

Human Capital and Retirement

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Abstract

This paper investigates the relation between human capital and retirement when the age of retirement is endogenous. This relation is examined in a life-cycle earnings model. An employee works full time until retirement. The worker accumulates human capital by training-on-the-job and by learning-by-doing. The human capital of an employee is subject to depreciation when knowledge of technologies becomes obsolete. After a shock in technology, the worker depreciates on his human capital. The lower human capital results in a lower life-time income, but also in a lower price of an earlier retirement.

Keywords: endogenous retirement, human capital, life-cycle models

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1. Introduction

Would Methusalem retire at the age of 65? Though life-expectancy has increased, labor participation of older employees has declined severely over recent decades (OECD (1998)). Early retirement schemes are becoming common practice and the institution of a mandatory pension at 65 might be ready for retirement. The incidence of early retirement differs over sectors and is lower for better educated workers (Blöndal and Scarpetta, 1998). In this paper I investigate the relation between human capital and retirement when the age of retirement is endogenous, and the effects of a change in technology on the preferred age of retirement.

Financial incentives are used to discourage work at older ages in most OECD countries (see Blöndal and Scarpetta (1998), and Gruber and Wise (1997)). Workers are induced to retire by opting for an early retirement scheme. The additional income of continuing work after the 'induced retirement' age (mostly 62 or 65) is low or even negative, because of the fall in the discounted total value of future pension and retirement benefits.¹ Labor market conditions and the increase in life-time wealth have resulted in an increase in the supply of early retirement schemes. The choice for early retirement schemes explains the exit behavior of older employees at the induced retirement ages. However, we observe that more and more employees leave the labor market before these ages.

In this paper, the incentives of employees to retire early are examined. The retirement decision of an employee can be seen as a trade-off between a longer period of retirement and a higher life-time income. The last decades wage levels and life-time wealth increased, which affected the preferred retirement age of employees. Because leisure is a form of consumption, higher life-time wealth results in a higher demand for leisure. Furthermore, changes in the wage level affect the opportunity costs of leisure, and the 'price' of retirement. Hence, to examine the shift in the age of retirement, we should also focus on endogenous wage profiles, which are determined by productivity levels.

Productivity levels are the result of investment in human capital in the past. The employee learns the newest technologies in school or on the job by learning-by-doing and

¹ These early retirement schemes are not paid by the workers or are not 'actuarially neutral' in the sense that prolonging the working period results in a decline in the total discounted value of the retirement benefits.

training-on-the-job. When the employee gets older, the return period for investment in human capital becomes shorter and incentives for human capital accumulation decrease. Because knowledge of older technologies becomes obsolete and because people forget, human capital is subject to depreciation. The short return period and the depreciation of human capital leads to a declining human capital at the end of the (working) life, unless the level of learning-by-doing is higher than the depreciation rate. Hence, declining productivity at older ages is the result of the simultaneous decisions on the amount of human capital investment and the intended date of retirement. According to Kotlikoff and Gokhale (1992), productivity peaks at age 45 and declines thereafter.

A change in technology affects productivity levels and the preferred age of retirement. The introduction of a new technology in a firm has opposite effects on wage levels. Firstly, it increases the marginal productivity of labor, which increases the wage level. And secondly, it makes knowledge of an older technology partly obsolete, which results in an instantaneous decline in human capital. The decline in human capital implies a lower price for leisure, and induces workers to substitute leisure for consumption and to advance the date of retirement. However, a lower human capital affects full income as well. When a worker is not keen on lowering his consumption level, he might prefer to work for a longer period, to maintain a higher consumption level.

Bartel and Sicherman (1993, p.174) find that *'when individuals experience an unexpected change in the rate of technological change, the older they are, they are more likely to retire'*. Moreover, they find that *'individuals in industries that have high "permanent" rates of technological change have longer working careers and this result is due to the effect of technological change on the amount of training as well as its effect on the slope of the investment profile'*.

Most models that explain retirement focus on financial incentives to induce workers into retirement. The dynamic life-cycle models do treat a wage profile over the working life and benefits from (early) retirement, but the wage profile is not endogenous. In Gustman and Steinmeier (1986) employees can choose part-time work, before entering retirement. In Stock and Wise (1990) postponement of retirement has an 'option' value and workers calculate the gain from postponing retirement versus immediate retirement. The employees intend to retire at the moment that the calculated gain is at its maximum. In the stochastic

dynamic programming models of Berkovec and Stern (1991) and Lumsdaine et al. (1992) the employee makes similar calculations, but takes into account that in the next year there will be new information which can be used to reoptimize the retirement decision. Rust and Phelan (1997) incorporate the effect of health status on the retirement decision in their dynamic programming model.

The literature on life-cycle earnings integrates human capital accumulation, work and leisure over the life-cycle, but retirement in these models is not treated satisfactory. Leisure increases smoothly and 'retirement' is defined as 'a period with few hours of work supplied to the market' (Heckman (1976), S15) or as in the Blinder-Weiss (1976) model, retirement is the result of a corner solution.

Blinder and Weiss (1976) and Ben-Porath (1967) have human capital functions that describe individual earnings profiles. Heckman (1976) uses the Ben-Porath human capital function and is able to perform analytical comparative statics. Killingsworth (1982) extends the model of Heckman, by taking learning-by-doing into account as well.

Sala-i-Martin (1996) extends the human capital literature by stressing the effect of changes in technology on the depreciation rate of human capital and its effect on the demand for older workers, but in his model the retirement decision is not modelled explicitly.

The model used in this paper stresses the relation between human capital accumulation and retirement. The model follows the literature on life-cycle earnings, but contrary to the model of Heckman (1976) and Killingsworth (1982) the age of retirement is an (explicit) endogenous decision variable. In the model an employee works full time until retirement. During his working period he divides his time between schooling and working. The worker increases his human capital by training-on-the job and by learning-by-doing. Human capital is subject to depreciation, because the worker forgets knowledge and loses physical capabilities. A second factor of depreciation is the result of changes in technology. Introduction of new technologies make older technologies obsolete, and the knowledge and experience the employee acquired of the replaced technology become partly useless. The decision to retire depends on the trade-off between higher living standards and a longer period of retirement.

The model enables comparative dynamics, which allows us to examine the effect of

changes in technology and wage levels on the intended moment of retirement when the retirement date is flexible. Moreover, the optimal pension date of people with different levels of learning and different ability levels can be determined. After a shock in technology employees have to weigh the additional utility of prolonging the working period, and enjoying a relatively higher level of consumption, against the utility of a longer period of retirement. The older is an employee, the stronger the substitution effect relative to the income effect will be. It is found that the income and substitution effects depend strongly on the intertemporal rate of substitution and the moment of the shock in the life cycle.

