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The Origin of Prospect Theory, or Testing the Intuitive Statistician

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The origin of prospect theory

or

Testing the intuitive statistician*

Floris Heukelom

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Abstract

The origin of prospect theory is the desire to test the intuitive statistician in the real world. The development of this theory by the cognitive psychologists Kahneman and Tversky can be traced to the former's work in cognitive psychophysics, in which deviations from average behavior are termed (statistical) errors; and the latter's work on decision theory, with its normative vs. descriptive framework. The combination of these two types of probabilistic psychology culminated in a new descriptive theory of human decision making in the real world coined Heuristics and Biases. The 1979 *Econometrica* article applies this new descriptive theory to economists' EUT. It equates the intuitive statistician with the rational economic man and shows how it descriptively fails.

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Introduction

In 2002 the cognitive psychologist Daniel Kahneman received the Nobel prize in economics "for having integrated insights from psychological research into economic science, especially concerning human judgment and decision-making under uncertainty"[1]. The big absent in Sweden was Amos Tversky, Kahneman's long-time collaborator on the research for which he received the prize, who died in 1996. Public opinion has it that their central contribution to economics in 1979 on prospect theory is the best cited article ever in *Econometrica*.

In this paper I offer an historical reading of this article by showing how the theoretical origins and developments of both authors culminated into prospect theory. In the first two sections an overview of their separate research is given, after which in the third section is set out how these were combined in their Heuristics and Biases theory. The fourth section extends this line to prospect theory. The fifth then infers what are the origins of prospect theory. Concluding remarks end the paper.

Kahneman: statistical inference and cognitive illusions

Statistical inference

Kahneman joined the Israeli army in 1951 at age seventeen and left it in 1956 in order to pursue his studies as a graduate student at Berkeley. While in the army he obtained an undergraduate degree in psychology. More important, he also obtained an extensive working experience as a psychologist. In his 2002 Nobel Prize Autobiography he recalls some of the work:

> "My assignment involved two tasks: first, to figure out whether there were personality dimensions that mattered more in some combat jobs than in others, and then to develop interviewing guidelines that would identify those dimensions."[2]

The work had a profound impact on Kahneman, it "was the beginning of a lifelong interest in the statistics of prediction and description."[3] If we look at the ten years between Kahneman's PhD thesis and his first publication with Tversky in 1971, it is a recurring theme. What Kahneman is interested in, is how to construct a measurement model in such a way that the (statistically) inferred conclusions indeed explain (i.e. can predict) the phenomenon

[1] The Royal Swedish Academy of Sciences (2002a)

[2] Kahneman (2002), p.4

[3] Kahneman (2002), p.4

investigated. We may distinguish two cases of (statistical) inference as it is used by Kahneman. On the one hand it refers to the statistical method of inference from data to hypothesis, or from data to a test of a hypothesis; on the other hand inference refers to the description of the human cognitive system. It is no coincidence, then, that the same language is used in both cases: the functioning of the human mind is understood to be the same as the scientific method of statistical inference. Kahneman, in other words, is a representant of what is broadly labeled the probabilistic approach to psychology[4]. Let me give two examples of what this probabilistic approach entails in the case of Kahneman.

Kahneman writes a few articles and his PhD thesis on the semantic differential (SD), part of the psychology of attitudes. The idea behind SD research is that words or concepts have a connotative meaning along with their denotative meaning. This connotative meaning, it is held, can be measured on any bi-polar scale. For example, on a 10-point scale ranging Good(1)-Bad(10), humans assign to *massacre* and *rape*, about the same connotation, say 9. But also *flower* and *sunny day* will be given roughly the same connotation, say 3. By thus constructing a list of connotative meanings associated with the concept, it is possible to derive the attitude of subjects in certain circumstances given the words they use.

Kahneman, however, is not interested in the theory itself. That is, he is not interested in questions whether it is meaningful to talk about attitudes and connotation in this way. The reason that Kahneman is interested in SD research is because of the statistical model that is used in its experiments. In Kahneman (1963), for example, he shows that the models commonly used in SD research to measure connotative meanings are not sufficiently sophisticated. They may give rise to wrong statistical inferences. It is not the experimental design Kahneman questions, but the statistical model that is used to infer conclusions from the data provided by the experiment. The solution he proposes is a model that, given the experimental setting, better distinguishes between the different theoretical concepts.

Kahneman's main research in the period 1961-1971, however, is on the psychophysics of vision. During the years 1962-1963 he sets up a vision lab at the Hebrew University at which he is working. Also here Kahneman's focus is mainly on constructing and improving models of statistical inference. In Kahneman and Wright (1971), he investigates the correlation between pupil size and short-term memory. Along with the testing of a specific hypothesis of rehearsing, it is shown that pupillary size provides a good measurement for mental activity. Kahneman et. al. (1969) compares the

[4] Danziger (1987, 1990), Gigerenzer (1987a,b,c), Murray (1987)

pupil diameter as a measurement for mental activity with measurements of the heart rate and of skin resistance. The results indicate that these three measurements are not well correlated. Pupil diameter measurement remains the preferred method.

