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Multi-Store Competition

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Multi-Store competition:

Market segmentation or Interlacing?

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Abstract. This paper develops a model for multi-store competition between firms. Using the fact that different firms have different outlets and produce horizontally differentiated goods, we obtain a pure strategy equilibrium where firms choose a different location for each outlet and firms' locations are interlaced. The location decisions of multi-store firms are completely independent of each other. Firms choose locations that minimize transportation costs of consumers. Moreover, generically, the subgame perfect equilibrium is unique and when the firms have an equal number of outlets, prices are independent of the number of outlets.

Keywords: multi-store competition, Hotelling, interlacing
JEL codes: I18, D10, Z13.

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1. Introduction

Retail chains, which operate a chain of stores or outlets, account for the majority of all retail sales (Jones and Simmons, 1990). Retail chains are common throughout the retailing industry. The largest retail chains are department stores and supermarkets. Taken together, all shops or outlets of a certain retail chain usually have a regional, national or international geographical coverage. The success of retail chains is due to their easy recognition by customers and the realization of economies of scale through market power in purchasing, more efficient and effective marketing and advertising, and lower costs in distribution. Stores within a chain share the same façade, shop format and pricing policy. Typically, if a retail chain owns its own shops, prices are set at the central firm level and are the same for all outlets within a certain geographical scope (e.g. national level). For example, IKEA uses a national catalogue for its furniture, where nationwide prices are quoted, and also clothing chains such as H&M and C&A have a uniform pricing policy for all their shops.[1,2]

Retail chains invest heavily in the attractiveness of their concept, and with some success. Consumers clearly have different preferences concerning shops belonging to different chains, although the products sold in these different chains may be very similar from a more technological point of view. Thus, outlets are homogeneous when owned by the same firm, but they are heterogeneous across firms.

The location of stores or outlets can be modeled as a linear or circular city problem (as in Hotelling (1929) or Salop (1979)), with three main differences. First, companies may have

[1] An exception to this general pricing rule is that individual shops may decide on the prices for their sale articles to clear their stock.

[2] Retail chains not always own all of their shops, with franchising as an important alternative. Contrary to retail chains that own their own shops, a franchisor (such as McDonalds or Shell) may legally not restrict the determination of sale prices by franchisees, but may recommend sale prices. (this legislation is stipulated for the European Union in Commission Regulation (EEC) No 4087/88 of 30 November 1988 on the application of Article 85(3) of the Treaty to categories of franchise agreements (Official Journal L 359, 28/12/1988, pp. 46-52). In practice, however, the result is that prices are still quite homogenous across outlets. Most customers will even not be aware of any difference in prices, especially due to the similarity in shop format and products.

several outlets; each with its own endogenously determined location. Second, within a chain outlets are homogenous, but across chains they are heterogeneous. Third, firms' prices are identical across outlets, i.e., all outlets of one firm charge the same price. In this paper, we modify the circular city model to accommodate these three features mentioned above in order to analyze firms' choice of outlet locations and pricing policy.

The analysis of multi-store location and competition issues has a troublesome history in the economics literature. Teitz (1968) introduced multi-store competition in Hotelling's original model, and showed that no pure strategy equilibrium exists in the firms' location decisions. Subsequently, Martinez-Giralt and Neven (1988) using the assumption of quadratic transportation cost as introduced by d'Aspremont *et al.* (1979), obtained an equilibrium in which firms agglomerate all their outlets at the same point and at opposite ends of the market.[3] Hence, in their model neither firm will open more than one store. Since competition between firms with multiple outlets is very common indeed, the outcomes of the horizontal differentiation models of Teitz (1968) and Martinez-Giralt and Neven (1988) are difficult to accept. Recently, Pal and Sarkar (2002) approached the issue of multi-store competition in a completely different way. Instead of having consumers choosing which outlet to visit taking the different prices and locations into account, they model a situation where firms choose the amount they want to sell at each point on the circle assuming Cournot competition at each point. Moreover, the firms bring the products to the consumers' doors[4] and the question they ask is where the firms will locate their stores to minimize transportation costs.

In analyzing the multiple store location decision issue, we go back to the original model of Salop and have consumers buy from the (nearest) outlet they prefer the most. Firms'

[3] Martinez-Giralt and Neven (1988) assume that each outlet can choose its own price. It is easy to see, however, that introducing the restriction in their model that all outlets of one firm charge the same price, does not affect the results they obtain.

[4] This implicitly assumes that consumers cannot choose to buy at different prices at different points on the circle. Hence, the Pal and Sarkar (2002) model is of a very different nature from the other models.

location and pricing decisions are modeled as a three-stage game where firms simultaneously choose the number of outlets in the first stage, their locations in the second stage and, subsequently, their prices in the third stage.[5] Apart from the fact that firms can choose multiple outlets, we recognize the fact that consumers have heterogeneous preferences across chains even if these chains' outlets charge identical prices and locate at the same spot. In modeling this second (exogenous) dimension of product differentiation, we follow De Palma *et al.* (1985).[6] There are different ways to interpret this assumption. One interpretation, due to De Palma *et al.* (1985) is that sellers are unable to establish the differences in customers' tastes and the underlying variables. Firms can at best determine the buying behavior of customers' up to a probability distribution. Another interpretation is that firms cannot adapt their product line (in the short-run) to take these differences in tastes into account. We assume that consumers' preferences for this second dimension of product differentiation are uniformly distributed. This second dimension in our case includes different tastes for specific store formats such as façade, design and layout, image and product collection.

