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Maarten C.W. Janssen

Tinbergen Institute
The Tinbergen Institute is the institute for economic research of the Erasmus Universiteit Rotterdam, Universiteit van Amsterdam and Vrije Universiteit Amsterdam.

Tinbergen Institute Amsterdam
Keizersgracht 482
1017 EG Amsterdam
The Netherlands
Tel.: +31.(0)20.5513500
Fax: +31.(0)20.5513555

Tinbergen Institute Rotterdam
Burg. Oudlaan 50
3062 PA Rotterdam
The Netherlands
Tel.: +31.(0)10.4088900
Fax: +31.(0)10.4089031

IMITATION OF COOPERATION IN PRISONER'S DILEMMA GAMES WITH SOME LOCAL INTERACTION

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Abstract. In this paper I study conditions for the emergence of cooperative behavior in a dynamic model of population interaction. The model has finitely many individuals located on a circle. The pay-off of each individual is partly based on the (local) interaction with neighbors and partly on (uniform) interaction with the whole population. The dynamics is driven by imitative behavior. I show that for a large class of parameters cooperation will emerge if the population is large; if the population is small, defection will prevail in the long run. The result contrasts with conventional wisdom which says that the larger the population, the less likely cooperation will be.

Key Words: Cooperation, Prisoner's Dilemma, Evolutionary Game Theory, Local Interaction

JEL-codes: C72, D62

Correspondence Address:
Maarten Janssen
Department of Economics/Micro
Erasmus University
P.O. Box 1738
3000 DR ROTTERDAM
The Netherlands

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1. Introduction

Different approaches exist to explaining cooperative behavior in the Prisoner's Dilemma Game. With perfectly rational agents cooperation is an equilibrium outcome if the chance that the game is repeated another time is sufficiently large. With boundedly rational agents in evolutionary game theory, cooperation may emerge if the interaction between agents is non-uniform, i.e., the chance that cooperative types meet each other is higher than the chance that they meet defective types. One such a case is when interaction is between siblings (see, e.g., Bergstrom and Stark, 1993). Another is when interaction is only between neighbors as in Eshel, Shaked and Samuelson (1998). It is commonly thought in both approaches that due to increasing anonymity cooperation is less likely to be observed in large populations than in smaller ones. In this paper I present a dynamic model that tell us that this is not necessarily so.

I consider the interaction between members of two different populations. The two populations have lived in isolation of each other and for reasons not analysed here, one population has evolved in the past towards cooperative behavior and the other towards defective behavior. Now that I see an increase in the interaction between different societies, I want to study the question whether the interaction between members of the two populations results in one type of behavior driving out the other or in the two types of behavior coexisting next to each other. This type of analysis has been done for coordination games in Goyal and Janssen (1997).

In the model each individual agent interacts with anyone in the population, but more so with their neighbors. Given some population configuration each individual receives a certain pay-off. The pay-off is a weighted average of the pay-offs under strict uniform and local interaction. The implications of *two types of imitation dynamics* are examined. Under *Imitating the Best Agent* (IBA), each agent observes the pay-off of each individual in the neighborhood and individuals imitate the agent who has received the highest pay-off in the previous period. Under imitating the best behavior (cooperation or defection), each agent only observes the average pay-off of cooperators and defectors in the neighborhood and they imitate the action that has yielded the highest average pay-off in the neighborhood (IBB). I will look at the development of the number of cooperative agents over time by investigating the behavior of the boundary between the two populations.

Roughly speaking, our main result is that for a wide range of parameter values and for any of the above mentioned dynamic processes there exists a threshold value (depending on the parameter values) of the population size: if the population size is larger than this threshold value cooperation will emerge, if on the other hand the population size is smaller than the threshold value cooperation will vanish. (For some other parameter values coexistence of the behaviors of the two populations is possible). The intuition for the main result can be grasped by comparing the pay-off of a cooperative type who is surrounded by other cooperative agents and the pay-off of a defective type who is surrounded by other defective agents. What matters in both classes of dynamic processes is *not* the *absolute* value of the pay-off, but the pay-off of a certain type *relative* to the pay-offs of others (cf., Vega-Redondo, 1997). The total pay-off difference between the cooperative and the defective type under consideration can be decomposed in a pay-off difference due to local interaction and a pay-off difference due to uniform interaction. The pay-off difference due to local interaction is independent of the size of the population and positive, i.e., the cooperative type receives a larger pay-off out of local interaction than the defective type. The pay-off difference due to uniform interaction itself can be decomposed into two parts. A first part is due to the fact that in a match with a given agent of the population, the defective type always receives a higher pay-off than the cooperative type. This part is also independent of the size of the population. A second part is due to the fact that under uniform interaction, a cooperative type meets relatively more defective agents than the defective agents themselves do. This is because a given agent will be matched with one of the *other* agents in the population.[1] This second part also favors the defective agents, but this part of the uniform interaction effect vanishes as the population size becomes large. Given the observations above it is then relatively easy to see that the uniform interaction effect may dominate the local interaction effect for small populations, whereas the reverse may hold true for large populations.