Again, the emphasis is not on the theory itself but on the investigation and comparison of different models of statistical inference. The question is what the best model is with which to infer information from the data. Kahneman is not interested in the psychological theories on which he is working, nor does he question the experimental designs that are used. Kahneman's focus is the statistical models employed to infer conclusions from the data provided by the experiments. In his psychophysics of vision the measurement of the pupil diameter remains the preferred method, not so much because of a theoretical conviction it is correlated with short term memory; it is used because an experiment on short term memory using pupil diameter as measurement instrument provides the data for the best possible statistical inference.

Cognitive illusions

The usefulness of statistical inference, however, is of vital importance. If statistical inference has little to do with the theory it is supposed to say something about, there is not much to be gained from statistical reasoning in psychology. The scientist may construct a sound and mathematically consistent model of statistical inference, if the inferred facts do not tell him what he wanted to know in the course of the theory under investigation, the statistical inference, however properly conducted, will be of no use. Kahneman links the absence of a relation between statistical inference and the theory of the scientist to psychological notions of 'error of judgment', 'illusion', or 'failure'. Of what I have labeled here a second characteristic in the early work of Kahneman a nice example is given his Nobel autobiography, again from the time he spent in the Israeli army.

> "Every month or so we had a "statistics day", during which we would get feedback from the officer-training school, indicating the accuracy of our ratings of candidates' potential. The story was always the same: our ability to predict performance at the school was negligible. But the next day, there would be another batch of candidates to be taken to the obstacle field, where we would face them with the wall and see their true natures revealed. I was so impressed by the complete lack of connection between the statistical information and the compelling

> experience of insight that I coined a term for it: "the illusion of validity." … It was the first cognitive illusion I discovered."[5]

Kahneman links this failed correspondence of statistical inference and theory to a theory of mind. In his cognitive theoretical conception of the mind as a system of statistical inference, the illusions are failures of the cognitive system. As (human) scientists we think we have inferred good information given our theory, the fact that this is an illusion is a failure of our cognition.

'Illusion', 'error' or 'failure' are not concepts coined by Kahneman. 'Error' for example is closely related with the rise of probabilistic theories and theories of measurement in psychology[6]. 'Illusion' bears close resemblance to visual illusions as shown by Gestalt psychology. Speaking of 'errors', 'illusions' etc, begs therefore for a closer exposition of what they constitute exactly in the work of Kahneman. The easiest way to do so is by posing the question of what is 'right' or 'good' in the cases of these 'errors', 'illusions', and 'failures'.

In the probabilistic approach to psychology as it is employed by Kahneman, the 'right' response of subjects is defined as the average response. All deviating responses by subjects are labeled 'errors'. This is the position Kahneman takes in his work on SD research. The average connotation given to concepts is defined as the 'true score', the deviation from this true score by individual subjects as 'error of judgment', or 'deviation'. Despite the negative connotation these terms have in everyday language the probabilistic psychologists employ these terms in a neutral way. It is not meant to surpass the statistical meaning.

This is different in the case of the psychophysics of vision, Kahneman's main research project in the period 1961-1971. In the well-known Gestalt psychological example of the two equal lines that appear to be of different size, the error the individuals make when thinking the lines are of unequal length (or the illusion into which they can be drawn), are what they are: an illusion or an error. What is the right response here is obvious. Kahneman uses the same concepts in both instances. In Kahneman et. al. (1967), for example, it is shown that the capacity to visually perceive of individuals substantially decreases when they are engaged in other mental tasks such as speech or calculation. Incorrect responses to the visual stimuli are labeled 'errors of judgment' in this research. And they really are errors of judgment as they might for example explain why drivers may miss a stop sign when engaged in conversation.

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[5] Kahneman (2002), p.3

[6] Gigerenzer (1987b)

The second main focus of Kahneman is what he labels (cognitive) illusions or errors (of judgment). These terms draw both on statistical language and on psychological research of vision. In the theoretical framework of Kahneman, incorrect statistical inferences therewith are the same phenomena as incorrect inferences from sensational stimuli. Hence, all human judgments can be analysed in the same terms, and are either 'right' or 'wrong'.

Tversky: man as an intuitive statistician, and the normative/descriptive distinction

Man as an intuitive statistician

Tversky is a few years Kahneman's junior. He is born in Haifa, Israel, fights in three Israeli wars and receives the highest citation possible in the Israeli army. As a psychologist he is widely recognized as man of great intellectual ability and superb mathematical skills. When the two start working together in 1971 Tversky's star is already rising rapidly as a result of his theoretical work in measurement theory and decision theory.

Tversky is a representant of mathematical psychology, a field which "may be characterized by the attempt to use mathematical methods to investigate psychological problems"[7]. This branch of psychology originates in the post-war period in the United States and counts among its founding fathers (and important contributors) such authors as Luce, Suppes, and Krantz. Important precursors are Thurstone and Brunswik, of whom the former coined the term as a replacement of his earlier "quantitative rational science"[8]. Mathematical psychology is thus to be understood as a part of the probabilistic approach to psychology. Not accidentally, an important part of the mathematical psychological research program constitutes of the theoretical work on theories of measurement. The theory of measurement developed by the mathematical psychologist, which predominantly elaborates on S. Stevens work on measurement, became known as the representational theory of measurement[9]. More specifically, then, mathematical psychology may be characterized as the attempt to improve (probabilistic) psychology's measurement models and psychological theories with the use of mathematics.