The resulting model generates a number of interesting outcomes. First, contrary to Teitz (1968) and Martinez-Giralt and Neven (1988), and due to the second dimension of heterogeneity, a pure strategy equilibrium where firms employ multiple outlets exists. More strikingly, the location decisions of multi-store firms are completely independent of each other. The spatial distribution of demand determines the specific locations. If the *distribution of demand along the circle is uniform*, a firm will choose to locate stores equidistantly. Any interlacing structure is an equilibrium, from head-to-head competition (where firms occupy the same locations) to perfect interlacing (where the difference between outlets belonging to

[5] It turns out that the analysis is not affected if the first two stages are analyzed as one stage; see also Section 3.

[6] They introduced this second dimension of product differentiation in order to restore Hotelling's equilibrium of minimal differentiation that was invalidated by D'Aspremont *et al.* (1979). Other literature that has analyzed two or more (endogenous) dimensions of product differentiation includes papers by Tabuchi (1994), Irmen and Thisse (1998) and Ansari *et al.* (1998).

different chains is maximal). This indeterminacy result is due to the indifference of each firm with regard to the distance between his shops and those of his competitor. If *the distribution of demand is non-uniform*, a firm differentiates the distance between stores according to the density of demand and, generally speaking, each chain has a unique optimal choice of locations. Consequently, if firms have the same number of stores, competition will be head-to-head. It follows that market segmentation where each firm has a "home base" of clustered outlets cannot be an equilibrium outcome.

We have two interesting findings in terms of the equilibrium prices that emerge. First, equilibrium prices reflect the dominance of firms in terms of the number of outlets within their chain. Being dominant in terms of the number of outlets, a firm is able to provide the nearest store to the larger part of the market. The resulting market power is reflected in a higher price than the competitor's price, the price difference being increasing in the difference in the number of outlets. Second, and more surprisingly, *provided firms have the same number of outlets* the total number of stores in the market appears to have no influence on the pricing decision, i.e., firms charge the same prices independent of the actual number of outlets. This finding, which may seem somewhat counterintuitive at first sight, is explained by the fact that when the outlets of a firm are located optimally, the number of outlets of the two firms determines the demand each firm faces. If both competitors have the same number of outlets their demand functions are identical and do not depend on that number.

Above we have already mentioned the literature that is most directly connected to the present paper. If we acknowledge that product line competition is similar to multi-store competition, then there is another related literature that comes to the fore. Brander and Eaton (1984) and Klemperer (1992) are important contributions in this field of product line competition that use similar models. The main difference with our paper lies in the fact that these papers only analyze and compare exogenously given product lines. On the other hand, contrary to us, they allow firms to charge different prices for the different brands.

Interestingly, Brander and Eaton (1984) show that market segmentation can be an equilibrium outcome. Since this is against the gist of our results, it can be inferred that non-uniform pricing (across outlets or product lines) is a necessary condition for a segmented market structure to arise.

The paper is organized as follows. Section 2 describes the model. The main results are given in Section 3 where we sequentially analyze location and pricing decisions for the model in the most general form. Section 4 analyzes location decisions in three special cases. This section provides more detailed results for the cases when distribution of consumers along the circle is uniform, when transportation costs are symmetrically linear and, finally, when both firms have chosen the same number of outlets in the first stage. Section 5 provides a discussion on how many outlets a firm wants to choose and Section 6 concludes. Proofs are contained in the Appendix.

2. The Model

Consider the following circular city model. There are two sellers (chain owners), each of whom can build a chain of outlets. The location of outlet *k* of firm *i* is denoted by x_i^k and the number of outlets of firm *i* is denoted by N_i. All locations of firm *i* on the circle are denoted by $\mathbf{x}_i = \left(x_i^1, \ldots, x_i^{N_i}\right)$, *i*=1,2. The length of the circle is normalized to be equal to 1. There is a unit measure of consumers distributed around the circle in accordance with a differentiable distribution function $\mu(x)$, $\mu(0)=0$, $\mu(1)=1$. All consumers are heterogeneous with respect to their preferences over the brands that are offered by the two different sellers. This type of heterogeneity is modeled by assuming that at any given location $x \in [0,1)$, where the density of buyers is $f(x) \equiv \mu'(x)$, consumers come in different types, denoted by *y*, and *y* is uniformly distributed over the range $[-\lambda, \lambda]$. The overall two-dimensional density function of a type y at a location x is, therefore, given by $h(x,y)=\frac{1}{2\lambda}f(x)$. A consumer j of type y_j whose location on the circle is x_j gets a utility $-p_1-t(d(x_j,\mathbf{x}_1))$ if he buys from seller 1 at price p_1, where $d(x_j,\mathbf{x}_i)$ is the distance the consumer has to travel from his location x_j to the closest location of seller i and $t(d)$ is the buyer's transportation costs.[7] If the consumer buys from seller 2, however, his utility is given by $-p_2-t(d(x_j,\mathbf{x}_2))-y_j$. Hence, a buyer of type y is willing to pay y (with y being positive or negative) more for the good of seller 1 than for the good of seller 2, *ceteris paribus*. We assume that every buyer ought to buy a good from either of the two sellers and they buy from the seller where the buyer's utility is maximized.

Firms' production costs are represented by the cost functions $C_i(D_i)$, where D_i is the demand for firm i and $C_i'>0$, $C_i''\geq 0$. In addition to the production costs, firms have to invest $I_i(N_i)$ in order to build a chain of N_i outlets, where $I_i'>0$, $I_i''\geq 0$. We assume that investment costs I_i and operational costs C_i are not very high such that both firms are always willing to build at least one outlet.[8]

Firms' location and pricing decisions are modeled as a three-stage game where in the first stage firms simultaneously decide how many outlets to build, in the second stage they choose their locations and, in the third stage, having observed each other outlets' locations, they simultaneously choose prices. Firms maximize their profits.

