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Exploitation and Cooperation in Networks

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Exploitation and Cooperation in Networks*

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Abstract

A fundamental question in social sciences is how trust emerges. We provide an answer which relies on the formation of social and economic relationships. We argue that behind trust lies the fact that individuals invest in connections taking into account the potential externalities networks produce. Once social ties are in place, these externalities shape the individuals' incentives to behave efficiently in their interactions and thereby efficient social norms are sustained.

We also show that the individual's incentives depend on the architecture of the network as well as on the position of the individual within the network. In particular, when an efficient interaction requires players to mutually cooperate, efficient social norms are easily sustained in symmetric networks. By contrast, when an efficient interaction requires players to play asymmetrically (one cooperates and the other free-rides), efficient social norms are best sustained in fully centralized architectures. We interpret these results indicating that a structural analysis is important to understand how individuals' incentives are shaped in many strategic contexts.

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1 Introduction

Why should I not free-ride on my coauthor in our scientific collaboration? Why should a firm not free-ride on another firm in the collaboration for the development of a new product? Why should I not order a very expensive meal when I go out with a friend for dinner with the implicit agreement that we will share the bill evenly? What does drive the economic man not to free-ride in such situations? A fundamental question in social sciences is how trust emerges. Our answer relies on the formation of social and economic relationships. We argue that individuals invest in connections taking into account the potential externalities networks produce. Once social ties are in place, these externalities shape the individuals' incentives to free-ride or cooperate with others and thereby efficient social norms are easily sustained. We also show that the individuals' incentives in their mutual interactions depend on the architecture of the social network as well as on the position of the individuals within the network.

We examine a framework where the investment in social or economic ties has a long run nature.[1] The formation of a link between two players brings benefits and costs to the two parties. The benefit results from the potential exchange of some valuable non-rival good, such as information. The cost arises because maintaining a collaboration requires to exert effort and spend time; a player may cooperate or not and a free-riding problem characterizes the cost side. As an illustration consider the following example. Two researchers form a scientific collaboration. The benefit to each researcher is the possibility of exchanging ideas, opinions and knowledge. Part of the knowledge is intrinsic to these two researchers, but another part is obtained as a result of the interaction with other agents in the social network. Further, maintaining the relationship is costly in terms of effort and time. If the two parties cooperate, the maintenance cost of the link will be lower as compared to the case in which they both free-ride on each other. The increase in the maintenance cost of the link reflects some sort of inefficiency, such as a delay in the project, which would not occur in case of cooperative behavior. However, given that one of the two parties cooperates, the other would prefer to free ride and save some time to develop other projects by his own. In this paper we ask how the endogenous formation of social and economic relationships may help to overcome free-riding problems.

We start by presenting the main features of the model. There is a finite set of individuals and each of them is endowed with some non-rival information. At the beginning of the game players propose links and this generates a network of relationships. Once the network is in place, every pair of linked players interacts for an infinite number

[1] The assumption that social ties have a long run nature captures the idea that the interaction between two acquaintances occurs more frequently than the formation of the relationship itself. For example, once a scientific collaboration is in place, the two parties meet and interact frequently before the project has been completed. Similarly, if two firms form a collaboration for the development of a new product, they typically interact frequently before the collaboration ends.

of periods. In the interaction phase, each player observes the entire network of relationships and the history of actions that each of his social acquaintances has taken in the interaction with him; neither the actions played by his acquaintances with third parties nor the actions played by non-acquaintances are observable.[2] In each period every pair of acquaintances, say i and j, play two simultaneous move games: an *Accessibility game* and a *Prisoner's Dilemma game*. In the Accessibility game player i (j) either withholds or conveys the information (the non-rival good) he has to player j (i). The outcome of this game across all pairs of social contacts determines how information flows in the network and therefore it defines the benefits to each player in that specific period. On the other hand in the Prisoner's Dilemma game, player i (j) decides whether to cooperate or defect with player j (i); this determines the cost of that particular link and how it is covered by the two parties.[3] Therefore, a strategy profile specifies a network and the way players act in the interaction phase. An equilibrium is a strategy profile such that the proposed network is pairwise stable and the strategy profile is a sequential equilibrium. In the analysis we first characterize the efficient outcomes and then we focus on efficient equilibria. We now discuss the main results.

We first characterize the efficient outcomes (Theorem 3.1). An efficient outcome is characterized either by the empty network or by any minimally connected network where players provide full accessibility. In this latter case two efficient outcomes arise for two distinct ranges of parameters. First, a *symmetric efficient outcome* where individuals cooperate with their acquaintances to maintain the cost of their relationships. Second, an *asymmetric efficient outcome* where for each pair of linked players, one individual cooperates and bears entirely the cost of the link, while the other player free-rides on him. Figure 1 depicts two possible efficient configurations in a society with 4 players.

We then turn to explore when the symmetric and asymmetric efficient outcomes can arise as a result of strategic considerations. We start by examining the existence of equilibria that sustain the asymmetric efficient outcome (exploitative efficient equilibria). We show that the exploitative efficient equilibrium which exists for the widest range of parameters has the following features. First, the social network has a star architecture. Second, each player i provides information and cooperates with player j if the amount of information player j accesses exclusively from player i is weakly

[2] The assumption that players fully observe the network is realistic when the network represents a physical infrastructure. When a link means a social relationship, it is hard to think that players observe the entire structure of the social network they belong to. We shall show that, for our results to hold, it is enough that each player has local information of the social network.

[3] In particular, whenever two linked players play symmetrically in the prisoner's dilemma game they share evenly the cost of that link (at the defection level if both players defect, or at the cooperative level otherwise); if they play asymmetrically the player who cooperates bears entirely the cost of that link (at the exploitative level).

less that the the amount of information player i accesses exclusively from player j; otherwise player i free-rides on player j. In other words, the way a player behaves with a social contact depends on how much valuable for the former is the relation with the latter. This equilibrium exists for a range of the discount factor which becomes wider as the size of the population increases.

We comment on the nature of this equilibrium. We first observe that the best way to sustain strategically the asymmetric efficient outcome is when players are embedded in the star network. The reason is that in the star architecture the central player detects directly any eventual deviation and his structural position allows him to punish any cheater with immediate social isolation (withholding all the information). Second, in the star network the central player always free-rides on his social contacts while the spoke players always cooperate. In other words, the spoke players sponsor their links with the central player; on the contrary the central player uses his strategic position to obtain private gains. Here, the role of the periphery-sponsored property is to transfer utility from poorly connected players (spoke players) to well connected players (central players). This aligns individuals and social incentives and therefore it enhances efficiency. Third, we show that the larger the population is the more likely an efficient equilibrium exists. This is due to the fact that the magnitude of the punishments is increasing in the amount of network externalities, ceteris paribus. These results suggest that centrality and periphery-sponsorship are crucial to sustain an asymmetric efficient outcome in information networks.

We finally explore the existence of equilibria that sustain the symmetric efficient outcome (cooperative efficient equilibria). Here, we show that the cooperative efficient equilibrium which exists for the widest range of parameters has the following features. First, the network has a line architecture. Second, players cooperate and provide information, while they punish deviations in the network stage as well as in the interaction phase by defecting and withholding information (cooperative strategy profile). This equilibrium exists for a range of the discount factor which depends on the size of the population.

We comment on this result. First, in sharp contrast with the exploitative case, players sustain the symmetric efficient outcome more easily when they are embedded in the line network, which is the most symmetric architecture within the class of minimally connected networks. The reason for this is that individual's incentives to deviate in the network formation stage are inversely related to the number of connections a player has. This indicates that symmetric distribution of links across players is important to sustain the symmetric efficient outcome. Second, we observe that similarly to the exploitative case when the exchange of information is used strategically individual players may credibly threat their social acquaintances by withholding information. This creates more severe punishments as compared to settings where information is not strategic and therefore also a relatively impatient society can sustain cooperative efficient equilibria.

Recently, it has been shown that network relationships play a role in shaping individuals behavior in a variety of strategic situations such as games of conflicts and coordination games.[4] The current paper is one of the first work which examines the interplay between endogenous strategic links formation and cooperative behavior. Seen in this perspective, it relates to different strands of the economic literature such as cooperation in repeated games, network theory and social capital and trust. We will discuss how our paper relates to these different branches after having present our main results. The rest of the paper is organized as follows. Section 2 presents the model. Section 3 characterizes efficient outcomes. Section 4 and 5 analyze exploitative and cooperative efficient equilibria, respectively. Section 6 reviews the related literature. Section 7 concludes. Proofs are developed in the Appendix.

2 Model

There is a finite set of players and each agent is endowed with some non-rival information which has a value, v. At the beginning of the game players form an undirected graph (network formation stage). Undirected graphs are used to model the network of relationships among players. A graph is composed by a set of nodes and a set of links; each node represents a player while each link indicates a bilateral relationship between two players. Once the network is formed, each pair of linked players play an infinitely repeated game, which consists on two simultaneous games: one, players may augment their information exchanging it with their social contacts (Accessibility game) and two, interacting players play a Prisoner's Dilemma game, which defines the cost a player has to pay for each link he has. The strategy of each player is therefore two-dimensional and this will play a crucial role in our further analysis. We now introduce the model formally.