The probabilistic approach to psychology of which also mathematical psychology forms part has some important implications for the way science

[7] Coombs, Dawes, and Tversky (1970), p.1

[8] Gigerenzer (1987c), p.56

[9] See for example the seminal *Foundations of Measurement I* by Krantz, Luce, Suppes and Tversky (1971).

is done and the theoretical view of man it employs. To start with the latter, the mathematical psychologists conceptualize the human being as an intuitive statistician. That is, the human mind is understood to be a system that has to make a choice on the basis of information that is of a statistical nature. Procesess by which man comes to a decision can hence be understood as intuitive statistics. The view of man as an intuitive statistician is taken explicitly by Tversky. In Rapoport and Tversky (1970) he remarks that in his research:

> "man is viewed as an intuitive statistician who acts in order to maximize some specified criteria while operating on the basis of probabilistic information."

At the same time, the general methodology of the mathematical psychologists is one in which average behavior is investigated by means of statistical analysis of experimental results. Thus, while the phenomenon investigated is conceived of as being an intuitive statitstician, it is also analysed using statistics. Mathematical psychology in other words, uses statistics to investigate the behavior of the average intuitive statistician. [10]

In Tversky's experiments the human being as an intuitive statistician is analyzed from the theoretical perspective of decision theory. Nevertheless, Tversky is not primarily interested in decision theory but in the intuitive statistician. Decision theory in the work of Tversky functions as the best among a number alternatives available to test the intuitive statistician. In mathematical psychology decision theory is traced back to the work of authors such as the Bernoullis and Laplace. However, also what are normally considered contributors to economic theorizing on utility and preferences, such as Edgeworth and Pareto, are referred to as important. In the tradition of decision theory, Tversky concentrates mainly on experimental and theoretical research on choice behavior in gambles and lotteries. Nevertheless, it should be noted that Tversky also investigates choice behavior, or decision making, more broadly. In the already mentioned Rapoport and Tverksy (1970), for example, he investigates *Choice Behavior in an Optimal Stopping Task,* which concerns the question whether humans in a sequence of choices choose the stopping point theory tells them they should choose. For Tversky, there is no principal difference between this

[10] Hence, individual deviations from average behavior become 'errors' in the statistical meaning of the word. This 'error' language, however, is a typical character of Kahneman's work. In Tversky's early work it is only mentioned a few times in passing.

choice and the choices between gambles and lotteries normally considered in decision theory. In explaining the problem he notes that "the player has to *decide* whether to continue or to stop sampling more offers"[11] (my emphasis). The disinction often made in theoretical microeconomics between the preference-based approach and the choice-based approach[12] is thus not made by Tversky.

Normative vs. Descriptive

The main body of experimental and theoretical work is nevertheless on gambles, always in combination with Tversky's work on the representational theory of measurement. Now, as said, man is considered to be an intuitive statistician. And as, especially in the case of gambles and lotteries, the decision problems the subjects in the experiments are confronted with can be formulated in terms of probabilities, this implies that the scientists can theoretically determine what the subject *should* choose. Because the mind is considered to function as an intuitive statistician, it is possible to determine how it behaves were it to work correctly. This distinction is explicitly labeled normative vs. descriptive by Tversky and the other mathematical psychologists. What is important to emphasize is that this is more than a distinction between theory and practice, or between theory and the real world. If man is considered to be an intuitive statistician, and if the world is considered to present itself to the individual in terms of probabilistic information of uncertain events, a decision that deviates from the theoretically optimal decision becomes a failure of the system, or an error of judgment. The theory thus determines how the individual *should* choose. In that sense the theory is normative.

Tversky does not, however, label deviations from the normative theory failures or errors of judgments. In his description of the intuitive statistician in terms of decision theory his focus is on (theoretical) problems of measurement and on the comparison of different descriptive models. Although it is theoretically clear how individuals should choose, it does not mean that this normative theory is also the best description of actual displayed decision behavior by individuals. The most prominent descriptive model Tversky investigates in his experimental research is the subjective expected utility (SEU) model. *Foundations of Statistics* (1954) by the mathematician Savage in which this model is first put forth, is one of the cornerstones in research on decision making in mathematical psychology. Briefly put, the model is a mix of Ramsey's theory of the logic of partial

[11] Rapoport and Tversky (1970), p.105

[12] See among others Mas-Colell et. al. (1995)

belief, de Finetti's subjective probality, and utility theory more generally as it developed since D. Bernoulli. Nevertheless, in its use of uttility it follows the interpretation as given by von Neumann and Morgenstern, in which utitlity is equated with the monetary value of an outcome. Hence, the SEU model supposes that every outcome of a choice has a certain utility for the individual, an utility that is an objective property of the outcome. Independent of this, the individual forms expectations about the probability of the different possible outcomes, labeled subjective probability. In the (experimental) case where the probability of the outcomes is known, this subjective probability should (normative!) be equal to the known objective probability.