This paper is structured as follows. In section 2 I first treat the model. In section 3 the effects of shocks on the retirement date are examined with comparative dynamics. Section 4 concludes.

2. The model

The model is based on the life-cycle models of Ben-Porath (1967) and Killingsworth (1982) and extends these models by taking the choice of the retirement age as an endogenous decision. Before the moment of retirement the employee has a full-time job. After retirement the individual does not work any more: all time after retirement is consumed as leisure. The option of part-time work is ruled out by assumption.

The value function is

$$\int_0^{T_r} U(c,0) e^{-\beta t} dt + \int_{T_r}^T U(c,1) e^{-\beta t} dt \quad (1)$$

where

$c(t)$	consumption at time t
$U(c,I)$	utility function of employee
T	moment of death
T_r	moment of retirement
β	rate of time preference

The utility function is strictly concave in consumption and the period of retirement ($T-T_r$). Though it is assumed that leisure is zero before retirement, the model can be easily extended for a fixed amount of leisure before retirement. Note that the utility function is continuous in T_r .

The employee optimizes his value function subject to the budget constraint. A perfect credit market is assumed: the employee can borrow and lend freely at the interest rate, r , though he is not allowed to die in debt. Because the retirement decision is endogenous, the timing of retirement affects the budget for consumption of goods of the employee. The accumulation of assets can be divided in a period before and after retirement. Before retirement the employee receives income from labor and capital. Labor income depends on the productivity level and hours of work.

$$\dot{a}(t) = wH(t)(1-s(t)) + ra(t) - c(t) \quad (t \leq T_r) \quad (2)$$

where a dot denotes the time derivative, $a(0)=a_0$, and

$a(t)$	assets
r	interest rate
w	effective net wage
$H(t)$	human capital
$s(t)$	schooling, $0 \leq s(t) \leq 1$

The net wage is $w=\hat{w}(1-\tau)$, where \hat{w} is the effective gross wage and τ the tax rate on wage income.

After retirement, the level of the assets, the interest income and the level of retirement benefits determine the available budget for consumption.

$$\dot{a}(t) = ra(t) + b(t) - c(t) \quad (t > T_r) \quad (3)$$

where

$a(T_r)=a_{T_r}$, $a(T)=0$	
$b(t)$	retirement benefits

The worker receives retirement benefits upon retirement from the labor force. Retirement

benefits are generally paid after the mandatory retirement date, but other forms of social security might be paid as well before the mandatory retirement date. These other social security payments are contained in the retirement benefits, though it is assumed that the retirement benefits are independent of the human capital level during the working life. Note that this set-up does not necessarily ignore payments of a capital-funded pension fund, which are dependent on wages during the working life. When the return to assets is equal for the worker and the pension fund, and the pension fund has an actuarial fair pension system, savings by pension funds can be seen as part of $a(t)$.

An employee decides about his human capital investment to maximize his income from his working life. During his working life the worker can use his time for schooling (training-on-the-job) or for working. Human capital is accumulated by training-on-the job along the lines of Ben-Porath (1967). Following Killingsworth (1982), the human capital function is extended for 'Learning-by-Doing'. During the hours spent on the labor market, the worker accumulates human capital by working with existing technologies. I assume that human capital which is accumulated by training-on-the-job and by learning-by-doing are perfect substitutes.

$$\dot{H}(t) = [g(s(t)H(t)) + kH(t)] - \delta H(t) \quad (4)$$

where $g(sH)$ is strict concave and

δ	rate of depreciation
k	rate of learning-by-doing

The level of human capital is subject to depreciation: people forget knowledge once learned, and their physical condition also deteriorates slowly. Moreover, human capital depreciates because of changes in technologies used. When a new technology replaces an older one, the knowledge of the older technology has become partly useless. It is assumed that δ is constant over the working period.

$$\delta = \gamma_0 + \gamma_1 \dot{\Phi} \quad (5)$$

where

$\Phi(t)$	level of technology
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The Langrangian can be formulated by taking the value function subject to the budget constraint

$$\begin{aligned} \mathfrak{L} = & \int_0^{T_r} \{ U(c,0) e^{-\beta t} - \mu [\dot{H} - g(sH) - Hk + \delta H] - \lambda [\dot{a} - wH(1-s) - ra + c] \} dt \quad (6) \\ & + \int_{T_r}^T \{ U(c,1) e^{-\beta t} - \lambda [\dot{a} - b - ra + c] \} dt \end{aligned}$$

Assuming an interior solution to the optimization problem, the first order conditions with respect to H , c , s and a are

$$\dot{\mu} = (\delta + \dot{\Phi}(t) - g's - k)\mu - \lambda w(1-s) \quad (t < T_r) \quad (7)$$

$$U_c e^{-\beta t} - \lambda = 0 \quad (8)$$

$$\mu g' - \lambda w = 0 \quad (t < T_r) \quad (9)$$

$$\dot{\lambda} = -r\lambda \quad (10)$$

$\lambda(t)$ gives the shadow price of assets and $\mu(t)$ is the shadow price of human capital. Because human capital is embodied in the worker, and labor income after retirement is zero, the shadow price of human capital, $\mu(T_r)$, is zero, at the moment of retirement.

The first order condition with respect to T_r is

$$(U(c(T_r),0) - U(c(T_r),1))e^{-\beta T_r} + \lambda(T_r)(wH(T_r)(1-s(T_r)) - b(T_r)) = 0 \quad (11)$$

At the moment of retirement, the utility of full-time leisure (retirement) equals the utility of the additional consumption of prolonging the working period. As long as the employee derives more utility from the additional consumption than from the additional leisure, he stays in the labor force. We observe that the preferred moment of retirement is affected by the human capital level over the life-cycle. Firstly, the human capital profile determines the level of consumption. Secondly, it determines the shadow price in terms of consumption of extending the working period.

To simplify the analysis further I assume that the utility function is separable in leisure and consumption.

$$U(c, 1) = u(c) + v(1) \quad (12)$$

I assume that the utility function of consumption is of the form

$$u(t) = \frac{1}{1 - 1/\rho} c(t)^{1 - 1/\rho} \quad (13)$$

where ρ is the rate of intertemporal substitution and $0 \leq \rho \leq \infty$. This leads to

$$c(t) = c(0)e^{\rho(r-\beta)t} \quad (14)$$

The consumption path of goods is determined by (14) and the slope depends on $\rho(r-\beta)$. When $r > \beta$, the consumption path slopes upwards.

