Financial Fragility and the Fiscal Multiplier

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April 2013

Abstract

We investigate the effectiveness of 'Keynesian' fiscal stimuli when government deficits and debt rollovers are financed by balance sheet constrained financial intermediaries. Because financial intermediaries operate under a leverage constraint, deficit financing of fiscal stimulus packages will cause interest rates to rise as private loans are crowded out by government debt in the credit provision channel. This lowers investment and (future) capital stocks, which affect output negatively for a prolonged period. Anticipations of these future consequences cause the price of capital to drop immediately when the policy is announced, inflicting capital losses on banks which leads to further tightening of leverage constraints and credit market conditions, thereby triggering a negative amplification cycle further lowering the fiscal multiplier. Moreover, current and anticipated future new debt issues associated with a Keynesian stimulus package cause prices of long term bonds to fall. Financial intermediaries then incur additional losses, with associated further tightening of leverage and credit constraints, and more negative output effects. When in addition sovereign default risk is introduced, additional capital losses may occur and outcomes deteriorate further after a deficit financed stimulus package, eventually implying a Keynesian multiplier close to zero or even negative. Of course we do not argue that multipliers are always negative; but financial fragility and sovereign risk problems may severely lower them, possibly to the point of becoming negative.

Keywords: 'Financial Intermediation; Macrofinancial Fragility; Fiscal Policy; Sovereign Default Risk'

JEL classification: E44; E62; H30

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1 Introduction

"of possibly great importance will be the effects of the crisis on (...) the ways in which economists analyze macroeconomic and financial phenomena" B. Bernanke (2011)

The current eurozone crisis has clearly exposed the close and troublesome interlinkages between the financial system, fiscal deficits and government debt. Southern European governments are struggling to finance their budget deficits even after draconic adjustments because of higher interest rates charged by investors. As interest rates increase, prices of government bonds decrease correspondingly, causing capital losses at financial intermediaries in the periphery and elsewhere holding the relevant sovereign debt on their balance sheet. These capital losses, in turn, cause the capital ratios of financial intermediaries to fall, restricting the flow of credit to non-financial firms, and increasing the cost of both new loans and loans that are extended. The subsequent tightening of credit conditions may lead output to decline, which in turn leads to further increases in deficits and so the vicious circle continues. Whether stimulating aggregate demand through expansionary fiscal policy is advisable, or for that matter, even possible, in such circumstances, has become a highly controversial issue in academic and policy circles alike. The interconnection between the effectiveness of fiscal policy and financial fragility is especially relevant because the European debt crisis is playing itself out in the aftermath of the global credit crisis from 2007-2008 which left in particular European banks severely undercapitalized. The link between financial intermediaries and fiscal policy is therefore likely to be significantly stronger in Europe than in the US because of the greater role of the banking system in financing both the private and the public sector. The same may hold for many emerging market economies where room for fiscal expansion may be more constrained than traditional macroanalysis would suggest if their banking system is undercapitalized and/or if commercial banks have substantial volumes of public bonds on their balance sheet.

The link between financial fragility and fiscal policy gives rise to two sets of questions, one of which we address in this paper. First, to what extent do conditions of financial fragility limit the impact of traditional policies of fiscal expansion, possibly to the extent of reversing even the sign of their impact? There is a variety of mechanisms at play, all of which are largely absent from modern macroeconomic tools of analysis. To begin with, if banks operate under a leverage constraint and governments rely partially or completely on banks to absorb their debt, a new crowding out channel arises: debt can crowd out loans to the private sector and so undermine the effectiveness of the initial stimulus (Kirchner and Van Wijnbergen (2012)). But it may not end there. If higher debt issue leads to higher interest rates in credit constrained markets, bond prices will fall, leading to capital losses on balance sheets of banks holding the sovereign's debt, further undermining the headroom banks have to finance private expenditure. A further twist may be added if higher interest rates in turn lead to more unfavorable debt dynamics and consequent doubts on the solvency of the government issuing the debt, which in turn will lead to higher financing costs for the government and higher capital losses in banks holding the sovereign's debt. This is especially relevant in the eurozone, since eurozone members cannot resort to monetary finance as a financing mode of last resort. The unholy interaction between financial fragility and government deficit financing is likely to be especially relevant because the balance sheets of many banks and other financial intermediaries in Europe still contain hidden losses (IMF: Global Financial Stability Report April 2011). A second question, which we will address in a different paper, concerns the impact of these mechanisms on the international spill-over of local fiscal problems (Lagarde's "chains of contagion", Lagarde (2011)). The questions we address in this paper concern the dynamic relation between expansionary fiscal policy, debt finance and financial fragility. Is additional government spending still effective to combat recessions in an environment where financial intermediaries have not cleaned up their balance sheets and are significantly exposed to sovereign debt holdings?

In order to investigate the dynamics described above, we build a general equilibrium model that incorporates balance sheet constrained financial intermediaries providing loans to the private sector, which uses them to purchase capital equipment for production, and to the government from whom the banks buy long term bonds. The government issues the debt to either roll over maturing debt or finance new deficits. We introduce price stickiness in standard Neo-Keynesian fashion in order to have more realistic output and inflation dynamics. The long term debt consists of (perpetual) consols[2]. The innovation of the current paper is the fact that it investigates the effectiveness of government spending stimulus in an economy where the financial intermediaries are balance sheet constrained and may suffer capital losses on their holdings of sovereign debt. We show that fiscal multipliers, the dynamic output effects of deficit financed fiscal expansions, deteriorate significantly when the financial intermediaries that finance the government operate under a balance sheet constraint and suffer capital losses on existing holdings of debt when interest rates rise. At the end of the paper we introduce sovereign default risk , whereby the government possiby defaults on (part) of its debt obligations. When defaults materialize, banks again suffer capital losses, which in turn initiates an additional perverse amplification cycle between sovereign distress and private credit tightness.

To set a benchmark, we first use the model to investigate the effects of fiscal stimuli after a financial crisis hits assuming past, current and future government deficits are financed by short term bonds only. The results are compared with a no policy base case to assess the size and time pattern of fiscal multipliers. We then introduce anticipation effects by analysing a fiscal stimulus that becomes known one (budget) year before its actual implementation. This is a more realistic policy experiment because of the time delays inherent in public sector budget processes. We then go on to the main body of the paper and investigate the same fiscal stimulus package, but now under the assumption that past, current and future deficits are financed by consols, which creates the possibility of banks incurring capital losses on their holdings of sovereign debt when interest rates rise. We finally introduce sovereign default risk in the long term debt set up.

Relation to existing literature. Since the start of the credit crisis, the theoretical literature with general equilibrium models containing financial frictions is growing. In fact the first paper to incorporate financial frictions in a general equilibrium setup dates from well before the credit crisis: Bernanke et al. (1999). There financial frictions arise because of asymmetric information problems between banks and their corporate borrowers. Gertler and Karadi (2011) introduce financial intermediaries that are balance sheet constrained because of a different agency problem, between deposit holders and bank owners/shareholders. This gives rise to an endogeneous leverage constraint, which becomes more binding when net worth is reduced by exogenous shocks, for example to the quality of the loans. See also Gertler and Kiyotaki (2010). Kircher and Van Wijnbergen (2012, henceforward KW (2012)) extend the Gertler-Karadi (2011) model by introducing asset choice for commercial banks: they now also finance government deficits and the rollover of existing government debt next to providing regular credit to the private sector. KW (2012) use this model to analyze the macroeconomic consequences of the way in which bank interventions and fiscal policy are financed. See Gertler and Karadi (2012) for a similar model variant, set up with a different focus: they analyze the asset purchase policies of the Federal Reserve ("Quantitative Easing") in the aftermath of the credit crisis in the US.

[2]The advantage of assuming that all debt consists of consols is that no new state variables are necessary to keep track of the maturity structure of existing debt.

We build on KW (2012) but extend their model structure by incorporating long term debt with the associated possibility of capital losses, and, in the last section of this paper, sovereign default default risk. Both long term debt and the possibility of sovereign default introduce new channels whereby increased government bond issuance, intended to stimulate the economy, may cause capital losses on existing debt held by commericial banks and thereby lead to increasing credit tightness and further crowding out of the private sector. Acharya et al. (2011) also investigate the negative amplification cycles triggered by the interaction between weak bank balance sheets and sovereign default, but their analysis occurs within a partial equilibrium setup. Homar and Van Wijnbergen (2013) demonstrate empirically the highly beneficial impact of bank recapitalization on the expected duration of recessions after a financial crisis has eroded bank balance sheets.

Sovereign default is analysed in a macromodel by Arellano (2008), which contains an endogeneous default mechanism. She finds a maximum level of debt conditional on the size of a negative income shock. Uribe (2006) introduces a government that continually defaults on part of its government debt each period in such a way that expected future tax receipts and liabilities match afterwards. We follow Schabert and Van Wijnbergen (2011) who introduce sovereign default risk by postulating a fiscal limit like in Leeper, assuming that there exists a maximum level of taxation that is politically feasible, posing an upper limit on the stock of debt that can be serviced without defaulting. As actual debt approaches the debt limit implied by the fiscal limit, the government defaults on an increasingly large share of its debt service obligations. The resulting relation between debt and default risk that emerges is very similar to the structure derived in a fully endogenous default setting by Arellano (2008).

2 Model description

In this section the model will be described. First we introduce the model of KW (2012) as a benchmark. Then we will extend that model by introducing long term government bonds, or 'consols', instead of short term government bonds. We will conclude by extending the 'consols' model by including a government that can default. The government issues debt to financial intermediaries and raises taxes in a lump sum fashion from the households to finance its expenditures and repay existing debt. The other part of the public sector is a central bank that is in charge of monetary policy. It sets the nominal interest rate on the deposits that the households bring to the financial intermediaries. The private sector consists of financial intermediaries and a non-financial sector that includes households and firms. The structure of the non financial private sector is relatively standard and consists of capital producing firms that buy investment goods and used capital, and convert these into capital that is sold to the intermediate goods producers. The intermediate goods producers use the capital as an input, together with labor, to produce intermediate goods for the retail firms. There is perfect competition in the intermediate goods market, and hence the ex-ante profits of the intermediate goods producers are zero in equilibrium. Each intermediate goods producer produces a differentiated product. The retail firms repackage and sell the retail products to the final good producers. Every retail firm is a monopolist and charges a markup for his product. The final good producers buy these goods and combine them into a single output good. The final good is purchased by the households for consumption, by the capital producers to convert it into capital, and by the government. The household maximizes life-time utility subject to a budget constraint, which contains income from deposits, profits from the firms, both financial and non-financial, and from labor. The income is used for consumption, lump sum taxes and investments in deposits.

The bankers running the financial intermediaries can steal a certain fraction of the assets. The depositors will then force the intermediary into bankruptcy, but can not recoup all the assets stolen by the banker. Depositors, therefore, will in equilibrium only provide deposits up till the level where the continuation value of the intermediaries is equal to the value of the assets that can be diverted by the bankers. This imposes an endogeneous leverage constraint on the financial intermediaries. The funds obtained from the depositors, together with the net worth of the intermediary, are used to provide loans to the intermediate goods producers and to purchase government bonds. The intermediate goods firms need the loans in order to buy capital with which they will be productive in the next period. If the capital deteriorates in quality after the loan has been provided, the intermediate goods firms will produce less, and might not be able to fully repay the loan. This causes the balance sheet of the financial intermediary to worsen, and the intermediary will not be able to provide as many loans as the production sector might demand without the leverage constraint. The government has the possibility to intervene by providing the financial sector with new net worth. This, however, increases government debt, which might crowd out private sector loans.

2.1 Household

The household sector consists of a continuum of infinitely lived households with identical preferences and asset endowments. A typical household consists of bankers and workers. Every period, a fraction f of the household members is a banker running a financial intermediary. A fraction $1-f$ of the household members is a worker. At the end of every period, all members of the household pool their resources, and every member of the household has the same consumption pattern. Hence there is perfect insurance within the household, and the representative agent representation is preserved. Every period, the household earns income from the labor of the working members and the profits of the firms that are owned by the household. In addition, households keep short term deposits in commercial banks, which are paid back with interest. The household uses these incoming cashflows to buy consumption goods which are consumed immediately upon purchase, and make new deposits into financial intermediaries.[3] The household members derive utility from consumption and leisure, with habit formation in consumption, in order to capture realistic consumption dynamics (Christiano et al. (2005)). Households then optimize expected discounted utility:

$$\max_{\{c_{t+s},h_{t+s},d_{t+s}\}_{s=0}^{\infty}} E_t\Big[\sum_{s=0}^{\infty}\beta^s\Big(\log\big(c_{t+s}-\upsilon c_{t-1+s}\big)-\Psi\frac{h_{t+s}^{1+\varphi}}{1+\varphi}\Big)\Big],\quad \beta\in(0,1),\quad \upsilon\in[0,1),\quad \varphi\geq 0$$

where c_t is consumption per household member, and h_t are hours worked by the members of the household that are workers. The utility function is subject to the following budget constraint:

$$c_t+d_t+\tau_t=w_t h_t+(1+r_t^d)d_{t-1}+\Pi_t$$

The household optimizes utility while respecting the budget constraints each period. Intermediary deposits d_t are posted at the financial intermediary in period t, and pay real interest r_{t+1}^d and principal at time $t+1$. w_t is the real wage rate, τ_t are lump sum taxes the household has to pay to the government, and Π_t are the profits from the firms owned by the households. The profits of the financial intermediary are net of the startup capital for new bankers, as will be explained below.