Savage, however, emphasizes that his theory is only applicable to 'small world' situations. A small world here is understood as describing only a part of the (real or grand) world, namely that part that is relevant for the decision to be made. Hence, if an individual has to choose between for example spending one thousand euros on a car or not spending one thousand euros on a car, a decision theoretic analysis in a small world only considers what is directly relevant for the decision (here the preferences of buying or not buying a car). One could, however very well argue that a number of other features of the world are also relevant for this decision, for example the uncertainty about possible future financial calamities. In that sense an analysis that only takes into account the directly relevant factors of the decision is labelled a 'small world' analysis, of which by the way Savage himself is quick to admit the problematic definition[13]

In Tversky (1967) the SEU model as description of choice behavior is compared with other descriptive models as for instance the expected utility (EU) model, that does not consider subjective probability, and the subjective expected value (SEV) model, which ignores utility. As in other studies the SEU model provides the best description. Tversky (1967) is also an illustration of how powerful the combination of Tversky's interests in both decision theory and measurement theory is. The main problem of the SEU model until Tversky starts working on it, is not the validity of the model as a description of actual choice behavior. No (mathematical) psychologist really doubts that it is the best description. The problem is that the outcome of the experiment, i.e. the actual choice made, is a combination of two (unknown variables):

[13] (Savage 1954, p.82-91). One of the main criticism from decision theorists on Kahneman and Tversky in later years has been that they apply Savage's theory to problems in the grand world without showing how this is possible given that the theory is devised for application in small worlds.

subjective probability and utility. Tversky is able to solve this problem with an advanced model based on the representational theory of measurement, known as conjoint measurement. It is a model specifically devised for the purpose of distinguishing two or more (theoretically determined) variables from one measurement.

To sum up, in Tversky's early work on decision theory and the representational theory of measurement, two main points stand out. Firstly, Tversky has a conception of man as an intuitive statistician. As a result (secondly), decision behavior in which the individual is faced with possible outcomes that can be analyzed in probabilistic terms, is investigated in a normative vs. descriptive framework.

Kahneman and Tversky: Heuristics and Biases

In the collaboration of Kahneman and Tverksy from the year 1971 onwards the different theoretical perspectives as described above are put together. Firstly, the cognitive illusions in the work of Kahneman, the result of an incorrect statistical inference, are linked with the decision theory used to test the intuitive statistician of Tversky. This connection enriches both theoretical approaches. On the one hand it provides the cognitive illusions of Kahneman with one general theory from which the cognitive illusions follow. Where in the early research of Kahneman cognitive illusions is a concept to describe a range of deviations from average or correct behavior, when it is linked with decision theory all cognitive illusions are the result of deviations from a normative decision theory. On the other side, it provides the normative versus descriptive dichotomy as it is employed in the investigation of the decision making by Tversky with an explanation of what are the deviations from normative decision theory. The normative versus descriptive distinction therewith becomes more pronounced. It is not just that there is a difference with how human beings in theory should make decisions and how in reality they do, when human beings do not make decisions according to how they should, the mistake is to be found in the individual.

Secondly, the intuitive statistician becomes the general concept on the basis of which all human behavior can be investigated. Thus, it is not only in analyses of lotteries and gambles that man can be understood as maximising some specified criteria while acting on the basis of probabilistic information, but also for instance in the case of the truck driver who is engaged in conversation and misses a stop sign. Thus, the intuitive statistician is no longer a theoretical concept that may be tested in some highly specified experimental settings but a concept with which all real-world behavior of humans can be understood. Therewith, Kahneman and

Tversky part with the theoretical restriction of Savage who emphasized that his theory is only applicable to 'small worlds'.

When Tversky and Kahneman start to collaborate, decision theory becomes the sole theory with which to investigate all human behavior. At the basis of this investigation of human behavior lies the concept the intuitive statistician; an intuitive statistician who does not always make decisions as he should. To put it in terms of the in decision theory often evoked concept of rationality: where normative decision theory determines the rational decision to take, all deviations from this theory are irrational. Cognitive illusions, in other words, are the result of irrational decision behavior.

Tversky and Kahneman (1971), their first, is entitled *Belief in the law of small numbers*[14]. In it they describe the results of an experiment that shows that even trained scientists at the annual conference of psychologists in the United Sates make systematic mistakes when faced with a decision that involves probabilities. The psychologists at the conference systematically overstate the probability with which a random sample is representational for the whole group, hence the title of the article.

Their second article, published in 1972, is a clear expression of the symbiosis of their work. It is entitled *Subjective Probability: A judgment of Representativeness*, and advances the thesis that "in many situations, an event A is judged more probable than an event B whenever A appears more representative than B."[15] The paper generalizes the findings of the 1971 paper by explaining that people judge the probability of events by it being similar in essential characteristics to its parent population, and hence the ease with which they can imagine the event. Individuals, in other words, overstate the more *representative* event. The 1972 also is the first paper in which the term heuristics is employed. Together with the normative/descriptive distinction this yields the following:

> "people do not follow the principles of probability theory in judging the likelihood of uncertain events……. Apparently, people replace the laws of chance by heuristics…….In the present paper, we investigate in detail one such heuristic called representativeness."[16]

The closely related Tversky and Kahneman (1973) for the first time bears heuristic in its title: *Availability: A Heuristic for Judging Frequency and*

[14] A direct reference to the law of large numbers in statistics, see for example Jorland (1987) or Stigler (1986).

