TI 2006-045/2
Tinbergen Institute Discussion Paper

Debt, Deficits, and Destabilizing Monetary Policy in Open Economies

*Andreas Schabert**
Sweder van Wijnbergen

Faculty of Economics and Econometrics, University of Amsterdam.

** Tinbergen Institute.*

Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam, and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

Please send questions and/or remarks of non-scientific nature to driessen@tinbergen.nl.
Most TI discussion papers can be downloaded at http://www.tinbergen.nl.

Debt, Deficits, and Destabilizing Monetary Policy in Open Economies[1]

Andreas Schabert[2]
University of Amsterdam

Sweder J.G. van Wijnbergen
University of Amsterdam

Revised Version: May 2006
[First Version: February 2006]

Abstract

Blanchard (2005) suggested that active interest rate policy might induce unstable dynamics in highly-indebted economies. We examine this in a dynamic general equilibrium model where Calvo-type price rigidities provide a rationale for inflation stabilization. Unstable dynamics can occur when the CB is aggressively raising the interest rate in response to higher expected inflation. The constraint on stabilizing interest rate policy is tighter the higher the primary deficit and the more open the economy is. If the government cannot borrow from abroad in its own currency, stability requires interest rate policy to be accommodating (passive). Inflation stabilization is nevertheless feasible if the CB uses an instrument not associated with default risk, e.g. money supply.

JEL classification: E52, E63, F41.
Keywords: Fiscal-monetary policy interactions, sovereign default risk, foreign debt, inflation targeting, original sin.

[1] The authors would like to thank Roel Beetsma, Leon Bettendorf, Massimo Giuliodori, Bernd Lucke, Charles van Marrewijk, Ivan Pastine, Jean-Marie Viaene, and other seminar participants at the Erasmus University Rotterdam and at the University of Amsterdam for helpful comments. This research is part of the RTN project "Macroeconomic Policy Design for Monetary Unions", funded by the European Commission (contract number HPRN-CT-2002-00237). The views expressed in this paper are those of the authors and do not necessarily reflect the views of any institution. The usual disclaimer applies.

[2] Address: University of Amsterdam, Department of Economics, Roeterstraat 11, 1018 WB Amsterdam, The Netherlands, Phone: +31 20 525 7159, Fax: +31 20 525 4254, E-mail: a.schabert@uva.nl.

1 Introduction

Inflation targeting has rapidly become the preferred modus operandi of central bankers across the world. The ECB has been downplaying its money targets, instituting a de facto if not yet de jure inflation targeting regime. The UK were early adopters, leaving the US as the only major hold out among OECD countries. And the practice is also rapidly gaining ground among emerging market economies (IMF 2005). Mishkin (2004), for example, stresses the importance of institutional arrangements, but otherwise concludes that "inflation targeting can be an effective tool for emerging market economies to manage their monetary policy".

Yet it is particularly in the latter context that doubts have been raised recently. Blanchard (2005) for example has brought up the possibility that explicit inflation targeting in highly indebted countries with a history of debt servicing problems can lead to "perverse effects on inflation" and may in fact destabilize the economy. The particular example given is inspired by the difficulties Brazil has had in bringing down inflation in spite of substantial fiscal and monetary belt-tightening, and is driven by the interaction between high interest rates, rising interest expenses and fears of debt default (see also Favero and Giavazzi, 2005).

The formal literature on inflation targeting has not yet focused on this issue, although it has obvious practical importance. The literature on inflation targeting is too large to survey even in summary; a recent overview is given by Svensson (2005). At the heart of its theoretical foundation is the idea that the central bank should minimize fluctuations in inflation and the output-gap which are costly because of the existence of price rigidities (see Svensson and Woodford, 2005). While the central idea is in principle not related to any particular policy instrument, inflation targeting is often associated with the use of interest rate feedback rules, also known as Taylor-rules. The idea is that a central bank should adjust this policy instrument in response to expected inflation in a way that reduces aggregate demand enough to stabilize inflation. The consensus view emerging from that literature is that macroeconomic stability of the regime will be assured as long as interest rates respond to inflation by more than one for one, such that the real interest rate behaves countercyclically.

In this paper, we set up a model of a small open economy, where a rigidity in domestic producer prices is the main macroeconomic distortion. This implies that the central bank should predominantly stabilize domestic producer prices instead of the CPI (Gali and Monacelli, 2005). In addition, we introduce the possibility that rising public debt levels lead to increasing fears of debt default. We view such a relation as a likely property of any reasonable specification of sovereign default, a property moreover with broad empirical support (see Edwards, 1984, Cantor and Packer, 1996, Min, 1998, Eichengreen and Mody, 1998, and Ferucci, 2003).[3]

For our analysis, we abstract from the precise origins of sovereign default and apply an ad-hoc specification for the default probability, following a common practice for open economy models with private debt (see Schmitt-Grohe and Uribe, 2003, for an overview). Within this environment, we analyze the stability implications of monetary policy under different rules governing fiscal policy and assuming different debt structures.

We start the analysis with the case where the government borrows from abroad in domestic currency and the central bank controls the short-run domestic interest rate, which determines the contractual price of government bonds. For this, it is shown that the conventional recommendation for interest rate policy might lead to unstable debt dynamics. Specifically, when the central bank raises the nominal interest rate strongly in response to a rise in (domestic producer price) inflation, it raises the cost of debt financing and can thereby feed expectations of sovereign default. Thus, the initial rise in the interest rate, which should have induced a real appreciation, can reduce foreign households' willingness to invest in domestic government bonds and, instead, lead to a real depreciation. The latter tends to increase aggregate demand for home goods and thus to raise rather than to lower domestic inflation, consistent with the effect described by Blanchard (2005). Thus, the latter effect might prevail, even though our model in principle accounts for the conventional "Fischer effect", i.e., that higher real interest rates tend to reduce domestic consumption. Finally, a central bank that further raises interest rates in response to the unintended rise in inflation induces a vicious circle, such that the economy evolves on an unstable path.

Yet, the central bank can avoid the perverse inflation effect and thus macroeconomic instability, by respecting an upper limit on the coefficient indicating the responsiveness of interest rates to inflation forecasts. The upper limit is shown to decrease the more higher interest payments are met through debt issue rather than through an increased primary surplus. Thus, high primary deficits raise the likelihood that an aggressive interest rate rule triggers unsustainable debt dynamics. In contrast, a rise in interest rates in response to higher inflation will only lead to a moderate increase in the stock of debt, if the additional costs of borrowing due to higher interest rates are mainly tax financed. At the same time, the real value of debt is reduced by higher domestic prices, which lowers the default probability. Concisely, a higher share of tax financing increases the range of monetary policy regimes under which the Fischer effect prevails over the Blanchard effect.

Moreover, it is not only the fiscal policy stance that affects the likelihood of macroeconomic instability. Since the probability of default depends on the real value of debt in terms of the aggregate consumption good, a real appreciation that tends to reduce the aggregate domestic price level – via an imported deflation – can raise the probability of

[3] Uribe (2005) for example derives a similar relation as an equilibrium outcome when fiscal policy is characterized by an exogenous primary surplus.

default. Like higher deficits, higher degrees of openness thus tend to narrow the stable range of monetary policy regimes, such that unstable equilibrium dynamics become more likely. If we go to the other extreme with respect to fiscal policy or consumption patterns, the central bank can safely adjust the interest rate in a highly reactive way in response to higher inflation without fears of destabilizing the economy. In particular, when the government runs a balanced budget policy and domestic demand for foreign goods is negligible, the upper limit on the stable range ceases to exist.[4] Thus, the Blanchard effect virtually disappears under a sound fiscal policy and with a strong consumption home bias.

Our analysis so far presumes that the government can borrow from abroad in its own currency, which is an unrealistic characterization of many less developed countries. Emerging market economies typically face difficulties when they attempt to borrow from abroad in domestic currency (Eichengreen and Hausmann, 1999, and Eichengreen et al., 2003). Due to this inability, which they refer to as "original sin", external debt of many less developed countries is mainly denominated in foreign currency. To account for this phenomenon, we consider the case where external debt is denominated in foreign currency.

Here, we obtain a stark result: with only foreign debt, and thus complete indexation with respect to domestic price level surprises, a non-accommodating interest rate policy will always lead to instability. Thus, whenever the central bank raises the real interest rate in response to higher domestic inflation, it causes a perverse inflation effect and destabilizes the economy, regardless of the current fiscal stance. Given that domestic inflation does not affect the real value of debt and thus the default probability, the Blanchard effect then prevails over the Fischer effect.

Of course these results do not imply that central banks in less developed countries should refrain from adopting an inflation targeting strategy altogether. First of all, inflation can be stabilized through active interest rate rules without running the risk of unstable dynamics, when monetary policy is safeguarded by a sound fiscal policy as long as a sufficiently large part of the debt is not indexed nor denominated in foreign currency. And even if full indexation of debt is unavoidable, the central bank can bypass the problems associated with sovereign risk if it implements the desired allocation (i.e., its optimal inflation targeting plan) by an instrument other than the interest rate on a risky asset. This principle is demonstrated in the last part of the paper, where we introduce money in our model in a simple way that avoids additional distortions stemming from money demand. When the central bank controls aggregate demand via the supply of nominal balances, it can in principle stabilize inflation and output in a way which is consistent

[4]Notably, local uniqueness of the rational expectations equilibrium is then ensured, implying that the well-known Taylor-principle does not apply. The reason is that the predetermined stock of nominal debt serves as a relevant nominal anchor, such that the usual (nominal) equilibrium indeterminacy result under interest rate policy does not hold. Similar determinacy result under interest rate policy and solvent fiscal policy can be found in Canzoneri and Diba (2005) and Linnemann and Schabert (2005), where debt non-neutrality is induced through liquidity services of debt.

with an optimal targeting plan derived from the minimization of a quadratic loss function without the instability problems interest rate rules may lead to.

The remainder of this paper is organized as follows. Section 2 develops the model. In section 3 we derive the stability implications of interest rate policy for the case where the government can borrow in its own currency. In the last part of section 3 we briefly examine the case where public debt is risk-free and reproduce conventional determinacy results (see Gali and Monacelli, 2005, and De Fiore and Liu, 2005). In section 4 we analyze macroeconomic stability for the case where the government borrows from abroad in foreign currency. Section 5 presents an example for a stable implementation of an inflation targeting strategy. Section 6 concludes.

2 A small open economy model

In this section we present a model of a small open economy.[5] There are two types of assets, which are both internationally traded, i.e., domestic public debt and risk-free privately issued securities. Public debt is associated with a premium due to expected non-zero sovereign default risk. For simplicity we neglect holdings of money and assume that the economy is cashless.[6] Nominal (real) variables are denoted by large (small) letters.

2.1 Public sector

The domestic public sector consists of two parts, the government and the central bank. The government levies lump-sum taxes $P_t\tau_t$ on domestic households (P_t denotes the price level of the aggregate consumption good) and issues one-period discounted bonds B_t. Domestic government debt is internationally traded and either held by domestic households $B_{H,t}$ or by foreign households $B_{F,t} : B_t = B_{H,t} + B_{F,t}$. The common price $1/R_t$ of government bonds in domestic currency is set by the central bank (see below). Each unit of debt B_{t-1} issued in $t-1$ leads to a payoff of one unit of the domestic currency in period t.

We allow for the possibility that an indebted government might not fulfill its debt repayment obligations in each period. In particular, we assume that the government defaults with a certain probability δ_t in each period. Since our analysis aims to explore the impact of sovereign risk on macroeconomic stability, we introduce a simple ad-hoc specification of a default probability that rises with the real value of debt,[7] a characteristic that is likely to emerge from any reasonable explicit derivation of default probabilities (see

[5] The basic framework builds on Gali and Monacelli (2005).

[6] See Woodford (2003).

[7] An endogenously derived default premium formula would possibly not just depend on debt but on debt scaled by a measure of payment capacity. To the extent that GDP is correlated with taxable capacity, the debt-to-output ratio rather than the level of debt is a plausible argument of the default premium function. In the last part of the appendix we show that the main results carry through with such a specification.

for instance Uribe, 2005, or Eaton and Gersovitz, 1981)

$$\delta_t = \delta\left(B_{t-1}/P_t\right) : R_+ \rightarrow (0,1)\,,\ \delta' > 0. \tag{1}$$

According to (1) the default probability rises with the total amount of debt outstanding. For the analysis of macroeconomic stability in the subsequent section, we will further assume that the government only borrows from abroad, which is typically the case for less developed countries. In this case the default probability will depend on foreign debt $\delta_t = \delta\left(B_{F,t-1}/P_t\right)$, which is consistent with the findings in several empirical studies on emerging market spreads (see e.g. Edwards, 1984, Cantor and Packer, 1996, Min, 1998, Eichengreen and Mody, 1998, and Ferucci, 2003). Analogous specifications for a risk premium can be found in several open economy models with private debt (see Schmitt-Grohe and Uribe, 2003, for a discussion and an overview).

One might think of (1) as a measure which accounts for cases where the government does not (fully) repay its debt obligations due to a lack of liquidity rather than due to intertemporal insolvency. Default might, for instance, occur if when government financing is constrained by lenders' unwillingness to borrow without limits or by natural or technical constraints to taxation. As an example, suppose that the government finances its debt obligations and stochastic goods purchases g partly by debt issuance and partly by tax revenues, e.g., according to a simple rule that ensures intertemporal solvency. Further, suppose that debt has to be issued at the beginning of each period. The amount of newly issued government bonds is thus based on its expected total expenditures. For a sufficiently broad support of random goods purchases there exists a non-zero probability of default, if tax revenues are not unbounded. The default probability would thereby rise with the tightness of the tax limit, with the variance of goods purchases, and, in particular, with the real value of debt B_{t-1}/P_t.

Given the specification (1), the period-by-period expected government budget constraint for any period t reads:

$$B_t R_t^{-1} + P_t \tau_t = P_t g_t + (1-\delta_t)\, B_{t-1}, \quad \text{where } B_t = B_{H,t} + B_{F,t}.$$

The government follows a simple tax rule. Specifically, taxes are raised to finance a constant fraction κ of total expenditures, which consist of goods purchases and interest rate costs of debt financing $(1-1/R_t)B_t$. Savings from partial debt repayment are assumed to be rebated randomly in every period, $P_t\widetilde{\tau}_t = \kappa \cdot [P_t g_t + (1-1/R_t)B_t]$, where $\kappa \in (0,1]$ and $P_t\widetilde{\tau}_t$ denotes lump-sum transfers consisting of taxes net of savings of default $P_t\widetilde{\tau}_t = P_t\tau_t - \delta_t B_{t-1}$. Thus, we abstract from the case where the government runs secondary surpluses (i.e. a primary surplus that more than covers nominal interest payments). Nevertheless, fiscal policy allows for a reduction of real debt due to inflationary erosion for sufficiently high shares of tax financing. Since our focus is on macroeconomic stability, government goods purchases do not affect the results and we assume that $g_t = 0$, for convenience. Thus, public debt evolves according to

$$B_t = R_t / [1 + \kappa(R_t - 1)]^{-1} B_{t-1}, \text{ where } \kappa \in (0, 1]. \tag{2}$$

Note that $\kappa = 1$ is the case of a budget that balances in every period, such that nominal government bonds are constant over time: $B_t = B_{t-1}$. For $\kappa < 1$, fiscal policy is characterized by a secondary deficit which leads to an accumulation of nominal debt. Whether real debt, which is decisive for sovereign default and thus for equilibrium outcomes, increases or decreases will actually depend on the inflation dynamics and thereby on monetary policy. Note also that we rule out a fiscal policy regime that does not respect government solvency, $\kappa = 0$, which is known as a "non-Ricardian" policy.[8] This particular case is considered in Uribe (2005), where sovereign default due to intertemporal insolvency is endogenously derived in a closed economy framework for a $\kappa = 0$ fiscal policy stance.