From the above it becomes clear that what is important for the result is *(i)* the presence of some neighborhood effect and *(ii)* some form of non-best reply dynamics. If interactions were uniform in the population, each agent will roughly meet the same agents as the others and in such a world, defection always yields a higher pay-off (as it is the dominant strategy). Also, over time best reply dynamics will result in defection as for any given behavior of a set of opponents,

[1] The fact that under uniform interaction you do not play a game with yourself is also used in a different context by Huck and Oechsler (1995).

defect is always the best reply. On the other hand, for the general idea of the results to hold, it is not so important whether pay-off comparisons are made locally or globally. In this paper I mainly focus on imitative behavior in which the pay-off comparisons are within the neighborhood.

There is a large and rapidly growing literature in evolutionary game theory explaining the emergence of cooperative behavior in populations. I provide a sketchy overview of some of the main results. Two branches can be distinguished: a large part of the literature considers the evolution of behavior in a *repeated* version of the prisoner's dilemma; a smaller part considers the one shot game. The literature on the repeated PD game started with investigations by Axelrod (1984) and Maynard Smith (1982). More recently, May and Nowak (1992, 1993) study how cooperation may emerge when agents interact in two-dimensional spatial structures. They use computer simulations showing that depending on the interaction structure complex patterns of cooperative and defective behavior may persist. Ashlock, Stanley and Tesfatsion (1994) allow players to choose their opponents in such a way that the interaction structure emerges endogenously. The evolution of cooperation in finitely and infinitely repeated prisoner dilemma games has also been studied by Bicchieri (1990), Fudenberg and Tirole (1990) and Nachbar (1992), among others. The main difference with the present paper is that I focus on the dynamics of cooperative behavior when each game is played as a one shot game.

Evolution of cooperation in one shot games has been studied by Bergstrom (1995), Bergstrom and Stark (1993) and Frank (1987), among others. What makes cooperation a feasible outcome in Frank's model is that before they play the game agents observe a noisy signal which informs them about the type of behavior to be expected from the opponent. This signal is supposed to be encoded in someone's genes. Players can make their one shot strategy conditional upon the observed signal. Bergstrom (1995) and Bergstrom and Stark (1993) also regard genes as the determinants of behavior and the object of selection. In their models, children inherit their genetical make-up from their parents. Cooperation emerges in their papers, because the prisoner's dilemma game is (mainly) played between siblings and as siblings have similar genes cooperative genes may do better than defective ones. The local interaction aspect of our model is the counterpart of the interaction between siblings in the papers by Bergstrom and Stark. Our paper is, however, closer in spirit to the main body of papers in evolutionary game theory in economics in the sense that the fundamental object on which the evolutionary or imitation process works is a strategy and *not* genes.

The paper closest to ours is Eshel et al. (1998). They show that under IBB with strict local interaction within a small neighborhood the majority of agents in a population will display cooperative behavior. There are a few differences between their paper and the present one. The present paper is simpler as at any point in time, there is one group of cooperators and one group of defectors. This is convenient as the dynamic process can be investigated by looking at the boundary between regions. Drawback of this assumption is that we can not allow for the possibility of mutants entering the population. On the other hand, the present paper is more general as different types of imitative behavior are considered and as a combination of local and uniform interaction is studied.

The paper is organized as follows. The next section describes the model that is used. Section 3 presents the results under imitating the best behavior dynamics and Section 4 briefly discusses the imitating the best agent dynamics. Section 5 concludes. Proofs are given in the appendix.

2. The Models

I consider a dynamic model where time is indexed by $t = 0, 1, 2, ...$ and the population is located on a circle. In each period, each member of the population plays the following Prisoner's Dilemma game with other people in the population, with $b > a > d > c$.

Pay-offs:		Strategy of player 2	
		C	D
strategy of player 1	C	(a,a)	(c,b)
	D	(b,c)	(d,d)

Table 1. Pay-offs in the Prisoner's Dilemma game

At t = 0, the population is divided into two groups, one group of cooperators and one group of defectors. The pay-off of individual i is composed of two parts. The *local interaction* part measures the pay-off out of interaction with the K immediate neighbors on either side. In most cases I will restrict the analysis for computational convenience to neighborhoods of two agents, one on each side. The *uniform interaction* part measures the pay-off out of interaction with anyone in the population. The average pay-off to individual i can be written as

$$U_i = \alpha \frac{\sum_{k=1}^{K} u(i, i-k) + u(i, i+k)}{2} + (1-\alpha) \frac{\sum_{j \neq i} u(i, j)}{(N-1)} \quad \textbf{(1)}$$

where U_i is the average pay-off of the individual agent on location i, α is the relative weight of local interaction in the average pay-off of individual i, $u_i(i,j)$ is the pay-off of individual i when interacting with individual j and N is the size of the population.