[15] Kahneman and Tversky (1972), p.431

[16] Kahneman and Tversky (1972), p.431

Probability. By availability is meant that people overstate the probability of events they can most easily bring to mind or recall. Consider a typical text in the English language of which all words of three letters and longer are taken. Given the thus collected words, is the letter R more likely to appear on the first position or on the third position? A highly significant majority of the subjects will answer that this must be the first position.[17] The opposite, however, is true. This is explained by the fact that people can more easily think of words that start with an R, than of words that have an R on the third position. Words with an R on the first position have a higher *availability*. Although this heuristic is again not presented as anything more than that, a heuristic, in passing we are given an explanation: "Availability is an ecologically valid clue for the judgment of frequency because, in general, frequent events are easier to recall or imagine than infrequent ones."[18] In itself this remark is not significant as it is not elaborated upon nor appears in later work. What it does show is that the direct reference to normative (probabilistic) decision theory as explanation of observed choice behavior is slightly fading. That is, the investigation of how humans in reality make their choices becomes a more independent research with respect to the normative theory, without however drifting from it in any fundamental way.

In *On the Psychology of Prediction* (1973), Kahneman and Tversky give the conclusions of their first three papers a more theoretical underpinning. They state it as follows:

> "In making predictions and judgments under uncertainty, people do not appear to follow the calculus of chance or the statistical theory of prediction. Instead, they rely on a limited number of heuristics which sometimes yield reasonable judgments and sometimes lead to severe and systematic errors."[19]

But failing to employ statistics is something different than not understanding statistics. The next question thus is what happens when people are faced with probabilistic information when having to make a judgment. Following their experiment, consider a group of 100 people, 30 of which are lawyers and 70 of which are engineers. Suppose further you are provided with a neutral description of a man. What do you consider the probability of this man to be a lawyer? A highly significant portion of the subjects answers 'about equally probable', where it should be 30 percent. Moreover, when the

[17] Tversky and Kahneman (1973), p.212
[18] Tversky and Kahneman (1973), p.209
[19] Kahneman and Tversky (1973), p.237

same description is given to subjects who are told that of the 100 people 70 are lawyer and 30 engineer, the same answer of 'about equal probable' is given by a highly significant number of subjects. In other words, it is shown that apart from the persistent failure of people to use statistics when facing a choice, they also systematically fail to use or understand the statistics contained in the information they have to base their decision upon.

The most well known article by Tversky and Kahneman in psychology is without any doubt their 1974 *Science* article entitled *Judgment under Uncertainty: Heuristics and Biases*. It is also their most cited article, 1851 times until 1996[20]. The article gives a brief overview of the experiments described in the previous three papers but most importantly sets out the theoretical framework in which their previous found heuristics can be placed and which will form the basis for their future research. Tversky and Kahneman state their conceptual framework:

> "This article shows that people rely on a limited number of heuristic principles which reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, these heuristics are quite useful, but sometimes they lead to severe and systematic errors."[21]

As said, Tversky and Kahneman herewith loosen the strict connection between normative and descriptive decision theory as it was employed by the mathematical psychologists in the 1950s and 1960s. That is, the theory of how people ought to decide no longer is equated with the theory of how people actually make decisions. Indeed, the reader is tempted to infer from the text that Tversky and Kahneman have concluded that people in fact do not at all employ statistics to base their decisions upon. This is at least what the above quote suggests when it speaks of reducing "the complex task of assessing probabilities and predicting to simpler judgmental operations". But this would be a wrong conclusion. Kahneman and Tversky firmly remain in the vein of the probabilistic approach to psychology. The "simpler judgmental operations" is equivalent with simplified statistics. For example, the representative heuristic described above simplifies the choice but it remains statistics nonetheless. When an event A is judged on the probability of it belonging to class B, the representative heuristic states that this is done on the basis of comparing the probabilistic characteristics of event A with

[20] Laibson and Zeckhauser (1998)

[21] Tversky and Kahneman (1974), p.1124

those of class B. In the availability heuristic the probability of an event is compared with a frequency distribution of similar events.

The Heuristics and Biases theory of Tversky and Kahneman thus combines the decision theoretic analysis of the intuitive statistician with the idea that deviations from normative theory are cognitive failures. Therewith it transforms Savage's small world decision theory into a tool which can be used to analyze every real world judgment and decision. Decision theory has become the tool with which to test the intuitive statistician in all its environments.

Prospect theory

After the work on their 1974 *Science* article Tversky and Kahneman decide to focus again on more traditional decision making problems as already extensively studied by Tversky. In the spring of 1975 they present their first results in this direction at a psychological conference. In his autobiography, Kahneman recalls how ninety percent of the content of the final article was already present at this stage and that the subsequent three years of numerous revisions were used to polish their argument in such a way that it would be acceptable for the most prestigious journal on decision making of the time, which happened to be an economic journal.[22] The final draft that appeared in *Econometrica* in 1979 was an explicit attempt to make their findings acceptable for the economic community.

The article consists of two parts of roughly equal lengths. In the first part Kahneman and Tversky put the economists' expected utility theory (EUT) in their own framework, show how it descriptively fails, and suggest three heuristics that may account for these systematic biases. The second part is a refinement of the Heuristics theory as a description of human decision making.

