A Social Network Analysis of Occupational Segregation

Sebastian Buhai¹,²
Marco van der Leij¹

¹ Erasmus University Rotterdam, and Tinbergen Institute;
² Aarhus School of Business.
Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam, and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

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Sebastian Buhai† and Marco van der Leij‡

November 7, 2006

Abstract

We develop a social network model of occupational segregation between different social groups, generated by the existence of positive inbreeding bias among individuals from the same group. If network referrals are important in getting a job, then expected inbreeding bias in the contact network structure induces different career choices for individuals from different social groups. This further translates into stable occupational segregation equilibria in the labour market. We derive the conditions for persistent wage and unemployment inequality in the segregation equilibria. Our framework is proposed as complementary to existing theories used to explain labour market inequalities between groups divided by race, ethnicity or gender.

JEL codes: A14, J31, Z13

Keywords: Social Networks, Inbreeding Bias, Occupational Segregation, Labour Market Inequality

1 Introduction

Occupational segregation between various social groups is an enduring and pervasive phenomenon with important implications for the labour market. Richard Posner recently pointed out that “a glance of the composition of different occupations shows that in many of them, particularly racial, ethnic, and religious

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*We are particularly grateful to Willemien Kets for carefully reading a previous version and providing us with very valuable comments. We also greatly benefited from useful discussions and suggestions, at different stages of this paper, by Michele Belot, Sanjeev Goyal, Maarten Janssen, James Montgomery, Jan van Ours, Gilles Saint-Paul, Ott Toomet, Fernando Vega-Redondo and audiences in seminars and conferences at University College London, Tinbergen Institute Amsterdam, Tinbergen Institute Rotterdam, WEHIA in Essex, SMYE in Geneva and EEA in Vienna. Any remaining errors are our sole responsibility.

†Tinbergen Institute, Erasmus University Rotterdam and Aarhus School of Business, email buhai@tinbergen.nl

‡Tinbergen Institute and Erasmus University Rotterdam; email mvanderleij@few.eur.nl
groups, along with one or the other sex and even groups defined by sexual orientation (heterosexual vs. homosexual), are disproportionately present or absent"\(^1\). There are countless empirical studies, particularly within sociology and economics, that document the extent of occupational segregation\(^2\). Most studies analyzing possible causes of occupational segregation agree that ‘classical’ theories such as taste or statistical discrimination by employers cannot explain alone occupational disparities and their remarkable persistence. While several meritorious alternative theories were to this date considered, some scientists with long-standing interest in the area, such as Kenneth Arrow (Arrow, 1998), particularly referred to modelling the social network interactions as promising avenue for further research in this context.

In this paper we apply social network theory to dynamically model occupational segregation in the labour market. Significant progress has been achieved in modelling labour market phenomena by means of social network analysis. Recent articles have investigated the effect of social networks on employment, wage inequality, labour market transitions, and social welfare.\(^3\) To the best of our knowledge this is the first attempt to model occupational segregation by social network analysis.

We construct a simple three-stage model of occupational segregation between two homogeneous, exogenously-given, mutually exclusive social groups, acting in a two-jobs labour market. In the first stage each individual chooses one of two specialized educations to become a worker. In the second stage individuals randomly form “friendship” ties with other individuals, with a tendency to form more ties with members of the same social group, what we call inbreeding bias. In the third stage workers use their networks of friendship contacts to search for jobs.

We show that with a positive inbreeding bias within social groups, a complete polarization in terms of occupations across the two groups can arise as a stable equilibrium outcome. These results are independent of the existence and extent of an eventual second inbreeding bias, by education. We extend the basic model by allowing for “good” and “bad” jobs, to analyze equilibrium wage and unemployment inequality between the two social groups. We show that with large differences in job attraction (wages), a natural outcome of the model is

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\(^1\) The quote is from a post in “The Becker-Posner Blog”, see http://www.becker-posner-blog.com. Posner goes on by giving a clear-cut example of gender occupational segregation: “a much higher percentage of biologists than of physicists are women, and at least one branch of biology, primatology, appears to be dominated by female scientists. It seems unlikely that all sex-related differences in occupational choice are due to discrimination”

\(^2\) See for instance Beller (1984), Alhelda (1986), Rich (1999), Charles and Grusky (2004). Some such papers, eg. Sorensen (2004), also discuss the extent of segregation between social groups by workplace or industry. Here we model segregation by occupation alone, which appears to be dominant at least relative to segregation by industry. Weedon and Sorensen (2001) convincingly show that occupational segregation in the USA is much stronger than segregation by industries and that if one wishes to focus on one single dimension, “occupation is a good choice, at least relative to industry”.

that one group fully specializes in the good job, while the other group mixes over the two jobs. The group that specializes in the good job always has a higher payoff and a lower unemployment rate in this partial segregation equilibrium. Furthermore, with a sufficiently large intra-group inbreeding bias, the fully specializing group also has a higher equilibrium wage rate and it largely crowds out the mixing group from the good jobs. Our model is thus able to explain typical empirical patterns of gender, race or ethnical labour inequality.

The paper is organized as follows. We review empirical evidence on the relevance of job contact networks and the existence of inbreeding bias in Section 2, we describe our model of occupational segregation in Section 3, and we discuss key results on the segregation equilibria in Section 4. We then derive results when jobs are not equally attractive in Section 5. A discussion on the timing and exogeneity of the contact network formation is presented in Section 6. We summarize and conclude the paper in Section 7.

2 Evidence on job contact networks and inbreeding bias

2.1 Importance of job contact networks

There is a well established set of stylized facts that show the importance of the informal job networks in searching and finding jobs. Firstly, it is known that on average about 50 percent of the workers obtain jobs through their personal contacts. Evidence in this sense starts back in the 1960’s and covers multiple countries, e.g. Rees (1966), Granovetter (1995), Holzer (1987), Montgomery (1991), Topa (2001). A second fact is that on average 40-50 percent of the employers use social networks of their current employees (i.e. they hire recommended applicants) to fill their job openings, e.g. Holzer (1987). It was also found that the employee-employer matches obtained through contacts appear to have certain common characteristics: those who found jobs through personal contacts were less likely to quit and had longer tenure e.g. Datcher (1983), Devine and Kiefer (1991), Simon and Warner (1992), Datcher Loury (2006). For a more detailed overview of studies on job information networks Ioannides and Datcher Loury (2004) is a good recent reference. Job contact networks are thus extremely relevant in the process of matching employees to employers.

2.2 Evidence on intra-group inbreeding biases

There is at the same time extensive empirical evidence on the existence of “inbreeding biases”, also called “assortative mixing” or “homophily” in the literature, among people with similar characteristics.