[3] but not in the ones owned by the family, in order to prevent self-financing.

The first order conditions are then given by:

$$
\begin{aligned}
c_t &: \quad \lambda_t = \left(c_t - \upsilon c_{t-1}\right)^{-1} - \upsilon\beta E_t\left[\left(c_{t+1} - \upsilon c_t\right)^{-1}\right] \quad (1)\\
h_t &: \quad \Psi h_t^{\varphi} = \lambda_t w_t \quad (2)\\
d_t &: \quad 1 = \beta E_t\Big[\Lambda_{t,t+1}(1 + r_{t+1}^d)\Big] \quad (3)
\end{aligned}
$$

where λ_t is the Lagrange multiplier of the budget constraint, and the stochastic discount factor $\Lambda_{t,t+i} = \lambda_{t+i}/\lambda_t$ for $i \geq 0$. The budget constraint is binding when the multiplier is larger than zero, i.e. $\lambda_t > 0$.

2.2 Financial intermediaries

The financial sector setup is similar to Gertler and Kiyotaki (2010), except that we allow the financial intermediary to invest both in private loans and government bonds. Thus financial intermediaries lend funds obtained from households to the intermediate goods producers and to the government. We assume that households do not bring their deposits to the bankers belonging to the same household, in order to prevent self-financing. Since we have a continuum of households with mass one, there are infinitely many other banks to which the household can lend its funds. The banker's intermediate period balance sheet is given by:

$$p_{j,t} = n_{j,t} + d_{j,t}$$

where $p_{j,t}$ are the assets on the balance sheet of bank j in period t, $n_{j,t}$ denotes the net worth of the bank, while $d_{j,t}$ denotes the deposits of the bank. The financial intermediary invests the funds obtained from the household in claims issued by the intermediate goods producer and in government bonds. In this section we set up the benchmark model with short term debt only, long term debt is introduced in Section 3. Hence the asset side of the bank's balance sheet has the following structure:

$$p_{j,t} \quad = \quad q_t^k s_{j,t}^k + s_{j,t}^b$$

where $s_{j,t}^k$ are the number of claims the financial intermediary j has acquired for a price q_t^k from the intermediate goods producers that pay a net real return r_{t+1}^k at the beginning of period $t+1$, and $s_{j,t}^b$ the number of government bonds acquired by intermediary j, for a net real return r_{t+1}^b at the beginning of period $t+1$. Financial intermediaries earn a return on their assets, and pay a return on the deposits. The difference between the two adds to the increase in net worth from one period to the next. The balance sheet of intermediary j then evolves according to the following law of motion:

$$
\begin{aligned}
n_{j,t+1} &= (1 + r_{t+1}^k)q_t^k s_{j,t}^k + (1 + r_{t+1}^b)s_{j,t}^b - (1 + r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g\\
&= (r_{t+1}^k - r_{t+1}^d)q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d)s_{j,t}^b + (1 + r_{t+1}^d)n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t}
\end{aligned}
$$

where $n_{j,t+1}^g = \tau_{t+1}^n n_{j,t}$ denotes the net worth provided by the government to the financial intermediary j, while $\tilde{n}_{j,t+1}^g = \tilde{\tau}_{t+1}^n n_{j,t}$ denotes the repayment of government support received in previous periods. We will describe later how the government can default.

The financial intermediary is interested in maximizing expected profits. There is a probability of $1 - \theta$ that the banker has to exit the industry next period, in which case he will bring the net worth $n_{j,t+1}$ to the household, while he is allowed to continue operating with a probability θ. The banker discounts these outcomes by the household's stochastic discount factor $\beta\Lambda_{t,t+1}$, since the banker is part of the household, the ultimate owner of the financial intermediary. The banker's objective is then given by the following Bellman recurrence relation:

$$V_{j,t} = \max E_t\Big[\beta\Lambda_{t,t+1}\big\{(1-\theta)n_{j,t+1} + \theta V_{j,t+1}\big\}\Big]$$

where $\Lambda_{t,t+1} = \lambda_{t+1}/\lambda_t$. We conjecture the solution to be of the following form, and later check whether this is the case:

$$V_{j,t} = \nu_t^k q_t^k s_{j,t}^k + \nu_t^b s_{j,t}^b + \eta_t n_{j,t}$$

We follow Gertler and Karadi (2011) and assume the banker can divert a fraction λ of the assets at the beginning of the period, and transfer these assets costlessly back to the household. The depositors will force the intermediary into bankruptcy, but will only be able to recover the remaining fraction $1-\lambda$ of the assets of the financial intermediary, since we assume that it is too costly for them to recoup all the assets. Hence lenders will only supply funds if the gains from stealing are lower than the continuation value of the financial intermediary. This gives rise to the following constraint:

$$V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + s_{j,t}^b) \quad \Rightarrow \quad \nu_t^k q_t^k s_{j,t}^k + \nu_t^b s_{j,t}^b + \eta_t n_{j,t} \geq \lambda(q_t^k s_{j,t}^k + s_{j,t}^b) \tag{4}$$

The optimization problem can now be formulated in the following way:

$$\max_{\{q_t^k s_{j,t}^k, s_{j,t}^b\}} V_{j,t}, \quad \text{s.t.} \quad V_{j,t} \geq \lambda(q_t^k s_{j,t}^k + s_{j,t}^b)$$

From the first order conditions we find that $\nu_t^b = \nu_t^k$. Hence the leverage constraint (4) can be rewritten in the following way:

$$\nu_t^k(q_t^k s_{j,t}^k + s_{j,t}^b) + \eta_t n_{j,t} \geq \lambda(q_t^k s_{j,t}^k + s_{j,t}^b) \Rightarrow q_t^k s_{j,t}^k + s_{j,t}^b \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda - \nu_t^k} \tag{5}$$

where ϕ_t denotes the ratio of assets to net worth, which can be seen as the leverage constraint of the financial intermediary. Substitution of the conjectured solution into the right hand side of the Bellman equation gives the following expression for the continuation value of the financial intermediary:

$$\begin{aligned} V_{j,t} &= E_t\Big[\Omega_{t+1} n_{j,t+1}\Big], \\ \Omega_{t+1} &= \beta\Lambda_{t,t+1}\big\{(1-\theta) + \theta[\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}]\big\} \end{aligned}$$

Ω_{t+1} can be thought of as a stochastic discount factor that incorporates the financial friction. Now we can substitute the expression for next period's net worth into the expression above:

$$\begin{aligned} V_{j,t} &= E_t\Big[\Omega_{t+1} n_{j,t+1}\Big] = E_t\Big[\Omega_{t+1}\big\{(1+r_{t+1}^k)q_t^k s_{j,t}^k + (1+r_{t+1}^b)s_{j,t}^b - (1+r_{t+1}^d)d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g\big\}\Big] \\ &= E_t\Big[\Omega_{t+1}\big\{\big(r_{t+1}^k - r_{t+1}^d\big)q_t^k s_{j,t}^k + \big(r_{t+1}^b - r_{t+1}^d\big)s_{j,t}^b + \big(1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n\big)n_{j,t}\big\}\Big] \end{aligned} \tag{6}$$

After combining the conjectured solution with (6), we find the following first order conditions:

$$\eta_t = E_t\Big[\Omega_{t+1}\big(1 + r^d_{t+1} + \tau^n_{t+1} - \tilde{\tau}^n_{t+1}\big)\Big] \quad (7)$$
$$\nu^k_t = E_t\Big[\Omega_{t+1}\big(r^k_{t+1} - r^d_{t+1}\big)\Big] \quad (8)$$
$$\nu^b_t = \nu^k_t = E_t\Big[\Omega_{t+1}\big(r^b_{t+1} - r^d_{t+1}\big)\Big] \quad (9)$$
$$\Omega_{t+1} = \beta\Lambda_{t,t+1}\big\{(1-\theta) + \theta[\eta_{t+1} + \nu^k_{t+1}\phi_{t+1}]\big\}$$

2.2.1 Financial sector support

Individual financial intermediaries are forward-looking, and hence incorporate financial sector support when forming expectations about their future net worth. We assume that the support provided to an individual intermediary is proportional to the intermediary's net worth in the previous period, in order to relate the support to the pre-crisis level of net worth. Hence individual financial support is given by:

$$n^g_{j,t} = \tau^n_t n_{j,t-1}, \quad \zeta \leq 0, \quad l \geq 0$$
$$\tau^n_t = \zeta(\xi_{t-l} - \xi)$$

When the financial sector support has to be paid back, some of the firms that have received the support will have gone out of business. Therefore the government levies a tax on the previous period net worth of the financial intermediaries in order to get paid back the full amount of support measures, which have to be paid by all previous period intermediaries, including the ones exiting:

$$\tilde{n}^g_{j,t} = \tilde{\tau}^n_t n_{j,t-1}$$

where $\tilde{\tau}^n_t$ is a scaling factor that may change over time.

2.2.2 Aggregation of financial variables

We can integrate over the balance sheets of the financial intermediaries to obtain the aggregate balance sheet of the financial sector:

$$p_t = n_t + d_t \quad (10)$$

Aggregation over the asset side of the balance sheet gives the composition of the aggregated financial system:

$$p_t = q^k_t s^k_t + s^b_t \quad (11)$$

Since ϕ_t does not depend on firm specific factors, we can aggregate the leverage constraint (5) across financial intermediaries to find the total number of assets:

$$p_t = q^k_t s^k_t + s^b_t = \phi_t n_t \quad (12)$$

where p_t denotes the aggregate quantity of assets that are on the balance sheets of the financial intermediaries, while n_t denotes the aggregate intermediary net worth. If capital k_t is hit with a quality shock between the moment the intermediary lends the funds to the goods producer and the production of the goods (see section 2.3 below), the gross return on the corporate assets will be below the return on deposits and the intermediary's capital will decline. This in turn induces him to deleverage, in order not to violate the leverage constraint. The share of assets invested in private loans is given by:

$$\omega_t = q_t^k s_t^k / p_t \tag{13}$$

At the same time, we know that at the end of the period, only a fraction θ of the current bankers will remain a banker, while the remaining fraction $1-\theta$ will (again) become a worker. We assume that current bankers only pay out dividends at the moment they quit the banking business. If they do not quit but continue as a banker, they retain their net worth, in order to be able to expand their balance sheet and make a larger profit. Thus net worth of existing bankers at the end of the period is equal to:

$$n_{e,t} = \theta\big[(r_t^k - r_t^d)q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d)s_{t-1}^b + (1+r_t^d)n_{t-1}\big]$$

At the same time, the exiting bankers bring back the net worth to the household income. We know that a fraction $1-\theta$ of the bankers has left the financial industry, which is equal to a fraction $(1-\theta)f$ of the households. The same fraction of the households will enter the financial industry next period by leaving their working job. We assume that the household will provide a starting net worth to the new bankers proportional to the assets of the old bankers, as in Gertler and Karadi (2011). Assume that the household transfers a fraction of $\chi/(1-\theta)$ of the assets of the old bankers to the new bankers. Then aggregate net worth of new bankers equals:

$$n_{n,t} = \chi p_{t-1}$$

At the beginning of the new period, random draws determine which bankers will leave the industry, taking their accumulated net worth with them. Total net worth of the bankers active in the new period (old bankers that continue plus new bankers) equals:

$$\begin{aligned} n_t &= n_{e,t} + n_{n,t} \\ &= \theta\big[(r_t^k - r_t^d)q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d)s_{t-1}^b + (1+r_t^d)n_{t-1}\big] \\ &+ \chi p_{t-1} + n_t^g - \tilde{n}_t^g \end{aligned} \tag{14}$$

where n_t^g and $\tilde{n}_t^g$ are aggregate financial sector support, respectively financial support payback. Since the only firm specific factor in individual support is the individual intermediary's net worth, it is straightforward to get aggregate financial sector support:

$$n_t^g = \zeta(\xi_{t-l} - \xi)n_{t-1} \tag{15}$$

Similarly, we can aggregate financial sector payback:

$$\tilde{n}_t^g = \tilde{\tau}_t^n n_{t-1} \Rightarrow \tilde{\tau}_t^n = \tilde{n}_t^g / n_{t-1} \tag{16}$$

We derive an expression for $\tilde{n}_t^g$ in section 2.4.

2.3 Production side

The production side of the economy is modeled in the same way as in KW (2012). There is a continuum of intermediate goods producers indexed by $i \in [0,1]$ that face perfect competition. They borrow funds from the financial intermediaries to purchase the capital necessary for production. With the proceeds from the sale of the output and the sale of the capital after it has been used, the firms will pay workers and pay back the loans to the financial intermediary. The output is sold to the retail firms. The capital producers buy and "recycle" the used (and partially depreciated) old capital; they also produce new capital (investment) from goods purchased from the final goods producers. The recycled old capital and the new investments combine to form the new capital stock at the beginning of the next period. This new capital is sold to the intermediate goods producers, which will use it for production next period. A continuum of retail firms, indexed by $f \in [0,1]$, repackage the products bought from the intermediate goods producers to produce a unique differentiated retail product. The retail firms sell their products to a continuum of final goods producers. Due to the fact that the products are differentiated, each individual retail firm faces a monopolistic competitive market, and can charge a markup. It can only change prices in a period, though, with a certain probability, which is equal to ψ and i.i.d. for every retail firm. Hence some firms are allowed to reset prices while others are not. The final goods producers have a technology to convert the inputs from the retail firms into final goods. Due to perfect competition, the profits will be zero in equilibrium, and the final goods are sold to the households, the government, and the capital producers, which use it as an investment to produce new capital for use in the next period by the intermediate goods producing firms.

