TI 2005-077/2
Tinbergen Institute Discussion Paper

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Ludger Linnemann[1]
Andreas Schabert[2]

[1] *University of Cologne,*
[2] *Department of Economics, University of Amsterdam, and Tinbergen Institute.*

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Tinbergen Institute Amsterdam
Roetersstraat 31
1018 WB Amsterdam
The Netherlands
Tel.: +31(0)20 551 3500
Fax: +31(0)20 551 3555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31(0)10 408 8900
Fax: +31(0)10 408 9031

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Ludger Linnemann
University of Cologne

Andreas Schabert[2]
University of Amsterdam

July 27, 2005

Abstract

We study the consequences of non-neutrality of government debt for macroeconomic stabilization policy in an environment where prices are sticky. Assuming transaction services of government bonds, Ricardian equivalence fails because public debt has a negative impact on its marginal rate of return and thus on private savings. Stability of equilibrium sequences requires a stationary evolution of real public debt, which steers inflation expectations and rules out endogenous fluctuations. Under anti-inflationary monetary policy regimes, macroeconomic fluctuations tend to decrease with the share of tax financing, which justifies tight debt constraints. In particular, a balanced budget policy stabilizes the economy under cost-push shocks, such that output and inflation variances can be lower than in a corresponding case where debt is neutral.

JEL classification: E32, E63, E52.
Keywords: Government debt, fiscal and monetary policy rules, stabilization policy, equilibrium uniqueness.

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[1] The authors would like to thank Roel Beetsma, Matthew Canzoneri, Jagjit Chadha, Alex Cukiermann, Roger Farmer, Jordi Gali, Robert Kollmann, Campbell Leith, and Massimiliano Marzo for useful suggestions. We further benefitted from comments by participants at the Workshop on "EMU Macroeconomic Institutions and Policies" 2003 in Milan, and the CEPR RTN Conference "Designing a Macroeconomic Policy Framework for Europe" 2004 in Barcelona. An ealier version of the paper circulated under the title "Fiscal-monetary policy interactions and macroeconomic stability". This research is part of the RTN project "Macroeconomic Policy Design for Monetary Unions", funded by the European Commission (contract number HPRN-CT-2002-00237). The views expressed in this paper are those of the authors and do not necessarily reflect the views of any institution. The usual disclaimer applies.

[2] Corresponding author. Address: University of Amsterdam, Department of Economics, Roeterstraat 11, 1018 WB Amsterdam, The Netherlands, Phone: +31 20 525 7159, Fax: +31 20 525 4254, E-mail: a.schabert@uva.nl.

1 Introduction

This paper studies the theoretical role of government debt policy for macroeconomic stabilization purposes. It has since long been noted that in intertemporal general equilibrium models the sequence of government bonds influences the sequences of endogenous variables like real output and inflation under certain conditions, namely when the neutrality requirements of the Ricardian equivalence theorem do not hold (Barro, 1974). Whether or not Ricardian equivalence fails, and government debt has a meaningful role in determining the economy's equilibrium, is primarily an empirical question. A large part of the earlier econometric literature is summarized in Elmendorf and Mankiw (1999), who conclude that most, though not all, studies find some relation between government debt and variables like output and real interest rates that point to a failure of Ricardian equivalence. The most recent available empirical results seem to converge on a set of compatible findings. Gale and Orszag (2004), Engen and Hubbard (2004), and Laubach (2003) all report a positive empirical relation between the level of government debt or deficits and the real interest rate in the U.S., a finding which Ardagna et al. (2004) corroborate for a panel of OECD countries. Thus, government debt indeed appears to be non-neutral.

In this paper we ask which implications the evolution of government debt has in a theoretical model of macroeconomic fluctuations that displays, in accordance with the empirical evidence, non-neutrality of debt. To do so, we introduce debt non-neutrality through the assumption that the current stock of government bonds directly affects aggregate consumption demand via an effect on the saving behavior of households. While infinite horizons and lump-sum taxation are assumed throughout, aggregate demand effects of public debt are brought about by a mechanism by which the total rate of return on government bonds decreases with the real value of its outstanding stock. In particular, holdings of government bonds are assumed to provide services that facilitate goods market transactions, which is – since Barro (1974) – known to break Ricardian equivalence.[1] The assumption of liquidity services of government bonds can be justified through the fact that (short-run) government bonds can easily be transformed into money, or through their role as collateral for many type of transactions. We introduce this assumption into a New Keynesian sticky-price model where the central bank sets the nominal interest rate and the government issues interest bearing debt. Our question is how the presence and the management of government debt then modifies the well-known trade-off between output and inflation stabilization associated with cost-push type shocks.

By assuming transaction costs to be decreasing and convex in both types of government

[1] This assumption has more recently been used by Bansal and Coleman (1996) and Canzoneri and Diba (2005). The former authors analyze the impact of liquidity services of debt on the equity premium puzzle and the risk-free rate puzzle, while the latter authors show (within a flexible price model) that the problem of price level indeterminacy under interest rate policy can be cured by allowing public debt to provide liquidity.

liabilities, money and bonds, the total rate of return on bonds consists of interest rate payments and of a real return from lowering transaction costs. As the marginal return from transaction services is decreasing in households' government bond holdings, real public debt is negatively related to its total rate of return.[2] Thus, an increase in the stock of outstanding government bonds requires a higher real interest rate in equilibrium, consistent with empirical evidence quoted above. Moreover, since private consumption will depend on the total rate of return on the savings instrument (bonds), an increase in public debt will ceteris paribus induce substitution of consumption from the future to the present.[3] It should be noted that our approach is different from models where fiscal policy matters because of the assumption that the government has only access to distortionary taxes, as in Benigno and Woodford (2003), Kollmann (2004), Schmitt-Grohe and Uribe (2004), and Canzoneri et al. (2005). In these models, government debt influences output and inflation only if it has an effect on the income tax rate, and thus on factor supply, while in the present paper there is a direct link between debt and aggregate demand.

We demonstrate how local equilibrium stability and the model's responses to shocks crucially depend on the interaction of monetary and fiscal policies, since the former (via the nominal interest rate) controls the price of government bonds, and the latter (via the budget deficit) their supply. As a consequence, both fiscal and monetary policy impact on the rate of return of bonds. The link between government debt and aggregate demand in our model has a crucial implication for stabilization policy: Under non-neutrality, the evolution of public debt introduces history dependence of equilibrium sequences, which steers the households' expectations about future prices and real activity. This link between the stock of outstanding government bonds and the allocation requires real public debt to be stationary in any locally stable equilibrium. Our results on the effects of stabilization policy in this model can be understood as following from the constraints that the debt exerts on the sequences of (expected) output and inflation.

In particular, we find the following results. First, equilibrium stability is guaranteed as long as there is a negative feedback from inflation to the real value of the stock of outstanding debt, which is the case when the nominal interest rate policy of the central bank is not too aggressively targeting inflation. Given this, any decline in real public debt tends to reduce aggregate demand and thus inflation, which in turn causes real debt to recover. This also rules out the possibility of local equilibrium multiplicity that is known to arise easily in models with nominal interest rate policy: Arbitrary expectations of higher inflation would raise the expectation of future decreases in the real value of government bonds, and thus of a higher rate of return, inducing current demand and prices to slump,

[2] Similar effects might arise when higher levels of public debt lead to a risk premium or when households face convex adjustment costs for bond holdings.

[3] In this sense, the mechanism studied here has similarities to the role of debt in overlapping generations models (like Leith and Wren-Lewis, 2000), although these models differ from our specification in that public debt is there also non-neutral in the long-run.

such that inflation expectations cannot be self-fulfilling. Hence, if public debt matters for aggregate demand, monetary policy is relieved from the task of avoiding sunspot equilibria, which is in contrast to the results obtained in corresponding models with neutral debt (see, e.g., Woodford, 2003).[4]

We then continue to analyze the contributions of monetary and fiscal policy to stabilizing fluctuations induced by cost-push shocks, which lead to an immediate rise in inflation, and a decline in output and public debt. A more aggressive interest rate setting reduces (raises) inflation (output) fluctuations, as would also be the case in a corresponding model with neutral debt. The new element here is, however, that a high degree of tax financing always reduces the inflation variance, while it may raise or reduce the output variance depending on the monetary stance. The reason is that a rise in inflation due to a cost-push shock tends to lower the real value of public debt, which raises the rate of return on debt and thus reinforces the output contraction. When this channel is more pronounced, the inflation variance declines, while the output variance rises for moderate interest rate policies. This, however, changes under aggressive anti-inflationary monetary policy regimes, where higher shares of tax financing also reduce output fluctuations. We further find that with tight debt control, and in particular under a balanced budget regime, both the variance of output and of inflation can be lower in the present model than in a standard New Keynesian model, where, other things equal, public debt is neutral. The reason is that tight constraints on public debt enhance macroeconomic stabilization, since fluctuations of inflation (and output) are limited by the requirement that the equilibrium sequence of real government debt must be stationary.

The rest of this paper is organized as follows. Section 2 presents the model and Section 3 the main results. The first part of section 3 derives the conditions for local equilibrium stability. The second part discusses the effects of monetary and fiscal policy measures. In the third part we isolate the effect of debt non-neutrality on inflation expectations and macroeconomic fluctuations. In the last part of section 3 we analyze monetary and fiscal policy effects on output and inflation volatility. Section 4 concludes.

2 The model

In this section a model in which the stock of government bonds affects their total rate of return is presented. Government bonds are – analogously to money – a means to

[4] It should be noted that this mechanism of equilibrium selection does not rely on a tax policy specification that could cause government insolvency, such as a 'non-Ricardian' fiscal policy (e.g. an exogenous primary surplus). Under a non-Ricardian regime, the households' transversality condition contributes to the determination of equilibrium (and of the price level sequence). In contrast, the transversality condition is irrelevant for equilibrium selection throughout this paper, since we restrict our attention to public policy regimes which guarantee the transversality condition to be satisfied even off equilibrium, i.e., regardless of the private sector behavior. For an analysis of the impact of non-Ricardian fiscal policy regimes on the determination of the price level and the equilibrium, see Leeper (1991), Woodford (2001a), or Benhabib et al. (2001).

lower the cost of market transactions. In so far as the transaction cost function is convex in bonds, the model becomes non-linear in public debt, and thus the households' savings/consumption decision is related to their holdings of government bonds.