The central bank controls the nominal interest rate R_t on government bonds. We assume that the central bank sets R_t in a state contingent way, i.e., according to a simple feedback rule. Given that a rigidity in the domestic producer price level is the main macroeconomic distortion, a welfare maximizing monetary policy should predominantly aim to stabilize the domestic price inflation rate at the target value $\overline{\pi}_H$ (see Gali and Monacelli, 2005). Specifically, we assume that the central bank sets the nominal interest rate on government bonds contingent on changes in (expected future) domestic producer price inflation $E_t \pi_{H,t+1}$:

$$R_t - \overline{R} = R(E_t \pi_{H,t+1} - \overline{\pi}_H), \qquad R' \geq 0, \ R_t > 1, \tag{3}$$

where $\overline{R}$ is the average interest rate and $\overline{\pi}_H$ is assumed to be consistent with the steady state. We interpret (3) as a simple specification of a monetary policy regime which aims to reduce welfare losses due to inefficient price setting, which serves as the main rationale for "inflation targeting" (see Svensson, 1999). Alternatively, an inflation targeting central bank might use CPI inflation as the indicator for interest rate adjustment. Notably, the stabilization of the price level of aggregate consumption is, as shown by Gali and Monacelli (2005) in a corresponding model without public debt, a suboptimal strategy.[9] Finally, we want to point out that the policy instrument R_t is an interest rate on an asset which exhibits a risky pay-off. Thus, even if one interprets the policy instrument as a short-run interest rate, it carries a risk component that will be reflected in equilibrium by a risk

[8] Fiscal policy regimes of this type (exogenous primary surpluses/deficits) have been used by the "Fiscal Theory of the Price Level" to determine the equilibrium price level when monetary policy fails to do so. See Kocherlakota and Phelan (1999) for an overview and Buiter (2002) or Niepelt (2004) for critical assessments of this approach. In our framework, governent solvency, which requires $\lim_{k\to\infty} b_k R_k^{-1} \prod_{i=1}^{k} \pi_i R_{i-1}^{-1}$ to be equal to zero, is guaranteed for $R_t > 1$, since (2) implies $\lim_{k\to\infty} b_k R_k^{-1} \prod_{i=1}^{k} \pi_i R_{i-1}^{-1} = \lim_{k\to\infty} (b_{t-1}/\pi_t) \prod_{i=0}^{k} (1 + \kappa \cdot (R_{t+i} - 1))^{-1} = 0 \ \forall t \geq 0$.

[9] Moreover, it might be a source of equilibrium multiplicity as shown by De Fiore and Liu (2005).

premium compared to a nominal risk-free interest rate on internationally traded bonds.[10]

2.2 Private sector

Households There is a continuum of infinitely lived domestic households. They have identical asset and time endowments, and exhibit identical preferences. They consume a consumption basket that is an aggregate of domestically produced goods c_H and foreign goods c_F:

$$c_t = \gamma c_{H,t}^{1-\vartheta} c_{F,t}^{\vartheta},$$

where $0 \leq \vartheta \leq 1$ and $\gamma = [\vartheta^{\vartheta}(1-\vartheta)^{1-\vartheta}]^{-1}$. For a given level of aggregate consumption, the cost minimizing demand for the goods of home and foreign origin are given by

$$c_{H,t} = (1-\vartheta)\left(\frac{P_{H,t}}{P_t}\right)^{-1} c_t, \qquad c_{F,t} = \vartheta\left(\frac{P_{F,t}}{P_t}\right)^{-1} c_t, \tag{4}$$

where $P_{H,t}$ and $P_{F,t}$ are the price indices of the domestically produced and foreign consumption goods, respectively. The price index of the aggregate consumption good (CPI) is defined as

$$P_t = P_{H,t}^{1-\vartheta} P_{F,t}^{\vartheta}. \tag{5}$$

Contemporaneous utility u_t of a representative domestic household rises with aggregate consumption and with leisure l_t, where $l_t \in [0,1]$ and $n_t = 1 - l_t$ is the working time. Its objective is given by

$$\max E_0 \left[\sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-n_t\right)\right], \quad \beta \in (0,1) \tag{6}$$

$$u\left(c_t, 1-n_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} + \zeta\frac{(1-n_t)^{1-\sigma_l}}{1-\sigma_l}, \quad \sigma > 0, \sigma_l \geq 0 \ ,$$

where β denotes the discount factor. The household earns labor income $P_t w_t n_t$, pays lump-sum taxes $P_t \tau_t$, and receives profits from monopolistically competitive firms indexed with $i \in [0,1]$.

Households have access to a complete set of contingent claims, which are internationally traded. Let $\Gamma_{t,t+1}$ denote the stochastic discount factor for one-period ahead nominal pay-off, i.e., the period t price of one unit of domestic currency in a particular state of period $t+1$ normalized by the probability of occurrence of that state, conditional on the information available in period t. Then, the time t price of a random payoff D_{t+1} in period $t+1$ is given by $E_t[\Gamma_{t,t+1} D_{t+1}]$. The budget constraint of a representative household in

[10]This point has been emphasized by Loyo (2005) in his discussion of Blanchard (2005). He argues that even an overnight rate (specifically, the Brazilian Selic) contains a risk premium. It should further be noted that the term "risk-free" refers to secure payments of units of currency.

terms of the domestic currency reads

$$E_t[\Gamma_{t,t+1}D_{t+1}] + (B_{H,t}/R_t) \le D_t + (1-\delta_t)\,B_{H,t-1} + P_t w_t n_t - P_t c_t - P_t \tau_t + \Sigma_t,$$

where Σ_t collects firms' profits. The household maximizes lifetime utility (6) subject to the budget constraint and a no-Ponzi-game condition, taking prices, taxes, dividends, the default probability and the initial wealth endowment F_0 and $B_{H,-1}$ as given. Its first order conditions are

$$\lambda_t = c_t^{-\sigma}, \tag{7}$$

$$\zeta(1-n_t)^{-\sigma_l} = w_t \lambda_t, \tag{8}$$

$$\beta E_t \left\{ (1-\delta_{t+1})\,\lambda_{t+1}\pi_{t+1}^{-1} \right\} = \lambda_t / R_t, \tag{9}$$

$$\beta \lambda_{t+1} \pi_{t+1}^{-1} = \lambda_t \Gamma_{t,t+1}, \tag{10}$$

where π_t denotes the gross inflation rate $\pi_t = P_t/P_{t-1}$. Further, the budget constraint holds with equality and the transversality condition is satisfied, $\lim_{k\to\infty} E_t(\Gamma_{t+k,t+1+k}D_{t+1+k} + B_{H,t+k}/R_{t+k})\Gamma_{t,t+1+k} = 0$. Combining the first order conditions leads to the following arbitrage condition $1/R_t = E_t\left\{(1-\delta_{t+1})\,\Gamma_{t,t+1}\right\}$. Thus, δ_t can be interpreted as a measure for a *sovereign risk premium* on the price of domestic public debt demanded by investors.

Production The production sector consists of two parts. Firstly, intermediate production is conducted by a continuum of monopolistically competitive firms, each producing a differentiated good being indexed on $i \in [0,1]$. Their technology is linear in labor,

$$y_{H,it} = n_{i,t}, \qquad \text{where } n_t = \int_0^1 n_{it} di.$$

Secondly, there are perfectly competitive firms producing the domestic consumption good by combining the differentiated intermediate goods as inputs. They use the CES technology

$$y_{H,t} = \left[\int_0^1 y_{H,it}^{\frac{\epsilon-1}{\epsilon}} di \right]^{\frac{\epsilon}{\epsilon-1}}, \ \epsilon > 1,$$

where ϵ denotes the elasticity of substitution. Firm i sets the price for the intermediate good $y_{H,it}$ in home currency $P_{H,it}$. For a given output level, the final good producer's cost minimizing demand is given by $y_{H,it} = (P_{H,it}/P_{H,t})^{-\epsilon}\, y_{H,t}$. Zero profits then imply that the price index of home produced goods is $P_{H,t}^{1-\epsilon} = \int_0^1 P_{H,it}^{1-\epsilon} di$.

The price setting decision of an intermediate domestic producer is modelled as in Calvo (1983). The fraction $\phi \in (0,1)$ of firms is assumed to adjust their prices with the steady state rate of domestic producer price inflation $\overline{\pi}_H$, where $\pi_{H,t} = P_{H,t}/P_{H,t-1}$, such that $P_{H,it} = \overline{\pi}_H P_{H,it-1}$. In each period a measure $1-\phi$ of randomly selected firms sets new prices $\widetilde{P}_{H,it}$ in order to maximize the expected sum of discounted future dividends $(P_{H,it} - P_{H,t} mc_{H,t})\, y_{H,it}$: $\max_{\widetilde{P}_{H,it}} E_t \sum_{s=0}^{\infty} \phi^s q_{t,t+s} (\widetilde{P}_{H,it} y_{H,it+s} - P_{H,t+s} mc_{H,t+s} y_{H,it+s})$, s.t. $y_{H,it+s} = (\overline{\pi}_H^s \widetilde{P}_{H,it})^{-\epsilon} P_{H,t+s}^{\epsilon} y_{H,t+s}$, where mc_H denotes real marginal costs. Suppose there exists a steady state where home prices grow at the rate $\overline{\pi}_H$, while all real variables are constant, e.g. $\overline{mc}_H = MC_H/P_H = (\epsilon - 1)/\epsilon$. Then one can derive the following marginal cost based Phillips curve (see Yun, 1995)

$$\widehat{\pi}_{H,t} = \chi \widehat{mc}_{H,t} + \beta E_t \widehat{\pi}_{H,t+1}, \tag{11}$$

where $\chi = (1-\phi)(1-\beta\phi)\phi^{-1} > 0$ and $\widehat{x}_t$ denotes the percent deviation of a generic variable x_t from its steady state value, $\widehat{x}_t = \log x_t/x$. Finally, labor demand in a symmetric equilibrium is given by

$$w_t = \frac{P_{H,t}}{P_t} mc_{H,t}. \tag{12}$$

2.3 Foreign households

The real exchange rate q_t is defined as $q_t = \frac{S_t P_t^*}{P_t}$, where P_t^* is the foreign consumption price index. The home country is assumed to be small in the sense that its exports are negligible in the foreign price indices. The foreign producer price level $P_{F,t}^*$ is then identical to the foreign consumption price index P_t^*,

$$P_t^* = P_{F,t}^*. \tag{13}$$

The law of one price holds (separately) for each good such that $P_{H,t} = S_t P_{H,t}^*$ and $P_{F,t} = S_t P_{F,t}^*$, where $P_{H,t}^*$ is the price of home produced goods expressed in foreign currency. Let the terms of trade z_t be defined as $z_t = P_{H,t}/P_{F,t}$. Thus, we get the following relation between the terms of trade and the real exchange rate,

$$z_t = \frac{1}{q_t} \frac{P_{H,t}}{P_t}. \tag{14}$$

Note that the ratio of domestic producer prices to the consumer price index, $P_{H,t}/P_t$, affects the marginal costs mc_H (see 12) We assume that preferences of foreign households exhibit the same qualitative structure as domestic households. Hence, their demand for domestically produced consumption goods $c_{H,t}^*$ and foreign consumption goods $c_{F,t}^*$ satisfies $c_{H,t}^* = \vartheta^* \left(P_t^*/P_{H,t}^* \right) c_t^*$ and $c_{F,t}^* = (1-\vartheta^*) \left(P_t^*/P_{F,t}^* \right) c_t^*$, where $\vartheta^* \in (0,1)$ and c_t^* is aggregate foreign consumption. Since the domestic economy is assumed to be small, ϑ^* has to be considerably smaller than ϑ. Using (13) and (14), foreign demand for domestic consumption goods can be rewritten as

$$c_{H,t}^* = \vartheta^* z_t^{-1} c_t^*.$$

Foreign households also have access to a complete set of contingent claims and they can invest in domestic public debt B_F, which is denominated in domestic currency. We assume that the instantaneous utility function of foreign households is similar to the one of home households (see 6) and that they exhibit the same discount factor β. Their first order conditions for investments in both assets are given by

$$\beta E_t \left\{ [1-\delta(t+1)] \left(c_{t+1}^*\right)^{-\sigma^*} \left(\pi_{t+1}^* S_{t+1}\right)^{-1} \right\} = \left(c_t^*\right)^{-\sigma^*} \left(R_t S_t\right)^{-1}, \tag{15}$$

$$\beta \left(c_{t+1}^*\right)^{-\sigma^*} \left(\pi_{t+1}^*\right)^{-1} S_t = \left(c_t^*\right)^{-\sigma^*} S_{t+1}\Gamma_{t,t+1}, \tag{16}$$

where $\pi_t^* = P_t^*/P_{t-1}^*$ and σ^* denotes the inverse of foreign households' intertemporal elasticity of substitution. Further note that the CPI definition (5) together with $P_{F,t} = S_t P_t^*$, imply $1 = (P_{H,t}/P_t)^{1-\vartheta} (q_t)^{\vartheta}$. Further using (14) gives $P_{H,t}/P_t = z_t^{\vartheta}$, and thus the following conditions for the terms of trade and the CPI inflation rate

$$z_t^{\vartheta-1} = q_t, \tag{17}$$

$$\pi_t = \pi_{H,t} \left(q_t/q_{t-1}\right)^{\frac{\vartheta}{1-\vartheta}} \quad \forall t \geq 1 \tag{18}$$

2.4 Market clearing

Goods market clearing for domestically produced final goods requires $y_{H,t} = c_{H,t} + c_{H,t}^*$. Further using that the demand for a differentiated good is given by $y_{H,i,t} = (P_{H,it}/P_{H,t})^{-\epsilon} y_{H,t}$, gives the following equilibrium relation for differentiated goods

$$y_{H,i,t} = (P_{H,it}/P_{H,t})^{-\epsilon} (c_{H,t} + c_{H,t}^*).$$

Domestic and foreign demand for the domestically produced final good satisfy $c_{H,t} = (1-\vartheta)(P_t/P_{H,t}) c_t$ and $c_{H,t}^* = \vartheta^*(P_t^*/P_{H,t}^*)c_t^*$. Hence, we get the following equilibrium relation between the supply of differentiated good i and aggregate domestic and foreign consumption

$$y_{H,i,t} = \left(\frac{P_{H,it}}{P_{H,t}}\right)^{-\epsilon} \left[(1-\vartheta) \left(\frac{P_{H,t}}{P_t}\right)^{-1} c_t + \vartheta^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-1} c_t^* \right].$$