To start with the informal job networks context, early studies by Rees (1966) and Doeringer and Piore (1971) showed that workers who had been asked for references concerning new hires, were in general very likely to refer people with ‘similar’ features to them. While these similar features can be anything, such as
ability, education, age, race and so on, the focus on our paper shall be on groups stratified along exogenous features (ie. one is born in such a group and cannot alter his/her group membership) such as those divided along the lines of gender, race or ethnicity. Indeed, most subsequent empirical evidence on inbreeding bias dealt with such groups. Hence, Marsden (1987) finds, using the U.S. General So-
cial Survey, that personal contact networks tend to be highly segregated by race, while other studies such as Brass (1985) or Ibarra (1992), using cross-sectional single firm data, find significant gender segregation in personal networks. Yet more direct evidence comes from the tabulations in Montgomery (1992), which unambiguously document the existence of large gender inbreeding biases within job contact networks: over all occupations in a US sample from the National Longitudinal Study of Youth, 87 percent of the jobs obtained through contacts by men were based on information received from other men and 70 percent of the jobs obtained informally by women were based on information from other women. This is true even for occupations and industries that are highly segre-
gated. Importantly, Montgomery (1992) shows that these outcomes hold even when looking at each narrowly defined occupation categories or one-digit industries separately, including traditionally male or female dominated occupations where job referrals for the minority group members were still obtained with a very strong inbreeding bias via their own gender group. For example, in male-
dominated occupations such as machine operators, 81 percent of the women who found their job through a referral, had a female reference. Such figures are in fact surprisingly large and- even more surprising- they are in fact likely to be lower bounds for magnitudes of the inbreeding biases within other exogenous social groups. Montgomery (1992) provides more in depth discussion and gives further references on the evidence on the gender inbreeding bias.

Perhaps the most recent addition to the pile of evidence here is an empirical study by Fernando and Sousa (2005) on a unique dataset documenting both the recruitment and the hiring stages for an entry-level job at a call centre of a large US bank. This study also finds unambiguously that contact networks contribute to the gender skewing of jobs and in addition documents directly that there is strong evidence of gender homophily in the refereeing process of the contacts: referees of both genders tend to strongly produce same sex referrals.

Indirect pieces of evidence come from other research areas. Extensive re-
search on homophily in general shows that people tend to be friends with sim-
ilar others, see for instance McPherson et al (2001) for a review, with ‘exoge-

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4 Weeden and Sorensen (2001) estimate a two-dimensional model of gender segregation, by industry and occupation, using a data sample generated from the 1990 Equal Employment Opportunity Supplemental Tabulations File. They find much stronger segregation across occupations than across industries: 86% of the total association in the data is explained by one dimensional model with only occupational segregation; this increases to about 93% once industry segregation is also accounted for. See also footnote 2.

5 The inbreeding bias by gender is likely to be smaller than inbreeding biases within social groups differentiated along race or ethnicity because of the frequent close-knit relationships between men and women. This is de facto verified for instance by Marsden (1988) who finds strong inbreeding biases in the contacts between individuals of the same race or ethnicity but less pronounced inbreeding bias within gender categories.
nous’ characters such as race, ethnicity or gender, being essential dimensions of homophily. The literature also shows that friendship patterns are more homophilous than would be expected by chance or availability constraints, even after controlling for the unequal distribution of races or sexes through social structure, e.g. Shrum, Cheek and Hunter (1988). There is furthermore a considerable body of evidence pointing towards "pure" same race preferences in marrying or dating (eg. the “mating taboo” in Wong, 2003 or the speed dating preferences in Fishman et al, 2006), among very young kids (eg. Hraba and Grant, 1970), among audiences of television shows (Dates, 1980, Lee, 2006) etc.

Finally, we address shortly the relative importance of inbreeding biases within ‘exogenously given’ versus ‘endogenously created’ social groups. As already mentioned, assortative mixing happens along a great variety of dimensions. However, the empirical literature so far documents that, within the society as a whole, the importance of the ‘natural’ inbreeding bias within exogenous groups such as those divided by race, ethnicity, gender and- to a certain extent- religion, outweighs inbreeding biases within endogenously formed groups such as those stratified by educational, political or economic lines. For example, using US data, Marsden (1988) finds strong inbreeding biases in contacts between individuals of the same race, ethnicity or religion and, less pronounced inbreeding bias by sex, age or education level. Another study by Tampubolon (2005), using UK data, finds that the dynamics of friendship is strongly affected by gender, marital status and age, but not by education and only marginally by social class.

3 A model of occupational segregation

Based on the stylized facts mentioned in Section 2, we build a parsimonious theoretical model of social network interaction, able to explain stable occupational segregation without a need for alternative theories. Our model should be seen as complementary to existing theories that explain observed occupational segregation patterns. While our framework has some common elements with group membership models used on a large scale in sociology for explaining general segregation patterns (neighborhood segregation, school segregation, workplace segregation etc), it markedly differs from these by explicitly modelling the dynamic network interaction.

Let us consider the following setup. A continuum of individuals with measure $N$ is equally divided into two social groups, Reds ($R$) and Greens ($G$). The individuals are homogeneous apart from their social color. They can work in

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6The precursor of many such studies is the seminal work by Schelling (1971) on the emergence of high levels of neighbourhood racial segregation from tiny differences in the tolerance threshold levels of members of each race group, regarding the presence of the other race. Our framework shares a few elements with Schelling’s, i.e. we operate with a two-category model and our group ‘inbreeding bias’ can be seen as the analogue of Schelling’s group “tolerance level”, but markedly differs otherwise, in assumptions, dynamics and results. A more closely related model to ours is the one by Benabou (1993); we address briefly the main similarities and differences between these two models in the beginning of our discussion section.
two occupations, A or B. Each occupation requires a corresponding thorough
specialized education (career track), hence a worker cannot work in an occu-
pation if she is not qualified to do so by having followed the educational track
corresponding to that occupation. We assume that it is too costly for individ-
uals to follow both educational tracks. Hence, individuals have to choose their
education before they enter the labour market. For example, graduating high
school students may face the choice of pursuing a medical career or a career in
technology. Both choices require several years of expensive specialized training,
which makes it unfeasible to follow both career tracks.

Consider now the following timing:
1. Individuals choose one education in order to specialize either in occupation
   A or in occupation B;
2. Individuals randomly establish “friendship” relationships, thus forming a
   network of contacts;
3. Individuals participate in the labour market. Individual i obtains a job
   with probability $s_i$ and if she has a job, she earns a wage $w_i$. If she does
   not have a job she obtains wage $0$.

We postpone a detailed discussion on the timing of the three stages and on
the exogeneity of the contact network formation in the second stage to Section
6, and proceed here with the implications of these assumptions. The choice of
education in the first stage involves strategic behavior. Workers choose the ed-
cuation that maximizes their expected payo

ff given the choices of other workers,
and we therefore look for a Nash equilibrium in this stage. A worker’s expected
payoff is determined by the network formation process in the second stage and
by the employment process in the third stage. We make these stages more
specific in what follows.

In the second stage the workers form a network of contacts. We assume
this network to be random with an inbreeding bias. Hence, we assume that the
probability for two workers to create a tie is $0 \leq p < 1$ when the two workers
are from different social groups and follow different education tracks; when the
two workers are from the same social group, the probability of creating a tie
increases with $\lambda > 0$. Similarly, if two workers choose the same education, then
the probability of creating a tie increases with $\kappa \geq 0$. As it will turn out below,
since the magnitude of $\kappa$ will be irrelevant for our results, we do not impose any
further restrictions on it vis-a-vis $p$ or $\lambda$, other than securing $p + \lambda + \kappa \leq 1$.
This leads to the tie formation probabilities given in Table 1. We will refer to
two workers that create a tie as “friends”\footnote{We do not consider the possibility that individuals have different inbreeding biases or
different probabilities of making friends in general, across the two homogeneous groups (hence $\lambda$ and/or $p$ are identical in the two groups). This could be an interesting extension for future research. There is for example some evidence in the gender homophily literature that men
tend to have better overall access to contact networks than women (suggestive of higher $p$) and
that women seem to use more their non-kin ties in ‘instrumental’ networks than in ‘expressive’
networks (suggestive of smaller $\lambda$); see for instance Moore (1990) or the introductory discussion
in Petersen, Saporta and Seidel (2000) and the references therein.}
Table 1: The probability of a tie between two individuals, depending on the group membership and education choice.

