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Consumers Networks and Search Equilibria

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Consumers Networks and Search Equilibria

Andrea Galeotti*

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Abstract

I examine a search model a la' Burdett and Judd (1983). Consumers are embedded in a consumers network, they may costly search for price quotations and the information gathered are non-excludable along direct links. This allows me to explore the effect of endogenous consumers externalities on market functioning. I first show that when search costs are low consumers randomize between searching for one price and two price quotations (high intensity search equilibrium). Otherwise, consumers randomize between searching for one price and not searching at all (low intensity search equilibrium). Second, in both equilibria consumers search less frequently in denser networks. Finally, when search costs are low the expected price and the social welfare increase as the consumers network becomes denser. These results are reverse when search costs are high.

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1 Introduction

A large body of empirical work shows that a variety of informal relationships complement the price system in coordinating the interaction among buyers and sellers in the market. For example, in marketing it is well established that consumers obtain much of their information via their social contacts (Feick and Price (1986, 1987)). In relation to this, firms have increasingly recognized the need for using informal channels as a way to market their products. The practice of consumers referral is an example;[1] according to the Direct Selling Association (1999), annual sales of firms that rely entirely on consumer referral grew from 13 billion to nearly 23 billion dollars between 1991 and 1998. Similarly, in the process of finding a job people rely heavily on their social contacts to obtain information about job opportunities (Granovetter (1974)). In medicine, and other specialized fields, professional networks shape the adoption of new technologies (Coleman 1966).[2]

These examples share a common feature: informal relationships between agents transform the information each individual privately obtains in a public good and this affects players' incentives as well as aggregate outcomes. This is the primary motivation for the development of a theory which studies the interplay between network relationships and market performance. This paper focuses on the role of social contacts in shaping the incentives to search, firms' pricing behavior and social welfare.

I examine a duopolistic version of Burdett and Judd (1983). On the supply side of the market there are two firms producing a homogeneous good and they set prices so as to maximize profits. Consumers have a common willingness to pay for the good and buy at most a single unit. The only way for a transaction to take place is that consumers have some information about prices. Consumers may individually search for price quotations and, in this case, they must pay a fixed search cost for each price quotation observed. In addition, consumers are embedded in a social network and they obtain information freely from their direct neighbors. To maintain symmetry on the consumers side I assume that each consumer holds the same number of connections, say k.[3] Once each consumer has searched, the information observed is freely provided to his direct neighbors and then transactions take place. The game is a one-shot simultaneous move game: firms set prices and consumers decide how many searches to make at the same moment. I focus on symmetric Nash equilibria.

I note that when the network is empty (or inactive), i.e. $k = 0$, I obtain the duopolistic version of Burdett and Judd (1983). By contrast, when k is positive, consumers strategically choose their search intensity taking into account that the information obtained is non-excludable across links. The non-excludability of price information

[1] Firms provide different sorts of benefits such as discounts to clients who bring new customers.

[2] I provide another example. Before competing in the market firms form agreements for the development of new products and they share knowledge (Hagedoorn (2002)).

[3] Thus, the consumer network is a regular graph with degree k.

across direct links creates information externalities across consumers, which clearly affect consumers and firms incentives and therefore the market equilibrium outcome. It is exactly the interplay between network relationships and market competitiveness that is the focus of the present paper. I shall show that in more dense networks consumers search less intensively and that in some instances firms charge on average higher prices when consumers hold more connections. Furthermore, I shall show that an increase in the density of the consumer network does not always enhance social efficiency as well as consumer surplus.

I start by noticing that, similarly to the findings of Burdett and Judd (1983), equilibria exhibit price dispersion. More interestingly, for any positive degree of the network there are two types of price dispersed equilibria. The first is a *high intensity search equilibrium*, where consumers randomize between searching for one price and two prices. This equilibrium exists for low search costs. The other is a *low intensity search equilibrium* where consumers randomize between searching for one price and not searching at all and it exists for high search costs. By contrast, when the consumers network is empty (or inactive) only the former equilibrium is strategically viable. In what follows I discuss the properties of these equilibria. In particular, I am interested in the effect of an increase in the degree of the network on the consumer search intensity, expected prices and social welfare.

I first elaborate on the high intensity search equilibrium. The first observation is that given that the degree of the network is strictly positive this equilibrium exists for sufficiently low search costs. Further, as the degree of the network increases, the existence region of this equilibrium shrinks. The intuition is that richer network relationships make the marginal gains of searching twice instead of once lower. Thus, for sufficiently high search costs (that is for sufficiently high marginal costs of searching twice instead of once) consumers cannot be indifferent between the two searching alternatives. Second, I note that expected prices are higher the lower is the density of the network, ceteris paribus. The reason is that the expected share of fully informed consumers is, ceteris paribus, increasing in the degree of the network. Third, I show that consumers search less frequently when network relationships are more dense. The second and the third observation imply that an increase in the number of connections has two opposite effects on the amount of information spread in the market. This leads me to investigate the relation between the degree of the network and expected prices.

Surprisingly, I show that *the expected price is higher when the network is more dense.* The intuition is the following. An increase in the number of connections leads consumers to free-ride more on each other. This reduces the expected share of fully informed consumers, despite the fact that the consumers hold more connections. Since firms compete less frequently, they charge on average higher prices. I also show that social welfare is higher, while consumer surplus is lower, when the consumers network is more dense. The increase in social welfare is due to the strategic substitutability between searching and network degree and the fact that consumers are active with probability one. The former effect reduces the waste in search costs, while the second assures that in equilibrium each possible transaction is indeed realized. The decrease in consumer surplus is due to the fact that firms price less aggressively when consumers hold more connections.

I finally turn to discuss the low intensity search equilibrium. I start by noticing that this equilibrium exists for moderate search costs. Interestingly, for a given network degree, the lowest search costs for which this equilibrium exists equals the highest search cost for which the high intensity search equilibrium exists. Secondly, as in the previous equilibrium, firms charge on average lower prices when the degree of the network increases, provided that consumers' behavior is fixed. Furthermore, also in this case consumers search less frequently in more dense networks.

Third, in sharp contrast with the previous equilibrium, an increase in the degree of the network lowers the expected equilibrium price. Therefore, when searching is relatively expensive, denser consumer networks enhance firms' competition. Interestingly, even if network relationships are beneficial for consumers, i.e. consumer surplus increases, they *decrease social welfare*. The reason is that as consumers free-ride more on each other, the number of realized transactions in equilibrium decreases. This negative effect offsets the saving on search costs.

This model relates to two branches of the economic literature: the theory of networks and market and the search theory. I start to discuss the relation of this paper with the theory of networks and market. The large empirical work on networks and market is the primarily motivation for the development of a theory which examines the effect of informal relationships on market competitiveness, e.g. Calvo-Armengol and Jackson (2004a, 2004b), Corominas-Bosch (1999), Goyal and Joshi (2003), Kranton and Minehart (2001). To the best of my knowledge the present paper is the first which studies systematically the role of consumers networks and market functioning. The works which come closer to mine are Bramoulle and Kranton (2004) and Goyal and Moraga (2001). The former examines a model of social learning where individuals search costly for new information and the results of their searching are non-excludable along links. While in Bramoulle and Kranton (2004) the benefit each consumer obtains by searching is exogenously given, in the present paper it is the outcome of firms' competition. This allows me to investigate not only the implication of social connections on consumers' search incentives, but also its indirect effect on strategic pricing behavior and market performances. Goyal and Moraga (2001) analyse a three-stages game where first firms form pairwise agreement for the development of new products, then they set an R&D effort which is costly and provide a reduction of the marginal production cost and finally they compete in the market. They find that the R&D effort a firm chooses in each agreement is decreasing in the effort that the partner firm sets and that this free-riding effect may lead to inefficient market outcomes. While Goyal and Moraga (2001) focus on the impact of network relationships on the supply side of the market, the current paper focuses on network relationships on the consumers side.

The consumer search literature is well established in economics, see for example Anderson and Renault (2000), Bester (1994), Braverman (1980), Burdett and Coles (1997), Burdett and Judd (1983), Morgan and Manning (1985), MacMinn (1980), Janssen and Moraga-Gonzalez (2000, 2003), Stahl (1989,1996). I have already discussed above the relation between the present paper and the model of Burdett and Judd (1983). Another paper which comes close to mine is Janssen and Moraga-Gonzalez (2003). They study a version of Burdett and Judd (1983) where consumers are ex-ante heterogeneous: one fraction of consumers are fully informed, while the remaining fraction must search costly to obtain price information. Increasing the fraction of fully informed consumers creates positive externalities for all consumers by boosting competitiveness and therefore lowering the expected price. The present paper provides a simple way of endogenizing externalities across consumers using network relationships and it shows that this may create negative consumers externalities which increase the expected price in equilibrium.