Equation (1) allows for two interpretations. First, one may consider a situation in which each individual meets only one other agent in the population in a certain period. The parameter α can then be interpreted as the probability that the individual will meet someone out of his neighborhood (and within this neighborhood, he will meet someone at random). In this interpretation, U_i is the expected pay-off of individual i and I have to allow agents to observe the *expected* pay-off of other agents (in the neighborhood) to retain a determinstic dynamics. A second and more favored interpretation is one in which agents meet everyone in the population, but meet neighbors more frequently than others. In this interpretation α measures the relative importance of local interaction for the average pay-off of an individual. Note that α is assumed to be independent of the population size.[2]

With neighborhoods of two agents, one can distinguish three subcategories of the cooperative, respectively defective, behavior depending on the neighbors. I will denote by D^+ and C^+ the individuals who defect, respectively, cooperate and who are surrounded by cooperative agents; D^0 and C^0 represent the individuals who have one cooperating neighbor and one defecting neighbor and D^- and C^- stand for the individuals who are surrounded by defecting individuals. When there is just one group of cooperators and one of defectors, the following statements hold. The '0' category of each type is the individual at the boundary between a cooperative region and a defective region. In any period in time there are maximally two individuals of each of these two '0' categories. The categories D^+ and C^- only come into play if there is one defective, respectively one cooperative, type left over in the population and these categories will de discussed only when appropriate.

What is important for the dynamics of the system is how the four different categories that remain (D^0, D^-, C^+ and C^-) are ranked in terms of average pay-off. Straightforward calculations show that the pay-offs of the four categories under consideration are as follows:

[2] If, on the other hand, the size of the neighborhood and the number of interactions with each agent in the neighborhood is fixed, then the importance of local interaction decreases with N. I think my way of comparing populations with different sizes, keeping α constant, is more natural.

category	*frequency*	*payoff*	**(2)**
D^{-}	$N-x-2$	$\frac{x(b-d)(1-\alpha)}{(N-1)}+d$	
D^{0}	2	$\frac{x(b-d)(1-\alpha)}{(N-1)}+d+\frac{\alpha(b-d)}{2}$	
C^{+}	$x-2$	$a-\frac{(N-x)(a-c)(1-\alpha)}{(N-1)}$	
C^{0}	2	$a-\frac{(N-x)(a-c)(1-\alpha)}{(N-1)}-\frac{\alpha(a-c)}{2}$	

where the number of cooperative (defective) agents is denoted by x (N-x). A few observations with respect to these pay-offs can be made. Obviously, the average pay-off of the '+' categories of each type is larger than the average pay-off of the '0' category and this in turn is larger than the average pay-off of the '-' category. Also, the average pay-off of the D^0 individuals is larger than that of the C^0 individuals. This is because the pay-off out of local interaction is the same for both categories, but the pay-off out of uniform interaction is larger for the D^0 individuals.

Under *imitation dynamics*, it is important to specify the information agents have about the pay-offs of other individuals. In this paper, I assume that agents observe the pay-offs of agents in their neighborhood, but not the pay-offs of agents outside the neighborhood. The basic idea here is that one needs a sufficient number of interactions with another agent to figure out what her pay-off actually is. By imitation I mean that an individual chooses the same action (cooperate or defect) that the type (or individual) has chosen who is imitated. If the neighborhood consists of individuals who take the same action, then this action is by definition the best in the neighborhood. Within this framework I analyze the dynamics stemming from "imitating the best agent in your neighborhood" (IBA) as well as from "imitating the behavior (cooperative or defective) with the highest average pay-off in your neighborhood" (IBB). According to the first dynamics, agents observe the pay-off of each individual in their neighborhood and choose, in the next period, the action the agents with the highest pay-off has chosen in this period. According to the second dynamics, agents only observe the average pay-off of cooperative and defective behaviors in their neighborhood and choose, in the next period, the behavior that has yielded the highest pay-off in this period.

3. Imitating the Best Behavior

In this section I consider a dynamic process based on "imitating the best behavior in your neighborhood" (IBB), where the best behavior is defined as the behavior (cooperative or defective) with the highest average pay-off. In what follows, I shall focus on the behavior of the boundary between the regions of cooperators and defectors. Note that the neighborhoods of the two individuals at the boundary differ: the neighborhood of the cooperative individual at one side of the boundary consists of one C^+ agent and one D^0 agent and that the neighborhood of the defector at the other side of the boundary consist of one C^0 agent and one D^- agent. Accordingly, I have to calculate the average pay-offs in the two neighborhoods separately.