<table>
<thead>
<tr>
<th>Social group</th>
<th>Education same</th>
<th>Education different</th>
</tr>
</thead>
<tbody>
<tr>
<td>same</td>
<td>( p + \kappa + \lambda )</td>
<td>( p + \lambda )</td>
</tr>
<tr>
<td>different</td>
<td>( p + \kappa )</td>
<td>( p )</td>
</tr>
</tbody>
</table>

The third stage we envision for this model is that of a dynamic labour process à la Calvó-Armengol and Jackson (2004) or Bramoullé and Saint-Paul (2006), in which employed workers randomly lose their jobs, while unemployed workers search for jobs. Unemployed workers receive job information either directly, or indirectly through their friends. The details of such a process are unimportant for our purposes. However, what is important is the assumption that unemployed workers have a higher propensity to receive job information when they have more friends with the same job background, that is, with the same choice of education. Denote the probability that individual \( i \) becomes unemployed by \( s^i \). Define \( s^i \equiv s(x_i) \), where \( x_i \) is the measure of friends of \( i \) with the same education as \( i \) has. We thus assume that \( s(x_i) \) is differentiable, \( 0 < s(0) < 1 \) (there is non-zero amount of direct job search) and \( s'(x_i) > 0 \) for all \( x > 0 \) (the probability of being employed increases in the number of friends with the same education).

It is instructive to show how \( s^i \) depends on the education choices of \( i \) and the choices of all other workers. Denote by \( \mu_R \) and \( \mu_G \) the fractions of Reds and respectively Greens that choose education \( A \). It follows that fractions \( 1 - \mu_R \) and \( 1 - \mu_G \) of groups \( R \) and respectively \( G \) choose education \( B \). Given the tie formation probabilities from Table 1 and some algebra, the employment rate \( s_A^X \) of \( A \)-workers in group \( X \in \{ R, G \} \) will be given by:

\[
s_A^X(\mu_R, \mu_G) = s((p + \kappa)\bar{\mu}N + \lambda \mu_X N/2) \tag{1}
\]

and similarly, the employment rate \( s_B^X \) of \( B \)-workers in group \( X \) will be

\[
s_B^X(\mu_R, \mu_G) = s((p + \kappa)(1 - \bar{\mu})N + \lambda (1 - \mu_X)N/2) \tag{2}
\]

where \( \bar{\mu} \equiv (\mu_R + \mu_G)/2 \).

Note that \( s_A^X > s_A^Y \) and \( s_B^X < s_B^Y \) for \( X, Y \in \{ R, G \}, X \neq Y \), iff \( \mu_X > \mu_Y \) and \( \lambda > 0 \). We will see in Section 4 that the ranking of the employment rates is crucial, as it creates a group-specific network effect. That is, keeping this ordering, if only employment matters (jobs are equally attractive), then individuals have an incentive to choose the same education as other individuals.

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This implicitly assumes that everybody has the same chance on the formal labour market or, in other words, that direct job search intensity is exogenously given for everybody.
in their social group. Importantly, it is straightforward to see that this ordering of the employment rates depends on $\lambda$, but it does not depend on $\kappa$. Therefore, only the inbreeding bias among members of the same social group is relevant to our results.

The eventual payo
tff of the workers depends on their employment probability and on the wage they receive. Without loss of generality we assume that an unemployed worker receives zero wage. We further assume that the wage rate decreases in the number of employed workers that choose a particular type of education. The assumption of a decreasing wage when the total number of employed workers increases can be easily explained using a simple classical model of a 2-goods economy with Cobb-Douglas utility functions, and a linear production function with labour as single input. Intuitively, when more workers are employed as $A$, the economy produces more $A$-products, which have to find their way to the consumer market. As the market price drops whenever production output increases, it follows that, in a competitive product and labour market, wages drop as well. Thus wages of $A$ ($B$)-jobs are negatively related to the number of workers that choose an $A$ ($B$)-education.

The assumption is formalized as follows. Let $L_A$ be the total measure of employed $A$-workers and $L_B$ be the total measure of employed $B$-workers. Hence,

$$L_A(\mu_R, \mu_G) = \mu_R N s^R_A(\mu_R, \mu_G)/2 + \mu_G N s^G_A(\mu_R, \mu_G)/2$$

and

$$L_B(\mu_R, \mu_G) = \mu_R N s^R_B(\mu_R, \mu_G)/2 + \mu_G N s^G_B(\mu_R, \mu_G)/2.$$

Then the wage of an $A$-job, $w_A(L_A)$, (and respectively of a $B$-job, $w_B(L_B)$) is differentiable and decreasing in $L_A$ (or $L_B$).

To ensure a unique labour market equilibrium, we make the following assumption on the wage function:

**Assumption 1** For the wage functions $w_A(L_A)$ and $w_B(L_B)$

(i) \[ \lim_{L_A \to 0} w_A(L_A) = \lim_{L_B \to 0} w_B(L_B) = \infty. \]

(ii) For $X \in \{R, G\}$, and for all $\mu_R, \mu_G \in [0, 1]$

\[ -\frac{\partial w_A}{\partial L_A} \frac{\mu_X N s_X^A}{w_A} > \frac{\partial s_X^A}{\partial \mu_X} \frac{\mu_X}{s_A} \]

and

\[ -\frac{\partial w_B}{\partial L_B} \frac{\mu_X N s_X^B}{w_B} > -\frac{\partial s_X^B}{\partial \mu_X} \frac{\mu_X}{s_B} \]

Part (i) of Assumption 1 implies that at least some workers choose education $A$ and some workers choose education $B$. In other words, everyone going for one of the two educations cannot be an equilibrium. In part (ii) we assume that the
education choice of an individual has a larger marginal effect on the wage rate than on the employment probability within a group. Parts (i) and (ii) guarantee the uniqueness of our results.

Assumption 1 above essentially states that the network effect on the wage rate is stronger than its effect on the employment rate, for each social group. This assumption is not restrictive in light of our model’s third stage. Thus, the assumption is obviously plausible in a competitive labour market without wage rigidity and random job separation rates, where wages react fully and instantaneously to shortages or increases in labour supply. Moreover, the employment probability of each individual in our model is bounded between \( s(0) > 0 \) and \( 1 \), with \( s(0) \) capturing the employment probability in the absence of any ties and thus induced only by the exogenously given direct job finding rate. Hence, a higher \( s(0) \) implies less of an impact of the network effect on the employment rate.

