SUBSTITUTION AND COMPLEMENTARITY UNDER COMPARATIVE ADVANTAGE AND THE ACCUMULATION OF HUMAN CAPITAL

by

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Abstract

The paper applies Ricardo's principle of comparative advantage to analyze the substitutability between types of labor. The problem of having to classify labor in a small number of types in e.g. standard CES models are avoided by applying a continuum of worker and job types, where better skilled workers have a comparative advantage in more complex jobs. The complementarity matrix, which is derived by inverting the substitution matrix, exhibits attractive features. The matrix depends only on the wage distribution and a single parameter, which is dubbed the complexity dispersion parameter.

A particularly intriguing application is the accumulation of human capital. An investment in human capital reduces the supply of low-skilled and increases the supply of highly skilled workers, therefore compressing wage differentials. The training of one skill group will therefore have (positive and negative) externalities to the wage of other skill groups. The complexity dispersion parameter measures the percentage compression in log wage differentials per percent accumulation of human capital. Empirical estimates suggest that this parameter is in the range of 2.4 - 3.8. This mechanism explains for example the massive compression of wage differentials in some Asian tigers during the seventies and the eighties. The inverse of the complexity dispersion parameter measures the maximum productivity gain that can be achieved by human capital accumulation.

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1 Introduction

Some 150 years ago Ricardo used his well known example of textile and wine in Britain and Portugal to lay out the principle of comparative advantage. Both countries should specialize in the production of that commodity in which they have a comparative advantage. Apparently, the factors of production in Britain and Portugal were not perfect substitutes.

Despite the natural relationship between the principle of comparative advantage and the analysis of the substitutability of factors of production, the link between both concepts has not been developed formally since Ricardo's day. Arrow, Chenery, Minhas and Solow's (1961) work on the CES function set the standard for the analysis of substitutability. Later work by Diewert (1971) and others provided new, more flexible specifications, but none of them started from the concept of comparative advantage. This paper fills this gap. It will be shown that there are simple and transparent closed-form relations between the structure of substitution and complementarity and the principle of comparative advantage, based on a continuum of factors of production, which have important advantages above the standard approach of a discrete number of factors of production.

Focussing on the labor market, the specification of the model makes it particularly suitable for the analysis of the distributional effects of policy interventions such as a general increase in human capital, training programs for specific skill groups, and minimum wages. The papers offers an explanation for wage compression in the United States during the fifties and the sixties following the high school revolution between 1910 and 1940, and similar, more recent experiences in some Asian tigers. Recent studies showing the large spill over effects of increases in the minimum wage to wage levels way above the minimum (DiNardo, Fortin, and Lemieux, 1996; Lee, 1999; Teulings, 1998) provide strong evidence for the empirical relevance of the model.

In previous work (Teulings, 1995), I have developed a handsome specification of comparative advantage in an assignment model of a continuum of worker types to a continuum of job types. Workers are indexed by their level of skill s, and jobs by their level of complexity c. This complexity index c measures the relative productivity gain of an additional unit of skill. The level of skill, therefore, matters more in complex jobs - and better skilled workers have a comparative advantage in these jobs. The equilibrium allocation can be described by a mapping of complexities to skills, denoted $c(s)$, $c'(s) > 0$. There are two types of substitution in this model. First, changes in relative prices for the output of various job types will affect the composition of product demand and therefore the skill composition of the demand for labor. This is the between-job substitution process, modelled by a standard CES function. The most interesting result will be derived for the case where the between-job elasticity of substitution is set equal to zero.

Second, changes in relative wages induce shifts in the assignment of worker-types to job-types. This is the within-job/between-worker-type substitution process. The model satisfies the DIstance Dependent Elasticity of Substitution (DIDES) characteristic: the shorter the skill distance between two worker types, the more substitutable they are. A crucial role is played here by the log wage function $w(s)$, in particular by its first and second derivatives. Its first derivative $w'(s)$ measures the return to the skill index (from the point of view of the worker) or the marginal relative cost of an additional unit of skill (from the point of view of the firm). Since the complexity index c measures the relative productivity gain of an additional unit of skill in that job type, a cost-minimizing employer offering jobs of type c will choose the optimal skill level such that $w'(s)$ is set equal to c. Were $w'(s)$ constant (and hence, were $w''(s)$ equal to zero), an employer would be indifferent between skill types because the higher wage of a better skilled worker would be exactly offset by her higher productivity. The higher $w''(s)$, the higher the additional cost of hiring a worker better or lower skilled than the optimal type. This implies that the second derivative must be a measure of the degree of substitutability between worker types: the higher $w''(s)$ is, the less substitutable they are. The formal analysis will confirm this idea.[1]