Given the sellers' locations $\mathbf{x}_i$ and prices p_i, for any location $x\in[0,1)$ we define a marginal type $y^*(x)$ as the consumer's type who is indifferent between buying from either of the sellers. All types $y>y^*(x)$ prefer buying from seller 1, while all types $y<y^*(x)$ prefer buying from seller 2. The marginal type itself is determined by

$$-p_1-t(d(x,\mathbf{x}_1))=-p_2-t(d(x,\mathbf{x}_2))-y^*(x),$$

[7] We could easily add a reservation price to this utility function assuming that the reservation price is high enough so that consumers will always buy one of the products.

[8] In Section 5, we will make this assumption more precise.

and takes the following form:

$$y^*(x) = p_1 - p_2 + t(d(x,\mathbf{x}_1)) - t(d(x,\mathbf{x}_2)).$$

We assume that $y^*(x) \in [-\lambda, \lambda]$, i.e., λ is sufficiently large in comparison with the transportation costs. This, in fact, implies that at every location *x*, there is an indifferent consumer. Then, the measure of buyers at location *x* who prefer to buy from seller 1 is

$$\int_{y^*(x)}^{\lambda} h(x,y)dy = \frac{1}{2\lambda} f(x)(\lambda - y^*(x)) = \frac{1}{2\lambda} f(x)(\lambda - (p_1 - p_2) - t(d(x,\mathbf{x}_1)) + t(d(x,\mathbf{x}_2))).$$

Hence, total demand for seller 1 becomes:

$$D_1(p_1, p_2, \mathbf{x}_1, \mathbf{x}_2) = \frac{1}{2} - \frac{p_1 - p_2}{2\lambda} - \frac{T_1 - T_2}{2\lambda}, \qquad (1)$$

where $T_i(\mathbf{x}_i) = \int_0^1 t(d(x,\mathbf{x}_i))f(x)dx$, i=1,2. T_i has a straightforward interpretation: it is simply the sum of the transportation cost of all consumers to travel to an outlet of firm *i*. Finally, operational profit firm *i* gets is given by

$$\pi_i(p_1, p_2, T_1, T_2) = p_i D_i - C_i(D_i). \qquad (2)$$

It should be noted that we have not made any specific assumptions about the shape of the transportation cost, the density of consumers along the circle and whether consumers can travel in both directions along the circle or there is a directional constraint.[9] In the next section, we analyze the model in this general form. The main assumption that is incorporated in this general model is the one with respect to the second dimension of consumer heterogeneity, namely that this heterogeneity of the preferences over brands is important enough (λ is large) and that consumers are distributed uniformly along this second dimension. Without these assumptions the analysis becomes technically very complicated.

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[9] In some applications, such as television news scheduling and bus and airline scheduling it is natural to assume that consumers can only move forward to the next 'selling point'. Literature on these applications with directional constraints is quite recent and involves Cancian *et al.* (1995), Nilssen (1997), Salvanes at al. (1997), Nilssen and Sørgard (1998), and Lai (2001).

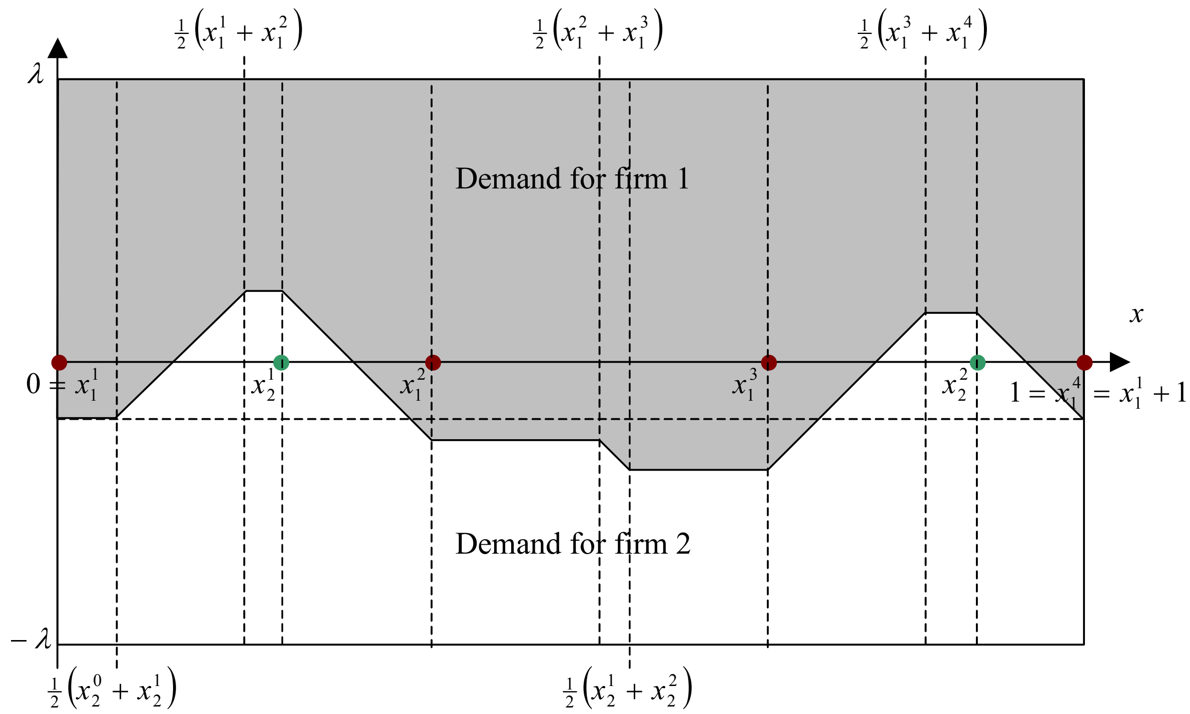

Figure 1. Division of the total demand over the firms for the linear bidirectional transportation costs.