We can now define the payoff of a worker. The payoff function of a \( A \)-educated worker from social group \( X \in \{R, G\} \) is

\[
\Pi^X_A(\mu_R, \mu_G) = w_A(L_A(\mu_R, \mu_G)) s^X_A(\mu_R, \mu_G). 
\]

(5)

Similarly,

\[
\Pi^X_B(\mu_R, \mu_G) = w_B(L_B(\mu_R, \mu_G)) s^X_B(\mu_R, \mu_G). 
\]

(6)

The derivatives of the payoff functions with respect to \( \mu_Y, Y \in \{R, G\} \), are given by

\[
\frac{\partial \Pi^X_A}{\partial \mu_Y} = N \frac{\partial w_A}{\partial L_A} \left( s^Y_A + \mu_R \frac{\partial s^R_A}{\partial \mu_R} + \mu_G \frac{\partial s^G_A}{\partial \mu_G} \right) s^X_A + w_A \frac{\partial s^X_A}{\partial \mu_Y} 
\]

(7)

\[
\frac{\partial \Pi^X_B}{\partial \mu_Y} = N \frac{\partial w_B}{\partial L_B} \left( -s^Y_B + (1 - \mu_R) \frac{\partial s^R_B}{\partial \mu_R} + (1 - \mu_G) \frac{\partial s^G_B}{\partial \mu_G} \right) s^X_B + w_B \frac{\partial s^X_B}{\partial \mu_Y}. 
\]

(8)

The first term of the derivatives \((7)\) and \((8)\) measures the effect of the education choice on the labour supply and on wages, whereas the second term measures the effect of education choice on the employment probability. It is now straightforward to see that Assumption 1 implies

\[
\frac{\partial \Pi^X_A}{\partial \mu_X} < N \frac{\partial w_A}{\partial L_A} \left( s^X_A \right)^2 + w_A \frac{\partial s^X_A}{\partial \mu_X} < 0 < -N \frac{\partial w_B}{\partial L_B} \left( s^X_B \right)^2 + w_B \frac{\partial s^X_B}{\partial \mu_X} < \frac{\partial \Pi^X_B}{\partial \mu_X}. 
\]

Hence, the payoff of an individual \( i \) decreases with the fraction of other individuals in the same group that choose the same education as \( i \) does.

3.1 Equilibrium

We would like to characterize the Nash equilibria in the model above. We are in particular interested in those equilibria in which there is segregation. We define segregation as follows:
Definition 1 Let \( \mu_X, X \in \{R,G\} \), be the fraction of workers in social group \( X \) that choose education \( A \). There is complete segregation if \( \mu_R = 0 \) and \( \mu_G = 1 \), or, vice versa, \( \mu_R = 1 \) and \( \mu_G = 0 \). There is partial segregation if for \( X \in \{R,G\} \) and \( Y \in \{R,G\}, Y \neq X: \mu_X = 0 \) but \( \mu_Y < 1 \), or, vice versa, \( \mu_X = 1 \) but \( \mu_Y > 0 \).

In a Nash equilibrium each worker chooses the education that gives the highest payoff, given the education choices of all other workers. Since workers of the same social group are homogenous, a Nash equilibrium implies that if some worker in a group chooses education \( A \) (\( B \)), then no other worker in the same group should prefer education \( B \) (\( A \)). With this idea in mind, we reformulate the equilibrium concept in a particularly useful way.

Definition 2 Let \( \mu_X, X \in \{R,G\} \), be the fraction of workers in social group \( X \) that choose education \( A \). A pair \( (\mu_R, \mu_G) \) is an equilibrium if and only if, for \( X \in \{R,G\} \), the following statements hold

\[
\begin{align*}
\Pi^X_A(\mu_R, \mu_G) &\leq \Pi^X_B(\mu_R, \mu_G) \text{ if } \mu_X = 0 \\
\Pi^X_A(\mu_R, \mu_G) &= \Pi^X_B(\mu_R, \mu_G) \text{ if } 0 < \mu_X < 1 \\
\Pi^X_A(\mu_R, \mu_G) &\geq \Pi^X_B(\mu_R, \mu_G) \text{ if } \mu_X = 1
\end{align*}
\]  

In the initial analysis adopting this definition we find multiple equilibria. However, some of these equilibria are not dynamically stable and we are not interested in them. We therefore use a simple stability concept based on a standard myopic adjustment process of strategies, which takes place before the education decision is made. That is, we think of the equilibrium as the outcome of an adjustment process. In this process, individuals repeatedly announce their preferred education choice, and more and more workers revise their education choice if it is profitable to do so, given the choice of the other workers. Concretely, we consider stationary points of a dynamic system in which

\[
\frac{d\mu_X}{dt} = k \left( \Pi^X_A(\mu_R(t), \mu_G(t)) - \Pi^X_B(\mu_R(t), \mu_G(t)) \right).
\]

The stability properties of stationary points in such dynamic systems are well-known in the literature. We base our definition on these properties, taking into account that the process might converge to a segregation equilibrium, thus to the boundaries of the solution space.

Definition 3 Let \( (\mu^*_R, \mu^*_G) \) be an equilibrium and define

\[
G(\mu_R, \mu_G) = \begin{bmatrix}
\frac{\partial (\Pi^R_A - \Pi^R_B)}{\partial \mu_R} & \frac{\partial (\Pi^G_A - \Pi^G_B)}{\partial \mu_R} \\
\frac{\partial (\Pi^R_A - \Pi^R_B)}{\partial \mu_G} & \frac{\partial (\Pi^G_A - \Pi^G_B)}{\partial \mu_G}
\end{bmatrix}
\]

9 One could think of such a process as the discussions students have before the end of the high school about their preferred career. An alternative with a longer horizon is an overlapping generations model, in which the education choice of each new generation partly depends on the choice of the previous generation.
The equilibrium is stable under the following conditions for $X \in \{R,G\}$:

(i) if $\Pi_X^R(\mu_R^*, \mu_G^*) = \Pi_X^B(\mu_R^*, \mu_G^*)$, then

$$\frac{\partial (\Pi_X^A - \Pi_X^B)}{\partial \mu_X}|_{\mu_R=\mu_R^*, \mu_G=\mu_G^*} < 0;$$

(iv) if $\Pi_R^A(\mu_R^*, \mu_G^*) = \Pi_R^B(\mu_R^*, \mu_G^*)$ and $\Pi_G^A(\mu_R^*, \mu_G^*) = \Pi_G^B(\mu_R^*, \mu_G^*)$, then

$$\det(G(\mu_R^*, \mu_G^*)) > 0.$$

The equilibrium is weakly stable if the above conditions only hold with weak inequality signs.

4 Occupational segregation

We initially assume that both jobs are equally attractive when the labor supply is the same. That is

**Assumption 2** For all $x \in (0, N]$ $w_A(x) = w_B(x)$.

We next characterize equilibria for two cases. In the benchmark case network effects are important, but there is no inbreeding bias in the social network, i.e. $\lambda = 0$. In the second case, we consider the full model including network effects and an inbreeding bias. Of course, in either case we allow for an arbitrary inbreeding bias by education, $\kappa$.

4.1 A labour market without intra-group inbreeding

We first consider a labour market in which there is no inbreeding bias in the social network. That is $\lambda = 0$. We obtain a standard result

**Proposition 1** Suppose $\lambda = 0$, Assumptions 1 and 2 hold. Then $(\mu_R^*, \mu_G^*)$ is a weakly stable equilibrium if and only if

$$\mu_R^* + \mu_G^* = 1.$$

In that case,

$$s_X^A(\mu_R^*, \mu_G^*) = s_B^X(\mu_R^*, \mu_G^*) = s (p + \kappa) N/2.$$

(14)

for $X \in \{R,G\}$, and

$$w_A(L_A(\mu_R^*, \mu_G^*)) = w_B(L_B(\mu_R^*, \mu_G^*)).$$

(15)

11
Proof. See Appendix. ■

This proposition simply restates the classical view that the price of labour, the wage, is equal to the value of the marginal product of labour. Since workers are homogenous with respect to their productivity, everyone earns the same wage and occupational segregation or social inequality does not occur.\(^\text{10}\) Note that Proposition 1 does not give a unique equilibrium, but a (convex) set of equilibria \((\mu_\ast^R, \mu_\ast^G)\) for which \(w_A(\cdot) = w_B(\cdot)\).\(^\text{11}\)

One may be surprised that network effects and inbreeding bias by education do not directly result in segregation. One has to remember however that there is no group-specific inbreeding bias in the social network and hence that the generated network effects, as well as the wages, are group-independent. Hence, the value of an \(A\)-education or \(B\)-education only depends on the total number of other workers that choose education \(A\) or \(B\), and not on the number of workers choosing \(A\) or \(B\) in each of the social group. It should then be clear that there is no reason to expect segregation, as the group identity does not matter in making an education choice.