To understand the main idea behind the Propositions below, I first concentrate on the special cases in which (a-c) = (b-d). The expressions for the average pay-off of the cooperator, resp. defector in the neighborhood of the defective agent (D^0) easily follow from the previous section:

$$\overline{U}_{D^0}(C) = a - \frac{(N - x)(a - c)(1 - x)}{N - 1} - \frac{\alpha(a - c)}{2}$$
$$\overline{U}_{D^0}(D) = \frac{x(b - d)(1 - \alpha)}{N - 1} + d + \frac{\alpha(b - d)}{4}$$

where $\overline{U}_{D^0}(C)$, resp. $\overline{U}_{D^0}(D)$, is the average pay-off of cooperators, respectively, defectors in the neighborhood of the D^0 individual. From these expressions it follows that in the special case under consideration $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$ if

$$N\left[\frac{\alpha(a - c)}{4} - (d - c)\right] > (a - d) - 3\alpha(a - c)/4. \qquad \textbf{(3)}$$

Similarly, the average pay-off of the cooperators, resp. defectors in the neighborhood of the cooperative individual C^0 is given by

$$\overline{U}_{C^0}(C) = a - \frac{(N - x)(a - c)(1 - \alpha)}{N - 1} - \frac{\alpha(a - c)}{4}$$
$$\overline{U}_{C^0}(D) = \frac{x(b - d)(1 - \alpha)}{N - 1} + d + \frac{\alpha(b - d)}{2}$$

where $\overline{U}_{C^0}(C)$, resp. $\overline{U}_{C^0}(D)$, is the average pay-off of cooperators, resp. defectors in the neighborhood of the C^0 individual. It is easily seen that in the special case of $(a\text{-}c) = (b\text{-}d)$ the following is true: $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$ if, and only if, $\overline{U}_{C^0}(C) > \overline{U}_{C^0}(D)$. Hence, when condition (3) is satisfied, the D^0 individuals will switch to cooperation and, as the above argument is independent of time and the number of cooperators in the population, cooperation will be the most observed behavior in the long run. In the context of proposition 2 and 3 it is shown in the appendix that the population will either have two defectors or will cycle between one and three defectors in the long run. On the other hand, when condition (3) is violated it is clear that $\overline{U}_{D^0}(C) < \overline{U}_{D^0}(D)$ and $\overline{U}_{C^0}(C) < \overline{U}_{C^0}(D)$ so that defection will prevail in the long run. Condition (3) also reveals that for cooperation to emerge in the long run N has to be larger than a certain cut-off value, which depends on the pay-off parameters and the value of α. This result is formally stated in proposition 1 and graphically illustrated in figure 1.

Proposition 1. Suppose $(a\text{-}c) = (b\text{-}d)$ and $x_0 = 2$. There is a time period T such that for all $t >, T$,

(i) if $N\left[\frac{\alpha(a-c)}{4} - (d-c)\right] > (a - d) - 3\alpha(a-c)/4$, then $x_t \geq N\text{-}3$.

(ii) if $N\left[\frac{\alpha(a-c)}{4} - (d-c)\right] < (a - d) - 3\alpha(a-c)/4$, then $x_t = 0$.

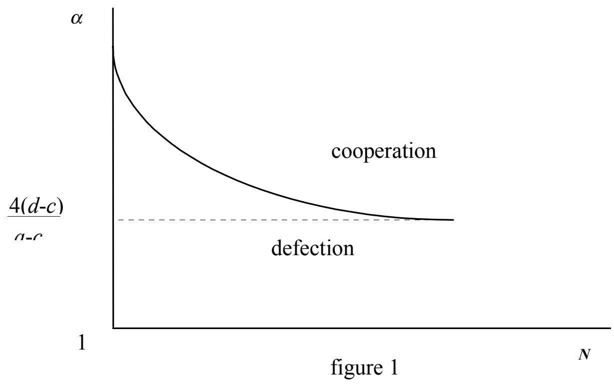

figure 1

The condition $(a\text{-}c) = (b\text{-}d)$ has two important implications. First, it implies that the ranking of pay-offs of cooperators and defectors at the boundary does not depend on the number of cooperators in the population and, hence, that this ranking does not change endogenously over time. Second, it implies that if cooperation is better on average than defection in the neighborhood of the defector at the boundary than it is also better in the neighborhood of the cooperator at the boundary, and vice versa. The two Propositions below show how the result indicated above generalizes to other values of the pay-off parameters. It turns out that for a given value of α the result generalizes for large and for small values of N. However, for intermediate values of N new phenomena emerge: if $(a\text{-}c) > (b\text{-}d)$, then the initial number of cooperators becomes important in co-determining the long-run outcome of the population (dependence on initial conditions). If, however, $(a\text{-}c) < (b\text{-}d)$, then the population will converge to a stable configuration with a group of cooperators and a group of defectors coexisting next to each other.