Using (8), (12), (13) and (14) the f.o.c. with respect to T_r can be written as

$$v(1)e^{(r-\beta)T_r} = c(0)^{-1/\rho} (wH(T_r) - b(T_r)) \quad (15)$$

When at $t=0$, the initial human capital implies a wage income that is below the level of the 'retirement' benefits, the 'employee' starts with a period of retirement. If $r > \beta$, the individual ends up in a corner solution, where he does not begin a working career at all. When $\beta > r$, employees might prefer to 'retire' first and to work afterwards. Though, some people like to travel after their study for some time, I assume that $\beta > r$ is a special case.²

For a worker to retire before his death sufficient is $wH(T) < b(T)$. Because, at the moment of death human capital is zero, the condition is satisfied when $b(T) > 0$.

The first order conditions with respect to H and s give

$$\dot{\mu} = (\delta - k)\mu - \lambda w \quad (16)$$

Define the ratio of the shadow price of human capital to the shadow price of non-human capital by $\sigma(t)$

² When people first 'retire', the time variables of the integrals in the Lagrangian have to be switched.

$$\sigma(t) = \frac{\mu(t)}{\lambda(t)} \quad (17)$$

From which follows that

$$\dot{\sigma}(t) = (r + \delta - k)\sigma(t) - w \quad (18)$$

Because $\mu(T_r)=0$ and $\lambda(T_r)\neq 0$, the relative shadow price of human capital at the moment of retirement is zero, $\sigma(T_r)=0$, and the general solution for $\sigma(t)$ is

$$\sigma(t) = \frac{w}{r + \delta - k} \left[1 - e^{(r+\delta-k)(t-T_r)} \right] \quad (19)$$

for $t \leq T_r$. $\sigma(t)$ declines monotonically with age. For reasons of analytical convenience, I assume that the human capital function is of the form $g(sH) = \alpha_0(sH)^{1/2}$. Because $g(sH)$ is strictly concave, we can invert the first order condition with respect to s . The demand for investment in human capital can be written as a function of the relative shadow price of human capital: $s(t)H(t) = (\sigma(t)/w)^2$, or using (19),

$$s(t)H(t) = \frac{\frac{1}{4} \alpha_0^2}{(r+x)^2} \left[1 - e^{(r+x)(t-T_r)} \right]^2 \quad (20)$$

where $x = \delta - k$. It is assumed that $r+x > 0$ ($r+\delta > k$), which guarantees a positive investment in human capital. If $r+x \leq 0$, the return to schooling is too low and investment in human capital is zero.

The investment in human capital is driven by the relative shadow price in human capital and declines monotonically with age. Because the shadow price upon retirement is zero, $\sigma(T_r)=0$, and $s(t)H(t) = (\sigma(t)/w)^2$, investment in human capital at T_r is zero, $s(T_r)H(T_r)=0$, and hence $s(T_r)=0$. Upon retirement the return to investment in human capital is zero. The curve of $s(t)H(t)$ is concave when $t < 1/(r+x) \ln(1/2) + T_r$, and is convex³ when $t > 1/(r+x) \ln(1/2) + T_r$ (see Figure 1). Because the return to investment in human capital is increasing in human capital, the incentives for human capital investment are relatively high in the beginning of the working period, though decrease strongly when

³ The turning point can be easily derived from the second order derivative of (20) to t .

the return period declines. This decline is counteracted by the concavity of the human capital function. At low levels of investment in human capital, the return to investment in human capital is relatively high.

Note that the investment in human capital is independent of the wage level, because both the return to investment in human capital and the opportunity costs depend linearly on the wage rate. Moreover, they are independent of the initial endowments of human and non-human capital, because the relative shadow price of human capital is independent of these endowments.

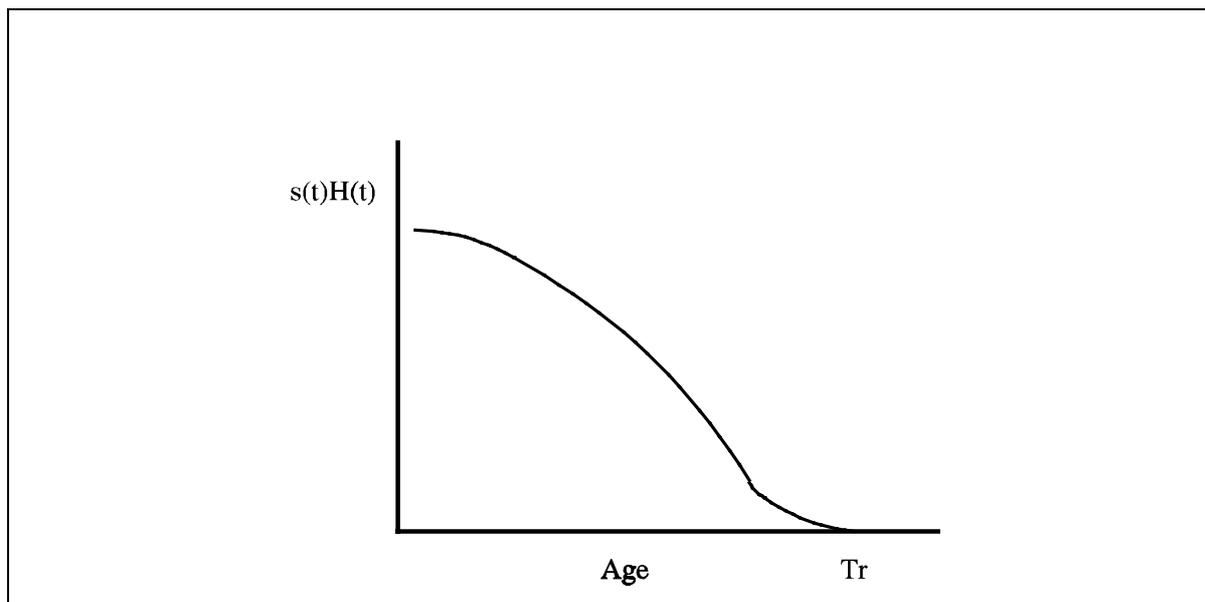


Fig. 1

(20) and (4) give $H(t)$

$$H(t) = e^{-xt} \int_0^t \frac{1}{r+x} \frac{\alpha_0^2}{2} [1 - e^{(r+x)(u-T_r)}] e^{xu} du + H(0)e^{-xt} \quad (21)$$

for $t \leq T_r$. In the beginning of the working period the accumulation of human capital by learning-by-doing and training-on-the job generally exceeds the depreciation of human capital and the human capital level increases. Because the investment in human capital decreases with age and is zero upon retirement, the accumulation of human capital declines and becomes negative when the rate of depreciation exceeds the rate of learning-by-doing (see Figure 2).