4.2 A labour market with intra-group inbreeding

We now consider the unrestricted version of our labour market model. A first observation is that the equilibrium changes drastically, even with a small amount of inbreeding bias.

Proposition 2 Suppose \(\lambda > 0\), Assumptions 1 and 2 hold. A weakly stable equilibrium \((\mu_\ast^R, \mu_\ast^G)\), in which \(0 < \mu_\ast^R < 1\) and \(0 < \mu_\ast^G < 1\), does not exist.

Proof. See Appendix. ■

This proposition shows that even with a small intra-group inbreeding bias, segregation by occupation is a natural outcome. At least one social group specializes fully in one type of occupation. The intuition is that an inbreeding bias in the social network among members of the same social group creates a group-dependent network effect. Thus, if slightly more Red workers choose \(A\) than Greens do, then the value of an \(A\)-education is higher for the Reds than for the Greens, while the value of a \(B\)-education is lower in the Reds’ group. Positive feedback then ensures that the initially small differences in education choices between the two groups widen and widen until at least one group segregates completely into one type of education.

While we have now shown that in a labour market model with network effects and inbreeding bias segregation is a natural outcome, the question remains...
what the segregation equilibria look like. Depending on the functional form of \( w_A(\cdot) \) and \( w_B(\cdot) \) and \( s(\cdot) \), there could be many equilibria. However, complete segregation is the most prominent outcome.

**Proposition 3** Suppose \( \lambda > 0 \), Assumptions 1 and 2 hold. Then \( (\mu_R, \mu_G) = (1,0) \) and \( (\mu_R, \mu_G) = (0,1) \) are the only stable equilibria.

**Proof.** See Appendix

Proposition 3 shows that complete segregation is always an equilibrium outcome when there is a positive inbreeding bias in the social network. That is, one social group specializes in one occupation, and the other group in the other occupation.

## 5 Social inequality

The discussion so far ignored eventual equilibrium differentials in wages and unemployment between the two types of jobs. In fact, since we have assumed that \( A \) and \( B \) jobs are equally attractive, it is easily seen that under complete segregation, there cannot be wage or unemployment inequality. However, not only is this in sharp contrast to observed gender and racial gaps in wages and unemployment, but it is also not obvious how our result of complete segregation can be sustained when there are large wage-induced incentives. Why would someone stick to the education choice of her social group when the wage benefits of choosing the other career track are very large? This motivates us to extend our framework in order to look at the robustness of our results under wage and employment inequality. We do this by making the following assumption on the wage function:

**Assumption 3** For every \( x \in (0, N) \), \( w_A(x) > w_B(x) \).

This says that if there are as many \( A \)-educated workers as \( B \)-educated workers, then the \( A \)-educated workers earn a higher wage. The implicit assumption behind Assumption 3 is that the marginal utility consumers derive from product \( A \) is larger than the marginal utility from product \( B \). This is a natural assumption as there is no a priori reason to expect that different products are equally appreciated.

We derive results on wage and unemployment inequality under Assumption 3. Our first observation is that the proof of Proposition 2 does not depend on the fact that \( w_A(x) = w_B(x) \) for every \( x \). Hence, this proposition also holds under Assumption 3.

**Proposition 4** Suppose \( \lambda > 0 \), Assumptions 1 and 3 hold. A weakly stable equilibrium \((\mu_R^*, \mu_G^*)\), in which \( 0 < \mu_R^* < 1 \) and \( 0 < \mu_G^* < 1 \), does not exist.

**Proof.** Same as Proposition 2

We next characterize the segregation equilibria. We consider two cases; either the difference between the attractiveness of \( A \) and \( B \)-jobs is relatively
small compared to the social network effect, or the difference is relatively large. We first consider the case in which the job difference is relatively small. In this case, complete segregation remains an equilibrium.

**Proposition 5** Suppose $\lambda > 0$, Assumption 1 and 3 hold. Define $s_H \equiv s((p + \kappa + \lambda)N/2)$ and $s_L \equiv s((p + \kappa)N/2)$ and suppose that

$$\frac{w_A(s_H N)}{w_B(s_H N)} < \frac{s_H}{s_L}. \quad (16)$$

Then $(\mu_R, \mu_G) = (1, 0)$ and $(\mu_R, \mu_G) = (0, 1)$ are the only two stable equilibria. In these equilibria,

$$w_A > w_B,$$

and, if $\mu_X = 1$ and $\mu_Y = 0$ for $X, Y \in \{R, G\}$, $X \neq Y$, then

$$\Pi^X_A > \Pi^Y_B > \Pi^X_A > \Pi^X_B. \quad (17)$$

**Proof.** See Appendix. ■

This proposition states that if the difference in wages is not too large, complete segregation is always an equilibrium outcome when there is a positive inbreeding bias in the social network. Thus one social group specializes in one occupation, and the other group in the other occupation. Since the social groups are of equal size, the employment probabilities in the two social groups are the same. However, since the wage rate of $A$—workers is higher in the equilibrium, the social group that specializes in occupation $A$ obtains a higher payoff than the other group. Hence, social inequality is a natural outcome of this model.

Interestingly, if some workers make mistakes in their education choice, then the workers that are the worst off are from the same social group as the workers that are the best off. Thus, if $\mu_R = 1$ and $\mu_G = 0$, then the Reds that choose $A$ receive the highest wage and have the best employment probabilities. However, if some of the Reds choose $B$ by mistake, then these Red $B$-workers are the most disadvantaged, as they earn the lowest wage and have the lowest employment chances. The payoffs of the workers who ‘make mistakes’ in their choice of education are materialized in the last two terms of inequality (17) above.

We turn next to the case in which wage differentials are large. We have the following proposition.

**Proposition 6** Suppose $\lambda > 0$, Assumption 1 and 3 hold. Define $s_H \equiv s((p + \kappa + \lambda)N/2)$ and $s_L \equiv s((p + \kappa)N/2)$ and suppose that

$$\frac{w_A(s_H N)}{w_B(s_H N)} > \frac{s_H}{s_L}. \quad (18)$$

(i) There is no equilibrium with complete segregation.
(ii) There are exactly two stable equilibria with partial segregation, in which either $\mu_R = 1 \text{ or } \mu_G = 1$. If $\mu_X = 1$ for $X \in \{R, G\}$, then for $Y \in \{R, G\}$,

$$\Pi^X_A > \Pi^Y_B = \Pi^Y_A > \Pi^X_B.$$  \hspace{1cm} (19)

**Proof.** See Appendix. \hfill $\blacksquare$

The proposition makes clear that complete segregation cannot be sustained if the wage differential is too large. Starting from complete segregation, a large wage differential gives incentives to the group specialized in $B$-jobs to switch to $A$-jobs. Interestingly, the unsustainable complete segregation equilibrium is then replaced by a partial equilibrium in which one group specializes in job $A$, while the other group has both $A$ and $B$-workers. As in the previous case of small wage differentials, the workers of the group specializing in $A$-jobs receive the highest payoffs, hence we have again a social inequality outcome. If these workers make a mistake in their education choice, they are again the worst off of everybody; their payoff is materialized in the last term of inequality (19).