To be able to formally state the results for intermediate values of N, I need to introduce three additional parameters, namely x^{1*}, x^{2*} and $x^{1\sim}$. These variables are defined as follows:

$$(a - d)(N - 1) = \alpha(N - 1)\frac{2(a - c) + (b - d)}{4} + (1 - \alpha)\left[N(a - c) - x^{1*}\left((a - c) - (b - d)\right)\right] \quad \textbf{(4)}$$

$$(a - d)(N - 1) = \alpha(N - 1)\frac{2(b - d) + (a - c)}{4} + (1 - \alpha)\left[N(b - d) - x^{2*}\left((a - c) - (b - d)\right)\right] \quad \textbf{(5)}$$

$$(a - d)(N - 1) = \frac{\alpha}{4}(N - 1)\left[2(b - d) + (a - c)\right] + (1 - \alpha)\left[N(a - c) - x^{1\sim}\left((b - d) - (a - c)\right)\right] \quad \textbf{(6)}$$

The parameter x^{1*}, resp. x^{2*}, may be interpreted for the case $(a-c) < (b-d)$ as the number of cooperators in the population such that the defector respectively cooperator, at the boundary is indifferent between defecting and cooperating. (Note that the number need not be a natural number). A similar interpretation holds true for $x^{1\sim}$ in case $(a-c) < (b-d)$.

I am now ready to state the two main propositions.

Proposition 2. Suppose $(a\text{-}c) > (b\text{-}d)$ and $x_0 > 2$. There is a time period T such that

(i) if $N\left[\alpha\frac{2(a-c)-(b-d)}{4}-(d-c)\right] > (a\text{-}d)-\frac{\alpha}{4}\left[2(a-c)+(b-d)\right]$

then $x_t \geq N\text{-}3$ for all $t > T$,

(ii) if $N\left[\alpha\frac{2(b-d)-(a-c)}{4}-(b-a)\right] > (a\text{-}d)-\frac{\alpha}{4}\left[2(b-d)+(a-c)\right]$

then $x_t = 0$ for all $t > T$,

(iii) if the conditions under (i) and (ii) are *not* satisfied,

then $x_t \geq N\text{-}3$ for all $t > T$ if $x_0 > x^{1*}$

$x_t = 0$ for all $t > T$ if $x_0 < x^{2*}$

$x_t = x_0$ for all t if $x^{1*} < x_0 < x^{2*}$.

Proposition 3. Suppose $(a\text{-}c) < (b\text{-}d)$ and $x_0 > 2$. There is a time period T such that

(i) if $N\left[\alpha\frac{2(b-d)-(a-c)}{4} - (b-a)\right] > (a-d)-\frac{\alpha}{4}\left[2(b-d)+(a-c)\right].$

then $x_t > N\text{-}3$ for all $t > T$;

(ii) if $N\left[\alpha\frac{2(a-c)-(b-d)}{4} - (d-c)\right] > (a-d)-\frac{\alpha}{4}\left[2(a-c)+(b-d)\right]$

then $x_t = 0$ for all $t > T$;

(iii) if the conditions under (i) and (ii) are *not* satisfied,

then $x_t = 0$ for all $t > T$ if $x^{1\sim} < 5$

x_t cycles around $x^{1\sim}$ with period 4 for all $t > T$ if $x^{1\sim} > 6$.

Two elements of the stated propositions deserve some further explanation. First, the initial number of cooperators, x_0 has to be larger than or equal to 2 in order for cooperation to have any chance of surviving. Second, the first part of both Propositions should be read as follows: depending on the initial number of defectors (whether it is odd or even) either the population converges to everyone cooperating in the long run or the population converges to a two-period cycle in which the number of defective agents alternates between 1 and 3.

4. Imitating the best Agent

I have argued in Section 3 that when local interaction is confined to meeting one individual on either side, then at the boundary the individual who defects receives a higher pay-off than the one (at the other side of the boundary) who cooperates. This implies that under IBA with local interaction in a neighborhood of two, the defective agent will keep on playing defect. On the other hand, the cooperative agent at the boundary may (if the D^0 agent receives a higher pay-off than the C^+ agent) or may not (if the D^0 agent receives a smaller pay-off than the C^+ agent) switch to defect. Hence, cooperation is not a feasible long-run outcome in the population at large; only coexistence or defection are to show that this particular result is sensitive to the neighborhood structure and that the general result also holds true under IBA I will now consider local interaction in a neighborhood of four (two on each side). As the main ideas of the analysis are quite similar to the analysis under IBB, I will only consider the case $(a\text{-}c) > (b\text{-}d)$ and concentrate on the condition on the parameters that makes cooperation the predominant long-run behavior in the population.