Using the latter we can relate the supply of the domestically produced final good, $y_{H,t}^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 (y_{H,i,t})^{\frac{\epsilon-1}{\epsilon}} di$, to aggregate domestic and foreign consumption

$$y_{H,t} = \left[(1-\vartheta) \left(\frac{P_{H,t}}{P_t}\right)^{-1} c_t + \vartheta^* \left(\frac{P_{H,t}^*}{P_t^*}\right)^{-1} c_t^* \right] \cdot P_{H,t}^{\epsilon} \left[\int_0^1 (P_{H,i,t})^{1-\epsilon} di \right]^{\frac{\epsilon}{\epsilon-1}}.$$

The price of the domestically produced final good satisfies $P_{H,t}^{1-\epsilon} = \int_0^1 P_{H,i,t}^{1-\epsilon} di$ and that foreign consumption demand can be written as $c_{H,t}^* = \vartheta^* z_t^{-1} c_t^*$. Hence, using $P_{H,t}/P_t = z_t^{\vartheta}$ and $z_t^{\vartheta-1} = q_t$, the goods market equilibrium for the domestically produced final good can be summarized by

$$y_{H,t} = (1-\vartheta) q_t^{\frac{\vartheta}{1-\vartheta}} c_t + \vartheta^* q_t^{\frac{1}{1-\vartheta}} c_t^*, \tag{19}$$

Further, the price $1/R_t^*$ of a risk-free privately issued one-period discounted bond F_t which pay one unit of foreign currency in period $t+1$, has to satisfy $1/R_t^* = E_t[\Gamma_{t,t+1}S_{t+1}/S_t]$. Thus, (10) and (16) imply the following pricing conditions

$$\beta E_t \left\{ \left(\frac{c_t}{c_{t+1}} \right)^{\sigma} \frac{q_{t+1}}{q_t} \frac{1}{\pi_{t+1}^*} \right\} = \frac{1}{R_t^*}, \quad \beta E_t \left\{ \left(\frac{c_t^*}{c_{t+1}^*} \right)^{\sigma} \frac{1}{\pi_{t+1}^*} \right\} = \frac{1}{R_t^*}, \tag{20}$$

The net foreign asset/debt position consists of holdings of internationally traded risk-free securities F_t and domestic public debt held by foreign investors B_{Ft}. Factor and asset market clearing implies that the net foreign debt position is determined by $P_{H,t}y_{H,t} - P_t c_t \leq S_t \{[(F_t/R_t^*) - F_{t-1}]\} - (B_{F,t}/R_t) + [1-\delta(t)] B_{F,t-1}$, or in real terms, by

$$q_t (f_t/R_t^*) - (b_{F,t}/R_t) = z_t^{\vartheta} y_{H,t} - c_t + q_t (f_{t-1}/\pi_t^*) - (1-\delta_t)(b_{F,t-1}/\pi_t), \tag{21}$$

where f_t denotes the real value of F_t in terms of the aggregate consumption good, $f_t = F_t/P_t$. In a rational expectations equilibrium (19), (21), the first order conditions of domestic households and firms, and of foreign households have to be satisfied for a domestic monetary and fiscal policy, and given sequences for the starred variables as well as initial asset endowments F_{-1}, $B_{H,-1}$, and $B_{F,-1}$ and initial price levels $P_{H,-1}$ and $P_{F,-1}$, where the latter is assumed to be consistent with (18) for $t = 0$.[11] The full set of equilibrium conditions is given in appendix 8.1.

3 Results

3.1 Macroeconomic stability under sovereign risk

In this section we examine the impact of monetary and fiscal policy on stability and uniqueness of the rational expectations equilibrium. In equilibrium, domestic households are indifferent between holding internationally traded risk-free private securities F_t and domestic public debt $B_{H,t}$. Since we are particularly interested in the role of foreign debt, we assume that domestic public debt is solely held by foreign investors, while domestic households only hold risk-free private debt. Hence, we focus on the case

$$B_{H,t} = 0 \Leftrightarrow B_t = B_{F,t}. \tag{22}$$

To derive the stability properties under different stances of fiscal and monetary policy, the equilibrium conditions are log-linearized at the steady state. In a neighborhood of the steady state the equilibrium sequences are then approximated by the solutions to the linearized equilibrium conditions. We consider an economy where the steady state is assumed to satisfy $q = 1$ by choice of units. This implies $z = 1$ and $\pi = \pi_H$ (see

[11] This assumption is made to facilitate the reduction of the model to a tractable set of equilibrium conditions. It should be noted that q_{t-1} will, nonetheless, not serve as a *relevant* predetermined variable in equilibrium.

17), and $1/\beta = R^*/\pi^*$. Given that the level of public debt might affect the equilibrium allocation, a steady state in general requires a constant real value of public debt (in terms of the aggregate consumption good). Since, we want focus on the case where the domestic government is indebted we only consider cases where the steady state satisfies $b_F > 0$. Thus, (2) leads to the following condition for the steady state inflation rate

$$\pi = R/\left[1 + \kappa\left(R - 1\right)\right], \tag{23}$$

where $\kappa \in (0,1)$ implies $\pi \geq 1$. Note that the interest rate rule (3) has been assumed to be consistent with the steady state, i.e., that steady state inflation rate equals the target inflation rate $\pi = \overline{\pi}_H$ and that $R = \overline{R}$. Then, (23) determines the steady state inflation rate as a function of the monetary policy parameter $\overline{R}$ and of the fiscal policy parameter κ. The steady state value of real debt b_F is then determined by $\beta = \left[1 - \delta\left(b_F/\pi\right)\right]R/\pi$. Substituting out the inflation rate shows that public debt and the interest rate are positively related by $R = 1 + ([(1-\delta)\,\beta]^{-1} - 1)/\kappa$. Finally, steady state domestic consumption is pinned down by $\kappa^{-1/\sigma}(1 - [(1-\vartheta)c + \vartheta^* c^*])^{\sigma\iota/\sigma} - [(\epsilon - 1)/\epsilon]c = 0$ and domestically produced goods by $y_H = n = (1-\vartheta)c + \vartheta^* c^*$.

Recall that domestic and foreign households are assumed to have access to a complete set of contingent claims, such that risk is internationally shared. Combining (7), (10), and (16) implies that the consumption growth rates are related as follows $\left(c_{t+1}/c_t\right)^{\sigma} = \left(c^*_{t+1}/c^*_t\right)^{\sigma^*}\left(q_{t+1}/q_t\right)$ $\forall t \geq 0$. This equilibrium condition on the growth rates of c_t, c^*_t, and q_t determines the relation between their levels up to a constant ξ, $c^{\sigma}_t = \xi q_t \left(c^*_t\right)^{\sigma^*}$, which depends on the pre-existing values c_{-1}, c^*_{-1}, and q_{-1}. Assuming that the latter are consistent with the steady state and log-linearizing the level relation, international risk sharing implies

$$\sigma\widehat{c}_t = \widehat{q}_t + \sigma^*\widehat{c}^*_t. \tag{24}$$

Throughout the analysis foreign macroeconomic variables (starred variables) are independent from domestic variables (see 13), i.e., they are exogenously determined. To simplify the analysis we assume that aggregate foreign consumption is constant, $c^*_t = c^*$, which implies that foreign monetary policy is conducted in a way that is consistent with a constant real interest rate $R^*_t/E_t\pi^*_{t+1} = \beta$ (see 16).

A *rational expectations equilibrium* for (22) and $\widehat{c}^*_t = 0$ is a set of sequences $\{\widehat{w}_t$,

$\widehat{\pi}_t, \widehat{\pi}_{H,t}, \widehat{c}_t, \widehat{q}_t, \widehat{y}_{H,t}, \widehat{R}_t, \widehat{b}_{F,t}\}_{t=0}^{\infty}$ satisfying

$$\widehat{w}_t - \sigma\widehat{c}_t = \sigma_n \widehat{y}_{H,t}, \tag{25}$$

$$\Phi\widehat{b}_{F,t} - \Phi E_t\widehat{\pi}_{t+1} = \widehat{R}_t - E_t\widehat{\pi}_{t+1} - \sigma(E_t\widehat{c}_{t+1} - \widehat{c}_t), \tag{26}$$

$$\sigma\widehat{c}_t = \widehat{q}_t, \tag{27}$$

$$\widehat{\pi}_{H,t} = \chi\left(\widehat{w}_t + [\vartheta/(1-\vartheta)]\,\widehat{q}_t\right) + \beta E_t\widehat{\pi}_{H,t+1}, \tag{28}$$

$$\widehat{y}_{H,t} = [(c/n)\,\vartheta + \vartheta^*\,(c^*/n)\,/(1-\vartheta)]\widehat{q}_t + (1-\vartheta)\,(c/n)\,\widehat{c}_t, \tag{29}$$

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + [\vartheta/(1-\vartheta)]\left(\widehat{q}_t - \widehat{q}_{t-1}\right), \tag{30}$$

where $\sigma_n = \sigma_l n/(1-n)$ and Φ denotes the default elasticity with respect to the real value of public debt $B_{F,t-1}/P_t$ at the steady state: $\Phi = \delta' \frac{b_F}{\pi}\frac{1}{1-\delta} > 0$, and the transversality conditions, for monetary and fiscal policy characterized by

$$\widehat{b}_{F,t} = \widehat{b}_{F,t-1} - \widehat{\pi}_t + \eta\widehat{R}_t, \tag{31}$$

$$\widehat{R}_t = \rho_\pi E_t\widehat{\pi}_{H,t+1}, \tag{32}$$

where $\rho_\pi = R'\pi_H/R \geq 0$ and $\eta = \frac{1-\kappa}{1+\kappa(R-1)} \in (0,1)$ for given initial values $\widehat{b}_{F,-1}$ and $\widehat{q}_{-1}$.

Internationally traded risk-free securities f_t do not affect the equilibrium allocation. For a uniquely determined equilibrium, the sequence $\{f_t\}_{t=0}^{\infty}$ can uniquely be determined by (21) for given initial values F_{-1}^* and P_{-1}^*. If there would be no risk-premium $\delta_t = 0$ or if the risk premium would be independent of the level of public debt $\delta' = \Phi = 0$, then the sequence of foreign holdings of government bonds b_F would be irrelevant for the equilibrium allocation and the model would be isomorphic to the one in Gali and Monacelli (2005). We will briefly refer to this case below.

3.2 Debt, Deficits and Macroeconomic Stability

Throughout the analysis we are mainly interested in the impact of public debt on macroeconomic stability for the benchmark case where the sovereign risk premium rises with foreign debt $\Phi > 0$.[12] For the analysis we focus on moderate values for the elasticity Φ, to avoid unreasonable debt dynamics. In particular, we will consider the case $\Phi < 1$, which ensures that lagged realizations of real debt exert a positive partial feedback on its current realizations (see 35). Otherwise, when a rise in real debt leads to a rise in the risk premium by more than one for one, this feedback can be negative, giving rise to oscillatory equilibrium sequences. Given that this type of equilibrium behavior is evidently neither realistic nor recommendable (given that macroeconomic fluctuations are associated with welfare costs), we focus on cases where equilibrium sequences are non-oscillatory.

Since we want to assess the stability implications of fiscal-monetary policy regimes, it suffices to focus on the structural part of the economy. The deterministic versions of the

[12] Below we briefly discuss the case where public debt is assumed to be risk-free: $\Phi = 0$ (see section 3.3).

equilibrium conditions can further be reduced to the following set of equilibrium conditions in $\widehat{b}_{F,t}$, $\widehat{q}_t$, $\widehat{R}_t$ and $\widehat{\pi}_{H,t}$:

$$\frac{1-\Phi\vartheta}{1-\vartheta}\left(\widehat{q}_{t+1}-\widehat{q}_t\right)=\widehat{R}_t-(1-\Phi)\,\widehat{\pi}_{H,t+1}-\Phi\widehat{b}_{F,t}, \tag{33}$$

$$\widehat{\pi}_{H,t}=\beta\widehat{\pi}_{H,t+1}+\psi\widehat{q}_t, \tag{34}$$

$$\widehat{b}_{F,t}=\frac{1}{1-\Phi\vartheta}\widehat{b}_{F,t-1}-\varphi\widehat{\pi}_{H,t}+\eta\widehat{R}_t, \tag{35}$$

and $\widehat{R}_t=\rho_\pi\widehat{\pi}_{H,t+1}$, where $\psi=\chi\left[\frac{\sigma_n}{n}\left(\frac{(1-\vartheta)c}{\sigma}+\frac{(1-\vartheta)c\vartheta+\vartheta^* c^*}{1-\vartheta}\right)+\frac{1}{1-\vartheta}\right]>0$ and $\varphi=\frac{1+\vartheta(1-\rho_\pi)}{1-\Phi\vartheta}$ (see appendix 8.2).

The equilibrium relation (33), which originates in the asset pricing condition for public debt, relates the real interest rate to the change in the real exchange rate in an almost conventional way: A rise in the real interest rate leads to an expected future depreciation for sufficiently small values for Φ. The implied instantaneous real appreciation ($\widehat{q}_t\downarrow$) leads to a decline in aggregate (domestic and foreign) demand for domestically produced goods (see 19), such that domestic producers tend to lower their goods prices, as can be seen from the aggregate supply relation (34). At the same time a rise in the nominal interest rate and the decline in inflation tends – for $\varphi>0$ – to raise real public debt $\widehat{b}_{F,t}$ (measured in units of the aggregate domestic consumption good) for a predetermined value for beginning-of-period real debt (see 35).[13]

A rise in the real debt $\widehat{b}_{F,t}$, however, tends to lower its expected total return, since it raises the default probability. This can be seen from the RHS of (33) which decreases with $\widehat{b}_{F,t}$. How the rise in public debt affects the previously described chain of events crucially depends on *i.)* monetary policy, which decides on the initial interest rate rise, on *ii.)* fiscal policy, which decides on the issuance of new debt, and, finally, *iii.)* on the endogenous price response which determines the real value of debt.

As suggested by Blanchard (2005) the negative feedback from public debt to its return, which originates in the sovereign risk premium, might cause unstable equilibrium dynamics. To get an intuition for this, suppose that inflation exceeds its steady state value due to some (unspecified) temporary fundamental shock. The central bank, which aims to stabilize inflation $\rho_\pi>0$, will then raise the nominal interest rate. If the tax share of government financing κ is very small (high η), a rise in the nominal interest rate can in principle cause an increase in real debt $\widehat{b}_{F,t}$. Sovereign default risk can thus rise, which reduces the foreign households' willingness to invest in public debt.

