TI 2006-069/1
Tinbergen Institute Discussion Paper

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Seeking Rents in the Shadow of Coase*

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July 4, 2006

Abstract

Trade opportunities are generally seen as valuable instruments to improve the allocation of resources in society. However, when the traded rights are secured through unproductive rent-seeking contests, the tradeability of the rents may provide stronger incentives to invest in rent-seeking activities, exacerbating rent-dissipation losses. In some cases the increase in rent dissipation may exceed the benefits of trade, rendering the opportunity to transfer rents socially undesirable. We consider a two-stage game in which the contestants have different valuations of the sought-after rent. In the first stage, parties invest to secure rights by participating in a rent-seeking contest. In the second stage, parties decide whether to reallocate the rights by entering in a Coasean exchange. We show that an opportunity for an ex post reallocation of the rights may have perverse ex ante effects. We consider the effect that such trading opportunities have on the parties' payoffs and evaluate the final outcome in terms of dissipation and misallocation costs, comparing our scenario with tradeable rents to the conventional case of non-tradeable rents.

JEL Classification: C72, D72

Keywords: Rent-seeking, asymmetric rent valuations, misallocation costs, rent dissipation.

*The authors would like to thank Ben Depoorter, Bruno Frey, Peter Menell, Ilia Rainer, Gordon Tullock, and the participants in the 2006 Erasmus Law and Economics Workshop at Hamburg University, the 2006 annual meeting of the Public Choice Society in New Orleans, and seminars at Ghent University, the University of Zurich and the Institute for Information Law at Universiteit van Amsterdam for helpful comments.

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1 Introduction

Imagine that two knights are taking part in a tournament to win the hand (and the love) of a princess. The knights are of equal strength, valor, and courage, but while one of them only aims at the kingdom, the other also secretly loves the princess. The literature on rent-seeking has analyzed these types of games and pointed to the fact that the participants will dissipate socially valuable resources in the attempt to win the prize. However, focused as it has been on contests with a commonly valued prize, this literature has failed to realize that there is an additional social cost associated with rent-seeking tournaments: the princess may marry the wrong knight.

When parties have equal valuations of the rent, the efficiency of rent-seeking outcomes is generally evaluated in terms of total rent dissipation. When parties have different valuations of the sought-after rent, a new source of inefficiency may come about, given the fact that rents are not necessarily appropriated by players that value them the most. Losses from inefficient misallocations of the rent should thus be considered in addition to the traditional problem of rent dissipation. Further, when parties attach different values to a prize, they will exert different levels of efforts, as their marginal benefits differ. Contests to win esteem, advance in social ranks, or obtain a non-tradable permit are of this type. We study the allocative efficiency of rent-seeking contests when heterogeneous players are involved.

In some cases, the allocative inefficiency of rent-seeking contests can be corrected ex post. Although in our example the knights cannot subsequently reallocate the princess, other sought-after rents can be traded ex post. In the case of tradable rights, the low-valuing party is likely to sell the prize to the other if he wins the contest. The race to obtain rights over the new Internet domain names '*.eu*' provides an example of a rent-seeking contest where an initial misallocation can be corrected ex post through Coasean bargaining.[1]

Ex post reallocation is desirable for low-valuing parties, since it allows them to resell at a higher price. High-valuing parties may or may not benefit from the prospect of ex post reallocation – it gives them an opportunity to purchase the right when they fail in the rent-seeking contest, but increases the competition that they may face from low-valuing contestants. The opportunity for ex post reallocation of the rent affects the total social cost of a rent-seeking contest in two different ways. First, it eliminates the risk of misallocations. Second, by making the prize more appealing for low-valuing players, it increases the stakes in the game and may occasion greater rent dissipation.

In *Avery Dennison Corp. v. Sumpton*,[2] for example, it emerged that a company had registered over 12,000 Internet domain names – among which the plaintiff's trademarks – for no other purpose than that of reselling them. Dissipation in the race to register the new '*.eu*' domain names is kept under check by Regulation (EC) No 874/2004 reserving or preventing registration of some names (artt. 7 ff.) and providing for an initial screening of claims involving, *inter alia*, trade marks, geografical indications or

[1] See the information available at *www.eurid.eu*.
[2] 999 F.Supp. 1337 C.D.Cal., 1998.

designations of origin (artt. 10 ff.). However, after this initial phase (the so-called *Sunrise period*), '*.eu*' domain names are allocated on a strict 'first come, first served' basis, which is likely to induce some residual rent-seeking activities.[3]

We study the overall impact of ex post bargaining on social cost and show that it is ambiguous and depends on the parties' bargaining power. If the low-valuing party has large bargaining power, he will be able to extract a large fraction of value from the high-valuing party. This prospect will in turn induce low-valuing parties to increase their rent-seeking investment up to the point that the surge in total rent dissipation might overcome the allocative advantage of having a market in the first place. From the study of the parties' rent-seeking incentives, it also emerges that, when the parties' rent valuations are sufficiently divergent, equilibrium levels of efforts may be decreasing in the marginal return to effort. This result runs contrary to the conventional wisdom in the literature according to which higher returns to effort induce an increase in rent-seeking expenditures and in the total rent dissipation.

This paper brings together insights and results from rent-seeking and property rights theory. Our key contribution is to show that the possibility for ex post Coasean bargaining affects the rent-seeking incentives for the original appropriation of property rights and, vice versa, rent-seeking outcomes affect the process of Coasean bargaining. The rent-seeking literature (Tullock, 1967; Krueger, 1974; Posner, 1975; Bhagwati, 1982)[4] generally considers parties competing for the appropriation of a commonly-valued rent. Asymmetries between the parties, when introduced, are modeled in terms of different returns to rent-seeking effort. We add to this literature by allowing the parties' valuations to diverge.[5] Further, we bring the Coase theorem to bear on our analysis (Coase, 1959 and 1960). According to the Coase theorem, ex post contractual negotiation will correct any possible initial misallocation of resources. It has been observed that rules of first possession (such as 'first come, first served' and 'finders, keepers') generate rent-seeking incentives in the initial allocation of property rights leading to wasteful expenditures in the appropriation of such rights also in the form of too fast exploitation (Barzel, 1968; Dasgupta and Stiglitz, 1980; Mortensen, 1982;[6] Anderson and Hill, 1983 and 1990; Lueck, 1995). We show that the possibility of ex post Coasean reallocation of the appropriated resources, while solving problems of misallocation, may exacerbate rent dissipation due to increased rent-seeking efforts.[7] The increase in rent dissipation may

[3]Most legal systems include similar rules of 'first come, first served' regulating how property on things that belong to nobody (*res nullius*) and abandoned things (*res derelictae*) is to be assigned. A well-know example is the 1862 Homestead Act, concerning the new territories of the west of the United States.

[4]For a review of the literature see Buchanan, Tollison and Tullock (1980); Congleton and Tollison (1995); Lockard and Tullock (2000); Rowley, Tollison and Tullock (1988); Tollison (2003).

[5]Other papers that study contests with heterogeneous private valuations are Baye, Kovenock and De Vries (1993, 1996), Amann and Leininger (1996), Krishna and Morgan (1997), Che and Gale (1998), Gavious, Moldovanu and Sela (2002), Goeree, Maasland, Onderstal, and Turner (2005), and Onderstal (2006). In contrast to our work, these papers assume a completely discriminatory contest, i.e., the highest bidder wins with probability equal to 1.

[6]Mortensen (1982) shows that the ex post distribution of the surplus affects ex ante incentives to invest in mating and racing games but does not discuss the issue of ex post trade.

[7]Literature on incomplete contracts (Grossman and Hart, 1986, and Hart and Moore, 1990) enphasizes the importance of rights that cannot be transferred by contract and the effects that the allocation of such rights has on the parties' investments in the relationship. Our perspective is different, since we analyze the investments of the parties prior to the allocation of such rights. In the context of property rules versus liability rules, Bebchuk (2001) observes that the choice of remedy affacts the ex post distribution of resources with ex ante effects on the levels of the parties' investments.

more than offset the allocative gains. In this case, Coasean bargaining results in a social loss. Under some conditions, foreclosing the opportunity for Coasean bargaining may actually improve social welfare.[8]

Our paper brings to light a contradiction with the existing literature. According to the Coase theorem, an efficient allocation of resources is achievable independently of the allocation of rights and distribution of surplus between the parties. The present paper shows an interesting twist of the argument revealing that the outcome depends on the distribution of the bargaining surplus − a distributive aspect of the Coase theorem, which is generally considered irrelevant for the efficient allocation of resources.