The rest of the paper proceeds as follows. In section 5.2 I define formally the model. Section 5.3 provides a preliminary equilibrium analysis. Section 5.4 and 5.5 characterize equilibria and Section 5.6 concludes. Proofs are relegated to the appendix.

2 The Model

I examine a model of non-sequential search where consumers are embedded in a network of connections. On the supply side there are $N = 2$ firms which produce a homogeneous good at constant returns to scale. I normalize their identical unit production cost to zero, without loss of generality.

On the supply side there is a finite number of consumers which I denote as m. All consumers are identical. They desire to buy a single unit of the product and their maximum willingness to pay is $\tilde{p} > 0$. For a transaction to take place consumers must observe at least one price quotation. A consumer may search simultaneously and the cost for each search is $c > 0$, where $c < \tilde{p}$. In addition, the price information each consumer obtains is freely provided to his neighbors. To maintain symmetry on the consumers side, I assume that the consumer network is a regular graph. Thus, the degree of the network, say k, may varies between 0 to $m - 1$, and it represents the number of connections each consumer holds.[4]

I note that when $k = 0$ the model is equivalent to a duopolistic version of Burdett and Judd (1983). By contrast, as k becomes positive, consumers strategically choose their search intensity taking into account that information gathered by their neighbors

[4] A regular graph may not exist when m is odd. Hence, in the paper we assume that m is even.

is non-excludable along direct links. This clearly affects individual incentives and therefore the market equilibrium outcomes. It is exactly the interplay between the externalities produced by the consumers network and market performance the focus of the present paper.[5]

Firms and consumers know the architecture of the network and play a simultaneous move game. An individual firm chooses its price taking price choices of the rivals as well as consumers' search behavior as given. I denote a firm's strategy by the price distribution $F(p)$ defined on a support σ; let $\underline{p}$ and $\bar{p}$ be the lowerbound and the upperbound of σ, respectively. Consumers form conjectures about the firms' price behavior and decide how many price observations to pay for. Once each consumer has searched, information is transmitted to the immediate neighbors. A strategy profile for a consumer is then a probability distribution over the set $\{0, 1, 2\}$. I denote as $q_{i,x}$ the probability of consumer i to search x time; thus a consumer's strategy is $\{q_x\}_{x=0}^{2}$. I will consider symmetric Nash equilibria. A symmetric equilibrium is a pair of strategies $s = \left\{F(p), \{q_x\}_{x=0}^{2}\right\}$ such that (a) $E\pi(s) = \bar{\pi}$ for any $p \in \sigma$ and (b) $\{q_x\}_{x=0}^{2}$ is an optimal search behavior given that the conjectures about firms' price behavior are correct.

3 Preliminary Analysis

The first observation is about the existence and characterization of equilibria in which consumers employ pure strategies.

Proposition 3.1. *For any $k \geq 0$ and $c > 0$, the only equilibria in which consumers use pure strategy take the following form: consumers never search, $q_0 = 1$, and firms charge a price $p \in [\tilde{p} - c, \tilde{p}]$.*

This proposition leads me to investigate equilibria in which consumers use a mixed strategy. The next Lemma shows the possible candidates for an equilibrium.

Proposition 3.2. *In any equilibrium in which consumers employ a mixed strategy firms price accordingly to an atomless price distribution, $F(p)$, defined on a convex support σ. Moreover, if $k = 0$ then $q_1 + q_2 = 1$, $q_1, q_2 \in (0, 1)$, while if $k > 0$, then either $q_1 + q_2 = 1$, $q_1, q_2 \in (0, 1)$ or $q_0 + q_1 = 1$, $q_0, q_1 \in (0, 1)$.*

There are two main observations which follow from Proposition 3.2. The first is that, despite the fact that consumers are fully homogenous, in any equilibrium price dispersion arises. Since consumers search randomly, ex-post in the market some consumers are more informed than others; in line with Burdett and Judd (1983), this allows

[5] I am assuming that a consumer provides the information to his neighbors surely. This represents a situation where local communication across consumers is perfect. More generally, I could relax this assumption by assuming that information is transmitted with some probability $\rho \in (0, 1)$. In this new setting the results of this paper will carry on qualitatively.

firms to extract profits by randomizing their prices. Second and more interestingly, when the network does not play any role, e.g. $k = 0$, for an equilibrium consumers must randomize between searching for one price and two prices. I refer to this as High Intensity search. However, when network externalities are taken into account another equilibrium candidate emerges where consumers randomize between searching once and not searching at all. I call this Low Intensity search. The possibility of a low intensity search equilibrium arises because the externalities produced in the consumer network allow even consumers who do not search on their own to observe some prices with strictly positive probability.

This preliminary analysis leads me to investigate the existence and the characterization of the high intensity and low intensity search equilibrium, respectively. As a presentation strategy, for each equilibrium candidate I first characterize firms' behavior, taken as exogenous consumers' strategy. This illustrates the direct effect that networks have on the way firms strategically price. Next, I endogenize consumers' behavior. Our main interest lies on the impact of network density on the consumers' search intensity, firms' pricing behavior and social welfare. Taken together, this analysis will clarify the effect of consumers' networks on market competitiveness.

4 High search intensity

Suppose consumers randomize between searching once and searching twice, i.e. $q_1 + q_2 = 1$, $q_1, q_2 > 0$. The expected number of consumers who observe only the price of firm i, say D_i, and the expected number of fully informed consumers, say $D_{i,j}$, can be written as follows,

$$D_i(k, q_1) = \frac{mq_1^{k+1}}{2^{k+1}} \tag{1}$$

$$D_{i,j}(k, q_1) = m\left(1 - \frac{q_1^{k+1}}{2^k}\right) \tag{2}$$

A consumer observes only the price of firm i (expression (1)) when he observes the price of firm i, $q_1/2$, and all his social contacts also observe only the price of firm i, $(q_1/2)^k$. Analogously, a consumer observes only the price of firm j with the same probability that a consumer observes only the price of firm i, i.e. $D_i = D_j$. Finally, with the remaining probability (expression (2)) a consumer observes both prices. It is readily seen that for a given q_1, the more a network is dense (the higher is k), the smaller is the fraction of partially informed consumers, expression (1), and the higher is the fraction of fully informed consumers, expression (2).

Using (1) and (2), the expected profit to firm i is

$$E\pi_i(p_i, p_j; k, q_1) = D_i(k, q_1)\, p_i + D_{i,j}(k, q_1)\, p_i\left[1 - F(p_i; k, q_1)\right] \tag{3}$$

The next Proposition summarizes equilibrium pricing by firms.

Proposition 4.1. *Assume* $q_1 + q_2 = 1,\ q_x \in (0,1)\,,\ x = 1,2.$ *In equilibrium:*

$$F\left(p;k,q_1\right) = 1 - \frac{q_1^{k+1}}{2\left(2^k - q_1^{k+1}\right)}\frac{\tilde{p}-p}{p},\ \forall p \in [\frac{q_1^{k+1}}{2^{k+1}-q_1^{k+1}}\tilde{p},\tilde{p}]$$

Furthermore, $F\left(p;k,q_1\right)$ *dominates in the first order stochastic sense* $F\left(p;k+1,q_1\right),$ $k = 0,...,m-1.$

Not surprisingly, Proposition 4.1 shows that it is possible to rank the price distributions with respect to k in the first-order stochastic sense: $F\left(p,k,q_1\right)$ first order stochastically dominates $F\left(p;k+1,q_1\right)$. Therefore, as k increases, firms charge on average lower prices. The intuition is as follows; when consumers hold more connections, the externalities in the consumers network are higher, ceteris paribus, which in turn enhances competition among firms. Figure 1 illustrates the equilibrium price distribution for different levels of network density.