The associated real depreciation $\widehat{q}_t\uparrow$ (see LHS of 33) then exerts an upward pressure on domestic prices through different channels. A rise in the real exchange rate q_t directly raises aggregate consumption $\widehat{c}_t$, as implied by international risk sharing (see 24 and 27). This rise in aggregate consumption tends to increase the demand for home goods. In

[13] We will refer to the case $\varphi<0$ below.

addition, expenditure switching of domestic and foreign households in response to the exchange rate change further increases the demand for domestically produced goods (see 29). This adds to the price pressure as producers incur higher marginal costs at higher output levels. Moreover, households will demand a higher nominal wage, since the price level of aggregate consumption will rise due to higher prices of imported goods (see 28). Hence, domestic producers will unambiguously raise their prices in response to the real depreciation (see 34), which reinforces the initial rise in inflation.

Thus, a rise in the nominal interest rate can actually lead to higher inflation if κ is small and ρ_π is high. Hence, the interaction of monetary and fiscal policy is decisive for the economy to evolve in a stable or in an unstable way. In order to derive the requirements for monetary and fiscal policy to ensure a stable equilibrium, we substitute out the interest rate with $\widehat{R}_t = \rho_\pi \widehat{\pi}_{H,t+1}$ in (33) and (35) to obtain a three-dimensional system, which features one predetermined (sluggish) variable $\widehat{b}_{F,t-1}$ and two forward-looking variables $(\widehat{\pi}_{H,t}, \widehat{q}_t)$. Thus, a stable set of equilibrium sequences requires one stable eigenvalue. The condition for the existence and the uniqueness of a stable and non-oscillatory equilibrium is given in the following proposition.[14]

Proposition 1 *Suppose that* $\Phi \in (0,1)$. *Then, there exist locally stable and non-oscillatory equilibrium sequences if and only if*

$$\rho_\pi < 1 + \Delta, \qquad \Delta = \frac{\kappa R (1 - \Phi\vartheta)}{1 - \kappa + [(1-\kappa)(2-\Phi) + \kappa 2R]\vartheta} > 0. \tag{36}$$

The equilibrium is locally unique, stable, and non-oscillatory if and only if $\rho_\pi < \min\{1 + \Delta, 1 + \widetilde{\Delta}\}$, *where* $\widetilde{\Delta} = (1 - \Phi\vartheta)\frac{2(1+\beta)(2-\Phi\vartheta) - \psi\Phi(1-\eta)(1-\vartheta)}{\psi(1-\vartheta)(\Phi^2\eta\vartheta + 2 - \Phi\eta)}$.

Proof. See appendix 8.3.

Proposition 1 shows that the existence of a stable set of equilibrium sequences depends on the particular monetary and fiscal policy stance, measured by the feedback parameters ρ_π and κ : $\rho_\pi < 1 + \Delta(\kappa, \vartheta)$. The main result is that a monetary policy which aims to stabilize inflation in an aggressive way (high ρ_π) can destabilize the economy by the Blanchard effect. This property is clearly at odds with the main principle known from many New Keynesian models of closed and open economies, which demands monetary policy to react strongly to changes in (domestic producer price) inflation in order to minimize welfare losses and to rule out equilibrium multiplicity (see Woodford, 2003, and Gali and Monacelli, 2005, respectively). When public debt is associated with a risk premium, which in turn is influenced by the level of public debt, a small or moderate feedback from inflation to the nominal interest rate is required, both, for stability and

[14] Note that the proposition does not cover the case of oscillatory equilibrium sequences. In fact, the equilibrium exhibits a unique, stable, and oscillatory solution if and only if $\rho_\pi > \max\{1 + \Delta, 1 + \widetilde{\Delta}\}$ (see proof of proposition 1).

uniqueness of (non-oscillatory) equilibrium sequences. Before we discuss the reason for the stability condition (36), we assess the impact of fiscal policy (κ) on the latter.

Corollary 1 *The likelihood for equilibrium sequences to be locally stable and non-oscillatory increases with the share of tax finance* κ

$$\frac{\partial \Delta}{\partial \kappa} = \frac{1 + (2 - \Phi)\vartheta}{(1 - \Phi\vartheta)\kappa^2 R}\Delta^2 > 0.$$

When the fiscal authority runs a balanced budget policy $\kappa = 1$*, the equilibrium is locally unique and stable if and only if* $\rho_\pi < 1 + \frac{1-\Phi\vartheta}{2\vartheta}$*. For the limiting case* $\kappa \to 0$*, a stable equilibrium exists only if* $\rho_\pi < 1$*.*

Corollary 1 shows that the upper bound on the inflation elasticity ρ_π rises with the share of tax financing. To get an impression about the magnitude of the upper bound, $1 + \Delta$, consider for example a small value for the premium elasticity $\Phi = 0.01$, a degree of openness ϑ equal to 1/4, and an average nominal interest rate $R = 1.02$ (implying an annualized net rate of 8%), which can be viewed as a reasonable value for the countries in question. Then, the coefficient Δ equals 0.2 for a tax share $\kappa = 1/4$, while an equal share of tax to debt financing ($\kappa = 1/2$) leads to $\Delta = 0.5$. The upper bound for the interest rate feedback coefficient $(1 + \Delta)$ then equals 1.5 and rises to 2 for $\kappa = 3/4$.

Hence, when debt service costs are mainly financed by taxes (high κ), the central bank can safely choose an inflation feedback ρ_π that exceeds one without causing the economy to evolve in an unstable way. The reason is that a rise in the nominal interest rate in response to an expected future inflation, which raises the costs of debt financing, leads to a less pronounced increase in debt. Thus, an increase in the nominal interest rate is less likely to reduce the total expected return from public debt (see above). If, however, the government finances its expenditures almost entirely by borrowing $\kappa \to 0$ ($\eta \to 1$), equilibrium stability requires monetary policy to be accommodating, $\rho_\pi < 1$.[15] Overall, the stability result qualifies the common view that a stabilizing monetary policy should be active (see section 3.3).

To get an intuition for the result consider the case where a temporary shock leads to a rise public debt. Since the default risk rises, investors are less willing to hold domestic public debt. The associated depreciation (see 33), leads to a rise in the demand for domestic goods and thus to an upward pressure on inflation (see 34). If the central bank aggressively raises the nominal interest in response to higher expected inflation, debt service costs will rise strongly, and will lead for small share of tax financing (small κ) to an even further increase in real debt and thus to unstable debt dynamics. If, however, the

[15] Note that only for the specific case $\kappa = 0$, which is excluded in our model (see 2), fiscal policy would tend to violate the government solvency constraint. Thus, for the limiting case $\kappa \to 0$, the stability condition (36) is consistent with Benhabib et al.'s (2001) result a non-Ricardian policy in a closed economy.

interest rate response is moderate (or passive), the real value of public debt can decrease due to the revaluation by a higher domestic price level if $\varphi > 0$ (see 35). For a high share of tax financing (small η), the central bank can thus actively ($\rho_\pi > 1$) react to the rise in inflation, without running the risk of unstable debt dynamics.

A closer look at the stability condition further reveals that the degree of openness, measured by ϑ, plays a non-negligible role. This can especially be seen from the stability condition for the balanced budget case $\kappa = 1$, which reads $\rho_\pi < 1 + \frac{1-\Phi\vartheta}{2\vartheta}$. According to the latter, the upper bound on the inflation feedback decreases with the degree of openness. This partial effect also holds for the general case $\kappa \in (0, 1]$.

Corollary 2 *The likelihood for equilibrium sequences to be locally stable and non-oscillatory decreases with the degree of openness* ϑ

$$\frac{\partial \Delta}{\partial \vartheta} = -2 \frac{1 + (R-1)\kappa}{(1 - \Phi\vartheta)^2 \kappa R} \Delta^2 < 0.$$

When households do not consume foreign goods $\vartheta = 0$*, the equilibrium is locally unique and stable if and only if* $\rho_\pi < 1 + \frac{\kappa R}{1-\kappa}$ *for* $\kappa \in (0, 1)$*. Under a balanced budget policy* $\kappa = 1$ *and* $\vartheta = 0$ *the equilibrium is locally unique and stable.*

Using the parameter values from above ($\Phi = 0.01$, $R = 1.02$) and $\kappa = 1/2$, a small degree of openness $\vartheta = 1/4$ leads to an upper bound of 1.5, while raising ϑ to $1/2$ lowers the bound to 1.33. Thus, openness to commodity trade contributes to instability, just like low tax shares.

Why does a higher degree of openness decrease the range of stabilizing monetary policy regimes? To answer this question we have to take a closer look at the composite parameter $\varphi = [1 + \vartheta(1 - \rho_\pi)]/(1 - \Phi\vartheta)$ in (35). Evidently, the sign of this coefficient, which governs the impact of domestic price inflation $\pi_{H,t}$ on real debt, is not unambiguous. In particular, if the inflation feedback ρ_π exceeds $1 + 1/\vartheta$, it has a negative sign. In this case, a rise in $\pi_{H,t}$ tends to raise real debt by (35). This seemingly counterintuitive effect is due to the property that the behavior of CPI inflation π_t can in principle differ from the dynamics in $\pi_{H,t}$. In particular, when monetary policy reacts to a rise in $\pi_{H,t}$ by a rise in the real interest rate, it triggers a real appreciation that tends to lower CPI by (30). Then, the real value of debt measured in the aggregate consumption bundle $b_{F,t} = B_{F,t}/P_t$ and therefore the risk premium tend to increase. It should be noted that openness can also be destabilizing (though to a smaller extent), when the risk premium is specified as a function of debt measured in units of domestically produced goods.

Thus, for a given stance of fiscal policy (including a balance budget regime), openness to commodity trade can be an other channel through which an active monetary policy can destabilize the economy. Notably, when households do not consume foreign goods, i.e., $\vartheta = 0$, such that they behave like in a closed economy, stability is ensured by a balanced budget policy, $\kappa = 1$. Then, any value for the monetary policy feedback coefficient ρ_π

leads to a uniquely determined, stable, and non-oscillatory equilibrium. The reason for the determinacy property is that the stock of nominal debt serves as a nominal anchor, which rules out nominal indeterminacy under interest rate policy like in Canzoneri and Diba (2005), where debt is non-neutral due to transactions services. Since prices are sticky, nominal determinacy is associated with a uniquely determined equilibrium allocation. Yet, fiscal policy can render equilibrium sequences unstable, which is ruled out by a balanced budget regime.[16]

3.3 A special case: Risk-free public debt

To relate our findings to existing results in the literature, consider the case where pubic debt exhibits no risk, such that $\Phi = 0$. The equilibrium allocation is then independent of fiscal policy and Ricardian equivalence holds. Since the level of public debt does not affect its rate of return, consumption growth depends solely on the nominal interest rate $\widehat{R}_t$, which is set by the central bank, and the inflation rate $\widehat{\pi}_{H,t+1} : \widehat{c}_{t+1} - \widehat{c}_t = (1-\vartheta)\,\sigma^{-1}(\widehat{R}_t - \widehat{\pi}_{H,t+1})$. Given that (34) can be written as $\widehat{\pi}_{H,t} = \psi\sigma^{-1}\widehat{c}_t + \beta\widehat{\pi}_{H,t+1}$, the equilibrium allocation can be determined independently from fiscal policy and, therefore, in an entirely forward-looking way, like in Gali and Monacelli (2005). Equilibrium stability and uniqueness then requires interest rate policy to be active $\rho_\pi > 1$, like in Gali and Monacelli (2005) and De Fiore and Liu (2005). The precise determinacy condition is derived in appendix 8.4.

Hence, it is solely the existence of a non-zero default elasticity – and, in particular, not its size – that is responsible for the stability condition in proposition 1 to stand in stark contrast to the well-known principles of stabilizing interest rate policies in models with risk-free debt. Due to the relevance of real debt for the equilibrium allocation, the predetermined stock of debt actually serves as an equilibrium selection criterion. When the tax share is sufficiently large, the central bank can, nevertheless, apply an active interest rate policy to stabilize inflation via the conventional Fischer effect. Under higher deficits (and openness), the Blanchard effect can prevail, leading to unstable dynamics.

4 Macroeconomic stability with indexed public debt

In this section we turn to the case where the domestic government cannot borrow from abroad in its own currency. Eichengreen and Hausmann (1999) call this inability the "Original Sin", which typically characterizes less developed countries. To account for this phenomenon, internationally traded public debt is thus issued in form of one-period government bonds denominated in foreign currency, $B^I_{F,t}$. Yet, the government is still assumed to be able to borrow from domestic households in terms of its own currency. We

[16] Under a balanced budget regime the evolution of debt satisfies $B_{F,t} = B_{F,t-1}$ and exerts a stabilizing impact like a (constant) money growth rate rule (see section 5).

view this as a reasonable assumption, since our model only considers short-term borrowing and problems in domestic borrowing typically emerge just for long-term borrowing (see Eichengreen et al., 2003). Hence, total nominal government liabilities are given by $B_t = B_{H,t} + S_t B_{F,t}$, and the government budget constraint changes to

$$\frac{B_{H,t}}{R_t} + S_t \frac{B^I_{F,t}}{R_{F,t}} + P_t \tau_t = (1-\delta_t) B_{H,t-1} + S_t (1-\delta_t) B^I_{F,t-1}, \tag{37}$$

where $1/R_{F,t}$ is the foreign currency price of domestic government bonds. In contrast to $1/R_t$, the latter price is not set by the domestic central bank and is endogenously determined in equilibrium. The risk premium (1) is accordingly given by $\delta_t = \delta([B_{H,t-1} + S_t B^I_{F,t-1}]/P_t)$ or

$$\delta_t = \delta \left(\frac{b_{H,t-1}}{\pi_t} + \frac{q_t b^I_{F,t-1}}{\pi^*_t} \right) \tag{38}$$

where $b^I_{F,t}$ is again defined as the real value of public debt in terms of the domestic (aggregate) consumption good. Thus, changes in the real exchange rate now also alter the risk premium on public debt. It should further be noted that changes in domestic prices indices can only change the value of public debt held by domestic households, as can be seen from (37) which can be rewritten as $\frac{b_{H,t}}{R_t} + q_t \frac{b^I_{F,t}}{R_{F,t}} + \tau_t = (1-\delta_t) \frac{b_{H,t-1}}{\pi_t} + q_t (1-\delta_t) \frac{b^I_{F,t-1}}{\pi^*_t}$. Correspondingly, the foreign households' first order conditions for the investment in domestic government bonds changes to

$$\beta E_t \left\{ [1-\delta_b(t+1)] \left(c^*_{t+1}\right)^{-\sigma^*} / \pi^*_{t+1} \right\} = \left(c^*_t\right)^{-\sigma^*} / R_{F,t}.$$