In the following section, we present the basic model of rent-seeking with asymmetric rent valuations. In section 3, we analyze rent-seeking without Coasean bargaining in a complete information framework. In section 4, we consider rent-seeking with Coasean bargaining. In section 5, we extend the analysis to a setting with incomplete information. Section 6 concludes the paper with some ideas for future research.

2 The model

We consider a rent-seeking contest between two parties, who have equal marginal returns to effort. Let $B_i \geq 0$, with $i = \{1, 2\}$, denote the parties' rent-seeking expenditures and V_i denote the value of the rent. Following the conventional Tullock framework,[9] party i's payoff is given by

$$U_i(B_i, B_j) = \frac{B_i^r}{B_i^r + B_j^r} V_i - B_i \tag{1}$$

with $\{i, j\} = \{1, 2\}$. The payoff function can be interpreted as an expected payoff function where greater effort by one party increases that party's probability to win the entire rent, rather than increasing the share of the appropriated rent.[10] As usual, r is an index of the parties' marginal productivity of effort.[11] In the following, we will focus on situations where the parties' participation constraint is always fulfilled. This is guaranteed when the players face constant or decreasing marginal returns to effort ($r \leq 1$), thus playing pure strategies.[12]

Players differ in their valuation of the rent V_i. For simplicity, we assume that there are two types of players: a low-valuing party L with value $V_L > 0$ and a high-valuing party H with value $V_H > V_L$. Previous contributions[13] study the pattern of the parties' rent-seeking efforts as a function of the rent value, but restrict the analysis to situations where the parties have an identical valuation of the rent, i.e., $V_H = V_L$. In the present paper, we extend the analysis considering the case in which the parties exhibit different valuations of the rent.

[8] Some previous contributions also emphasize the perverse effects of trading possibilities. For example, in a context different from ours, Jacklin (1987) shows that the presence of liquid equity markets undermines the role of banks as reducing liquidity risk.

[9] In reality, the splitting of a prize between two contestants may not follow Tullock's sharing rule and may take up a whole range of possible functional forms. We follow Tullock's traditional framework since it allows for an easier comparison of our results with those of the existing rent-seeking literature and for an explicit calculation of equilibrium values.

[10] Assuming risk neutrality, these two interpretations are functionally equivalent. Thus, hereinafter, when referring to the parties' share of the rent, we could alternatively refer to the parties' probability to win the entire rent.

[11] Alternatively, r can be seen as the discriminative power of the auction.

[12] Note that this is a sufficient but not a necessary condition.

[13] For a survey see Lockard and Tullock (2000).

As known from the previous literature, if parties were symmetric and attached the same value to the rent, they would play the same strategies and consequently split the rent in equal shares in equilibrium. When parties' valuations differ, their strategies and shares of the rent are also expected to differ. In the following, we investigate the effect of these asymmetries on the parties' strategies. We subsequently consider the allocative function of rent-seeking contests and the interesting problems associated with the efficient final allocation of the rent.

With parties with equal valuations of the rent, the efficiency of rent-seeking outcomes is generally evaluated in terms of total rent dissipation through the parties' effort. When the parties have different valuations of the rent, a new source of inefficiency may come about, given the fact that rents are not necessarily appropriated by players who value them the most. Put differently, when parties have heterogeneous valuations of the rent, losses from inefficient misallocations of the rent should be considered in addition to the traditional problems of rent dissipation. The misallocation losses can be viewed as an opportunity cost due to the fact that the rent could have been put to a higher-value use.

Accordingly, we define two loss variables: the rent dissipation, given by the sum of the parties' efforts,[14]

$$D = B_i + B_j \tag{2}$$

and the rent misallocation, given by the share of the rent appropriated by the low-valuing player scaled by the valuation difference (with $V_j < V_i$ and relabel otherwise)

$$M = \frac{B_j^r}{B_i^r + B_j^r} (V_i - V_j) \tag{3}$$

When parties have heterogeneous valuations of the rent, the misallocation loss is given by the difference between the parties' valuations multiplied by the share of the rent appropriated by the low-valuing party. Obviously, in the special case of parties with equal valuations of the rent, the misallocation is equal to zero, irrespective of whether both parties are low-valuing or high-valuing players.

In the following two sections, we study this model under complete information, i.e., both parties are completely informed about each other's value. First, we consider the case in which the parties cannot resell the rent after the contest. We subsequently consider a scenario in which parties can enter an additional stage of the game, where rents can be reallocated (sold) by a low-valuing player to a high-valuing player. Through such Coasean bargaining, rent misallocations are corrected, but optimal strategies may be altered, with remaining rent-dissipation losses. Finally, we consider an incomplete information case where parties know their own valuation but do not know the valuation of the other player.

[14] Note that we assume throughout that the parties' rent-seeking efforts have purely redistributive effects. For an analysis of rent-seeking contests where rent-seeking effort is socially productive see Dari-Mattiacci and Parisi (2005) and the concluding remarks at the end of this paper.

3 Rent-seeking without Coasean bargaining

Consider the rent-seeking contest with complete information, in which one party has valuation V_L and the other V_H.

3.1 Parties' equilibrium efforts and payoffs

Parties maximize their respective payoffs in (1) according to the following first order condition:

$$\frac{\partial U_i}{\partial B_i} = r \frac{B_i^{r-1} B_j^r}{\left(B_i^r + B_j^r\right)^2} V_i - 1 = 0 \tag{4}$$

It is well known that, if parties have equal valuations of the rent (with $V_L = V_H = V$), the equilibrium effort levels are $B_L^* = B_H^* = \frac{r}{4}V$, and the total rent dissipation is $D = \frac{r}{2}V$. In this scenario, there is no possible misallocation resulting from the game. If parties have heterogeneous valuations, misallocation costs are possible and, in general, the outcome of the game will also depend on the difference between the parties' valuations. To simplify notation, we introduce a variable measuring the level of asymmetry between the parties' valuations, $\gamma \equiv \frac{V_L}{V_H}$. The value of γ ranges $0 < \gamma < 1$, approaching 1 if the parties have similar valuations of the rent and approaching 0 if the parties' valuations differ substatially.

Proposition 1 *The parties' equilibrium levels of efforts are directly proportional to their valuations of the rent:* $\frac{B_L^*}{B_H^*} = \gamma$*. The equilibrium payoffs* U_L^* *and* U_H^* *are instead less than proportional to the parties' valuations:* $\frac{U_L^*}{U_H^*} < \gamma$*.*

In equilibrium, the party with higher valuation makes larger rent-seeking expenditures compared to the other party. The ratio of the parties' efforts is equal to the ratio of their valuations. The high-valuing party has greater incentives to invest in rent-seeking and, thus, will appropriate a larger share of the rent. Additionally, he gives greater value to the rent. The combination of these two effects gives the high-valuing party a more-than-proportional advantage on his opponent.[15]

This framework enables us to discuss changes in the parties' relative valuations as separate from changes in the absolute value of the prize. If the value of the rent increases without affecting the valuation ratio γ, both parties' efforts obviously increase. In our model, this may occur when both V_H and V_L increase in the same proportion (e.g., both of them double).

Performing some simple comparative-statics analysis, it is possible to study how the parties' efforts and payoffs vary when the parameters of the game, r and γ, change. We define a change in γ as a change in V_L alone, i.e., we keep V_H constant.

Corollary 2 $\frac{\partial B_L^*}{\partial \gamma} > 0$*,* $\frac{\partial B_H^*}{\partial \gamma} > 0$*,* $\frac{\partial U_L^*}{\partial \gamma} > 0$ *and* $\frac{\partial U_H^*}{\partial \gamma} < 0$*.*

This corollary yields that both parties' efforts increase as their respective valuations approach each other (γ increases). From a different perspective, when their valuations are further apart, the players

[15] It is easy to show that $U_i^* > 0$ for both parties.

participate in an asymmetric rent-seeking contest, with lower rent-seeking investments. Not surprisingly, it follows that the high-valuing party gains (his expected payoff increases) when the parties' valuations get further apart (γ decreases), while the low-valuing party loses in the same scenario.

Corollary 3 *$\frac{\partial B_L^*}{\partial r}$ and $\frac{\partial B_H^*}{\partial r}$ are positive if the parties' valuations are similar and negative otherwise, $\frac{\partial U_L^*}{\partial r} < 0$ and $\frac{\partial U_H^*}{\partial r}$ is negative if the parties' valuations are very similar and positive if they are very different.*

Studying how the parties' equilibrium levels of efforts vary when their strength r changes yields an interesting result that runs against the conventional wisdom in the public choice literature. It is commonly believed that when the marginal productivity of effort rises, parties will exert more effort, up to the point where their decreasing payoffs approach zero. When the parties' valuations differ, we find a counter-intuitive result. When the parties' valuations are sufficiently far apart (γ close to 0) their levels of efforts may decrease (instead of increasing) as a result of an increase in r. In this scenario, the payoff of the high-valuing party may increase (instead of decreasing), while the payoff of the other party will always decrease.