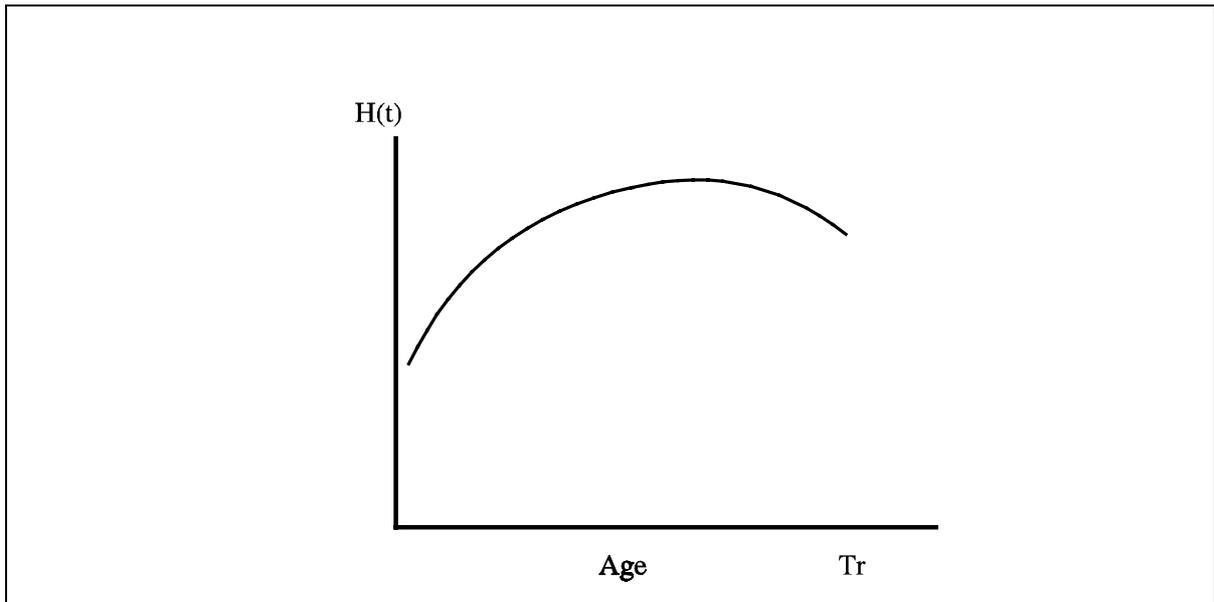


Fig. 2

The return to schooling declines when the return period becomes shorter. The level of training-on-the-job declines monotonically with age and is zero upon retirement (see Figure 3). Hence, the number of hours of work increases over time. Earnings increase initially because of the increase in human capital and because of the increase in the hours of work. As result of a decline in human capital, earnings might decline at the end of the working life, though the peak of the earnings is at a higher age than the peak of the human capital function.

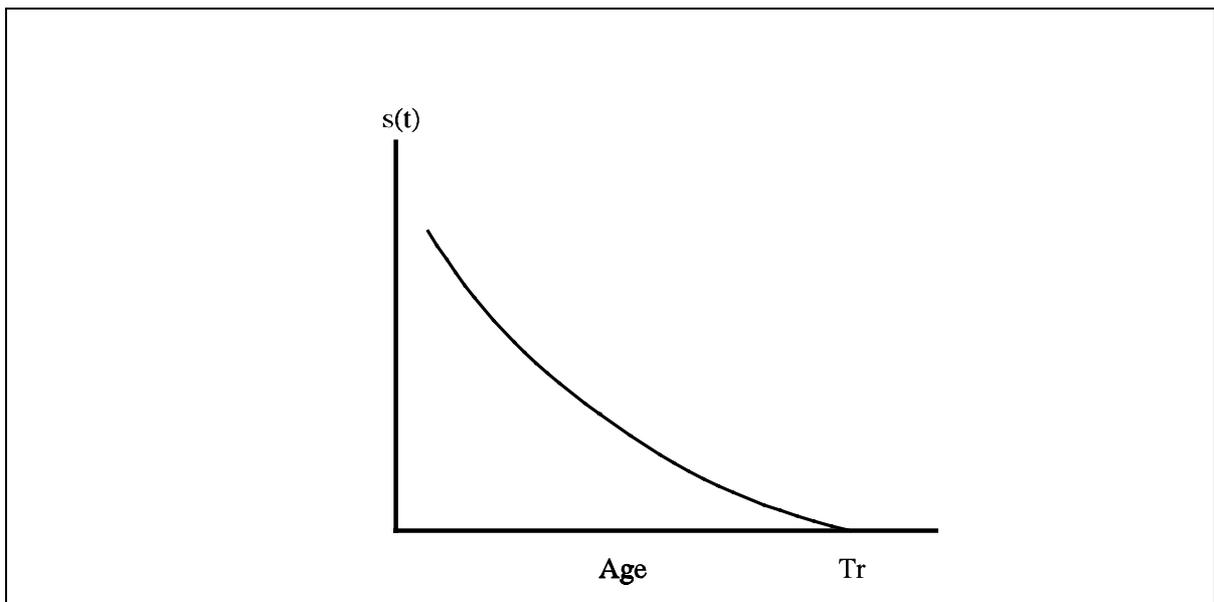


Fig. 3

Using (2), (3) and (21) the total budget of the individual can be rewritten to

$$\frac{c(0)}{r - \rho(r - \beta)} [1 - e^{(\rho(r - \beta) - r)T}] = \int_0^{T_r} w \left[H(t) - \frac{\frac{1}{4} \alpha_0^2}{(r + x)^2} [1 - e^{(r+x)(t - T_r)}]^2 \right] e^{-rt} dt + \int_{T_r}^T b(t) e^{-rt} dt + a(0) \quad (22)$$

(15) and (22) give a system of equations in T_r and $c(0)$. By substitution of (22) in (15), we have one equation in one variable, T_r .

The function $f(T_r, c(0))$ is defined as

$$f(T_r, c(0)) = c(0)^{-1/\rho} (wH(T_r) - b(T_r)) - v(1)e^{(r - \beta)T_r} \quad (23)$$

The sign of $f(T_r, c(0))$ determines whether the employee is retired. When $f(T_r, c(0)) > 0$, the marginal utility of consumption of prolonging the working period is higher than the marginal utility of a longer period of retirement.

Proposition 1

Let

$$\frac{wH(0) - b(0)}{\left[\frac{r - \rho(r - \beta)}{1 - e^{(\rho(r - \beta) - r)T}} \left[\int_0^T b(t) e^{-rt} dt + a(0) \right] \right]^{1/\rho}} > v(1) \quad (24)$$

then there exist a T_r for which $f(T_r, c(0)) = 0$. In this point $\partial f(T_r, c(0)) / \partial T_r < 0$.

Proof. See Appendix.

When (24) is satisfied the employee starts working at $t=0$. The employee retires at the point that $f(T_r, c(0)) = 0$. Because $\partial f(T_r, c(0)) / \partial T_r < 0$, an upward shift in $f(T_r, c(0))$ means that retirement is delayed, see Figure 4.