Figure 1 gives, for some arbitrarily chosen parameter values, an illustration how demand is divided between the firms at given locations and prices for linear transportation costs when buyers can travel in either direction along the circle.[10]

3. Location and Pricing Decisions: General Results

In this section we analyze the last two stages of the model in its general form using backwards induction. We first analyze the last, price competition stage of the game and derive Nash equilibrium prices for given N_1 and N_2 and given location choices. Then we study the second stage of the game, where firms choose their locations.

From the previous section we know that in the 3rd stage of the game demands are given by expression (1), where T_i depends only on the locations of firm i. In a lemma stated and

[10] Non-linear transportation costs will generate similar pictures but with curved segments instead of straight lines.

proved in the Appendix we show that every subgame in the 3rd stage has a unique Nash equilibrium provided that λ is large enough.

The main result of our paper can then be formulated as follows.

Proposition 1. *For any number of outlets N_1 and N_2 chosen in the first stage the corresponding subgame has a subgame perfect Nash equilibrium. This equilibrium is unique for all generic spatial distributions of buyers and has the following properties:*

a) Each firm chooses its locations in order to minimize the sum of the transportation costs of all consumers to travel to its outlets.

b) The location choices of the two firms are in dominant strategies. Moreover, equilibrium locations of a firm depend only on the number of outlets it has chosen in the first stage and do not depend on the number of outlets the other firm has.

c) All locations of a firm are distinct, i.e., no two locations of the same firm coincide.

d) The equilibrium price and profit of a firm are strictly increasing and bounded functions of its own number of outlets and strictly decreasing functions of the number of outlets of its rival.

As it follows from equation (1), the number of outlets firms have and their locations affect firms' profits only through T_1 and T_2. It turns out that the profit of firm i monotonically decreases with respect to T_i. Consequently, the firm chooses locations of its outlets that minimize the sum of transportation costs of all consumers T_i, as stated in part (a) of Proposition 1. In order to understand the monotonic influence of T_i on profit π_i, it is useful to disentangle the way in which a firm's choice of locations affects its own profit. First, firm's profit π_i directly and negatively depends on T_i. We label this the direct effect. Second, the choice of T_i strategically affects the second stage equilibrium prices p_1^* and p_2^* which, in turn, also affect firm's profit π_i. This strategic or indirect effect of T_i on profit is positive.

It turns out that the direct effect always dominates the indirect effect. Indeed, the direct effect accounts for extra demand a firm can get by shifting its demand curve upwards. The indirect effect, on the other hand, accounts for a loss of demand due to the price reaction of the competing firm in order to *partially* recover the initial allocation of demand between the firms.

An interesting consequence of having an equilibrium in dominant strategies (part (b) of Proposition 1) is that firms do not need to observe the choice of the number of outlets N_1 and N_2 made in the first stage. Hence, even if both firms did not observe the number of outlets chosen in the first stage of the game, they would still choose the same locations. This non-observability of the outcomes is equivalent to simultaneity of choosing both the number of outlets and all their locations. Consequently, Proposition 1 remains valid for the corresponding changes in the game structure.

Similarly, even if one firm were able to observe the location choice of the other firm before choosing its own locations, it would still choose the same locations. This availability of extra information is equivalent to making the second stage of the game sequential. Thus, Proposition 1 remains valid also for this change in the game structure.

The fact that generically no two locations coincide (i.e., part (c) of Proposition 1) is, at an intuitive level, a consequence of the fact that firms want to minimize the transportation costs of the consumers. Two separate locations will in this sense always be better than two outlets on one location. Part (d) of Proposition 1 is mainly explained by the fact that if a firm has more locations, consumers are (generally speaking) more keen to buy from that firm as transportation costs will be lower. This increase in a firm's demand curve translates itself into higher equilibrium prices and profits.

It is difficult to characterize the location choices any further on the current level of generality. In the next section we derive more detailed results in three special cases under more restrictive assumptions.

4. Equilibrium Locations and Prices: Three Special Cases

We can derive some interesting properties of location choices and price decisions by making some specific assumptions with regard to distribution of consumers, transportation costs and number of outlets. We begin the investigation of the equilibrium locations by assuming linear transportation cost. Then we look at location decisions when consumers are uniformly distributed around the circle while keeping transportation costs general. Finally, we show what happens if both firms have decided to build the same number of outlets for any distribution of consumers along the circle.

Linear transportation costs

When transportation costs are linear equilibrium locations exhibit the following local property: firms choose more outlets where more consumers agglomerate. This result is formally stated and proved in the next proposition.