These findings can be explained considering that an increase in r makes competition more fierce. If γ is sufficiently low and r increases, the low-valuing party may prefer to give way to his opponent, decreasing his effort and obtaining a lower payoff. Consequently the high-valuing party can respond reducing his own effort, and yet gain a larger payoff thanks to the partial withdrawal of his opponent.

3.2 Dissipation and misallocation

The social cost of rent-seeking is given by the sum of rent dissipation and rent misallocation. Using the results of the previous section it is possible to calculate these costs, expressed in (2) and (3), and to assess the effects of changes in r and γ on total rent-seeking costs.

Proposition 4 *The total rent dissipation is $0 < D^* < \frac{1}{2}V_H$, with $\frac{\partial D^*}{\partial \gamma} > 0$ while $\frac{\partial D^*}{\partial r}$ is positive if the parties' valuations are similar and negative otherwise. The rent misallocation is $0 < M^* < \frac{1}{2}V_H$, with $\frac{\partial M^*}{\partial \gamma}$ negative if the parties' valuations are similar and positive otherwise, and $\frac{\partial M^*}{\partial r} < 0$. The total social loss is $0 < D^* + M^* < V_H$, with $\frac{\partial}{\partial \gamma}(D^* + M^*)$ positive if the parties' valuations are similar and negative otherwise, and $\frac{\partial}{\partial r}(D^* + M^*)$ negative if the parties' valuations are similar and positive otherwise.*

When the parties' rent valuations approach each other, the rent dissipation grows, a result that follows directly from the fact that each party's effort increases with γ. However, contrary to previous literature, we find that the rent dissipation may actually decrease when the parties' productivity of effort, r, increases. In fact, as shown above, if the parties have highly asymmetric valuations, their efforts may decrease in r, and hence total rent dissipation may also decrease.

As the gap between the parties' valuations widens, however, an additional cost arises due to the potential misallocation of the rent. In our formulation, rent misallocation occurs whenever party L

appropriates the rent, while the magnitude of the allocative loss depends on the difference between the parties' valuations, as in (3).

The misallocation may either increase or decrease when γ varies. Two opposite forces are responsible for this result. On the one hand, if the parties' valuations move closer to each other, the effort of the low-valuing player tends to approach that of his opponent. Thus, with more homogeneous valuations the probability of a misallocation becomes greater. On the other hand, the magnitude of the misallocation decreases, since the gap between the parties' valuations becomes narrower. For low values of γ the former effect dominates ($\frac{\partial M^*}{\partial \gamma} > 0$), while for larger values of γ the latter effect dominates ($\frac{\partial M^*}{\partial \gamma} < 0$). Instead, the misallocation unambiguously decreases as the parties become stronger ($\frac{\partial M^*}{\partial r} < 0$). This can be explained considering that when the parties' strength increases, the magnitude of the misallocation remains unchanged, but the probability that the low-valuing player actually wins the contest decreases. This follows from the fact that with an increase in r the incentives for the high-valuing party to invest in rent-seeking increase more rapidly than for his competitor, due to his larger stakes in the game. Obviously, when the parties' valuations are the same, the misallocation loss disappears.

Total rent-seeking losses are given by $D^* + M^*$. Such a total loss has two components: dissipation, D^*, increasing in γ, and misallocation, M^*, which initially increases and then decreases in γ. The final effect is that for lower levels of γ total rent-seeking losses increase as the parties' valuations approach each other ($\frac{\partial}{\partial \gamma}(D^* + M^*) > 0$). With higher values of γ, parties' valuations become more homogeneous and total rent-seeking costs start decreasing, as the reduction in misallocation losses more than compensates for the increase in dissipation ($\frac{\partial}{\partial \gamma}(D^* + M^*) < 0$). At the limit, when the parties' valuations converge, the total social loss is reduced to the sole dissipation.

The result is opposite to the former if we consider the variation of the total rent-seeking losses in r. When parties have heterogeneous valuations, total rent-seeking losses decrease in r ($\frac{\partial}{\partial r}(D^* + M^*) < 0$). This is due to the fact that both D (dissipation) and M (misallocation) decrease in this case. When the parties' valuations converge, total rent-seeking losses increase, as the increase in dissipation dominates the decrease in misallocation ($\frac{\partial}{\partial r}(D^* + M^*) > 0$).

Given the fact that D^* and M^* mostly vary in opposite directions, the maximum social loss is not the sum of the maximum D^* plus the maximum M^*, however we find that the total social loss may be above $\frac{1}{2}V_H$.[16] This is an interesting result, showing that, when we account for the misallocation costs created by heterogeneous rent valuations, total rent-seeking losses exceed the maximum level of rent-dissipation obtained in the standard Tullock game.

4 Rent-seeking with Coasean bargaining

As shown in the previous analysis, the presence of asymmetric rent valuations creates the risk of allocative inefficiencies. In some situations, possible misallocations of the rent can however be corrected through ex

[16] For example, this is the case when $r = 1$ and γ is close to 1.

post Coasean bargaining. Low-valuing winners can transfer the rent to their high-valuing opponents.

The results of the previous analysis should thus be revisited in light of the possibility of ex post reallocations of the rent. We consider the general case in which the winner of the rent and the party who wishes to buy it bargain with one another and reallocate the rent, splitting the contractual surplus according to their respective bargaining power, α for the low-valuing party and $1-\alpha$ for the high-valuing party, with $0 \leq \alpha \leq 1$. Whenever a reallocation of the rent takes place, the price paid by the high-valuing party to the low-valuing party is $P \equiv \alpha V_H + (1-\alpha) V_L$. In this section, we study how the opportunity for such ex post bargaining affects the parties' payoffs and ex ante incentives. Several interesting results are derived from this analysis.

Intuitively, the possibility of Coasean bargaining transforms an asymmetric-valuation into a symmetric-valuation game. The opportunity to transfer the rent to the high-valuing contestant, induces low-valuing contestants to take into account the valuation of the other party.

4.1 Parties' equilibrium efforts and payoffs

In a complete information setting, low-valuing parties will choose their effort levels considering the potential price obtainable if the rent is won and subsequently transferred to a high-valuing contestant. If the parties play pure strategies, the possibility of ex post Coasean bargaining affects their payoffs as follows:

$$\begin{aligned} U_L(B_H, B_L) &= \frac{B_L^r}{B_H^r + B_L^r} P - B_L \\ U_H(B_H, B_L) &= \frac{B_H^r}{B_H^r + B_L^r} V_H + \frac{B_L^r}{B_H^r + B_L^r} (V_H - P) - B_H \end{aligned}$$

Rearranging, we have:

$$U_L(B_H, B_L) = \frac{B_L^r}{B_H^r + B_L^r} P - B_L \tag{5a}$$

$$U_H(B_H, B_L) = \frac{B_H^r}{B_H^r + B_L^r} P - B_H + (V_H - P) \tag{5b}$$

The above formulations of the parties' payoffs highlight the interesting features of rent-seeking with Coasean bargaining. The parties' payoffs remain asymmetric, because the parties' actual gains from the game depend on their respective bargaining power. Unless the low-valuing party has full bargaining power and is able to extract the entire surplus from the other party, the high-valuing party will still have a larger payoff from the game. This is due to the fact that the high-valuing party appropriates the difference between his valuation and the price paid to his opponent.

However, despite this persistent asymmetry, the parties' incentives to invest in rent-seeking effort become symmetric. This is an interesting result because parties with asymmetric payoffs end up playing symmetrically. The intuition is that the high-valuing party will always obtain his high-valued rent V_H, whether through direct appropriation or ex post reallocation. Holding V_H constant, the difference for a high-valuing player between success and failure only depends on the price to be paid to the low-valuing party if the latter successfully appropriates the rent. If the high-valuing party wins the rent-seeking game, he avoids the payment of P. The same is true for the low-valuing party, for whom winning the rent-seeking game means securing a payment P from his opponent.