2.3.1 Capital Producers

In this section we describe the capital producers. At the end of period t, when the intermediate goods firms have produced, they sell what remains of the capital stock after depreciation δ to the capital producers at a price q_t^k. The capital producers also buy i_t final goods from the final good producers; these purchases (investment) are an input in the capital production process: they are used to produce additional capital. Capital producers combine this additional capital with the old, partially depreciated stock bought earlier from the intermediate goods producers and so produce the new capital stock. This new capital is being sold to the intermediate goods producers at a price q_t^k. We assume that the capital producers face convex adjustment costs whenever investment $i_t$deviates from previous period investment i_{t-1}. These adjustment costs are the reason that one unit of investment goods cannot be transformed into one unit of capital, unless $i_t = i_{t-1}$. Hence we have the following capital production technology:

$$k_t \quad = \quad (1-\delta)\xi_t k_{t-1} + (1-\Psi(\iota_t))i_t, \quad \Psi(x) = \frac{\gamma}{2}(x-1)^2, \quad \iota_t = i_t/i_{t-1} \tag{17}$$

The capital producers are profit maximizing, and the profits are passed on to the households, who are the owners of the capital producers. The profit in period t is given by:

$$\Pi_t^c \quad = \quad q_t^k k_t - q_t^k (1-\delta)\xi_t k_{t-1} - i_t$$

The capital producers' optimization problem is then given by:

$$\max_{\{i_{t+i}\}_{i=0}^{\infty}} E_t\Big[\sum_{i=0}^{\infty} \beta^i \Lambda_{t,t+i}\Big(q_{t+i}^k\big(1-\Psi(\iota_{t+i})\big)i_{t+i} - i_{t+i}\Big)\Big]$$

Differentiation with respect to investment gives the first order condition for the capital producers:

$$q_t^k\big(1-\Psi(\iota_t)\big) - 1 - q_t^k \iota_t \Psi'(\iota_t) + \beta E_t \Lambda_{t,t+1} q_{t+1}^k \iota_{t+1} \Psi'(\iota_{t+1}) = 0$$

This equation can be rewritten to find the price of capital to be:

$$\frac{1}{q_t^k} = 1 - \frac{\gamma}{2}\Big(\frac{i_t}{i_{t-1}} - 1\Big)^2 - \frac{\gamma i_t}{i_{t-1}}\Big(\frac{i_t}{i_{t-1}} - 1\Big) + \beta E_t\Big[\Lambda_{t,t+1}\frac{q_{t+1}^k}{q_t^k}\Big(\frac{i_{t+1}}{i_t}\Big)^2 \gamma\Big(\frac{i_{t+1}}{i_t} - 1\Big)\Big] \tag{18}$$

2.3.2 Intermediate Goods Producers

There exists a continuum of intermediate goods producers indexed by $i \in [0,1]$. Each of these firms produce a differentiated good. The intermediate goods producers borrow from the financial intermediaries against future profits. We assume that there are no financial frictions between the financial intermediaries and the intermediate goods producers. Hence there are no monitoring costs for the financial intermediaries, and the intermediate goods producers can commit next period's profits to the financial intermediaries. The securities issued by the intermediate goods producers are therefore perfectly state-contingent debt , see also Gertler and Kiyotaki (2010)[4]. The production technology of the intermediate goods producers is given by:

$$
\begin{aligned}
y_{i,t} &= a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha}, \\
& \log(a_t) = \rho_a \log(a_{t-1}) + \varepsilon_{a,t} \\
& \log(\xi_t) = \varrho_\xi \log(\xi_{t-1}) + \varepsilon_{\xi,t}
\end{aligned}
$$

where a_t equals total factor productivity and ξ_t capital quality. The innovations $\varepsilon_{x,t}$ are distributed as $\varepsilon_{x,t} \sim N(0,\sigma_x^2)$ for $x = a, \xi$. The intermediate goods producer acquires the capital at the end of period $t-1$, while the production only occurs after the capital quality shock ξ_t has hit at the beginning of period t. Hence $\xi_t k_{i,t-1}$ denotes the effective capital in our model. We see that if a negative realization of $\varepsilon_{\xi,t}$ occurs, the quality of the capital deteriorates immediately. Hence the firm will not be able to produce as much as when the shock does not occur. Remember that the number of claims ($s_{i,t}^k$) is equal to the number of units of capital purchased ($k_{i,t}$); hence the return on the claims of the financial intermediary will be lower. We can think of the shock ξ_t as accelerated economic depreciation or obsolescence of capital. The intermediate goods producer decides at the end of period $t-1$ how much capital to purchase. At the moment the intermediate goods producer purchases the capital, he does not know the realization of ξ_t in period t yet. To finance his purchase at the end of period $t-1$, he needs to issue claims $s_{i,t-1}^k$, with the number of claims $s_{i,t-1}^k$equal to the number of capital units ($k_{i,t-1}$) acquired. The price at which the claims are sold equals q_{t-1}^k, and they pay a state-contingent net real return r_t^k in period t. The intermediate goods producer also hires labor $h_{i,t}$ for a wage rate w_t after the shock (ξ_t) has been realized. When the firm has produced in period t, the output is sold for a relative price m_t to the retail firms. m_t is the relative price of the intermediate goods with respect to the price level of the final goods, i.e. $m_t = P_t^m/P_t$. After production, the intermediate goods producing firms sell back what remains of the effective capital to the capital producers at a price of q_t^k. The effective capital stock is also subject to regular depreciation δ during production. Hence the intermediate goods producer receives $q_t^k(1-\delta)\xi_t k_{i,t-1}$ for his end of period capital stock. So real profits in period t are given by:

$$
\Pi_{i,t} = m_t a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} + q_t^k(1-\delta)\xi_t k_{i,t-1} - (1+r_t^k)q_{t-1}^k k_{i,t-1} - w_t h_{i,t}
$$

The objective of the intermediate goods producing firms is to maximize gross profits. They take the relative output price (m_t), and the input prices q_t^k, r_t^k and w_t as given when maximizing profits.

[4] The claims of financial intermediaries can therefore be better thought of as equity. Occhino and Pescatori (2010) explicitly model loans to producers with a fixed face value, and include the possibility of a default by the goods producers. We refrain from explicitly modelling the producers' default, and note the equity characteristics of debt in the real world when firms have not enough funds to pay off the loan.

The first order condition with respect to capital and labor are given by:

$$
\begin{aligned}
k_{i,t} &: \quad E_t\Big[\beta\Lambda_{t,t+1}q_t^k(1+r_{t+1}^k)\Big] = E_t\Big[\beta\Lambda_{t,t+1}\big(\alpha m_{t+1}y_{i,t+1}/k_{i,t} + q_{t+1}^k(1-\delta)\xi_{t+1}\big)\Big] \\
h_{i,t} &: \quad w_t = (1-\alpha)m_t y_{i,t}/h_{i,t}
\end{aligned}
$$

Firms pay out residual revenues to the financial intermediaries. By substituting the first order condition for the wage rate into the zero-profit condition $\Pi_{i,t} = 0$, we can find an expression for the ex-post return on capital:

$$
r_t^k \quad = \quad (q_{t-1}^k)^{-1}\big(\alpha m_t y_{i,t}/k_{i,t-1} + q_t^k(1-\delta)\xi_t\big) - 1
$$

The first order condition for labor and the expression for the ex-post return on capital can be rearranged to derive factor demands. These are given by:

$$
\begin{aligned}
k_{i,t-1} &= \alpha m_t y_{i,t}/[q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t] \\
h_{i,t} &= (1-\alpha)m_t y_{i,t}/w_t
\end{aligned}
$$

Finally the relative intermediate output price m_t can be obtained by substituting the factor demands into the production function:

$$
m_t \quad = \quad \alpha^{-\alpha}(1-\alpha)^{\alpha-1}a_t^{-1}\Big(w_t^{1-\alpha}\big[q_{t-1}^k(1+r_t^k)\xi_t^{-1} - q_t^k(1-\delta)\big]^{\alpha}\Big) \qquad (19)
$$

2.3.3 Retail firms

Retail firms purchase goods ($y_{i,t}$) from the intermediate goods producing firms for a nominal price P_t^m, and convert these into retail goods ($y_{f,t}$). These goods are sold for a nominal price $P_{f,t}$ to the final goods producer. We assume that it takes one intermediate goods unit to produce one retail good ($y_{i,t} = y_{f,t}$). All the retail firms produce a differentiated retail good. The individual retail firm therefore operates in a market characterized by monopolistic competition, and can therefore charge a markup over the input price P_t^m, so the nominal profit of the retail firm is given by $(P_{f,t} - P_t^m)y_{f,t}$.

We assume that in each period a fraction $1-\psi$ of retail firms is allowed to reset their prices optimally, while the ψ remaining firms are not allowed to do so. This is set up like in Calvo (1983) and Yun (1996), so we assume that this probability is independent of the probability that other producers are allowed to reset their prices, and independent of whether or not the firm was allowed to adjust its prices the previous period. Hence the probability that a firm is not allowed to reset its prices is ψ, where ψ is i.i.d. When a firm is allowed to reset its prices, it will do it in such a way that the expected sum of discounted profits is maximized. Again, the retail firms are owned by the households, and therefore the stochastic discount factor for the nominal pay-outs to the household is given by $\beta^s\Lambda_{t,t+s}(1/P_{t+s})$. The relevant part of the optimization problem of the typical retail firm is now given by:

$$
\max_{P_{f,t}} E_t\Big[\sum_{s=0}^{\infty}(\beta\psi)^s\Lambda_{t,t+s}(1/P_{t+s})[P_{f,t} - P_t^m]\Big]y_{f,t+s}
$$

where $y_{f,t} = (P_{f,t}/P_t)^{-\epsilon}y_t$ is the demand function. y_t is the output of the final good producing firms, and P_t the general price level. The expression for the demand function for the retail firms products will be derived in the next section. Since all the retail firms have access to the same technology, all the firms that are allowed to reset their prices will choose the same new price (P_t^*) for their goods. We remember that the relative price m_t is equal to $m_t = P_t^m / P_t$. Differentiation with respect to $P_{f,t}$ gives the first order condition for the price the retail firms will charge for their products:

$$\frac{P_t^*}{P_t} = \frac{\epsilon}{\epsilon - 1} \frac{E_t \sum_{s=0}^{\infty} (\beta\psi)^s \lambda_{t+s} P_{t+s}^{\epsilon} P_t^{-\epsilon} m_{t+s} y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta\psi)^s \lambda_{t+s} P_{t+s}^{\epsilon-1} P_t^{1-\epsilon} y_{t+s}}$$

We define the relative price of the firms that are allowed to reset their prices to be equal to $\pi_t^* = P_t^*/P_t$, while gross inflation is defined to be equal to $\pi_t = P_t/P_{t-1}$. The above first order condition can now be rewritten in the following form:

$$\pi_t^* = \frac{\epsilon}{\epsilon - 1} \frac{\Xi_{1,t}}{\Xi_{2,t}} \tag{20}$$

$$\Xi_{1,t} = \lambda_t m_t y_t + \beta\psi E_t \pi_{t+1}^{\epsilon} \Xi_{1,t+1} \tag{21}$$

$$\Xi_{2,t} = \lambda_t y_t + \beta\psi E_t \pi_{t+1}^{\epsilon-1} \Xi_{2,t+1} \tag{22}$$

The aggregate price level will be given by the following law of motion:

$$(1-\psi)(\pi_t^*)^{1-\epsilon} + \psi \pi_t^{\epsilon-1} = 1 \tag{23}$$

2.3.4 Final Good Producers

The final good firms purchase intermediate goods which have been repackaged by the retail firms in order to produce the final good. The technology that is applied in producing the final good is given by $y_t^{(\epsilon-1)/\epsilon} = \int_0^1 y_{f,t}^{(\epsilon-1)/\epsilon} df$, where $y_{f,t}$ is the output of the retail firm indexed by f. ϵ is the elasticity of substitution between the intermediate goods purchased from the different retail firms. We assume that the final good firms operate in an environment of perfect competition, and hence they maximize profits by choosing $y_{f,t}$ such that $P_t y_t - \int_0^1 P_{f,t} y_{f,t} df$ is maximized. The final good producer takes P_t and $P_{f,t}$ as given. Taking the first order conditions with respect to $y_{f,t}$ gives the demand function of the final good producers for the retail goods. Substitution of the demand function into the technology constraint gives the relation between the price level of the final good and the price level of the individual retail firms:

$$y_{f,t} = (P_{f,t}/P_t)^{-\epsilon} y_t$$

$$P_t^{1-\epsilon} = \int_0^1 P_{f,t}^{1-\epsilon} df$$

2.3.5 Aggregation

First recall that $y_{f,t} = y_{i,t} = y_t \big(P_{f,t}/P_t\big)^{-\epsilon}$, for all f and i. Hence we can write the factor demands by firm i as:

$$h_{i,t} = (1-\alpha) m_t y_{f,t}/w_t, \quad k_{i,t-1} = \alpha m_t y_{f,t} / [q_{t-1}^k (1 + r_t^k) - q_t^k (1-\delta)\xi_t]$$