This multifactor model of comparative advantage is the starting point of the analysis, with one important adjustment. In order to be able to calculate a matrix of elasticities of substitution, the continuum of skill types, where s can take any real number, is replaced by a stepfunction, where $s_i = s_{i-1} + \Delta s$. By considering the limit for $\Delta s \rightarrow 0$, the case of continuous variation in s can be approximated arbitrarily close. Where an analytical expression for cost function of this economy is available, the substitution matrix can be derived straightforwardly. However, the complementarity matrix can only be constructed by inversion of the bordered substitution matrix, since the production function cannot be specified explicitly.

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[1]There is an alternative way to see this. The first-order condition for cost minimization by the firm implies that $c(s) = w'(s)$. Since this relation applies identically for all s, the following can be differentiated: $c'(s) = w''(s)$. Hence, $w''(s)$ is a measure of the dispersion of job complexity among workers. The greater the difference between job types, the more difficult it will be to substitute worker types.

The paper's greatest challenge is to find an analytical description of the inverse of a matrix, which is of infinite dimensions in the limit for $\Delta s \to 0$.

The matrices of substitution and complementarity of this model have two peculiarities: i) the substitution matrix is close to diagonal, and ii) the complementarity matrix is governed by a second-order difference equation. The reader will find it helpful to gain a basic intuition for these characteristics. In this analysis, between-job substitution will be temporarily ignored for expositional convenience.

In this model, a price increase (or, for the labor market: a wage increase) for one skill type leads to substitution only to the two adjacent skill types. The demand for other skill types remains unaffected because the adjacent skill types are better substitutes than all other skill types, which are at a longer skill-distance. As long as the price of the best substitutes remains constant (which is the basic assumption underlying the concept of elasticities of substitution), there is no need for substitution to other, less adequate substitutes. Hence, the matrix of elasticities of substitution **H** is close to diagonal: minusses at the main diagonal and plusses at the elements directly adjacent to the main diagonal. Since substitution effects along a column sum to zero, the two plusses adjacent to the main diagonal should each be half the size of the minusses at the main diagonal. Hence, the i-th vector $\underline{h}_i$ of the substitution matrix shows a pattern like [...0,0,1,-2,1,0,0,...], where -2 is the i-th element. This feature renders **H** a less than useful description of the structure of substitution when the number of skill types goes to infinity because the within-job/between-worker-type substitution is zero almost everywhere. **H** measures only the between-job type substitution. I focus therefore on the complementarity matrix.

Whereas a substitution matrix documents the effect of price changes on quantities, the complementarity matrix **E** documents the effect of changes in quantities on prices. Loosely speaking, the one is therefore the inverse of the other: **H E** = **I**. The j-th vector $\underline{e}_j$, with elements $\{e_{ij}\}$ of **E**, should therefore satisfy (by the symmetry of **H** and **E**, columns are indentical to rows): $\underline{h}_i'\underline{e}_j = 0$. By the pattern of $\underline{h}_i$, this implies: $e_{i-1,j} - 2\, e_{ij} + e_{i+1,j} \equiv \Delta^2 e_{i+1,j} = -$ *a constant*, where the constant comes from the fact that we have to invert not simply **H**, but the bordered substitution matrix. The pattern of the substitution implies, therefore, that the complementarity matrix is governed by a second-order difference equation.[2]

[2]Some results in this paper have been conjectured for a special case in Teulings (1999): the importance of the second derivative of the log wage locus, the substitution matrix being zero almost everywhere, a second- order differential equation governing the trajectory of a row vector of **E** and the non-differentiability at the main

The model has important implications for the relation between the accumulation of human capital and wage dispersion. In general, investments in human capital reduce wage differentials: the supply of highly skilled workers goes up (reducing the wages of these workers), while the supply of low-skilled workers goes down (raising the wages of this group). The comparative-advantage model allows a formal characterization of the relation between the accumulation of human capital and the compression of wage differentials.