Private sector Throughout the paper, nominal variables are denoted by upper-case letters, while real variables are denoted by lower-case letters. A bar over a variable denotes a constant steady state value, and a caret operator marks a logarithmic deviation from steady state, $\widehat{x}_t = \log(x_t/\overline{x})$. There is a continuum of households indexed with $j \in [0,1]$. Households have identical asset endowments and identical preferences. Household j maximizes the expected sum of a discounted stream of instantaneous utilities u :

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{jt}, l_{jt}\right), \tag{1}$$

where E_0 is the expectation operator conditional on the time 0 information set, and $\beta \in (0,1)$ is the subjective discount factor. The instantaneous utility u is assumed to be increasing in consumption c, decreasing in working time l, strictly concave, twice continuously differentiable, and to satisfy the usual Inada conditions. For simplicity, instantaneous utility u is further restricted to be separable in private consumption c and working time $l : u(c_{jt}, l_{jt}) = \upsilon(c_{jt}) - \mu(l_{jt})$.

At the beginning of period t household j is endowed with holdings of money M_{jt-1} and government bonds B_{jt-1}, which are carried over from the previous period. Purchases of the consumption good are assumed to be associated with real transaction costs. While it is commonly assumed that only money provides transaction services, here also holdings of government bonds reduce transaction costs. We view this assumption, which is for example also applied Bansal and Coleman (1996), Lahiri and Vegh (2003), and in Canzoneri and Diba (2005), as reasonable, since government bonds can in general easily be transformed into money and serve as collateral for many types of transactions.[5] We assume that the goods market opens before the asset market, such that households rely on the beginning-of-period holdings of government liabilities to reduce transaction costs.[6]

Assumption 1 *The transaction cost function* $h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t)$ *satisfies: i)* h *is non-negative, increasing in* c*, strictly decreasing in* M_{jt-1}/P_t *and in* B_{jt-1}/P_t*, and twice continuously differentiable in all arguments, ii)* $h_{cc} \geq 0$, $h_{mm} > 0$, $h_{bb} > 0$, $\lim_{m \to 0} h_m = -\infty$, $\lim_{b \to 0} h_b = -\infty$*, and iii)* $h_{cm} = h_{cb} \,(= h_{mb}) = 0$.

Part *iii)* implies that the transaction cost function is separable in all arguments (as in Lahiri and Vegh, 2003). We further assume that transaction costs are private costs that

[5] For example, Lacker (1997) and Schabert (2004) have shown that short-run government securities exhibit a liquidity value if they are (in contrast to private debt) eligible as collateral in open market operations.

[6] The partial derivative of h with respect to the real value of beginning-of-period t money (bond) holdings M_{jt-1}/P_t (B_{jt-1}/P_t) is denoted by h_m (h_b).

are paid to a particular sector whose only function is to rebate its receipts immediately to the household sector through lump-sum transfers, such that transaction costs do not show up in the aggregate resource constraint. Both assumptions ensure that there is no direct (wealth) effect of money and bond holdings on consumption. Nonetheless, there is an effect of government bond holdings on consumption that operates through intertemporal substitution, which will be explained below.

In order to introduce supply side disturbances, we assume that households monopolistically supply differentiated labor services. Differentiated labor services l_j are transformed into aggregate labor input l_t, which can be employed for the production of the final good. The transformation is conducted via the aggregator $l_t^{1-1/\vartheta_t} = \int_0^1 l_{jt}^{1-1/\vartheta_t} dj$. The elasticity of substitution between differentiated labor services $\vartheta_t > 1$ varies exogenously over time. Cost minimization then leads to the following demand for differentiated labor services l_{jt},

$$l_{jt} = \left(\frac{w_{jt}}{w_t}\right)^{-\vartheta_t} l_t, \qquad \text{with} \qquad w_t^{1-\vartheta_t} = \int_0^1 w_{jt}^{1-\vartheta_t} dj, \tag{2}$$

where w_{jt} and w_t are the individual and the aggregate real wage rate, respectively. Household j faces a lump-sum tax $P_t\tau_t$ (where P is the aggregate price level), labor income $P_t w_{jt} l_{jt}$ and dividends $D_{j,it}$ from monopolistically competitive firms indexed by $i \in [0, 1]$. After the goods market is closed, the financial market opens where households can either invest in nominal bonds B_{jt} at the price $1/R_t$, in money holdings M_{jt}, or in nominal state contingent claims. Household j's flow budget constraint reads

$$\begin{aligned} & M_{jt} + B_{jt}/R_t + P_t c_{jt} + P_t h(c_{jt}, M_{jt-1}/P_t, B_{jt-1}/P_t) \\ \leq\ & P_t w_{jt} l_{jt} + B_{jt-1} + M_{jt-1} - P_t\tau_t + \int_0^1 D_{j,it} di. \end{aligned} \tag{3}$$

It maximizes (1) subject to (2), (3), and borrowing constraints for given initial values $M_{j(-1)} = M_{-1} > 0$, and $B_{j(-1)} = B_{-1} > 0$. The first order conditions for the household's problem are given by

$$\lambda_{jt}(1 + h_c(c_{jt})) = v'(c_{jt}), \tag{4}$$

$$\xi_t^{-1} w_{jt} \lambda_{jt} = \mu'(l_{jt}), \tag{5}$$

$$E_t\left\{\frac{\lambda_{jt+1}}{\lambda_{jt}}\left[\left(1 - h_b(b_{jt}\pi_{t+1}^{-1})\right)\frac{R_t}{\pi_{t+1}}\right]\right\} = 1/\beta, \tag{6}$$

and $\beta E_t\left[\lambda_{jt+1}\pi_{t+1}^{-1}\left(1 - h_m(m_{jt}\pi_{t+1}^{-1})\right)\right] = \lambda_{jt}$, where λ_{jt} is the Lagrange multiplier on the budget constraint, $\pi_t = P_t/P_{t-1}$ is the gross inflation rate, $m_{jt} \equiv M_{jt}/P_t$ and $b_{jt} \equiv B_{jt}/P_t$ are real cash and government bond holdings, respectively, and $\xi_t \equiv \vartheta_t/(\vartheta_t - 1)$ denotes the wage mark-up, which is assumed to follow an exogenous stochastic process (see below). Further, the transversality conditions $\lim_{t\to\infty} E_0\beta^t\lambda_{jt}b_{jt} = 0$ and $\lim_{t\to\infty} E_0\beta^t\lambda_{jt}m_{jt} = 0$ hold in the household's optimum.

Equations (4) and (5) are first order conditions for consumption and labor supply. The central model element can be seen in equation (6), which is the first order condition for bond holdings. Here, the growth rate of the shadow price of wealth λ_{jt} is related to the expected total rate of return on government bonds (given in the square brackets), consisting of the real interest rate R_t/π_{t+1}, and the marginal benefit from transaction services $-h_b(b_{jt}\pi_{t+1}^{-1})$. By the assumption $h_{bb} > 0$ the latter is decreasing in the stock of real bonds. Thus, a higher stock of bonds reduces their total rate of return. This either causes an intertemporal reallocation of consumption, since it requires the growth rate $\lambda_{jt+1}/\lambda_{jt}$ to rise, or raises the real interest rate R_t/π_{t+1}. Since bonds are assets that yield an additional non-pecuniary benefit through their ability to facilitate transactions, monetary policy which will be assumed to set the nominal interest rate R_t interacts with the supply of public debt, since both jointly affect its total rate of return by (6).

The final consumption good is an aggregate of differentiated goods produced by monopolistically competitive firms indexed with $i \in [0,1]$. The CES aggregator of differentiated goods is defined as $y_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y_{it}^{\frac{\epsilon-1}{\epsilon}} di$, with $\epsilon > 1$, where y_t is the number of units of the final good, y_{it} the amount produced by firm i, and ϵ the constant elasticity of substitution between these differentiated goods. Let P_{it} and P_t denote the price of good i set by firm i and the price index for the final good. The demand for each differentiated good is $y_{it} = (P_{it}/P_t)^{-\epsilon} y_t$, with $P_t^{1-\epsilon} = \int_0^1 P_{it}^{1-\epsilon} di$. A firm i produces good y_i employing a technology which is linear in the labor input: $y_{it} = l_{it}$ (note that $l_t = \int_0^1 l_{it} di$). Hence, labor demand satisfies: $mc_{it} = w_t$, where mc denotes real marginal costs. Nominal stickiness is present in form of staggered price setting as developed by Calvo (1983). Each period firms may reset their prices with the probability $1-\phi$ independently of the time elapsed since the last price setting. The fraction $\phi \in [0,1)$ of firms are assumed to adjust their previous period's prices according to the simple rule $P_{it} = \overline{\pi} P_{it-1}$, where $\overline{\pi}$ denotes the average inflation rate.[7] Firms are assumed to maximize their market value, which equals the expected sum of discounted dividends $E_t \sum_{s=0}^{\infty} q_{t,t+s} D_{it+s}$, where $D_{it} \equiv (P_{it} - P_t mc_{it})\, y_{it}$ and $q_{t,t+s}$ is a stochastic discount factor which will be discussed below. In each period a measure $1-\phi$ of randomly selected firms set new prices $\widetilde{P}_{it}$ as the solution to $\max_{\widetilde{P}_{it}} E_t \sum_{s=0}^{\infty} \phi^s q_{t,t+s} (\overline{\pi}^s \widetilde{P}_{it} y_{it+s} - P_{t+s} mc_{t+s} y_{it+s})$, s.t. $y_{it+s} = (\overline{\pi}^s \widetilde{P}_{it})^{-\epsilon} P_{t+s}^{\epsilon} y_{t+s}$. The first order condition for the optimal price setting of re-optimizing producers is given by

$$\widetilde{P}_{it} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} \phi^s \left[q_{t,t+s} y_{t+s} P_{t+s}^{\epsilon+1} \overline{\pi}^{-\epsilon s} mc_{t+s} \right]}{E_t \sum_{s=0}^{\infty} \phi^s \left[q_{t,t+s} y_{t+s} P_{t+s}^{\epsilon} \overline{\pi}^{(1-\epsilon)s} \right]}, \tag{7}$$

where we used $mc_{it} = mc_t$. The linear approximation of this first order condition and $P_t^{1-\epsilon} = \phi\,(\overline{\pi} P_{t-1})^{1-\epsilon} + (1-\phi)\, \widetilde{P}_t^{1-\epsilon}$ at the steady state for a given initial price level

[7] Due to this assumption, different values for the steady state inflation rate, which depend on monetary as well as fiscal policy, do not affect the structure of the (log-linearized) aggregate supply constraint (see Ascari, 2003).

$P_{-1} > 0$ is known to lead to $\widehat{\pi}_t = \chi\widehat{mc}_t + \beta E_t \widehat{\pi}_{t+1}$, with $\phi\chi = (1-\phi)(1-\beta\phi)$, while aggregate output is given by $y_t = (P_t^*/P_t)^{\epsilon} l_t$, where $(P_t^*)^{-\epsilon} = \int_0^1 P_{it}^{-\epsilon} di$ and thus $(P_t^*)^{-\epsilon} = \phi\left(\overline{\pi}P_{t-1}^*\right)^{-\epsilon} + (1-\phi)\widetilde{P}_t^{-\epsilon}$ (see Yun, 1996).