The symmetry between the parties' incentives is emerges the fact that the first order conditions for (5a) and (5b) are identical. Both parties participate in the rent-seeking game with the prospect of appropriating a rent that can be sold (or should be bought) for the price P. The surplus $V_H - P$ is always appropriated by the high-valuing player, regardless of who appropriates the rent in the first place. This yields that the parties' equilibrium levels of efforts are the same and can be calculated (as in a traditional Tullock game) by reference to the price P rather than to the different valuations of the contested rent; a double asterisk denotes equilibrium values with Coasean bargaining:

$$B_L^{**} = B_H^{**} = \frac{r}{4}P \tag{6}$$

A consequence of the equivalence between the parties' incentives is that the total rent dissipation can also be calculated as in a traditional Tullock game and expressed as a function of the price P as follows:

$$D^{**} = B_L^{**} + B_H^{**} = \frac{r}{2}P \tag{7}$$

Proposition 5 *If rents can be reallocated ex post at a price P, the parties' equilibrium levels of efforts are the same $B_L^{**} = B_H^{**}$, with $\frac{\partial B_L^{**}}{\partial \gamma} = \frac{\partial B_H^{**}}{\partial \gamma} \geq 0$, $\frac{\partial B_L^{**}}{\partial r} = \frac{\partial B_H^{**}}{\partial r} > 0$ and $\frac{\partial B_L^{**}}{\partial P} = \frac{\partial B_H^{**}}{\partial P} > 0$. The equilibrium payoffs are instead different $U_L^{**} < U_H^{**}$, with $\frac{\partial U_L^{**}}{\partial \gamma} \geq 0$, $\frac{\partial U_H^{**}}{\partial \gamma} \leq 0$, $\frac{\partial U_L^{**}}{\partial r} < 0$, $\frac{\partial U_H^{**}}{\partial r} < 0$, $\frac{\partial U_L^{**}}{\partial P} > 0$, and $\frac{\partial U_H^{**}}{\partial P} < 0$. The rent dissipation is $0 < D^{**} \leq \frac{1}{2}V_H$, with $\frac{\partial D^{**}}{\partial \gamma} \geq 0$, $\frac{\partial D^{**}}{\partial r} > 0$, and $\frac{\partial D^{**}}{\partial P} > 0$. The rent misallocation is $M^{**} = 0$.*

4.2 Dissipation and misallocation

The opportunity for ex post reallocations affects the ex ante parties' incentives to expend in rent-seeking. In the presence of an opportunity for ex post reallocation of the contested rent, the parties behave as rent-seekers for the expected price P, price at which the rent will be reallocated ex post. As a consequence, the social loss of rent-seeking depends on the parties' bargaining power in the negotiation phase. Low-valuing parties with greater bargaining power are able to extract a higher price. Therefore, as the balance of the bargaining power shifts from the high- to the low-valuing party, parties exert more effort and the social loss of rent-seeking increases.

It further emerges that those events that benefit the low-valuing party have an adverse effect on the high-valuing party's payoff. For instance, if the parties' valuations approach each other, the low-valuing party will benefit from an implicit increase in his bargaining power, as the minimum price he can accept shifts up, and hence his payoff increases. This causes the payoff of the high-valuing party to decrease accordingly. To the same effect, a direct increase in the price improves the position of the low-valuing party and worsens the position of the high-valuing party. A change in the parties' strength has however the same effect on both, as when r increases, the payoffs of both parties decreases. All of these changes in the respective advantages of the parties are largely redistributive, as we have noticed that any of these changes increase the rent dissipation thereby reducing social welfare.

It is interesting to compare the outcomes obtained under Coasean bargaining with the results of the previous section.

Corollary 6 $B_L^{**} > B_L^*$, $U_L^{**} > U_L^*$; *when the low-valuing party has little bargaining power, we have* $B_H^{**} < B_H^*$, $U_H^{**} > U_H^*$, $D^{**} < D^* < D^* + M^*$; *when the low-valuing party has strong bargaining power, we have* $B_H^{**} > B_H^*$, $U_H^{**} < U_H^*$, *and* $D^{**} > D^*$; *for even greater values of the low-valuing party's bargaining power and asymmetric valuations, we have* $D^{**} > D^* + M^*$ $(M^{**} = 0)$.

Coasean bargaining unambiguously improves the position of the low-valuing party, who exerts more effort and is able to appropriate a larger share of the rent through ex post negotiations with his high-valuing opponent. A variation in the relative bargaining power of the parties will only quantitatively affect this result, determining the price to be paid for the ex post reallocation.

The position of the high-valuing party, instead, crucially depends on the parties' bargaining power and resulting price. If the price is low, also the high-valuing party will benefit from the possibility of Coasean bargaining, receiving a larger payoff. However, under different circumstances, the high-valuing player may suffer a prejudice from the possibility of Coasean bargaining. Namely, the effect of Coasean bargaining on the high-valuing party's payoff may vary due to the presence of two countervailing effects. First, Coasean bargaining makes it possible for the rent to be purchased ex post by the high-valuing party. This lowers the high-valuing party's need to invest in rent-seeking contest for a direct appropriation of the rent. Second, the possibility of ex post reallocations makes the low-valuing party behave more aggressively. The net impact of these two countervailing effects depends on the parties' bargaining power. If the high-valuing party has a sufficiently strong bargaining position, the price will be low and the first effect will dominate over the second, improving his payoff. If the low-valuing party has a strong bargaining power, the price will be high and closer to the full rent valuation of the prospective buyer. In this case, the second effect prevails and we have the paradoxical result that giving the high-valuing party the opportunity to buy the rent if he failed to win worsens his position, causing a decrease in his payoff.

By the same token, Coasean bargaining may increase or decrease rent dissipation, depending on the parties' bargaining power. Consider two limit cases: When the high-valuing party has full bargaining power ($\alpha = 0$), the price is equal to V_L and the two contestants compete as if they were both low-valuing parties. Hence, the opportunity for ex post reallocation of the rent reduces the equilibrium level of dissipation. On the contrary, when the low-valuing party has full bargaining power ($\alpha = 1$), the price is equal to V_H and both parties compete for the same higher value. Evidently, in this case the dissipation is larger in the Coasean environment. Although misallocation costs are totally eliminated by

Coasean bargaining, the possibility for ex post reallocation may actually increase the total social cost of rent-seeking.

We can identify an iso-dissipation boundary where the rent dissipation under Coasean bargaining is equal to the rent dissipation without Coasean bargaining. To illustrate, consider the case of constant returns to scale, $r = 1$. We can easily identify a threshold level of α (the bargaining power of the low-valuing party), that will generate equal levels of dissipation:

$$\alpha^D = \frac{\gamma}{1+\gamma}$$

Likewise, we can identify the threshold level of α, such that the rent dissipation under Coasean bargaining is equal to the total social loss without Coasean bargaining (rent dissipation plus rent misallocation):

$$\alpha^{D+M} = 3\frac{\gamma}{1+\gamma}$$

When $0 \leq \alpha < \alpha^{D+M}$, Coasean bargaining leads to a reduction in total rent-seeking losses. Ex post reallocation of the rent eliminates misallocation costs. These savings more than compensate the possible increase in rent dissipation, which will occur for $\alpha^D < \alpha < \alpha^{D+M}$. On the contrary, when $\alpha^{D+M} < \alpha \leq 1$, the opportunity for ex post reallocation of the rent increases the total social loss.

5 Incomplete information

In this section, we relax the assumption of complete information used in the previous analysis. Here players do not observe their opponent's rent valuation before making their effort decisions, but only know their own valuation. We assume that low-valuing and high-valuing players are equally likely.

5.1 Rent-seeking without Coasean bargaining

If the parties cannot resell the rent, a party with value $V_i \in \{V_L, V_H\}$ exerts effort B_i and obtains an expected payoff $U_i(V_i, B_i, B_L, B_H)$, which depends on the type and effort expended by the other player. The other player will choose effort B_L or B_H according to whether he is a low- or high-valuing individual. This generates expected payoffs equal to:

$$U_i(V_i, B_i, B_L, B_H) = \frac{1}{2}\frac{B_i^r}{B_i^r + B_L^r}V_i + \frac{1}{2}\frac{B_i^r}{B_i^r + B_H^r}V_i - B_i. \tag{8}$$

The first and second terms on the right-hand side of (8) represent the equally-likely rent appropriations, depending on whether the other player has a low or high valuation of the rent. The parties maximize their payoffs taking into account the probability that their opponent is of a certain type according to the following first order condition:

$$\frac{\partial U_i(V_i, B_i, B_L, B_H)}{\partial B_i} = \frac{1}{2}\left(\frac{rB_i^{r-1}B_L^r}{\left(B_i^r + B_L^r\right)^2} + \frac{rB_i^{r-1}B_H^r}{\left(B_i^r + B_H^r\right)^2}\right)V_i - 1 = 0 \tag{9}$$

We will use the symbol ‡ to denote equilibrium values with incomplete information.