To derive this result, the differential equation governing the matrix of complementarity is solved for the case of a zero between-job elasticity of substitution. The resulting complementarity matrix can be fully described by the log wage distribution (which can be observed directly) and a variable dubbed the complexity dispersion parameter. The latter parameter, which is estimated to be in the range 2.4 - 3.8, is a charateristic of the wage function $w(s)$, depending on its first and second derivatives. It turns out that this parameter can also be interpreted as a compression elasticity: it measures the percentage reduction in log wage differentials per percent increase in the stock of human capital. Each percent investment in human capital reduces wage differentials by 2.4 - 3.8%. The accumulation of human capital reduces the wage differentials (and therefore the return on human capital), thereby reducing the scope for further investment. The model allows us to calculate the maximum productivity gain to be achieved by investment in human capital, which is equal to the inverse of the complexity dispersion parameter. Beyond that point, the return on further investment in human capital is zero. The estimated values for the complexity dispersion parameter suggest a maximum gain of between 26% and 42% of total labor product.

The organization of the paper is as follows. Section 2 sets out the structure of the model. Section 3 provides a general framework for the analysis of elasticities of substitution and complementarity, which will serve as a background for the subsequent analysis. Sections 4 and 5 deal with, respectively, the derivation of the elasticities of substitution and complementarity. In Section 6, the complexity dispersion parameter is introduced. Section 7 deals with a somewhat more structured specification, which allows the characterization of the relation between the accumulation of human capital and wage dispersion. The paper closes with some final remarks in Section 8.

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diagonal. These results will be proven in the more general framework of this paper. Moreover, the argument will be more intelligible than that attempted in Teulings (1999).

2 The structure of the economy

Consider an economy producing a single output by means of the input of labor in an infinite number of job types. Production does not require any other input, like for example capital. Job types are indexed by an index c, $c \in [c^-, c^+]$, $c^- \geq 0$. The index c will be referred to as the level of job complexity. The relation between inputs and output is given by a continuous-type CES production function with constant returns to scale. One can think of this CES function either as the production function of a firm that uses all inputs to produce a single output, or as a consumption function, that describes the way in which heterogeneous outputs of various job types are combined in a single composite consumption good. As long as the commodity markets for these heterogeneous outputs are perfectly competitive, both interpretations are equivalent.

It is convenient for our purpose to specify directly the log cost function per unit of output associated with this CES production function:

$$p_x = \frac{1}{1-\eta} \ln \int_{c^-}^{c^+} \exp[\delta(c) + (1-\eta) p(c)] dc \qquad \textbf{(1)}$$

where:

p_x = the log price per unit of output;

$p(c)$ = the log price per efficiency unit of labor of type c;

$\delta(c)$ = an exogenous, twice differentiable function of share parameters;

η = the elasticity of substitution between job types.

Labor in a job of type c can be provided by workers with different skill levels, where skill types differ in their productivity. There are I types of workers in the economy, each type endowed with a skill level s_i, i=1,...,I. The labor markets for types are perfectly competitive. Let: $s_0 = s^-$ and let: $s_i = s_{i-1} + \Delta s$, where Δs is a constant that satisfies: $\Delta s = (s^+ - s^-)/I$. Hence: $s_I = s^+$. The domain of the skill variable is therefore divided into a number of intervals of equal width and s jumps stepwise. The case of continuous variation in the skill level s will be approximated by considering the sequence of economies: $I = I_0$, I_0+1, I_0+2, ..., keeping all other parameters of the economy except Δs fixed. Hence, in the limit: $I \to \infty$, or equivalently: $\Delta s \to 0$.

Assumption A:

productivity of worker type s_i in job type c = exp(s_i c).

This specification captures the notion of comparative advantage. The higher the job complexity c, the higher is the relative productivity gain of the marginal unit of skill. Hence, it is easy to show that in a competitive equilibrium highly skilled workers will be assigned to more complex jobs. Let w_i be the log wage for type i. A competitive firm offering jobs of type c will therefore choose the worker type i to minimize exp(w_i - s_i c). In the initial market equilibrium, these wage rates are assumed to be points on a locus $w(s)$: $w_i = w(s_i)$. For the evaluation of elasticities of substitution, we shall consider slight variations of these wage rates around this initial equilibrium.

Assumption B:

B0: $w(.)$ is twice differentiable;

B1: $w'(s^-) = c^-$;

B2: $w'(s^+) = c^+$;

B3: $w''(.) > 0$.