Public sector The public sector consists of the fiscal authority and the central bank. The fiscal authority issues risk-less one-period bonds B_t at the price $1/R_t$ paying B_t units of currency in period $t+1$, receives a transfer τ_t^c from the central bank, and collects lump-sum taxes τ_t from households,

$$B_{t-1} = B_t/R_t + P_t\tau_t + P_t\tau_t^c. \tag{8}$$

Government expenditures on goods are normalized to zero, such that the services on outstanding debt are the only flow that needs to be financed, either by issuing new debt or by raising taxes. We assume that the fiscal authority sets taxes according to a simple feedback rule. In particular, we specify the level of taxes as a fraction of debt service costs $\frac{i_t}{1+i_t}B_{t-1}$ net of central bank transfers,

$$P_t\tau_t = \kappa_t \frac{i_t B_{t-1}}{1+i_t} - P_t\tau_t^c, \tag{9}$$

where $i_t = R_t - 1$. We further assume that κ_t satisfies $\kappa_t = \kappa\exp(\varepsilon_{\kappa t})$, where $\kappa \in (0,1]$ and $\varepsilon_{\kappa t}$ is an i.i.d. random variable with mean zero. These fiscal policy shocks are solely introduced for the purpose to isolate the macroeconomic effects of changes in public finance. The average feedback κ thus specifies how actively the government seeks to collect funds from the private sector to finance its debt burden. Using (8) and (9), the evolution of government debt can be summarized by

$$B_t = (1 + (1-\kappa_t)i_t)\, B_{t-1}. \tag{10}$$

Note that $\kappa_t = 1$ is the case of a budget that balances in every instant, such that nominal government bonds are constant over time: $B_{t-1} = B_t$. Thus, κ_t measures the share of government expenditures that are financed through taxation as opposed to debt. It is crucial to note that this specification of tax policy, in particular the property $\kappa > 0$, ensures that government debt grows on average with a rate which is strictly smaller than the interest rate $E_0 B_t/B_{t-1} < R_t$, implying $\lim_{t\to\infty} B_t \Pi_{v=1}^t R_v^{-1} = 0$. Hence, for any non-explosive sequence of real money and inflation, this policy guarantees government solvency $\lim_{s\to\infty}(b_{t+s}R_{t+s}^{-1} + m_{t+s})\Pi_{v=1}^s \pi_{t+v}/R_{t-1+v} = 0$. Further, $\kappa > 0$ implies that the fiscal policy regimes considered in our analysis do not exhibit an exogenous primary surplus, which typically characterizes a so-called non-Ricardian policy regime (see Benhabib et al., 2001).

The central bank transfers seigniorage to the fiscal authority, $P_t\tau_t^c = M_t - M_{t-1}$, and controls the nominal interest rate R_t on government bonds. We assume that the central bank sets R_t in a simple way contingent on current inflation, subject to a monetary policy shock,

$$R_t = R(\pi_t, \varepsilon_{rt}) = \mathcal{R}\pi_t^{\rho_\pi} \exp(\varepsilon_{rt}), \qquad \rho_\pi > 0,\ R_t \geq 1, \tag{11}$$

where ε_{rt} is assumed to be i.i.d. with mean zero. We further assume the support of all shocks to be small enough such that there exists a constant $\mathcal{R}$ that the central bank can choose to ensure that $R_t \geq 1$ holds.

Rational expectations equilibrium All households are identically endowed and behave symmetrically in equilibrium, such that we drop the individual index j to denote aggregate variables. The stochastic discount factor in the firms' maximization problem $q_{t,t+s}$ is assumed to satisfy $q_{t,t+s} = \beta^s \left(\lambda_{t+s}P_{t+s}^{-1}/\lambda_t P_t^{-1}\right)$. This can be rationalized by the (implicit) assumption that the manager of a firm is some randomly chosen household. Since money enters the transaction cost function in a separable way, it is irrelevant for for the analysis of the equilibrium behavior of the remaining variables and will therefore be neglected in what follows. We will focus on the properties of (locally) stable equilibria, such that sequences of real money balances that are consistent with these equilibria will never allow government insolvency. Using that transaction costs are private (implying $y_t = c_t$), a rational expectations equilibrium can be defined as follows.

Definition 1 *A rational expectations equilibrium is a set of sequences* $\{y_t,\ l_t,\ \pi_t,\ P_t^*,\ P_t,\ \widetilde{P}_t,\ mc_t,\ w_t,\ b_t,\ R_t\}_{t=0}^\infty$ *satisfying the firms' first order conditions* $mc_t = w_t$, *(7) with* $\widetilde{P}_{it} = \widetilde{P}_t$, *and* $P_t^{1-\epsilon} = \phi\left(\overline{\pi}P_{t-1}\right)^{1-\epsilon} + (1-\phi)\widetilde{P}_t^{1-\epsilon}$, *the households' first order conditions* $\mu'(l_t)\upsilon'(y_t)^{-1}\left(1 + h_c(y_t)\right) = \xi_t^{-1}w_t$, *and*

$$\beta E_t \left[\left[\upsilon'(y_{t+1})\left(1 + h_{c,t+1}\right)\right]^{-1} \pi_{t+1}^{-1}\left(1 - h_b(b_t\pi_{t+1}^{-1})\right)\right] R_t = \left[\upsilon'(y_t)\left(1 + h_{c,t}\right)\right]^{-1}, \tag{12}$$

and $\pi_t = P_t/P_{t-1}$, *the aggregate resource constraint* $y_t = (P_t^*/P_t)^\epsilon l_t$, *where* $(P_t^*)^{-\epsilon} = \phi\left(\overline{\pi}P_{t-1}^*\right)^{-\epsilon} + (1-\phi)\widetilde{P}_t^{-\epsilon}$, *and the transversality condition* $\lim_{t\to\infty}\beta^t E_0 \frac{\upsilon'(c_t)}{1+h_c(c_t)} b_t = 0$, *for fiscal and monetary policy satisfying* $b_t = (1 + (1-\kappa_t)i_t)b_{t-1}\pi_t^{-1}$ *and (11), and given sequences of* $\{\varepsilon_{\kappa t}\}_{t=0}^\infty$, $\{\varepsilon_{rt}\}_{t=0}^\infty$, *and* $\{\xi_t\}_{t=0}^\infty$ *and initial values* $P_{-1} > 0$, $P_{-1}^* > 0$, *and* $b_{-1} \equiv B_{-1}/P_{-1} > 0$.

As implied by definition 1, the equilibrium sequence of public debt cannot separately be determined from the equilibrium sequences of the other variables, which leads to the failure of Ricardian equivalence. This property is due to the assumption that the level of debt affects its total rate of return $h_b < 0$. Below, we will compare the results coming from this model with one in which government debt is neutral. Debt neutrality in our model is implied by the limiting case $h_b \to 0$. However, a cautionary note is in order here. In the model with $h_b = 0$ the equilibrium would look very different. In this case, different sequences of public debt would leave the behavior of households and firms unchanged (another way of stating that Ricardian equivalence would hold). Thus, when $h_b = 0$ a rational expectations equilibrium would be defined only in terms of $\{y_t,\ l_t,\ \pi_t,\ P_t^*,\ P_t,\ \widetilde{P}_t,$

$mc_t,\ w_t,\ R_t\}_{t=0}^{\infty}$, but excluding public debt. These sequences have to satisfy the firms' and the households' first order conditions and the market clearing conditions, but are independent of the fiscal policy regime (10). Hence, if $h_b = 0$, *any* sequence of real public debt would be consistent with a particular rational expectations equilibrium.

Remark 1 *A rational expectations equilibrium of a model version with $h_b = 0$ is consistent with infinitely many sequences for real government debt $\{b_t\}_{t=0}^{\infty}$.*

Thus, in the limiting case of debt neutrality ($h_b = 0$), debt does not enter the equilibrium definition, since it does not affect the other equilibrium variables. Debt would thus not be part of a fundamental equilibrium, in the sense of an equilibrium solution relying on the minimal set of state variables. Thus, if $h_b = 0$ explosive debt sequences would be consistent with a stable equilibrium (defined exclusively of debt), as long as fiscal solvency is not touched. When we turn to the analysis of locally stable equilibria, this difference becomes crucial: the debt sequence then has to be stationary in a fundamental equilibrium only in the model with $h_b < 0$. In this case, equilibrium sequences of inflation and output are constrained accordingly. In contrast, there is no corresponding restriction on the evolution of debt that affects the equilibrium sequences when debt is neutral $h_b = 0$.

On a side note, it may be observed that a similar type of non-neutrality of public debt as in the model with $h_b < 0$ could in principle be derived in a model without transactions services from bonds (i.e. with $h_b = 0$), but in which the fiscal authority defaults on its debt obligations with a positive probability. In this case, if the level of real outstanding bonds raises the probability of default, an interest rate premium on bond emissions that take place at high levels of indebtedness would be expected to result. Such a default risk premium would provide a similar link between debt and its rate of return as in our model.[8]

Steady state A deterministic steady state ($\varepsilon_{rt} = \varepsilon_{\kappa t} = 0$ and $\xi_t = \overline{\xi}$) of the model is characterized by constant values for output, inflation, and government bonds. Due to the assumption that transaction costs are private and separable, the first order conditions on consumption and labor, and the aggregate resource constraint uniquely determine steady state output by

$$\mu'(\overline{y})\upsilon'(\overline{y})^{-1}(1 + h_c(\overline{y})) = (\vartheta - 1)(\epsilon - 1)/(\vartheta\epsilon), \tag{13}$$

where bars indicate steady state values of endogenous variables. A steady state further requires $b_t = \overline{b}$ and $\pi_t = \overline{\pi}$ (see 10 and 12). The fiscal and monetary policy specification

[8] As a simple example, assume that a fraction $0 < \delta < 1$ per unit of debt will not be repaid in every period, and that $\delta(B_{t-1}/P_t)$ holds with $\delta' > 0$. Then the typical household's flow budget constraint displays a return on bonds $(1 - \delta(B_{t-1}/P_t))\, B_{jt-1}$, which implies a first order condition $\lambda_t = R_t \beta E_t(\lambda_{t+1} \frac{1-\delta(b_t/\pi_{t+1})}{\pi_{t+1}})$. The public sector budget constraint then reads $P_t g_t + M_{t-1} + (1-\delta)\, B_{t-1} = P_t \tau_t + B_t/R_t + M_t$. The fiscal authority rebates the savings from partial debt repayment in a lump-sum way, such that taxes satisfy $P_t \tau_t = \kappa_t (i_t \frac{B_{t-1}}{1+i_t}) - (P_t \tau_t^c + \delta B_{t-1})$. As a consequence, public debt again evolves according to (10). A rational expectations equilibrium is then characterized as in definition 1, except that (12) is replaced by $\beta E_t[(\upsilon'(y_{t+1})\,(1 + h_{c,t+1}))^{-1} \pi_{t+1}^{-1} (1 - \delta(b_t/\pi_{t+1}))] R_t = [\upsilon'(y_t)\,(1 + h_{c,t})]^{-1}$, which implies the same local behavior as (12).