- **Network Formation**

Let $N = \{1, 2, ..., n\}$ be a set of players and let i be a typical member of this set. To avoid trivialities, we shall assume throughout that $n \geq 3$. In period zero, each player i proposes a set of links, i.e. $\omega_i^0 = \left(\omega_{i,1}^0, \omega_{i,2}^0, ..., \omega_{i,n}^0\right)$, where $\omega_{i,j}^0 \in \{0, 1\}$, $\forall j \in N \backslash \{i\}$. If $\omega_{i,j}^0 = 1$ we say that player i wants to form a link with j. A link between two agents, say i and j, is formed if both players agree on it, i.e. $\omega_{i,j}^0 = \omega_{j,i}^0 = 1$. These decisions are summarized in $\omega^0 = (\omega_1^0, \omega_2^0, ..., \omega_n^0)$ and results in an undirected network $g(\omega^0) = (g_1(\omega^0), g_2(\omega^0), ..., g_n(\omega^0))$, where $g_i(\omega^0) = (g_{i,1}(\omega^0), g_{i,2}(\omega^0), ..., g_{i,n}(\omega^0))$, $g_{i,j}(\omega^0) = \omega_{i,j}^0 \cdot \omega_{j,i}^0$ $\forall j \in N \backslash \{i\}$ and $g_{i,i}(\omega^0) = 0$

[4]See Kosfled (2004) for a survey of experimental work on networks. In section 6 we briefly discuss some of these studies.

$\forall i \in N$. When there is no confusion we will use g^0 instead of $g(\omega^0)$. We say that players i and j have a direct link if $g^0_{i,j} = 1$, otherwise $g^0_{i,j} = 0$.[5] Let $\mathcal{G}$ be the set of all possible undirected networks on N. For a network $g \in \mathcal{G}$, the set $N^d_i(g) = \{j \in N \setminus \{i\} : g_{i,j} = 1\}$ defines the bilateral relationships (social contacts) of player i. Let $\mu^d_i(g)$ be its cardinality.

- **Infinitely Repeated Game**

  - **Prisoner's Dilemma Game (PDG)**

In any period $t \geq 1$, each pair of acquaintances (i, j) plays a PDG represented in table 1. Let us denote $\alpha^t_{i,j} \in \{C, D\}$ as the action chosen by player i in the interaction with j, where C means cooperation and D defection.

$i \backslash j$	C	D
C	c, c	f, e
D	e, f	d, d

Table 1

We shall assume throughout the paper that $e > c > d > f$, $2d < f$ and we normalize, without loss of generality, $e = 0$. We denote as $\phi^t_{i,j}(\alpha_{i,j}, \alpha_{j,i})$ the cost player i faces when interacting with player j, which is represented in table 1. In words, each pair of interacting players may either share the cost of the link symmetrically (either at the cooperative level, c, or at the defection level, d) or only one of the two players bears entirely the cost of that link at the exploitative level, f.

- - **Accessibility Game (AG)**

In any period $t \geq 1$, simultaneously to the prisoner's dilemma game, each player $i \in N$ decides either to withhold or provide (at no cost) the information to each of the other players $j \in N \backslash \{i\}$ (both his own information and the information he acquires from other agents). We denote by $\lambda^t_{i,j} \in \{0, 1\}$ this decision, where $\lambda^t_{i,j} = 1$ indicates that player i transmits the information to player j. For example, when the link is a scientific collaboration, an R&D collaboration or a social tie providing accessibility means to share ideas, opinions, knowledge. If a link is a collaboration to construct an infrastructure like a bridge between two cities, then providing accessibility means to allow the other player to access the other city using that bridge.

[5] We note that $g^0_{i,j} = g^0_{j,i}$, $\forall \{i, j\} \in N$.

Let us define $\lambda_i^t = \left\{\lambda_{i,j}^t\right\}_{j\in N\setminus\{i\}}$, then the pattern $\lambda^t = \Pi_{i\in N}\lambda_i^t$ determines the flow of information within a network $g \in \mathcal{G}$ and the combination of g and λ^t results in an directed network g^{λ^t}, to which we will refer as "flow network", where $g_{i,j}^{\lambda^t} = g_{i,j} \cdot \lambda_{j,i}^t$. We say that the information flows from j to i if $g_{i,j}^{\lambda^t} = 1$; otherwise $g_{i,j}^{\lambda^t} = 0$. A *flow path* from j to i in g^{λ^t} is denoted as $j \overset{g^{\lambda^t}}{\to} i$ where either $g_{i,j}^{\lambda^t} = 1$ or there exists a sequence of agents $j_1, ..., j_m$ different from i and j such that $g_{i,j_1}^{\lambda^t} = g_{j_1,j_2}^{\lambda^t} = ... = g_{j_{m-1},j_m}^{\lambda^t} = g_{j_m,j}^{\lambda^t} = 1$. Thus, given a flow network g^{λ^t}, the set of players that i accesses is $N_i\left(g^{\lambda^t}\right) = \left\{j \in N\setminus\{i\} : j \overset{g^{\lambda^t}}{\to} i\right\}$ and we denote as $\mu_i\left(g^{\lambda^t}\right)$ the cardinality of this set. For simplicity, we assume that information flows across links without decay.

The following notation is important to define the strategy profiles of the game. Given g^{λ^t}, for any $g_{i,j}^{\lambda^t} = 1$ the set $\mathcal{I}_{i,j}(g^{\lambda^t}) = \left\{k \in N : \left(\exists k \overset{g^{\lambda^t}}{\to} i\right) \wedge \left(j \notin k \overset{g^{\lambda^t}}{\to} i\right)\right\}$ indicates the set of players agent j accesses exclusively via a path containing i and its cardinality, i.e. $I_{i,j}(g^{\lambda^t}) \equiv \left|\mathcal{I}_{i,j}(g^{\lambda^t})\right|$, represents the benefit player j obtains in the specific interaction with player i.[6] The same definition applies at period $t = 0$ once we impose that $g^{\lambda^0} = g$.

Combining the two games, the action space of each player with regard to each of his social contact is $A \equiv \{(\alpha, \lambda)\}_{\alpha_j\in\{C,D\},\lambda\in\{0,1\}}$. Let us define $\mathcal{A}_i \equiv A^{n-1}$ $\forall i \in N$ and $\mathcal{A} = \Pi_{i\in N}\mathcal{A}_i$; we note that any element in $\mathcal{A}_i$ is a vector of tuples $a_i = (a_{i,1}, ..., a_{i,i-1}, a_{i,i+1}, ..., a_{i,n})$, which represents the action played by agent i with the remaining players in the constituent game of the infinitely repeated game.

- **Strategy Profiles**

We shall focus on pure strategy profiles. We assume that at each period $t \geq 1$ each player i observes the social network, the past actions of his social contacts in their specific collaboration, and the information received by each of their social contacts, i.e. $a^t(i; g) = \{(a_{j,i}^\tau, a_{i,j}^\tau), \left(I_{j,i}(g^{\lambda^t}), I_{i,j}(g^{\lambda^t})\right)\}_{j\in N_i^d(g),\tau\in\{1,...,t-1\}}$. We also assume that players neither observe the behavior of their social contacts with third parties nor the behavior of non-acquaintances. Let $\Psi^t(i; g)$ be the space of observable actions for player $i \in N$.[7] Then, the observed history at period t of player i is $h^t(i) = \{g, a^t(i; g)\}$ and the set of histories of player i at time t is $\mathcal{H}^t(i) \equiv \left\{\{g, \psi\}_{\psi\in\Psi^t(i;g)}\right\}_{g\in\mathcal{G}}$. We refer to $s = \{\omega^0, \omega^1, ..., \omega^t, ...\}$ as a pure strategy profile of this game, and s_i is a pure

[6] To illustrate this, in a star network, the strategic information of the center in respect to any other player, say j, is $(n-1)$, while the strategic information of j in respect to the center is 1.

[7] With some abuse of notation, we can define in an arbitrary period $t \geq 1$, the space of observable actions for player $i \in N$ as $\Psi^t(i; g) \equiv A^{2(t-1)\mu_i^d(g)} \times \{0, 1, ..., n-1\}^{2(t-1)\mu_i^d(g)}$.

strategy of player i which consists on a set of link proposals, ω_i^0, and a sequence of functions, $\omega_i^1, ..., \omega_i^t, ...$, where $\omega_i^t : \mathcal{H}^t(i) \rightarrow \mathcal{A}_i$, $\forall t \geq 1$. Let player $i's$ strategy set be denoted as S_i, and let $S \equiv \prod_{i \in N} S_i$ be the set of pure strategy profiles.

It is important to note that a strategy profile $s = \{\omega^0, \omega^1, ..., \omega^t, ...\}$ results in an undirected network g^0 and in an infinite sequence of directed networks $\{g^{\lambda^1}, g^{\lambda^2}, ..., g^{\lambda^t}, ...\}$, one for each period t. In the analysis we will focus on strategy profiles which are *stationary,* i.e players play the same action in the equilibrium path (at every period).

- **Payoff structure**

We are now ready to define the payoff structure of the game. Given a strategy profile $s = \{\omega^0, \omega^1, ..., \omega^t, ...\}$, the total value generated at each period t, $v^t(s)$, and the utility player i obtains at that period, $u_i^t(s)$, can be written as:

$$\begin{aligned} v^t(s) &= \sum_{i \in N} u_i^t(s), \quad \text{where} \\ u_i^t(s) &= \mu_i\left(g^{\lambda^t}\right) \cdot v + \sum_{j \in N_i^d(g^0)} \phi_{i,j}\left(\alpha_{i,j}^t, \alpha_{j,i}^t\right) \end{aligned}$$

Therefore, the value generated by a strategy profile s in the entire game, $V(s)$, and the utility to player i in the entire game, $u_i(s)$, may be represented as follows:

$$V(s) = \sum_{t=1}^{\infty} \delta^{t-1} v(s) = \sum_{i \in N} u_i(s) \tag{1}$$

$$u_i(s) = \sum_{t=1}^{\infty} \delta^{t-1} u_i^t(s) \tag{2}$$

where $\delta \in (0,1)$ is the common discount factor displayed by all agents. In words, the utility to player i is given by a discounted sum of infinite earnings derived from the information i accesses, i.e. $\mu_i^t(g^{\lambda^t}) \cdot v$, and the cost player i bears in his interactions, *i.e.* $\sum_{j \in N_i^d(g^0)} \phi_{i,j}\left(\alpha_{i,j}, \alpha_{j,i}\right)$.