In the figure below the boundary between a region of cooperators and a defectors is depicted and individuals 1 and 2 have chosen to cooperate in the past and agents 3 and 4 have chosen to defect. I will investigate the long-run behavior of the population by looking at the behavior of the boundary in the next period.

C	C	C	D	D	D
0	1	2	3	4	5

The pay-offs of the four individuals is respectively:

$$U_1 = \alpha \frac{3a + c}{4} + (1-\alpha)\frac{(x-1)a + (N-x)c}{N-1} \qquad \textbf{(7)}$$

$$U_2 = \alpha \frac{2a + 2c}{4} + (1-\alpha)\frac{(x-1)a + (N-x)c}{N-1}$$

$$U_3 = \alpha \frac{2b + 2d}{4} + (1-\alpha)\frac{(x\,b) + (N-x-1)d}{N-1}$$

$$U_4 = \alpha \frac{b + 3d}{4} + (1-\alpha)\frac{(x\,b) + (N-x-1)d}{N-1}$$

Comparing the pay-offs of individuals 1 and 3 reveals that $U_1 > U_3$, if and only if,

$$\frac{\alpha}{4}\left[(a - d) + (c - d) + 2(a - b)\right] (N - 1) \ > \ (1 - \alpha) \ \left[(d - c)N + (a - d) + x(b - d - (a - c))\right].$$

This inequality holds for all x if it holds for $x = 0$. Rewriting yields

$$N\left[\alpha \frac{3(a - c) - 2(b - d)}{4} \ - \ (d - c)\right] \ > \ (a - d) \ - \ \frac{\alpha}{4}\left[2(a - c) + 3(b - d)\right]. \qquad \textbf{(8)}$$

As agent 1 is in 3's neighborhood, agent 3 observes 1's pay-off. Also, 3 observes the pay-off of agents 4 and 5, but these pay-offs are lower than his. Hence, individual 3 will switch to cooperation in the next period. Hence, if the above inequality holds, the boundary between the regions of cooperators and the defectors will be between individuals 3 and 4. As the above argument is independent of time, it can be used to prove that in the long run the population consists predominantly of cooperators.[3]

[3] Note that depending on the number of cooperators in the first period in relation to the size of the population, the population may converge to all agents cooperating or to a two-period cycle in which N-1 (or N-2) cooperate in period t and N-5 (or N-6) cooperate in period t+1.

5. Conclusion

In this paper, I have investigated conditions under which cooperation will emerge in a population in which individuals are matched to play a Prisoner's Dilemma game. Part of the interactions between individuals are local and part of them are global. It is shown that for a large class of parameter values and for different kinds of dynamic processes, cooperation will emerge if, and only if, the population size is large enough.

The main principle on which the result is based is that only relative performance matters in evolution and relative performance of defectors becomes poorer when the population size increases (due to a smaller difference in pay-offs out of uniform interaction). It has been shown that this principle holds for different types of imitative behavior.

Appendix

Proof of Proposition 1

(i) Suppose $(a - c) > (b - d)$ In this case the following is true: if $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$, then also $\overline{U}_{C^0}(C) > \overline{U}_{C^0}(D)$. Moreover, it is easily seen that if $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$ for $x = 0$, then it holds for all values of x. Finally, $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$ for $x = 0$ if

$$N\left[\alpha\frac{2(a-c)-(b-d)}{4}-(d-c)\right] > (a - d) - \frac{\alpha}{4}[2(a - c) + (b - d)].$$

Hence, if this condition holds, the D^0 agent at the boundary will switch to cooperation, while the C^0 agent will stick to cooperation. Hence, the boundary between the regions shifts and more agent cooperate. This process continues until either there is no defector left or there is only one D^+ agent left in the population. This agent will be imitated by its neighbors, but they will switch back to cooperation the next period. Hence, for some T, we have that for all $t > T$, $x_t \geq N - 3$.

(ii) When $(a - c) > (b - d)$, we have that $\overline{U}_{C^0}(D) > \overline{U}_{C^0}(C)$ implies $\overline{U}_{D^0}(D) > \overline{U}_{D^0}(C)$. Moreover, it is easily seen that if $\overline{U}_{C^0}(D) > \overline{U}_{C^0}(C)$ for $x = N$, it also holds for any $x < N$. Finally, $\overline{U}_{C^0}(D) > \overline{U}_{C^0}(C)$ for $x = N$ if

$$N\left[\alpha\frac{2(b-d)-(a-c)}{4}-(b-a)\right] > (a - d) - \frac{\alpha}{4}[2(b-d)+(a-c)]$$

Hence, when this condition holds, the C^0 agent at the boundary will switch to defection and the D^0 agent will continue to defect. This process of C^0 agents switching to defection continues until all agents defect. (The last cooperative agent may belong to the C^{-} category and he will switch to defection as well.)