Remark that it is the higher employment rate of the group specializing in $A$ that makes the difference in the payoff inequality from Proposition 6. The employment rate of group $X$ fully specializing in $A$-jobs is given by $s_X^A(1, x)$, where $x$ is the fraction of $A$ workers in group $Y$ that mixes over the jobs. On the other hand, the employment rate of group $Y$ is $xs_A^Y(1, x) + (1 - x)s_B^Y(1, x)$. Hence, the group that fully specializes in the $A$-job has a lower unemployment rate than the other group.

We now know that, in a partial segregation equilibrium, the ’advantaged’ group fully specializes in the good job and also has a higher employment rate. But does this group also earn a higher wage rate in equilibrium? The following proposition shows that this depends on the amount of group inbreeding and the labour supply ratio that equalizes wages.

**Proposition 7** Suppose $\lambda > 0$, Assumption 1 and 3 hold. Define $s_H \equiv s((p + \kappa + \lambda)N/2)$ and $s_L \equiv s((p + \kappa)N/2)$ and suppose that $w_A(s_HN) > w_B(s_LN)$. Define $\hat{\mu} \in (0, 1)$, such that

$$w_A(L_A(1, \hat{\mu})) = w_B(L_B(1, \hat{\mu})), \hspace{1cm} (20)$$

and let $\mu^*$ be such that $(1, \mu^*)$ and $(\mu^*, 1)$ are the only two equilibria.

(i) If

$$\hat{\mu} < \frac{\lambda}{2(p + \kappa + \lambda)},$$

then $0 < \mu^* < \hat{\mu}$ and $w_A(L_A(1, \mu^*)) > w_B(L_B(1, \mu^*))$.

(ii) If

$$\hat{\mu} > \frac{\lambda}{2(p + \kappa + \lambda)},$$

then $\hat{\mu} < \mu^* < 1$ and $w_A(L_A(1, \mu^*)) < w_B(L_B(1, \mu^*))$. 

15
Proof. See Appendix.

Proposition 7 says that, if the group inbreeding bias $\lambda$ is large relative to $p$ and $\kappa$ (in fact $p + \kappa$) and there is a big difference between the good and the bad jobs (case (i) above), then not only does one group fully specialize in one job and benefit from a higher employment rate (as seen already in Proposition 6), but at the same time its wage rate is higher in the equilibrium. In contrast, the group that mixes over jobs is to a large extent excluded ('crowded out') from the good jobs. Moreover, the latter social group not only earns on average less than the fully specializing social group, but also its unemployment rate is higher in the equilibrium. If, on the contrary, the inbreeding bias $\lambda$ is small relative to $p$ and $\kappa$ and there is a big difference between the good and the bad jobs (case (ii) above), then the employment rate (and the payoff) of the group fully specializing in job $A$ remains higher than the employment rate of the mixing group, but the average wage inequality in the equilibrium reverses. Since the wage rate of the $A$-workers is now lower in the equilibrium, the average wage rate of workers from the social group fully specializing in $A$ will be lower than the average wage rate of the workers mixing over the two jobs.

There are some interesting considerations to be made here. The conclusions above concerning average wage differentials between the workers in the two groups, in the partial segregation equilibrium, are hinging on the relative size of $\lambda$ relative to $p + \kappa$, in the term $\frac{\lambda}{2(p + \kappa + \lambda)}$ from the inequality conditions in Proposition 7. It is interesting to go beyond the theoretical aspect and highlight the practical relevance of this result. First, as seen from the empirical evidence put forward in Section 2, the inbreeding bias by education, $\kappa$, is typically found to be much lower relative to the inbreeding bias along racial, ethnical or gender lines. The second interesting situation is a scenario where the probability of making contacts in general, $p$, were already extremely high relative to the the intra-group inbreeding bias. However, given the surprisingly large size of intra-group inbreeding biases in personal networks of contacts found empirically, this is also unlikely. Hence, the likelihood is very high that in practice $\lambda$ would dominate the other parameters in the cutoff term $\frac{\lambda}{2(p + \kappa + \lambda)}$.

We sum up the findings from the last two propositions. The fully specializing group is always being better off in terms of unemployment rate and payoff, independent of either relative or absolute sizes of $\lambda$, $p$ and $\kappa$ (as long as $\lambda > 0$, of course), as shown in Proposition 6. Furthermore, given the observed patterns of social networks discussed in Section 2, the condition of $\lambda$ dominant relative to $p$ and $\kappa$ is likely to be met. This ensures that the group fully specializing in the good job always has a higher wage in the equilibrium than the group mixing over the two jobs, as proved in Proposition 7; in addition, a very large $\lambda$ also means that the group that mixes over jobs is in fact largely crowded out from the good job. Note that this partial segregation equilibrium is in remarkable agreement with observed occupational, wage and unemployment disparities in the labour market. We conclude that our simple model offers a plausible explanation for major empirical patterns of labour market inequality.
6 Discussion

The model we presented in this paper is similar in structure to that of Roland Benabou (1993). In our model the social groups play the role of the neighborhoods from Benabou’s framework, while our social networks with inbreeding bias are the analogue of his local externalities. An essential difference between our paper and Benabou’s is that we focus on the persistence of wage and unemployment inequality, and on how this depends on the degree of inbreeding bias. In contrast, in Benabou (1993) the wage of highly educated individuals is always higher than the wage of individuals with low education. The model in this paper provides a plausible explanation for the persistence of occupational segregation and wage inequality.

Our model relies on two crucial assumptions that we did not so far discuss in depth: the timing and the exogeneity of the network formation.

6.1 Timing of network formation

We assumed that individuals first choose an education, and then form a network of job contacts. As a consequence, individuals have to make expectations about the network they could form, and base their education decisions on these expectations. This is in contrast to earlier work on the role of networks in the labour market. In former research, the network was supposed to be already in place, or the network was formed in the first stage (Montgomery, 1991; Calvó-Armengol & Jackson, 2004).

Our departure from the earlier frameworks raises questions about the assumed timing of the education choice. Are crucial career decisions made before or after job contacts are formed? One might be tempted to answer: both. Of course everyone is born with family ties, and in early school and in the neighborhood children form more ties. It is furthermore known that peer-group pressure among children has a strong effect on decisions to, for instance, smoke or engage in criminal activities and, no doubt, family and early friends do also form a non-negligible source of influence when making crucial career decisions. However, we argue that most job-relevant contacts (so called ‘instrumental ties’) are made later, for instance at the university or early at the workplace or simply when starting to look for a suitable job, hence after a specialized career track has been chosen. Even if those ties are typically not as strong as family ties, they are more likely to provide relevant information on vacancies to job seekers. Granovetter (1973, 1985) provides convincing evidence that job seekers more often receive crucial job information from acquaintances (“weak ties”) instead of family or close friends (“strong ties”). If the vast majority of such instrumental ties are formed after the individual embarked on a (irreversible) career and we strongly believe so- then it is justified to consider a model in which the job contact network is formed after making a career choice.
6.2 Exogeneity of network formation

One of our assumptions is that individuals form a stochastic contact network (with an inbreeding bias). This means that the probability that an individual $i$ forms a tie with individual $j$ is exogenously given and constant. Critics might see this as a problematic assumption since establishing a friendship between two individuals typically involves a rational choice from the part of each of those individuals, as well. It is therefore plausible that individuals try to optimize their job contact network in order to maximize their chances on the labour market. In particular, individuals from the disadvantaged social groups should have an incentive to form ties with individuals from the advantaged group.