Aggregation over all firms i gives us aggregate labor and capital:

$$h_t = (1-\alpha) m_t y_t \Delta_t / w_t, \quad k_{t-1} = \alpha m_t y_t \Delta_t / [q_{t-1}^k (1 + r_t^k) - q_t^k (1-\delta)\xi_t]$$

where $\Delta_t = \int_0^1 \left(P_{f,t}/P_t\right)^{-\epsilon} df$ denotes the price dispersion. It is given by the following recursive form:

$$\Delta_t = (1-\psi)(\pi_t^*)^{-\epsilon} + \psi \pi_t^{\epsilon} \Delta_{t-1} \tag{24}$$

Now we calculate the aggregate capital-labor ratio, and see that it is equal to the individual capital-labor ratio:

$$k_{t-1}/h_t \quad = \quad \alpha(1-\alpha)^{-1} w_t / [q_{t-1}^k(1+r_t^k) - q_t^k(1-\delta)\xi_t] = k_{i,t-1}/h_{i,t} \tag{25}$$

Calculate aggregate supply by aggregating $y_{i,t} = a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha}$:

$$\int_0^1 a_t(\xi_t k_{i,t-1})^{\alpha} h_{i,t}^{1-\alpha} di \quad = \quad a_t \xi_t^{\alpha} \left(\frac{k_{t-1}}{h_t}\right)^{\alpha} \int_0^1 h_{i,t} di = a_t(\xi_t k_{t-1})^{\alpha} {h_t}^{1-\alpha}$$

while aggregation over $y_{i,t}$ gives:

$$\int_0^1 y_{i,t} df = y_t \int_0^1 \left(P_{f,t}/P_t\right)^{-\epsilon} df = y_t \Delta_t$$

Hence we get the following relation for aggregate supply y_t:

$$y_t \Delta_t = a_t(\xi_t k_{t-1})^{\alpha} {h_t}^{1-\alpha} \tag{26}$$

2.4 The Government

To finance its (exogeneous) expenditures g_t, the government levies lump sum taxes on households and issues bonds. The government also uses the proceeds to repay bonds that come due this period (in the case of short term debt financing, which is what we consider in this section, all bonds coming due this period equal all bonds issued the previous period). We also allow the government to aid the financial sector when the quality of the capital, and hence the value of the financial assets deteriorates due to a negative capital quality shock, similar to the way Gertler and Karadi (2011) and KW (2012) implement this policy. The funds that the government spends on increasing the net worth of the financial sector was derived earlier and is given by:

$$\begin{aligned} n_t^g \quad &= \quad \tau_t^n n_{t-1}, \quad \zeta \leq 0, \quad l \geq 0 \\ \tau_t^n \quad &= \quad \zeta(\xi_{t-l} - \xi) \end{aligned} \tag{27}$$

Thus the government provides funds to the financial sector if $\zeta < 0$, and a negative shock to the quality of capital occurs. Depending on the value of l, the government can provide support instantaneously ($l = 0$), or with a lag ($l > 0$). In addition, the government can demand the financial sector to pay back the aid that was given. In that case the government receives a percentage of the funds provided earlier:

$$\tilde{n}_t^g \quad = \quad \vartheta n_{t-e}^g, \quad \vartheta \geq 0, \quad e \geq 1 \tag{28}$$

The case where $\vartheta = 0$ corresponds to the case where the net worth is a gift from the government to the financial intermediaries. In case $\vartheta = 1$, the government aid can be viewed as a zero interest loan, while a $\vartheta > 1$ implies that the financial intermediaries have to pay a penalty for the support received earlier[5]. The parameter e denotes the number of periods after which the government aid has to be paid back (i.e. the maturity of the loans extended)[6].

Finally, consider the government's purchases of final goods in more detail. Government purchases are partially driven by an exogenous stochastic process, and in addition can possibly respond to deteriorations in the quality of capital. The latter component can be interpreted as a Keynesian stimulus package. As the economy deteriorates due to a shock to capital quality, the government attempts to stimulate the economy by increasing government purchases. Combining these two components yields the actual time path for government expenditures/purchases of final goods in period t:

$$g_t = \tilde{g}_t + \varsigma(\xi_{t-l} - \xi), \quad \varsigma < 0, \quad l \geq 0 \tag{29}$$

$$\log(\tilde{g}_t/\bar{g}) = \rho_g \log(\tilde{g}_{t-1}/\bar{g}) + \varepsilon^u_{g,t} + \varepsilon^a_{g,t-4} \tag{30}$$

where $\varepsilon^x_g \sim N(0, \sigma^2_x)$ with $x = (u, a)$. We assume that the autocorrelation coefficient $\rho_g \in [0, 1)$, and the steady state value of government purchases to be larger than zero ($\bar{g} > 0$). This way we can study the effects of surprise shocks to government spending ($\varepsilon^u_{g,t}$), but also the effects of shocks that are announced one year in advance ($\varepsilon^a_{g,t}$). The parameter ς determines the size of the response to a deterioration in the quality of capital. If $\varsigma = 0$, the government does not respond to a deterioration in the quality of capital. $\varsigma < 0$ implies that the government reacts to a deterioration in the quality of capital by increasing government spending above the steady state value. The case where $l = 0$ implies that the government reacts instantaneously to a capital quality shock, while $l > 0$ implies that the government reacts with some lag. Whereas it might be preferable in general to model the government response as an endogeneous optimizing feedback from output, we choose to model government intervention as an exogeneous process because of our focus on the size of fiscal multipliers. This allows us to make policy impact comparisons that are not "polluted" by second round effects triggered by the macroeconomic response to government expenditure shocks leading to subsequent rounds of government interventions. The government budget is given by:

$$b_t + \tau_t + \tilde{n}^g_t = g_t + n^g_t + (1 + r^b_t)b_{t-1} \tag{31}$$

We assume that the government follows a simple fiscal rule for its core tax policy τ_t, as in Bohn (1998):

$$\tau_t = \bar{\tau} + \kappa_b(b_{t-1} - \bar{b}) + \kappa_g(g_t - \bar{g}) + \kappa_n n^g_t, \quad \kappa_b \in (0, 1], \quad \kappa_g, \kappa_n \in [0, 1] \tag{32}$$

where $\bar{b}, \bar{g}$ are the steady state levels of debt, respectively government spending. The Bohn (1998) policy rule formulation guarantees that the real value of public debt eventually grows at a rate smaller than the gross real rate of interest. Bohn (1998) proves that following this rule is a sufficient condition for government solvency. If we set $\kappa_n = 0$, the additional government transfers to the financial sector are completely financed by issuing new debt. $\kappa_n = 1$ implies that the additional spending is completely financed by increasing lump sum taxes. Similarly $\kappa_g = 0$ implies that all government spending above the steady state value is completely deficit financed, whereas $\kappa_g = 1$ implies a tax-financed government spending stimulus.

[5]The case where $\vartheta > 1$ happened in the Netherlands, where the big financial intermediaries were forced to accept government aid, which had to be paid back with a penalty rate of 50 percent on top of the loan.

[6]See Kirchner and Van Wijnbergen (2012) for an extensive analysis of different ways of recapitalizing distressed banks.

2.5 The Central Bank

The Central Bank sets the nominal interest rate on deposits r_t^n according to the following Taylor rule, in order to minimize output and inflation deviations:

$$r_t^n = (1-\rho)\big(r^n + \kappa_\pi(\pi_t - \bar{\pi}) + \kappa_y \log(y_t/y_{t-1})\big) + \rho r_{t-1}^n + \varepsilon_{r,t} \tag{33}$$

where $\varepsilon_{r,t} \sim N(0, \sigma_r^2)$, and $\kappa_\pi > 0$ and $\kappa_y > 0$. ρ is a smoothing parameter. The parameter $\bar{\pi}$ is the target inflation rate or the natural inflation rate. In order to satisfy the Taylor principle, we choose $\kappa_\pi > 1$ (leaning against the wind). The values of κ_π and κ_y determine the strength with which the authorities react to deviations from the natural rate of inflation and output. The nominal and the real interest rate on deposits are linked through the following Fisher relation:

$$1 + r_t^d = (1 + r_{t-1}^n)/\pi_t \tag{34}$$

Hence monetary policy is executed through the control of interest rates on deposits rather than the interest rates on the government bonds: the latter are endogeneously determined in equilibrium.

2.6 Market clearing

Equilibrium requires that the number of claims owned by the financial intermediaries (s_t^k) is equal to aggregate capital (k_t), while the number of government bonds owned by the financial sector (s_t^b) must equal the number of bonds issued by the government (b_t):

$$s_t^k = k_t \tag{35}$$

$$s_t^b = b_t \tag{36}$$

Goods market clearing requires that the aggregate demand equals aggregate supply:

$$c_t + i_t + g_t = y_t \tag{37}$$

This completes the description of the model.

3 Introducing Long Term Government Debt

Consider next an alternative way of structuring the public debt. Instead of just issuing short term (one period) bonds, we go to the other extreme and assume that the government issues perpetual bonds or consols only. Capital losses on holdings of sovereign debt have played an important role in the current (2010-2013) European debt problems; in order to capture that element of the complicated interactions between financial fragility in the commercial banking system and fiscal policy, we introduce longer term government debt than the one period bonds considered in Section 2.4. For analytical convenience and clarity, we in fact go to the other extreme and assume that the government issues consols, instruments entitling the holder to a perpetual flow of (interest) payments fixed in real terms r_c indefinitely, or until the consols are bought back by the government. Thus the consols represent a claim on an infinite stream of payments fixed in real terms. The consols are traded every period, and have a price q_t^b. If the government buys back the consols at time t', it buys them out of the market and has to pay the then current market price $q_{t'}^b$.The introduction of consols changes the equations governing the financial sector as well as the government budget constraint.

3.1 Financial intermediaries

The balance sheet of the typical financial intermediary is now given by:

$$p_{j,t} = q_t^k s_{j,t}^k + q_t^b s_{j,t}^b \tag{38}$$

An investor who purchases a consol in period t for a price q_t^b, receives a fixed interest payment in real terms of r_c in period $t+1$, and earns a return r_{t+1}^b if he sells the consol in period $t+1$ for a price q_{t+1}^b:

$$1 + r_{t+1}^b = \frac{q_{t+1}^b + r_c}{q_t^b} \tag{39}$$

This changes the equation for the net worth of individual intermediary j in period $t+1$:

$$\begin{aligned} n_{j,t+1} &= (1 + r_{t+1}^k) q_t^k s_{j,t}^k + q_{t+1}^b s_{j,t}^b + r_c s_{j,t}^b - (1 + r_{t+1}) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (r_{t+1}^k - r_{t+1}^d) q_t^k s_{j,t}^k + (r_{t+1}^b - r_{t+1}^d) q_t^b s_{j,t}^b + (1 + r_{t+1}^d) n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t} \end{aligned}$$

The leverage constraint is now given by:

$$q_t^k s_{j,t}^k + q_t^b s_{j,t}^b \leq \phi_t n_{j,t}, \quad \phi_t = \frac{\eta_t}{\lambda - \nu_t^k} \tag{40}$$

The first order conditions are given by similar expressions as in the case of short term government debt:

$$\eta_t = E_t \Big[\Omega_{t+1} \big(1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n \big) \Big] \tag{41}$$

$$\nu_t^k = E_t \Big[\Omega_{t+1} \big(r_{t+1}^k - r_{t+1}^d \big) \Big] \tag{42}$$

$$\nu_t^b = \nu_t^k = E_t \Big[\Omega_{t+1} \big(r_{t+1}^b - r_{t+1}^d \big) \Big] \tag{43}$$

$$\Omega_{t+1} = \beta \Lambda_{t,t+1} \big\{ (1 - \theta) + \theta [\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}] \big\}$$

While aggregate net worth evolves according to the following law of motion:

$$\begin{aligned} n_t &= \theta \big[(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b + (1 + r_t^d) n_{t-1} \big] \\ &+ \chi p_{t-1} + n_t^g - \tilde{n}_t^g \end{aligned} \tag{44}$$

3.2 The Government Budget

The government budget changes due to the introduction of the consols. Taxation and government spending are given by the same process as in the case of short term bond financing, and so are the recapitalization process of the financial intermediaries and the payback of the government support. Assuming there are b_{t-1} consols outstanding at the beginning of period t, the government can issue new consols in period t for a market price q_t^b. The amount of debt raised is then equal to $q_t^b(b_t - b_{t-1})$, where b_t is the number of consols outstanding at the end of period t. Consols always entitle the buyer to the same flow of real payments r_c, although the valuation of those payment streams, q_t^b, will in general vary over time. The government has to pay a total coupon payment to the 'old' consol holders equal to $r_c b_{t-1}$ in period t. Hence the government budget becomes:

$$\begin{aligned} q_t^b (b_t - b_{t-1}) + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + r_c b_{t-1} \Rightarrow \\ q_t^b b_t + \tau_t + \tilde{n}_t^g &= g_t + n_t^g + (1 + r_t^b) q_{t-1}^b b_{t-1} \end{aligned} \tag{45}$$

where r_t^b is the real return on a consol in period t, as defined in (39).