The remainder of the model remains unchanged. In particular, government solvency is again guaranteed by (2) and the government is assumed not to issue debt domestically, such that domestic households do not hold public debt, $B_{H,t} = 0$. Log-linearizing the equilibrium conditions at the steady state, leads for (22) to a set of equilibrium conditions in $\widehat{w}_t$, $\widehat{\pi}_{t+1}$, $\widehat{\pi}_{H,t}$, $\widehat{c}_t$, $\widehat{q}_t$, $\widehat{y}_{H,t}$, $\widehat{R}_t$, $\widehat{b}^I_{F,t}$, and $\widehat{R}_{F,t}$, given by

$$\widehat{R}_{F,t} - E_t \widehat{\pi}^*_{t+1} = \Phi \left[E_t \widehat{q}_{t+1} + \widehat{b}^I_{F,t} - E_t \widehat{\pi}^*_{t+1} \right] + \sigma E_t \widehat{c}^*_{t+1} - \sigma \widehat{c}^*_t, \tag{39}$$

$$\widehat{b}^I_{F,t} = \widehat{b}^I_{F,t-1} - \widehat{\pi}^*_t + \eta \widehat{R}_{F,t}, \tag{40}$$

and the set of previous equilibrium condition (25)-(32) excluding (31). Compared to the equilibrium in the benchmark case, there exists one additional endogenous variable $\widehat{R}_{F,t}$ and an additional equilibrium condition (39). In fact, the asset pricing conditions in the previous version, (9)-(10) and (15)-(16), have been non-singular. As shown in appendix 8.5, the deterministic version of the linearized model can be reduced to the following system in $\widehat{q}_t$, $\widehat{R}_t$, $\widehat{R}_{F,t}$, $\widehat{\pi}_{H,t}$, and $\widehat{b}^I_{F,t}$ (for an exogenously given sequence for $\widehat{\pi}^*_t$)

$$\left(\Phi+\frac{1}{1-\vartheta}\right)\widehat{q}_{t+1}-\frac{1}{1-\vartheta}\widehat{q}_t=\widehat{R}_t-\widehat{\pi}_{H,t+1}-\Phi\widehat{b}^I_{F,t}, \quad (41)$$

$$\widehat{R}_t-\widehat{\pi}_{t+1}-(\widehat{q}_{t+1}-\widehat{q}_t)=\widehat{R}_{F,t}-\widehat{\pi}^*_{t+1}, \quad (42)$$

$$\widehat{b}^I_{F,t}=\widehat{b}^I_{F,t-1}-\widehat{\pi}^*_t+\eta\widehat{R}_{F,t}, \quad (43)$$

(34), and $\widehat{R}_t=\rho_\pi\widehat{\pi}_{H,t+1}$. The main difference to the previous version can immediately be seen from (43). In contrast to the benchmark case where public debt (held by foreigners) is denominated in domestic currency, the domestic producer price inflation does not directly affect the evolution of real public debt (measured in units of the aggregate consumption good). Real debt endogenously changes over time only via changes in the foreign currency interest rate on public debt $\widehat{R}_{F,t}$. As a consequence, the stability condition on monetary policy is more restrictive than in the benchmark case.

Proposition 2 *Suppose that $\Phi\in(0,1)$. Then, there exist locally stable and non-oscillatory equilibrium sequences only if*

$$\rho_\pi<1. \quad (44)$$

The equilibrium is locally unique, stable, and non-oscillatory if (but not only if) $\kappa<\frac{\vartheta}{(R-1)(1-\vartheta)+1}$ and $\rho_\pi<\min\{1,1+2(\beta+1)\frac{\Phi(1-\vartheta)+2-\Phi\eta}{\psi(1-\vartheta)(2-\Phi\eta)}\}$. If the government runs a balance budget policy $\kappa=1\Leftrightarrow\eta=0$, there exists a unit root.

Proof. See appendix 8.6.

According to proposition 2, macroeconomic stability requires a passive policy when public debt is indexed. Thus, any attempt of the central bank to stabilize inflation by an active interest rate policy, $\rho_\pi>1$, will lead to unstable (non-converging) sequences regardless of the fiscal stance, if public debt is indexed, i.e., denominated in foreign currency.

To get an intuition, consider the case where real indexed debt is higher than in the steady state, for example due to a temporary decrease in foreign prices. The risk premium will then rise such that investors are less willing to hold public debt. Thus, there will be a real depreciation ($\widehat{q}_t\uparrow$), which tends to raise inflation as before (see 34). If the central bank responds to the rise in inflation by raising the interest rate in an active way, the foreign currency interest rate $\widehat{R}_{F,t}$ tends to increase (see 42), leading to a more pronounced rise in real debt $\widehat{b}^I_{F,t}$ (see 43). The latter further feeds the risk premium and therefore the real depreciation, accelerating the initial temporary shock effect. Thus, the economy will be destabilized, i.e., it will not converge back to the steady state. If interest rate policy is however passive, then the increase in debt due to higher interest rate costs is less pronounced. When foreign inflation rises back to its steady state level, the associated real appreciation leads to a decline in the foreign currency interest rate and tends to reduce the real value of debt.

Notably, a balance budget policy, $\kappa = 1 \Leftrightarrow \eta = 0$, leads to an evolution of public debt which is completely independent of domestic events. Thus, explosive debt dynamics due to aggressive interest rate adjustments cannot occur. Yet, the fact that the sequence of real indexed debt is isolated from domestic developments implies that it exhibits a unit root. This can be seen from (43) or from the non-linearized condition $b^I_{F,t} = b^I_{F,t-1}/\pi^*_t$. Hence, any transitory (exogenously induced) change in the foreign inflation rate leads to a permanent shift in real indexed debt $b^I_{F,t}$. Since the latter affects its rate of return, this would also cause a permanent shift in consumption and inflation, which is evidently undesirable when public policy aims to stabilize macroeconomic aggregates.

To summarize, when public debt solely consists of indexed bonds, the real debt burden and thus the risk premium do not respond to changes in domestic prices. In particular, a rise in domestic price inflation does not reduce the real value of debt, as in the previous case. As a consequence, the central bank should refrain from setting the nominal interest rate policy in an active way. Moreover, fiscal policy cannot relax this constraint on monetary policy, like in the previous case where the upper bound for the inflation feedback was increasing in the share of tax financing (see corollary 1).

Overall, the results derived so far are in stark contrast to the conventional recommendation for central banks to stabilize inflation via interest rate adjustments, the Taylor-principle. Of course, they do not imply that the central bank should refrain from macroeconomic stabilization altogether, just that under the circumstances that often prevail in many LDCs, i.e. debt that is denominated in foreign currency or indexed to the price level, interest rates are not the appropriate policy instrument.

5 Successful inflation stabilization

In the previous sections we have shown that an aggressive interest rate policy, which might – in the first place – be designed to stabilize inflation, can lead to undesirable outcomes, namely, perverse effects on inflation and unstable equilibria. Yet, this does not imply that inflation stabilization is infeasible or unwanted. Far from it, the rigidity in domestic producer prices is – despite the possibility of default – the main distortion in our economy, implying that a central bank that aims to maximize welfare should predominantly stabilize domestic producer prices, like in Gali and Monacelli's (2005) model with risk free debt. The main conclusion of the analysis in the previous sections is that the central bank should not use an interest rate associated with risk as its instrument for this purpose. Thus, even though short-run nominal interest rates are widely viewed as the favorable instrument (see Taylor, 1993, or Woodford, 2003), they might be less suited for the implementation of monetary policy in less developed countries. As stressed by Taylor (2002), the use of nominal interest rates as the policy instrument are a less appropriate instrument when risk premia are "*high and variable*" (Taylor, 2002, p. 444).

Thus, there might be a case for alternative monetary policy instruments, which can be used by the central bank of such economies to implement an inflation stabilizing policy, which avoids problems associated with external borrowing. Taylor (2002) continues this line of arguments by recommending money supply rules: "*Thus, policy makers in emerging market economies might want to give greater consideration to policy rules with monetary aggregates, even if rules with the interest rate become the preferred choice.*" (Taylor, 2002, p. 445).

In this section we act on his suggestions and demonstrate that an inflation stabilizing policy is feasible in our framework, even under full debt indexation, when the central bank applies an alternative instrument. For this we assume that the central bank issues money, such that domestic households can accumulate wealth in form of non-interesting bearing money M_t and debt instruments (as before). For the purpose of this analysis, we want to abstract from any distortion other than the nominal price rigidity. In particular, we follow King and Wolman (2004) and introduce the following simplified money demand specification which avoids "wealth and substitution effects that normally arise in optimizing models of money demand":

$$M_t = P_t c_t. \tag{45}$$

Thus, households hold money for their expenditures on aggregate consumption.[17] This money demand function can in principle also been derived from a cash-in-advance constraint, which typically leads to an additional cash-credit good distortion between consumption and leisure. Allowing for this would complicate the analysis in an unnecessary way,[18] but it would not change the fact that the central bank can bypass problems stemming from weak fiscal policy by controlling aggregate demand and inflation via an alternative instrument. Due to the simplified money demand specification (45), the equilibrium conditions listed in section 3.1 are not affected.

Suppose that the central bank wants to implement an inflation targeting policy. To be more precise, it aims to minimize the expected sum of future discounted losses $E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} L_t$, where the period-by-period loss rises with the variances of the domestic producer price inflation and of the output gap $\widehat{x}_{H,t} : L_t = \hat{\pi}_{H,t}^2 + \lambda \widehat{x}_{H,t}^2$, where $\widehat{x}_{H,t} = \widehat{y}_{H,t} - \widehat{y}_{H,t}^{eff}$ and $y_{H,t}^{eff}$ denotes the efficient level of output. As shown by Gali and Monacelli (2005), this loss function can approximate welfare losses in our framework (see appendix 8.7 for a discussion).[19] Thus, this form of inflation targeting can in principle be interpreted as a policy regime that aims at maximizing social welfare in our economy.

[17] Evidently, the households' budget constraint and the consolidated budget constraint of the private sector have to be adjusted accordingly.

[18] The opportunity costs of money holdings can either be measured by the interest rate on risky government bonds or by the risk-free interest rate on internationally traded bonds. Thus, even when interest rate changes affect the equilibrium allocation due to a money demand distortions, sovereign risk concerns would then just residually determine the interest rate differential between public debt and risk-free internationally traded bonds.

[19] Specifically, this will be the case for permanently balanced trade, which requires a unitary intertemporal elasticities of substitution $\sigma = \sigma^* = 1$ (see Corsetti and Pesenti, 2001).

Coming back to Taylor's (2002) suggestion, we examine the case where the central bank controls the growth rate of money,

$$\mu_t = M_t/M_{t-1},$$

in a way that the average gross money growth rate equals one, $\overline{\mu} = 1$.

In this case, fiscal policy (as described in section 2.1) does not affect the equilibrium allocation, i.e., the sequence of taxes and, in particular, of public debt are irrelevant for the remaining macroeconomic variables.[20] Put differently, the conditions for the applicability of the Ricardian equivalence principle are satisfied in this case. In contrast to the case where the central bank sets the interest rate on risky public debt, the reduced set of equilibrium conditions now contains the money demand condition (45), but excludes (26) and (31). As a further implication of Ricardian equivalence, steady state inflation is uniquely pinned down by the average money growth rate, $\pi = \overline{\mu}$ and $\pi_H = \overline{\mu}$, such that (23) does not apply. Thus, by setting $\overline{\mu}$ equal to one the central bank avoids long-run welfare costs due to the price rigidity.

Now suppose that the central bank wants to implement its optimal inflation targeting plan. Specifically, we assume that the central bank minimizes the expected sum of future discounted losses subject to the private sector equilibrium condition. It can then be shown that a contingent money growth reaction function of the following form is able to implement the central bank's optimal commitment plan under a timeless perspective (see appendix 8.7)

$$\mu_t - \mu = \mu_\pi \left(\pi_{H,t} - \overline{\pi}_H\right), \quad \text{where } \mu_\pi < 1, \tag{46}$$

such that $\widehat{m}_t - \widehat{m}_{t-1} + \widehat{\pi}_t = \widehat{\mu}_t = \mu_\pi \widehat{\pi}_{H,t}$. The state contingent money supply rule (46) implies that nominal balances rise by less than one for one with domestic producer price inflation. Using the relation between the latter and CPI inflation (18), as well as (27) and (45), which implies $\widehat{m}_t = \widehat{c}_t$, we can rewrite (46) as $\widehat{m}_t - \widehat{m}_{t-1} = -(1-\mu_\pi)(1 + \frac{\vartheta}{1-\vartheta}\sigma)^{-1}\widehat{\pi}_{H,t}$. Hence, the growth rate of real balances decreases with inflation, and money supply is non-accommodating.

Given the monetary policy specification (46), we can easily examine macroeconomic stability. Due to the simple (non-distortionary) money demand specification (45), the set of equilibrium condition can be reduced to a two dimensional system in inflation and domestic producer price inflation. It turns out that a money supply satisfying (46) ensures local stability and uniqueness of the equilibrium sequences $\{\widehat{m}_t,\ \widehat{\pi}_{H,t}\}_{t=0}^{\infty}$.[21] The equilibrium sequences $\{\widehat{w}_t, \widehat{\pi}_t, \widehat{c}_t, \widehat{n}_t, \widehat{q}_t, \widehat{y}_{H,t}\}_{t=0}^{\infty}$ are then also uniquely determined and stable.

[20] This property further relies on the assumption $\kappa > 0$ (see 2) which – together with the assumption that nominal balances are constant in the long-run ($\overline{\mu} = 1$) – ensures intertemporal solvency of the consolidated public sector.

[21] This finding is consistent with McCallum's (1999) stability results for a money supply rule with a negative feedback from (lagged) inflation to the current money growth rate.

Proposition 3 *Suppose that money demand satisfies (45). Then the equilibrium under a money supply reaction function (46) is locally stable and uniquely determined.*

Proof. See appendix 8.8.

According to the result summarized in proposition 3, the central bank can safely implement a stable and uniquely determined equilibrium, which is characterized by a stabilized inflation sequence, by applying a money supply reaction function (46) with a small or negative value for μ_π (see also appendix 8.7).[22] This analysis of course neglects additional problems and distortions associated with money demand and the central bank's ability to control aggregate demand by money supply. Nevertheless, it shows that money supply based inflation stabilization policy, or, an inflation targeting strategy, is in principle feasible under risky public debt.

6 Conclusion

Is interest-rule based inflation targeting an appropriate monetary policy strategy in countries with high external debt? Recent evidence for emerging market economies indicates that "inflation targeting can have perverse effects" (Blanchard, 2005): Higher interest rates can lead to increased fears of debt default, a real depreciation, and higher domestic goods prices, such that active interest rate policy might induce unstable dynamics. We examine this mechanism in a dynamic general equilibrium model of a small open economy where a goods price rigidity provides a rationale for inflation stabilization.