(24) gives a relation between value of non-human capital (initial assets and retirement benefits), the wage rate and utility of leisure. The numerator on the LHS gives the

gain in income of working to retirement. We might assume $b(0) \rightarrow 0$: retirement benefits are low or zero when someone retires at $t=0$. A lower level of benefits increases the relative gain of working to retirement. Moreover, the additional utility of earnings from labor depends on the level of non-human capital, the denominator on the LHS. The incentives to work are high, when the budget of an individual is low. When $a(0) \rightarrow 0$ and $b(t) \rightarrow 0$, the budget of an individual that does not work is close to zero, which implies that his consumption level is close to zero. In this case, the utility of leisure is low relative to consumption and hence the individual will not 'retire' at $t=0$. On the other hand, for high levels of non-human capital, for instance as a result of high bequests, the incentives for work are low. Furthermore, the condition shows that the probability of not working increases when utility of leisure is higher. Someone very keen on leisure is reluctant to join the labor force.

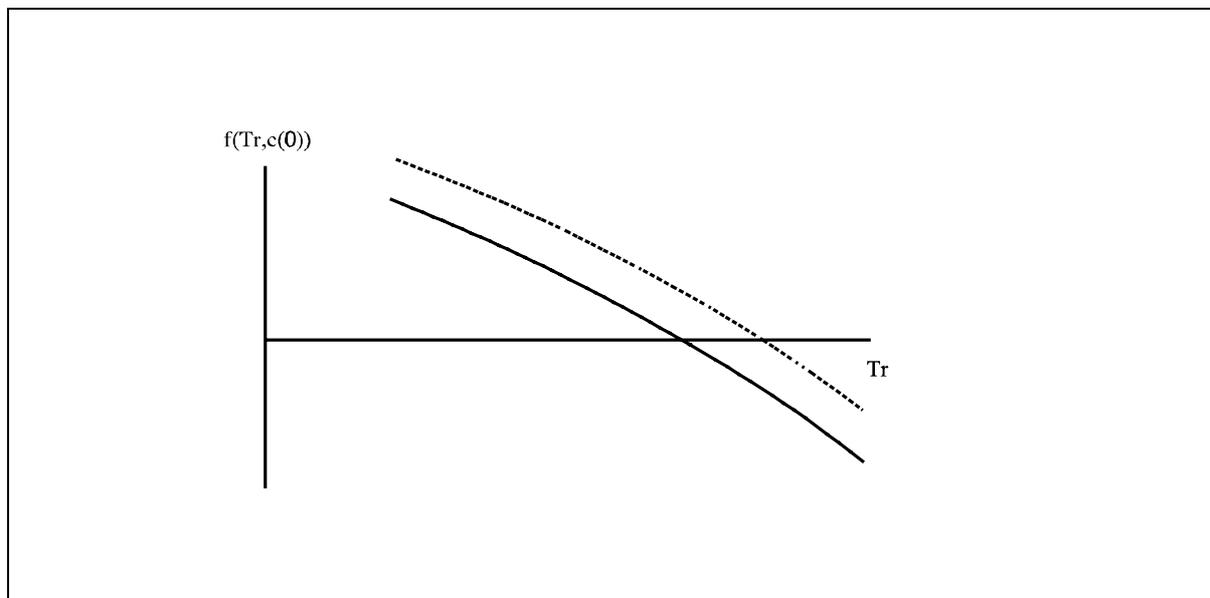


Fig. 4

3. Comparative dynamics

The effects of unexpected exogenous shocks in technology and wage levels on the moment of retirement are analyzed. Note that both shocks can be interpreted as components of one and the same technology shock. On the one hand, a shock in technology affects human capital: new technologies make older technologies obsolete and knowledge of these technologies is only partly useful after the shock. On the other hand, a higher

level of technology can result in a higher marginal productivity of labor, which yields higher wages.

3.1 *Non-anticipated one-off shock in technology*

At $t=\tau$ the economy is hit unexpectedly by a positive shock in technology. A new technology is introduced, which makes part of the knowledge of older technologies obsolete. The effect on human capital (when T_r is constant) after an unexpected shock in $\Phi(t)$ is⁴

$$\frac{\partial H(t)}{\partial \Phi(\tau)} = -\gamma_1 H(\tau) e^{(\tau-t)x} \quad (25)$$

for $t \geq \tau$, where γ_1 denotes the level of depreciation as result of the shock. Note that the shock in technology does not affect $s(t)H(t)$ when the age of retirement is unchanged. Because the investment in human capital is independent of initial conditions, the investment in human capital only changes as a result of an adjustment in the moment of retirement.

The effect of a shock in technology on $f(T_r, c(0))$ is

$$\frac{\partial f}{\partial \Phi(\tau)} = -1/\rho \frac{\frac{\partial c(0)}{\partial H(t)} \gamma_1 H(\tau) e^{(\tau-t)x}}{c(0)} c(0)^{-1/\rho} (wH(T_r) - b(T_r)) - \gamma_1 wH(\tau) e^{(\tau-T_r)x} c(0)^{-1/\rho} \quad (26)$$

The first term of the RHS of (26) gives the effect of a decline in human capital on the life-time earnings. The decline in life-time earnings decreases the consumption profile, and results in a lower shadow price of consumption, which stimulates prolonging of the working period at $t=T_r$. The second term gives the effect of the decline in human capital on the wage level at the intended moment of retirement. A decline in human capital decreases the opportunity costs of leisure, which induces the individual to retire earlier. The sum of both effects is ambiguous.

⁴ The derivation of (25) is shown in the Appendix.

Calibration

To obtain an idea of the effect of the shock in technology, Φ , on retirement behavior, some numerical calculations are made. For the benchmark case of the calculations, which will be used for all simulations, I normalize w to one, and I take $H(0)=60$ and $b(t)=15$. These values for $H(0)$ and $b(t)$ are comparable to the income of a higher educated employee and the social security payments in the Netherlands⁵. Estimates of the intertemporal rate of substitution are generally low, between 0 and 0.4, see for instance Hall (1988). Some studies do find higher levels of the intertemporal rate of substitution. Mankiw, Rotemberg and Summers (1985), and Hansen and Singleton (1983) obtained estimates above one and for the benchmark case I take $\rho=0.5$. There is no empirical estimate available for x , (depreciation - learning-by-doing). I take $x=0.02$, which implies that when the working period is 40 years, 45% of the initial human capital of the employee is left upon retirement. To get a realistic wage profile, I take $\alpha_0=0.9$. The interest rate $r=0.05$, which is between return on assets and return on risk-free bonds, and $T=55$. It is assumed that one starts working at the age of twenty. This means that life-expectancy is 75 years. The value of ν is calibrated to give a retirement at the age of 60 years for the parameters in the benchmark case.

