	e	0.0014	0.0305	0.1277	-0.1314	0.0000	0.1445***	353
	m	0.0013	0.0233	0.0570	-0.0785	0.0456	0.0659	353
	y	-0.0011	0.0143	0.0679	-0.0703	0.0000	0.1726***	353
	i	-0.0006	0.0360	0.1183	-0.1342	0.0000	0.1425***	353
	r	0.0254	1.5130	6.8200	-9.2300	0.0000	0.3079***	353
	p	0.0017	0.0064	0.0376	-0.0149	0.0000	0.3720***	353

In this table, all statistics are based on monthly rates of change of exchange rates, money supply (M2), industrial production, interest rates and consumer price index. Exchange rates are local currency prices per one US dollar. The fundamental series are the rates of change of the domestic variables relative to the foreign (US) variables.