- **Equilibrium and Efficiency Notions**

We are interested in determining a strategy profile $s = \{\omega^0, \omega^1, ..., \omega^t, ...\}$ such that the proposed network $g(\omega^0)$ is stable given the prescriptions of the strategy s in the continuation game, i.e. $\{\omega^1, ..., \omega^t, ...\}$; and this strategy is a sequential equilibrium (i.e. prescribes to play an equilibrium for any possible observed network $g' \in \mathcal{G}$ and any history of play in the infinitely repeated game). To determine the stability of the network we use the notion of pairwise stability.[8] Formally, given $\{\omega^1, ..., \omega^t, ...\}$, the network g^0 is pairwise stable if no pair of players wants to form an additional link and no individual player wants to delete any set of his links.[9]

Definition 2.1 *The strategy profile* $s = \{\omega^0, \omega^1, ..., \omega^t, ...\}$ *is an equilibrium if* $\{\omega^1, ..., \omega^t, ...\}$ *is a sequential equilibrium, and* g^0 *is pairwise stable.*

To complete, we define the notion of efficiency.

Definition 2.2 *A strategy* s *is efficient if* $V(s) \geq V(\hat{s})$ *for any* $\hat{s} \in S$.

We note that if a strategy is socially efficient, it is also Pareto efficient. The reverse does not hold.

3 Efficient Outcomes

We start by characterizing the efficient outcomes. We shall then proceed to investigate strategy profiles which enable to sustain efficient equilibria. We will focus on maximal punishment strategy profiles. This will clarify the role networks play on the emergence of efficient social norms and the effect of efficient social norms in shaping the incentives to invest in connections. Taken these effects together, the analysis will also clarify the tension between individual and social incentives.

We first introduce some notation. Given a network g, we say there is a *path of links* from j to i, denoted as $j \overset{g}{\longleftrightarrow} i$, if either $g_{i,j} = 1$ or there exists a sequence of players $j_1, ..., j_m$ not including i and j such that $g_{i,j_1} = g_{j_1,j_2} = ... = g_{j_m,j} = 1$. A set $C(g) \subset N$ is a component of g if for any $i, j \in C(g)$ there is a path between them, and it does not exist a path between an agent in $C(g)$ and $N \backslash C(g)$. A component is *minimal* if there exists only one path between any pair of players $i, j \in C(g)$. A network g is connected if it has a unique component. If a network is connected and its unique component is minimal, we say that it is minimally connected. A player i in a network g is said to be an end-agent if he has a unique link. A network is empty if there are no links across players. A network g has a star architecture and i is the central player if $g_{i,j} = 1$, $\forall j \in N \backslash \{i\}$, and there are no other links. A line is a minimally connected network where only two end-agents exist.

The following result characterizes the efficient outcomes.

[8] The concept of pairwise stability has been introduced by Jackson and Wolinsky (1996). In the current paper we will use a modified version of this notion introduced by Goyal and Joshi (2003)

[9] We propose that g^0 must be pairwise stable, instead of requiring ω^0 to be a Nash equilibrium. In the latter case, given $\{\omega^1, ..., \omega^t, ...\}$, we would obtain a multiplicity of equilibria of the type: whenever $\omega^0_{i,j} = 0\ \forall \{i, j\} \in N$, then player j is indifferent between setting $\omega^0_{j,i} = 1$ or $\omega^0_{j,i} = 0$. The former case avoids this problem and stresses the architecture of the equilibrium network, g^0.

Theorem 3.1 *Suppose (1) and (2) hold.* (*a*) *If* $nv + \max\{2c, f\} > 0$ *then* $s = \{\omega^0, \omega^1, ..., \omega^t, ...\}$ *is efficient if and only if the following conditions hold: (i)* g^0 *is minimally connected; (ii) each player provides accessibility in every period; (iii) the total cost for each link in every period is* $\max\{2c, f\}$. (*b*) *If* $nv + \max\{2c, f\} < 0$, s *is efficient if and only if* g^0 *is the empty network.*

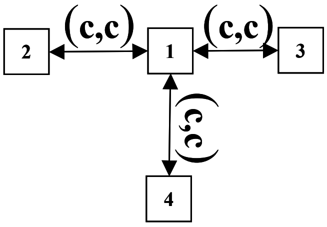
We sketch here the main steps of the proof of Theorem 3.1. First, since exchanging information is costless and it (weakly) increases social welfare, a social planner will prescribe to mutually exchange information. Second, the no-decay assumption implies that an efficient network is minimal. Third, since linking up two players otherwise disconnected creates positive network externalities an efficient network is either empty or connected. Finally, in any minimally connected network social welfare is maximized whenever the cost for each link is as low as possible.

Theorem 3.1 shows that efficiency requires that players form a minimally connected network and they exchange information. Furthermore, two possibilities may arise. First, in some settings, i.e. $2c > f$, players must mutually cooperate.[10] We will refer to this case as the *symmetric efficient outcome*; it is readily seen that a symmetric efficient outcome generates a social welfare equal to $(n-1)(nv+2c)/(1-\delta)$. Second, in other settings, efficiency requires that for each link a player cooperates and bears entirely the cost of that link, while the other player free-rides on him. We will refer to this case as the *asymmetric efficient outcome*, which generates a social welfare equal to: $(n-1)(nv+f)/(1-\delta)$.[11]

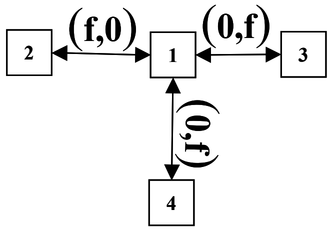
Figure 1 depicts one possible symmetric and asymmetric efficient outcome in a society composed of 4 players. In the figure, a link is represented by an edge connected two players and an arrowhead pointed to one player indicates that information flows in the direction of that player; finally, the way the cost of a link is covered by the two players is indicated with the letters above the edge.

[10] For example, assume that to maintain the link agents must exert effort and that the cost to exert effort for a player is convex. In this case splitting the mainteinance tasks between the two parties is more efficient than letting only one player to take care of them.

[11] For example, when the performance of a task (the maintenance of the link) requires a lot of coordination across players or the opening of different bureaucratic procedures, it may be more efficient leaving the task to be solved unilaterally as compared to solve it bilaterally. In the scientific collaboration example, if the two reaserchers belong to two different universities, it is generally more costly for the two researchers to meet in a conference and both of them paying the plane fare as compared to the case that only one of the two researchers visits the other one in his own university.



Symmetric Efficient Outcome, 2c>f



Asymmetric Efficient Outcome, 2c<f

In what follows, we shall study under which circumstances is possible to sustain strategically the asymmetric and the symmetric efficient outcome, respectively. We will restrict our analysis to the case in which $\max\{2c, f\} + nv > 0$, which would eventually be the case in large societies, even if the individual cost of a link is high.

4 Exploitative Efficient Equilibria

We start by examining the existence of equilibria which sustain the asymmetric efficient outcome. We will refer to these equilibria as *exploitative efficient equilibria.* We first analyse a benchmark case where social ties do not play any strategic role in the game. To do this we assume that the communication of information is not strategic and that players' behavior in the interaction phase does not depend on the realization of the network stage.[12] The following remark shows that in this setting exploitative efficient equilibria do not exist.

Remark 4.1. *Suppose social ties do not play any role, then an exploitative efficient equilibrium does not exist.*

In what follows we shall show that when strategic communication is allowed this inefficiency fades away. To show this we characterize the efficient exploitative equilibrium which exists for the widest range of parameters. This equilibrium is characterized by two features: one, the network has a star architecture, and two, the central player always free rides while the peripheral players always cooperate. Therefore, in equilibrium the cost of each connection is paid by the player who values it more.[13] As a presentation strategy we will first define a strategy profile, called exploitative strategy, which sustains the asymmetric efficient outcome. We then show that such strategy

[12] Recall we are restricting our analysis to stationary strategy profiles.

[13] In a different setting, Meléndez-Jiménez (2002) obtains that when two agents bargain on the cost sharing of a link, the agent who values more the link bears a higher part of the cost of the link, and when both value the link equally, they split evenly the cost.

sustains the exploitative efficient equilibrium for the widest range of parameters and we characterize it.

The *exploitative strategy* prescribes players to form a set of links, which generates a social network, say g^0. If players observe a network which is different from the prescribed one, they defect and withhold the information. Differently, if players observe g^0, each agent i provides information and cooperates with player j if the amount of information player j accesses exclusively from player i is weakly less that the the amount of information that player i accesses exclusively from player j; otherwise player i free-rides on player j. Furthermore, a deviation in the interaction phase is punished by withholding information and defecting. It is important to note that even if players only observe the behavior of their direct neighbors in their mutual interaction, they can infer deviations of their social contacts with third parties and deviations of non-acquaintances by observing the flow of information they access in their interactions. In other words, even if players have only local information, social punishments are indirectly implementable. To define the strategy profile formally, we need to introduce some additional notation.