Numerical results

The results of the calculations are shown in Figure 5⁶. The five lines represent the effects of a positive shock in technology at $t=\tau$ as function of the intended moment of retirement, where $\tau = 0, 10, 20, 30$ and 40 . The x-axis gives the planned moment of retirement before the economy was hit by a technology shock. A positive value for dT_r on the y-axis denotes a delay in retirement. Note that when $T_r=30$, the retirement age is 50.

The results confirm that a positive unanticipated shock in technology has an ambiguous effect on the moment of retirement. Figure 5 shows that the results depend strongly on the length of the period between the planned moment of retirement and the moment of the shock. When the employee is older at τ , the lines get flatter and the probability of earlier retirement increases. With a planned retirement age of 60, a shock in

⁵ $H(0)$ is not the income level, because part of his time is invested in training-on-the-job.

⁶ Tables and figures are shown in the Appendix.

technology that results in a decline of 10% in the human capital level at the age of 20, leads to a prolonging of the working period of 2.82 years. A young employee with a long working horizon loses a relatively high level of consumption because of the decline in human capital over his working life, though the effect on the level of human capital at the intended moment of retirement is relatively small. The knowledge of the 'new' technology will be partly replaced by newer innovations, and the investment in human capital during his working life reduces the effect of the initial decline in human capital on the productivity level at the intended moment of retirement. Hence, for a young employee the negative effect of the decline in income on the demand for leisure is likely to dominate the substitution effect of the decline in the price of leisure upon retirement. If the new technology is introduced when the employee is 50, 10 years before his retirement, the employee will *shorten* his working life with 1.91 years. When the worker is closer to his retirement, the worker has already accumulated assets from wage income in the past, and his willingness to work for a lower wage is reduced. His consumption level depends less on the remaining working period and therefore his working period shortens. This result confirms the hypothesis of Bartel and Sicherman (1993) that the probability of an earlier retirement after a shock in technology increases, the older the worker gets.

Table 1 shows that the sensitivity of the results depends in particular on ρ and r . The willingness to smooth consumption over time decreases for higher levels of ρ . This implies that with a higher ρ , the level of consumption at the moment of retirement is higher, and therefore the willingness of the employee to substitute leisure for consumption after a decline in the human capital level is higher. A boost in the interest rate stimulates savings and leads to a decline in the level of investment in education. As a result, a larger share of total income is determined by the initial human capital level. Hence, with a higher interest rate the effect of a decline in human capital on income is relatively smaller and therefore the employee retires earlier.⁷

3.2 Shock in technical progress or learning-by-doing

Sectors and jobs differ in technological change and the rate of learning-by-doing.

⁷ The results of the sensitivity analysis strongly depends on the length of the remaining working period. When this period is shorter than 20 years, the difference between the benchmark case and the case with a higher interest rate is very small.

Moreover, it can be assumed that after the introduction of a new technology the employee who works with the new technology is learning at an earlier stage of the learning curve, implying a higher level of learning-by-doing. To examine differences in learning in sectors we have to examine the effects of a positive shock in x ($x=\delta-k$) at $t=0$, *i.e.* a lower level of learning-by-doing or a higher rate of depreciation for the whole working life.

The effect of a negative shock in the rate of learning-by-doing, which is equivalent to a positive shock in x , is

$$\frac{\partial f(T_r, c(0))}{\partial x} = -\frac{1}{\rho} \frac{\int_0^{T_r} w \frac{\partial \left[H(t) - \frac{\frac{1}{4} \alpha_0^2}{(r+x)^2} [1 - e^{(r+x)(t-T_r)}]^2 \right]}{\partial x} e^{-rt} dt}{\int_0^{T_r} w H(t) (1-s(t)) e^{-rt} dt + \int_{T_r}^T b(t) e^{-rt} dt + a(0)} + \frac{\frac{\partial H(T_r)}{\partial x}}{H(T_r)} \quad (27)$$

The first term of the RHS of (27) represents the effect of a shock in the rate of learning on total life-time earnings, which determines the income effect on the demand for leisure. The second term gives the marginal effect on labor income of an extension of the working period at the intended moment of retirement. A decline in the marginal income upon retirement implies that the price for leisure decreases, which stimulates substitution of leisure for consumption. The total effect of a shock in the rate of x on the retirement age is ambiguous.

Numerical results

In Figure 6, the results are shown of the effects of a positive change in x on the moment of retirement in the benchmark case. The difference between planned retirement and retirement after the shock increases until the planned date of retirement is in approximately 20 years. If the intended moment of retirement is later, the shortening of the working period declines again. The positive shock in x results in a lower accumulation of human capital. The relatively lower human capital decreases the price of leisure, which results in a higher demand for leisure. Because a short period until retirement implies a relatively small effect on life-time earnings and hence on consumption levels, the effect of the shock in technical change on the marginal income level upon retirement dominates the effect of

income on the demand for leisure, and the employee retires earlier. When the planned retirement age is further away, the negative effect on human capital increases. At first the negative effect of the decline in the marginal income at retirement becomes even stronger relative to the effect on life-time income. However, with higher planned retirement ages and longer (working) horizons, the continuous lower human capital results in an increasing negative effect on income and a decline in the shortening of the working period.

We observe that with an intended retirement date at 60, the employee chooses to retire 0.88 years earlier, after a positive shock of $x=0.01$. Hence, for the parameters in the benchmark case, employees that work in sectors with a higher net rate of learning-by-doing (learning-by-doing minus depreciation of knowledge) delay retirement. This result is very variable though. The sensitivity analysis shows that the results depend strongly on ρ and r . Because the exact rate of the intertemporal rate of substitution is unknown, the level and even the sign of the effect on the retirement date are uncertain.

To examine the second hypothesis of Bartel and Sicherman (1993), stating that individuals in industries that have high 'permanent' rates of technological change have longer working careers, the effect of technical change in the form of higher labor productivity, and therefore higher wages, should be taken into account as well. Furthermore, the net level of learning-by-doing should be estimated. Sectors with higher levels of technical change often have a higher rate of depreciation, but also a higher level of learning-by-doing.