Proposition 7 *With incomplete information, the parties' equilibrium levels of efforts are directly proportional to the parties' valuations of the rent:* $\frac{B_L^{\ddagger}}{B_H^{\ddagger}} = \gamma$*. The equilibrium payoffs are instead less than proportional to the parties' valuations:* $\frac{U_L^{\ddagger}}{U_H^{\ddagger}} < \gamma$*.*

These results are similar to those derived for the case of complete information. Parties with higher valuations make larger rent-seeking expenditures and earn larger payoffs. While the ratio of the parties' efforts is equal to the ratio of their valuations, the high-valuing party gains a more-than-proportional advantage on his opponent with respect to payoffs.

We can now proceed to consider the social costs of rent-seeking in the incomplete information setting.

Proposition 8 *With incomplete information, the total rent dissipation is* $0 < D^{\ddagger} < \frac{1}{2}V_H$ *and the rent misallocation is* $0 < M^{\ddagger} < \frac{1}{2}V_H$*. The total social loss is* $0 < D^{\ddagger} + M^{\ddagger} < V_H$*.*

These findings are analogous to the ones in the complete information case.

5.2 Rent-seeking with Coasean bargaining

Let us now consider the an imperfect-information rent-seeking contest followed by Coasean bargaining. We assume that after the rent-seeking game, the parties' valuations are revealed. Ex ante, the parties know that, if the rent is won by a low-valuing contestant facing a high-valuing opponent, they will be able to engage in ex post bargaining. They also know that in all other scenarios (in which parties have the same valuations or in which the rent is originally appropriated by the high-valuing player) no reallocation will take place. The parties' payoffs can thus be rewritten in expected terms, taking into account the various possible outcomes of the game and the possible ex post reallocation at price P:

$$U_L(V_L, B_i, B_L, B_H) = \frac{1}{2}\frac{B_i^r}{B_i^r + B_L^r}V_L + \frac{1}{2}\frac{B_i^r}{B_i^r + B_H^r}P - B_i \tag{10a}$$

$$U_H(V_H, B_j, B_L, B_H) = \frac{1}{2}\frac{B_j^r}{B_j^r + B_H^r}V_H + \frac{1}{2}\left(V_H - \frac{B_L^r}{B_j^r + B_L^r}P\right) - B_j. \tag{10b}$$

Although there is no closed-form solution for the equilibrium bidding strategies $B_H^{\ddagger\ddagger}$ and $B_L^{\ddagger\ddagger}$, the following results can be obtained:

Proposition 9 *With incomplete information, if rents can be reallocated ex post at a price* P*,* $B_H^{\ddagger\ddagger} > B_L^{\ddagger\ddagger}$ *with* $\frac{B_L^{\ddagger\ddagger}}{B_H^{\ddagger\ddagger}} > \gamma$*. The rent dissipation is* $0 < D^{\ddagger\ddagger} < \frac{1}{2}V_H$ *and the rent misallocation is* $M^{\ddagger\ddagger} = 0$*.*

Corollary 10 $B_L^{\ddagger\ddagger} > B_L^{\ddagger}$*,* $U_L^{\ddagger\ddagger} > U_L^{\ddagger}$*, when the low-valuing party has little bargaining power, we have* $B_H^{\ddagger\ddagger} <> B_H^{\ddagger}$*,* $U_H^{\ddagger\ddagger} <> U_H^{\ddagger}$*,* $D^{\ddagger\ddagger} <> D^{\ddagger}$*, and* $D^{\ddagger\ddagger} <> D^{\ddagger} + M^{\ddagger}$ *(*$M^{\ddagger\ddagger} = 0$*).*

The results of Coasean bargaining under incomplete information are qualitatively similar to those derived for the complete information case. Most importantly, also in this case the opportunity for ex post Coasean reallocations has an indeterminate effect on the overall social cost of rent-seeking.

6 Conclusions

New dimensions of the rent-seeking problem emerge when the rent-seekers have different valuations of the sought-after rent. In this paper, we have analyzed these new dimensions, showing how rent-seeking incentives are affected by such valuation asymmetries. Unlike winner-takes-all auctions, rent-seeking does not guarantee the efficient allocation of the rents. This paper highlights an interesting interrelationship between these misallocation costs and other rent-seeking costs due to the dissipation of valuable resources. We derive a result that runs contrary to the conventional wisdom in the literature: when the parties' rent valuations are sufficiently divergent, equilibrium levels of efforts may be decreasing in the marginal returns to effort. Further, we find that rent-seeking contests between parties with different valuations always yield less rent dissipation than the same contests with equally-valuing parties. The more parties are alike, the larger the final dissipation of valuable resources will be.

When parties have unequal valuations of the rent, rent-seeking contests may hence serve as valuable mechanisms to force parties to reveal their preferences through investment choices. High-valuing parties will fight more aggressively and will thus have higher probabilities to appropriate the rent. In this sense, rent-seeking contests may play a valuable allocative role. Given the mechanics of rent-seeking contests, however, this allocative role is imperfect and misallocations remain possible.

In this paper, we thus extend the analysis to study the effect of ex post reallocation on the parties' incentives and total rent dissipation. According to the Coase theorem, ex post contractual solutions correct possible initial misallocation of resources. Whenever the sought-after prize is transferable, the opportunity for ex post reallocation eliminates any possible misallocation. We show however that the possibility of ex post transfers, while correcting misallocation problems, may exacerbate rent dissipation due to an increase in the stakes of the game. The increase in rent dissipation may in some situations exceed the allocative gains, rendering the alienability of the rent socially undesirable. These results shed light on important policy questions, and provide important foundations for the design of rent-seeking contests.

Further studies should consider the relationship between asymmetries in the parties' valuation of the rent and other forms of asymmetries, such as differences in the parties' returns to effort or rent-seeking costs, and the fact that some rent-seeking contests are not purely redistributive of existing resources but enhance the value of such resources.

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A Appendix

Proof of proposition 1. Each party chooses his level of effort B_i in order to maximize his payoff from participation in (1). Straightforward calculations on the FOCs in (4) yield the following result:

$$B_L^* = r\frac{\gamma^r}{(1+\gamma^r)^2}V_L \tag{11a}$$

$$B_H^* = r\frac{\gamma^r}{(1+\gamma^r)^2}V_H \tag{11b}$$

It follows that in equilibrium, $\frac{B_L^*}{B_H^*} = \frac{V_L}{V_H} = \gamma$. The SOCs are

$$\frac{\partial^2 U_L}{\partial B_L^2} = -\frac{rB_L^r B_H^r}{B_L^2 (B_H^r + B_L^r)^3} V_L \left[r\left(B_L^r - B_H^r\right) + B_H^r + B_L^r\right] < 0 \tag{12a}$$

$$\frac{\partial^2 U_H}{\partial B_H^2} = -\frac{rB_L^r B_H^r}{B_H^2 (B_H^r + B_L^r)^3} V_H \left[r\left(B_H^r - B_L^r\right) + B_H^r + B_L^r\right] < 0 \tag{12b}$$

It is evident that (12a) holds true because $r\left(B_L^r - B_H^r\right) + B_H^r + B_L^r > 0$ for all B_L and B_H and $r \leq 1$. Likewise (12b) holds true. Thus, (B_L^*, B_H^*) is the unique Nash equilibrium of the game. Substituting (11) into (1), we obtain:

$$U_L^* = \frac{\gamma^{2r} + (1-r)\gamma^r}{(1+\gamma^r)^2}V_L \tag{13a}$$

$$U_H^* = \frac{1 + (1-r)\gamma^r}{(1+\gamma^r)^2}V_H \tag{13b}$$

which yields $\frac{U_L^*}{U_H^*} = \frac{\gamma^{2r}+(1-r)\gamma^r}{1+(1-r)\gamma^r}\gamma$. Noting that $0 < \frac{\gamma^{2r}+(1-r)\gamma^r}{1+(1-r)\gamma^r} < 1$, we have $\frac{U_L^*}{U_H^*} < \gamma$. *QED*