(iii) Suppose that the conditions under (i) and (ii) are both *not* satisfied. For any given x, N and pay-off parameters, there are x^{1*} and x^{2*}, defined by equations (4) and (5). From these equations it is clear that $x^{1*} > x^{2*}$. We also know that $x^{1*} > 0$ and $x^{2*} < N$. I will argue that if $x_0 > x^{1*}$, the analysis under (i) remains valid, if $x_0 < x^{2*}$ the analysis under (ii) remains valid and if $x^{2*} < x_0 < x^{1*}$ there will be coexistence and the boundary remains where it is. Suppose $x_0 > x^{1*}$. It is easily seen that in period 0, $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$. Following the argument under (i) reveals that in period 1 x has increased by 2. However, when $(a - c) > (b - d)$. more x favors cooperation, i.e., $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$ in period 1,2,... Hence, the analysis under (i) holds true. Similarly, if $x_0 < x^{2*}$, $\overline{U}_{C^0}(D) > \overline{U}_{C^0}(C)$ in period 0 and the decrease in x favors defection in subsequent periods. Finally, consider $x^{2*} < x_0 < x^{1*}$. In this case in period 0, $\overline{U}_{D^0}(C) < \overline{U}_{D^0}(D)$ and $\overline{U}_{C^0}(C) > \overline{U}_{C^0}(D)$. Hence, nobody will switch to another behavior in period 1 or in any subsequent period. This implies coexistence of the two regions.

Q.E.D.

Proof of Proposition 2

(i) If $(a - c) < (b - d)$, then $\overline{U}_{C^0}(C) > \overline{U}_{C^0}(D)$ implies $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$. Also, $\overline{U}_{C^0}(C) > \overline{U}_{C^0}(D)$ holds for all x if it holds for $x = N$. The inequality holds for $x = N$ if

$$(a - d)(N - 1) > \frac{\alpha}{4}(N - 1)[2(b - d) + (a - c)] + N(1 - \alpha)(b - d),$$

which can be rewritten as

$$N\left[\alpha\frac{2(b - d) - (a - c)}{4} - (b - a)\right] > (a - d) - \frac{\alpha}{4}[2(b - d) + (a - c)].$$

For the rest of the argument, I refer to the proof of Proposition 1 (i).

(ii) In the same way as above: $\overline{U}_{D^0}(D) > \overline{U}_{D^0}(C)$ holds for all x if it holds for $x = 0$. Moreover, $\overline{U}_{D^0}(D) > \overline{U}_{D^0}(C)$ implies $\overline{U}_{C^0}(D) > \overline{U}_{C^0}(C)$ and $\overline{U}_{D^0}(D) > \overline{U}_{D^0}(C)$ holds for $x = 0$ if $N\left[\alpha\frac{2(a - c) - (b - d)}{4} - (d - c)\right] > (a - d) - \frac{\alpha}{4}[2(a - c) + (b - d)]$.

For the rest of the argument, I refer to the proof of Proposition 1 (ii).

(iii) Suppose that the conditions under (i) and (ii) are not satisfied. For any given α and N, there are $x^{1\sim}$ and $x^{2\sim}$ defined by the following equalities:

$$(a - d)(N - 1) = \frac{\alpha}{4}(N - 1)[2(b - d) + (a - c)] + (1 - \alpha)\left[N(a - c) - x^{1\sim}\left((b - d) - (a - c)\right)\right]$$

and

$(a - d)(N - 1) = \frac{\alpha}{4}(N - 1)\left[2(a - c) + (b - d)\right] + N(1 - \alpha)(a - c) + x^{2\sim}(1 - \alpha)$
$\left[(b - d) - (a - c)\right]$.

From the above equations it is clear that $x^{1\sim} < x^{2\sim}$. Also, I know that $x^{1\sim} < N$ and $x^{2\sim} > 0$, but it may be that $x^{1\sim} < 0$ and $x^{2\sim} > N$.

Consider first an arbitrary $x_t > x^{2\sim}$. As in this case $\overline{U}_{C^0}(D) > \overline{U}_{C^0}(C)$ and $\overline{U}_{D^0}(D) > \overline{U}_{D^0}(D)$, $x_{t+1} = x_t - 2$. On the other hand, for an arbitrary $x_t \in [3, x^{1\sim})$, $\overline{U}_{C^0}(C) > \overline{U}_{C^0}(D)$ and $\overline{U}_{D^0}(C) > \overline{U}_{D^0}(D)$ so that $x_{t+1} = x_t + 2$.