Definition 4.1 *We say that the action taken by player i against j at period t, $a^t_{i,j}$, is well-behaved, WB, with respect to the relative flow of information between i and j, if and only if*

$$a^t_{i,j} = \begin{cases} (C,1) \ \textit{if } I_{i,j}(g^{\lambda^t}) = I_{i,j}(g^0) \leq I_{i,j}(g^0) = I_{j,i}(g^{\lambda^t}) \\ (D,1) \ \textit{otherwise} \end{cases}$$

The exploitative strategy profile is then defined as $s^E = \left\{\omega^0_i, \omega^{E,1}_i, ..., \omega^{E,t}_i, ...\right\}_{i \in N}$, where $\omega^E_i = \left\{\omega^0_i, \omega^{E,1}_i, ..., \omega^{E,t}_i, ...\right\}$ is such that for any $\hat{g} \in \mathcal{G}$ and any $j \in N$:

$$\omega^{E,1}_{i,j} = \begin{cases} (D,1) \text{ if } \hat{g} = g^0 \text{ and } I_{i,j}(g^0) > I_{j,i}(g^0) \\ (C,1) \text{ if } \hat{g} = g^0 \text{ and } I_{i,j}(g^0) \leq I_{j,i}(g^0) \\ (D,0) \text{ otherwise} \end{cases}, \text{ and } \forall t \geq 2:$$

$$\omega^{E,t}_{i,j} = \begin{cases} (D,1) \text{ if } a^{t-1}_{k,i} \text{ and } a^{t-1}_{i,k} \text{ are } WB, \forall k \in N^d_i(g^0), \text{ and } I_{i,j}(g^0) > I_{j,i}(g^0) \\ (C,1) \text{ if } a^{t-1}_{k,i} \text{ and } a^{t-1}_{i,k} \text{ are } WB, \forall k \in N^d_i(g^0), \text{ and } I_{i,j}(g^0) \leq I_{j,i}(g^0) \\ (D,0) \text{ otherwise} \end{cases}$$

It is worth noting that the exploitative strategy profile prescribes that each player plays the Nash equilibrium $(D,0)$ in any out-of-equilibrium path. This implies that, to define the conditions for existence of a sequential equilibrium, we only need to focus on individuals' incentives in the equilibrium path. We are now ready to provide the main result of this section. Let us denote $\bar{n} = \frac{(v-2d)(d-f)}{v^2} + 1$.

Theorem 4.1 *Suppose (1) and (2) hold and assume* $2c < f$. *An efficient equilibrium exists if and only if (i)* $f + (n-1)v \geq 0$ *and (ii)* $\delta \geq \frac{d-f}{(n-1)v}$. *Furthermore, if* $n > \bar{n}$ *then the unique network part of the efficient equilibrium which exists for the widest range of parameters is the star network.*[14]

The proof of the theorem is based on Lemmas 1 and 2 which are provided in the appendix. We sketch here the main arguments. In Lemma 1 we start by showing that the exploitative strategy sustains the exploitative efficient equilibrium for the widest range of parameters in a star network. The reason is that in the star network the center may punish any deviations directly by withholding all the information; this implies that any cheater would be socially isolated just after one period of his deviation. On the contrary, in any other minimally connected network there exists some player who could deviate and yet enjoy some information for some period after the deviation. Next, conditions (i) and (ii) follow by solving the equilibrium conditions for the exploitative strategy profile in the case where the initial network has a star architecture.

In Lemma 2 we show that if conditions (i) and (ii) are not satisfied then there does not exist a strategy profile which sustains as equilibrium the asymmetric efficient outcome. To prove this we first show that any strategy profile which sustains the asymmetric efficient outcome for the parameter range in which condition (i) holds should impose that each player pays at most for one of the links he has. We then turn to the equilibrium condition imposed by the sequential rationality notion, i.e. condition (ii). Here we note that when condition (ii) binds, i.e. $\delta = \frac{d-f}{(n-1)v}$, for an equilibrium any end-agent must pay for his link. For otherwise, there would be a player which pays for at least two links and this player would deviate at that level of the discount factor. Using this fact, the proof follows by noticing that the exploitative strategy prescribes a maximal punishment to an end-agent who deviates. Finally we prove that, if the population is high enough ($n > \bar{n}$), when conditions (i) and (ii) are binding any strategy profile different from the exploitative strategy profile is not an equilibrium.

We first comment on the existence region of exploitative efficient equilibria. Two remarks are worth making. One, both condition (i) and (ii) become weaker as the size of the population increases. Asymptotically as the size of the society becomes infinite an exploitative efficient equilibrium always exists. This is so because the higher is the size of the population, the higher are the network externalities which are produced, which in turn increase the magnitude of punishments. This suggests that the conflict between individual and social incentives is less severe in large societies. Two, we also note that the pairwise stability notion leads to an equilibrium condition (condition (i)), which is invariant with respect to the architecture of the network, provided that

[14] We note that even if $n < \bar{n}$, the star network is an efficient equilibrium for the widest range of parameters, but it is not the unique one.

each player pays at most the cost of one link. Therefore, the pairwise stability notion does not have any bite on the architecture of the network. On the contrary, the requirement of sequential equilibrium crucially depends on the architecture of the network. Hence, in our setting, individuals' incentives are embedded in their social network.

The second set of observations concerns with the nature of efficient exploitative equilibria. Here, we would like to emphasize three remarks. First, the star architecture allows to sustain strategically the asymmetric efficient outcome more likely as compared to other minimally connected networks. The reason is that in the star network the central player has the ability to maximally punish occurring deviations. This suggests that centrality is a crucial structural property to monitor individuals' behavior in information networks. Second, centrality is accompanied by the periphery-sponsored property: the cost of each link is unilaterally met by the player who values it more, which is the peripheral player in that specific interaction. Here, the role played by the periphery-sponsored property is to transfer utility from poor connected players to well connected players. In other words, well connected players use strategically their structural position to obtain private gains. This is in line with the theory of structural holes developed by Burt (1983), which has been successfully tested in many empirical analyses.

Third, we note that exploitative efficient equilibria are characterized by hierarchical structures. The following example illustrates this point. Consider a society composed of 9 players, arranged in the network depicted in Figure 2, who follow the exploitative strategy profile.

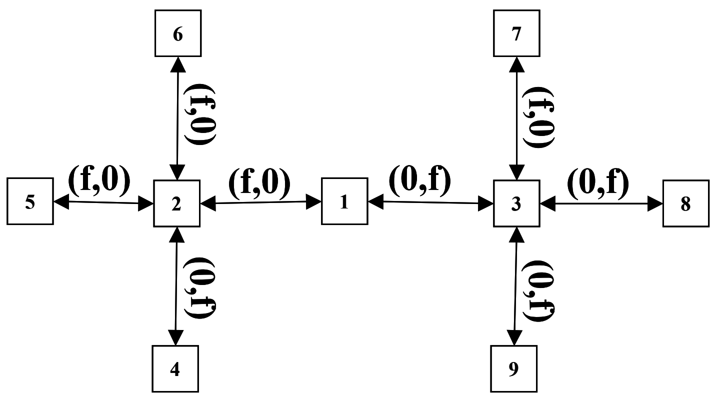


Figure 2

We note that three types of players emerges. Player 1 is the *exploitative player* in the sense that he free-rides on every of his neighbors (players 2 and 3). The role of the exploitative player is to connect two star components, which would be otherwise disconnected. Players 2 and 3 are hybrid players in the sense that they cooperate with the central player and they exploit all their other social contacts.

The remaining players are always exploited. Note that hybrid players have higher incentives to deviate than exploited players because, as compared to the end-agents, hybrid players could defect with their exploiter (the center) and yet they would enjoy the information from the end-agents for some period after that the deviation has occurred.

5 Cooperative Efficient Equilibria

We now explore the existence of equilibria which sustain the symmetric efficient outcome. We shall refer to these equilibria as cooperative efficient equilibria. Similarly to the previous section, we first analyse a benchmark case where social ties do not play any strategic role. The next remark shows that in these cases the trade-off between individual and social incentives is substantial.

Remark 5.1. *Suppose that social ties do not play any role and that* $2c > f$. *An efficient equilibrium exists if and only if* $v + c \geq 0$ *and* $\delta \geq c/d$

We note that given a minimally connected network the continuation game degenerates, for each period $t \geq 1$, in a finite number of bilateral prisoner's dilemma games which are strategically independent. As a consequence the equilibrium condition to sustain mutual cooperation in any link is obtained by applying the standard Folk theorem, i.e. $\delta \geq c/d$. Once players are sufficiently patient, the condition $v + c \geq 0$ is necessary and sufficient so that a minimally connected network is pairwise stable (in this case the critical agent would be the one linked with an end-agent).

We now examine the effect of allowing players to use the information strategically. We shall show that when strategic communication is allowed the conflict between individual and social incentives is less severe. As in the previous section, we analyse the cooperative efficient equilibrium which exists for the widest range of parameters. This equilibrium is characterized by two features: the network has a line architecture and every player cooperates.

We start by introducing the cooperative strategy profile. This strategy prescribes players to form a set of links, which generates a network of relationships, say g^0. In the interaction phase each player defects and withholds information whenever a network which is different from the prescribed one is observed. Otherwise, each player cooperates and provides information to his neighbors, while he punishes an eventual deviation by defecting and withholding the information in every period onwards in all his links. Also in this case, each player detects directly the deviations of their social contacts in his interactions and indirectly (via the information flow) the deviations of his social contacts with third parties and the deviations of non-acquaintances.