Proposition 2. *Suppose that transportation costs are linear: $t(d)=\tau d$. Then, for any two given outlet locations of firm **i**, x_i^{k-1} and x_i^{k+1}, with exactly one intermediate outlet x_i^k, the intermediate outlet is located closer to x_i^{k-1} than to x_i^{k+1} if, and only if, the average density of buyers in the interval $\left(x_i^k - \frac{1}{2}\left(x_i^k - x_i^{k-1}\right), x_i^k\right)$ is higher than in the interval $\left(x_i^k, x_i^k + \frac{1}{2}\left(x_i^{k+1} - x_i^k\right)\right)$.*

Proposition 2 is a consequence of the fact that in the general model firms choose locations so as to minimize the aggregate transportation costs of the consumers. When transportation costs are linear this intuitively implies that firms locate their outlets where there are more consumers. If transportation costs were not linear, the equilibrium locations would have had a similar flavor, but then we would have to compare equilibrium location decisions across different distributions of consumers, $\mu_1(x)$ and $\mu_2(x)$: if under $\mu_1(x)$ a firm would have chosen the location of two of its outlets at certain spots and if the distribution density under $\mu_2(x)$ would be higher between these two locations than under $\mu_1(x)$, then the distance between these two locations would be smaller under $\mu_2(x)$ than under $\mu_1(x)$.

Uniform spatial distribution

We now shift our attention to the special case where consumers are uniformly distributed over the circle. We will show that in this case, firms choose equidistant location structures.

Proposition 3. *If consumers are uniformly distributed over the circle then all the subgame perfect equilibria have firms spread their outlets evenly over the circle. This equilibrium is unique up to the choice of the first outlet of each firm.*

Since firms choose locations that minimize the sum of consumers' transportation costs (see part (a) of Proposition 1), this 'equidistant' result for a uniform distribution is straightforward. It also follows from Proposition 3 that if both firms have more than two outlets, a form of market segmentation where each firm has its own "home base" where they cluster their outlets together cannot be an equilibrium outcome. When both firms have an equal number of outlets, outlets of the two firms have to alternate so that an interlacing structure emerges.

Equal chain sizes

We finally return to the generic distributions considered in the previous section and consider the special case where both firms have the same number of outlets, i.e., $N_1=N_2=N$. Proposition 1 implies that the two firms then choose the same locations for their outlets. Thus, firms will be competing head-to-head, i.e., $x_1^k = x_2^k$ and by the definition of T_i, $T_1=T_2$.

This, in turn, has an important implication as in this case the expression for demand in equation (1) simplifies into

$$\begin{cases} D_1 = \dfrac{1}{2} - \dfrac{p_1 - p_2}{2\lambda} \\ D_2 = \dfrac{1}{2} - \dfrac{p_2 - p_1}{2\lambda} \end{cases}$$

This shows that if both competitors have the same number of outlets their demand functions are identical and, more importantly, do not depend on that number. Thus, the corresponding equilibrium prices and profits are insensitive to changes in the number of outlets of both firms *N*. Firms charge the same prices if they have the same number of outlets, regardless of that number.

5. On the Number of Outlets

The only question that remains open now is the choice of the number of outlets in the first stage. Substituting the optimal (minimized) $T_i = T_i^*(N_i)$ into the profit functions $\pi_i^*(T_1, T_2)$ we get the reduced-form profit functions in the first stage: $\pi_i^{**}(N_1, N_2) \equiv \pi_i^*\left(T_1^*(N_1), T_2^*(N_2)\right)$. In Section 2 we assumed that investment costs I_i and operational costs C_i are small enough so that both firms are always willing to build at least one outlet. We can now make this assumption more precise. Formally, we assume that $\pi_i^{**}(1, N_{-i}) - I_i(1) > 0$ for all N_{-i} and i=1,2.

Proposition 1 states that π_i^{**} decreases with N_{-i} and increases and is bounded with respect to N_i. Thus:

$$\pi_i^{**}(N_i, N_{-i}) \le \pi_i^{**}(N_i, 1) < \lim_{N_i \to \infty} \pi_i^{**}(N_i, 1) = \pi_i^*\left(0, T_{-i}^*(1)\right).$$

In other words, π_i^{**} is bounded uniformly. On the other hand, investment costs $I_i(N_i)$ are convex and increasing, thus unbounded. This implies that

$$\lim_{N_i \to \infty} \left(\pi_i^{**}(N_i, N_{-i}) - I_i(N_i) \right) = -\infty ,$$

where the convergence is uniform. That is, no firm is going to build infinitely many outlets. Hence, there exists a number $\hat{N}$ such that building more than $\hat{N}$ outlets and getting negative pay-off, is strictly dominated by building 1 outlet and getting positive pay-off. This implies, in turn, that the strategy space can be safely assumed to be finite and, consequently, the reduced form game always has a Nash equilibrium (possibly in mixed strategies). This is the content of the following proposition.

Proposition 4. *The game always has a subgame perfect Nash equilibrium that can involve mixed strategies in the first stage. In equilibrium both firms build a finite number of outlets.*

Unfortunately, we cannot be more specific than this about the number of outlets chosen by firms in equilibrium. There may be asymmetric equilibria where one firm has more locations than the other. Also, due to the fact that N_i has to be an integer number, it may happen that a pure strategy equilibrium does not exist and that the only equilibrium number of locations is in mixed strategies.

6. Conclusion

In this paper, we have analyzed a model where firms choose multiple outlets and uniform prices across outlets to compete in the market place. The products the firms produce are horizontally differentiated. Contrary to conventional wisdom in this field (see, e.g, Teitz (1968) and Martinez-Giralt and Neven (1988)), we obtain that a pure strategy subgame perfect Nash equilibrium where firms choose different locations for each outlet exists. Moreover, for all generic distribution functions, this equilibrium is unique. Firms, independent from each other, choose locations that minimize transportation costs. When firms choose an equal number of outlets, they choose identical locations. Consequently, market segmentation can never be an equilibrium. Firms that dominate in terms of number of locations charge higher prices and if the two firms have an identical number of outlets, equilibrium market prices are independent of the number of outlets chosen.