Assumptions B1 and B3, combined with the assumption that $c^- \geq 0$, imply that $w(s)$ is an increasing function. This is motivated by the assumption of absolute advantage: whatever the job type to which a worker is assigned, the higher s is, the more productive the worker is.[3] Assumption B3 is justified by the assumption of comparative advantage (see Sattinger (1975) and Teulings (1995)).[4] Were assumption B3 not satisfied for some interval of s, then the skill types in an even larger interval would not be employed in any job type in market equilibrium. Assumption B3 is therefore equivalent to the assumption that employment in all skill types is

[3]The assumption of absolute advantage is not required for the present paper, but keeps it in line with Sattinger (1975) and Teulings (1995).

[4]The concept of comparative advantage is related but different from the concept of supermodularity, see for example Shimer and Smith (1997). Let $f(s,c)$ be the output of worker type s in job type c, then comparative advantage implies $d[f_s/f]/dc > 0$, or equivalently $f\, f_{sc} > f_s f_c$, while supermodularity requires $f_{sc} > 0$. A direct comparison is troubled by the fact that output is heterogeneous in c in this paper, while it is homogeneous in Shimer and Smith. However, both concepts yield equilibrium allocations where there is a positive association between s and c.

strictly positive. Alternatively, assumption B3 can be interpreted as the second- order condition for optimal worker assignment. If B3 were not satisfied, a firm could increase profits by either hiring a less skilled worker (because the productivity loss would more than offset the reduction in the wage bill) or by hiring a better skilled worker (for a similar argument).

We have by previous definitions:[5]

$\lim_{\Delta s \to 0} \Delta w_i/\Delta s = w'(s_i)$;

$\lim_{\Delta s \to 0} \Delta^2 w_i/(\Delta s)^2 = w''(s_i)$,

where Δ is the first difference operator. Without proof, I state that consecutive skill types s_{i-1}, s_i will be employed in consecutive, connected but non-overlapping, intervals of the job type index c, where higher skill types are employed in the more complex jobs. This result is due to the fact that, except for the borderlines between two consecutive intervals of c, the cost per efficiency unit of labor, $\exp(w_i - s_i c)$, has a unique minimum for each c. Let c_i be the borderline between the intervals of c employing skill type s_{i-1} and skill type s_i. By cost minimization, the cost of employing both skill types should be equal at this borderline job type. Hence: $s_i c_i - w_i = s_{i-1} c_i - w_{i-1}$, or:

$c_i(w_{i-1}, w_i) = \Delta w_i/\Delta s, \quad i = 2, I$;

$c_1 \equiv c^-$;

$c_{I+1} \equiv c^+$.

Hence:

$\lim_{\Delta s \to 0} c_i(w_{i-1}, w_i) = w'(s_i)$;

$\lim_{\Delta s \to 0} \Delta c_i(w_{i-1}, w_i)/\Delta s = w''(s_i)$.

The first equality is equivalent to the first-order condition for the case where s varies continuously: $w'(s) = c$. This condition is referred to in Section 1. The marginal relative cost of an additional unit of skill, $w'(s)$, should be equal to the relative productivity gain, c. From assumptions A and B, there exists a $c_i(.,.)$ within $[c^-, c^+]$; moreover, $\Delta c_i > 0$ for each i=1,I-1. All job types in the interval $[c_i, c_{i+1}]$ will therefore be occupied by workers of type i. Substituting these relations in equation (1) yields:

[5]Throughout the paper I apply a somewhat inadequate, but convenient notation. Since $s_i = s^- + i\,\Delta s$, strictly speaking: $\lim_{\Delta s \to 0} f(s_i) = f(s^-)$, where I mean: $\lim_{\Delta s \to 0} f(s_{i/\Delta s})$. In other words, where the notation suggests that i is kept constant (and hence s_i goes to s^-), in fact s is kept constant (and hence i goes to ∞).

$$p_x(\underline{w}) = \frac{1}{1-\eta}\ln Q(\underline{w})$$

$$Q(\underline{w}) \equiv \sum_{i=1}^{I} \int_{c_i(w_{i-1},w_i)}^{c_{i+1}(w_i,w_{i+1})} \exp[\delta(c) + (1-\eta)(w_i - s_i c)]dc \qquad \textbf{(2)}$$

where $Q(\underline{w})$ is an auxiliary function and where $\underline{w}$ is the (Ix1) vector with elements w_i (all vectors will be underlined throughout the paper). Equation (2) defines log cost per unit of output as a function of log wage rates of all worker types. Wages of worker types enter along two channels. First, they enter by the integrand: production cost per job type, taking the assignment of workers as given. Second, they enter via the upper and lower bounds of the integration intervals. Each interval denotes the job types that employ a particular type of workers s_i in market equilibrium. Note that each boundary c_i depends on the wage rates of only two worker types: type i and the preceeding type i-1. Changes in relative wages will change the optimal assignment, which will be an important source of substitution effects. Whereas an explicit production function for cost function (1) can be specified, no explicit expression is available for the production function associated with cost function (2). This cost function will therefore be the starting point of the analysis.