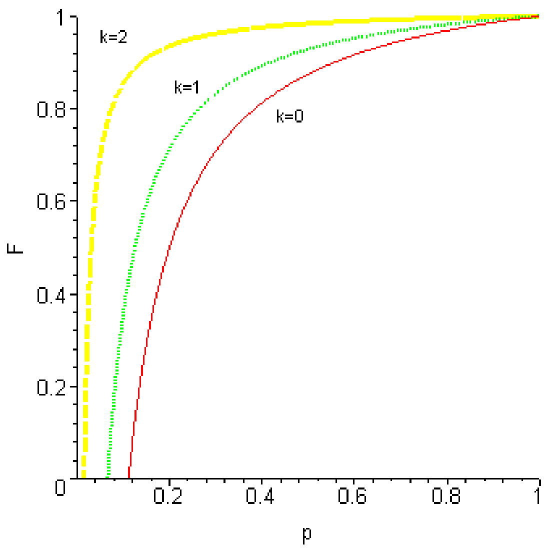

Figure 1. $F(p,k)$ for $q_1 = 0.5$ and $\tilde{p} = 1$.

I now endogenize the consumers side. I denote as $E(p)$ the price expected by a consumer who searches only for one price quotation, while $E_{\min}(p)$ indicates the price expected by a consumer who searches for two price quotations.[6] The expected utilities

[6]Formally, $E\left(p\right)$ $\left(E_{\min}\left(p\right)\right)$ is the expected value of one (two) draw from the price distribution.

to a consumer from the two distinct searching alternatives are:[7]

$$Eu\left(q_1=1\right) = \tilde{p}-\frac{q_1^k}{2^k}E\left(p\right)-\left(1-\frac{q_1^k}{2^k}\right)E_{\min}\left(p\right)-c \quad (4)$$

$$Eu\left(q_2=1\right) = \tilde{p}-E_{\min}\left(p\right)-2c \quad (5)$$

In words, an arbitrary consumer j who searches once, expression (4), observes only one price quotation when all his social contacts are searching once as well, q_1^k, and each of them observes the same price quotation that j observes, $1/2^k$. With the remaining probability consumer j is fully informed. In equilibrium a consumer should be indifferent between the two different search alternatives, i.e. $Eu\left(q_1=1\right)=Eu\left(q_2=1\right)$. This leads to the following equilibrium condition:

$$\frac{q_1^k}{2^k}\left[E\left(p\right)-E_{\min}\left(p\right)\right]=c \quad (6)$$

Each consumer trades-off the marginal cost of searching once more, c, with its marginal gain. The marginal gain of searching twice instead of once is the difference between buying at the expected price and at the expected minimum price, i.e. $E(p)-E_{\min}(p)$, weighted for the probability with which a consumer who searches for one price will indeed observe only one price quotation, i.e. $q_1^k/2^k$. When the network is empty, i.e. $k=0$, this marginal gain simply becomes the difference between the expected price and the expected minimum price. The next result provides the full characterization of the high intensity search equilibrium for any given $k=0,...,m-1$. Let $\bar{c}\left(k\right)=\frac{1}{2^k\left(2^{k+1}-2\right)}\left(\frac{2^{k+1}}{2^{k+1}-2}\ln\left(2^{k+1}-1\right)-2\right)$.

Theorem 4.1. *If $k=0$ there exists a $\tilde{c}>0$ such that for any $c\in(0,\tilde{c})$ a stable high intensity search equilibrium exists where firms behave according to Proposition 4.1 and q_1^* is the smallest solution of (6). If $k>0$, there exists $\bar{c}\left(k\right)<\tilde{c}$ such that for any $c\in\left(0,\bar{c}\left(k\right)\right)$ a high intensity search equilibrium exists where firms behave according to Proposition 4.1 and q_1^* is the unique solution of (6). Furthermore, this equilibrium is stable and $\bar{c}\left(k\right)$ is decreasing in k.*

I first elaborate on the existence condition of this equilibrium. Figure 2 below illustrates the equilibrium condition for different level of k. In the Figure I plot the LHS of expression (6) for different level of k as a function of q_1.

[7] More precisely expression 4 (resp. 5) indicates the expected utility of a consumer i who searches for one price quotation (resp. for two price quotations), given that all other consumers are searching for one price quotation with probability q_1, and for two prices with the remaining probability, $1-q_1$.

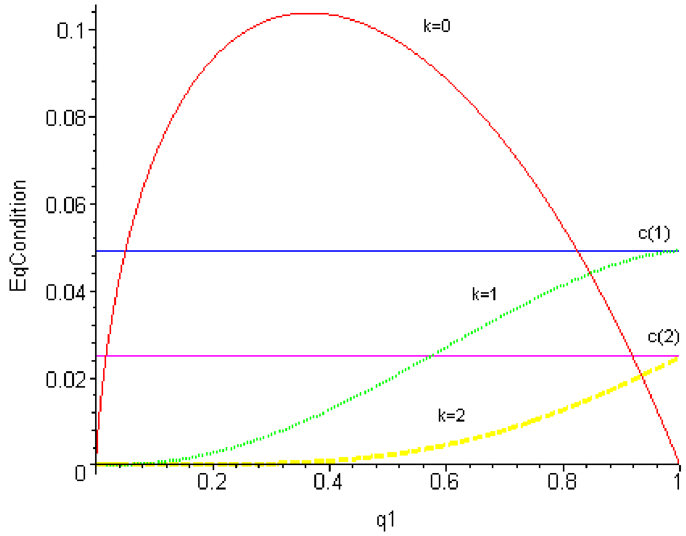

Figure 2. Equilibrium condition for k=0,1,2.

As already discussed, when $k = 0$ the model is equivalent to Burdett and Judd (1983). In this case, for a given c there are at most two equilibria, but only one is stable. Differently, when network externalities are allowed, there is a unique solution of the equilibrium condition (6), which is also stable. Interestingly, for this equilibrium to exist searching must be relatively inexpensive and as k increases the equilibrium exists for smaller and smaller search costs. The intuition is as follows. Network externalities reduce the marginal gains of searching twice instead of once and therefore for search costs sufficiently high a consumer cannot be indifferent between the two searching alternatives. The decrease in the marginal gains of searching twice is due to two effects. The first is that the probability of a consumer who searches once to be ex-post fully informed increases, and the second is that the difference between expected price and expected minimum price decreases in k.

I now turn to analyse the effect of consumers network on search incentives, expected prices, consumer surplus and social welfare. The next proposition summarizes the findings.

Proposition 4.2. *Suppose we move from k to $k+1$, $k \in [1, ..., m-2]$ and assume that $c < \bar{c}(k+1)$. Then: (a) consumers search less frequently, i.e. q_2 decreases, (b) expected price increases (c) social welfare increases and (d) consumer surplus decreases.*

I would like to elaborate on three aspects of this comparative static result. The first is that a consumer searches less frequently when the relationships in the network become denser: an increasing in the network degree leads consumers to free-ride more on each other. Secondly, this has a somewhat surprising effect on the equilibrium pricing behavior of firms: *expected price is higher in settings where consumers ex-*

change information more frequently. The intuition is the following. An increase in the degree of the consumer network induces two effects. The first is highlighted in Proposition 4.1 and it tells us that, keeping constant the consumers' behavior, an increase in the degree of the network increases the share of consumers which are ex-post fully informed. The second is a free-riding effect: more connections lead players to search less intensively and this results in a decreasing of expected fully informed consumers. When consumers search intensively, the free-riding effect offsets the former effect. Thus, an increase in the degree of the network decreases the equilibrium fraction of consumers which are fully informed; as a consequence firms compete less intensively and the expected price increases. In Figure 3a below I plot the probability of a consumer who searches once to be fully informed in equilibrium. In line with the intuition above, figure 3a shows that for a given search cost the information in the market decreases when the degree of the network increases. Figure 3b shows how the expected price varies with respect to the degree of the network in equilibrium.

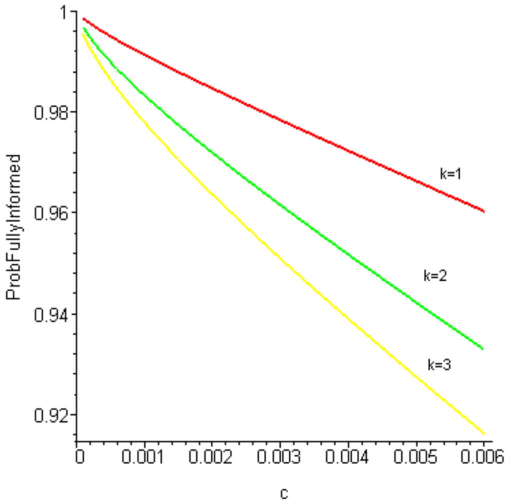

Figure 3a

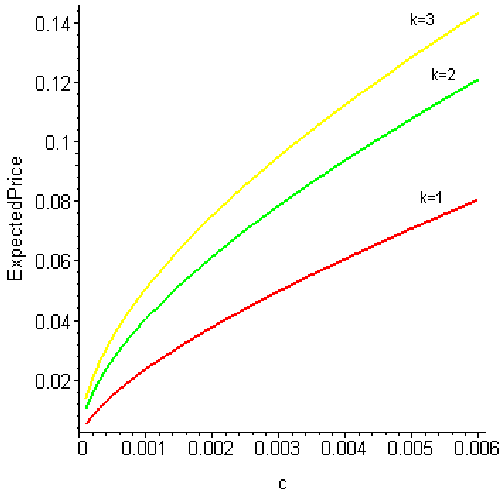

Figure 3b

Next, I note that consumer surplus decreases. This follows by noting that not only the expected price decreases but the same holds for the expected minimum price. Finally, I show that an increase in the degree of the network enhances social efficiency. This is due to the fact that the free-riding effect leads to saving on the total search cost, yet, since consumers search surely, each possible transaction is realized in equilibrium.