Formally, let us define the cooperative strategy profile as $s^C = \{\omega_i^0, \omega_i^{C,1}, ..., \omega_i^{C,t}, ...\}_{i \in N}$, where $\omega_i^{C,t} = \left\{\omega_{i,1}^{C,t}, ..., \omega_{i,n}^{C,t}\right\}$ is such that for any $\hat{g} \in \mathcal{G}$ and any $j \in N$:

$$\omega_{i,j}^{C,1} = \begin{cases} (C,1) & \text{if } \hat{g} = g^0 \\ (D,0) & \text{otherwise} \end{cases} \quad \text{and } \forall t \geq 2:$$
$$\omega_{i,j}^{C,t} = \begin{cases} (C,1) & \text{if } a_{k,i}^{t-1} = a_{i,k}^{t-1} = (C,1) \text{ and } (I_{k,i}(g^{\lambda^{t-1}}), I_{i,k}(g^{\lambda^{t-1}})) = (I_{k,i}(g^0), I_{i,k}(g^0)), \forall k \in N_i^d(g) \\ (D,0) & \text{otherwise} \end{cases}$$

It is worth noticing that the cooperative strategy profile prescribes to play the Nash equilibrium $(D,0)$ in any out-of-equilibrium path. Thus, to define the existence conditions of a sequential equilibrium it is enough to focus on individuals' incentives in the equilibrium path. The Theorem below provides the main result of this section. Let δ^* be the solution of the following equation $\delta^*[v + (n-2)\,v\delta^* + c - d - d\delta^*] + c = 0$.

Theorem 5.1 *Suppose (1) and (2) hold and assume that* $2c > f$. *An efficient equilibrium exists if and only if* (i) $(n-1)v + 2c \geq 0$ *and* (ii) $\delta \geq \delta^*$. *Furthermore, the unique network which is part the cooperative efficient equilibrium which exists for the widest range of parameters is the line network.*

The Theorem follows from lemmas 3 and 4 in appendix. We sketch here the main arguments. In the first lemma we show that the cooperative strategy profile sustains the cooperative efficient equilibrium which exists for the widest range of parameters when the initial network has a line architecture. To prove this we first observe that, given that players follow the cooperative strategy in the interaction phase, in the network formation stage players' incentives to deviate are increasing in the number of links they have. Furthermore, we show that in any other network different from the line, in the interaction phase players have at least the same incentives to deviate as compared to players embedded in a line network. The equilibrium conditions (i) and (ii) are obtained by imposing pairwise stability and sequential rationality, respectively. In the second Lemma we show that both conditions (i) and (ii) should hold in order that a symmetric efficient outcome can be sustained as an equilibrium. This follows by noticing that the cooperative strategy profile prescribes maximal punishments.

We would like to elaborate on some aspects of the results presented in Theorem 5.1. We first observe that when players can strategically decide whether to convey or withhold information to their neighbors, cooperative efficient equilibria exist for parameter ranges where they would not exist, otherwise (see Remark 5.1). The reason is that players may credibly commit to convey information, conditionally on having inferred that each player has cooperated in his interactions. Similarly to the exploitative case analysed in the previous section, network externalities are higher in a larger society, which enhances efficiency. The second observation is about the nature of the equilibrium which exists for the widest range of parameters. Figure 3 illustrates this equilibrium in a society composed of 4 players.

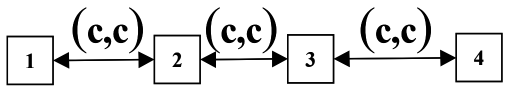


Figure 3: Line network

The main feature of this equilibrium is that players are embedded in a very symmetric network: the line network. This suggest that a symmetric distribution of connections across players (fairly compatible networks) is crucial to sustain strategically the symmetric efficient outcome.

6 Related Literature

The prevalence of situations in which networks play a role and their crucial importance in shaping the final outcome of these interactions has been extensively studied empirically. In particular, in the context of games of conflicts, a recent experiment by Rield and Ule (2002) examine the role of endogenous network formation in the way rational players' play a repeated prisoners' dilemma game. When comparing a treatment where the network is exogenously given with treatments where the network is formed endogenously, they observe that in the latter case cooperation rates are significantly higher as compared to the former case. Cassar (2002) analyzes the cooperative behavior in three classes of networks: random networks, small world networks and regular networks. The main result is that small world networks exhibit the smaller level of cooperation among different network structures considered. This suggests that the way players are connected shapes their willing to cooperate. The current paper provides a theoretical account of the effects of network externalities in the strategic formation of informal relationships and individuals' incentives. Our paper relates to three strands of the economic literature, that is network formation, cooperation in repeated games and social capital and trust. We will refer to each of them in turn.

The first contribution refers to the body of literature which studies how networks form when players have the discretion of forming links, see Aumann and Myerson (1989), Bala and Goyal (2000) and Jackson and Wolinsky (1996).[15] Our model borrows from that literature the basic elements of network formation models: forming a link with another individual requires to costly exert effort and it allows access, in part and in due course, to the benefits available to the latter via her own links. However, we depart

[15] Other examples are Dutta, Nouweland and Tijs (1995), Galeotti, Goyal and Kamphorst (2003), Jackson and Watts (2001) and Kranton and Minehart (2001).

from the existing literature as players can decide whether to exchange information and their behaviors determine the cost of the links. Therefore, players' strategies are two-dimensional and this is the primary difference between the current paper and the existing literature on network formation. The main effect of this multidimensionality in the strategy space is that players can strategically relate the decision of providing information with the action that determines the cost of the link. We have shown that this fact mitigates the trade-off between individual and social incentives and that the trade-off vanishes as the size of the society is large enough.

Secondly, our paper relates to the literature of cooperation in repeated games. To some extent it relates to the work of Kandori (1992) and Ellison (1994). They analyse a setting where players, belonging to a community, are repeatedly and randomly matched to play a Prisoner's dilemma game. The main result is that cooperation can be sustained if players react to a deviation by punishing subsequent partners. Therefore, the community has a positive effect in the enforcement of cooperative behavior: to free-ride on one player causes sanction by others. The main difference in our approach is that players' interaction takes place in a fixed pattern of play (the community is endogenously structured) and this allows us to investigate the effect of network externalities on the enforcement of efficient long-run and stable relationships.

A paper which shares the same spirit of ours is Haag and Lagunoff (2000); they analyse a prisoners' dilemma game where players are ex-ante heterogeneous with respect to their discount factor and they play with all the population (i.e. the complete network) and each player is restricted to play the same action with all agents. They show that the more symmetric the distribution of the discount factor is, the higher it is the maximum achievable degree of cooperation. Moreover, the effects of the size of the population on cooperation depends on the parameters of the model. Differently, in the current paper, even if players are ex-ante homogeneous, heterogeneity comes from the different positions in the network once it is (endogenously) determined.

Finally, we relate our paper to the theory of social capital and trust.[16] Social capital is a relational concept and its existence is inherent to socioeconomic networks. Social capital affects individuals' behavior as well as aggregate economic phenomena. An individual player can use his social capital, which depends on the nature of his connections, to obtain private economic gains. From a societal perspective, social capital represents the basis of trust in repeated interactions. Sociologists have widely studied this subject. Coleman (1988) emphasizes the role of redundant links for the emergence of trust introducing the so-called *closure* argument. Consider, for simplicity, a society with three players. Coleman (1988) argues that social capital is higher when players are embedded in a cycle network as compared to a star network. The reason is that in a cycle players can monitor rivals' deviations more efficiently than in a star:

[16] See J. Sobel (2002) for an extensive discussion on the notion of social capital.

in a cycle if a friend cheats on me I could communicate this to a common friends (in the star there are no common friends), which would eventually react by punishing the cheater. By contrast, Burt (1992) emphasizes the importance of non-redundant connections introducing the so-called *structural hole* argument. Structural holes are players who connected networks by linking different components, which would be otherwise disconnected. These players on the one hand integrate additive sources of information, which in turn increases the value generated by the network and on the other hand they may use strategically their structural position to obtain private gains. These two theories should be seen as complementary: the closure argument explains how the benefits of network externalities can be realized in a community, while the structural hole argument explains how network externalities come about, and the structural properties of players who are crucial for the emergence of these externalities.

In economics, the notion of social capital has been mainly used to study issues related to economic development, criminality and education.[17] However, a theoretical analysis of how social capital emerges is still at a preliminary stage. The first attempt to address this issue is Vega-Redondo (2002). Agents interact according to a collection of infinitely repeated prisoner's dilemma games played on the current social network. The strategic effects of networks result from the fact that players can communicate via their links behavioral information about their acquaintances. This allows for the formation of stable and dense networks in which players can monitor efficiently other players' behavior and this mitigates the incentives to free-ride. By contrast, in our model social capital emerges because players invest in connections that generates network externalities, and players use these externalities to punish possible deviations. We would like to emphasize that while the work of Vega-Redondo (2002) is more in the spirit of the closure argument, our paper follows the idea of structural holes. In this perspective, the current paper can be seen as complementary to the work of Vega-Redondo (2002).

7 Discussion and Conclusion

Free-riding problems are often solved in many economic and social interactions. We have showed that the mere fact that economic actions are embedded in the social structure allows players to internalize network externalities to a great extent. Furthermore, players may use strategically these externalities to overcome free-riding problems. These effects result from the fact that players can strategically use social channels to exchange information and this serve to shape individuals' incentives.