Proof of corollary 2. For $\gamma < 1$, we have:

$$\frac{\partial B_L^*}{\partial \gamma} = r\gamma^r \frac{1 + r\left(1-\gamma^r\right) + \gamma^r}{(\gamma^r+1)^3}V_H > 0$$

$$\frac{\partial B_H^*}{\partial \gamma} = r\gamma^{r-1} \frac{r\left(1-\gamma^r\right)}{(\gamma^r+1)^3}V_H > 0$$

$$\frac{\partial U_L^*}{\partial \gamma} = \gamma^r \frac{(1+\gamma^r)^2 - r^2\left(1-\gamma^r\right)}{(\gamma^r+1)^3}V_H > 0$$

$$\frac{\partial U_H^*}{\partial \gamma} = -r\gamma^{r-1} \frac{1 + r\left(1-\gamma^r\right) + \gamma^r}{(\gamma^r+1)^3}V_H < 0. \quad \textit{QED}$$

Proof of corollary 3. Let $g(r,\gamma) \equiv 1 + \gamma^r + r\left(\log\gamma\right)\left(1-\gamma^r\right)$. Then

$$\frac{\partial B_L^*}{\partial r} = g(r,\gamma)\frac{\gamma^{r+1}}{(\gamma^r+1)^3}V_H$$

$$\frac{\partial B_H^*}{\partial r} = g(r,\gamma)\frac{\gamma^r}{(\gamma^r+1)^3}V_H$$

These derivatives have the same sign as $g(r,\gamma)$. It is readily verified that $\lim_{\gamma\downarrow 0} g(r,\gamma) < 0$ and $\lim_{\gamma\uparrow 1} g(r,\gamma) > 0$. Since $\frac{\partial g(r,\gamma)}{\partial\gamma} > 0$, there is a $\bar{\gamma}$ such that $\frac{\partial B_L^*}{\partial r}$ and $\frac{\partial B_H^*}{\partial r}$ are negative for $\gamma < \bar{\gamma}$ and positive for $\gamma > \bar{\gamma}$. It can be shown that $\frac{\partial U_H^*}{\partial r} > 0$ for γ close to 0 and $\frac{\partial U_H^*}{\partial r} < 0$ for γ close to 1. Finally, we have:

$$\frac{\partial U_L^*}{\partial r} = -\frac{\gamma^{r+1}}{(\gamma^r+1)^3} V_H \left[1+\gamma^r - (\log\gamma)(1+\gamma^r - r(1-\gamma^r))\right] < 0. \ QED$$

Proof of proposition 4. Substituting (11) in (2) yields:

$$D^* = B_L^* + B_H^* = r\frac{\gamma^r}{(1+\gamma^r)^2}(1+\gamma)V_H \tag{14}$$

with $\frac{\partial D^*}{\partial\gamma} = \frac{\partial B_L^*}{\partial\gamma} + \frac{\partial B_H^*}{\partial\gamma} > 0$ and $\frac{\partial D^*}{\partial r} = \frac{\partial B_L^*}{\partial r} + \frac{\partial B_H^*}{\partial r}$, which is negative for γ close to 0 and positive for γ close to 1. Because $\frac{\gamma^r}{(1+\gamma^r)^2} < \frac{1}{4}$, we have $0 < D^* < \frac{1}{2}V_H$.

Substituting (11) in (3) yields:

$$M^* = \frac{(B_L^*)^r}{(B_L^*)^r + (B_H^*)^r}(V_H - V_L) = \frac{\gamma^r}{1+\gamma^r}(1-\gamma)V_H$$

so that:

$$\frac{\partial M^*}{\partial\gamma} = h(r,\gamma)\frac{\gamma^{r-1}}{(1+\gamma^r)^2}V_H$$

with $h(r,\gamma) \equiv r(1-\gamma) - \gamma(1+\gamma^r)$. Because $\lim_{\gamma\downarrow 0} h(r,\gamma) > 0$, $\lim_{\gamma\uparrow 1} h(r,\gamma) < 0$ and $\frac{\partial}{\partial\gamma}h(r,\gamma) < 0$, there is a $\bar{\gamma}$ such that $\frac{\partial M^*}{\partial\gamma}$ is positive for $\gamma < \bar{\gamma}$ and negative for $\gamma > \bar{\gamma}$. Moreover, we have:

$$\frac{\partial M^*}{\partial r} = \log\gamma\frac{\gamma^r(1-\gamma)}{(1+\gamma^r)^2}V_H < 0$$

Finally, it is readily verified that $0 < M^* < \frac{1}{2}V_H$.

The total social loss due to rent-seeking is:

$$D^* + M^* = \frac{\gamma^r}{(1+\gamma^r)^2}V_H\left[r(1+\gamma) + (1-\gamma)(1+\gamma^r)\right] \tag{15}$$

The total social loss varies as follows:

$$\begin{aligned}\frac{\partial}{\partial\gamma}(D^*+M^*) &= \frac{\partial D^*}{\partial\gamma} + \frac{\partial M^*}{\partial\gamma} \\ &= l(r,\gamma)\frac{\gamma^{r-1}}{(\gamma^r+1)^3}V_H\end{aligned}$$

where $l(r,\gamma) \equiv r(1+\gamma^r) - \gamma(1+\gamma^r)^2 + r^2(1+\gamma)(1-\gamma^r)$. $\frac{\partial}{\partial\gamma}(D^*+M^*)$ is positive for γ close to 0, because $\frac{\partial D^*}{\partial\gamma}$ and $\frac{\partial M^*}{\partial\gamma}$ are positive. $\frac{\partial}{\partial\gamma}(D^*+M^*)$ is negative for γ close to 1, since $\lim_{\gamma\uparrow 1} l(r,\gamma) < 0$. Furthermore:

$$\begin{aligned}\frac{\partial}{\partial r}(D^*+M^*) &= \frac{\partial D^*}{\partial r} + \frac{\partial M^*}{\partial r} \\ &= m(r,\gamma)\frac{\gamma^r}{(\gamma^r+1)^3}V_H\end{aligned}$$

where $m(r,\gamma) \equiv (1+\gamma^r)(1+\gamma) + (\log\gamma)(\gamma^r+1)(1-\gamma) + r(\log\gamma)(1-\gamma^r)(1+\gamma)$. $\frac{\partial}{\partial r}(D^* + M^*)$ is negative for γ close to 0, because $\frac{\partial D^*}{\partial r}$ and $\frac{\partial M^*}{\partial r}$ are negative; $\frac{\partial}{\partial r}(D^* + M^*)$ is positive for γ close to 1, since $\lim_{\gamma \uparrow 1} m(r,\gamma) > 0$.*QED*

Proof of proposition 5. The following claims derive from straightforward manipulation of (6): $B_L^{**} = B_H^{**}$, $\frac{\partial B_L^{**}}{\partial \gamma} = \frac{\partial B_H^{**}}{\partial \gamma} \geq 0$, $\frac{\partial B_L^{**}}{\partial r} = \frac{\partial B_H^{**}}{\partial r} > 0$ and $\frac{\partial B_L^{**}}{\partial P} = \frac{\partial B_H^{**}}{\partial P} > 0$. Substituting (6) into (5), we have $U_H^{**} = U_L^{**} + V_H - P$, which yields $U_L^{**} < U_H^{**}$. We further have:

$$U_L^{**} = \frac{2-r}{4} P = \frac{2-r}{4} (\alpha + (1-\alpha)\gamma) V_H$$

with $\frac{\partial U_L^{**}}{\partial \gamma} \geq 0$, $\frac{\partial U_L^{**}}{\partial r} < 0$, $\frac{\partial U_L^{**}}{\partial P} > 0$. Likewise, we have:

$$U_H^{**} = V_H - \frac{2+r}{4} P = \left[1 - \frac{2+r}{4}(\alpha + (1-\alpha)\gamma)\right] V_H$$

with $\frac{\partial U_H^{**}}{\partial \gamma} \leq 0$, $\frac{\partial U_H^{**}}{\partial r} < 0$, $\frac{\partial U_L^{**}}{\partial P} < 0$. For the rent dissipation

$$D^{**} = \frac{r}{2} P = \frac{r}{2}(\alpha + (1-\alpha)\gamma) V_H \leq \frac{1}{2} V_H$$

we have $\frac{\partial D^{**}}{\partial \gamma} \geq 0$, $\frac{\partial D^{**}}{\partial r} > 0$ and $\frac{\partial D^{**}}{\partial P} > 0$. With Coasean bargaining we have $M^{**} = 0$ by hypothesis. *QED*

Proof of corollary 6. Using (6) and (11a) we have $B_L^{**} = \frac{r}{4} P > r \frac{\gamma^r}{(1+\gamma^r)^2} V_L = B_L^*$, because $P \geq V_L$ and $\frac{\gamma^r}{(1+\gamma^r)^2} < \frac{1}{4}$. Using (5a), (6), and (13a), we obtain $U_L^{**} = \frac{2-r}{4} P > \frac{\gamma^{2r} + (1-r)\gamma^r}{(1+\gamma^r)^2} V_L = U_L^{**}$, because $V_L \leq P$ and $\frac{2-r}{4} > \frac{\gamma^{2r}+(1-r)\gamma^r}{(1+\gamma^r)^2}$. Using (6) and (11b) we obtain $B_H^{**} = \frac{r}{4} P < r \frac{\gamma^r}{(1+\gamma^r)^2} V_H = B_H^*$ if $P < \frac{4\gamma^r}{(1+\gamma^r)^2} V_H$ and $B_H^{**} \geq B_H^*$ otherwise (note that $\frac{4\gamma^r}{(1+\gamma^r)^2} < 1$). Using (5b), (6), and (13b), we obtain $U_H^{**} = V_H - \frac{2+r}{4} P > \frac{1+(1-r)\gamma^r}{(1+\gamma^r)^2} V_H = U_H^*$ if $P < \frac{4\gamma^r}{(1+\gamma^r)^2} \frac{3-r+\gamma^r}{2+r} V_H$ and $U_H^{**} \leq U_H^*$, otherwise.