5 Low Intensity Searching

I now analyse the case in which consumers randomize between searching once and not searching at all, i.e. $q_0 + q_1 = 1$, $q_0, q_1 > 0$. I start by considering consumers' behavior as exogenously given. The expected fraction of consumers who observe only the price of firm i, say D_i, and the expected fraction of fully informed consumers, say

$D_{i,j}$, are:

$$D_i(k,q_0) = \frac{m(1-q_0)}{2}\sum_{x=0}^{k}\binom{k}{x}\frac{q_0^{k-x}(1-q_0)^x}{2^x} + \tag{7}$$
$$+mq_0\sum_{x=1}^{k}\binom{k}{x}\frac{q_0^{k-x}(1-q_0)^x}{2^x}$$
$$D_{i,j}(k,q_0) = m(1-q_0)\left(1-\sum_{x=0}^{k}\binom{k}{x}\frac{q_0^{k-x}(1-q_0)^x}{2^x}\right)+ \tag{8}$$
$$+mq\left(1-q_0^k-2\sum_{x=1}^{k}\binom{k}{x}\frac{q_0^{k-x}(1-q_0)^x}{2^x}\right)$$

The interpretation of expression (7) is as follows: the first term denotes the fraction of consumers who have searched once on their own and found firm i, i.e. $m(1-q_0)/2$, and that they have either received the same information or no information from their neighbors; the second term indicates the fraction of consumers who did not search, but that have received the price information of firm i from some of their social contacts. Expression (8) as a similar interpretation. Expressions (7) and (8) can be rewritten as follows:

$$D_i(k,q_0) = \frac{m\left[(1+q_0)^{k+1}-2^{k+1}q_0^{k+1}\right]}{2^{k+1}} \tag{9}$$
$$D_{i,j}(k,q_0) = \frac{m\left[2^k\left(1+q_0^{k+1}\right)-(1+q_0)^{k+1}\right]}{2^k} \tag{10}$$

Thus, the expected profit of firm i is:

$$E\pi(p_i,p_j;k,q_0) = D_i(k,q_0)\,p_i + D_i(k,q_0)\,p_i\left[1-F(p_i;k,q_0)\right] \tag{11}$$

The next Proposition summarizes the firms' price behavior in equilibrium.

Proposition 5.1. *Assume* $q_0+q_1=1$, $q_x\in(0,1)$, $x=0,1$. *In equilibrium:*

$$F(p;k,q_0) = 1-\frac{(1+q_0)^{k+1}-2^{k+1}q_0^{k+1}}{2\left(2^k\left(1+q_0^{k+1}\right)-(1+q_0)^{k+1}\right)}\frac{\tilde{p}-p}{p},\ \forall p\in\left[\frac{(1+q_0)^{k+1}-2^{k+1}q_0^{k+1}}{2^{k+1}-(1+q_0)^{k+1}}\tilde{p},\tilde{p}\right]$$

Furthermore, $F(p;k,q_0)$ *dominates in the first order stochastic sense* $F(p;k+1,q_0)$.

As in the high intensity search equilibrium, an increase in the degree of the network has a direct effect on the way firms price: the more dense a network is, the lower the expected price. This is illustrated in the Figure 4 below.

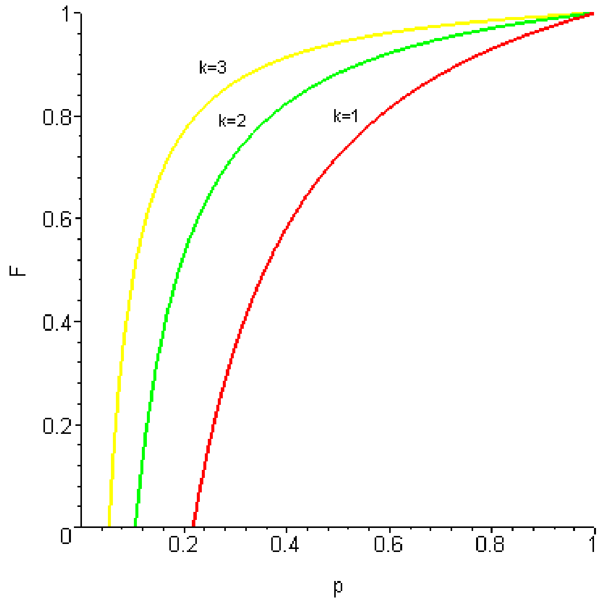

Figure 4. $F(p,k)$ for $q_0 = 0.5$ and $\tilde{p} = 1$.

I now endogenize the consumers' search behavior. Let $\alpha(q_0,k) = \sum_{x=0}^{k} \binom{k}{x} \frac{q_0^{k-x}(1-q_0)^x}{2^x}$ and $\beta(q_0,k) = \sum_{x=1}^{k} \binom{k}{x} \frac{q_0^{k-x}(1-q_0)^x}{2^{x-1}}$; the utility a consumer gets from the two distinct search alternatives is:

$$Eu(q_1 = 1) = \tilde{p} - \alpha(q_0,k) E(p) - (1 - \alpha(q_0,k)) E_{\min}(p) - c \quad (12)$$

$$Eu(q_0 = 1) = \tilde{p}\left(1 - q_0^k\right) - \beta(q_0,k) E(p) - \left(1 - q_0^k - \beta(q_0,k)\right) E_{\min}(p) \quad (13)$$

The interpretation of expression (12) is the following. Since a consumer searches once on its own he always buys: he buys at the expected price whenever his neighbors provide redundant or no information; otherwise he buys at the expected minimum price. Differently, a consumer who does not search, expression (13), buys only when at least one of his social contact searches, $\left(1 - q_0^k\right)$. The expressions (12) and (13) can be rewritten as follows:

$$Eu(q_1 = 1) = \tilde{p} - \frac{(1+q_0)^k}{2^k} E(p) - \left(\frac{2^k - (1+q_0)^k}{2^k}\right) E_{\min}(p) - c \quad (14)$$

$$Eu(q_0 = 1) = \tilde{p}\left(1 - q_0^k\right) - \left(\frac{(1+q_0)^k - 2^k q_0^k}{2^{k-1}}\right) E(p) - \quad (15)$$

$$- \left(\frac{2^{k-1}\left(1 + q_0{}^k\right) - (1+q_0)^k}{2^{k-1}}\right) E_{\min}(p)$$

In equilibrium every consumer must be indifferent between searching once and not searching at all, i.e $Eu(q_1 = 1) = Eu(q_0 = 1)$. This condition is satisfied if and only

if:
$$\frac{(1+q_0)^k - 2^{k+1}q_0^k}{2^k}\left[E(p) - E_{\min}(p)\right] + q_0^k\left(\tilde{p} - E_{\min}(p)\right) = c \tag{16}$$
The interpretation of (16) is similar to the interpretation of (6). The next result shows that for moderate value of search costs there exists at least a stable low intensity search equilibrium.

Theorem 5.1. *For any $k > 0$ there exists a $\tilde{c}$ such that for any $c \in (\bar{c}(k), \tilde{c})$ a stable low intensity search equilibrium exists where firms behave according to Proposition 5.1 and q_0^* is the smallest solution of (16).*

Theorem 5.1 tells us that for moderate search costs there exists at least a stable solution of the equilibrium condition (16). The proof is in the appendix and also shows that there always exists at least another solution of the equilibrium condition (16), which however is not stable. Further, numerical simulations reveal that these are the only two possible solutions.[8] In what follows I focus on the stable equilibrium. I start with a discussion of the existence of the low intensity search equilibrium. In Figure 5 below I plot the LHS of the equilibrium condition (16) with respect to q_0 for different levels of k.