3.3 *Change in the effective net wage, w*

To examine the effects of a positive shock in w or a negative shock in the tax rate, it is assumed that the retirement benefits grow at the same rate as the effective net wage level, *i.e.* $b(T_r) = b_r w$. The effect of a shock in w on $f(T_r, c(0))$ is

$$\frac{\partial f}{\partial w} = -1/\rho \frac{\frac{\partial c(0)}{\partial w}}{c(0)} c(0)^{-1/\rho} (wH(T_r) - b(T_r)) + \left(H - \frac{b(T_r)}{w} \right) c(0)^{-1/\rho} \quad (28)$$

When $a(0)=0$ and the shock in w is at $t=0$, $(\partial c(0)/\partial w)/c(0)=1/w$, the individual retires earlier when $\rho < 1$. In this case the income effect of the higher wage will be stronger than the incentive to work longer for a higher wage. If $\rho > 1$, the substitution effect will be

stronger.⁸ The probability of a delay in retirement will be higher when initial assets are substantial or when the employee is longer in the labor force, and has already accumulated assets⁹. In these cases the effect of the extra income relative to the total budget is smaller, and the effect of an increase in the marginal income level becomes relatively stronger. Upon retirement the income effect is minimal and an employee can be induced to delay retirement when wages are increased.

3.4 Heterogeneity in learning skills

In the model there are two forms of heterogeneity in learning skills: employees differ in efficiency in learning during years of formal schooling (initial human capital), and the efficiency in learning-by-doing. For ease of computation I assume the higher effect of learning efficiency during the schooling period is equal to the higher learning efficiency during the working period. Hence, $H(0) \sim (\alpha_0)^2$.¹⁰ The effect on $f(T_r, c(0))$ is

$$\begin{aligned} \frac{\partial f}{\partial \alpha_0} = & -1/\rho \frac{\frac{2}{\alpha_0} \int_0^{T_r} wH(t)(1-s(t))e^{-rt} dt}{\int_0^{T_r} wH(t)(1-s(t))e^{-rt} dt + \int_{T_r}^T b(t)e^{-rt} dt + a(0)} c(0)^{-1/\rho} (wH(T_r) - b(T_r)) \\ & + \frac{2}{\alpha_0} wH(T_r) c(0)^{-1/\rho} \end{aligned} \quad (29)$$

Higher skilled people work longer, $\partial f/\partial \alpha_0 > 0$, when

⁸ The effect of a higher intertemporal rate of substitution on the retirement date resembles the sensitivity analysis in 5.3.1: a higher intertemporal rate of substitution implies a relatively higher level of consumption upon retirement, and therefore the willingness to substitute leisure for consumption is higher.

⁹ A severe shock in the wage level might induce individuals even to re-enter the labor force. Moreover, when the shock is anticipated a worker might substitute time intertemporally.

¹⁰ A shock of $H(0)$ in isolation is equal to the case where a shock in technology resulted in a decline in human capital, when the worker is hit at $\tau=0$.

$$-1/\rho \frac{\int_0^{T_r} wH(t)(1-s(t))e^{-rt} dt}{\int_0^{T_r} wH(t)(1-s(t))e^{-rt} dt + \int_{T_r}^T b(t)e^{-rt} dt + a(0)} + \frac{wH(T_r)}{wH(T_r) - b(T_r)} > 0 \quad (30)$$

Condition (30) holds surely for $\rho \geq 1$. The condition holds as well for lower levels of ρ , when retirement benefits or initial assets are positive. Higher skilled people have a higher full income and as a result they might have a higher demand for leisure. But because their wage growth is higher as well, this effect is counteracted by the high human capital at retirement, which makes retirement costly in terms of consumption.

Blöndal and Scarpetta (1998) show that the incidence of early retirement is significantly higher for employees with basic education than for employees with tertiary education. Berkovec and Stern (1991) find that a lack of education increases the probability of an early retirement.

3.5 *Shock in the level of retirement benefits*

A shock in b results in earlier retirement, which can be observed directly in (23). Firstly, the income increases, which decreases the value of extra consumption, and secondly, the incentives for working are decreased. This result is consistent with the findings of Crawford and Lilien (1981) and Hurd (1997).

3.6 *Heterogeneity in initial assets*

Because of higher initial assets, income from labor has a smaller share in full income, and the utility of additional consumption of working relatively declines. As a result, an employee will retire earlier.

4. **Conclusion**

In this paper the effects of a shock in technology on the retirement decision are examined. An unambiguous effect of an unanticipated positive one-off shock in technology on the retirement age cannot be obtained. The effect on the retirement age depends on the decline in human capital on the intended moment of retirement and on the decline in life-time

earnings. The employee weighs this income effect against the incentives to substitute leisure for consumption: the lower human capital makes leisure less expensive in terms of consumption. It is shown that the income effect depends strongly on the intertemporal rate of substitution, the interest rate and the moment of the shock in technology. The shorter the period until retirement, the stronger the substitution effect will be. Because the effects of a shock in technology are hard to disentangle in effects on human capital, effects on the rate of learning-by-doing and increases in labor productivity, future research should address the relative importance of these effects for different ages of employees.

The model shows that the preferred working period depends on the net rate of learning-by-doing, which is job- and sector specific, and on the skill level of workers. This calls for a more flexible choice of the pension date and for a change of pension systems, which are often based on the mandatory pension age of 65.

Future research should pay more attention to labor market imperfections, *e.g.* the effect of relative and absolute wage rigidities on human capital accumulation and the retirement age. Firms have difficulties to lay off employees without reputation costs or to demote them. This might provide incentives for employees to decrease investment in human capital at the end of the working life.

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Appendix

Proof of Proposition 1

To prove that there exists at least one interior solution for T_r , it is sufficient to prove that $f(T_r, c(0)) > 0$ at $T_r = 0$, and $f(T_r, c(0)) < 0$ at $T_r = T$. Sufficient condition for $f(T_r, c(0)) > 0$ at $T_r = 0$ is condition (24). For $f(T_r, c(0))$ to be negative at $T_r = T$, it is sufficient that $wH(T) < b(T)$. It is assumed that $\delta(t) \rightarrow \infty$ for $t \rightarrow T$, at the moment of death human capital will be 0. Hence, $wH(T) < b(T)$ for $b(T) > 0$.