We now discuss how the results presented in the paper are robust to the main as-

[17]See Dasgupta and Sarageldin (1999) for a discussion of the main contributions on social capital in economics.

sumptions. We first elaborate on the impact of relaxing the assumption that players observe fully the structure of the network. It is easy to see that for our results to hold it is enough that players are aware about the connections held by their direct social contacts and the information their social contacts can potentially provide. Indeed, this information is sufficient to allow players to employ the strategy profiles used in our analysis.

Second, we have showed that efficient equilibria are best sustained when the size of the society is large. It is worth noticing that this result holds as far as the payoffs functions are increasing in the amount of information a player accesses. Thus, the fact that we consider linear payoff functions is not crucial. A weaker case can be made if we relax the assumption of frictionless information flow. Suppose we introduce a small amount of decay then an efficient outcome is characterized by a star network where players provide information and the cost of each link is either shared at the cooperative level (when 2c>f) or it is borne unilaterally at the exploitative level (otherwise). In the former case, since the central player bears the cooperative cost for each link, the existence conditions for a cooperative efficient equilibrium will be independent of the size of the population. On the contrary, in the latter case, the results presented in section 4 carry on qualitatively.

Third, we have explored a model where investment in links is sunk and players cannot change their network over time. We note that the strategy profiles use also apply on a repeated game where the network formation and players interaction occurs simultaneously. Finally, we elaborate on the possibility of time-preference heterogeneous players. Our analyses shows that in equilibrium players having different position in the networks have different incentives. Therefore it is not crucial that players discount the features evenly. A formal analysis of these topics is left for future research.

8 Appendix

Efficient Outcomes

We start by proving Theorem 3.1, which characterizes the efficient outcomes of the game.

Proof of Theorem 3.1. We start by proving part (a), i.e. $nv + \max\{2c, f\} > 0$. We first claim that if s is efficient then conditions (i)-(iii) hold. First, the requirement that g^0 is minimal follows from the no-decay assumption. Second, we note that given a minimal network, to provide information strictly increases social welfare. Thus, condition (ii) follows. Third, condition (iii) assures that the cost of each link at any period is minimized. Fourth, we note that g^0 must be connected. For a contradiction let assume that g^0 is minimal but not connected. Consider an end-agent belonging to a component $C(g^0)$ of cardinality $k > 1$; the social welfare produced by the link with the end-agent is $2(k-1)v+\max\{2c,f\} \geq 0$, which is positive since, by assumption, s is efficient. Let us consider a strategy $\hat{s}$ which prescribes a network $\hat{g}^0$, which differs from the original network in the fact that $\hat{g}^0$ has an additional link, say between i and j, where $i \in C(g^0)$, $j \in N \backslash C(g^0)$, and the information is exchanged in the new link. It is readily seen that $V(\hat{s}) - V(s) \geq 2kv + \max\{2c,f\}$. Since $2kv + \max\{2c,f\} > 2(k-1)v + \max\{2c,f\} \geq 0$, it follows that $V(\hat{s}) - V(s) > 0$. This contradicts the fact that s is efficient. Hence, the claim follows. We now observe that any minimally connected network in which condition (ii) and (iii) are satisfied produces a social welfare equals to $V(s) = \frac{(n-1)(nv+\max\{2c,f\})}{1-\delta} > 0$. This proves the part (a) of the Theorem. Part (b) follows trivially. Hence, the proof is completed.■

Exploitative Efficient Equilibria

Proof of Theorem 4.1. The proof of the Theorem is based on the next two lemmas. Let $\bar{n} = \frac{(v-2d)(d-f)}{v^2} + 1$

Lemma 1 *Suppose (1) and (2) hold and assume $2c < f$. The strategy profile s^E is an efficient equilibrium if and only if (i) $\delta \geq \frac{d-f}{(n-1)v}$ and (ii) $(n-1)v + f \geq 0$. Further, if $n > \bar{n}$ then the strategy s^E is an efficient equilibrium for the widest range of parameters when g^0 is a star network.*

Proof. Consider the strategy $s^E = \left\{\omega^0, \omega^{E,1}, ..., \omega^{E,t}, ...\right\}$ where g^0 is minimally connected. We first observe that, given s^E, in order to obtain an asymmetric efficient outcome, we need to focus on minimally connected network where $I_{i,j}(g^0) \neq I_{j,i}(g^0)$ for any $g_{i,j} = 1$. Next, we observe that, given s^E, in any minimally connected network each player pays at most the cost of one link at the exploitative level. It is readily seen that g^0 is pairwise stable if and only if $(n-1)v + f \geq 0$.

Second, we analyse the conditions for the discount factor δ (relative to the interaction stage). We start by noticing that, since as soon as players realize a deviation via the information flow they reverse their behavior to the Nash equilibrium $(D, 0)$, it follows that s^E is optimal, regardless of the players' beliefs. This implies that to determine the parameter conditions for a sequential equilibrium we simply need to focus on the players' incentives on the equilibrium path.

Third, let us assume that g^0 is the star network. Here, we start by noticing that the central player, say j, does not have any incentives to deviate from s^E, since he obtains the maximum achievable payoff in this game, i.e. $u_j^f\left(s^E\right) = \frac{(n-1)v}{1-\delta}$. Furthermore, every agent $i \in N \backslash \{j\}$ faces the same problem; select then an arbitrary player i in this set. Next, we show that i does not deviate if and only if $\delta \geq \frac{d-f}{(n-1)v}$. To see this we note that the utility agent i obtains following the strategy s^E is $u_i\left(s^E\right) = \frac{(n-1)v+f}{1-\delta}$, and the utility if he deviates is $u_i^d\left(s_i^d, s_{-i}^E\right) = (n-1)v + \frac{d}{1-\delta}$. Therefore an equilibrium requires that $u_i\left(s^E\right) \geq u_i^d\left(s_i^d, s_{-i}^E\right)$, which is satisfied if and only if $\delta \geq \frac{d-f}{(n-1)v}$. Hence, if $(n-1)v + f \geq 0$ and $\delta \geq \frac{d-f}{(n-1)v}$ the strategy s^E where g^0 is a star network is an equilibrium. Differently, if g^0 is not a star network, we note that there always exist at least two end-agents; it is readily seen that an end-agent does not deviate from s^E only if $\delta \geq \frac{d-f}{(n-1)v}$.

Fourth, we show that if $n > \bar{n}$ the strategy s^E is an equilibrium for the widest range of parameters only if g^0 is a star network. Suppose not, then when $n > \bar{n}$, $(n-1)v+f \geq 0$ and $\delta = \frac{d-f}{(n-1)v}$, the strategy s^E is an equilibrium for some minimally connected network g^0 different from the star. We note that in any minimally connected network g^0 it exists an agent, say j, who has k links with k end agents $(k \geq 1)$ and one additional link with a non end-agent, i.e. $\mu_j^d\left(g^0\right) = k+1$. The utility this player obtains following the strategy is $u_j\left(s^E\right) = \frac{(n-1)v+f}{1-\delta}$. Assume player j deviates with the k end-agents; the utility from such deviation is $u_j^d\left(s_j^d, s_{-j}^E\right) = (n-1)v + \frac{d}{1-\delta} + kv\delta + \frac{kd\delta^2}{1-\delta}$. Since g^0 is part of an equilibrium, it must be the case that the incentives to deviate of an arbitrary end-agent i are weakly higher than the incentives of player j, i.e. $u_i^d\left(s_i^d, s_{-i}^E\right) \geq u_j^d\left(s_j^d, s_{-j}^E\right)$.[18] This is satisfied if and only if $(n-1)v + \frac{d}{1-\delta} \geq (n-1)v + \frac{d}{1-\delta} + kv\delta + \frac{kd\delta^2}{1-\delta}$, which can be rewritten as $v - \delta(v-d) \leq 0$. We now note that when $\delta = \frac{d-f}{(n-1)v}$ the condition $v - \delta(v-d) = v - \frac{(d-f)(v-d)}{(n-1)v} \leq 0$ if and only if $n \leq \frac{(d-f)(v-d)}{v^2} + 1$. Since $\frac{(d-f)(v-d)}{v^2} + 1 < \bar{n}$, this contradicts the assumption that $n > \bar{n}$. Hence if $n > \bar{n}$ the star network uniquely allows the strategy s^E to be an equilibrium for the widest parameter range.

This completes the proof of the Lemma.■

We now prove that, given any strategy profile an efficient equilibrium exists only if the conditions (i) and (ii) in Theorem 4.2 are satisfied and that given that these conditions are binding then a strategy $s = (g^0, \omega^1, ..., \omega^t, ...)$ different from the exploitative strategy profile is not an efficient equilibrium.