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Appendix

Lemma 1. *For any locations* $\mathbf{x_1}$ *and* $\mathbf{x_2}$ *chosen in stage 2 and for any cost functions* C_i *the third stage subgame has a unique Nash equilibrium in prices if* λ *is taken to be large enough. Equilibrium prices are continuously differentiable functions of* T_1 *and* T_2.

Proof of Lemma 1

Maximizing firm i's profit (2) with respect to p_i yields $2\lambda D_i(\tilde{p}_i) - \tilde{p}_i + C_i'(D_i(\tilde{p}_i)) = 0$. This first order condition defines a unique reaction function $\tilde{p}_i(p_{-i}, T_1, T_2)$ of firm i. To this end define a function $F_1(p_1)$ as $F_1(p_1) \equiv 2\lambda D_1(p_1) - p_1 + C_1'(D_1(p_1))$. It is easy to see that $F_1(0) = 2\lambda D_1(0) + C_1'(D_1(0)) > 0$ as $D_1(0)$ must be positive at zero prices for the firm to make non-negative profits. Then, as the demand D_1 is linear in p_1, there exists a price $\overline{p_1} > 0$ such that $D_1(\overline{p_1}) = 0$. Hence, it must be true that $F_1(\overline{p_1}) = C_1'(0) - \overline{p_1} < 0$, otherwise the firm is again would make losses. Finally, F_1 is a decreasing function as $F_1' = -2 - \frac{1}{2\lambda} C_1''(D_1) < 0$. Thus, there exists a unique reaction function $\tilde{p}_1(p_2, T_1, T_2) \in (0, \overline{p_1})$ satisfying $F_1(\tilde{p}_1) = 0$.

Similarly, the second reaction function is given by $F_2(\tilde{p}_2) = 0$, where $F_2(p_2) \equiv 2\lambda D_2 - p_2 + C_2'(D_2)$. For every pair ($T_1$, T_2) the third stage Nash equilibrium in prices (p_1^*, p_2^*) is determined by the following system:

$$\begin{cases} p_1^*(T_1, T_2) = \tilde{p}_1(p_2^*, T_1, T_2) \\ p_2^*(T_1, T_2) = \tilde{p}_2(p_1^*, T_1, T_2) \end{cases} \Rightarrow \begin{cases} p_1^* = 2\lambda D_1 + C_1'(D_1) \\ p_2^* = 2\lambda D_2 + C_2'(D_2) \end{cases} \tag{A.1}$$

We will show that for all sufficiently large values of λ this system has a unique solution. To this end we rewrite the system using $\gamma = \frac{1}{\lambda}$, $z_i = \gamma p_i^*$ and the definition of the demands D_i:

$$\begin{cases} z_1 = \frac{1}{2}(1+z_2) - \frac{1}{2}\gamma(T_1 - T_2 - C_1'(D_1)) = \frac{1}{2}(1+z_2) - \frac{1}{2}\gamma R_1(z_1, z_2) \\ z_2 = \frac{1}{2}(1+z_1) - \frac{1}{2}\gamma(T_2 - T_1 - C_2'(D_2)) = \frac{1}{2}(1+z_1) - \frac{1}{2}\gamma R_2(z_1, z_2) \end{cases},$$

where R_i and all their first order partials are finite. At γ=0 the system has a unique solution $z_1(0) = z_2(0) = 1$. On the other hand, the solution z_i continuously depends on γ at γ=0 as:

$$\begin{cases} z_1'(0) = -\frac{1}{3}(R_2(1,1) + 2R_1(1,1)) \\ z_2'(0) = -\frac{1}{3}(R_1(1,1) + 2R_2(1,1)) \end{cases}$$

Hence, there exits a Λ such that for all $\lambda > \Lambda$ the system has a unique solution.

In order to show that equilibrium prices $p_1^*(T_1, T_2)$ are continuously differentiable one can differentiate (A.1) and solve for partials $\frac{\partial p_1^*}{\partial T_1}$ and $\frac{\partial p_2^*}{\partial T_1}$:

$$\begin{cases} \dfrac{\partial p_1^*}{\partial T_1} = -\dfrac{2\lambda + C_1''(D_1)}{6\lambda + C_2''(D_2) + C_1''(D_1)} \\ \dfrac{\partial p_2^*}{\partial T_1} = \dfrac{2\lambda + C_2''(D_2)}{6\lambda + C_2''(D_2) + C_1''(D_1)} \end{cases}, \tag{A.2}$$

that ends the proof. ■

Proof of Proposition 1

We first show that if equilibrium exists it must satisfy properties (a), (b) and (c) of Proposition 1. Then, the existence of dominant strategies for both firms guarantees the existence of subgame perfect Nash equilibrium. Finally, we prove its uniqueness and establish part (d) of Proposition 1.

In any subgame (N_1, N_2) the locations $\mathbf{x_1}$ and $\mathbf{x_2}$ hence, T_1 and T_2 as well, are chosen in the second stage so as to maximize the reduced-form profit functions

$$\pi_i^*(T_1, T_2) \equiv \pi_i(p_1^*(T_1, T_2), p_2^*(T_1, T_2), T_1, T_2).$$

Differentiating π_1^* w.r.t. T_1 and taking into account (A.1) and (A.2) yields:

$$\frac{\partial \pi_1^*}{\partial T_1} = -\frac{4\lambda + C_1''(D_1^*)}{6\lambda + C_2''(D_2^*) + C_1''(D_1^*)} D_1^* < 0 .$$

It follows that each firm minimizes the sum of the transportation cost of all consumers to travel to one of its outlets. Each firm thus chooses its locations to minimize the corresponding T_i and part (a) of Proposition 1 is proven.