leads to the restriction

$$\overline{\pi} = 1 + (1-\kappa)(\mathcal{R}\overline{\pi}^{\rho_\pi} - 1), \tag{14}$$

on the steady state inflation rate. Whether condition (14) has a unique or multiple solutions for the steady state inflation rate depends on both policy parameters. The equilibrium condition for bond holdings (12) can be used to uniquely determine the steady state level of government bonds for a given steady state inflation rate,

$$h_b(\overline{b}/\overline{\pi}) = 1 - \overline{\pi}/\left[\beta\mathcal{R}\overline{\pi}^{\rho_\pi}\right]. \tag{15}$$

The steady state inflation rate and, thus, the steady state level of government bonds, is determined by (14). As policy satisfies $\kappa \in (0,1]$ and $\overline{R} = \mathcal{R}\overline{\pi}^{\rho_\pi} \geq 1$, we know that $G(\overline{\pi}) \equiv (1 + (1-\kappa)(\mathcal{R}\overline{\pi}^{\rho_\pi} - 1)) - \overline{\pi}$ is strictly positive for $\overline{\pi} \to 0$. Hence, $G(\overline{\pi}) = 0$ has a unique solution if $G'(\overline{\pi}) < 0 \Leftrightarrow \rho_\pi < [\left(\mathcal{R}\overline{\pi}^{\rho_\pi - 1}\right)(1-\kappa)]^{-1}$. Using that (15) and $h_b < 0$ imply $\mathcal{R}\overline{\pi}^{\rho_\pi - 1} < 1/\beta$, a sufficient condition for the existence and uniqueness of a steady state inflation rate is

$$\rho_\pi < \beta/(1-\kappa). \tag{16}$$

If (16) is satisfied, the model further exhibits a unique steady state level of government bonds. Appendix 5.1 gives a formal discussion of the properties of the steady state. It may be noted that the existence of a steady state relies on the two effects of inflation on public debt. On the one hand, the real value of nominal debt decreases with inflation. On the other hand, higher inflation induces the central bank to raise the nominal interest rate such that the fiscal authority might issue new debt to finance additional interest rate payments. If $G'(\overline{\pi}) < 0$, then there exists an inflation rate where both effects exactly offset each other, such that real public debt is constant.

3 Results

This section consists of four parts. In the first part, we examine the requirements for stability and uniqueness of local equilibrium sequences. Then, we briefly assess the transmission of monetary and fiscal policy shocks in the second part. In the third part, we show how public debt contributes to forecasts of macroeconomic aggregates and thereby affects the inflation variance, assuming that cost push shocks, which imply a trade-off for monetary policy with respect to the stabilization of inflation and output, are the only source of macroeconomic fluctuations. In the final and main part of this section, we (numerically) examine how the stance of monetary and fiscal policy affect the policy trade-off under cost push shocks, highlighting the role of government debt non-neutrality for macroeconomic stabilization policy.

Throughout the analysis, we restrict our attention to the case where the effect of public debt on its rate of return and thus on consumption growth is small. This facilitates comparisons with corresponding models where debt is neutral. In particular, for the rest of the paper we assume that the steady state elasticity of the transaction costs with regard to public debt is smaller than one, $\Psi = \frac{\overline{h_{bb}}}{1-\overline{h}_b}\frac{\overline{b}}{\overline{\pi}} < 1$ (this condition is sufficient for several analytical results to follow). Log-linearizing the equilibrium conditions given in definition 1 at the steady state leads to the following set of approximate equilibrium conditions in $\widehat{y}_t$, $\widehat{\pi}_t$, $\widehat{b}_t$, and $\widehat{R}_t$:

$$\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \widehat{R}_t + (1-\Psi)\, E_t\widehat{\pi}_{t+1} + \Psi\widehat{b}_t, \qquad \Psi \in (0,1), \tag{17}$$

$$\widehat{\pi}_t = \beta E_t\widehat{\pi}_{t+1} + \omega\widehat{y}_t + \widehat{\varphi}_t, \tag{18}$$

$$\widehat{b}_t = \widehat{b}_{t-1} + \eta\widehat{R}_t - \widehat{\pi}_t - \varepsilon_{\kappa t}, \qquad \eta \in [0,1) \text{ and } \partial\eta/\partial\kappa < 0, \tag{19}$$

$$\widehat{R}_t = \rho_\pi\widehat{\pi}_t + \varepsilon_{rt}, \tag{20}$$

together with an exogenous stochastic process for the cost-push shock assumed to follow $\widehat{\varphi}_t \equiv \chi\widehat{\xi}_t = \rho_c\widehat{\varphi}_{t-1} + \varepsilon_{ct}$, with $\rho_c \in (0,1)$, where ε_{ct} is a white noise innovation, $\sigma \equiv -\frac{\overline{u_{cc}c}}{\overline{u_c}} + \frac{\overline{h_{cc}c}}{1+\overline{h}_c} > 0$, $\omega \equiv \chi(\sigma + \frac{\overline{u_{ll}l}}{\overline{ul}}) > 0$, $\eta \equiv \frac{(1-\kappa)\overline{R}}{1+(1-\kappa)(\overline{R}-1)}$, and $\Psi \equiv \frac{\overline{h_{bb}}}{1-\overline{h}_b}\frac{\overline{b}}{\overline{\pi}}$, given sequences for $\varepsilon_{\kappa t}$, ε_{rt}, and ε_{ct}. Equation (17) specifies the evolution of real aggregate demand as a function of the nominal interest rate and inflation. If debt were neutral, consumption growth would only depend on the real interest rate; crucially, this is different here as real debt $\widehat{b}_t$ enters the demand equation. Equation (19), which stems from the log-linearized flow budget constraint of the composite government sector, the describes the evolution of real debt. Note that the composite parameter $\eta(\kappa)$ is strictly decreasing in κ. Finally, equation (20) gives the log-linearized nominal interest rate feedback rule of the central bank.

3.1 Equilibrium stability and uniqueness

We first look for the conditions under which the rational expectations equilibrium sequences are locally stable and unique, such that the steady state is a saddle point. Note that real debt is a predetermined state variable, which evolves in a history dependent way. Since the evolution of this state variable is affected by the realizations of inflation, the requirement of equilibrium stability imposes a restriction on the equilibrium inflation sequence. Hence, the model under debt non-neutrality appears to behave in a fundamentally different way compared to a model with debt neutrality, where the sequences of inflation and output are not related to the evolution of real debt (see remark 1). Thus, this difference does not rely on a quantitatively strong effect of debt on output growth, which is measured by the elasticity $\Psi > 0$. As long as $h_b < 0$, the sequences of inflation and output are history dependent, since past realizations of real debt are relevant in equilibrium (see 19). As a consequence, the conditions for stability and uniqueness of local equilibria differ from the well-known Taylor-principle, which characterizes the version of the model with debt neutrality (see Woodford, 2001b).

Proposition 1 (Local dynamics) *Suppose that there exists a steady state. Then, the model's local approximation (17)-(20) has a unique equilibrium converging to the steady state if and only if* $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$.

Proof. See appendix 5.2.

The condition in proposition 1 departs from the one known from the determinacy condition for the corresponding model with debt neutrality (e.g. Clarida et al., 1999), where the interest rate policy would have to be active ($\rho_\pi > 1$) to ensure local equilibrium uniqueness. Here, stability requires that the central bank does not raise the nominal interest rate too much in response to inflation, and the precise meaning of what is too much depends on the fiscal parameter κ. Essentially, both policy authorities must ensure that the partial effect of inflation on future debt, $\partial\widehat{b}_t/\partial\widehat{\pi}_t = \eta\rho_\pi - 1$ (from equations 19 and 20) is negative, which in rearranged form gives the condition stated in the proposition.

To see why, recall that debt is positively related to aggregate demand (17). Hence, if debt were temporarily higher than in the steady state, this would tend to raise output and inflation. To bring debt back to its steady state, higher inflation must reduce the real value of outstanding bonds. But policy interferes with this stabilization mechanism. If the central bank raises the nominal interest rate in response to higher inflation, this increases the burden of public debt service costs on the fiscal budget, since the government would have to finance additional interest payments on existing debt. If $\kappa < 1$, not all of this additional expenditure is financed through taxation, but a fraction is covered by issuance of new debt. Since these add to existing debt holdings, they trigger the "rate of return effect" of real debt: higher real debt reduces the marginal return from transaction services such that savings are ceteris paribus reduced and consumption is shifted to the present. Thus, an aggressive interest rate policy (high ρ_π) can induce the economy to evolve on a divergent path, unless $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}} \Leftrightarrow \partial\widehat{b}_t/\partial\widehat{\pi}_t < 0$.

It should further be noted that indeterminacy cannot occur in this model if this condition is fulfilled. Arbitrary expectations of rising inflation then imply that the future real value of debt is reduced, which tends to lower demand and, therefore, prices, such that inflation expectations cannot be self-fulfilling. Some notable implications of this result are summarized in the following corollary.[9]

Corollary 1 *(1) The model exhibits a unique and saddle point stable steady state if monetary and fiscal policy satisfy (16). (2) This is the case if fiscal policy runs a balanced budget in the sense* $\kappa = 1 \Rightarrow B_t = B_{t-1}$, *or if (3) monetary policy pegs the nominal interest rate at an arbitrary constant* $R_t = \overline{R} > 1$.

[9] This property closely relates to the main result in Canzoneri and Diba (2005), namely that the price level can be determined under an interest rate peg in a flexible price model if government bonds provide transaction services.

Part (1) states that the condition for uniqueness of the steady state, $\rho_\pi < \beta/(1-\kappa)$ from (16), is sufficient for the saddle path stability condition $\rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$ from proposition 1 to hold. Thus, there is local equilibrium determinacy if the steady state is unique. Parts (2) and (3) of the corollary state that both a balanced budget fiscal rule and a nominal interest rate peg are sufficient for a unique and saddle point stable steady state to prevail. In these cases stability is ensured by either monetary or fiscal policy alone. If the interest rate is pegged, an inflation increase unambiguously reduces the real interest rate, such that debt emissions by the government decline, and the resulting demand slump stabilizes inflation. If the budget is balanced in nominal terms, it can be seen from (19) that $\eta = 0$ and thus the evolution of real government debt is not directly affected by the nominal interest rate. Hence, in this case higher than average inflation automatically reduces the real value of debt, increases (by $h_{bb} < 0$) the return from transaction services, and induces postponement of consumption to the future such that inflation is kept down.