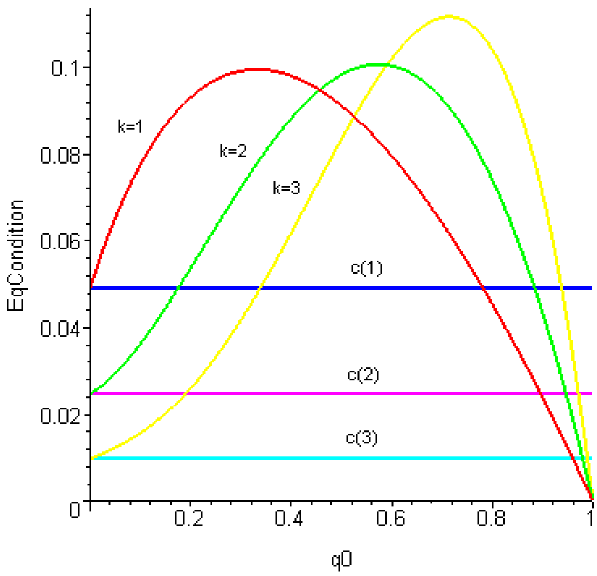

Figure 5. $F(p,k)$ for $q_0 = 0.5$ and $\tilde{p} = 1$.

I note that for any positive k the stable low intensity search equilibrium exists for search costs which are higher than $\bar{c}(k)$. Moreover, when the search cost is exactly equal to $\bar{c}(k)$ in equilibrium consumers search once with probability one and the low

[8] I have run numerical simulations of the equilibrium condition (16) for $k = 1, ..., 100$. The simulations reveal that there are at most two solutions, among which only the smaller one is stable.

intensity and high intensity search equilibrium coincide.[9] In contrast with the findings of Burdett and Judd (1983), when the search costs are sufficiently high, consumers free-ride on each other intensively and an equilibrium where consumers are always active is not strategically viable. Furthermore, as k increases this equilibrium exists for a wider range of parameters.

I now turn to examine the comparative statics with respect to k. The intractability of the equilibrium condition (16), leads me to rely on simulations. The findings are summarized in the following remark. [10]

Remark 5.1. *Suppose we move from k to $k+1$ and assume that $c \in (c(k), \tilde{c})$. Then: (a) consumers search less frequently, i.e. q_0 increases, (b) expected price decreases, (c) social welfare decreases and (d) consumer surplus increases.*

The numerical simulations confirm that consumers free-ride more on each other in denser networks. However, in sharp contrast with the high intensity search equilibrium, the effect on the way firms strategically price is reverse: *the higher the density of consumers network, the lower the expected price is.* Figure 6a below shows the expected probability for a consumer to be fully informed in equilibrium. Figure 6b illustrates the expected price for different network degrees in equilibrium.

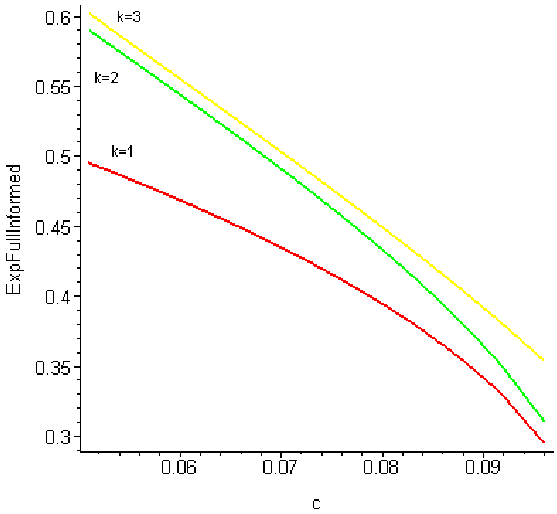

Figure 6a

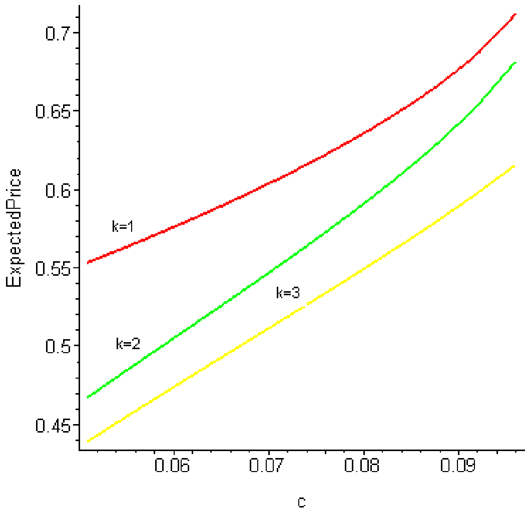

Figure 6b

Next, I note that total social welfare is decreasing in the density of the network. The reason is that when the search cost is moderate, a consumer who completely relies on his connections takes the risk to be ex-post completely ignorant about prices; in such a

[9]It is readily seen that the price distributions in Proposition 4.1 and 5.1 and the equilibrium conditions (6) and (16) coincide when $q_1 = 1$.

[10]I have run simulations for $k = 1, ..., 100$. For any k I first determine the range of the search costs for which the stable equilibrium exists (the smaller solution of (16)), say $[c_1(k), c_2(k)]$. Next, for each $c \in [c_1(k), c_2(k)]$, I derive the stable solution of equation (16), $q_0(k, c)$. Finally, using this value I compute the expected price and the social welfare.

case a transaction does not take place. When the degree of the network increases this negative effect, due the temptation of consumers to be inactive, dominates the realized savings in search cost and therefore the overall social welfare decreases. Finally, even if social welfare decreases, consumers surplus increases in the density of the network because both the expected price and the expected minimum price decrease.

6 Conclusion

I have developed a search model which examines the effect of endogenous externalities, inherent in consumers network, in the consumers' search incentives, firms' price behavior, consumer surplus and social welfare. I have shown that consumers search less frequently in denser networks due to a free-riding effect. This has a somewhat surprising implication on the firms' price behavior as well as on the overall performance of the market. In particular, when the search costs are sufficiently low the equilibrium expected price is higher and the consumers welfare is lower in settings where consumers networks are denser. Furthermore, when search costs are moderate, the market outcome becomes more inefficient as networks density rises.

To the best of my knowledge, this is the first model which examines the interplay between consumer networks and competition. There are many extensions which may be of interest for further research. The first is to examine the implication of asymmetric connections across players. Even if an analysis for any network architecture may be unfeasible, one could focus on a particular class of networks which matches much empirical evidence on social networks such as the star and variants of this architecture. For example, in a star network a natural question which arises is whether the center may use its structural position to extract private gains and what the implications for the market outcomes are. A second extension would be to endogenize the formation of the consumer network. A natural way of doing so is to consider a two-stage game where in the first stage consumers invest in connections and in the second stage the search game is played. A third extension would be to consider the impact of network relationships in a sequential search model.

Appendix

Proof of Proposition 3.1.
First, it is easy to verify that the strategy profile $\{p, q_0 = 1\}$, where $p \in [\tilde{p} - c, \tilde{p}]$ is a Nash equilibrium. I now prove that these are the only (generic) equilibria in which consumers employ pure strategies. There are two possibilities, which I analyse in turn. First, suppose $q_1 = 1$; if $k = 0$, then each consumer will observe only one price and as a consequence firms will charge $p = \tilde{p}$. However, as far as $c > 0$, a consumer strictly gains by not searching at all. Consider then that $k > 0$; I claim that if this were an equilibrium then firms will price according to an atomless price distribution $F(p)$ defined on a convex support σ. The reason is that since

$k > 0$ and $q_1 = 1$ there is a fraction of consumers which will observe both firms' prices with a strictly positive probability. Therefore, if firms charge a price p with a mass point, they will tight at that price with strictly positive probability, but then an individual firm has a strict incentive to undercut the atom. I now show that, given $k > 0$, an equilibrium where consumers search once with probability one, i.e. $q_1 = 1$, exists for a unique value of the search cost (it is not generic). The utility to a consumer is $Eu(q_1 = 1) = \tilde{p} - \frac{1}{2^k}E(p) - \left(1 - \frac{1}{2^k}\right)E_{\min}(p) - c$. In equilibrium it must be the case that $Eu(q_1 = 1) \geq Eu^d(q_x = 1)$, $x = 0, 2$, where $Eu^d(q_0 = 1) = \tilde{p} - \frac{1}{2^{k-1}}E(p) - \left(1 - \frac{1}{2^{k-1}}\right)E_{\min}$ and $Eu^d(q_2 = 1) = \tilde{p} - E_{\min}(p) - 2c$. Solving the two inequality I obtain that:

$$c = \frac{1}{2^k}\left(E(p) - E_{\min}\right)$$

Second, suppose $q_2 = 1$ and $k \geq 0$. It is easy to see that each consumer will observe always two prices. If this were an equilibrium firms would charge the competitive price, $p = 0$. However, a consumer is strictly better-off by searching only once. This completes the proof of the Proposition.■

Proof Proposition 3.2.