4 Government default

In the main body of the paper we do not incorporate defaults by the public sector; capital losses on public debt arise from the impact of higher interest rates on the value of long term debt holdings only. However in a final variant, we also introduce the possibility for the government to default on its debt obligations. The government debt is still financed by the financial intermediaries, and hence they must now incorporate the fact that they can possibly loose funds when the government decides to forgo on its debt obligations. This changes the first order condition for consols, as well as the law of motion for aggregate financial net worth. The government can decide each period whether or not it will default on the payment of the consols. This decision is denoted by Δ_t, where $\Delta_t = 1$ indicates that the government forgoes the coupon payment on the consols, while $\Delta_t = 0$ implies that all debt obligations are honored. We assume that the government will (partially) default if the debt level is larger than a maximum level of debt to be introduced shortly.

Default process

The fact that governments can default of course has an impact on the expectations of investors and the returns they require before buying government paper. We introduce the possibility of default by assuming that in any period there is a maximum level of tax revenues (scaled by GDP) that the government can devote to its debtservice $r_c b_t$, similar to the fiscal limit introduced in Davig, Leeper and Walker (2011). Because r_c is fixed over time, this maximum level of tax revenues translates in a maximum level of debt that the government can sustain. If debt arises above that level, the government is forced to (partially) default on its debt service obligations in that period. Following Schabert and Van Wijnbergen (2011), one could interpret this default rule as the outcome of a preemptive game between investors in government paper and the government, with a mixed strategy as an outcome, like in Benabou (1989) or Pastine (2002). This would make the maximum taxation level (and its associated maximum sustainable debt level) a random variable, the probability density function of which is known to investors. Default probabilities are a positive function of debt-to-GDP ratios in such a set up (see Schabert and van Wijnbergen (2011)). Less ambitiously, we can simply assume investors do not know the true value of the maximum level of sustainable debt but have beliefs on it that can be summarised by a similar pdf. Either way we can map the maximum level of taxation to a maximum level of debt, which then also becomes a random variable with associated pdf. We assume that the maximum level of debt that is politically sustainable is given by the following expression:

$$b_t^{max} = \bar{b}_{max} + \rho_b(b_{t-1}^{max} - \bar{b}_{max}) + \varepsilon_{b,t} \tag{46}$$

where $\varepsilon_{b,t} \sim N(0, \sigma_b^2)$. Arellano (2008), who fully endogenizes the default process in an unavoidably much simpler model, similarly finds a maximum debt level which depends on the particular shocks to the income process in the economy. Even though the current setup is a simplified version, we capture the important features of a default. Reinhart, Rogoff and Savastano (2003) find in an empirical study that a debt limit indeed exists. Higher levels of debt then necessarily decrease the distance to the maximum level of government debt, thereby increasing the probability of default and sovereign debt problems. A next step could involve working out the default process in more detail, but for reasons of tractability we currently refrain from doing so.

4.1 Default Risk and Financial Intermediaries

The equations governing the financial intermediaries change when the possibility of a government default is introduced. The default is taken into account by the intermediaries through Δ_{t+1}. The balance sheet of a financial intermediary is now given by:

$$p_{j,t} = q_t^k s_{j,t}^k + q_t^b s_{j,t}^b \tag{47}$$

Consider first the case of no default. An investor who purchases a consol in period t for a price q_t^b, receives payments r_c per consol held in period $t+1$, and earns a return r_{t+1}^b if he sells the consol in period $t+1$ for a price q_{t+1}^b, assuming the government does not default:

$$1 + r_{t+1}^b = \frac{q_{t+1}^b + r_c}{q_t^b} \tag{48}$$

The possibility of a (partial) government default changes the equation for the net worth of individual intermediary j in period $t+1$:

$$\begin{aligned} n_{j,t+1} &= (1 + r_{t+1}^k) q_t^k s_{j,t}^k + q_{t+1}^b s_{j,t}^b + (1 - \Delta_{t+1}) r_c s_{j,t}^b - (1 + r_{t+1}) d_{j,t} + n_{j,t+1}^g - \tilde{n}_{j,t+1}^g \\ &= (r_{t+1}^k - r_{t+1}^d) q_t^k s_{j,t}^k + (r_{t+1}^b - \frac{\Delta_{t+1} r_c}{q_t^b} - r_{t+1}^d) q_t^b s_{j,t}^b + (1 + r_{t+1}^d) n_{j,t} + \tau_{t+1}^n n_{j,t} - \tilde{\tau}_{t+1}^n n_{j,t} \end{aligned}$$

The leverage constraint is still given by (40), while only the first order condition for the portfolioshare held as consols is adjusted with respect to the no default case:

$$\begin{aligned} \eta_t &= E_t\Big[\Omega_{t+1}\big(1 + r_{t+1}^d + \tau_{t+1}^n - \tilde{\tau}_{t+1}^n\big)\Big] && (49) \\ \nu_t^k &= E_t\Big[\Omega_{t+1}\big(r_{t+1}^k - r_{t+1}^d\big)\Big] && (50) \\ \nu_t^b &= \nu_t^k = E_t\Big[\Omega_{t+1}\big(r_{t+1}^b - \frac{\Delta_{t+1} r_c}{q_t^b} - r_{t+1}^d\big)\Big] && (51) \\ \Omega_{t+1} &= \beta \Lambda_{t,t+1}\big\{(1-\theta) + \theta[\eta_{t+1} + \nu_{t+1}^k \phi_{t+1}]\big\} \end{aligned}$$

Aggregate net worth now evolves according to the following law of motion:

$$\begin{aligned} n_t &= \theta\big[(r_t^k - r_t^d) q_{t-1}^k s_{t-1}^k + (r_t^b - r_t^d) q_{t-1}^b s_{t-1}^b - \Delta_t r_c s_{t-1}^b + (1 + r_t^d) n_{t-1}\big] \\ &+ \chi p_{t-1} + n_t^g - \tilde{n}_t^g \end{aligned} \tag{52}$$

4.2 Defaults and the Government Budget

First consider the budget constraint in the absence of a government default. In that case, the government would need to issue an amount of debt equal to $q_t^b \tilde{b}_t$, as seen from the following expression for the government budget constraint:

$$\begin{aligned} q_t^b(\tilde{b}_t - b_{t-1}) &+ \tau_t + \tilde{n}_{g,t} = g_t + n_{g,t} + r_c b_{t-1} \\ &\Rightarrow q_t^b \tilde{b}_t = g_t + n_{g,t} + (1 + r_t^b) q_{t-1}^b b_{t-1} - \tau_t - \tilde{n}_{g,t} \end{aligned} \tag{53}$$

where the last step follows from substitution of (48). We next assume that the government defaults on its debt service obligations when the level of debt $\tilde{b}_t$ necessary to avoid default exceeds the maximum level of debt b_t^{max}. Hence the budget constraint is given by:

$$q_t^b(b_t - b_{t-1}) + \tau_t + \tilde{n}_{g,t} \quad = \quad g_t + n_{g,t} + r_c(1 - \Delta_t)b_{t-1} \tag{54}$$

In order to be able to solve the model, we assume the default indicator is a continuous function, and approximate it by the cdf of a normal distribution with mean μ zero, and a very small variance:

$$\Delta_t = \Phi\big(\tilde{b}_t - b_t^{max}, \mu = 0, \sigma\big) \tag{55}$$

where σ denotes the curvature of the default function. This implies that the fraction of debt on which the government defaults increases as the amount of consols $\tilde{b}_t$ necessary to prevent a default gets closer to the maximum level of debt. As the debt is low (far away from the debt limit), the default elasticity is very low, but as the maximum debt level is approached, the default elasticity (sharply) increases. Because defaulting on current debt obligations may not be enough to close the gap between $\tilde{b}_t$ and b_t^{max}, actual debt may in fact exceed the "maximum" level of debt. Aizenman and Park (2013) confirm empirically that default fears increase in debt-output ratios of debtor governments.

5 Results

In this section we use the model just presented to trace out dynamic multipliers after Keynesian demand stimuli. We start off with an economy that has been shocked by a financial crisis, modeled as a sudden unanticipated downward shock to the value of commercial banks' assets, without any government repsonse. Following Gertler and Karadi (2011), we implement the downward shock to bank assets as a downward shock to the "quality" of capital. We then analyse the output response to an expansionary shift in government expenditure as a response to the shock. In the first set of policy experiments, we assume existing government debt is short term only, and thus continuously rolled over as needed, and the government stimulus package is entirely financed by short term debt too. In this case the only source of capital losses on commercial banks' balance sheets is the decline in the value of capital q_t^k after the quality shock. Then we introduce long term government debt and thereby a second source of capital losses for commercial banks, and compare the outcome with the short term debt only case. We then investigate the more realistic case where a stimulus package is announced at the onset of the financial crisis, but implemented only four quarters (one budget year) after its announcement, and the dynamic patterns that arise then for the Keynesian multiplier, again assuming consol financing of past, current and future deficits. Finally we introduce a third amplification cycle by introducing the possibility of government default on debt service payments.

5.1 Calibration

The parameter values can be found in table 1. Most of the parameters are common in the literature on DSGE models, or frequently used in models containing financial frictions. We mostly follow the calibration of Gertler and Karadi (2011). This is the case for the subjective discount factor β, the degree of habit formation υ, the Frisch elasticity of labor supply φ^{-1}, the elasticity of substitution among intermediate goods ϵ, the price rigidity parameter ψ, the effective capital share α, and the investment adjustment cost parameter γ. The calibration of the financial variables is also taken from Gertler and Karadi (2011). The steady state leverage ratio is set to 4, while the credit spread Γ is set to 100 basis points annually (which amounts to $\Gamma = 0.0025$), which coincides with the pre-2007 spreads in US financial data between BAA corporate and government bonds. The parameter θ is calibrated by taking the average survival period ($\Theta = 1/(1-\theta)$) to be equal to 36 quarters, or $\theta = 0.9722$. The parameters in the Taylor rule are set to conventional values.

The feedback from government debt on taxes is set to a value such that both the model with and without default are stable. In order to match US macroeconomic data, we calibrate the steady state ratios of investment and government spending over GDP, $\bar{i}/\bar{y}$ and $\bar{g}/\bar{y}$ to 20 percent by calibrating the depreciation parameter δ. The debt over GDP ratio $\bar{b}/\bar{y}$ is set equal to 2.4, implying an annual debt-to-GDP ratio of 60 percent. The steady state default probability is set to $\bar{\Delta} = 0.1587$, and a curvature parameter of the default indicator equal to $\sigma = 0.100$. The default probability is relatively large, but the default only occurs on the coupon payments of the consols. Hence it is reasonable to set the steady state default probability larger. Even though government financing by financial intermediaries accounts for only a small part in the US, most financial friction models have been calibrated on US data; for comparability with the existing literature, we follow the conventional calibration, leaving calibration on European data for the future. We assume more aggressive monetary policy in the face of a credit crisis, ssetting $\rho_r = 0.4$ in times of crisis. We do not set it to zero, due to the fact that we might get below the zero bound. A fnancial crisis is triggered by a negative shock to the capital quality ξ_t of 5 percent on impact, with an autocorrelation coefficient $\rho_\xi = 0.66$, as in Gertler and Karadi (2011). We refrain in the current paper from investigating a shock to the debt limit. Hence our simulations have a standard deviation $\sigma_b = 0$, or a fixed maximum level of debt b_t^{max}.

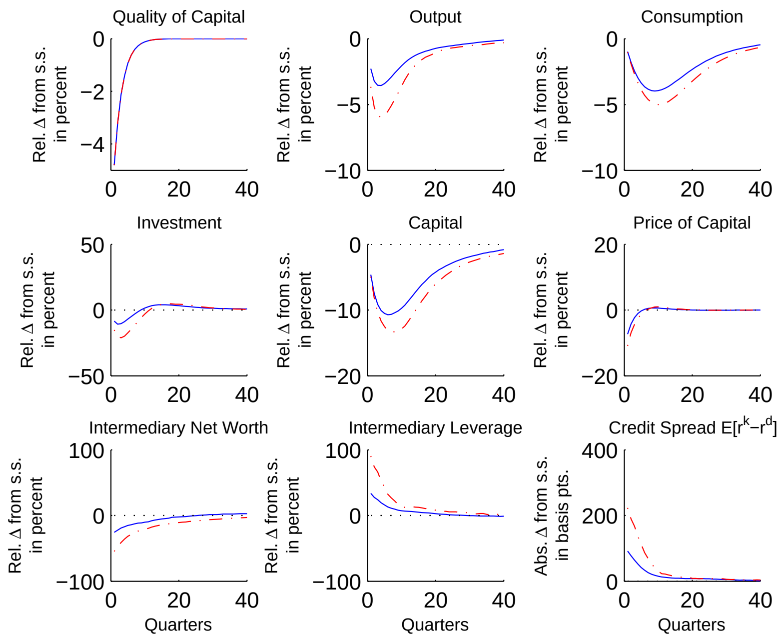
5.2 Short term government debt vs. consols, no government policy response to the financial crisis

In this section we will investigate the effect of a credit quality shock when the government is financed through short term debt, and compare this with the consequences of the same shock when the government is financed through long term government debt, or 'consols'. This introduces the possibility of capital losses on bank holdings of sovereign debt. We only compare the impact of the crisis under the two different assumptions about the maturity of public debt and assume for the time being that the government does not engage in any additional policy. The results can be seen in figure 1.