3.2 Transmission of policy shocks

In this section, we briefly discuss the impact responses of core variables to fiscal and monetary policy shocks, $\varepsilon_{\kappa t}$ and ε_{rt}, as well as to cost-push shocks, $\widehat{\varphi}_t$. The former shocks are studied to assess the impact of policy measures on macroeconomic aggregates, while the latter are analyzed to pave the ground for the discussion of policy trade-offs and the performance of differently parameterized rules in the next section. To this end, we derive the state space representation of the log-linearly approximated model (17) to (20) in the endogenous variables $\widehat{b}_t$, $\widehat{\pi}_t$, and $\widehat{y}_t$, given the state variables $\widehat{b}_{t-1}$, ε_{rt}, $\varepsilon_{\kappa t}$, $\widehat{\varphi}_t$. Throughout this section it is assumed that condition (16) is satisfied, such that there is a unique and saddle point stable steady state and that the model's fundamental solution is the unique solution of the model. Let $\delta_{yb} \equiv \partial\widehat{y}_t/\partial\widehat{b}_{t-1}$, $\delta_{yr} \equiv \partial\widehat{y}_t/\partial\widehat{\varepsilon}_{rt}$, $\delta_{y\kappa} \equiv \partial\widehat{y}_t/\partial\widehat{\varepsilon}_{\kappa t}$, and $\delta_{yc} \equiv \partial\widehat{y}_t/\partial\widehat{\varphi}_t$ be the solution coefficients describing the impact of the state variables on output (analogous definitions apply for the solution coefficients with respect to inflation and bonds).

Proposition 2 (Impulse responses) *Suppose (16) is satisfied. Then, the unique solution of the model (17)-(20) is characterized by*

1. $\partial\widehat{b}_t/\partial\varepsilon_{\kappa t} = \delta_{b\kappa} < 0$, $\partial\widehat{\pi}_t/\partial\varepsilon_{\kappa t} = \delta_{\pi\kappa} < 0$, and if $\rho_\pi < \widetilde{\rho_\pi} : \partial\widehat{y}_t/\partial\varepsilon_{\kappa t} = \delta_{y\kappa} < 0$,

2. $\partial\widehat{b}_t/\partial\varepsilon_{rt} = \delta_{br} > 0$, and if $\eta < \widetilde{\eta} : \partial\widehat{\pi}_t/\partial\varepsilon_{rt} = \delta_{\pi r} < 0$ and $\partial\widehat{y}_t/\partial\varepsilon_{rt} = \delta_{yr} < 0$,

3. $\partial\widehat{b}_t/\partial\widehat{\varphi}_t = \delta_{bc} < 0$, $\partial\widehat{\pi}_t/\partial\widehat{\varphi}_t = \delta_{\pi c} > 0$ and $\partial\widehat{y}_t/\partial\widehat{\varphi}_t = \delta_{yc} < 0$,

and $\partial\widehat{b}_t/\partial\widehat{b}_{t-1} = \delta_b \in (0,1)$, $\partial\widehat{\pi}_t/\partial\widehat{b}_{t-1} = \delta_{\pi b} > 0$, *and* $\partial\widehat{y}_t/\partial\widehat{b}_{t-1} = \delta_{yb} > 0$, *where* $\widetilde{\rho_\pi} \equiv (1-\Psi)/\beta > 0$ *and* $\widetilde{\eta} \equiv \left(\Psi + \rho_\pi + (1-\delta_b)\left[\frac{\sigma}{\omega}(1+\beta-\beta\delta_b) + 1 - \Psi\right]\right)^{-1} > 0$.

Proof. See appendix 5.3.

In response to a temporary rise in taxes ($\varepsilon_{\kappa t} > 0$), public debt and, by the rate of return effect, also inflation declines (see part 1.). As the central bank reacts to the latter by lowering the nominal interest rate, the output response crucially depends on monetary policy. If the central bank is not too aggressive, for which $\rho_\pi < \widetilde{\rho_\pi}$ is sufficient, the expansionary impact of the decline in the nominal interest rate is dominated by the contractionary effect of lower debt, such that output decreases.

A contractionary monetary policy shock, i.e. a positive innovation to the nominal interest rate rule ($\varepsilon_{rt} > 0$), raises the interest rate burden on outstanding bonds, which leads to a future rise in debt. The response of output and inflation is generally ambiguous (see part 2.). The reason is that if a sufficiently large portion of government expenditures is tax financed ($\eta < \widetilde{\eta}$), then inflation and output decline in response to a monetary contraction. Otherwise, with heavy deficit finance the implied large rise in public debt can cause an increase in inflation and output due the positive intertemporal substitution effect of debt on private consumption. Finally, part 3. of the proposition states that a cost-push shock leads to a decline in output and a rise in inflation, while the latter causes a reduction in real public debt.

Summing up, for monetary and fiscal policy feedback rules which do not feature extreme parameter values, in the sense that ρ_π is not too high and κ is not too low, the model's predictions about impact responses to interest rate and tax shocks qualitatively accord to the evidence, based on vector autoregressions, provided by Christiano et al. (1999) for federal funds rate shocks and by Mountford and Uhlig (2002) for tax cut shocks.

3.3 Public debt and inflation expectations under flexible prices

We now turn to the impact of public debt non-neutrality on the cyclical behavior of the model under cost-push shocks. The focus here is on the modification that debt non-neutrality and fiscal policy brings about in this respect. Thus, for the rest of the analysis, the policy innovations are set equal to zero, $\varepsilon_{rt} = \varepsilon_{\kappa t} = 0$, such that the fiscal policy stance is constant $\kappa_t = \kappa$ and the interest rate feedback rule reduces to $\widehat{R}_t = \rho_\pi \widehat{\pi}_t$. When debt is non-neutral, the fiscal policy stance is relevant for the cyclical properties of macroeconomic variables. Thus, the interaction between fiscal and monetary policies affects the variances of output and inflation in our model, whereas only monetary policy is responsible for macroeconomic fluctuations when debt is neutral. The distinguishing feature of debt non-neutrality is that the evolution of public debt has to follow a stationary path, which imposes a restriction on feasible equilibrium sequences of inflation and output. This raises the question how macroeconomic fluctuations are altered due to the relevance of public debt.

To disclose the role of public debt on macroeconomic fluctuations, we apply the flexible price version of the model. Thus, we assume that the probability of firms not receiving a price signal equals zero $\phi = 0$, such that the equilibrium conditions for the log-linearized version of the model are given by (17), (19), (20), and the static condition $\widehat{y}_t = -(\sigma + \overline{u_{ll}}\bar{l}/\overline{u_l})^{-1}\widehat{\xi}_t$. Public debt does, evidently, not affect the equilibrium behavior of output regardless whether debt is neutral or not. Comparing both versions, it turns out that inflation is less volatile under debt non-neutrality when the cost-push shock is not too strongly autocorrelated. The following proposition summarizes the particular condition for this result for the example of an interest rate peg, which facilitates the isolation of the debt effect on the inflation variance.

Proposition 3 (Variances under $\phi = 0$) *Suppose that prices are flexible and that the central bank pegs the nominal interest rate, $\rho_\pi = 0$. Then, the inflation variance is smaller under debt non-neutrality ($\Psi > 0$) than under debt neutrality ($\Psi = 0$) if $\rho_c < (1 - \Psi + \sqrt{2})^{-1}$.*

Proof. See appendix 5.4.

The reason for this result is that a stable rational expectations equilibrium path must – under debt non-neutrality – be associated with a stationary sequence of real public debt, which requires a sequence of inflation that induces the debt sequence to be mean reverting. To see this, consider, first, a cost increasing shock for the case where debt is neutral. This leads to an immediate decline in output, which has to be accompanied by a rise in the real rate of return from government bonds and, thus, by a decline in expected future inflation. Under debt non-neutrality this cannot be an equilibrium outcome, as a decline in inflation would tend to raise real public debt that, on the other hand, would tend to lower the total real marginal return from bond holdings and would therefore even amplify the increase in expected future inflation (see 17). Thus, stability requires a positive and mild inflation response, such that real public debt can decline in a way that is consistent with the expected rise in future inflation. Once the cost-push shock has died out, inflation has, by (17), to be lower than in the steady state, inducing the real value of public debt to converge back to its steady state value. Thus, debt non-neutrality induces a history dependence by which inflation depends on the real value of beginning-of-period debt. This is responsible for a reduction of the inflation variance if the autocorrelation of cost-push shocks is sufficiently small, e.g. $\rho_c \leq 0.4$ $(< (1 - \Psi + \sqrt{2})^{-1})$. Otherwise, the recovery of real debt and, thus, of inflation are extended in a way that causes its variances to be higher than in the case where debt is neutral.

3.4 Policy interactions and macroeconomic stabilization

We now return to the sticky-price case and investigate the role of government debt for macroeconomic fluctuations by means of calculating variances for empirically plausible parameter values. The impact of different (fiscal and monetary) policy parameters on macroeconomic volatility is assessed by comparing the variances of output and inflation relative to the variance of their source, i.e., the cost-push shock process φ_t. Using the