While the above argument may be true, the harsh reality is that this simply does not happen on a large scale. In Section 2 we provide an abundance of evidence that strong inbreeding bias exists even within groups that have strong labour market incentives not to preserve such an inbreeding bias in forming their ties. The reason could be that, even though individuals decide rationally on the friendships they make, the payoff of forming a tie is mainly determined by various social and cultural factors, and only for a smaller part by benefits from the potential transmission of valuable job information. On top of that, studies such as, for instance, Granovetter (2002) also note that many people would feel exploited if they find out that someone befriends them for the selfish reason of obtaining job information. These elements might hinder the role of labour market incentives when forming ties. Hence, while we do not doubt that incentives play a role when forming ties, we believe that such incentives are not sufficient to undo the effects of the significant intra-group inbreeding biases. In order to keep the model simple, we assume in this paper that network formation is random with an inbreeding bias.

7 Summary and conclusions

We have investigated a parsimonious social network framework where jobs are obtained through a network of contacts formed stochastically, after career decisions have been made. We have shown that even with a very small amount of inbreeding bias within each social group, dynamically stable occupational segregation equilibria will arise. If the wage differential across the occupations is not too large, complete segregation will always be sustainable. If the wage differential is large, complete segregation cannot be sustained, but a partial segregation equilibrium in which one of the group fully specializes in one education while the other group mixes over the career tracks, is sustainable. Furthermore, our model is able to explain sustained unemployment and wage differences between the social groups.

While our social interaction model can describe empirical patterns of occupational segregation and wage inequality between gender, racial or ethnical groups, other factors are also documented to play a significant role in this context. This model should thus be seen as complement to alternatives such as
taste discrimination or rational bias by employers, which are still present in the market despite their (predicted) erosion over time, due to both competitive pressure and institutional instruments. It is therefore pertinent to directly investigate how relevant are the mechanisms described in this paper and to assess their relative strength in explaining observed occupational segregation, vis-à-vis other proposed theories. Another avenue for future research is to extend this framework to issues such as the position of minority versus majority groups, by looking at the interaction between social groups of unequal sizes.

References


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A Proofs for the propositions in the text

A.1 Proposition 1

**Proof.**

For \( \lambda = 0 \) and \( X \in \{R, G\} \), we have \( s_X^X(\mu_R^*, \mu_G^*) = s((p + \kappa)\bar{\mu}N) \), and \( s_B^X(\mu_R^*, \mu_G^*) = s((p + \kappa)(1 - \bar{\mu})N) \). With \( \mu_R = \mu_R^* \) and \( \mu_G = \mu_G^* = 1 - \mu_R^* \), we directly observe that (14) holds. Further, \( L_A(\mu_R^*, \mu_G^*) = L_B(\mu_R^*, \mu_G^*) > 0 \), and (15) follows. Therefore, \( \Pi^X_A(\mu_R^*, \mu_G^*) = \Pi^X_B(\mu_R^*, \mu_G^*) \). Hence, \( (\mu_R^*, \mu_G^*) \) is an equilibrium.

Moreover,

\[
\frac{\partial s_X^X(\mu_R, \mu_G)}{\partial \mu_R} = \frac{\partial s_X^X(\mu_R, \mu_G)}{\partial \mu_G} = (p + \kappa)N s'((p + \kappa)\bar{\mu}N)/2, \tag{21}
\]

and

\[
\frac{\partial s_B^X(\mu_R, \mu_G)}{\partial \mu_R} = \frac{\partial s_B^X(\mu_R, \mu_G)}{\partial \mu_G} = -(p + \kappa)N s'((p + \kappa)(1 - \bar{\mu})N)/2. \tag{22}
\]

With \( \mu_R = \mu_R^* \) and \( \mu_G = \mu_G^* = 1 - \mu_R^* \), it follows that

\[
\frac{\partial s_X^X}{\partial \mu_R} = \frac{\partial s_X^X}{\partial \mu_G} = \frac{-\partial s_B^X}{\partial \mu_R} = \frac{-\partial s_B^X}{\partial \mu_G}. \tag{23}
\]

Equation (23) and Assumption 1 imply that \( \frac{\partial (n_X^X - n_B^X)}{\partial \mu_R} = \frac{\partial (n_X^X - n_B^X)}{\partial \mu_G} < 0 \), and therefore the determinant of \( G(\mu_R^*, \mu_G^*) \) from expression (13) equals zero. Hence, the equilibrium \( (\mu_R^*, \mu_G^*) \) is weakly stable.

Finally, Assumption 1 implies that no other \( (\mu_R, \mu_G) \) can be an equilibrium, since \( \Pi^X_A > \Pi^Y_B \) for \( \mu_R + \mu_G < 1 \), and \( \Pi^X_A < \Pi^Y_B \) for \( \mu_R + \mu_G > 1 \). ■

A.2 Proposition 2

**Proof.** Suppose \( (\mu_R^*, \mu_G^*) \) is a stable equilibrium, and \( \mu_R^* \in (0,1) \) and \( \mu_G^* \in (0,1) \). By condition (11)

\[
\Pi^R_A(\mu_R^*, \mu_G^*) = \Pi^R_B(\mu_R^*, \mu_G^*) \quad \text{and} \quad \Pi^C_A(\mu_R^*, \mu_G^*) = \Pi^C_B(\mu_R^*, \mu_G^*) \tag{24}
\]
Substituting (5)-(6) into (24) and rewriting, these equations become

\[
\frac{w_A(L_A(\mu_R^*, \mu_G))}{w_B(L_B((\mu_R^*, \mu_G))} = \frac{s_B^R(\mu_R^*, \mu_G)}{s_B^A(\mu_R^*, \mu_G)} = \frac{s_B^G(\mu_R^*, \mu_G)}{s_B^A(\mu_R^*, \mu_G)}.
\]

(25)

Since \( \lambda > 0 \), \( \mu_R^* > \mu_G^* \) implies \( s_A^R > s_A^G \) and \( s_B^R < s_B^G \). But this means that if \( \mu_R^* > \mu_G^* \), then

\[
\frac{s_B^R(\mu_R^*, \mu_G^*)}{s_B^A(\mu_R^*, \mu_G^*)} < \frac{s_B^G(\mu_R^*, \mu_G^*)}{s_B^A(\mu_R^*, \mu_G^*)},
\]

which contradicts (25). The same reasoning holds for \( \mu_R^* < \mu_G^* \). Hence, it must be that \( \mu_R^* = \mu_G^* \).

However \( (\mu_R^*, \mu_G^*) \) with \( \mu_R^* = \mu_G^* \) cannot be a weakly stable equilibrium. To see this, suppose that \( (\mu^*, \mu^*) \) with \( \mu^* \in (0, 1) \) is a weakly stable equilibrium. Hence \( \Pi_A^X(A; \mu^*, \mu^*) = \Pi_A^X(B; \mu^*, \mu^*) \) for \( X \in \{ R, G \} \) and \( \frac{\partial (\Pi_A^X - \Pi_B^X)}{\partial \mu_Y} \leq 0 \) at \( \mu_R = \mu_G = \mu^* \), and \( \det(G(\mu^*, \mu^*)) \geq 0 \), where \( G \) is defined in (13).