We first consider the case where the government is assumed to borrow from abroad in its own currency. For this case, we show that unstable equilibrium dynamics can indeed occur when interest rates are strongly raised in response to higher domestic inflation. The higher the primary deficit (or the lower the primary surplus) and the higher the degree of openness, the more stringent are the constraints on interest rate policy if run away unstable equilibria are to be avoided. With rising shares of tax financing the constraint on monetary policy is relaxed, implying that the central bank can apply an active interest rate policy if it is supported by a sufficiently sound fiscal policy.

Turning to the more realistic case where the government is unable to borrow abroad in its own currency, macroeconomic stability requires interest rate policy to be accommodating (passive). Notably, with indexed debt an active interest rate policy leads to a non-stationary sequences for macroeconomic aggregates even if the government runs primary surpluses high enough to cover interest payments.

Further analysis may qualify our finding; but the results from this analysis at least suggest caution before a switch to inflation targeting – that is aimed to be implemented

[22] Evidently, a constant money growth rule guarantees local stability and uniqueness in this model. A comprehensive analysis of the stability implications and the equivalence of money growth and interest rate rules in a closed economy can be found in Schabert (2005).

by interest rate adjustments – is considered when substantial parts of the public debt are indexed or denominated in foreign currency. Yet, successful inflation stabilization is still feasible in such an environment, if the central bank does not apply an interest rate policy. In particular, a domestic inflation targeting policy, which can maximize social welfare in our framework, can safely be implemented by money supply adjustments, which avoid interactions with weak fiscal policy regimes.

7 References

Benhabib, J., Schmitt-Grohe, S., and M. Uribe, 2001, Monetary Policy and Multiple Equilibria, American Economic Review 91, 167-185.

Benigno, G. and P. Benigno, 2003, Price Stability in Open Economies", Review of Economic Studies 70, 743–764.

Blanchard, O.J., 2005, Fiscal Dominance and Inflation Targeting: Lessons from Brazil, in Giavazzi, F., I. Goldfajn and S. Herrera, Inflation Targeting, Debt and the Brazilian experience 1999 to 2003, MIT Press.

Blanchard, O.J., and C.M. Kahn, 1980, The Solution of Linear Difference Models under Rational Expectations, Econometrica 48, 1305-1313.

Buiter, W. H., 2002, The Fiscal Theory of the Price Level: A Critique', Economic Journal 112, 459-480.

Calvo, G., 1983, Staggered Prices in a Utility-Maximizing Framework, Journal of Monetary Economics 12, 383-398.

Cantor, R., and F. Packer, 1996, Determinants and Impact of Sovereign Credit Ratings, Federal Reserve Bank New York, Economic Policy Review, October 37-54.

Carlstrom, C.T., and T.S. Fuerst, 2001, Timing and Real Indeterminacy in Monetary Models, Journal of Monetary Economics 47, 285-298.

Clarida, R., J. Gali, and M. Gertler, 2001, Optimal Monetary Policy in Open vs. Closed Economies: An Integrated Approach, American Economic Review 91 P. & P., 248-252.

Clarida, R., J. Gali, and M. Gertler, 2002, A Simple Framework for International Monetary Policy Analysis, Journal of Monetary Economics 49, 879-904.

Corsetti, G. and P. Pesenti, 2001, Welfare and Macroeconomic Interdependence, Quarterly Journal of Economics 116 , 421–446.

De Fiore, F., and Z. Liu, 2004, Does Trade Openness Matter for Aggregate Instability?, Journal of Economic Dynamics and Control, forthcoming.

Eaton, J., and M. Gersovitz, 1981, Debt with Potential Repudiation: Theoretical and Empirical Analysis, Review of Economic Studies 48, 289-309.

Edwards, S., 1984, LDC Foreign Borrowing and Default Risk: An Empirical Investigation, American Economic Review 74, 726-734.

Eichengreen, B. and R. Hausmann, 1999, Exchange Rates and Financial Fragility, in New Challenges for Monetary Policy, Kansas City: Federal Reserve Bank of Kansas City, 329-368.

Eichengreen, B., R. Hausmann, and U. Panizza, 2003, The Pain of Original Sin, in Barry Eichengreen and Ricardo Hausmann (eds.), Debt Denomination and Financial Instability in Emerging-Market Economies, Chicago: University of Chicago Press, forthcoming.

Eichengreen, B. and A. Mody, 1998, What Explains Changing Spreads on Emerging-Market Debt: Fundamentals or Market Sentiment?, NBER Working Paper No. 6408.

Favero, G., and F. Giavazzi, 2005, Inflation Targeting and Debt: Lessons from Brazil, in Giavazzi, F., I. Goldfajn and S. Herrera, Inflation Targeting, Debt and the Brazilian experience 1999 to 2003, MIT Press.

Ferrucci, G., 2003, Empirical Determinants of Emerging Market Economies' Sovereign Bond Spreads, Bank of England Working Paper no. 205.

Gali, J., and T. Monacelli, 2005, Monetary Policy and Exchange Rate Volatility in a Small Open Economy, Review of Economic Studies 72, 707–734.

IMF 2005, Does Inflation Targeting Work in Emerging Markets?, in IMF: World Economic Outlook, September 2005, Washington DC.

King, R.G., and A.L. Wolman, 2004, Monetary Discretion, Pricing Complementarity and Dynamic Multiple Equilibria, Quarterly Journal of Economics 119, 1513-1553.

Kocherlakota, N., and C. Phelan, 1999, Explaining the Fiscal Theory of the Price Level, Federal Reserve Bank of Minneapolis Quarterly Review 23, 14-23.

Linnemann, L. and A. Schabert, 2005, Debt Non-neutrality, Policy Interactions, and Macroeconomic Stability, Tinbergen Institute Discussion Paper No. 05-077/2.

Loyo, E., 1999, Tight Money Paradox on the Loose: A Fiscalist Hyperinflation, J.F. Kennedy School of Government, Harvard University.

Loyo, E., 2005, Comment on Blanchard's "Fiscal Dominance and Inflation Targeting: Lessons from Brazil", in Giavazzi, F., I. Goldfajn and S. Herrera, Inflation Targeting, Debt and the Brazilian experience 1999 to 2003, MIT Press.

Niepelt, D., 2004, The Fiscal Myth of the Price Level, Quarterly Journal of Economics 119, 277-299.

McCallum, B.T., 1999, Role of the Minimal State Variable Criterion in Rational Expectations Models, International Tax and Public Finance 6, 621-639.

Min, H., 1998, Determinants of Emerging Market Bond Spreads: Do Economic Fundamentals Matter?, World Bank Working Paper No. 1899.

Mishkin, F., 2004, Can Inflation Targeting Work in Emerging Market Economies?, NBER, WP 10646.

Reinhart, C.M., K.S. Rogoff, and M.A. Savastano, 2003, Debt Intolerance, Brookings Papers on Economic Activity 1:2003, 1-74.

Schabert, A., 2005, Money Supply and the Implementation of Interest Rate Targets, CEPR Discussion Paper DP5094.

Schmitt-Grohe, S., and M. Uribe, 2003, Closing Small Open Economy Models, Journal of International Economics 61, 163-185.

Sims, C.A., 2005, Limits to Inflation Targeting, by Christopher A. Sims, in Bernanke, B.S., and M. Woodford, eds., The Inflation-Targeting Debate, University of Chicago Press.

Svensson, L.E.O., 1999, Inflation Targeting as a Monetary Policy Rule, Journal of Monetary Economics 43, 607-654.

Svensson, L.E.O., 2005, Optimal Inflation Targeting: Further Developments of Inflation Targeting, Princeton University.

Svensson, L.E.O., and M. Woodford, 2005, Implementing Optimal Policy through Inflation-Forecast Targeting, in Bernanke, B.S., and M. Woodford, eds., The Inflation-Targeting Debate, University of Chicago Press, 19-83.

Taylor, J.B., 1993, Discretion versus Policy Rules in Practice, Carnegie-Rochester Conference Series on Public Policy 39, 195-214.

Taylor, J.B., 2002, Using Monetary Policy Rules in Emerging Market Economies, Stabilization and Monetary Policy, Banco de Mexico, 441-457.

Uribe, M., 2005, A Fiscal Theory of Sovereign Risk, Journal of Monetary Economics, forthcoming.

Woodford, M., 2003, Interest and Prices: Foundations of a Theory of Monetary Policy, Princeton: Princeton University Press.

Yun, T., 1996, Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles, Journal of Monetary Economics 37, 345-370.

8 Appendix

8.1 Set of equilibrium conditions

A rational expectations equilibrium is a set of sequences $\{mc_{H,t}, w_t,\ \pi_t,\ \pi_{H,t}, c_t, n_t,\ q_t, z_t, y_{H,t},\ R_t, R_t^*, f_t, b_{H,t}, b_{F,t}\}_{t+0}^{\infty}$ satisfying

$$\zeta(1-n_t)^{-\sigma_l} = w_t c_t^{-\sigma}, \tag{47}$$

$$\beta E_t\left\{\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{q_{t+1}}{q_t}\frac{1}{\pi_{t+1}^*}\right\} = \frac{1}{R_t^*}, \tag{48}$$

$$\beta E_t\left\{\left[1-\delta\left(\frac{b_{H,t-1}}{\pi_t}+\frac{b_{F,t-1}}{\pi_t}\right)\right]\left(\frac{c_t}{c_{t+1}}\right)^{\sigma}\frac{1}{\pi_{t+1}}\right\} = \frac{1}{R_t}, \tag{49}$$

$$\beta E_t\left\{\left[1-\delta\left(\frac{b_{H,t-1}}{\pi_t}+\frac{b_{F,t-1}}{\pi_t}\right)\right]\left(\frac{c_t^*}{c_{t+1}^*}\right)^{\sigma}\frac{1}{\pi_{t+1}}\frac{q_t}{q_{t+1}}\right\} = \frac{1}{R_t}, \tag{50}$$

$$w_t = (q_t \cdot z_t)\, mc_{H,t}, \tag{51}$$

$$z_t^{\vartheta-1} = q_t, \tag{52}$$

$$\frac{1+\kappa(R_t-1)}{R_t}b_{H,t} + \frac{1+\kappa(R_t-1)}{R_t}b_{F,t} = \frac{b_{H,t-1}}{\pi_t}+\frac{b_{F,t-1}}{\pi_t}, \tag{53}$$

$$(1-\vartheta)\left(z_t^{\vartheta}\right)^{-1}c_t + \vartheta^*\left(z_t\right)^{-1}c_t^* = y_{H,t}, \tag{54}$$

$$\pi_t = \pi_{H,t}\left(q_t/q_{t-1}\right), \tag{55}$$

$$\left[\frac{b_{F,t}}{R_t} - \left[1-\delta\left(\frac{b_{H,t-1}}{\pi_t}+\frac{b_{F,t-1}}{\pi_t}\right)\right]\frac{b_{F,t-1}}{\pi_t}\right] + z_t^{\vartheta}y_{H,t} = c_t + q_t\left[\frac{f_t}{R_t^*}-\frac{f_{t-1}}{\pi_t^*}\right], \tag{56}$$

the aggregate supply constraint that combines the price setting conditions of firms, and the constraint on aggregate domestic production (which will in linearized form given by $\widehat{\pi}_{H,t} = \chi\widehat{mc}_{H,t} + \beta E_t\widehat{\pi}_{H,t+1}$ and by $\widehat{y}_{H,t} = \widehat{n}_t$), the transversality condition, and a monetary policy (3) for given sequences $\{c_t^*,\ R_t^*, \pi_t^*\}_{t=0}^{\infty}$ satisfying $\beta E_t\{\left(c_t^*/c_{t+1}^*\right)^{\sigma}/\pi_{t+1}^*\} = 1/R_t^*$, initial asset endowments F_{-1}, $B_{H,-1}$, and $B_{F,-1}$, and an initial price level $P_{H,-1}$.

8.2 The linearized benchmark model

For the case where domestic households' holdings of public debt equals zero, $B_{H,t}=0$, the set of linearized equilibrium conditions is summarized by (25)-(32). Eliminating $\widehat{w}_t$ and $\widehat{y}_{H,t}$ with (25) and (29), the aggregate supply constraint (28) can be rewritten as

$$\widehat{\pi}_{H,t} = \chi\left[\left(\sigma_n\alpha + \frac{\vartheta}{1-\vartheta}\right)\widehat{q}_t + \left(\sigma_n\frac{(1-\vartheta)c}{n}+\sigma\right)\widehat{c}_t\right] + \beta E_t\widehat{\pi}_{H,t+1}$$

where $\alpha = \frac{(1-\vartheta)c\vartheta+\vartheta^{*}c^{*}}{n(1-\vartheta)}$. Further, eliminating $\widehat{\pi}_t$ and $\widehat{c}_t$ with (27) and (30) the set of equilibrium conditions can be reduced to the following system in $\{\widehat{\pi}_{H,t}, \widehat{q}_t, \widehat{R}_t, \widehat{b}_{F,t}\}_{t=0}^{\infty}$

$$i.) \quad -\Phi\widehat{b}_{F,t} = -\widehat{R}_t + (1-\Phi)\, E_t\widehat{\pi}_{H,t+1} + \frac{1-\Phi\vartheta}{1-\vartheta}\left(E_t\widehat{q}_{t+1} - \widehat{q}_t\right)$$

$$ii.) \quad \widehat{b}_{F,t} = \widehat{b}_{F,t-1} - \widehat{\pi}_{H,t} - \frac{\vartheta}{1-\vartheta}\left(\widehat{q}_t - \widehat{q}_{t-1}\right) + \eta\widehat{R}_t$$

$$iii.) \quad \widehat{\pi}_{H,t} = \chi\left[\frac{\sigma_n}{\sigma}\frac{(1-\vartheta)c}{n} + 1 + \sigma_n\alpha + \frac{\vartheta}{1-\vartheta}\right]\widehat{q}_t + \beta E_t\widehat{\pi}_{H,t+1}$$

$$iv.) \quad \widehat{R}_t = \rho_\pi E_t\widehat{\pi}_{H,t+1}$$

Finally, we use that $i.)$ implies $E_{t-1}\widehat{q}_t - \widehat{q}_{t-1} = \frac{1-\vartheta}{1-\Phi\vartheta}(-\Phi\widehat{b}_{F,t-1} + (\rho_\pi - (1-\Phi))\, E_{t-1}\widehat{\pi}_{H,t})$ $\forall t \geq 1$. Hence, we can rewrite $ii.)$ in the following way

$$b_{F,t} = \frac{1}{1-\Phi\vartheta}\widehat{b}_{F,t-1} - \frac{\vartheta\,(\rho_\pi - 1) + 1}{1-\Phi\vartheta}\widehat{\pi}_{H,t} + \eta\rho_\pi E_t\widehat{\pi}_{H,t+1} + \xi_{t-1,t},$$

where $\xi_{t-1,t}$ collects the stochastic terms stemming from the expectation errors $E_{t-1}\widehat{\pi}_{H,t} - \widehat{\pi}_{H,t}$ and $E_{t-1}\widehat{q}_t - \widehat{q}_t$. Under certainty, we thus end up with the system (33)-(35).