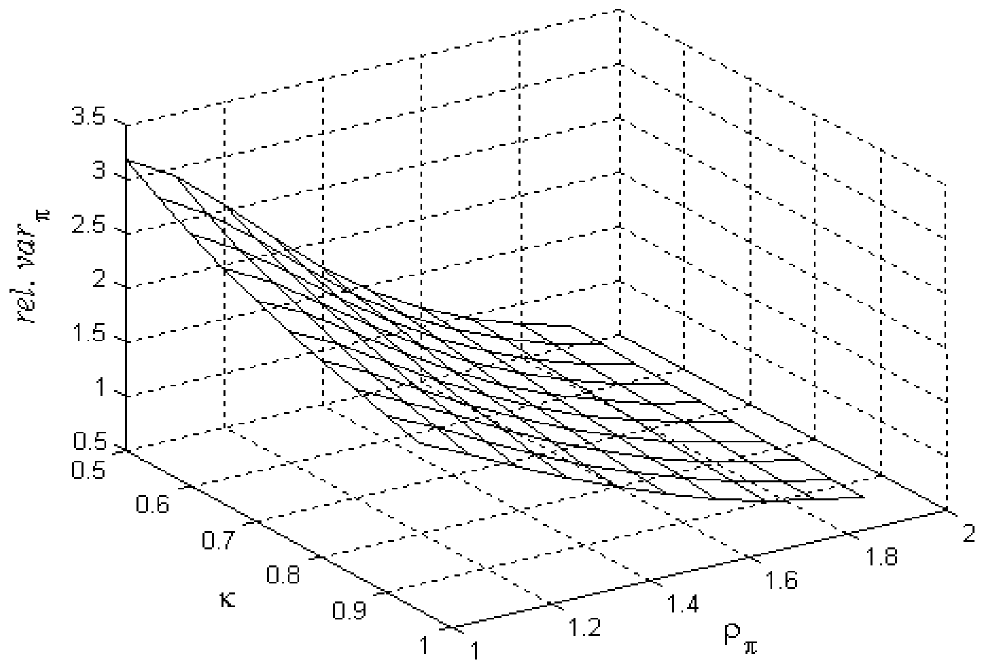

Figure 1: Relative inflation variance

coefficients of the model's solution discussed in proposition 2 (see the proof to proposition 2 for the exact definitions), the variances of the model's endogenous variables output (var_y), inflation (var_π) and government debt (var_b) can be expressed in relation to the exogenous variance of the cost-push shock, $var_\varphi = (1-\rho_c^2)^{-1} var_{\varepsilon_c}$, where var_{ε_c} is the variance of the white noise innovation to φ_t. The resulting expressions are $var_b/var_\varphi = (1-\delta_b^2)^{-1}\delta_{bc}^2$, $var_\pi/var_\varphi = \left(\delta_{bc}^2\delta_{\pi b}^2\right) / \left(1-\delta_b^2\right) + \delta_{\pi c}^2$, and $var_y/var_\varphi = \left(\delta_{bc}^2\delta_{yb}^2\right) / \left(1-\delta_b^2\right) + \delta_{yc}^2$.

For convenience, we present the results in graphical form applying a set of deep parameters in accordance with values often found in the literature. In particular, we set preference parameters equal to $\sigma = \vartheta = 2$ and $\beta = 0.99$, the average (quarterly) gross nominal interest rate to $\overline{R} = 1.01$, the autocorrelation of cost-push shocks to $\rho_c = 0.9$, and the fraction of non-optimally price adjusting firms to $\phi = 0.8$, where the latter value accords to estimates in Galí and Gertler (1999). We set the transaction cost elasticity equal to $\Psi = 0.05$ for the benchmark specification. To assess the impact of this value on the variances of inflation and output, a sensitivity analysis with respect to its influence is presented in the last part of this section. These parameter values are then applied to compute the solution coefficients δ_b, δ_{bc}, $\delta_{\pi b}$, $\delta_{\pi c}$, δ_{yb}, and δ_{yc} for varying policy parameters ρ_π and κ.

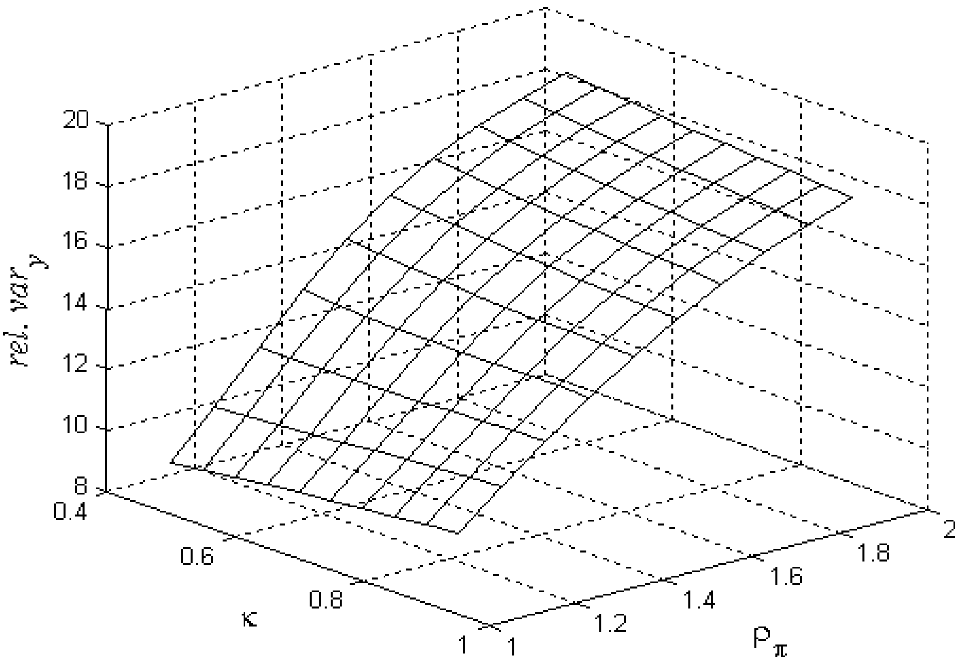

Figure 2: Relative output variance

Figure 1 displays the relative variance of inflation var_π/var_φ, and figure 2 the relative variance of output var_y/var_φ, each for various values of the fiscal feedback parameter κ and the inflation elasticity ρ_π of the nominal interest rate on government bonds. Evidently, interest rate policy faces the usual trade-off when the model is driven by cost-push shocks, in that higher values of ρ_π lower the variance of inflation, but increase the variance of output. What is new here is the influence of the fiscal policy parameter κ: a higher value of κ, i.e. a higher share of tax financing, reduces changes in public debt and is generally associated with a lower inflation variance, while it has an ambiguous (but generally small) influence on the output variance. The lowest inflation volatility is achieved with a balanced budget policy.

Before turning to explanations, it is useful to compare the performance of different stabilization policies to the case where debt is neutral. Therefore, figure 3 shows the relative output and inflation variances for selected monetary and varying fiscal policy parameters in comparison to the latter case, which is labelled DN (for *d*ebt *n*eutral).[10] Recall that the DN model can be summarized by (20), $\sigma\widehat{y}_t = \sigma E_t\widehat{y}_{t+1} - \rho_\pi\widehat{R}_t + E_t\widehat{\pi}_{t+1}$,

[10] In this case, $\Psi = 0$, steady state inflation is determined by $\overline{\pi} = \overline{R}\beta$ and is, thus, independent of fiscal policy.

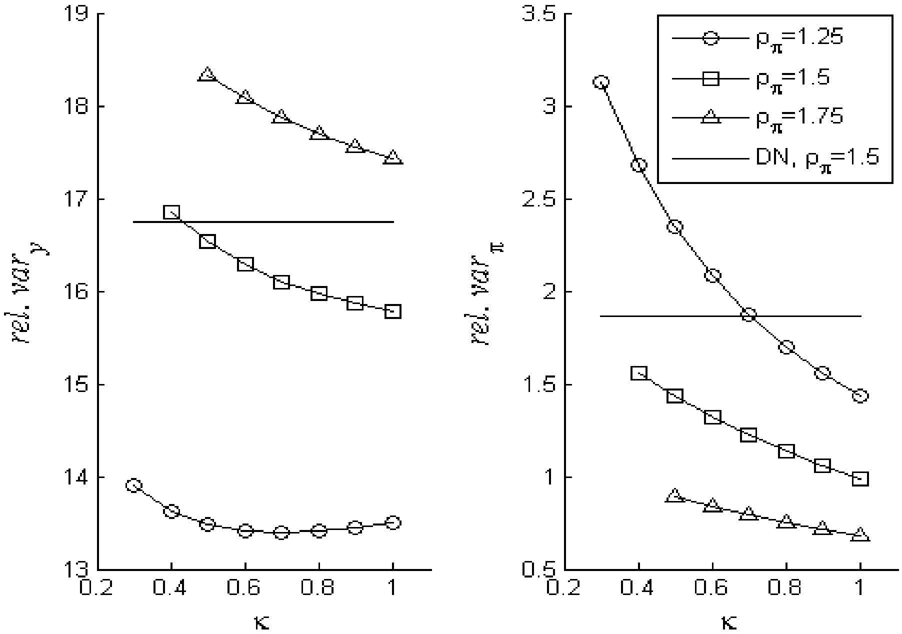

Figure 3: Relative inflation and output variances, comparison to debt neutral (DN) case.

and $\widehat{\pi}_t = \beta E_t \widehat{\pi}_{t+1} + \omega \widehat{y}_t + \widehat{\varphi}_t$ (which accords to the prototype New Keynesian model). Given that public debt is irrelevant in this model, the fundamental solution exhibits no endogenous state variable and is characterized by the following coefficients on cost-push shocks: $\widetilde{\delta}_{yc} = -\frac{\rho_\pi - \rho_c}{\omega(\rho_\pi - \rho_c) + (1-\beta\rho_c)(1-\rho_c)\sigma}$ and $\widetilde{\delta}_{\pi c} = \frac{\sigma(1-\rho_c)}{\omega(\rho_\pi - \rho_c) + (1-\beta\rho_c)(1-\rho_c)\sigma}$. The relative output and inflation variances, which are given by $var_y/var_\varphi = \widetilde{\delta}_{yc}^2$ and $var_\pi/var_\varphi = \widetilde{\delta}_{\pi c}^2$, are displayed for $\rho_\pi = 1.5$ by the solid horizontal lines in figure 3.

Figure 3 further displays relative variances of the model with non-neutral debt for three different values of the monetary policy feedback parameter, $\rho_\pi \in \{1.25,\ 1.5,\ 1.75\}$, which ensure a comovement of the nominal and the real interest rate (only points where the parameter combination entails equilibrium stability are shown). Not surprisingly, higher ρ_π values reduce the inflation variance and raise the output variance; this effect is already well known from the DN case. What is new here is seen by comparing the unmarked solid lines in figure 3 with the lines marked with squares. These show that for a given monetary policy stance – in this case for the example value $\rho_\pi = 1.5$ – the variances of both output and inflation are lower for $\Psi > 0$ (lines marked with squares) than in the DN case (solid line without markers) if the share of tax financing κ is sufficiently high. Thus, the relevance of government debt for demand determination appears to stabilize inflation and output fluctuations when tax policy contributes to a relatively smooth evolution of real debt (through a high κ value). The reason is that if a cost-push shock hits the economy, output declines while inflation rises. As has been shown above, the inflation increase reduces the real value of public debt (despite the positive partial effect from a higher real interest rate). The debt reduction exerts – via a higher marginal rate of return – a negative impact on consumption, which tends to exacerbate the output contraction and to mitigate the rise in inflation caused by cost-push shocks. In equilibrium, the dampening effect on (future) inflation is strong enough to limit the real interest rate increase so much that, in the end, the output variance can be even lower than in the DN case.