Part one: formulating the problem

In the first three sentences of the article, Kahneman and Tversky link their theory with the model of decision making that dominates economics, that is with EUT. In economics, EUT is conceived as a model of rational decision making, which links well with their normative decision theory. Hence,

> "Expected Utility Theory has dominated the analysis of decision making under risk. It has generally been accepted as a normative model of rational choice, and widely applied as a descriptive model of economic behavior. Thus, it is assumed that all reasonable people

[22] Kahneman (2002), p.14, Royal Swedish Academy of Sciences (2002b)

> would wish to obey the axioms of the theory, and that most people actually do, most of the time."[23]

The economists' conception of EUT as the model of rational choice is thus interpreted by Kahneman and Tversky both as a normative decision theory, and as a description of the rational man's decision behavior. Therewith, the conception of rational economic man is equated with the conception of man as an intuitive statistician. This equation is possible for two reasons. First of all, decision theory in economics and cognitive psychology resemble one another to a very large extent. They only differ in the language they employ and in the models of decision theory they emphasize (EUT or SEU). But they do, secondly, differ in one important respect, namely in the use of the normative-descriptive framework. In economics this distinction is not made, EUT in economics is simply how people behave. Kahneman and Tversky are hence free to use the framework and to employ it to their liking. Thus economists' EUT can be understood to be both a descriptive and a normative decision theory at the same time. The fact that, from a psychology point of view, economists describe their subjects in the same way as how they should behave, makes it possible to equate the rational economic man with the intuitive statistician.

After this, but still on the first page, the authors acknowledge their adherence to probabilistic psychology.

> "Decision making under risk can be viewed as a choice between prospects or gambles. A prospect $(x_1,p_1;\ldots;x_n,p_n)$ is a contract that yields outcome x_i with probability p_i, where $p_1 + p_2 + \ldots + p_n = 1$"[24]

In line with their theoretical position and their earlier experimental research, Kahneman and Tversky conceive of EUT as an integration of probability and utility. Therewith, their conception of EUT strongly resembles Savage's SEU model, with the difference that in the case of EUT the probabilities are considered objectively given. The choice made by the individual is made on the basis of a combination of the utility of the outcome and the probability of the outcome. And as the overall utility of a prospect is the expected utility of its outcomes, this formally yields the following (still on the first page):

$$U(x_1,p_1;\ldots;x_n,p_n) = p_1u(x_1) + \ldots + p_nu(x_n)$$

[23] Kahneman and Tversky (1979), p.263

[24] Kahneman and Tversky (1979), p.263

Furthermore, Kahneman and Tversky assume utility to be a concave function with respect to money. This is a central assumption throughout the history of probabilistic thinking and can be traced back to the work of Daniel Bernoulli, who needed it to offer a solution to the Sint-Petersburg paradox.[25] Also economists generally employ this assumption, although there are some notable exceptions. Von Neumann and Morgenstern (1944/1954), for example, assume for reasons of simplicity the relation of utility and money to be linear. In the probabilistic tradition in which Kahneman and Tversky stand it is, however, of central importance. In this case it is labeled the assumption of risk aversion.

In what follows Kahneman and Tversky give an overview of experiments that show how EUT systematically fails as a descriptive theory of human decision making. They do so with the use of three heuristics, which in this article are labeled 'effects': the certainty effect, the reflective effect, and the isolation effect. All three can be best explained using Kahneman and Tversky's own examples. For the certainty effect, consider the following choice:

A: (4000; 0.8), or B: (3000) (1)

The authors show that most subjects (they refrain from talking of the average individual) prefer B to A, this despite the fact that the expected pay-off of A is higher than the expected pay-off of B. This choice can be explained with risk aversion and implies that $u(3000)/u(4000) > 0{,}8$. Now consider the following choice:

C: (4000; 0.2), or D: (3000;0,25)[26] (2)

In this case, it is shown, most subjects prefer C to D, which implies $u(3000)/u(4000) < 0{,}8$, a contradiction with the preference of B over A . It implies that a reduction of the probability from 1,0 to 0,25 has a larger impact than reducing the probability from 0,8 to 0,2. This phenomenon is labeled the certainty effect. The certainty effect can be seen as an amplification of risk-aversion for certain outcomes.

For losses this effect is exactly the opposite, which is coined the reflection effect. If subjects are proposed the following choice

[25] Jorland (1987)

[26] Note that the difference between A,B and C,D is a difference of the probabilities of a factor 4.

E: (–4000; 0.8), or F: (–3000) (3)

the majority of them prefer E. However, when faced with the choice

G: (–4000; 0.2), or H: (–3000;0,25) (4)

they choose H. Firstly, this implies that where individuals are risk-averse when the choice involves gains, they are risk-seeking when the choice involve losses. Secondly, where a certain outcomes amplifies the risk-aversion in the case of gains, it amplifies the risk-seeking in case of losses.

The third bias discussed by Kahneman and Tversky is the isolation effect, which is an explanation of systematic deviations from EUT in the case of complex choices. It states that individuals reduce a complex choice problem into components with which they can deal. Consider as an example a two-stage game in which in the first round the probability of staying in the game is 0,25, and 0,75 of leaving it with nothing. If the second stage is reached, the individual is given the following choice.