Lemma 2 *Suppose (1) and (2) hold and assume* $2c < f$. *An efficient equilibrium exists only if* (i) $(n-1)v+f \geq 0$ *and* (ii) $\delta \geq \frac{(d-f)}{(n-1)v}$. *Further, given that* $(n-1)v+f = 0$ *and* $\delta = \frac{(d-f)}{(n-1)v}$, *if* $n > \bar{n}$ *every strategy* $s = (g^0, \omega^1, ..., \omega^t...)$ *different from the exploitative strategy is not an efficient equilibrium.*

Proof. Let $f > 2c$. Assume that the outcome of $s = \{\omega^0, ..., \omega^t, ...\}$ is efficient, i.e. g^0 is a minimally connected network, all links are paid at the exploitative level and there is complete flow of information. We first claim that s is pairwise stable only if $(n-1)v+f \geq 0$. In order to get the lower bound condition to attain pairwise

[18] This is true because the utility to player i and j by following the strategy profile s^E coincides.

stability we shall assume that the strategy s prescribes a maximal punishment in the network formation stage, i.e. if a network $\hat{g} \neq g^0$ is observed then agents play $(D, 0)$ in all the interactions. Then, given that players follow s in the interaction phase the payoff of an agent when the network g^0 is formed is $\frac{(n-1)v+\mu_i^{d,p}\left(g^0,s\right)f}{1-\delta}$, where $\mu_i^{d,p}\left(g^0, s\right) \leq \mu_i^d\left(g^0\right)$ represents the links agent i pays (at the exploitative level) given s. The best deviation of player i in the network formation stage would be to delete all his links and obtain a payoff 0. Hence the condition for pairwise stability is obtained when $(n-1)\, v + \max_{i \in N} \mu_i^{d,p}\left(g^0, s\right) f \geq 0$. Given s, for any g^0, $\min\{\max_{i \in N} \mu_i^{d,p}\left(g^0, s\right)\} \geq 1$, thus, s is pairwise stable only if $(n-1)\, v + f \geq 0$ and the claim follows.

Second, we claim that s is a sequential equilibrium only if $\delta \geq \frac{d-f}{(n-1)v}$. We note that, in order to get a lower bound on δ, we shall consider that the strategy profile s involves maximal punishments in case of any deviation in the interaction stage. Such strategy should prescribe that if at some period t an agent $i \in N$ deviates in his interaction with $j \in N_i^d\left(g^0\right)$ then j plays $a_{j,i}^{\tau} = (D, 0)\ \forall \tau \geq t+1$. Therefore, given s, two possibilities may occur: (1) there is at least one end agent, say j, who pays for his link, and (2) no end-agent pays for his link. We start considering case (1); the payoff to the end-agent j, if he follows the strategy is $u_j\left(s\right) = \frac{(n-1)v+f}{1-\delta}$, and the payoff if he deviates in his interaction is $u_j^d\left(s_j^d, s_{-j}\right) = (n-1)\, v + \frac{d}{1-\delta}$. Agent j does not want to deviate whenever $\delta \geq \frac{d-f}{(n-1)v}$. Hence, in this case, our claim follows.

We now consider case (2), i.e. no end-agent pays for his link. Let $E_0\left(g^0\right) \in N$ represent the set of end-agents in g^0. We claim that there exists some player $i \in M_0\left(g^0\right) \equiv N \backslash E_0\left(g^0\right)$ who is paying all his direct links. Assume for a contradiction that no agent pays for all his links, i.e. $\mu_i^{d,p}\left(g^0, s\right) < \mu_i^d\left(g^0\right)\ \forall i \in N$. Since the end-agents are not paying for their links, any agent $k_0 \in M_0\left(g^0\right)$ linked to an agent $j_0 \in E_0\left(g^0\right)$ is paying for the link $\{j_0, k_0\}$. This implies that in case g^0 has a star architecture, the center pays for all his direct links; this is a contradiction. Therefore, let g^0 be a minimally connected network different from the star. Let $g^{0,1}$ be a network obtained by removing from g^0 all agents belonging to $E_0\left(g^0\right)$ and their corresponding links. We note that since g^0 is minimally connected, also $g^{0,1}$ is minimally connected. Let $E_1\left(g^0\right) \in M_0\left(g^0\right)$ be the set of end-agents in $g^{0,1}$. We note that each player $j_1 \in E_1\left(g^0\right)$ had some link with some end-agent in g^0 and he was paying for that particular link; since j_1 is an end-agent in $g^{0,1}$ and no agent pay for all his links in g^0, it follows that j_1 does not pay for the link in $g^{0,1}$, i.e. there exists some player $k_1 \in M_1\left(g^0\right) \equiv M_0\left(g^0\right) \backslash E_1\left(g^0\right)$ linked to some agent $j_1 \in E_1\left(g^0\right)$ and such that k_1 pays for the link $\{j_1, k_1\}$.

We can proceed with the same reasoning defining the network $g^{0,2}$ as the resultant network from removing from $g^{0,1}$ all agents in $E_1\left(g^0\right)$ and their corresponding links. We note that $g^{0,2}$ is also minimally connected. Let $E_2\left(g^0\right) \in M_1\left(g^0\right)$ be the set of end-agents in $g^{0,2}$. Since each $j_2 \in E_2\left(g^0\right)$ had some link with some end-agent in $g^{0,1}$, he was paying for that particular link and this agent just have one link in $g^{0,2}$ it follows that any agent $k_2 \in M_2(g^0) \equiv M_1(g^0) \backslash E_2(g^0)$ linked to an agent $j_2 \in E_2(g^0)$ is paying for that link $\{j_2, k_2\}$. Since the number of players is finite, by induction we obtain that at some finite iteration period τ, the cardinality of the set $M_\tau(g^0)$ is either 1 or 2. Consider the case $M_\tau(g^0) = \{i_1\}$. Then this agent pays for all his links, which contradicts our initial assumption. Now consider the case $M_\tau(g^0) = \{i_1, i_{,2}\}$; note that i_1 and i_2 must be necessarily linked in g^0 and, therefore, one of these agents pays for all his links, a contradiction. This proves the claim.

This claim implies that there exists some player i such that $\mu_i^{d,p}(g^0, s) = \mu_i^d(g^0)$. Since player i is not an end agent in g^0, $\mu_i^d(g^0) \geq 2$. The payoff of such agent from following the strategy s is $u_i(s) = \frac{(n-1)v+\mu_i^d(g^0)f}{1-\delta}$, and the payoff from deviating in all his interactions, s_i^d, is $u_i(s_i^d, s_{-i}) = (n-1)v + \frac{\mu_i^d(g^0)d}{1-\delta}$. For an equilibrium it must be the case that $u_i(s) \geq u_i(s_i^d, s_{-i})$, i.e. $\delta \geq \frac{\mu_i^d(g^0)(d-f)}{(n-1)v}$. We observe that since $\mu_i^d(g^0) \geq 2$, $\frac{\mu_i^d(g^0)(d-f)}{(n-1)v} > \frac{d-f}{(n-1)v}$. The argument developed so far shows that conditions (i) and (ii) are necessary for a strategy s to be an efficient equilibrium.

We now prove that if these two conditions are binding and $n > \bar{n}$, any strategy s different from s^E is not an efficient equilibrium. Assume $s = \{g^0, ..., \omega^t, ...\}$ is an efficient equilibrium; we start by noting that since $(n-1)v + f = 0$ and s is an efficient equilibrium it must be the case that each player $i \in N$ pays at most for one link, i.e. each player cooperates with at most one of his social contacts. Next, consider now an arbitrary pair of players, say i and j, who are directly linked, $g_{i,j}^0 = 1$, and, without loss of generality, let us assume that $I_{i,j}(g^0) > I_{j,i}(g^0)$. We have two possibilities, which we analyse in turn.

I.) Suppose the strategy profile s prescribes in the equilibrium path that player j cooperates and player i defects. In this case, since s^E and s are equivalent in the equilibrium path and s^E prescribes maximal punishments for every deviation which eventually occurs, it follows that the incentive of player j (i) to follow s with i (j) cannot be higher than to follow s^E. In this case, we can use Lemma 4.1 to prove the claim.

II.) Suppose that the strategy profile s prescribes in the equilibrium path that player i cooperates and player j defects. The utility of player i to follow s is $u_i(s) = \frac{(n-1)v+f}{1-\delta}$. If player i deviates (using his best deviation) against player j at some period t, the utility he obtains in the continuation game is $u_i^d(s_i^d, s_{-i}) = (n-1)v + (n-1-I_{j,i}(g^0))v\delta + \frac{d}{1-\delta} + \frac{(\mu_i^d-1)d\delta^2}{1-\delta}$. Now we claim that player i have always incentives to deviate (for $\delta = \frac{d-f}{(n-1)v}$). To see this, let us consider the case where player i

would have the lowest incentives to deviate[19], i.e. $I_{j,i}(g^0) = n/2$ and $\mu_i^d = n-2$.[20] In this case, the utility of the best deviation of player i becomes $\bar{u}_i^d\left(s_i^d, s_{-i}\right) = (n-1)v + \left(\frac{n-2}{2}\right)v\delta + \frac{d}{1-\delta} + \frac{(n-2)d\delta^2}{1-\delta}$. We now note that $u_i(s) \geq \bar{u}_i^d\left(s_i^d, s_{-i}\right)$ if and only if $(n-1)v\delta - \left(\frac{n-2}{2}\right)v\delta(1-\delta) - (n-2)d\delta^2 > d-f$. Since $\delta = \frac{d-f}{(n-1)v}$, we can rewrite this condition as $-v + \delta(v-2d) = -v + \frac{d-f}{(n-1)v}(v-2d) > 0$, which is satisfied if and only if $n < \bar{n}$. This contradicts the fact that $n > \bar{n}$ and completes the proof of the Lemma.■

The two lemmas prove the Theorem.■

Cooperative Efficient Equilibria

Proof of Theorem 5.1. We first observe that the cooperative strategy profile prescribes players to play the Nash equilibrium $(D, 0)$ in any possible out-of-equilibrium path. This implies that, to define the existence conditions of a sequential equilibrium, we just need to focus on individuals' incentives in the equilibrium path. Using this fact, the proof of the theorem is based on two Lemmas which are stated and proved below.