This mechanism corresponds to the one outlined for the flexible price case in proposition 3 for an interest rate peg in the presence of price flexibility. As there, the central point here is that if government debt is relevant for the determination of the equilibrium values of inflation and output, the equilibrium response of inflation is constrained by the requirement that real debt must return to its steady state value subsequent to a shock. With an active monetary policy, this implies that during the adjustment process future real rates of interest must be lower than in steady state, which – with a large enough value for ρ_π – also reduces the impact of the shock on consumption and thus can mitigate output volatility. Thus, the history dependence of the equilibrium sequences introduced by fiscal policy can smooth fluctuations, which in the sticky-price case discussed here also holds for high shock autocorrelations. This property is related to the history dependence of optimal monetary policy in the debt neutral case. There, an optimal commitment policy is known to be backward-looking and thereby steers private sector expectations in a favorable way (see Woodford, 2003).

Figure 3 further shows that the inflation variance is always declining in the tax financing share κ, while the output variance is ambiguously linked to κ for a lower inflation elasticity of the interest rate rule, $\rho_\pi = 1.25$. In fact, the inflation variance reaches a minimum in the balanced budget case $\kappa = 1$. The reason is that with a nominally balanced budget the negative influence of inflation on the real value of debt is strongest, and the mechanism described above is maximal. The effects on the output variance are ambiguous, since debt reduction on the one hand reduces output partially, but the resulting inflation decrease makes room for lower real interest rates. Given an aggressive monetary policy (high ρ_π), however, there is no trade-off involved in fiscal policy: both the output and inflation variance decrease in κ and are minimized by a balanced budget policy ($\kappa = 1$).[11]

Finally, figure 4 shows the same numerical experiment for different values of the coefficient Ψ which parameterizes the aggregate demand effect of public debt, assuming a monetary policy coefficient of $\rho_\pi = 1.5$. As Figure 4 reveals, the attenuation of the inflation variance attributable to the non-neutrality of debt is stronger when the rate of return effect of debt is quantitatively more pronounced, i.e. with higher Ψ. The effect on the output variance is ambiguous. When the share of tax financing is small, the output

[11] It should noted that the latter fiscal policy regime further minimizes the average distortion from the nominal rigidity as it implies the aggregate price level to be constant in the steady state, $\overline{\pi} = 1$.

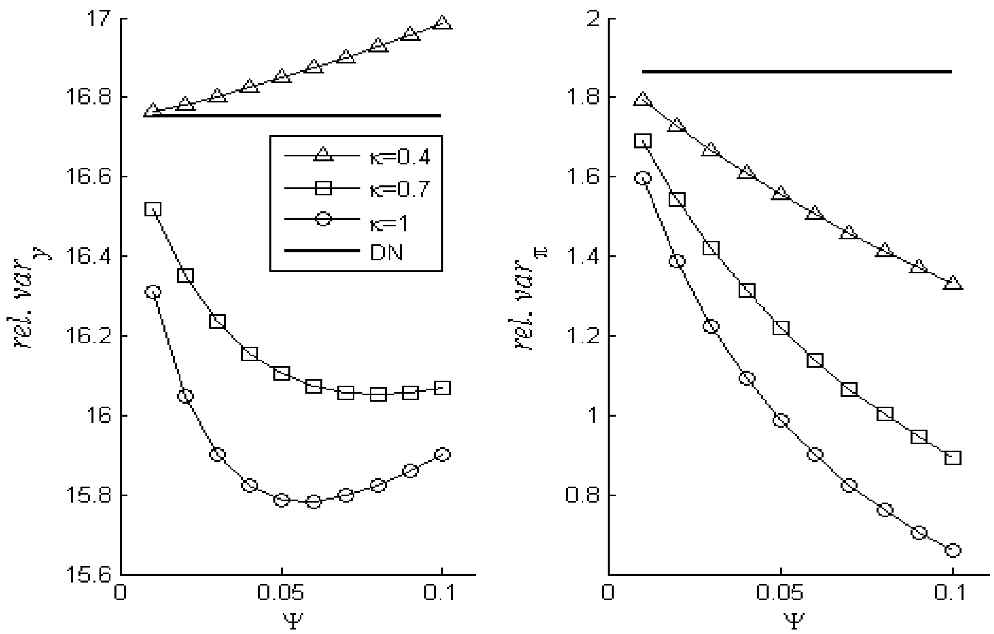

Figure 4: Influence of Ψ on relative output and inflation variances ($\rho_\pi = 1.5$).

variance can even rise with the elasticity as demonstrated for $\kappa = 0.4$. Provided that the government finances a sufficiently large share of its outlays through taxation, the output and inflation variances are lower than in the corresponding debt neutral case.

4 Conclusion

This paper has explored the consequences of public debt non-neutrality for the short-run dynamics of a sticky-price business cycle model, and examined the interaction of fiscal and monetary policy. Government debt matters for aggregate demand determination through assumptions that imply the (negative) dependence of the rate of return on government bonds on the real value of their outstanding stock. This is the case when government bonds yield transactions services. In this case, a rise in public debt leads to a decline in the total rate of return, exerting an expansionary intertemporal substitution effect on (consumption) demand, thus implying a tendency for rising inflation. There is fiscal-monetary policy interaction, in that the central bank's interest rate reaction to changes in inflation influences the amount of payments on existing debt that the government has to finance. The composition of government finance among taxation and debt issuance in turn feeds back on the equilibrium values of output and inflation.

It is shown that even small effects of public debt on consumption growth can lead to results which substantially depart from those known for comparable business cycle models with neutral debt. Non-neutrality of debt implies that the sequence of real bonds must be stationary in a locally stable equilibrium, which constrains the admissible equilibrium solutions of inflation. As a consequence, the central bank's reaction to inflation must not be too large if the steady state is to be unique and saddle point stable, or the tax financing share used by the government may not be too low. Otherwise, if the expansionary impact of higher debt were associated with aggressive interest rate increases, the resulting surge in debt service costs would require the government to issue more debt, which might cause equilibrium sequences to become divergent. However, a balanced budget fiscal policy that keeps the nominal stock of bonds constant inevitably leads to equilibrium uniqueness and stability, since higher inflation then reduces the real value of debt irrespective of the monetary policy stance.

Further, debt non-neutrality is found to have an impact on the performance of public policy in stabilizing the inflation and output volatility arising from cost-push shocks. The well known trade-off that these impart on monetary policy, which can only lower inflation variance at the cost of augmenting output variance, is existent here as well. Whether fiscal policy also faces a trade-off depends on the monetary policy stance. For an aggressive anti-inflationary monetary policy regime, inflation and output variances are simultaneously minimized for high shares of tax finance and, in particular, for a balanced budget regime. Inflation and output variances can even be lower than in the debt neutrality case, as the relevance of public debt imposes a restriction on admissible equilibrium values, introducing a history dependence in an otherwise forward looking environment. Thus, provided that public policy is conducted under tight debt constraints, macroeconomic fluctuations might be smaller due to a fiscal policy induced history dependence, which tends to reduce the forecast error variance for macroeconomic aggregates.

5 Appendix

5.1 Properties of the steady state

The properties of the steady state are summarized in the following proposition.

Proposition A1 *Assume that fiscal and monetary policy satisfy (16). Then a steady state of the model described in definition 1 exists and is uniquely determined. It is characterized by i)* $\overline{y} > 0$, $\overline{\pi} \geq 1$, *and* $\overline{b} > 0$; *ii)* $\partial\overline{y}/\partial\kappa = 0, \partial\overline{\pi}/\partial\kappa < 0$, *and* $\partial\overline{b}/\partial\kappa \lesseqgtr 0 \Leftrightarrow \rho_\pi \gtreqless 1 - \Psi$, *where* $\Psi \equiv \frac{\overline{h_{bb}}}{1-\overline{h_b}}\frac{\overline{b}}{\overline{\pi}} > 0$; *and iii)* $\partial\overline{y}/\partial\rho_\pi = 0, \partial\overline{\pi}/\partial\rho_\pi > 0$, *and* $\partial\overline{b}/\partial\rho_\pi \gtreqless 0 \Leftrightarrow \rho_\pi \gtreqless 1 - (\Psi + \Upsilon)$, *where* $\Upsilon \equiv \frac{\overline{\pi}(1+\ln\overline{\pi})}{\partial\overline{\pi}/\partial\rho_\pi} > 0$.

Proof. The steady state condition (13) determines $\overline{y}$ independently of the policy parameters, such that $\partial\overline{y}/\partial\kappa = \partial\overline{y}/\partial\rho_\pi = 0$. Condition (14) implies that $\partial\overline{\pi}/\partial\kappa = (\overline{R} - 1)/G'(\overline{\pi}) < 0$ and $\partial\overline{\pi}/\partial\rho_\pi = -[(1-\kappa)\overline{R}\ln\overline{\pi}]/G'(\overline{\pi}) > 0$, given that (16) ensures $G'(\overline{\pi}) < 0$. Condition (15) can then be used to derive the impact on $\overline{b}$. As $\partial\overline{b}/\partial\kappa = \left(\partial\overline{b}/\partial\overline{\pi}\right)\left(\partial\overline{\pi}/\partial\kappa\right)$ and $\left(\partial\overline{b}/\partial\overline{\pi}\right) = \overline{b}/\overline{\pi} - \overline{\pi}\left(1-\rho_\pi\right)/\left(\overline{h_{bb}}\beta\overline{R}\right) \gtreqless 0 \Leftrightarrow \frac{\overline{h_{bb}}}{1-\overline{h_b}}\frac{\overline{b}}{\overline{\pi}} \gtreqless (1-\rho_\pi)$, we can conclude that $\partial\overline{b}/\partial\kappa \gtreqless 0 \Leftrightarrow \rho_\pi \lesseqgtr 1 - \Psi$. From (15), we obtain $\partial\overline{b}/\partial\rho_\pi = \overline{b}\overline{\pi}^{-1}\left(\partial\overline{\pi}/\partial\rho_\pi\right) + \overline{\pi}\left(\overline{h_{bb}}\beta\overline{R}\right)^{-1}\left[\overline{\pi}\left(1+\ln\overline{\pi}\right) - \left(1-\rho_\pi\right)\left(\partial\overline{\pi}/\partial\rho_\pi\right)\right] \gtreqless 0 \Leftrightarrow \rho_\pi \gtreqless 1 - (\Psi + \Upsilon)$. ■

Output and (equivalently) consumption are not affected by monetary or fiscal policy measures in the steady state. Assuming that (16) is satisfied, steady state inflation unambiguously rises with the reactiveness of monetary policy and declines with a permanent rise in the fiscal policy parameter κ governing the proportion of tax financing. The effects on public debt are not unambiguous. A rise in κ leads to a decline in real public debt if and only if the inflation elasticity of the interest rate rule is sufficiently aggressive, $\rho_\pi > 1 - \Psi$. The latter has a further (direct) impact on public debt via (15), making real public debt increase with ρ_π for $\rho_\pi > 1 - (\Psi + \Upsilon)$. It should be noted that the results summarized in proposition A1 also apply for version B, where $\rho_\pi < \frac{\beta(1-\overline{\delta})}{1-\kappa}$ replaces condition (16) and the composite parameter Ψ is defined as $\Psi \equiv \frac{\overline{\delta'}}{1-\overline{\delta}}\frac{\overline{b}}{\overline{\pi}} > 0$.