A: (4000; 0.8), or B: (3000) (5)

Kahneman and Tversky show that although this game amounts to the same choice as the choice in problem (2), in which subjects preferred C to D, the subjects in the two-stage problem prefer B over A, as they did with choice problem (1). Subjects, in other words, do not see the two-stage problem as one big choice problem, but as two different choices, in each case of which they make a decision independent of the other stage.

Therewith, the economists' EUT is formulated in terms of Kahneman and Tversky's probabilistic psychology, and the systematic failures of EUT as a descriptive theory for individual decision making are exposed. Up to this point, the article has provided an overview of known choice behavior of individuals and related this to the relevant economic theory of individual decision making. It has related the failures of EUT as descriptive theory to the theory advanced by Kahneman and Tversky. Although perhaps disturbing from an economic point of view, for the probabilistic psychologists this is merely a well documented summary of known facts. Also Kahneman and Tversky seem to consider it only an introduction to the second theoretical part, witness the first sentence of this second part: "The preceding discussion reviewed several empirical effects which appear to invalidate the expected utility theory as a descriptive model."[27]

[27] Kahneman and Tversky (1979), p.274

Part 2: refining the heuristics

Prospect theory, the theory Kahneman and Tversky offer as a descriptive model in EUT's stead is a refinement of their heuristics theory. In a way, prospect theory describes the underlying mechanism out of which result the different heuristics. The heuristics (or effects) that have been described in the different publications all account for a part of human decision making as described by prospect theory. With EUT, the authors also do away with utility as a descriptive variable.

Firstly, the authors distinguish two phases in the decision process: an editing phase and an evaluation phase. The editing phase consists of different operations, of which four are labeled 'major'. 1) Coding, in which a reference point is chosen. The result of this operation is that all outcomes are interpreted as either losses or gains. 2) Combination, in which choices are combined that can be simplified by doing so. 3) Segregation, in which riskless and risky components are segregated. 4) Cancellation, in which irrelevant choices are cancelled. The isolation heuristic/effect described above can be seen as largely the result of segregation in the editing phase. Kahneman and Tversky furthermore note that many biases (or 'anomalies' as they are coined here) are the result of this editing phase. During the evaluation phase the decision maker evaluates the edited prospects and chooses according to the prospect of highest 'value'.

This 'value', V, of Kahneman and Tversky is a variable that is determined by two components. The first is the 'weight' π given to the probability p of each outcome, hence $\pi(p)$. It is tempting to understand this 'weight' as the subjective probability given to the objective probability in the sense of Savage's SEU model. And indeed, there is little to refute such a claim. The authors stress, however, that π is not a probability measure and that the total weight need not add up to 1. On the basis of their experiments they derive the following (average) weighting function.

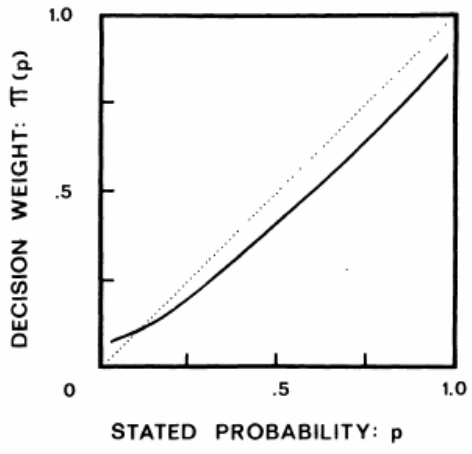

Figure 1: The weighting function, Kahneman and Tversky (1979), p.283

The second component, υ, is the subjective value of each outcome *x*, hence *υ(x)*. Although not stated explicitly, there are two main differences between subjective value and utility according to Kahneman and Tversky. First of all, utility is interpreted, as an objective attribute of the outcome. In other words, it is independent of the individual. In this sense, subjective value could be understood as a 'subjective utility' attached to the outcome. Secondly, the subjective value is a valuation that is made with respect to a reference point, where utility is considered to be an absolute attribute of the outcome. Given this definition of 'value', and on the basis of their experimental results, Kahneman and Tversky draw the following, well-known, curve.