It is easily seen now that due to the monotonicity of π_1^* w.r.t. T_1 firm 1 has a dominant strategy, namely choosing locations in such a way that T_1 is minimized irrespective of T_2. The existence of the optimal location structure follows from the facts that, first, the reduced-form profit function $\pi_1^*(T_1, T_2)$ is continuous in T_1, which, in turn, continuously depends on $\mathbf{x_1}$ and, second, the feasible set for $\mathbf{x_1}$ is compact. Thus, part (b) of Proposition 1 is proven.

In order to show that all equilibrium locations are distinct we setup the problem of minimization of T_1 and derive the first order conditions. In case of no directional constraints they take the following form:

$$\int_{x_i^k - \frac{1}{2}\left(x_i^k - x_i^{k-1}\right)}^{x_i^k} t'\left(x_i^k - x\right) f(x)\,dx = \int_{x_i^k}^{x_i^k + \frac{1}{2}\left(x_i^{k+1} - x_i^k\right)} t'\left(x - x_i^k\right) f(x)\,dx\,,\ k=1,\ldots,N_i, \qquad \text{(A.3)}$$

It is easily seen that if $x_i^k = x_i^{k+1}$ for some k then it must hold for all k=1,…,N_i, which is impossible. In case of directional constraints the arguments are the same. Thus, no two locations coincide and part (c) of Proposition 1 is proven.

In order to proof the uniqueness of the subgame perfect Nash equilibrium for all generic distributions we proceed in two steps. First, we show that every solution to (A.3) is an isolated solution, i.e., the solution is locally unique.[11] Thus, there can be generically only a finite number of solutions. Then, we show that if a given distribution μ is such that (A.3) has multiple solutions, each one generating the same profit level to the firm, then the profit generated by each solution has different sensitivity to all generic changes of the distribution.

[11] The arguments can be easily adjusted for the directional constraints case.

This implies that in case of multiple local maxima, all of them yield different profits and, therefore, generically there exists a unique location pattern that maximizes the firm's profit.

The F.O.C. (A.3) forms a system of equations $\mathbf{F}(\mathbf{x}_i) = \mathbf{0}$. The Jacobian matrix of $\mathbf{F}$ has the following structure:

$$\frac{\partial F_j}{\partial x_i^k} = \begin{cases} 2t'(0)f\left(x_i^k\right) - \frac{1}{2}\left(t'\left(\frac{x_i^k - x_i^{k-1}}{2}\right)f\left(\frac{x_i^{k-1}+x_i^k}{2}\right) + t'\left(\frac{x_i^{k+1}-x_i^k}{2}\right)f\left(\frac{x_i^k + x_i^{k+1}}{2}\right)\right) + \int\limits_{\frac{1}{2}\left(x_i^{k-1}+x_i^k\right)}^{\frac{1}{2}\left(x_i^k+x_i^{k+1}\right)} t''\left(\left|x_i^k - x\right|\right)d\mu, \text{if } j = k \\ -\frac{1}{2}t'\left(\frac{x_i^k - x_i^{k-1}}{2}\right)f\left(\frac{x_i^{k-1}+x_i^k}{2}\right), \text{if } j = k-1 \\ -\frac{1}{2}t'\left(\frac{x_i^{k+1}-x_i^k}{2}\right)f\left(\frac{x_i^k+x_i^{k+1}}{2}\right), \text{if } j = k+1 \\ 0, \text{otherwise} \end{cases}$$

It is clearly seen that $f\left(x_i^k\right)$, i.e., the distribution density at the exact location x_i^k, affects only the corresponding diagonal entry in the Jacobian matrix of $\mathbf{F}$ and has no influence on any other entries. Hence, the Jacobian generically has full rank and, therefore, every solution of $\mathbf{F}(\mathbf{x}_i) = \mathbf{0}$ is locally unique.

Now let us suppose that there are multiple solutions of $\mathbf{F}(\mathbf{x}_i) = \mathbf{0}$. In particular, let $\hat{\mathbf{x}}_i$ and $\hat{\hat{\mathbf{x}}}_i$ be two solutions, i.e., $\mathbf{F}(\hat{\mathbf{x}}_i) = \mathbf{F}\left(\hat{\hat{\mathbf{x}}}_i\right) = \mathbf{0}$ provided $\hat{\hat{\mathbf{x}}}_i \neq \hat{\mathbf{x}}_i$. Both locations generate the following total transportation costs (in case of no directional constraint):

$$T_i(\hat{\mathbf{x}}_i) = \int\limits_0^1 t(d(x, \hat{\mathbf{x}}_i))f(x)dx \text{ and } T_i\left(\hat{\hat{\mathbf{x}}}_i\right) = \int\limits_0^1 t\left(d\left(x, \hat{\hat{\mathbf{x}}}_i\right)\right)f(x)dx.$$