Lemma 3. *Suppose (1) and (2) hold and assume* $2c > f$. *The cooperative strategy profile,* $s^C = \left\{\omega^0, \omega^{C,1}, ..., \omega^{C,t}, ...\right\}$ *is an equilibrium for the widest range of parameters when* g^0 *is a line network. In such case,* s^C *is an equilibrium if and only if* $(n-1)v + 2c \geq 0$ *and* $\delta \geq \delta^*$, *where* $\delta^*\left[v + (n-2)v\delta^* + c - d - d\delta^*\right] + c = 0$.

Proof.
Let us consider the strategy profile $s^C = \{\omega^0, \omega^{C,1}, ..., \omega^{C,t}, ...\}$, where g^0 is a minimally connected network. We first show that the network which is pairwise stable for the widest range of parameters is the line network. The utility a player i obtains from following the cooperative strategy is $u_i\left(s^C\right) = \frac{(n-1)v + \mu_i^d\left(g^0\right)c}{1-\delta}$, where $\mu_i^d\left(g^0\right) \in \{1, 2, ..., n-1\}$. Suppose player i deviates in the network stage. Since players play defection and withhold information in the interaction phase if a network different from g^0 is observed, it is clear that the best deviation of player i is to delete any link he has and doing so player i obtains zero utility. Thus, player i follows s^C if and only if $u_i\left(s^C\right) \geq 0$, which is equivalent to $(n-1)v + \mu_i^d\left(g^0\right)c \geq 0$. This implies that in any minimally connected network g^0 the player who has the highest incentive to deviate in the network stage is player j such that $\mu_j^d\left(g^0\right) = \max_{i\in N}\mu_i^d\left(g^0\right)$. We now observe that in the line network $\mu_j^d\left(g^{line}\right) = 2 < \mu_j^d\left(g^0\right)$ for any minimally connected

[19] Note that player i's incentives to deviate are decreasing in $I_{j,i}\left(g^0\right)$, because it represents the amount of information he looses when deviating (with a lag of only one period), and these incentives are also decreasing in μ_i^d, since when player i deviates at some period t he will have to pay a cost d in his relationship with each of his social contacts from period $t+2$ onwards.

[20] Note that this situation is not possible, but we use it to get a lower bound in the incentives to deviate.

network g^0 different from the line. It is readily seen that, given s^C, where g^0 is the line network, the network g^0 is pairwise stable if and only if $(n-1)v + 2c \geq 0$.

We now analyse the conditions for the discount factor δ (relative to the interaction stage). Let us assume that g^0 is the line network. We first show that the player who has the highest incentive to deviate in the interaction phase is either an end-agent, say i, or a player linked with an end agent, say j. Consider an end agent i, then the utility this player obtains following the strategy is $u_i\left(s^C\right) = \frac{(n-1)v+c}{1-\delta}$. If player i deviates in the interaction stage, his utility would be $(n-1)v + \frac{c}{1-\delta}$; thus a deviation is not profitable when $\delta \geq \bar{\delta} \equiv \frac{-c}{(n-1)v-d}$. Next, consider a player j linked with an end-agent; we note that player j has two links: one with an end-agent, say i, and one with a non-end agent, say j'. The utility of player j from following the strategy $u_j\left(s^C\right) = \frac{(n-1)v+2c}{1-\delta}$. At any period t, player j has two relevant possible deviations. One, player j may deviate only with player i at period t and deviate with player j' in period $t+1$; let us denote this deviation strategy as s_j^{d1}, then $u_j^{d1}\left(s_j^{d1}, s_{-j}^C\right) = (n-1)v + (n-2)v\delta + c + \frac{d\delta}{1-\delta} + \frac{d\delta^2}{1-\delta}$. Two, player j may deviate both with player i and j' at period t; let us denote this deviation strategy as s_j^{d2}, then $u_j^{d2}\left(s_j^{d2}, s_{-j}^C\right) = (n-1)v + \frac{2d\delta}{1-\delta}$.[21] We now observe that any other player who is neither an end-agent nor a player linked with an end-agent, say j', has a link with two non end-agents and he may deviate similarly to player j: to deviate only with the agent who is closest to an end agent of the line, say $s_{j'}^{d1}$, and to deviate with both of his social contacts, say $s_{j'}^{d2}$. It is readily seen that $u_j^{d1}(s_j^{d1}, s_{-j}^C) > u_{j'}^{d1}(s_{j'}^{d1}, s_{-j'}^C)$ and $u_j^{d2}(s_j^{d2}, s_{-j}^C) = u_{j'}^{d2}(s_{j'}^{d2}, s_{-j'}^C)$. This proves the claim.

Second, we claim that agent j has a higher incentive to deviate as compared to player i. Above we have shown that player i follows s^C if and only if $\delta > \bar{\delta}$. We now investigate the incentive of player j. Assume $\delta > \bar{\delta}$, then $u_j^{d1} \geq u_j^{d2}$ if and only if $\delta \geq \hat{\delta} \equiv \frac{-c}{(n-2)v-d} > \bar{\delta}$. Player j follows s^C if and only if $u_j\left(s^C\right) \geq u_j^{d1}\left(s_j^{d1}, s_{-j}^C\right)$, which is equivalent to $\delta[v + (n-2)v\delta + c - d - d\delta] + c \geq 0$. Let us define the following function: $\Upsilon(\delta) = \delta[v + (n-2)v\delta + c - d - d\delta] + c$. We note that $\frac{\partial \Upsilon(\delta)}{\partial \delta} > 0$ and using the fact that $c = d\hat{\delta} - (n-2)v\hat{\delta}$. we observe that $\Upsilon\left(\hat{\delta}\right) = -(n-3)v\hat{\delta} < 0$. Hence for $\delta \leq \hat{\delta}$ player j has incentives to deviate. This proves the claim. Moreover we can state the condition for player j not to deviate. Since when $\delta > \hat{\delta}$, the best deviation of player j is s_j^{d1}, for an equilibrium we need that $\delta \geq \delta^*$, where $\delta^* > \hat{\delta}$ is such that $\Upsilon(\delta^*) = 0$.

We now claim that in any other minimally connected network g^0 different from the line, say g^0, if $\delta < \delta^*$, s^C is not an equilibrium. We observe that in any minimally

[21] Player j may also deviate only with player i at period t. However, this deviation is strictly dominated by the deviation s_j^{d1}. Finally, player j may deviate with player j' at period t and either deviate with i at period $t+1$ or not. These two possibilities are strictly dominated by s_j^{d2}.

connected network it must be the case that there exist a player, say j'' who has k links with k end agents ($k \geq 1$) and one additional link, which may be either with a non end-agent, or with an end-agent (this last case would only be possible with the star network). Here we have two possibilities. One, if $k = 1$ then the incentives to deviate of player j'' are the same that the incentives of a player linked with an end-agent in a line network (player j above); in this case the claim follows. Two, $k \geq 2$; by construction $\mu^d_{j''}(g^0) = k + 1$ and the utility player j'' obtains following s^C is $u_{j'}\left(s^C\right) = \frac{(n-1)v+(k+1)c}{1-\delta}$. Let us assume that player j'' deviates in his interactions with the k end-agents at some period t and with the remaining player at period $t+1$, $s^{dk}_{j''}$. The utility from such deviation is $u^{dk}_{j''}\left(s^{dk}_{j''}, s^C_{-j''}\right) = (n-1)\,v + (n-1-k)\,v\delta + c + \frac{kd\delta+d\delta^2}{1-\delta}$. Therefore player j'' follows s^C if and only if $u_{j''}\left(s^C\right) \geq u^{dk}_{j''}\left(s^{dk}_{j''}, s^C_{-j''}\right)$, which is analogous to $\delta[kv + (n-1-k)\,v\delta + c - kd - d\delta] \geq -kc$. We now show that when $\delta = \delta^*$, player j'' deviates. To see this we note that $\Upsilon(\delta^*) = 0$ implies that $-c = \delta^*[v + (n-2)\,v\delta^* + c - d - d\delta^*]$. Assume for a contradiction that player j' does not want to deviate at δ^*, i.e. $\delta^*[kv + (n-1-k)\,v\delta^* + c - kd - d\delta^*] \geq -kc$. If, in the RHS, we substitute $-c$ from the equation $\Upsilon(\delta^*) = 0$ we obtain $\delta^*[kv + (n-1-k)\,v\delta^* + c - kd - d\delta^*] \geq -k\delta^*[v + (n-2)\,v\delta^* + c - d - d\delta^*] \Leftrightarrow \delta^* \leq \frac{-c}{(n-1)v-d}$, which is a contradiction since we have already shown that $\delta^* > \hat{\delta} > \bar{\delta} = \frac{-c}{(n-1)v-d}$.This proves the claim.

This completes the proof of the Lemma.■

Lemma 4. *Suppose (1) and (2) hold and assume $2c > f$. Consider the set of strategies $S^* \subset S$ which result in the asymmetric efficient outcome. If a strategy $s \in S^*$ is an equilibrium, then s^E with $g^0 = g$ is also an equilibrium.*

Proof.

Assume $2c > f$. To prove this, it is enough to show that s^C is a maximal punishment strategy profile. To see this note that if a player, say i, deviates in the network formation stage, he receives the worst possible outcome from period 1 onwards (because $a^{\tau}_{j,i} = (D, 0)$, $\forall j \in N^d_i(g), \tau \geq 1$). If player i deviates in the interaction stage, at some period t then the players with whom player i deviates directly realize the deviation, and hence they play $(D, 0)$ from period $t + 1$, while the remaining social contacts realize the deviation at $t + 1$ and hence they play $(D, 0)$ from period $t + 2$ onwards. Clearly, given the informational structure, player i receives the maximum punishment when he deviates. This completes the proof.■

Lemmas 4 and 5 prove the Theorem.■

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