3 A general framework for the derivation of elasticities

This section offers a general discussion on the derivation of elasticities of complementarity and substitution in the case of constant returns to scale. The results are standard, but an explicit discussion is included because the logs of quantities and prices are applied, instead of their levels. This simplifies the derivation in Section 4.

Let $C[\underline{P}]$ be production cost per unit of output as a function of a vector of input prices $\underline{P}$. Its first derivative, $\underline{C}_P[\underline{P}]$, is equal to the demand for input per unit of output $\underline{X}[\underline{P}]$:

$\underline{C}_P[\underline{P}] = \underline{X}[\underline{P}]$.

Consider the effect of a change $d\underline{P}$ in $\underline{P}$. For simplicity, $C[\underline{P}]$ is normalized to unity. Let $d\underline{P}^+$ be the vector $[d\underline{P} \mid \lambda]$, where λ is the Lagrange multiplier of the constraint that $dC[\underline{P}]$ is dC; let $d\underline{X}^+$ be the vector $[d\underline{X} \mid dC]$; and let $\mathbf{C}_{PP}$ be the matrix of second derivatives of the cost function (throughout the paper, bolds denote matrices). Then:

$d\underline{X}^+ = \mathbf{C}_{PP}{}^+ \, d\,\underline{P}^+$,

where $\mathbf{C}_{PP}{}^+ = [\mathbf{C}_{PP}\ \underline{C}_P \mid \underline{C}_p{}'\ 0]$ is the bordered Hessian matrix. The matrix of elasticities of substitution, which measures the effect of a change in the price of one input on the demand for other inputs, is defined as:

$\mathbf{H} = \mathbf{C}_P{}^{-1}\mathbf{C}_{PP}\mathbf{C}_P{}^{-1}$,

where $\mathbf{C}_P$ is a diagonal matrix with diagonal $\underline{C}_P$.

Calculations can be simplified by using log prices instead of prices as argument of the cost function (lower cases denote the log of the corresponding upper cases):

$c(\underline{p}) = \ln C(\exp[\underline{p}])$.

The first derivatives of this function are thus equal to the value shares of each input:

$$\underline{c}_p = C^{-1}\mathbf{C}_P \exp[\underline{p}] = \underline{V} \qquad \mathbf{(3)}$$

where $\underline{V}$ is the vector of value shares. Likewise:

$\mathbf{c}_{pp',\text{ off diagonal}} = C^{-1}\ \mathbf{P}\ \mathbf{C}_{PP}\ \mathbf{P} - C^{-1}\ \underline{V}\ \underline{V}'$,

where $\mathbf{P}$ is a diagonal matrix with diagonal $\underline{P}$. Hence:

$$\mathbf{H}_{\text{off diagonal}} = 1 + \mathbf{V}^{-1}\mathbf{c}_{pp}\mathbf{V}^{-1} \qquad \mathbf{(4)}$$

where $\mathbf{V}$ is a diagonal matrix with $\underline{V}$ on the diagonal. This equation will be applied in Section 4.

The diagonal elements are solved as a residual item, since substitution effects add up to zero:

$\mathbf{H}\,\underline{V} = \underline{0}$,

where $\underline{0}$ is a vector with all elements equal to zero.

Let $F(\underline{X})$ be production as a function of input $\underline{X}$ that goes along with the cost function analyzed above (like C, $F[\underline{X}]$ is normalized to unity). Its first derivative $\underline{F}_X[\underline{X}]$ is equal to input prices $\underline{P}[\underline{X}]$:

$\underline{F}_X[\underline{X}] = \underline{P}[\underline{X}]$.

Let $\mathbf{F}_{XX}$ be the matrix of second derivatives and define $\mathbf{F}_{XX}^{+} = [\mathbf{F}_{XX}\ \underline{F}_X \mid \underline{F}_X'\ 0]$. The effect of a change $d\underline{X}$ in $\underline{X}$ subject to the constraint that $F[\underline{X}]$ remains unchanged is:

$d\underline{P}^{+} = \mathbf{F}_{XX}^{+}\, d\,\underline{X}^{+}$.