8.3 Proof of proposition 1

In order to establish the claims made in the proposition, the model (33)-(35) and $\widehat{R}_t = \rho_\pi E_t\widehat{\pi}_{H,t+1}$ is rewritten as

$$\begin{pmatrix} \widehat{b}_{F,t} \\ \widehat{q}_{t+1} \\ \widehat{\pi}_{H,t+1} \end{pmatrix} = \mathbf{A}\begin{pmatrix} \widehat{b}_{F,t-1} \\ \widehat{q}_t \\ \widehat{\pi}_{H,t} \end{pmatrix}, \quad \mathbf{A} = \begin{pmatrix} \Phi & \frac{1-\Phi\vartheta}{1-\vartheta} & 1-\Phi-\rho_\pi \\ 1 & 0 & -\eta\rho_\pi \\ 0 & 0 & \beta \end{pmatrix}^{-1}\begin{pmatrix} 0 & \frac{1-\Phi\vartheta}{1-\vartheta} & 0 \\ \frac{1}{1-\Phi\vartheta} & 0 & -\varphi \\ 0 & -\psi & 1 \end{pmatrix}.$$

Given that there is one predetermined state variable $(\widehat{b}_{F,t-1})$, while the other two variables can jump, a stable and uniquely determined equilibrium requires the matrix $\mathbf{A}$ to exhibit exactly one eigenvalue with modulus smaller than one (see Blanchard and Kahn, 1980). The characteristic polynomial of $\mathbf{A}$ is given by

$$\begin{aligned} Q(X) = {} & X^3 + \frac{2\beta - \Phi\vartheta(1+\beta) + 1 + (\rho_\pi - 1)(\psi\vartheta - \psi) + (\rho_\pi\eta - 1)(\Phi\psi - \Phi\psi\vartheta)}{(\Phi\vartheta - 1)\,\beta}X^2 \\ & + \frac{(\Phi\vartheta - 1)(\Phi\vartheta - \beta - 2) + (\rho_\pi - 1)\left(\psi\vartheta - \psi - \Phi\psi\vartheta + \Phi\psi\vartheta^2\right)}{(\Phi\vartheta - 1)^2\,\beta}X - \frac{1}{\beta\,(1-\Phi\vartheta)}. \end{aligned}$$

Suppose that $\Phi \in (0,1)$. Then, the determinant of $\mathbf{A}$, which is given by $\det(\mathbf{A}) = -Q(0) = \frac{1}{\beta(1-\Phi\vartheta)} > 1$, is strictly larger than one. Thus, $\mathbf{A}$ exhibits at least one unstable eigenvalue and either one or three positive eigenvalues. To establish the existence of a stable non-oscillatory equilibrium solution, which demands a stable positive eigenvalue, we examine

$Q(X)$ at $X = 1$, which is given by

$$Q(1) = \frac{(1-\vartheta)\,\Phi\psi}{(\Phi\vartheta - 1)^2\,\beta}\,(2\vartheta - \Phi\vartheta + 1 + (\Phi\eta\vartheta - \eta - 2\vartheta)\,\rho_\pi)\,.$$

Hence, there exists at least one stable eigenvalue between zero and one if $Q(1) > 0 \Leftrightarrow$

$$\rho_\pi < 1 + \Delta, \;\; \text{where } \Delta = \frac{(1-\Phi\vartheta)\,(1-\eta)}{(1-\Phi\vartheta)\,\eta + 2\vartheta}$$

Next, we want to establish that there exists no other stable eigenvalue. Given that $\det(\mathbf{A}) > 1$, we have to rule out the existence of a stable root of $Q(X) = 0$, which lies between minus one and zero. Since there is at least one unstable eigenvalue and $Q(0) < 0$, there exists a stable negative eigenvalue if and only if $Q(-1) > 0$, where

$$Q(-1) = -\frac{1}{(\Phi\vartheta - 1)^2\,\beta}\begin{pmatrix} (\Phi\vartheta - 1)\,(2\,(1+\beta)\,(2-\Phi\vartheta) - \psi\Phi\,(1-\eta)\,(1-\vartheta)) \\ +\,(\rho_\pi - 1)\,\psi\,(\vartheta - 1)\left(\Phi^2\eta\vartheta - \Phi\eta + 2\right) \end{pmatrix}.$$

and the stable and non-oscillatory solution is the unique stable solution, if $Q(-1) < 0 \Leftrightarrow$

$$\rho_\pi < 1 + \widetilde{\Delta}, \;\; \text{where } \widetilde{\Delta} = (1-\Phi\vartheta)\,\frac{2\,(1+\beta)\,(1-\Phi\vartheta) + (2\,(1+\beta) - \psi\Phi\,(1-\eta)\,(1-\vartheta))}{\psi\,(1-\vartheta)\,(\Phi^2\eta\vartheta + 2 - \Phi\eta)}$$

Hence, there exist exactly one stable and positive eigenvalue if and only if $\rho_\pi < \min\{1 + \Delta, 1 + \widetilde{\Delta}\}$, while there exist exactly one stable and negative eigenvalue (oscillatory equilibrium sequences) if and only if $\rho_\pi > \max\{1 + \Delta, 1 + \widetilde{\Delta}\}$. ■

8.4 Appendix to the special case of risk-free debt

When public debt is assumed to exhibit no default risk, $\delta = \Phi = 0$, fiscal policy is neutral and the equilibrium can be summarized to a set of sequences $\{\pi_{H,t}, c_t, R_t\}_{t+0}^{\infty}$ satisfying $\widehat{c}_{t+1} - \widehat{c}_t = (1-\vartheta)\,\sigma^{-1}(\widehat{R}_t - \widehat{\pi}_{H,t+1})$, $\widehat{\pi}_{H,t} = \psi\sigma^{-1}\widehat{c}_t + \beta\widehat{\pi}_{H,t+1}$, and $\widehat{R}_t = E_t\widehat{\pi}_{H,t+1}$. Uniqueness of the equilibrium sequences then solely depends on the monetary policy stance, while equilibrium instability is then be ruled out due to the lack of history dependence. The condition for local equilibrium uniqueness is summarized in the following proposition.

Proposition A1 *Suppose that public debt exhibits no default risk, $\delta = \Phi = 0$. Then, the equilibrium is locally unique only if $\rho_\pi > 1$. A necessary and sufficient condition for local equilibrium uniqueness is $\rho_\pi \in (1, 1 + \widetilde{\Delta}_{\Phi=0})$, where $\widetilde{\Delta}_{\Phi=0} = 2\,(1+\beta)\,/[\psi\,(1-\vartheta)]$. The equilibrium is then locally stable.*

Proof. When $\Phi = 0$, the equilibrium sequences for $\widehat{q}_t$ and $\widehat{\pi}_{H,t}$ are determined by $(\widehat{q}_{t+1}\;\;\widehat{\pi}_{H,t+1}) = \mathbf{A}_{\Phi=0}\,(\widehat{q}_t\;\;\widehat{\pi}_{H,t})$ where the characteristic polynomial of $\mathbf{A}_{\Phi=0}$ is $X^2 + \frac{1}{\beta} + \frac{X}{\beta}\,(\psi\,(1-\vartheta)\,(\rho_\pi - 1) - 1 - \beta)$, such that $\text{trace}(\mathbf{A}_{\Phi=0}) = \beta^{-1}\,(\psi\,(\vartheta - 1)\,(\rho_\pi - 1) + 1 + \beta)$ and $\det(\mathbf{A}_{\Phi=0}) = \beta^{-1} > 1$. Further given that $\det(\mathbf{A}_{\Phi=0}) - \text{trace}(\mathbf{A}_{\Phi=0}) = -(1 +$

$\psi(1-\vartheta)(\rho_\pi - 1)/\beta)$ and $\det(\mathbf{A}_{\Phi=0}) + \text{trace}(\mathbf{A}_{\Phi=0}) = \beta^{-1}(2+\beta-\psi(1-\vartheta)(\rho_\pi - 1))$, we can conclude that $\mathbf{A}_{\Phi=0}$ exhibits two unstable eigenvalues (indicating equilibrium uniqueness) if and only if $\rho_\pi > 1$ (implying $\det(\mathbf{A}_{\Phi=0}) - \text{trace}(\mathbf{A}_{\Phi=0}) > -1$) and $\rho_\pi < 1 + \frac{2(1+\beta)}{\psi(1-\vartheta)}$ (implying $\det(\mathbf{A}_{\Phi=0}) + \text{trace}(\mathbf{A}_{\Phi=0}) > -1$). ■

Hence, when public debt is risk-free interest rate policy should not be too aggressive in order to rule out multiple equilibrium solutions. The reason that uniqueness requires the central bank to respect the upper bound $1+\widetilde{\Delta}_{\Phi=0}$ is that interest rate policy is specified in a forward-looking way. As shown by De Fiore and Liu (2005) or Carlstrom and Fuerst (2001) in the related closed economy model, forward-lookingness is an independent source for equilibrium multiplicity.

8.5 The linearized model with indexed debt

Combining (26) and (39), and using that international risk sharing implies, $\sigma\widehat{c}_t = \widehat{q}_t + \sigma^*\widehat{c}_t^*$, we get the following relation between the domestic price $\widehat{R}_t$ and the foreign price $\widehat{R}_{F,t}$ of public debt $\widehat{R}_t - E_t\widehat{\pi}_{t+1} = \widehat{R}_{F,t} - E_t\widehat{\pi}^*_{t+1} + E_t\widehat{q}_{t+1} - \widehat{q}_t$. Thus, a rational expectations equilibrium of the model with indexed public debt and with $B_{H,t} = 0$ is a set of sequences for $\widehat{\pi}_{t+1}, \widehat{\pi}_{H,t}$, $\widehat{c}_t$, $\widehat{q}_t$, $\widehat{y}_{H,t}$, $\widehat{R}_t$, $\widehat{R}_{F,t}$, and $\widehat{b}^I_{F,t}$ satisfying (27), (29), (30), and (32)

$$\widehat{R}_t = \Phi\left[E_t\widehat{q}_{t+1} + \widehat{b}^I_{F,t} - E_t\widehat{\pi}^*_{t+1}\right] + E_t\widehat{\pi}_{t+1} + \sigma E_t\widehat{c}_{t+1} - \sigma\widehat{c}_t, \tag{57}$$

$$\widehat{R}_t - E_t\widehat{\pi}_{t+1} = \widehat{R}_{F,t} - E_t\widehat{\pi}^*_{t+1} + E_t\widehat{q}_{t+1} - \widehat{q}_t, \tag{58}$$

$$\widehat{b}^I_{F,t} = \widehat{b}^I_{F,t-1} - \widehat{\pi}^*_t + \eta\widehat{R}_{F,t}, \tag{59}$$

$\widehat{\pi}_{H,t} = \chi\left(\sigma_n\widehat{y}_{H,t} + \sigma\widehat{c}_t + \frac{\vartheta}{1-\vartheta}\widehat{q}_t\right) + \beta E_t\widehat{\pi}_{H,t+1}$, the transversality condition, for given sequences of the starred variables and initial conditions. The aggregate supply constraint (34) can be derived as before. Eliminating $\widehat{R}_{F,t}$ with (58), we can rewrite (57) and (59) as

$$-\Phi\left[E_t\widehat{q}_{t+1} + \widehat{b}^I_{F,t} - E_t\widehat{\pi}^*_{t+1}\right] = -\widehat{R}_t + E_t\widehat{\pi}_{t+1} + \sigma\widehat{c}_{t+1} - \sigma\widehat{c}_t,$$
$$\widehat{b}^I_{F,t} - \left(\widehat{b}^I_{F,t-1} - \widehat{\pi}^*_t\right) = \eta E_t\widehat{\pi}^*_{t+1} + \eta\left(\widehat{R}_t - E_t\widehat{\pi}_{t+1} - E_t\widehat{q}_{t+1} + \widehat{q}_t\right)$$

Finally, eliminating $\widehat{c}_t$ and $\widehat{R}_t$ by (27) and (32), gives

$$-\Phi E_t\widehat{q}_{t+1} - \Phi\widehat{b}^I_{F,t} + \Phi E_t\widehat{\pi}^*_{t+1} = (1-\rho_\pi)E_t\widehat{\pi}_{H,t+1} + \frac{1}{1-\vartheta}\left(E_t\widehat{q}_{t+1} - \widehat{q}_t\right)$$
$$\widehat{b}^I_{F,t} = \widehat{b}^I_{F,t-1} - \widehat{\pi}^*_t + \eta\left(\Phi E_t\widehat{q}_{t+1} + \Phi\widehat{b}^I_{F,t} - \Phi E_t\widehat{\pi}^*_{t+1}\right) + \eta E_t\widehat{\pi}^*_{t+1}$$

which – together with (58) – can under certainty be summarized by (41)-(43).