The slope of $f(T_r, c(0))$ at T_r determines the effect of a change in $f(T_r, c(0))$ on the moment of retirement, which is used in the comparative dynamics. To prove that $\partial f(T_r, c(0)) / \partial T_r < 0$ in the optimum, we have to show that

$$\frac{\partial f}{\partial T_r} + \frac{\partial f}{\partial c(0)} \frac{\partial c(0)}{\partial T_r} = f_1 - \frac{1}{\rho} c(0)^{-1/\rho-1} \frac{\partial c(0)}{\partial T_r} (wH(T_r) - b(T_r)) < 0 \quad (31)$$

When a maximum exists, the second derivative to T_r of the maximization problem is negative, and hence $f_1 < 0$ in the optimum (the subscript denotes differentiation to the first argument). Furthermore, if an interior solution exists, then $wH(T_r) - b(T_r) > 0$ (see (15)), and the second term is negative when $\partial c(0) / \partial T_r > 0$. Because the growth in consumption does not depend on the pension date (see (14)), the consumption profile shifts up when income increases as a result of a longer working period. Hence, we have to show that an extension of the working period results in an increase in the income level of the employee. Total income increases, and $\partial c(0) / \partial T_r > 0$, when

$$\int_0^{T_r} w \frac{\partial \left[H(t) - \frac{1}{4} \alpha_0^2 \frac{[1 - e^{(r+x)(t-T_r)}]^2}{(r+x)^2} \right]}{\partial T_r} e^{-rt} dt + (wH(T_r) - b(T_r)) e^{-rT_r} > 0 \quad (32)$$

The individual has to maximize his income over the period until retirement. When the working period extends, the employee can always mimic the original path of investment in human capital (in which case the first term of (32) is zero). Hence, the income of the employee always increases after an extension of the working period when the wage level exceeds the level of retirement benefits at the moment of retirement. \square

Proof of (25)

To examine an one-off at $t=\tau$ shock in technology, we define the function $\Delta(t)$

$$\Delta(t) = \int_0^t \delta(u) du = \gamma_0 t + \gamma_1(\Phi(t) - \Phi(0)) \quad (33)$$

where $\Delta(t)$ can be seen as the survival fraction of initial human capital, $H(0)$, at t .

Because

$$e^{-xt} = e^{(kt - \int_0^t \delta(u) du)} = e^{(kt - \Delta(t))} \quad (34)$$

the effect of a shock in technology on e^{-xt} is

$$\frac{\partial e^{-xt}}{\partial \Phi(\tau)} = \gamma_1 e^{-x\tau} \quad (t = \tau) \quad , \quad \frac{\partial e^{-xt}}{\partial \Phi(\tau)} = 0 \quad (t \neq \tau) \quad (35)$$

as $\Phi(t)$ jumps at $t=\tau$. Using (19), the effect on human capital (when T_r is constant) after an unexpected shock in $\Phi(t)$ is

$$\frac{\partial H(t)}{\partial \Phi(\tau)} = -\gamma_1 H(\tau) e^{(\tau-t)x} \quad (36)$$

for $t \geq \tau$.

Table 1: Sensitivity analysis for different parameters of the effects of a negative shock of 0.10 in $H(\tau)$ at $\tau=0$ on the intended date of retirement.

τ	x	r	b	α_0	ρ	dT_r (years)
0	0.02	0.05	15	0.9	0.5	2.83 ¹
	0.01					2.89
	0.03					2.66
		0.03				3.51
		0.07				1.24
			10			2.85
			20			2.79
				0.7		2.82
				1.1		2.40
					0.3	3.62
					0.7	2.23

¹ Benchmark case.

Table 2: Sensitivity analysis for different parameters of the effects of a positive shock of 0.01 in technical change at $\tau=0$ on the intended date of retirement.

τ	x	r	b	α_0	ρ	dT_r (years)
0	0.02	0.05	15	0.9	0.5	-0.88 ¹
	0.01					-0.49
	0.03					-1.34
		0.03				0.27
		0.07				-0.83
			10			-0.79
			20			-0.97
				0.7		-2.28
				1.1		-0.20
					0.3	2.89
					0.7	-3.70

¹ Benchmark case.

dTr
[years]

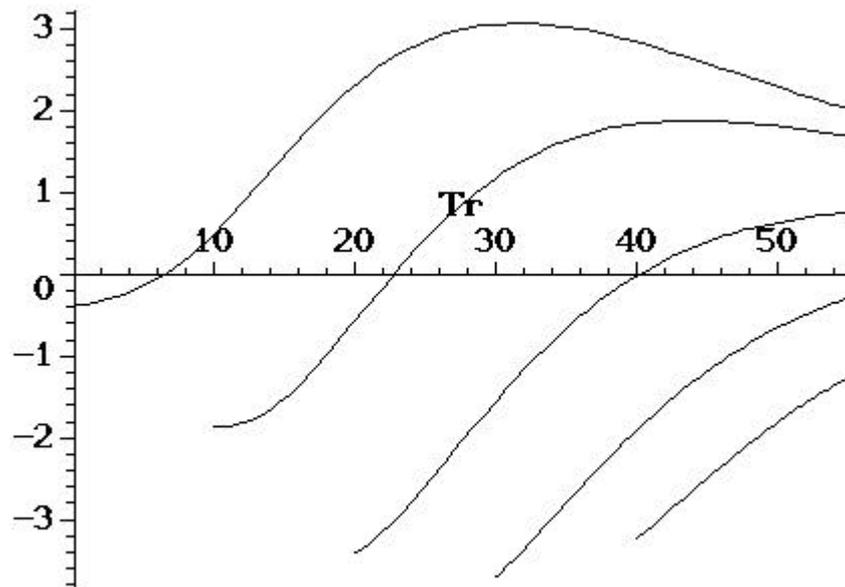
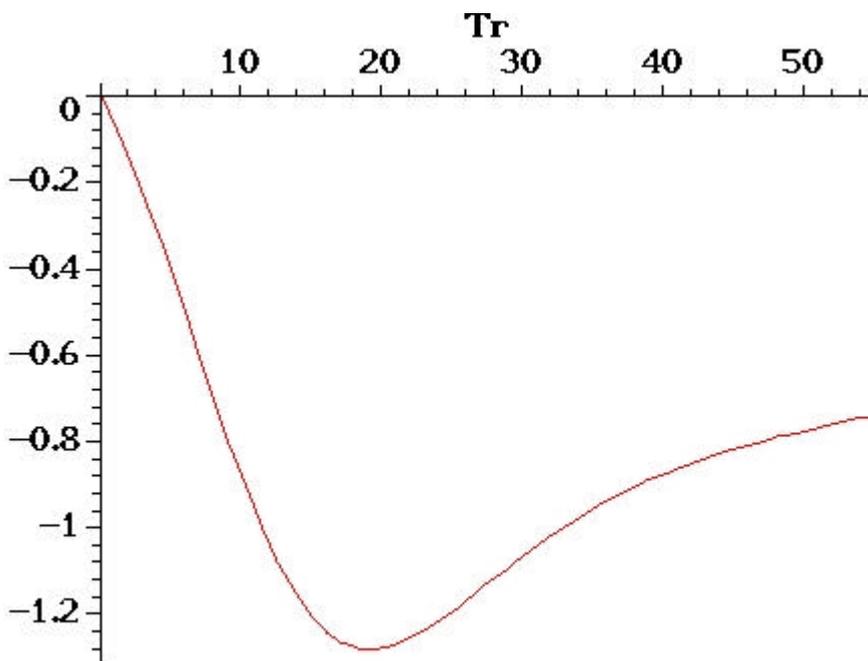


Fig. 5



dTr
[years]

Fig. 6