Hence: $\mathbf{F}_{XX}^{+} = \mathbf{C}_{PP}^{+\ -1}$. Elasticities of complementarity, measuring the effect of a change in one input on the prices of other inputs, are defined as:

$\mathbf{E} = \mathbf{F}_X^{-1}\mathbf{F}_{XX}\mathbf{F}_X^{-1}$.

When a production function is not available, these elasticities should be derived from the cost function by matrix inversion. Define:

$\mathbf{H}^{+} = [\mathbf{H}\ \underline{1} \mid \underline{1}'\ 0]$,

where $\underline{1}$ is a vector with all elements equal to unity. Then:

$\mathbf{H}^{+} = \mathbf{C}_P^{+\ -1}\mathbf{C}_{PP}^{+}\mathbf{C}_P^{+\ -1}$,

where $\mathbf{C}_P^{+}$ is a diagonal matrix with diagonal $[\underline{C}_P \mid 1]$. Then:

$\mathbf{E} = \mathbf{F}_X^{-1}\{\mathbf{C}_{PP}^{+\ -1}\}\mathbf{F}_X^{-1} = \mathbf{V}^{-1}\{\mathbf{G}^{+}\}\mathbf{V}^{-1}$,

where $\mathbf{G}^{+} \equiv \mathbf{H}^{+\ -1}$, and where curly brackets around a matrix denote an operation that drops its final column and row. The second equality is due to the fact that $\underline{F}_X = \underline{P}$, $\underline{C}_P = \underline{X}$, and $F = C = 1$. Hence, we shall use equation ? to calculate $\mathbf{H}^{+}$ and then calculate the elasticities of complementarity by applying:

$$e_{ij} = \frac{g_{ij}}{v_i v_j} \qquad \textbf{(5)}$$

where $\{e_{ij}\}$ and $\{g_{ij}\}$ are elements of the matrices $\mathbf{E}$ and $\mathbf{G}^{+}$ respectively and where $\{v_i\}$ is a diagonal element of $\mathbf{V}$.

4 The matrix of the elasticities of substitution

The first derivative of log cost function (2) reads:

$$\frac{\partial p_x(\underline{w})}{\partial w_i} = \frac{1}{Q(\underline{w})} \int_{c_i}^{c_{i+1}} \exp[\delta(c) + (1-\eta)(w_i - s_i c)]dc \quad +$$

$$\frac{1}{(1-\eta)Q(\underline{w})\Delta s} \quad \times \tag{6}$$

$$\Big(\quad -\exp[\delta(c_i)+(1-\eta)(w_i - s_i c_i] \quad -\exp[\delta(c_{i+1})+(1-\eta)(w_i - s_i c_{i+1})]$$

$$+\exp[\delta(c_i)+(1-\eta)(w_{i-1} - s_{i-1} c_i)] \quad +\exp[\delta(c_{i+1})+(1-\eta)(w_{i+1} - s_{i+1} c_{i+1})]$$

where the arguments of $c_i(.)$ are omitted for notational convenience. The first term refers to the between-job substitution, the second term to the within-job/between-worker-type substitution. The second term cancels due to the envelope theorem: it is equal to the first- order conditions for the optimal assignment of workers to jobs. This term, however, does not cancel for the second derivative, which will be discussed below. By equation (3):

$$v_i = \frac{1}{Q(\underline{w})} \int_{c_i}^{c_{i+1}} \exp[\delta(c) + (1-\eta)(w_i - s_i c)]dc \tag{7}$$

$$v(s_i) \equiv \frac{w''(s_i)}{Q(\underline{w})} \exp[\delta(c_i) + (1-\eta)(w_i - s_i c_i)] \tag{8}$$

where v_i is the value share of worker type i. Define:

We have, by equation (7), definition (8) and $\lim_{\Delta s \to 0} \Delta c_i/\Delta s = w''(s_i)$:

$$\begin{aligned}
\lim_{\Delta s \to 0} v_i/\Delta s \quad &= \exp[\delta(c_i)+(1-\eta)(w_i - s_i c_i)]/Q(\underline{w}) \lim_{\Delta s \to 0} (c_{i+1} - c_i)/\Delta s \\
&= \exp[\delta(c_i)+(1-\eta)(w_i - s_i c_i)]/Q(\underline{w})\, w''(s_i) \\
&= v(s_i).
\end{aligned}$$

This allows a nice interpretation of $v(s_i)$: it is the density function associated with the distribution of value over s along the support $[s^-, s^+]$.