Summing up, while the fiscal authority can reduce nominal debt by raising the share of tax financing (see 10), its influence on the real value of outstanding debt crucially relies on the reaction of inflation and thus on the stance of monetary policy. Note that in the balanced budget case $\kappa = 1$, from (14) steady state inflation is $\overline{\pi} = 1$.

5.2 Proof of proposition 1

The deterministic version of the model (17)-(20) can be summarized as

$$\begin{pmatrix} \widehat{y}_{t+1} \\ \widehat{\pi}_{t+1} \\ \widehat{b}_t \end{pmatrix} = \begin{pmatrix} -\left(\sigma\beta\right)^{-1}\omega\left(\Psi - 1\right) + 1 & \Xi & -\Psi/\sigma \\ -\frac{1}{\beta}\omega & \frac{1}{\beta} & 0 \\ 0 & \eta\rho_\pi - 1 & 1 \end{pmatrix} \begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix} = A \begin{pmatrix} \widehat{y}_t \\ \widehat{\pi}_t \\ \widehat{b}_{t-1} \end{pmatrix},$$

where $\Xi \equiv \frac{1}{\sigma}\left(\rho_\pi - \Psi\left(\eta\rho_\pi - 1\right) + \left(\Psi - 1\right)/\beta\right)$. Since there is one predetermined state variable $(\widehat{b}_{t-1})$, while the other two variables can jump, a saddle path configuration obtains if the matrix A has exactly one eigenvalue with modulus smaller than one. The characteristic polynomial of A reads $H(X) = X^3 - (\sigma\beta)^{-1}(\sigma+\omega+2\sigma\beta-\Psi\omega)X^2 - (\sigma\beta)^{-1}(\Psi\eta\omega\rho_\pi - \omega-\sigma\beta-\omega\rho_\pi-2\sigma)X - (\sigma\beta)^{-1}(\sigma+\omega\rho_\pi)$. The determinant of A is strictly larger than one, $\det(A) = -H(0) = (\sigma\beta)^{-1}(\sigma+\omega\rho_\pi) > 1$, indicating that A exhibits at least one unstable eigenvalue. Given that

$$H(1) = (\sigma\beta)^{-1}\left(1-\eta\rho_\pi\right)\Psi\omega,$$

there is at least one stable (and positive) eigenvalue lying between zero and one if $1 > \eta\rho_\pi$. As $H(-1) = \omega\left(\sigma\beta\right)^{-1}\left[\Psi\left(1+\eta\rho_\pi\right) - 2\left(1+\rho_\pi\right) - 4\frac{\sigma}{\omega}\left(1+\beta\right)\right]$, we know that $H(-1) < 0$ for $\Psi \leq 2$ and $\eta < 1$, and that the third eigenvalue is unstable. Hence, the model exhibits exactly one stable and positive eigenvalue if and only if $1-\eta\rho_\pi > 0 \Leftrightarrow \rho_\pi < 1 + \frac{\kappa}{(1-\kappa)\overline{R}}$. ■

5.3 Proof of proposition 2

To derive qualitative properties of the impulse response of the endogenous variables $X_t = (\widehat{b}_t,\ \widehat{\pi}_t,\ \widehat{y}_t)'$ to policy and cost-push shocks, we apply the fundamental solution of the model which features the state variables $S_t = (\widehat{b}_{t-1},\ \varepsilon_{rt},\ \varepsilon_{\kappa t},\ \widehat{\varphi}_t)'$. In what follows we assume that (16) is satisfied, such that the fundamental solution is the unique solution to (17)-(20). The model is then solved applying the method of undetermined coefficients for the elements of Δ defined by

$$X_t = \begin{pmatrix} \delta_b & \delta_{br} & \delta_{bk} & \delta_{bc} \\ \delta_{\pi b} & \delta_{\pi r} & \delta_{\pi k} & \delta_{\pi c} \\ \delta_{yb} & \delta_{yr} & \delta_{\pi k} & \delta_{\pi c} \end{pmatrix} \cdot S_t = \Delta \cdot S_t.$$

Given that (16) is assumed to be satisfied, we already know from proposition 1 that $\delta_b \in (0,1)$. Hence, we aim at deriving the solutions for the remaining elements of Δ as functions of δ_b. The two other coefficients describing the structural part of the solution are given by

$$\delta_{\pi b} = \frac{1-\delta_b}{1-\eta\rho_\pi} > 0, \qquad \delta_{yb} = \frac{1-\beta\delta_b}{\omega}\frac{1-\delta_b}{1-\eta\rho_\pi} > 0,$$

which are unambiguously positive as (16) ensures $\eta\rho_\pi < 1$ (see proof of proposition 1). The coefficients governing the impact responses to the fiscal policy shocks ($\varepsilon_{\kappa t}$) are

$$\delta_{bk} = -\frac{\rho_\pi\omega+\sigma}{\left(1-\eta\rho_\pi\right)\Gamma+\rho_\pi\omega+\sigma} \in (-1,0), \qquad \delta_{\pi k} = -\frac{\Gamma}{\left(1-\eta\rho_\pi\right)\Gamma+\rho_\pi\omega+\sigma} < 0,$$

$$\delta_{yk} = -\frac{\Psi+\delta_{\pi b}(1-\Psi-\rho_\pi\beta)+\delta_{yb}\sigma}{\left(1-\eta\rho_\pi\right)\Gamma+\rho_\pi\omega+\sigma},$$

where $\Gamma \equiv \omega(\Psi+\delta_{yb}\sigma) + \delta_{\pi b}\left[\omega(1-\Psi)+\beta\sigma\right] > 0$. Inspecting the solution for δ_{yk}, immediately reveals that $\rho_\pi < (1-\Psi)/\beta$ is sufficient for $\delta_{yk} < 0$. The coefficients on the monetary policy shock (ε_{rt}), are given by

$$\delta_{br} = \frac{\omega\left(1-\eta\rho_\pi\right)+\left(\rho_\pi\omega+\sigma\right)\eta}{\left(1-\eta\rho_\pi\right)\Gamma+\rho_\pi\omega+\sigma} > 0, \qquad \delta_{\pi r} = -\frac{\omega-\eta\Gamma}{\left(1-\eta\rho_\pi\right)\Gamma+\rho_\pi\omega+\sigma},$$
$$\delta_{yr} = -\frac{1}{\Gamma}\frac{\left(\omega-\eta\Gamma\right)\left[\delta_{\pi b}(1-\Psi)+\Psi+\delta_{yb}\sigma\right]+\beta\delta_{\pi b}\left(\Gamma+\sigma\right)}{\left(1-\eta\rho_\pi\right)\Gamma+\rho_\pi\omega+\sigma},$$

Thus, a low value for η (high κ) satisfying $\eta < \widetilde{\eta}$, where $\widetilde{\eta} \equiv (\Psi+\rho_\pi+(1-\delta_b)\left[\frac{\sigma}{\omega}\left(1+\beta-\beta\delta_b\right)+1-\Psi\right])^{-1} > 0$, is sufficient to ensure $\omega - \eta\Gamma > 0 \Leftrightarrow \delta_{\pi r} < 0$ and also guarantees $\delta_{yr} < 0$. Finally, the coefficients on the cost-push shocks ($\widehat{\varphi}_t$) are given by

$$\delta_{bc} = -\frac{(1-\eta\rho_\pi)(1-\rho_c)\sigma}{\Theta} < 0, \qquad \delta_{\pi c} = \frac{(1-\rho_c)\sigma}{\Theta} > 0,$$
$$\delta_{yc} = -\frac{\omega\left(\rho_\pi(1-\eta\Psi)+1-(\delta_b+\rho_c)(1-\Psi)\right)+\sigma(1-\delta_b)(1-\beta\delta_b)}{\omega\Theta} < 0,$$

where $\Theta \equiv -\omega[(\delta_b+\rho_c)(1-\Psi)+\rho_\pi(\eta\Psi-1)-1]+\sigma[(1-\beta\rho_c)(2-\delta_b-\rho_c)+\beta(1-\delta_b)^2] > 0$, given that $1 < \rho_\pi < 1/\eta$ and $\Psi < 2$. These properties of the solution coefficients are summarized in the proposition. ■

5.4 Proof of proposition 3

Under an interest rate peg and flexible prices, the model with $\Psi > 0$ can be reduced to the following conditions in inflation and real public debt: $-\gamma\left(1-\rho\right)\widehat{\xi}_t = \left(1-\Psi\right)E_t\widehat{\pi}_{t+1} + \Psi\widehat{b}_t$ and $\widehat{b}_t = \widehat{b}_{t-1} - \widehat{\pi}_t$, where $\gamma = \sigma(\sigma + \overline{u_{ll}}\overline{l}/\overline{ul})^{-1} > 0$. Applying the method of undetermined coefficients for a generic solution form featuring real public debt as a state variable $\widehat{b}_t = \delta_b\widehat{b}_{t-1} + \delta_{be}\widehat{\xi}_t$ and $\widehat{\pi}_t = \delta_{\pi b}\widehat{b}_{t-1} + \delta_{\pi e}\widehat{\xi}_t$, leads to the following fundamental solution $\delta_b = 0$, $\delta_{\pi b} = 1$, $\delta_{be} = -\delta_{\pi e}$ and $\delta_{\pi e} = \gamma\left(1-\rho_c\right)/(1-\rho_c+\Psi\rho_c)$, and to an inflation variance satisfying $var_\pi = 2\delta_{\pi e}^2 var_\xi$. When debt is neutral ($\Psi = 0$) the fundamental solution for inflation reads $\widehat{\pi}_t = \delta_{\pi e}\widehat{\xi}_t$ where $\delta_{\pi e} = -\gamma\left(1-\rho_c\right)/\rho_c$, and its variance is $var_\pi = \delta_{\pi e}^2 var_\xi$. Hence, inflation is more volatile in the latter case if $2\left[\gamma\left(1-\rho_c\right)/\left(1-\rho_c+\Psi\rho_c\right)\right]^2 < \left[\left(1-\rho_c\right)\rho_c^{-1}\gamma\right]^2 \Leftrightarrow \rho_c^{-1} > 1-\Psi+\sqrt{2}$. ■

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