Let us then consider an $x_t \in [x^{1\sim}, x^{2\sim}]$ and suppose there is one region with cooperative behavior. It is clear that both the D^0 and the C^0 agent will switch actions in period t+1 so that $x_{t+1} = x_t$. At each of the two boundaries on the circle between cooperators and defectors the sequence of categories at t+1 is as follows:

$$C^+ - C^0 - D^+ - C^{-} - D^0 - D^{-},$$

where the D^+ (C^{-}) agent is the C^0 (D^0) agent of period t. It is clear that at t+2, both the C^0 and C^{-} agent of t+1 will switch to defection. Hence, taken the two boundaries together $x_{t+2} = x_t - 4$.

Suppose then that $x^{1\sim} > 6$. Consider $x_t > x^{1\sim}$. From the above, it is clear that $x_{t+2} = x_t - 4$. If $x_{t+2} > x^{1\sim}$, then $x_{t+4} = x_{t+2} - 4$. If, on the other hand $x_{t+2} < x^{1\sim}$, then $x_{t+3} = x_{t+2} + 2$. If $x_{t+3} < x^{1\sim}$, then $x_{t+4} = x_{t+2} + 4$ and I have a cycle of period 4 around $x^{1\sim}$. If $x_{t+3} > x^{1\sim}$, then $x_{t+5} = x_{t+3} - 4$. As $x^{1\sim} > 6$, and as x_t is an integer x_{t+5} is either 3 or 4. This implies that $x_{t+6} = x_{t+5} + 2$ and $x_{t+7} = x_{t+6} + 2 = x_{t+3}$, establishing (again) a cycle of period 4. A similar analysis applies if $3 \leq x_t \leq x^{1\sim}$.

Suppose then that $x^{1\sim} < 5$. If $x_t > x^{1\sim}$, then $x_{t+2} = x_t - 4$. If x_{t+2} is equal to 1 or 2, then $x_{t+3} = 0$. If on the other hand, x_{t+2} equals 3 or 4, then $x_{t+3} = x_{t+2} + 2$ and $x_{t+3} > x^{1\sim}$. In this case, however, $x_{t+5} = x_{t+3} - 4$ and x_{t+5} is equal to 1 or 2.

Q.E.D.

Literature

Ashlock, D. A. Stanley, L. Tesfatsion. 1994. Iterated Prisoner's Dilemma with Choice and Refusal of Partners. In C. Langton (ed.) *Artifical Life*. Addison-Wesley: 131-67.

Axelrod, R. 1984. *The Evolution of Cooperation*, New York: Basic Books.

Bergstrom, T. 1995. On the Evolution of Altruistic Ethical Rules for Siblings. *American Economic Review* **85**: 58-81.

Bergstrom, T., and O. Stark, 1993. How Altruism can prevail in an Evolutionary Environment, *AEA Papers and Proceedings* **83**: 149-155.

Bicchieri, C. Norms of Cooperation. *Ethics* **100**: 838-861.

Eshel, I., L. Samuelson and A. Shaked. 1998. Altruists, Egoists and Hooligans in a Local Interaction Model. *American Economic Review*. **88**: 157-179.

Goyal, S. 1996. Interaction Structure and Social Change. *Journal of Institutional and Theoretical Economics*. **152**: 472-495.

Goyal, S. and M. Janssen. 1997. Non-Exclusive Conventions and Social Coordination. *Journal of Economic Theory*. **77**: 34-57.

Huck, S. and J. Oechssler. 1995. The Indirect Evolutionary Approach to Explaining Fair Allocations. Department of economics, Humboldt University Berlin, Mimeo.

Frank, R.H. 1987. If Homo Economicus Could Choose His Own Utility Function, Would He Want One with a Conscience? *American Economic Review* **77**: 593-604.

Fudenberg, D. and E. Maskin. 1990. Evolution and Cooperation in Noisy Repeated Games. *American Economic Review (Papers and Proceedings)* **80**: 274-9.

Maynard Smith, J. 1982. *Evolution and the Theory of Games*. Cambridge: CUP.

Nachbar, J.H. 1992. Evolution in the Finitely Repeated Prisoner's Dilemma, *Journal of Economic Behavior and Organization*, **19:** 307-326.

Nowak, M., Bonhoeffer, S., and May, R. 1994. Spatial games and the maintenance of cooperation, *Proceedings of the National Academy of Science*, 91: 4877-4881.

Nowak, M., Bonhoeffer, S., and May, R. 1994b. More Spatial Games. *International Journal of Bifurcation and Chaos*, **4**: 33-56.

Vega-Redondo, F. 1997. The Evolution of Walrasian Behavior. *Econometrica* **65**: 375-84.