I first show that firms price according to an atomless price distribution $F(p)$. If $k = 0$ the model degenerates to the duopoly version of Burdett and Judd (1983) and the claim follows. Next, assume $k > 0$ and suppose there exists some price p^* with a mass point. Since consumers search at least once with some positive probability and $k > 0$, it follows that a fraction of consumers observe two prices with strictly positive probability. Therefore, firms would tie at the price p^* with strictly positive probability; in such a case a firm gains by undercutting p^*. This is a contradiction. I finally show that for any $k \geq 0$ the support σ must be convex. Suppose not, i.e. $\exists \tilde{\sigma} \subsetneq \sigma : F(p) = c \; \forall p \in \tilde{\sigma}$. Let $p^* = \inf \tilde{\sigma}$, then a firm charging p^* gains by increasing such price. This completes the proof of the first part of the Lemma.

I now show that if $k > 0$ in any equilibrium either $q_1 + q_2 = 1$, $q_1, q_2 \in (0, 1)$ or $q_0 + q_1 = 1$, $q_0, q_1 \in (0, 1)$. I start by claiming that $q_0 + q_2 = 1$ cannot be part of an equilibrium. Suppose it is, then firms would set the competitive price with probability one. The reason is that the expected demand of a firm derives from two sources: consumers who search on their own and consumers who do not search but obtain information from their social contacts. The former would always observe two prices, while the latter either do not observe any price or they also observe two prices. Using a standard undercutting argument it follows that firms must charge the competitive price. Since firms charge the competitive price with probability one a consumer strictly benefits by searching only once.

Next I show that $q_0 + q_1 + q_2 = 1$ cannot be part of an equilibrium. Suppose it is an equilibrium; the same argument above implies that $F(p)$ is atomless and it is defined on a convex support σ. In equilibrium it must be the case that $Eu(q_x = 1) =$

$Eu\left(q_y=1\right), x, y=0,1,2$. Let $\alpha\left(k\right)=\sum_{x=1}^{k}\binom{k}{x}\frac{q_0^{k-x}q_1^x}{2^x}$, then I obtain that:

$$
\begin{aligned}
Eu\left(q_0=1\right) &= \tilde{p}\left(1-q_0^k\right)-\alpha\left(k\right)E\left(p\right)-\left(1-q_0^k-\alpha\left(k\right)\right)E_{\min}\left(p\right)\\
Eu\left(q_1=1\right) &= \tilde{p}-\left(q_0^k+\frac{\alpha\left(k\right)}{2}\right)E\left(p\right)-\left(1-q_0^k-\frac{\alpha\left(k\right)}{2}\right)E_{\min}\left(p\right)-c\\
Eu\left(q_2=1\right) &= \tilde{p}-E_{\min}\left(p\right)-2c
\end{aligned}
$$

Solving for the equilibrium conditions it follows that:

$$q_0^k\left[\tilde{p}-E\left(p\right)\right]=q_0^k\left[E\left(p\right)-E_{\min}\left(p\right)\right]$$

Given that $q_0>0$ this condition is satisfied if and only if $\tilde{p}-E\left(p\right)=E\left(p\right)-E_{\min}\left(p\right)$. I now show that this is impossible. To see this I note that

$$E_{\min}\left(p\right)=2E\left(p\right)-\int_{\underline{p}}^{\tilde{p}}2pf(p)F(p)dp$$

Therefore:

$$E\left(p\right)-E_{\min}\left(p\right)=\int_{\underline{p}}^{\tilde{p}}2pf(p)F(p)dp-E\left(p\right)=$$

Integrating by parts I can show that:

$$\int_{\underline{p}}^{\tilde{p}}2pf(p)F(p)dp=\tilde{p}-\int_{\underline{p}}^{\tilde{p}}\left[F(p)\right]^2dp$$

which implies that:

$$E\left(p\right)-E_{\min}\left(p\right)=\left[\tilde{p}-E\left(p\right)\right]-\int_{\underline{p}}^{\tilde{p}}\left[F(p)\right]^2dp<\left[\tilde{p}-E\left(p\right)\right] \tag{17}$$

This is a contradiction and therefore the claim follows. Hence, the proof for the case $k\geq 1$ is complete.

I finally consider the case where $k=0$. The same argument used for $k\geq 1$, shows that $q_0+q_2=1$ and $q_0+q_1+q_2=1$ cannot be part of an equilibrium. Therefore, the only possibility left is $q_0+q_1=1$. If this were an equilibrium firms would charge the monopolist price. However, in such a case consumers cannot be indifferent between not searching and searching once, i.e. $Eu\left(q_0=1\right)=0>-c=Eu\left(q_1=1\right)$. This completes the proof of the Proposition.■

High Search Intensity Equilibrium

Proof of Proposition 4.1.

I first note that the upper bound of the price distribution must be the reservation price, $\tilde{p}$; for otherwise a firm charging $\overline{p}<\tilde{p}$ strictly gains by increasing it. This implies that the expected equilibrium profit is: $E\pi_i^*\left(\tilde{p}, p_j; k, q_1\right) = \frac{m q_1^{k+1}}{2^{k+1}}\tilde{p}$. In equilibrium a firm i must be indifferent between charging any price in the support σ, i.e. $E\pi_i\left(p_i, p_j; k, q_1\right) = E\pi_i^*\left(\tilde{p}, p_j; k, q_1\right), \forall p \in \sigma$. Solving this condition I obtain the expression of $F\left(p; k, q_1\right)$ and the expression of the lowerbound of the support is obtained by solving for $E\pi_i\left(\underline{p}_i, p_j; k, q_1\right) = E\pi_i^*\left(\tilde{p}, p_j; k, q_1\right)$. Finally, let $\psi = \frac{q_1^{k+1}}{2\left(2^k - q_1^{k+1}\right)}$, then it is easy to see that $\frac{\partial F(p;k,q_1)}{\partial k} > 0$ if and only if $\frac{\partial \psi}{\partial k} = \frac{q_1^{k+1} 2^{k-1}}{\left(2^k - q_1^{k+1}\right)^2} \ln \frac{q_1}{2} < 0$. This completes the proof.■

Proof Theorem 4.1.
The proof of the case $k = 0$ is the same as Burdett and Judd(1983) and therefore it is omitted. I focus instead in the case $k > 0$.

Let us define the RHS of expression (6) as $\phi\left(p; k, q_1\right) = \frac{q_1^k}{2^k}\left[E\left(p\right) - E_{\min}\left(p\right)\right]$. I start by showing that $\frac{\partial \phi(p;k,q_1)}{\partial q_1} > 0$. Suppose, without loss of generality that $\tilde{p} = 1$. Using the expression of the price distribution $F\left(p; k, q_1\right)$ defined in proposition 5.3. I can invert it to obtain:

$$p\left(z; k, q\right) = \frac{1}{g\left(z; k, q_1\right)} \tag{18}$$

where

$$g\left(z; k, q_1\right) = 1 + \frac{2\left(2^k - q^{k+1}\right)}{q^{k+1}}\left(1 - z\right) \tag{19}$$

I now note that:

$$\frac{2^k}{q_1^k}\phi\left(p; k, q_1\right) = 2\int_{\underline{p}(k,q_1)}^{1} pf(p; k, q_1)\left(1 - F(p; k, q_1)\right) dp - \int_{\underline{p}(k,q_1)}^{1} pf(p; k, q_1) dp$$

Integrating by parts yields,

$$\frac{2^k}{q_1^k}\phi\left(p; k, q_1\right) = \int_{\underline{p}(k,q_1)}^{1}\left[F(p; k, q_1)\left(1 - F(p; k, q_1)\right)\right] dp$$

Using the inverse function $p\left(z; k, q_1\right)$, I can write this expression as:

$$\frac{2^k}{q_1^k}\phi\left(z; k, q_1\right) = \int_0^1\left[p\left(\sqrt{z}; k, q_1\right) - p\left(z; k, q_1\right)\right] dz$$