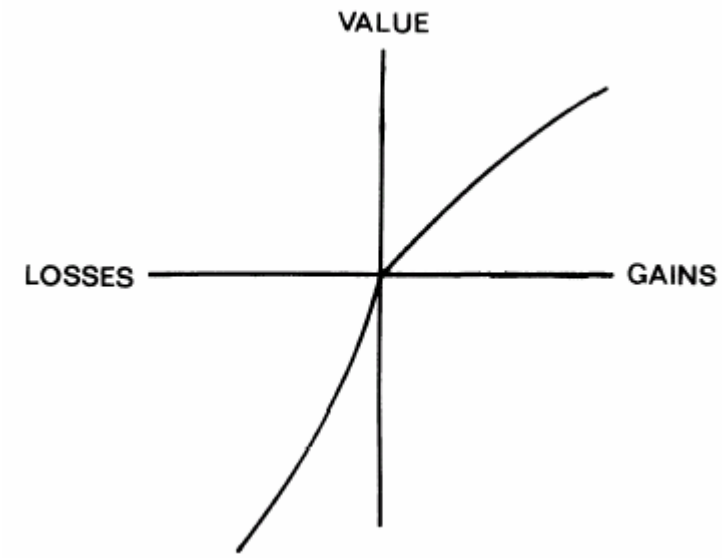

Figure 2: The value function, Kahneman and Tversky (1979), p.279

The origins of prospect theory

Prospect theory should first of all be understood as a part of the probabilistic approach to psychology. Both Kahneman and Tversky are from the onset of their professional careers in this vein of psychological research. Their early work is however far from the same. Kahneman is working mainly in the field of what may be labeled 'cognitive psychophysics'. He theorizes about his psychophysical research of the visual system in cognitive science terms in the sense that he gives explanations of the working of the visual system in terms of the working of the cognitive system. The probabilistic approach shows up in his work on the places where he builds or applies models of statistical inference. Kahneman's concern is the average response of the subjects and the (systematic) deviations of individual subjects with respect to the average. Using the statistical language of the probabilistic psychologists, these deviations are labeled 'errors', 'error of judgment', 'cognitive failures', or 'cognitive illusions'. These last two labels show how in Kahneman's work a

connection is laid between the visual illusions as first shown by Gestalt psychology, and the 'error' language of statistics[28].

Tversky's main occupations are the representational theory of measurement and the theory of decision making. Theorizing on individual decision making in mathematical psychology bears great resemblance to, and draws on much of the same work as, economics. In differs from it in important respects as well. In mathematical psychology decision making is understood in probabilistic terms, a tradition that is traced back to the early eighteenth century theorists of probability. By the time of Tversky, these psychologists have taken on a view of man as an intuitive statistician. This conception of man that is also taken explicitly by Tversky has as a result that the working of the human cognitive system is understood in the same terms as how the world presents itself to the human being. The world presents itself in terms of probabilistic information of uncertain outcomes, and man is conceived of as a cognitive system of statistical inference. Given that humans do not act according to statistics, this implies that a distinction can (and should) be made between normative decision theory and descriptive decision theory. Probabilistic psychology in Tversky's work thus shows up in two ways. First, statistics provides the basis upon which mathematical models of measurement are based. Secondly, statistics provides a conception of how to understand the human mind and a norm by which to understand his behavior.

When Kahneman and Tversky start to collaborate in 1971, these different research projects within the same probabilistic vein are combined. First of all, decision theory in the tradition of Savage as it was practiced by Tversky is extended from gambles and lotteries to every choice problem that involves uncertain outcomes. From a small world theory that can only be applied to specific (laboratory) circumstances, it is transformed into a theory with which every human decision can be analyzed. Secondly, the deviations from normative decision theory in these real-world situations come to be understood as cognitive failures.

Therewith, Kahneman and Tversky construct a new, complete approach to the understanding of the human psyche. The intuitive statistician is taken out of his laboratory and tested in all kinds of decisions he has to take in the real world. As normative decision theory fails to describe most of these real world decisions problems, Kahneman and

[28] Years later these illustrations of visual failures are used by Kahneman and Tversky to illustrate what they mean when they insist on studying cognitive failures to understand the working of the system. See Tversky and Kahneman (1986), pp. S266-S267.

Tversky propose an alternative descriptive decision theory: Heuristics and Biases.

In the 1979 *Econometrica* article, Kahneman and Tversky confront their new descriptive theory with the small world decision theory that had initially provided the norm for the decision behavior of the intuitive statistician. As their work of the early seventies had done for cognitive psychology, the first half of the article shows, in the economists' language, that the Savage-type decision theory descriptively fails. That is, small world decision theory fails to describe the decision behavior of the intuitive statistician in reality. The second half of the article provides a few heuristics[29] that may account for the deviations from the norm as described in the first half It also provides a refinement, or extension, of the Heuristics and Biases theory in the form of an editing and an evaluation phase. These phases can be understood both as generalizations of the heuristics and as a description of an underlying process from which the heuristics derive. The article is not definitive in this respect.

For the main message this does not matter. The purpose of the article is to show that what decision theory calculates as the optimal decision, and thus as the norm, does not provide a good description of how the intuitive statistician in reality behaves. The intuitive statistician here is equated with the rational economic man of the economists. Therewith, Kahneman and Tversky introduce a normative vs. descriptive distinction in the economists' theorizing of decision behavior. The origin of prospect theory, thus, is the wish to test the intuitive statistician in the real world.

Conclusion

In this paper I have given an historical reading of prospect theory. That is, I have given an explanation of the article in terms of the theoretical development of both authors, as well as in terms of the scientific tradition in which these authors can be placed. Hence, I have not made, nor want to suggest to have made, any claim about the validity of Kahneman and Tversky's work for understanding human decision making. I merely hope to have pointed out some important aspects for the understanding and interpretation of Kahneman and Tversky's most important contribution to economics. Future research could perhaps proceed on this track by investigating how the intuitive statistician continued to influence economics through behavioral economics.

[29] Labelled 'effects' in this paper, which is somewhat confusing as also biases are sometimes refered to as 'effects'.

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