8.6 Proof of proposition 2

To establish the claim in the proposition, we follow the strategy of the proof of proposition 1 and rewrite the equilibrium conditions under certainty as

$$\begin{pmatrix} \widehat{b}^I_{F,t} \\ \widehat{q}_{t+1} \\ \widehat{\pi}_{H,t+1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{b}^I_{F,t-1} \\ \widehat{q}_t \\ \widehat{\pi}_{H,t} \end{pmatrix}, \quad \mathbf{A}= \begin{pmatrix} \Phi & \Phi + \frac{1}{1-\vartheta} & 1-\rho_\pi \\ 1-\eta\Phi & -\eta\Phi & 0 \\ 0 & 0 & \beta \end{pmatrix}^{-1} \begin{pmatrix} 0 & \frac{1}{1-\vartheta} & 0 \\ 1 & 0 & 0 \\ 0 & -\psi & 1 \end{pmatrix},$$

where the characteristic polynomial of $\mathbf{A}$ is given

$$F(X) = X^3 + X \frac{(\Phi+\beta+\psi-\Phi\eta-\Phi\vartheta-\psi\vartheta-\psi\rho_\pi+\psi\vartheta\rho_\pi+2)}{\beta(\Phi-\Phi\eta-\Phi\vartheta+1)} - \frac{1}{\beta(\Phi-\Phi\eta-\Phi\vartheta+1)}$$
$$+X^2 \frac{\Phi(\beta+1)(\eta+\vartheta-1)-(1+2\beta)+(\rho_\pi-1)\psi(\vartheta-1)(\Phi\eta-1)}{\beta(\Phi-\Phi\eta-\Phi\vartheta+1)}.$$

The determinant of $\mathbf{A}$, $\det(\mathbf{A}) = -F(0)$ is strictly positive, since $F(0) = -\frac{1}{\beta(1-\Phi\eta+\Phi(1-\vartheta))} < 0$, and can be smaller or larger than one. The existence of a stable and positive eigenvalue is ensured when $F(1) = (1-\rho_\pi)(1-\vartheta)\Phi\eta\psi \cdot [-F(0)]$ is strictly positive, which requires $\eta > 0 \Leftrightarrow \kappa < 1$ and

$$F(1) > 0 \Leftrightarrow \rho_\pi < 1.$$

Then, there is no stable negative root if $F(-1) = \frac{(1-\rho_\pi)\psi(\vartheta-1)(2-\Phi\eta)+2(\beta+1)(\Phi\eta-\Phi+\Phi\vartheta-2)}{(\Phi-\Phi\eta-\Phi\vartheta+1)\beta}$ is negative, which requires

$$F(-1) < 0 \Leftrightarrow \rho_\pi < 1 + 2(\beta+1)\frac{\Phi(1-\vartheta)+2-\Phi\eta}{\psi(1-\vartheta)(2-\Phi\eta)}.$$

Yet, indeterminacy (equilibrium multiplicity) is still not ruled out, since there might be three stable and positive eigenvalue. This is evidently not the case if $\det(\mathbf{A}) > 1$, which is ensured by $1 + \Phi(1-\vartheta-\eta) < 1 \Leftrightarrow \kappa < \frac{\vartheta}{(R-1)(1-\vartheta)+1}$. If $\kappa = 1 \Leftrightarrow \eta = 0$ then the characteristic polynomial of $\mathbf{A}$ reads

$$F(X)|_{\eta=0} = \left(X^2 - X\frac{1+\beta+\Phi(1-\vartheta)+\psi(\rho_\pi-1)(\vartheta-1)}{\beta(1+\Phi(1-\vartheta))} + \frac{1}{\beta(1+\Phi(1-\vartheta))}\right)(X-1),$$

indicating the existence of a unit root $X_1 = 1$. ■

8.7 Appendix to money supply and inflation targeting

Despite the assumption of a non-zero default probability, the model does not exhibit any capital market imperfection. Like in Gali and Monacelli (2005) the nominal rigidity in domestic producer prices is therefore the main macroeconomic distortion in our model. Yet, it is well-known that pure domestic producer price inflation targeting is in general not welfare maximizing in an open economy framework. The reason is that welfare can – compared to the flexible price allocation (even without an average price mark-up) – be improved by terms of trade adjustments (see Corsetti and Pesenti, 2001). As shown by Benigno and Benigno (2003) the welfare gain from the latter depends on the preference specification, and vanishes under unitary intertemporal elasticities of substitution, here $\sigma = \sigma^* = 1$.

For the latter parameter values (4), (19), and perfect international risk sharing imply permanently balanced trade (where initial values are normalized to satisfy $\xi = \vartheta^*/\vartheta$). If one further disregards the long-run distortions due to the average mark-up and average inflation (or if they are assumed to be eliminated), social welfare can be approximated by a second-order Taylor expansion at the steady state leading to the following quadratic loss function specification (see Gali and Monacelli, 2005, Appendix C).

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} L_t, \quad L_t = \hat{\pi}_{H,t}^2 + \lambda \widehat{x}_{H,t}^2, \tag{60}$$

where $\lambda > 0$ and $\widehat{x}_{H,t}$ denotes the domestic output-gap, i.e., the deviation of the current level of domestically produced goods from its efficient value, $\widehat{x}_{H,t} = \widehat{y}_{H,t} - \widehat{y}_{H,t}^{eff}$. The weight in the loss function λ is further determined by the structural parameters σ_n, ϵ, and χ, and equals $\lambda = \chi(1 + \sigma_n)/\epsilon$. Now consider the problem for a central bank which aims to minimize (60) with respect to $\hat{\pi}_{H,t}$ and $\widehat{x}_{H,t}$ subject to the private sector equilibrium conditions. Further, suppose for convenience that some (unspecified) cost-push shocks that are entirely distortionary – for instance, shocks to the substitution elasticity ϵ like in Clarida et al. (2002) – are the only source of uncertainty, and that the steady state is undistorted. Then, the efficient level of output equals its steady state value such that $\widehat{x}_t = \widehat{y}_{H,t}$. The supply constraint (34), which is the single relevant constraint for the policy problem, can, by using $\widehat{x}_{H,t} = \varrho \widehat{q}_t$ where $\varrho = [(c/n)\,\vartheta + \vartheta^*\,(c^*/n)\,(1-\vartheta)^{-1} + (1-\vartheta)\,(c/n)\,\sigma^{-1}] > 0$ (see 27 and 29), be rewritten as

$$\widehat{\pi}_{H,t} = \beta E_t \widehat{\pi}_{H,t+1} + (\psi/\varrho)\,\widehat{x}_{H,t},$$

where $\psi/\varrho = \chi\,(1 + \sigma_n)$ for $\sigma = \sigma^* = 1$. The first order conditions for the commitment solution for this problem under the timeless perspective (see Woodford, 2003) can then be combined to the following first order condition, which is also known as a "targeting rule" (Svensson, 2005):

$$\hat{\pi}_{H,t} = -\,(\lambda\varrho/\psi)\,(\widehat{x}_{H,t} - \widehat{x}_{H,t-1})\,, \tag{61}$$

which holds for all $t \geq 0$. It is then easy to show that a money growth rule satisfying (46) can implement the policy plan. To see this, replace the output-gap by $\widehat{x}_{H,t} = \varrho\sigma\widehat{m}_t$, to rewrite the targeting rule (61) in terms of real balances:

$$\widehat{m}_t - \widehat{m}_{t-1} = -\frac{\psi}{\lambda\varrho^2\sigma}\hat{\pi}_{H,t}. \tag{62}$$

Further, use $\widehat{\pi}_t = \widehat{\pi}_{H,t} + \frac{\vartheta}{1-\vartheta}(\widehat{q}_t - \widehat{q}_{t-1})$ and $\widehat{q}_t = \sigma\widehat{c}_t = \sigma\widehat{m}_t$, to transform the money growth rule (46) into a rule for the growth rate for real balances, $\widehat{m}_t - \widehat{m}_{t-1} = \frac{\mu_\pi - 1}{1+\frac{\vartheta}{1-\vartheta}\sigma}\widehat{\pi}_{H,t}$. A comparison with (62) then immediately shows that the policy plan can be implemented by a money growth rule (46) satisfying $\mu_\pi = \widetilde{\mu}_\pi$, where $\widetilde{\mu}_\pi = 1 - \psi\left[1 + \sigma\vartheta/(1-\vartheta)\right] / \left(\lambda\varrho^2\sigma\right) < 1$.

8.8 Proof of proposition 3

Combining the monetary policy rule $\widehat{m}_t - \widehat{m}_{t-1} = (1 + \frac{\vartheta}{1-\vartheta}\sigma)^{-1}(\mu_\pi - 1)\widehat{\pi}_{H,t}$ with the aggregate supply constraint (34) which can – by $\widehat{m}_t = \sigma\widehat{q}_t$ – be rewritten as $\widehat{\pi}_{H,t} = \beta E_t\widehat{\pi}_{H,t+1} + \psi\sigma\widehat{m}_t$, the deterministic version of the model in $\widehat{m}_t$ and $\widehat{\pi}_{H,t}$ reads

$$\begin{pmatrix} E_t\widehat{\pi}_{H,t+1} \\ \widehat{m}_t \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ \beta & \psi\sigma \end{pmatrix}^{-1} \begin{pmatrix} \frac{\mu_\pi - 1}{1+\vartheta\sigma/(1-\vartheta)} & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} \widehat{\pi}_{H,t} \\ \widehat{m}_{t-1} \end{pmatrix} = \mathbf{A} \begin{pmatrix} \widehat{\pi}_t \\ \widehat{m}_{t-1} \end{pmatrix}.$$

The characteristic polynomial of $\mathbf{A}$ is $S(X) = X^2 + (\beta^{-1}\frac{\psi\sigma(\mu_\pi - 1)}{1+\sigma\vartheta/(1-\vartheta)} - \beta^{-1} - 1)X + \frac{1}{\beta}$ with $S(0) = \beta^{-1} > 1$ and $S(1) = -\frac{1}{\beta}\frac{\psi\sigma}{1+\sigma\vartheta/(1-\vartheta)}(1 - \mu_\pi)$. Thus, there is one eigenvalue between zero and one and the other eigenvalue is larger than one (indicating equilibrium uniqueness and stability) for $\mu_\pi < 1$. ■

8.9 Appendix to an alternative default probability function

Suppose that the probability of sovereign default is a function of the debt-to-GDP ratio (which often serves as an indicator for debt sustainability in empirical studies, e.g. Reinhart et al., 2003) rather than solely a function of debt (see 1). Throughout this appendix we assume that the default probability satisfies

$$\delta_t = \delta\left(\frac{B_{t-1}/P_t}{y_{H,t}}\right) : R_+ \to (0,1)\,,\ \delta' > 0,$$

or $\delta_t = \delta\left(\frac{b_{t-1}/\pi_t}{y_{H,t}}\right)$. Since the remainder of the model is unchanged, the set of equilibrium conditions for the benchmark case (with $B_{H,t} = 0$) just changes with regard to (26), which now reads

$$\Phi\left(\widehat{b}_{F,t} - E_t\widehat{\pi}_{t+1} - E_t\widehat{y}_{H,t+1}\right) = \widehat{R}_t - E_t\widehat{\pi}_{t+1} - (E_t\widehat{q}_{t+1} - \widehat{q}_t),$$

where Φ is now defined as $\Phi = \delta'\frac{b_F}{y_H\pi}\frac{1}{1-\delta} > 0$. Using that $\widehat{y}_{H,t} = \varpi\widehat{q}_t$, where $\varpi = (c/n)\,\vartheta + \vartheta^*\,(c^*/n)\,(1-\vartheta)^{-1} + (1-\vartheta)\,(c/n)\,/\sigma > 0$, the equilibrium conditions can – like in appendix 8.2 – be reduced to (32), (34), and

$$\Phi\varpi E_t\widehat{q}_{t+1} = \widehat{R}_t - (1-\Phi)\,\widehat{\pi}_{H,t+1} - \Phi\widehat{b}_{F,t} - \frac{1-\Phi\vartheta}{1-\vartheta}\,(\widehat{q}_{t+1} - \widehat{q}_t)\,,$$
$$\widehat{b}_{F,t} = \frac{1}{1-\Phi\vartheta}\widehat{b}_{F,t-1} - \varphi\widehat{\pi}_{H,t} + \eta\widehat{R}_t + \frac{\vartheta}{1-\Phi\vartheta}\Phi\varpi E_t\widehat{q}_t + \widetilde{\xi}_{t-1,t},$$

where $\widetilde{\xi}_{t-1,t}$ collects the stochastic terms from expectation errors. The deterministic versions of the equilibrium conditions can then be written as $\left(\widehat{b}_{F,t}\,\widehat{q}_{t+1}\,\widehat{\pi}_{H,t+1}\right)' = \mathbf{A}\left(\widehat{b}_{F,t-1}\,\widehat{q}_{t}\,\widehat{\pi}_{H,t}\right)'$, where

$$\mathbf{A} = \begin{pmatrix} \Phi\left(-\Phi\varpi + \frac{1-\Phi\vartheta}{1-\vartheta}\right) & 1-\Phi-\rho_\pi \\ 1 & 0 & -\eta\rho_\pi \\ 0 & 0 & \beta \end{pmatrix}^{-1} \begin{pmatrix} 0 & \frac{1-\Phi\vartheta}{1-\vartheta} & 0 \\ \frac{1}{1-\Phi\vartheta} & \frac{-\vartheta}{1-\Phi\vartheta}\Phi\varpi & -\varphi \\ 0 & -\psi & 1 \end{pmatrix}.$$

Evidently, for $\varpi = 0$ we get the same structure as in the benchmark case (see proof of proposition 1). To examine local stability and uniqueness of the equilibrium, we assess the characteristic polynomial of $\mathbf{A}$, which reads

$$\begin{aligned} F(X) &= -\frac{1/\beta}{(1-\Phi\vartheta)-\Phi\varpi(1-\vartheta)} + X^3 \\ &+X^2\frac{(\vartheta+\varpi(1-\vartheta))\,\Phi(1+\beta)-(1+2\beta)-(\psi(1-\Phi)+\rho_\pi\psi(\Phi\eta-1))(1-\vartheta)}{((1-\Phi\vartheta)-\Phi\varpi(1-\vartheta))\,\beta} \\ &+X\frac{\Phi\vartheta(3+\beta-\Phi\vartheta)-(2+\beta)+(1-\vartheta)(\psi(\Phi\vartheta+1)(\rho_\pi-1)-\varpi\Phi(\Phi\vartheta-1))}{((1-\Phi\vartheta)-\Phi\varpi(1-\vartheta))\,\beta(\Phi\vartheta-1)}. \end{aligned}$$

The determinant of $\mathbf{A}$ is $\det(\mathbf{A}) = -F(0) = \frac{1/\beta}{(1-\Phi\vartheta)-\Phi\varpi(1-\vartheta)}$, and $F(X)$ at $X=1$ equals

$$F(1) = -F(0)\left[(1-\vartheta)\,\Phi\psi/(1-\Phi\vartheta)\right](2\vartheta-\Phi\vartheta+1+(\Phi\eta\vartheta-\eta-2\vartheta)\,\rho_\pi).$$

Like in the benchmark case, a necessary condition for stability and uniqueness of non-oscillatory equilibrium sequences is therefore $2\vartheta-\Phi\vartheta+1+(\Phi\eta\vartheta-\eta-2\vartheta)\,\rho_\pi > 0 \Leftrightarrow$

$$\rho_\pi < 1+\Delta \tag{63}$$

where $\Delta = \frac{(1-\Phi\vartheta)(1-\eta)}{(1-\Phi\vartheta)\eta+2\vartheta}$. Then, there exists at least one stable eigenvalue. To ensure that there is only one stable and non-oscillatory solution, it is sufficient when $\det(\mathbf{A}) = \frac{1/\beta}{(1-\Phi\vartheta)-\Phi\varpi(1-\vartheta)} > 1$ (implying the existence of at least one unstable eigenvalue) is further satisfied, where $\det(\mathbf{A}) > 1$ if $(1-\Phi\vartheta) > \Phi\varpi(1-\vartheta)$, or, replacing ϖ, if

$$\Phi < \frac{(1-\vartheta)c+\vartheta^* c^*}{\left(2\vartheta+\frac{1-\vartheta}{\sigma}\right)(1-\vartheta)\,c+(1+\vartheta)\,\vartheta^* c^*} \tag{64}$$

Thus, for small values for the default elasticity Φ, satisfying (64), the equilibrium is locally stable, unique and non-oscillatory if the condition (63) is satisfied. Evidently, this closely relates to the stability result for $\Phi \in (0,1)$ in the case where the default probability is just a function of real debt (see proposition 1). For $\sigma \geq 1$ and the limiting case $\vartheta \to 0$, the RHS of (64) does not exceed one, and the condition for local stability and uniqueness of non-oscillatory solution in proposition 1 exactly applies. Concisely, for small values for Φ or ϑ, the change in the functional form for the default probability does not qualitatively affect the results for the benchmark case, as described in section 3.2.