Or,

$$\frac{2^k}{q_1^k}\phi\left(z; k, q_1\right) = \int_0^1 p\left(z; k, q_1\right)\left(2z - 1\right) dz$$

Let $a = q^{k+1}$, $b = 2\left(2^k - q^{k+1}\right)$ and $c = 2^{k+1}(2k+1)$, then,

$$
\begin{aligned}
\frac{2^k}{q_1^{2k}} \frac{\partial \phi(z;k,q_1)}{\partial q_1} &= \int_0^1 \frac{[ka(2z-1)+c(1-z)](2z-1)}{[a+b(1-z)]^2} dz \\
&= -\int_0^{1\backslash 2} \frac{[ka(2z-1)+c(1-z)](1-2z)}{[a+b(1-z)]^2} dz + \\
&\quad + \int_{1\backslash 2}^1 \frac{[ka(2z-1)+c(1-z)](2z-1)}{[a+b(1-z)]^2} dz
\end{aligned}
$$

I note that $\frac{[ka(2z-1)+c(1-z)]}{[a+b(1-z)]^2}$ is positive and increasing in z for any $z \in (0, 1/2)$ and that $[a+b(1-z)]^2$ is decreasing in z. Therefore:

$$
\frac{2^k}{q_1^{2k}} \frac{\partial \phi(z;k,q_1)}{\partial q_1} > -\int_0^{1\backslash 2} \frac{2^k(2k+1)(1-2z)}{2^{2k}} dz + \int_{1\backslash 2}^1 \frac{[ka(2z-1)+c(1-z)](2z-1)}{2^{2k}} dz
$$

I now note that $[ka(2z-1)+c(1-z)]$ is positive and it is decreasing in z, which implies that:

$$
\begin{aligned}
\frac{2^k}{q_1^{2k}} \frac{\partial \phi(z;k,q_1)}{\partial q_1} &> -\int_0^{1\backslash 2} \frac{2^k(2k+1)(1-2z)}{2^{2k}} dz + \int_{1\backslash 2}^1 \frac{q_1^{k+1}k(2z-1)}{2^{2k}} dz \\
&> \left(\frac{2^k(2k+1)+q_1^{k+1}k}{2^{2k}}\right) \int_0^1 (2z-1)\, dz \\
&> \left(\frac{2^k(2k+1)+q_1^{k+1}k}{2^{2k}}\right) \left(\frac{(2z-1)^2}{4}\right)_0^1 = 0
\end{aligned}
$$

Next, I note that $\lim_{q\to 0} \phi(p;k,q_1) = 0$ and that $\lim_{q\to 1} \phi(p;k,q_1) = \overline{c}(k)$. The facts that $\frac{\partial \phi(z;k,q_1)}{\partial q_1} > 0$, $\lim_{q\to 0} \phi(p;k,q_1) = 0$ and $\lim_{q\to 1} \phi(p;k,q_1) = \overline{c}(k)$ imply that for any $c \in (0, \overline{c}(k))$ there exists a unique solution, say $q_1^* \in (0,1)$, of the equilibrium condition (6), i.e. $\phi(p;k,q_1^*) = c$.

I finally show that consumers do not want to deviate. Given that all consumers randomize between searching once and twice, the expected utility to a consumer who deviates by not searching at all is:

$$
Eu^d(q_0 = 1) = \tilde{p} - \frac{q_1^k}{2^{k-1}} E(p) - \left(1 - \frac{q_1^k}{2^{k-1}}\right) E_{\min}(p)
$$

For an equilibrium it must be the case that $Eu^d(q_0 = 1) \leq Eu(q_1 = 1)$. Using the expression (4) it follows that this deviation is not profitable if and only if:

$$
c \leq \frac{q_1^k}{2^k} [E(p) - E_{\min}(p)]
$$

This condition is always satisfied because in equilibrium $c = \frac{q_1^k}{2^k}[E(p) - E_{\min}(p)]$. This completes the proof of the Theorem. ■

Proof Proposition 4.2.
I recall that the RHS of the equilibrium condition (6) may be written as:

$$\phi(z; k, q_1) = \int_0^1 \frac{q_1^k}{2^k} p(z; k, q_1)(2z - 1)\, dz$$

First, I show that $\frac{\partial \phi(z;k,q_1)}{\partial k} < 0$. The derivative of $\phi(z; k, q_1)$ with respect to k is:

$$\begin{aligned} \frac{\partial \phi(z; k, q_1)}{\partial k} &= -\frac{q_1^{2k+1}}{2^k} \ln \frac{2}{q_1} \int_0^1 \left(\frac{q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)}{\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]^2} \right)(2z-1)\, dz + \\ &+ \frac{q_1^{2k+1}}{2^k} \ln q_1 \int_0^1 \frac{q_1^{k+1}(2z-1)^2}{\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]^2} dz \end{aligned}$$

I note that the second term of this expression is weakly negative and that $-\frac{q_1^{2k+1}}{2^k} \ln \frac{2}{q_1}$ is also weakly negative. Therefore it is sufficient to show that:

$$\xi = \int_0^1 \left(\frac{q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)}{\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]^2} \right)(2z-1)\, dz > 0$$

To see this note that:

$$\begin{aligned} \xi &= -\int_0^{1/2} \left(\frac{q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)}{\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]^2} \right)(1-2z) + \\ &+ \int_{1/2}^1 \left(\frac{q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)}{\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]^2} \right)(1-2z) \end{aligned}$$

Since $\left(\frac{q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)}{\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]^2} \right)$ is increasing in z for $z \in (0, 1/2)$ and $\left[q_1^{k+1} + 2\left(2^k - q_1^{k+1}\right)(1-z)\right]$ is decreasing in z, then:

$$\xi > -\int_0^{1/2} \left(\frac{2^{k+1}}{2^{2k}} \right)(1-2z) + \int_{1/2}^1 \left(\frac{q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)}{2^{2k}} \right)(1-2z)$$

Furthermore, $q_1^{k+1} + 2\left(2^{k+1} - q_1^{k+1}\right)(1-z)$ is also decreasing in z, which implies that

$$\begin{aligned}
\xi &> -\int_0^{1/2} \frac{2^{k+1}}{2^{2k}}(1-2z) + \int_{1/2}^{1} \frac{q_1^{k+1}}{2^{2k}}(1-2z) = \\
&= \frac{2^{k+1}+q_1^{k+1}}{2^{2k}} \int_0^1 (2z-1)\,dz \\
&= \frac{2^{k+1}+q_1^{k+1}}{2^{2k}} \left(\frac{(2z-1)^2}{4}\right)_0^1 = 0
\end{aligned}$$

The fact that $\frac{\partial\phi(z;k,q_1)}{\partial k} < 0$ and that $\frac{\partial\phi(z;k,q_1)}{\partial q_1} > 0$ implies that if k increases then q_1 must also increase.

Second, I show that if k increases, then expected prices increase as well. Let $\psi(k,q_1) = \frac{q_1^{k+1}}{2\left(2^k - q_1^{k+1}\right)}$, then the expression of the price distribution defined in Proposition 4.1 can be rewritten as:

$$F(p) = 1 - \psi\frac{\tilde{p}-p}{p}$$

To prove the claim it is enough to show that:

$$\frac{d\psi}{dk} = \frac{\partial\psi}{\partial k} + \psi\frac{\partial q_1}{\partial k} > 0$$

I denote $\phi_k(k,q_1) = \partial\phi(k,q_1)/\partial k$ and $\phi_{q_1}(k,q_1) = \partial\phi(k,q_1)/\partial q_1$. Using the equilibrium condition $\phi(k,q_1) - c = 0$ and applying the implicit function theorem I can derive

$$\frac{\partial q}{\partial k} = -\frac{\phi_k(\cdot)}{\phi_{q_1}(\cdot)}$$

where

$$\begin{aligned}
\phi_k(k,q_1) &= -\frac{q_1^k}{2^k}\ln\frac{2}{q_1}\psi[(1+2\psi)\ln\frac{1+\psi}{\psi} - 2] + \frac{q_1^k}{2^k}\psi_k[(1+4\psi)\ln\frac{1+\psi}{\psi} - \frac{3+4\psi}{1+\psi}] \\
\phi_{q_1}(q_1,k) &= \frac{kq_1^{k-1}}{2^k}\psi[(1+2\psi)\ln\frac{1+\psi}{\psi} - 2] + \frac{q_1^k}{2^k}\psi_{q_1}[(1+4\psi)\ln\frac{1+\psi}{\psi} - \frac{3+4\psi}{1+\psi}]
\end{aligned}$$