The underlying mechanism that drives the results is similar to what happens in Kirchner and Van Wijnbergen (2012). A decrease in the capital quality ξ_t induces losses at the financial intermediaries on the loans provided to the intermediate goods producers. Due to these losses, the net worth of the financial intermediaries decreases, and the intermediaries become more balance sheet constrained. This can be seen from the increase in the credit spread by almost 100 basis points. The lower quality of capital also decreases the expected future productivity of the capital that is purchased with the loans. Because of the lower net worth, the financial intermediaries have to cut back on lending, which drives up interest rates and further reduces the value of capital, with higher returns on the loans expected in the future because of overshooting in q_t^k. Due to arbitrage between private loans and government bonds, the expected interest rate on government bonds also increases, which makes it more expensive for the government to borrow. The collapse in the value of capital leads to

Parameter	Value	Definition
Households		
β	0.990	Discount rate
υ	0.815	Degree of habit formation
Ψ	3.409	Relative utility weight of labor
φ	0.276	Inverse Frisch elasticity of labor supply
Financial Intermediaries		
λ	0.3863	Fraction of assets that can be diverted
χ	0.0021	Proportional transfer to entering bankers
θ	0.9722	Survival rate of the bankers
Intermediate good firms		
ϵ	4.176	Elasticity of substitution
ψ	0.779	Calvo probability of keeping prices fixed
α	0.330	Effective capital share
Capital good firms		
γ	1.728	Investment adjustment cost parameter
δ	0.0494	Steady state depreciation rate
Autoregressive components		
ρ_z	0.95	Autoregressive component of productivity
ρ_ξ	0.66	Autoregressive component of capital quality
ρ_g	0.800	Persistence in government spending
ρ_b	0.800	Autoregressive component of maximum level of non default debt
ρ_r	0.800	Interest rate smoothing parameter
Policy		
κ_b	0.050	Tax feedback parameter from government debt
κ_π	1.500	Inflation feedback on nominal interest rate
κ_y	0.125	Output feedback on nominal interest rate
Default parameters		
$\bar{\Delta}$	0.1587	Steady state share of default indicator
Shocks		
σ_z	0.010	Standard deviation productivity shock
σ_ξ	0.050	Standard deviation capital quality shock
σ_u	0.050	Standard deviation unannounced government spending shock
σ_a	0.050	Standard deviation pre-announced government spending shock
σ	0.100	Standard deviation default indicator function
σ_r	0.0025	Standard deviation interest rate surprise shock

Table 1: Table with model parameters.



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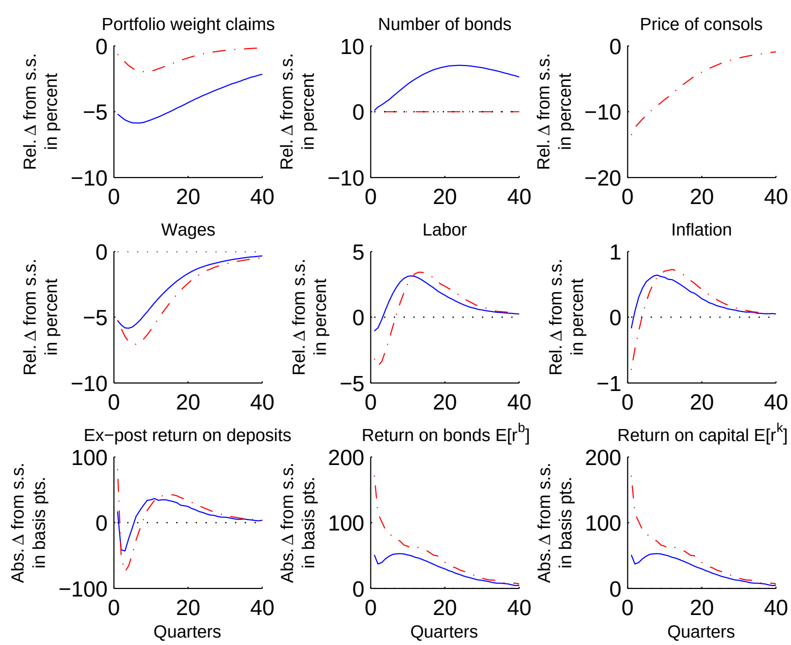


(b)

Figure 1: Plot of the impulse response functions for economy with short term gov't debt financing (blue) and consols (red,-.-.) after a negative capital quality shock of 5 percent relative to the steady state.

further losses on the private loans, which in turn trigger additional declines in financial net worth, once again higher interest rates and from there again further reduction in demand for capital. The intermediaries' balance sheet continues to deteriorate, with subsequent consequences for borrowing costs and so on. So we see a pro-cyclical amplification cycle with a substantial impact: investments and capital drop by more than 10 percent.

A third balance sheet effect was highlighted by KW (2012) and is due to direct crowding out by government debt. Government spending g_t is fixed; then government borrowing is primarily affected by the interest rate r_t^b. Since the increase in the (expected) interest rates on private loans pushes up the (expected) interest rates on government debt, the absolute amount of government debt to be issued gradually increases to finance the higher interest payments. Hence a smaller proportion of the intermediaries' balance sheets is available for financing the capital purchases of the intermediate goods producers. This effect is amplified because the size of the balance sheet is reduced as well, due to a further tightening of the balance sheet constraint.

The lower capital quality also leads to lower wages, as the marginal product of labor falls too with effectively less capital at its disposal. Lower wages and reduced profits translate into lower consumption: output and consumption are reduced by more than 3 percent. We see that even after after 40 quarters the economy still has not recovered completely from the initial shock. This model therefore highlights the point made by Reinhart and Rogoff (2009): financial crises lead to long lasting declines in output.

The policy experiments also show that the maturity structure of the government debt has a major impact on the effects of a financial shock. Like in the short term debt case, interest rates go up, but now banks incur losses on their portfolio of long term sovereign debt, introducing another negative balance sheet effect of a financial crisis. The price of consols goes down by more than 10%. As a consequence, bank capital is reduced much more and credit tightness increases correspondingly: the credit spread doubles when the government is financed through consols instead of through short term debt. When the government finances itself through short term debt only, no such capital losses on government bonds arise. The introduction of long term government debt thus introduces a new negative amplification channel through capital losses on government bonds held in commercial banks' portfolios. This additional amplification channel leads to a more than twice as large a decrease in net worth at the financial intermediaries significantly tightening balance sheet constraints further, and lowering the price of capital even more. This causes (expected) interest rates to shoot up to more than twice as high a level as in the short term debt only case. This also more than doubles the reduction in investment, pushing down future capital stocks significantly more. But debt levels go up less than in the short term debt only case because old debt does not need to be rolled over, as it does in the case of short term government debt. The perverse link between higher interest rates and higher debt service costs on existing debt that plagues the *short term debt only* case of course does not arise when all debt consists of perpetuals only. But although the higher interest rates may not in themselves trigger increases in government debt as debt services on old debt does not go up, the trough in output is doubled nevertheless, since the reduction in investment is more than twice as high as in the case of short term debt finance only. Accordingly, negative effects on consumption and wages are aggrevated as well through the introduction of long term debt.

5.3 Fiscal policy experiments

In this section we look at deficit financed fiscal policy whereby the government borrows additional funds from financial intermediaries in order to stimulate the economy after the financial crisis has caused a recession.

5.3.1 Short term government debt: no additional policy vs. additional fiscal stimulus

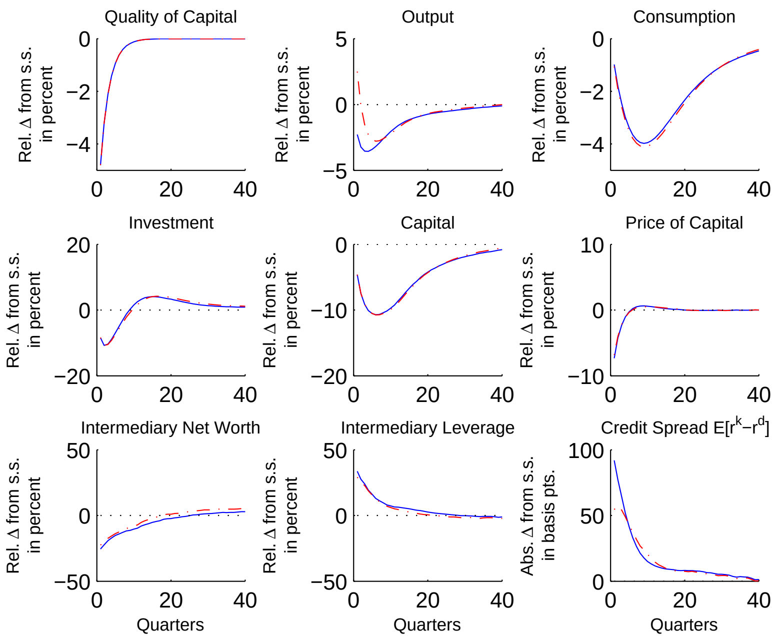
In this section we first assume all debt is again short term, as a benchmark. Consider the effect of an immediate increase in government spending of 5 percent of steady state output through increased deficit financing (figure 2) in response to the crisis, in comparison with the *no policy response* case. The first thing to notice is the fact that the proportion of intermediaries' assets invested in government bonds increases in order to finance the additional government spending. But the financial intermediaries are balance sheet constrained, so the increase in government bonds crowds out private investment, leaving a (slightly) lower capital stock than in the no-response case. Interestingly, we also see a negative impact of the stimulus package on private consumption, a point we will come back to below.

With private investment and consumption declining, it should not be a surprise that the output increase in response to the stimulus package is only 4,5 percentage points relative to the steady state on impact. Because of the negative response of private investment and consumption, the Keynesian impact multiplier is smaller than 1. It is anticipated that government spending is only temporary, and hence most of the additional output is delivered through an increase in labor supply rather than through increased capital accumulation. The increase in labor supply is triggered by increased wages with respect to the case without government stimulus. The extra stimulus, therefore, slows down the drop in output, but only temporarily. The trough of the recession is half a percentage point above the no stimulus case, but occurs later. Higher taxes are anticipated in order to service the additional government debt, leaving less room for future consumption. As this is anticipated by intermediate goods producers, demand for capital drops, and we see investment drop slightly with respect to the no intervention case after the initial quarters of the crisis. Financial intermediaries are also more balance sheet constrained, as indicated by a higher credit spread. Interest rates are increased as a consequence of the stimulus package, reducing the demand for capital further. This causes investment to be below the no intervention case for a prolonged period, pushing output eventually below the no stimulus case: eventually the multiplier turns negative! So the stimulus package is characterized by a contemporaneous Keynesian multiplier smaller than 1 and a dynamic multiplier that eventually even turns negative.

5.3.2 Fiscal stimuli and long term debt

Consider now the impact of long term government debt. The fiscal stimulus is again implemented immediately as the crisis hits (figure 3). The fiscal stimulus again amounts to an increase in government spending of 5 percent of steady state output. The results of the simulations show that lengthening the maturity of the government debt has a dramatic impact.

The credit spread goes up approximately 150 basis points more than in the short term debt only case, clearly indicating a much stronger tightening of the balance sheet constraint of the financial intermediaries. The reason is clear: now not only the price of capital drops, causing capital losses for banks, but bond prices drop as well, by almost 15 percent, causing the initial drop in net worth of financial intermediaries to be more than twice as high. The ensuing tightening of



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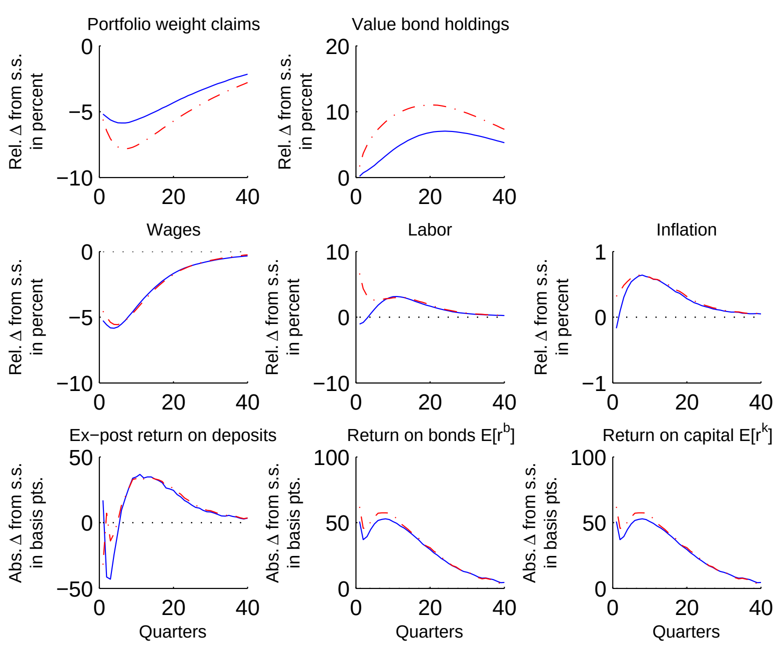

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Figure 2: Plot of the impulse response functions for economy with short term gov't debt financing comparing no additional government policy in case a financial crisis hits (blue), and the case where the government engages in immediate additional government spending of 5 percent of steady state output through deficit financing (red,-.-.). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

	No default	No default	Default	Default
Type of stimulus	Immediate	Delayed	Immediate	Delayed
Discounted cumulative multiplier	0.90	0.37	0.31	-0.51

Table 2: Table displaying the discounted cumulative multiplier as defined in the text, when the government is financed through consols.

the leverage constraint causes loan interest rates to increase dramatically as well, thereby further reducing demand for capital, so the decline in investment is also about twice as high as it is in the case of short term government debt only. The net impact on output is therefore much less than in the short term debt only case, both initially and as time goes by. Thus both impact and dynamic Keynesian multipliers are substantially lower in the consol case than they are in the short term debt only case. The impact multiplier remains positive although smaller than one for the first few quarters, and then actually turns negative.