Concerning the social loss, using (14) we have $D^{**} = \frac{r}{2} P < r \frac{\gamma^r}{(1+\gamma^r)^2} (1+\gamma) V_H = D^*$ if $P < \frac{2\gamma^r}{(1+\gamma^r)^2} (1+\gamma) V_H$ and $D^{**} \geq D^*$, otherwise. Using (15), for γ close to 0 and r close to 1, it is readily verified that $D^{**} + M^{**} < D^* + M^*$ for α close to 0 , and $D^{**} + M^{**} > D^* + M^*$for α close to 1. *QED*

Proof of proposition 7. Straightforward calculations on the FOCs in (9) yield the following result:

$$\frac{1}{2} V_L \left(\frac{r}{4B_L^\ddagger} + \frac{r\left(B_L^\ddagger\right)^{r-1} \left(B_H^\ddagger\right)^r}{\left(\left(B_L^\ddagger\right)^r + \left(B_H^\ddagger\right)^r\right)^2} \right) = 1 \tag{16a}$$

$$\frac{1}{2} V_H \left(\frac{r}{4B_H^\ddagger} + \frac{r\left(B_H^\ddagger\right)^{r-1} \left(B_L^\ddagger\right)^r}{\left(\left(B_L^\ddagger\right)^r + \left(B_H^\ddagger\right)^r\right)^2} \right) = 1 \tag{16b}$$

Substituting $B_L = \zeta B_H$ in (16a) and (16b) yields (after some straightforward manipulations):

$$\begin{aligned} B_L^{\ddagger} &= \frac{r}{2}\left(\frac{1}{4} + \frac{\zeta^r}{(1+\zeta^r)^2}\right) V_L \\ B_H^{\ddagger} &= \frac{r}{2}\left(\frac{1}{4} + \frac{\zeta^r}{(1+\zeta^r)^2}\right) V_H \end{aligned}$$

so that it is readily established that $\zeta \equiv \frac{B_L^{\ddagger}}{B_H^{\ddagger}} = \frac{V_L}{V_H} = \gamma$. Finally, observe that for $0 < r \leq 1$, and $B_i > 0$,

$$\frac{\partial^2 U_i(V_i, B_i, B_L, B_H)}{\partial B_i^2} < 0$$

This ensures that the second-order condition is satisfied as well.

Substituting into (8) we obtain that the parties' payoffs in equilibrium are:

$$\begin{aligned} U_L^{\ddagger} &= \frac{1}{4} V_L + \frac{1}{2} \frac{\left(B_L^{\ddagger}\right)^r}{\left(B_L^{\ddagger}\right)^r + \left(B_H^{\ddagger}\right)^r} V_L - B_L^{\ddagger} \\ U_H^{\ddagger} &= \frac{1}{4} V_H + \frac{1}{2} \frac{\left(B_H^{\ddagger}\right)^r}{\left(B_L^{\ddagger}\right)^r + \left(B_H^{\ddagger}\right)^r} V_H - B_H^{\ddagger} \end{aligned}$$

so that:

$$\begin{aligned} \frac{U_L^{\ddagger}}{U_H^{\ddagger}} &= \frac{(\gamma^r + 1)\gamma\left(\frac{1}{4}V_H - B_H^{\ddagger}\right) + \frac{1}{2}\gamma^{r+1} V_H}{(\gamma^r + 1)\left(\frac{1}{4}V_H - B_H^{\ddagger}\right) + \frac{1}{2} V_H} \\ &< \frac{(\gamma^r + 1)\left(\frac{1}{4}V_H - B_H^{\ddagger}\right) + \frac{1}{2} V_H}{(\gamma^r + 1)\left(\frac{1}{4}V_H - B_H^{\ddagger}\right) + \frac{1}{2} V_H}\gamma = \gamma. \;\; \textit{QED} \end{aligned}$$

Proof of proposition 8. $D^{\ddagger}$ is the expected value of the sum of the parties' efforts:

$$\begin{aligned} D^{\ddagger} &= \frac{1}{4}\left(B_L^{\ddagger} + B_L^{\ddagger}\right) + \frac{1}{4}\left(B_H^{\ddagger} + B_H^{\ddagger}\right) + \frac{1}{2}\left(B_L^{\ddagger} + B_H^{\ddagger}\right) \\ &= B_L^{\ddagger} + B_H^{\ddagger} = \frac{r}{2}(1+\gamma)\left(\frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2}\right) V_H \end{aligned}$$

Since $D^{\ddagger}$ increases both in γ and in r, it can be easily shown that $0 < D^{\ddagger} < \frac{1}{2}V_H$. $M^{\ddagger}$ is as follows:

$$\begin{aligned} M^{\ddagger} &= \frac{1}{2} \frac{\left(B_L^{\ddagger}\right)^r}{\left(B_L^{\ddagger}\right)^r + \left(B_H^{\ddagger}\right)^r} (V_H - V_L) \\ &= \frac{1}{2}(1-\gamma)\frac{\gamma^r}{1+\gamma^r} V_H. \end{aligned}$$

Since $\frac{\gamma^r}{1+\gamma^r} < \frac{1}{2}$, it is easy to show that $0 < M^{\ddagger} < \frac{1}{2}V_H$. Finally, $0 < D^{\ddagger} + M^{\ddagger} < V_H$ because $D^{\ddagger} < \frac{1}{2}V_H$ and $M^{\ddagger} < \frac{1}{2}V_H$. *QED*

Proof of proposition 9. Differentiating (10) yields:

$$\begin{aligned}\frac{\partial U_L(V_L,B_i,B_L,B_H)}{\partial B_i} &= \frac{1}{2}\frac{rB_i^{r-1}B_L^r}{(B_i^r+B_L^r)^2}V_L+\frac{1}{2}\frac{rB_i^{r-1}B_H^r}{(B_i^r+B_H^r)^2}P-1\\ \frac{\partial U_H(V_H,B_j,B_L,B_H)}{\partial B_j} &= \frac{1}{2}\frac{rB_j^{r-1}B_H^r}{\left(B_H^r+B_j^r\right)^2}V_H+\frac{1}{2}\frac{rB_j^{r-1}B_L^r}{\left(B_L^r+B_j^r\right)^2}P-1\end{aligned}$$

The FOCs in equilibrium, in which player L exerts effort $B_i = B_L$ and player H exerts effort $B_j = B_H$, are:

$$B_L = \frac{r}{8}V_L+\frac{1}{2}r\frac{B_L^rB_H^r}{(B_L^r+B_H^r)^2}P \tag{17}$$

$$B_H = \frac{r}{8}V_H+\frac{1}{2}r\frac{B_L^rB_H^r}{(B_H^r+B_L^r)^2}P \tag{18}$$

Observe that for $r \leq 1$, all $B_i > 0$, and $B_H > B_L$,

$$\frac{\partial^2 U_i(V_i,B_i,B_L,B_H)}{\partial B_i^2}<0.$$

Therefore, the second-order condition is satisfied as well. Substituting $B_L = \xi B_H$ in (17) and (18) yields:

$$\begin{aligned}B_L^{\ddagger\ddagger} &= \frac{r}{8}V_L+\frac{1}{2}r\frac{\xi^r}{(\xi^r+1)^2}P\\ B_H^{\ddagger\ddagger} &= \frac{r}{8}V_H+\frac{1}{2}r\frac{\xi^r}{(\xi^r+1)^2}P.\end{aligned}$$

From $B_L^{\ddagger\ddagger} - \xi B_H^{\ddagger\ddagger} = 0$, it is readily established that ξ is a solution to

$$\begin{aligned}\gamma &= \frac{(1+\xi^r)^2\xi-4\alpha(1-\xi)\xi^r}{(1+\xi^r)^2+4(1-\alpha)(1-\xi)\xi^r}\\ &= \xi-\frac{4(1-\alpha)(1-\xi)\xi^{r+1}+4\alpha(1-\xi)\xi^r}{(1+\xi^r)^2+4(1-\alpha)(1-\xi)\xi^r}\end{aligned}$$

Note that $\xi < 1$ for all $\gamma < 1$ because $\xi \geq 1$ implies $\gamma \geq 1$, which is not possible by hypothesis. Therefore, $B_H^{\ddagger\ddagger} > B_L^{\ddagger\ddagger}$. Moreover, $\gamma < \xi = \frac{B_L^{\ddagger\ddagger}}{B_H^{\ddagger\ddagger}}$.