Plugging the expressions $\frac{\partial q_1}{\partial k}$ in $\frac{d\psi}{dk}$, I obtain that

$$\frac{d\psi}{dk} = \frac{\psi}{\phi_{q_1}}\left(\left((1+2\psi)\ln\frac{1+\psi}{\psi} - 2\right)\left(\psi_k\frac{kq^{k-1}}{2^k} + \psi_{q_1}\frac{q_1^k}{2^k}\ln\frac{2}{q_1}\right)\right)$$

Since ϕ_{q_1} and $\psi(q_1,k)$ are strictly positive it follows that $\frac{d\psi}{dk} > 0$ if and only if

$$\left((1+2\psi)\ln\frac{1+\psi}{\psi} - 2\right)\left(\psi_k\frac{kq_1^{k-1}}{2^k} + \psi_{q_1}\frac{q_1^k}{2^k}\ln\frac{2}{q_1}\right) > 0$$

Computing the derivatives $\psi_k = -\frac{q_1^{k+1}2^k}{2\left(2^k-q_1^{k+1}\right)^2}\ln\frac{2}{q_1}$ and $\psi_{q_1} = \frac{(k+1)q_1^k 2^k}{2\left(2^k-q_1^{k+1}\right)^2}$ it follows that:

$$\left(\psi_k \frac{kq_1^{k-1}}{2^k} + \psi_{q_1}\frac{q_1^k}{2^k}\ln\frac{2}{q_1}\right) = \frac{q_1^{2k}}{2\left(2^k-q_1^{k+1}\right)^2}\ln\frac{2}{q_1} > 0$$

Furthermore, using the expression of $\psi(k, q_1)$ I obtain that:

$$\begin{aligned}(1+2\psi)\ln\frac{1+\psi}{\psi} - 2 &= \frac{2^k}{2^k - q_1^{k+1}}\ln\left(\frac{\left(2^{k+1}-q_1^{k+1}\right)}{q^{k+1}}\right) - 2 > \\ &> \frac{2^k}{2^k-1}\ln\left(2^{k+1}-1\right) - 2 > \\ &> -2 + 2\ln 3 > 0\end{aligned}$$

This proves the claim.

Third, I show that social welfare increases as k increases. To see this note that for a given k, the social welfare is $SW(k, q_1, c) = \tilde{p} - q_1 c - (1-q_1)2c = \tilde{p} - 2c + q_1 c$; since when k increases, q_1 increases then social welfare increases as well.

Finally, I show that the consumer surplus decreases as k increases. To see this note that the consumer surplus is $CS = Eu(q_1 = 1) = Eu(q_2 = 1) = \tilde{p} - E_{\min}(p) - 2c$. Given he price distribution $F(p; k, q_1)$, the distribution of the minimum price is $F_{\min}(p; , k, q_1) = F(p; k, q_1)(2 - F(p; , k, q_1))$. Using the expression for $F(p; k, q_1)$ illustrated in proposition 5.3. I obtain $F_{\min}(p; , k, q_1) = 1 - \psi^2\left(\frac{\tilde{p}-p}{p}\right)^2$. Therefore $\frac{\partial F_{\min}(p;,k,q_1)}{\partial k} < 0$ if and only if $\frac{d\psi}{dk} > 0$, which follows from above. This completes the proof.■

Low Search Intensity Equilibrium

Proof of Proposition 5.1.

I first note that for an equilibrium $\overline{p} = \tilde{p}$; for otherwise a firm charging $\overline{p} < \tilde{p}$ strictly gains by increasing such price. Using expression 11 and the fact that $\overline{p} = \tilde{p}$, it follows that the expected equilibrium profit is $E\pi^* = \left(m\left((1+q)^{k+1} - 2^{k+1}q^{k+1}\right)/2^{k+1}\right)\tilde{p}$. In equilibrium it must be the case that $E\pi(p) = E\pi^*\ \forall\, p \in \sigma$. Solving the equilibrium conditions I obtain the expression for $F(p; k)$. Similarly, the expression of the lower-bound is the solution of $E\pi\left(\underline{p}\right) = E\pi^*$. Finally, let $\psi(k, q_0) = \frac{(1+q_0)^{k+1} - 2^{k+1}q_0^{k+1}}{2\left(2^k\left(1+q_0^{k+1}\right) - (1+q_0)^{k+1}\right)}$; then to prove the first order stochastic dominance relation it is enough to see that $\frac{\partial\psi(k,q_0)}{\partial k} = \frac{\left[2^k\left(1-q_0^{k+1}\right)(1+q_0)^{k+1}(\ln(1+q_0) - \ln 2) + 2^k q_0^{k+1}\left((1+q_0)^{k+1} - 2^{k+1}q_0^{k+1}\right)\ln(q_0)\right]}{2\left(2^k\left(1+q^{(k+1)}\right) - (1+q)^{k+1}\right)^2} < 0$.■

Proof of Theorem 5.1.

Without loss of generality let $\tilde{p} = 1$. Using the expression of the price distribution

$F(p;k,q_0)$ defined in proposition 5.4, I can invert it to obtain:

$$p(z,k,q_0)=\frac{1}{g(z;k,q_0)} \tag{20}$$

where

$$g(z;k,q_0)=1+\frac{2^{k+1}\left(1+q_0^{k+1}\right)-2\left(1+q_0\right)^{k+1}}{\left(1+q_0\right)^{k+1}-2^{k+1}q_0^{k+1}}(1-z) \tag{21}$$

Using (20) the equilibrium condition (16) can be rewritten as follows:

$$\frac{\left(1+q_0\right)^k-2^{k+1}q_0^k}{2^k}\int_0^1 p(z,k,q_0)(2z-1)\,dz+q_0^k\left(1-2\int_0^1 p(z,k,q_0)(1-z)\,dz\right)=c \tag{22}$$

I denote as $\rho(z;k,q_0)$ the LHS of 22 and I note that:

$$\begin{aligned}\lim_{q_0\to 0}\rho(z;k,q_0) &= \bar{c}(k)\\ \lim_{q_0\to 1}\rho(z;k,q_0) &= 0\end{aligned}$$

Furthermore, I note that limit when q_0 goes to zero of the derivative of $\rho(z,k,q)$ is positive:[11]

$$\lim_{q_0\to 0}\frac{\partial\rho(z;,k,q_0)}{\partial q_0}=1$$

Hence, since $\rho(q_0,k)$ is positive at $q_0=0$, increasing in the neighbor of $q_0=0$ and it is zero at $q_0=1$ it follows that for any $k>0$ there exists a $\tilde{c}>\bar{c}(k)$ such that for any $c\in[\bar{c}(k),\tilde{c}]$ There exists at least two solutions of the equilibrium condition (16). It is easy to see that among these two solutions only the smaller one is stable.

I finally show that a consumer does not have an incentive to deviate. The expected utility to a consumer who deviates by searching twice is:

$$Eu^d(q_2=1)=\tilde{p}-E_{\min}(p)-2c$$

For an equilibrium it must be the case that this deviation is not profitable, i.e. $Eu^d(q_2=1)\le Eu(q_1=1)$. Using the expression (14) I obtain that $Eu^d(q_2=1)\le Eu(q_1=1)$ if and only if:

$$c\ge\frac{\left(1+q_0\right)^k}{2^k}\left[E(p)-E_{\min}(p)\right]$$

[11] I develop the result using the program Mathematica. To do this I compute the following transformation. Let $\rho_{q_0}(q_0,k)=\frac{\partial\rho(q_0,k)}{q_0}$, then $\lim_{q_0\to 0}\rho_{q_0}(q_0,k)=e^{\lim_{q_0\to 0}\ln\left(\rho_{q_0}(q_0,k)\right)^{q_0}}$. The computation is avaialable upon request of the author.

Using the equilibrium condition (16), I can rewrite this inequality as

$$E(p) - E_{\min}(p) \leq \tilde{p} - E(p)$$

In the proof of proposition 5.2 I have shown that:

$$E(p) - E_{\min}(p) = [\tilde{p} - E(p)] - \int_{\underline{p}}^{\tilde{p}} [F(p)]^2 \, dp < [\tilde{p} - E(p)]$$

Hence, given that all consumers randomized between searching once and not searching at all, a consumer does not want to deviate by searching twice. This completes the proof.■

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