Of course the increase in debt is less in the consol case because the old debt does not need to be refinanced at higher rates, as is necessary in the short term debt only case, but this mitigating effect is not enough to reverse the negative impact of capital losses on sovereign debt holdings of commercial banks. Again bond prices drop precipitously, now by some 15% upon impact. Output and labor demand go up less and wages fall more commensurately.

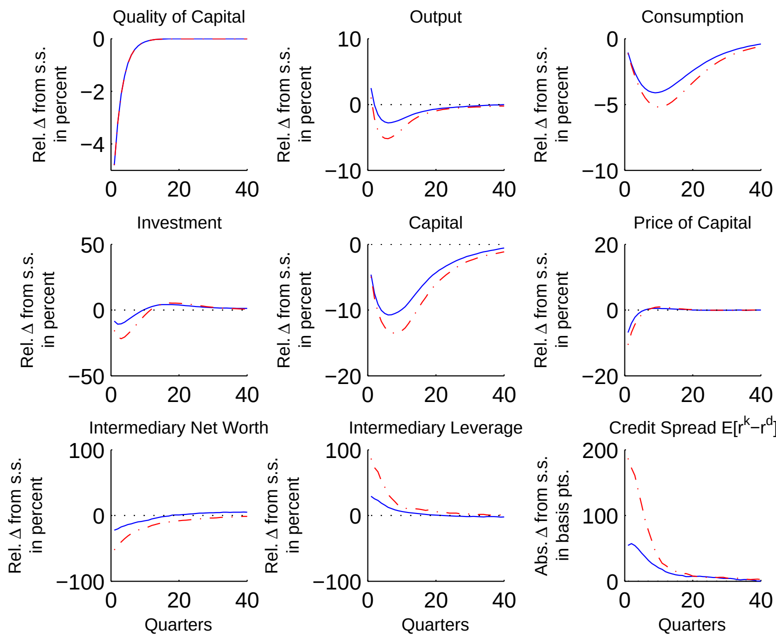
Next we look at the more realistic case where additional government spending (equal to 5 percent of steady state GDP) is announced when the crisis hits, but will be implemented only 4 quarters later, a delay that conforms to the normal budget cycle. Once again the impact effects of a stimulus are more subdued in the consol case. The initial output response is less, as is the response on initiation of the package four quarters later. Credit spreads go up more as the balance sheet tightens because of the additional losses triggered by the fall in the price of consols.

Arbitrage brings the drop in asset prices forward, so credit conditions tighten immediately although the package is only implemented after four quarters. Hence leverage goes up immediately as net worth falls, and investment falls more on impact in the consol case than it does in the short term debt case, with commensurately more negative impact on the time path of output, just as in the case of immediate stimulus.

We can also evaluate the effectiveness of the stimulus packages by looking at the discounted cumulative multiplier[7] from table 2. We see that the effects for immediate stimulus are better than for delayed stimulus. Both multipliers, though, are smaller than 1 and imply therefore that additional government spending as a policy to combat the effects of a financial crisis is ineffective. We also observe that the multiplier is smaller for the delayed stimulus due to the negative anticipation effects mentioned earlier.

[7] The discounted cumulative multiplier is calculated through the following formula, where x^{st} denotes a variable under stimulus, while x^{np} denotes a variable under no additional government policy:

$$\sum_i \frac{\beta^i(y^{st}_{t+i} - y^{np}_{t+i})}{\beta^i(g^{st}_{t+i} - g^{np}_{t+i})}$$



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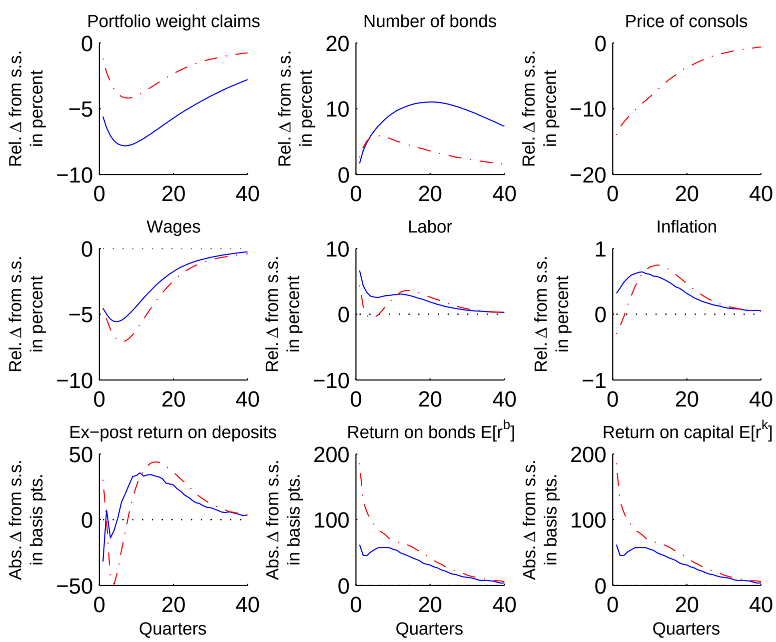

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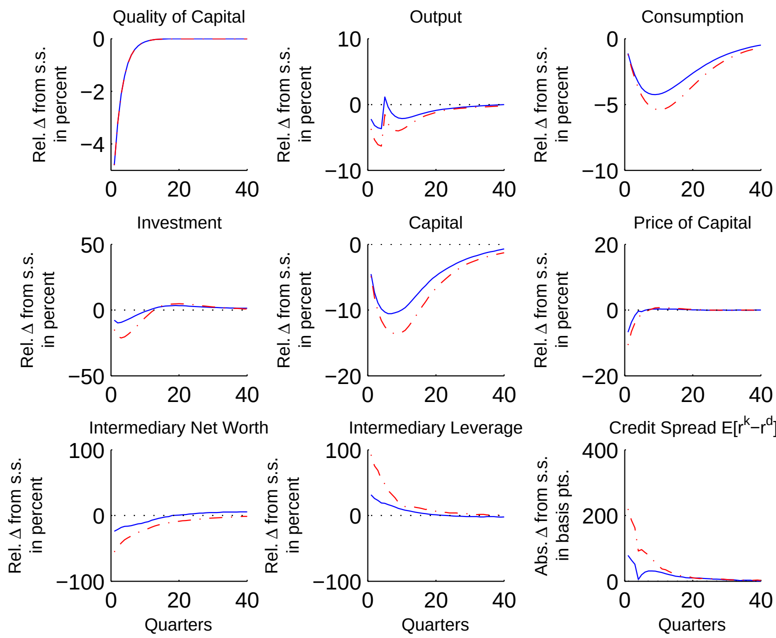
Figure 3: Plot of the impulse response functions for economy with short term gov't debt financing (blue) and consols (red,-.-.) after a negative capital quality shock of 5 percent relative to the steady state, where the government engages in additional government spending of 5 percent of steady state output through deficit financing.



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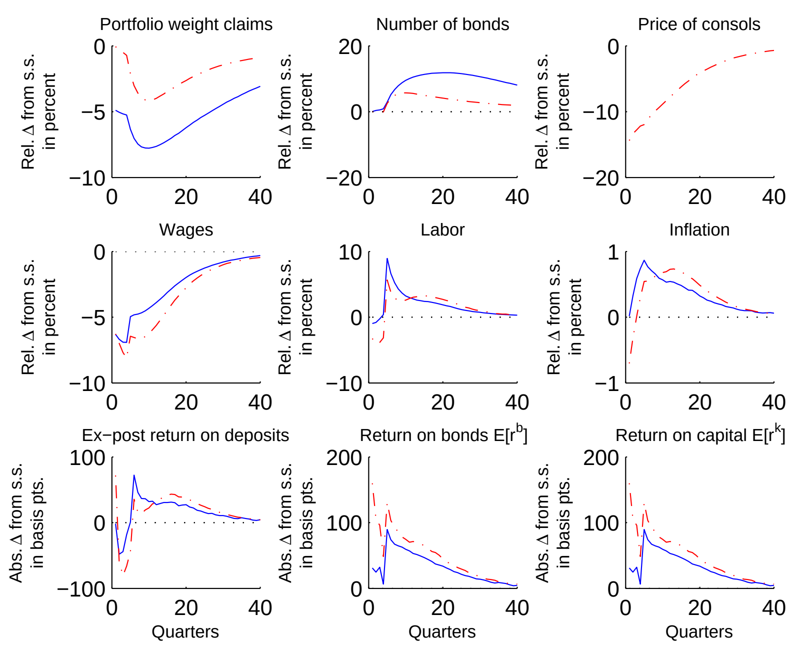

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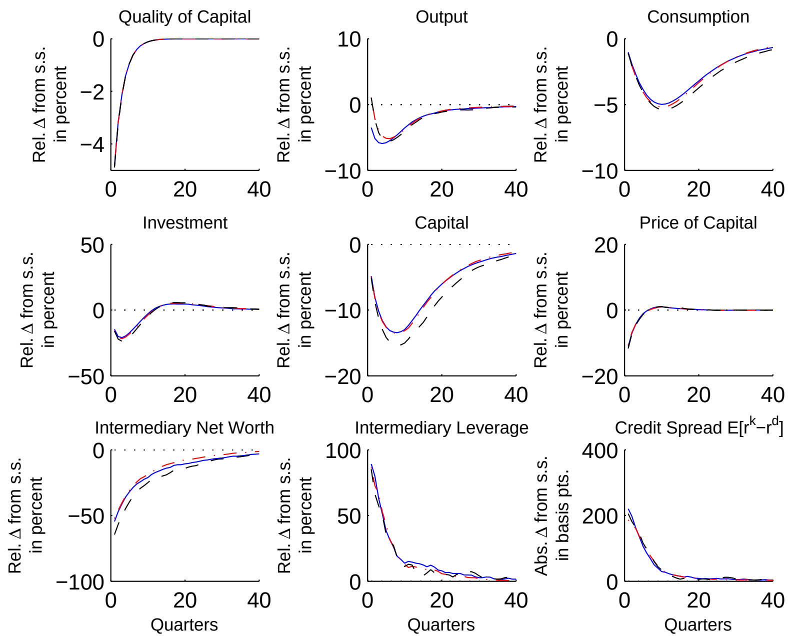
Figure 4: Plot of the impulse response functions for economy with short term gov't debt financing (blue) and consols (red,-.-.) after a negative capital quality shock of 5 percent relative to the steady state, where the government engages in additional government spending of 5 percent of steady state output through deficit financing. The government spending is announced at the onset of the crisis, but implemented 4 quarters later.