Since \( \lambda > 0 \), it must be that

\[
\frac{\partial s_A^X}{\partial \mu_X} > \frac{\partial s_A^X}{\partial \mu_Y}
\]

(26)

for \( X, Y \in \{ R, G \} \) and \( Y \neq X \). Furthermore, \( s_A^X = s_A^Y \) for \( \mu_R = \mu_G = \mu^* \). Substituting (26) into (7) and (8), and considering Assumption 1, it follows that, at \( \mu_R = \mu_G = \mu^* \),

\[
\frac{\partial (\Pi_A^X - \Pi_B^X)}{\partial \mu_Y} < \frac{\partial (\Pi_A^X - \Pi_B^X)}{\partial \mu_X} < 0.
\]

for \( X, Y \in \{ R, G \} \), \( X \neq Y \). But then it is straightforward to see that \( \det(G(\mu^*, \mu^*)) < 0 \). This contradicts weak stability. ■

### A.3 Proposition 3

**Proof.** Consider \((\mu_R, \mu_G) = (1, 0)\). Then \( s_A^R = s_B^G = s((p + \kappa + \lambda)N/2) \) and \( s_B^R = s_A^G = s((p + \kappa)N/2) \). Hence \( L_A(1, 0) = L_B(1, 0) \), and we have

\[
\Pi_A^R(1, 0) = w_A(L_A(1, 0))s_A^R(1, 0) > w_B(L_B(1, 0))s_B^R(1, 0) = \Pi_B^R(1, 0)
\]

and

\[
\Pi_A^G(1, 0) = w_A(L_A(1, 0))s_A^G(1, 0) < w_B(L_B(1, 0))s_B^G(1, 0) = \Pi_B^G(1, 0).
\]

This is clearly a stable equilibrium. The same is true for \((\mu_R, \mu_G) = (0, 1)\).

We only have to show that there are no other stable equilibria. From Proposition 2 we already know that \((\mu_R, \mu_G)\) with \( 0 < \mu_R < 1 \) and \( 0 < \mu_G < 1 \) cannot be a stable equilibrium. So consider \( \mu_R = 1 \). We know that \( \Pi_A^G(1, 0) < \Pi_B^G(1, 0) \). Moreover, by Assumption 1 we know that

\[
\frac{\partial \Pi_A^G(1, \mu_G)}{\partial \mu_G} < \frac{\partial \Pi_B^G(1, \mu_G)}{\partial \mu_G}.
\]

23
Hence, $\Pi^G_A(1, \mu_G) < \Pi^G_B(1, \mu_G)$ for all $\mu_G \in [0, 1]$, and therefore $(1, \mu_G)$ can only be a stable equilibrium if $\mu_G = 0$. Similarly, if $\mu_G = 0$ then only $\mu_R = 1$ can be a stable equilibrium. ■

A.4 Proposition 5

Proof. If (16) holds, then

$$\Pi^R_A(1, 0) > \Pi^R_B(1, 0) \text{ and } \Pi^G_A(1, 0) < \Pi^G_B(1, 0).$$

Hence, $(\mu_R, \mu_G) = (1, 0)$ is clearly a stable equilibrium. Further, as $L_A(1, 0) = L_B(1, 0) = s_BN$, it holds that $w_A > w_B$ at $(\mu_R, \mu_G) = (1, 0)$. Finally, at $(\mu_R, \mu_G) = (1, 0)$

$$w_A s_A^R > w_B s_B^G > w_A s_A^G > w_B s_B^R,$$

and this is equivalent to (17). The same is true for $(\mu_R, \mu_G) = (0, 1)$.

Proposition 4 and Assumption 1 ensure that these are the only two equilibria. ■

A.5 Proposition 6

Proof. (i) If (18) is true, then

$$\Pi^G_A(1, 0) > \Pi^G_B(1, 0).$$

Thus, if $(\mu_R, \mu_G) = (1, 0)$ then Greens would like to deviate by choosing education $A$, and therefore $(\mu_R, \mu_G) = (1, 0)$ cannot be an equilibrium. The same holds for $(\mu_R, \mu_G) = (0, 1)$.

(ii) From Assumption 1 we know that

$$\frac{\partial}{\partial \mu_G} \left( \Pi^G_A(A; 1, \mu_G) - \Pi^G_B(1, \mu_G) \right) < 0.$$ 

It follows from Assumption 1 and equation (27) that there must be a unique $\mu^*$, such that

$$\Pi^G_A(1, \mu^*) = \Pi^G_B(1, \mu^*).$$

Moreover, $s_A^R(1, \mu^*) > s_B^G(1, \mu^*)$ and $s_B^G(1, \mu^*) > s_A^R(1, \mu^*)$, so we have at $(\mu_R, \mu_G) = (1, 0)$

$$w_A s_A^R > w_B s_B^G = w_A s_A^G > w_B s_B^R,$$

and this is equivalent to (19) for $X = R$ and $Y = G$. As

$$\Pi^R_A(1, \mu^*) > \Pi^R_B(1, \mu^*),$$

it is also clear that $(\mu_R, \mu_G) = (1, \mu^*)$ is a stable equilibrium. The same is true for $(\mu_R, \mu_G) = (\mu^*, 1)$.

To show that there is no other equilibrium, note that by (18) $\Pi^R_P(1, 0) > \Pi^R_B(1, 0)$. Assumption 1 then implies that $\Pi^R_F(\mu, 0) > \Pi^R_B(\mu, 0)$ for all $\mu \in [0, 1]$. Hence, $(\mu, 0)$ and, similarly, $(0, \mu)$ cannot be an equilibrium. By Proposition 4 we also know that there is no mixed equilibrium. ■
A.6 Proposition 7

Proof. Consider the stable equilibrium at $(1, \mu^*)$. Since it is an equilibrium we know that
\[ \Pi_A^{G}(1, \mu^*) = \Pi_B^{G}(1, \mu^*). \]

Further, by Assumption 1 we know that \( \Pi_A^{G}(1, \mu_G^*) - \Pi_B^{G}(1, \mu_G) \) is strictly monotonically decreasing in \( \mu_G \).

(i) If \( \hat{\mu} < \frac{\lambda}{2(p + \kappa + \lambda)} \), then \( s_A^{G}(1, \hat{\mu}) < s_B^{G}(1, \hat{\mu}) \). As \( w_A(L_A(1, \hat{\mu})) = w_B(L_B(1, \hat{\mu})) \) it must be that \( \Pi_A^{G}(1, \hat{\mu}) < \Pi_B^{G}(1, \hat{\mu}) \).

But then it also must be that \( \mu^* < \hat{\mu} \). By Proposition 6 we know that \( \mu^* > 0 \). Hence, \( 0 < \mu^* < \hat{\mu} \). Moreover, since \( L_A(1, \mu^*) < L_A(1, \hat{\mu}) \) and \( L_B(1, \mu^*) > L_B(1, \hat{\mu}) \), it must be that \( w_A(L_A(1, \mu^*)) > w_B(L_B(1, \mu^*)) \).

(ii) If \( \hat{\mu} > \frac{\lambda}{2(p + \kappa + \lambda)} \), then \( s_A^{G}(1, \hat{\mu}) > s_B^{G}(1, \hat{\mu}) \) and \( \Pi_A^{G}(1, \hat{\mu}) < \Pi_B^{G}(1, \hat{\mu}) \). But then \( \mu^* > \hat{\mu} \). By Assumption 1 we know that \( \mu^* < 1 \). Hence, \( \hat{\mu} < \mu^* < 1 \). Moreover, since \( L_A(1, \mu^*) > L_A(1, \hat{\mu}) \) and \( L_B(1, \mu^*) < L_B(1, \hat{\mu}) \), it must be that \( w_A(L_A(1, \mu^*)) < w_B(L_B(1, \mu^*)) \) \( \blacksquare \)