Moreover, $D^{\ddagger\ddagger}$ can be written as

$$\begin{aligned}D^{\ddagger\ddagger} &= B_L^{\ddagger\ddagger}+B_H^{\ddagger\ddagger}=\frac{r}{8}(V_L+V_H)+r\frac{\xi^r}{(\xi^r+1)^2}P\\ &< \frac{1}{2}V_H.\end{aligned}$$

Finally, $M^{\ddagger\ddagger} = 0$. *QED*

Proof of corollary 10. The parties' equilibrium levels of efforts can be written as:

$$\begin{aligned}B_L^{\ddagger\ddagger} &= \frac{r}{8}V_L+\frac{1}{2}rp_L^{\ddagger\ddagger}\left(1-p_L^{\ddagger\ddagger}\right)P\\ B_H^{\ddagger\ddagger} &= \frac{r}{8}V_H+\frac{1}{2}rp_L^{\ddagger\ddagger}\left(1-p_L^{\ddagger\ddagger}\right)P\end{aligned}$$

where

$$p_L^{\ddagger\ddagger} = \frac{(B_L^{\ddagger\ddagger})^r}{(B_L^{\ddagger\ddagger})^r + (B_H^{\ddagger\ddagger})^r}$$

is the equilibrium probability that a low-valuing party wins against a high-valuing one. Recall that

$$\begin{aligned} B_L^{\ddagger} &= \frac{1}{2}r\left(\frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2}\right)V_L \\ &= \frac{1}{2}r\left(\frac{1}{4} + p_L^{\ddagger}\left(1 - p_L^{\ddagger}\right)\right)V_L \end{aligned}$$

where $p_L^{\ddagger}$ is the equilibrium probability that a low-valuing party wins against a high-valuing one if there is no possibility of ex post reallocation of the rent. We have observed that

$$\frac{B_L^{\ddagger\ddagger}}{B_H^{\ddagger\ddagger}} > \gamma = \frac{B_L^{\ddagger}}{B_H^{\ddagger}}$$

which implies that

$$\frac{1}{2} > p_L^{\ddagger\ddagger} > p_L^{\ddagger}$$

so that

$$p_L^{\ddagger\ddagger}\left(1 - p_L^{\ddagger\ddagger}\right) > p_L^{\ddagger}\left(1 - p_L^{\ddagger}\right)$$

and in turn

$$B_L^{\ddagger\ddagger} > B_L^{\ddagger}.$$

Moreover, if $\gamma = 0$ and $\alpha < 1$, $B_L^{\ddagger\ddagger} > 0$ so that $p_L^{\ddagger\ddagger} > 0$ and in turn, $B_H^{\ddagger\ddagger} > \frac{r}{8}V_H = B_H^{\ddagger}$. Then, if $\gamma = \frac{9}{26}$, $r = 1$, and $\alpha = 0$, $B_H^{\ddagger\ddagger} = \frac{1}{8}V_H + \frac{1}{26}V_H < \frac{1}{8}V_H + \frac{234}{1225}V_H = B_H^{\ddagger}$.

Analogously,

$$\begin{aligned} U_L^{\ddagger\ddagger} &= \left(\frac{1}{4} - \frac{r}{8}\right)V_L + \frac{1}{2}p_L^{\ddagger\ddagger}P\left[1 - r\left(1 - p_L^{\ddagger\ddagger}\right)\right] \\ &> \left(\frac{1}{4} - \frac{r}{8}\right)V_L + \frac{1}{2}p_L^{\ddagger}V_L\left[1 - r\left(1 - p_L^{\ddagger}\right)\right] \\ &= U_L^{\ddagger} \end{aligned}$$

Moreover, if $\alpha > 0$,

$$\lim_{\gamma\downarrow 0} U_H^{\ddagger} = \frac{3}{4}V_H - \lim_{\gamma\downarrow 0} B_H^{\ddagger} > \frac{3}{4}V_H - \lim_{\gamma\downarrow 0} B_H^{\ddagger\ddagger} > \lim_{\gamma\downarrow 0} U_H^{\ddagger\ddagger}.$$

The first inequality follows from $\lim_{\gamma\downarrow 0} B_H^{\ddagger} < \lim_{\gamma\downarrow 0} B_H^{\ddagger\ddagger}$, the second follows from the fact that in rent-seeking with Coasean bargaining the high-valuing party may have to buy the rent from the low-valuing party at a positive price. With some manipulations it is straightforward to show that if $\gamma = \frac{9}{26}$, $r = 1$, and $\alpha = 0$, $U_H^{\ddagger\ddagger} \approx 0.529 > 0.267 \approx U_H^{\ddagger}$.

Moreover, $D^{\ddagger\ddagger}$ can be written as

$$D^{\ddagger\ddagger} = B_L^{\ddagger\ddagger} + B_H^{\ddagger\ddagger} = \frac{r}{8}(V_L + V_H) + rp_L^{\ddagger\ddagger}\left(1 - p_L^{\ddagger\ddagger}\right)P.$$

If $\alpha \geq \frac{1}{2}$, then $P \geq \frac{1}{2}(V_H + V_L)$, so that

$$D^{\ddagger\ddagger} \geq \frac{1}{2} r (V_L + V_H) \left(\frac{1}{4} + p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger} \right) \right)$$

Recall that

$$\begin{aligned} D^{\ddagger} &= \frac{1}{2} r (V_L + V_H) \left(\frac{1}{4} + \frac{\gamma^r}{(1+\gamma^r)^2} \right) \\ &= \frac{1}{2} r (V_L + V_H) \left(\frac{1}{4} + p_L^{\ddagger} \left(1 - p_L^{\ddagger} \right) \right) \end{aligned}$$

Because

$$p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger} \right) > p_L^{\ddagger} \left(1 - p_L^{\ddagger} \right)$$

it immediately follows that $D^{\ddagger\ddagger} > D^{\ddagger}$.

Finally, we prove that $D^{\ddagger\ddagger} + M^{\ddagger\ddagger} < D^{\ddagger} + M^{\ddagger}$ for α close to 0. We can write

$$D^{\ddagger} + M^{\ddagger} - D^{\ddagger\ddagger} - M^{\ddagger\ddagger} > D^{\ddagger} - D^{\ddagger\ddagger}$$

Moreover, for $\alpha = 0$,

$$\begin{aligned} D^{\ddagger} - D^{\ddagger\ddagger} &= r \left[\frac{1}{2} (V_L + V_H) p_L^{\ddagger} \left(1 - p_L^{\ddagger} \right) - p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger} \right) P \right] \\ &= r \left[\frac{1}{2} (V_L + V_H) p_L^{\ddagger} \left(1 - p_L^{\ddagger} \right) - p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger} \right) V_L \right] \\ &> r \left[\frac{1}{2} (V_L + V_H) p_L^{\ddagger} \left(1 - p_L^{\ddagger} \right) - \frac{1}{4} V_L \right] \\ &= r V_H \left[\frac{1}{2} (1+\gamma) \frac{\gamma^r}{(1+\gamma^r)^2} - \frac{1}{4} \gamma \right] \\ &> r V_H \left[\frac{1}{2} (1+\gamma) \frac{\gamma}{(1+\gamma)^2} - \frac{1}{4} \gamma \right] \\ &> 0. \end{aligned}$$

The first inequality follows from

$$p_L^{\ddagger\ddagger} \left(1 - p_L^{\ddagger\ddagger} \right) < \frac{1}{4},$$

and the second from the fact that

$$\frac{\gamma^r}{(1+\gamma^r)^2}$$

is decreasing in r. *QED*