Suppose that μ is such that $T_i(\hat{\mathbf{x}}_i) = T_i\left(\hat{\hat{\mathbf{x}}}_i\right)$, i.e., this two solutions $\hat{\mathbf{x}}_i$ and $\hat{\hat{\mathbf{x}}}_i$ generate the same level of transportation costs, thus, profits as well. Let us consider the following perturbation of the distribution density function: $f(x) + \alpha h(x)$, where h is an arbitrary function satisfying $\int\limits_0^1 h(x)dx = 0$. Then, both T_i become functions of α. Their derivatives α are given by:

$$\begin{cases} \dfrac{d}{d\alpha} T_i(\hat{\mathbf{x}}_i) = \int_0^1 t(d(x, \hat{\mathbf{x}}_i))h(x)dx, \\ \dfrac{d}{d\alpha} T_i\left(\hat{\hat{\mathbf{x}}}_i\right) = \int_0^1 t\left(d\left(x, \hat{\hat{\mathbf{x}}}_i\right)\right)h(x)dx \end{cases}$$

This is so because $\hat{\mathbf{x}}_i$ and $\hat{\hat{\mathbf{x}}}_i$ are maximizors of $T_i(\mathbf{x}_i)$ (envelope theorem). Then,

$$\frac{d}{d\alpha}\left(T_i(\hat{\mathbf{x}}_i) - T_i\left(\hat{\hat{\mathbf{x}}}_i\right)\right) = \int_0^1 \left(t(d(x, \hat{\mathbf{x}}_i)) - t\left(d\left(x, \hat{\hat{\mathbf{x}}}_i\right)\right)\right)h(x)dx$$

The last expression generically is not equal to zero as $\hat{\hat{\mathbf{x}}}_i \neq \hat{\mathbf{x}}_i$ and h is an arbitrary function. Thus, every locally optimal location pattern generates generically different profit levels and, therefore, there exists a unique equilibrium location that maximizes profit.

Finally, we derive the relations between the size of a chain, prices and profits. One may verify that $T_i(\hat{\mathbf{x}}_i)$ strictly decreases with N_i. Indeed, adding up one additional outlet to the firm's locations leads to a strictly higher profit due to the possibility of imitating the "old" pattern with two coinciding outlets, which is strictly sub-optimal. Then, in the proof of Lemma 1 we already derived that $\frac{\partial p_1^*}{\partial T_1} < 0$ and $\frac{\partial p_2^*}{\partial T_1} > 0$, hence, p_i^* strictly increases and p_{-i}^* strictly decreases with N_i. Similarly,

$$\begin{cases} \dfrac{\partial \pi_1^*}{\partial T_1} = \left(\dfrac{\partial p_2^*}{\partial T_1} - 1\right) D_1^*(T_1, T_2) = -\dfrac{4\lambda + C_1''(D_1^*)}{6\lambda + C_1''(D_1^*) + C_2''(D_2^*)} D_1^*(T_1, T_2) < 0 \\ \dfrac{\partial \pi_2^*}{\partial T_1} = \left(\dfrac{\partial p_1^*}{\partial T_1} + 1\right) D_2^*(T_1, T_2) = \dfrac{4\lambda + C_2''(D_2^*)}{6\lambda + C_1''(D_1^*) + C_2''(D_2^*)} D_2^*(T_1, T_2) > 0 \end{cases},$$

hence, π_i^* strictly increases and π_{-i}^* strictly decreases with N_i.

Finally, as the F.O.C. (A.3) implies that $\lim_{N_i \to \infty}\left(x_i^k - x_i^{k-1}\right) = 0$ for all k, the following limits can be readily shown: $\lim_{N_i \to \infty} T_i(\hat{\mathbf{x}}_i) = 0$, $\lim_{N_i \to \infty} p_i^*(T_i(\hat{\mathbf{x}}_i), T_{-i}) = p_i^*(0, T_{-i})$ and $\lim_{N_i \to \infty} \pi_i^*(T_i(\hat{\mathbf{x}}_i), T_{-i}) = \pi_i^*(0, T_{-i})$, that ends the proof of Proposition 1. ■

Proof of Proposition 2

Rewriting (A.3) for $t(d)=\tau d$ yields:

$$\int_{x_i^k-\frac{1}{2}\left(x_i^k-x_i^{k-1}\right)}^{x_i^k} f(x)dx = \int_{x_i^k}^{x_i^k+\frac{1}{2}\left(x_i^{k+1}-x_i^k\right)} f(x)dx .$$

Writing $\langle f\rangle_{(a,b)}$ for the average density over an interval (a,b), results in

$$\left(x_i^k - x_i^{k-1}\right)\langle f\rangle_{\left(x_i^k-\frac{1}{2}\left(x_i^k-x_i^{k-1}\right),x_i^k\right)} = \left(x_i^{k+1} - x_i^k\right)\langle f\rangle_{\left(x_i^k,x_i^k+\frac{1}{2}\left(x_i^{k+1}-x_i^k\right)\right)} .$$

The statement of the proposition then follows immediately. ■

Proof of Proposition 3

Rewriting (A.3) for $f(x)=1$ yields $t'\left(\frac{1}{2}\left(x_i^k - x_i^{k-1}\right)\right) = t\left(\frac{1}{2}\left(x_i^{k+1} - x_i^k\right)\right)$. It can be easily verified that in the case of the directional constraint, when buyers have to travel only *clockwise* to the nearest outlet of the seller, (A.3) takes the following form: $t\left(x_i^{k+1} - x_i^k\right) = t\left(x_i^k - x_i^{k-1}\right)$.

As $t' > 0$, it is easy to see that $x_i^k = \frac{1}{2}\left(x_i^{k+1} - x_i^{k-1}\right)$ in both cases. Hence, an equidistant location structure is the unique optimum for any given N_i. ■