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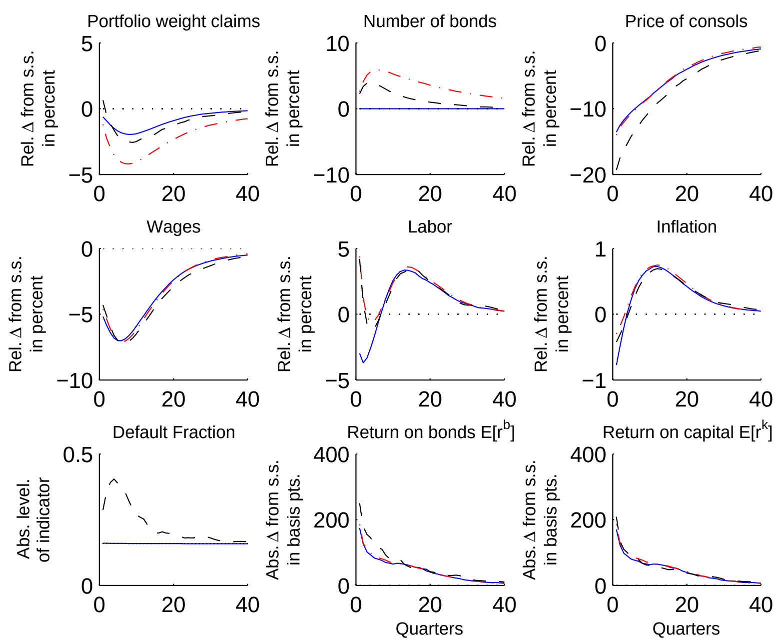


(b)

Figure 5: Plot of the impulse response functions for economy with consol financing comparing the case with sovereign default risk (blue), the case with immediate stimulus spending and no sovereign default risk (red,-.-.), and the case with immediate stimulus spending in the presence of sovereign default risk (black,- - - -). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

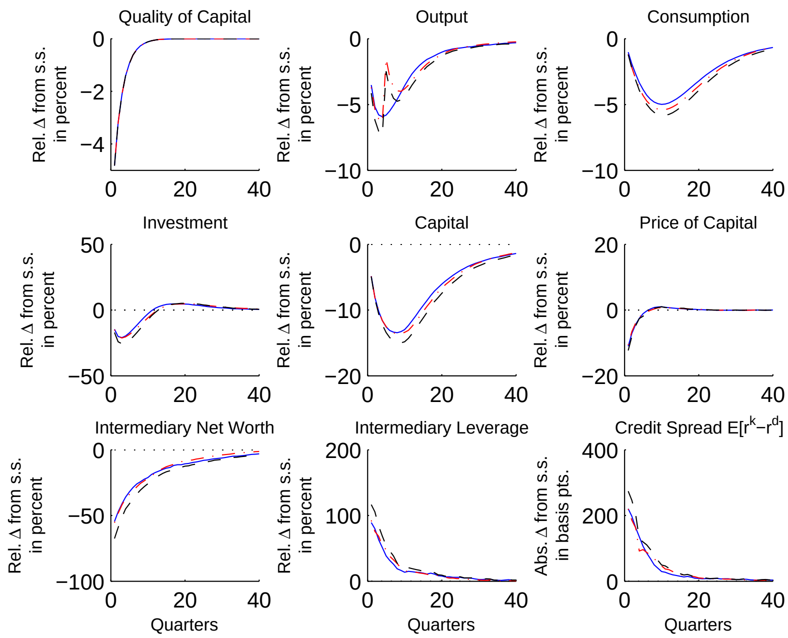
5.4 Fiscal policy in the presence of sovereign default risk

Consider now the impact of sovereign default risk, along the lines of the default model introduced in Section 4.2. Introducing sovereign risk in the no-policy-repsonse case does not change much because the time path of public debt does not materially change without policy response. We therefore immediately go to the analysis of the stimulus package, and compare the cases with and without default risk. Figure 5 displays the case where the government engages in immediate stimulus spending, while figure 6 displays the case where the government immediately announces that government stimulus spending will be implemented 4 quarters from announcement.

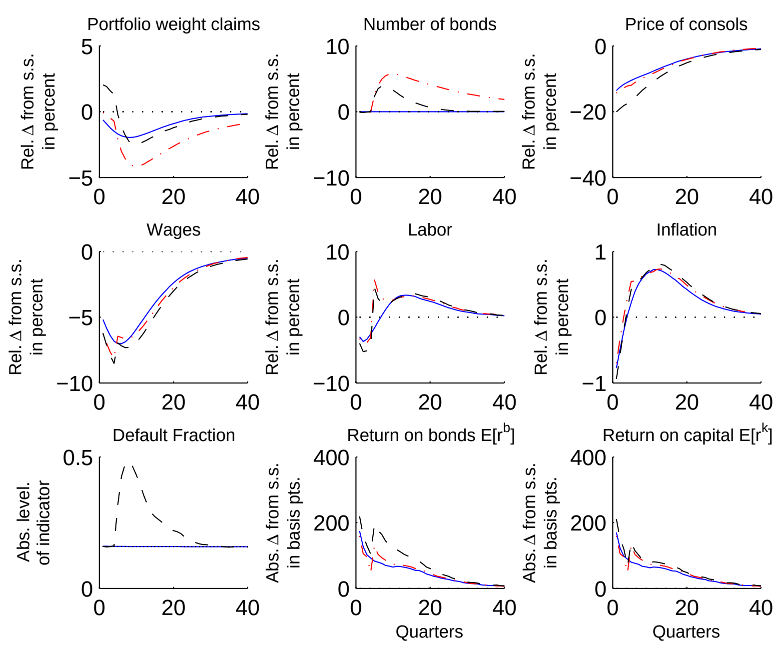
Incorporating sovereign default risk in the analysis of a deficit financed fiscal stimulus leads to a significant deterioration compared to the no default risk outcome. The credit spread increases, as well as the leverage ratio, but especially the net worth of the financial intermediaries decreases by an additional 10 percent with respect to the steady state value. The reason is the fact that the government needs to issue new consols in order to finance the additional spending. Due to the fact that the financial intermediaries have to take losses on the private loans, as described earlier, they become more balance sheet constrained. The leverage constraint becomes more binding, which pushes up (expected) interest rates, reducing the demand for new capital. Due to abritrage between private loans and consols, the (expected) return on consols is driven up as well, driving down the price of consols. This in turn increases the number of consols the government needs to issue in order to finance the additional government spending. As a consequence, the government therefore, defaults on a larger fraction of the coupon payments to the consol holders, to reduce the debt to sustainable levels. The financial intermediaries financing the government anticipate the higher default fraction, which leads to a further fall in their price, which drops with more than 5 percent compared with the no default case. This in turn, leads to larger additional capital losses for financial intermediaries on their holdings of 'old' consols. Therefore aggregate financial net worth goes down by more than 10 percent of steady state net worth.

The impact on the real economy is substantial. Even though output is initially hardly affected, the trough of the recession is half a percentage point deeper, and remains below the no default case output time path. Households become more constrained through lower wages, and lower profits from both the financial and non-financial firms they own, so consumption goes down. The deterioration in net worth causes (expected) interest rates to go up, driving down investment and the capital stock, thereby pushing down output in the long run. It should be noted, though, that the number of consols increases more slowly in the presence of sovereign default risk because the government is defaulting on part of the coupon payments to the holders of the consols.

We see from figure 6 that the effects are even stronger when the fiscal stimulus is implemented 4 quarters after the financial crisis starts. The credit spread increases further, and even though the default fraction does not increase on impact, the upcoming consols issue, and the accompanying anticipated increase in the default fraction drives down the consol price more than in the case of immediate stimulus, causing net worth to fall further down. This drives up (expected) interest rates, reducing demand further: investment declines more than in the case of immediate stimulus. At the moment the stimulus is implemented, the consols price is lower than in the case of immediate stimulus. Compared with the no default case, output is lower by a percentage point in the quarter before implementation of the stimulus, and this difference only dies out slowly. There are significant output losses before and after the stimulus is implemented in comparison to the case of no additional fiscal stimulus, with the fiscal stimulus pushing output above the no fiscal stimulus case only during the first three quarters immediately follwoing implementation, as can be seen from figure 7. For later periods, the Keynesian multplier is actually negative. The deterioration with respect to the



(a)



(b)

Figure 6: Plot of the impulse response functions for economy with consol financing comparing the case with sovereign default risk (blue), the case with preannounced stimulus spending implemented 4 quarters after announcement and no sovereign default risk (red,-.-.), and the case with immediate stimulus spending in the presence of sovereign default risk (black,- - - -). The financial crisis is initiated through a negative capital quality shock of 5 percent relative to the steady state.

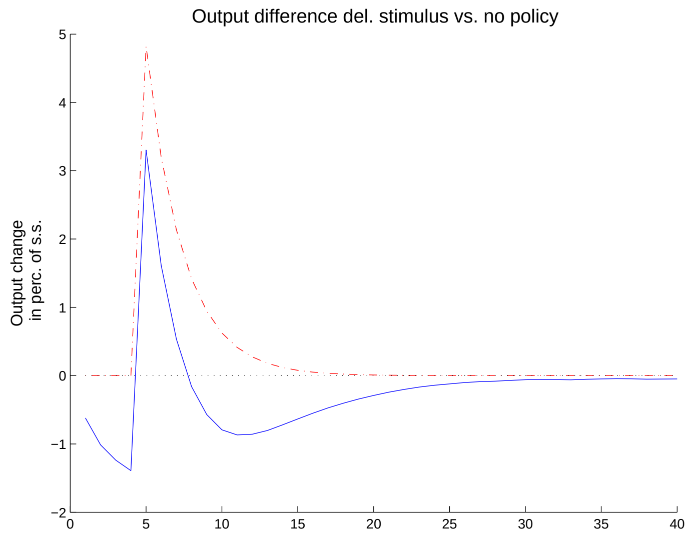


Figure 7: Figure showing the difference in output between a delayed fiscal stimulus and no additional government policy in the presence of sovereign default risk, expressed as percentages of steady state output. The red dotted line displays the fiscal stimulus itself expressed as a percentage of steady state output.

no default case becomes especially clear from table 2, where we see that the discounted cumulative multiplier decreases for both immediate and delayed stimulus by 0.50, even generating a negative multiplier for the delayed fiscal stimulus package!

6 Conclusion

In this paper we investigate the impact of fiscal stimulus packages in the presence of financial fragility, such as when a financial crisis leaves the banking system weakly capitalized. A key aspect of our model structure is that commercial banks play a role in the financing of government debt; they act as placement agents for sovereign debt, collecting deposits on the one hand and investing the proceeds in loans to the government and the private sector on the other. Banks act under a leverage constraint due to agency problems between depositors and bank management (and its shareholders), building on an approach introduced by Gertler and Karadi (2011). We extend their approach by introducing asset choice in bankportfolios (banks now both lend to governments and to the private sector), long term sovereign debt and, in the last section of the paper, sovereign default risk. The latter two extensions introduce a crucial feature of the current euro crisis in the framework: capital losses by commercial banks on their holdings of sovereign debt.

In the benchmark case with short term debt only and without default risk, we already find that Keynesian stimulus packages will see much reduced effectiveness in conditions of financial fragility. We find that the contemporaneous fiscal multiplier of an immediate spending stimulus is slightly smaller than 1, with consumption, investment and output *below* the no intervention case after the initial stimulus has died out: so dynamically, multipliers actually turn negative. We also analyze the realistic case of a pre-announced but delayed stimulus, announced immediately when the financial crisis hits but implemented only four quarters (one budget cycle) later. In the run up to the implementation, consumption, output and investment barely move with respect to the no intervention case, even though future demand is expected to be higher once the spending package is implemented. This is due to the anticipation of the credit tightening that is also foreseen once the package is implemented. The effects on the economy are worse than without the intervention within three quarters after the initial implementation of the stimulus package. The impact (instantaneous) Keynesian multiplier is smaller than 1, but anticipation effects leading up to the package and credit tightening afterwards actually cause negative output effects of the stimulus before the start of the program and after the intial positive impact: before and after the quarters in which the stimulus measures are initiated, multipliers are actually negative.

We find that outcomes worsen significantly when we incorporate long term government debt into the model and the associated posibility of capital losses on commercial banks' holdings of sovereign debt. For sharper focus we introduce consols, which admittedly exaggerates the maturity of outstanding government debt we see in practice. But the mechanism captures a very real aspect of the euro crisis, capital losses on holdings of sovereign debt by commercial banks. The stress tests conducted by the European Banking Authority revealed that in the eurozone as a whole, sovereign debt constituted no less than 16% of commercial bank assets, with much higher numbers in the Southern periphery countries where banks play a much bigger role in the financing of fiscal deficits than they do in countries like Germany or the Netherlands. Tightening credit conditions and rising interest rates on public sector debt then cause bond prices to drop; the subsequent capital losses further erode the capital ratios of commerial banks, amplifying the initial credit tightness. Consumption and output decrease by multiple percentage points compared with the case of short term bond financing, while the decrease in investment is more than doubled, which in turn leads to declines in the capital stock as time passes, and associated reductions in the growth rate of potential output. When we also introduce sovereign default risk, the effects of fiscal stimulus become significantly worse. The trough of consumption drops by more than a percentage point, while the (discounted) cumulative output multiplier is close to zero or even negative; negative effects on investment and subsequently the capital stock are substantial. So short term output gains, if there are any at all, are bought at the expense of future output as potential output growth actually slows down in response to the stimulus measures. Capital losses on long term sovereign debt have played an important role in the current euro crisis limiting the scope for expansionary fiscal policy, since financial sector problems feed into the budgetary problems of sovereigns, which in turn feed back to the financial sector in the form of capital losses on long term sovereign debt holdings. Of course we do not argue that multipliers are always negative; but financial fragility and sovereign risk problems may severely lower them, possibly to the point of becoming negative.

The negative balance sheet effects of deficit financed Keynesian stimulus packages are likely to be less if banks are better capitalized; thus a recapitalization of banks after a financial crisis (as analyzed in Van der Kwaak and Van Wijnbergen (2013)) may well provide more room for fiscal stimulus measures. That would also explain why fiscal expansion seems to have worked well in the United States, where financial intermediaries were forced to clean up their balance sheets in 2009, and where banks anyhow play a smaller role in the provision of public and private debt finance. Our model indicates that in Europe, though, additional government stimulus might backfire, given the weak balance sheets of financial intermediaries. In spite of extensive government support measures, financial intermediaries still carry extensive unrecognized losses on their balance sheets in the aftermath of the financial crisis (IMF 2011). Additional government spending is more likely to crowd out private credit in such an environment, and cause further financial intermediary problems through feedback loops through bond prices, possibly much exacerbated if there is significant sovereign default risk. A more aggressive approach to restoring bank capital ratios might thus widen the scope for expansionary fiscal policy and so promote recovery in debt ridden economies, rather the opposite of what many fear the higher capital ratios envisaged under Basel-III will bring about. An obvious extension of our current framework would focus on international spillovers through cross border holdings of sovereign debt by commercial banks, Lagarde's "chains of contagion".

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