The second derivatives that are not adjacent to the main diagonal satisfy the following:

$$\frac{\partial^2 p_x}{\partial w_i \partial w_j}\Big|_{j \neq i-1,i,1+1} = -\frac{1-\eta}{Q(\underline{w})^2}\int_{c_i}^{c_{i+1}} \exp[\delta(c)+(1-\eta)(w_i - s_i c)]dc \int_{c_j}^{c_{j+1}} \exp[\delta(c)+(1- \qquad (9)$$

$$= -(1-\eta)v_i v_j$$

The terms reflecting the within-job/between-worker substitution cancel, because neither c_i and c_{i+1}, nor c_j and c_{j+1} depend on both w_i and w_j. However, for the elements $\{i\text{-}1,i\}$ adjacent to the main diagonal, c_i depend on both w_i and w_{i-1}. Hence, the term reflecting the within-job/between-worker-type substitution does not cancel. Basically, when w_i goes up, firms offering jobs of type c_i can save costs by shifting from worker type i to type i-1. Using definition (8), we have:

$$\frac{\partial^2 p_x}{\partial w_{i-1} \partial w_i} = -(1-\eta)v_{i-1}v_i + \frac{1}{Q(\underline{w})\Delta s}\exp[\delta(c_i)+(1-\eta)(w_i - s_i c_i)] = -(1-\eta)v_i \qquad \textbf{(10)}$$

Equations (4), (9) and (10) motivate Proposition I:

Proposition I:

The elements $\{h_{ij}\}$ of the matrix of elasticities of substitution of the model discussed in Section 2 read as follows:

I-1) $h_{ij,\, j \neq i-1,i,i+1} = \eta$;

I-2) $h_{i-1,i} = \eta + q_i$,

where:

$q_i \equiv v(s_i)/[v_{i-1}v_i w^{//}(s_i)\Delta s]^{-1}$.

I-3) h_{ii} solves: $\mathbf{H}\, \underline{V} = \underline{0}$

The entries not adjacent to the main diagonal satisfy the standard constant elasticitity of substitution result for the between-job substitution. This is the consequence of applying a CES function (see equation (1)). For the entries adjacent to the main diagonal, a second term shows up, reflecting the change in the assignment of workers to jobs when relative wages change. The factor q_i reveals the crucial importance of the second derivative of the wage function, as was alluded to in the introduction of the paper. Since: $\lim_{\Delta s \to 0} \Delta c_i/\Delta s = w''(s_i)$, this second derivative is a measure of the dispersion of job complexity. This is exactly the reason that this factor enters in definition (8). The higher the second derivative, the greater the differences are between job types (that is, the greater the differences are in productivity ratios of worker types across jobs). Note that in the limit $I \to \infty$, the number of entries adjacent to the main diagonal relative to the total number of entries goes to zero. The **H** converges to the standard CES result almost everywhere. Hence, elasticities of substitution are not very useful in a model of comparative advantage.

5 The characterization of elasticities of complementarity

Elasticities of complementarity will be derived from the inverse of the bordered matrix of elasticities of substitution in the way described in Section 3. The number of types of labor (and therefore the dimensions of the bordered matrix $\mathbf{H}^+$) will tend to infinity when we let Δs approach zero. We shall be able to give an analytical description of its inverse matrix. At the limit, the inverse $\mathbf{G}^+$ can be described by a function of two continuous arguments reflecting the row and column indices:

$g(s_i,s_j) \equiv \lim_{\Delta s\to 0} g_{ij}\,(\Delta s)^{-2}$.

Due to equation (5), elasticities of complementarity are equal to $g_{ij}/[v_iv_j]$. Hence, where $\lim_{\Delta s\to 0} v_i/\Delta s = v(s_i)$, these elasticities converge to:

$\lim_{\Delta s\to 0} g_{ij}/[v_iv_j] = g(s_i,s_j)/[v(s_i)v(s_j)]$.

The main ingredient of the characterization of $g(.)$ will be a second-order differential equation in its first argument, keeping the second argument constant. The intuition for this result is discussed in Section 1. This differential equation is derived in two steps. First, $\mathbf{G}^+$ is characterized in general. Next, the differential equation for $g(.)$ is derived by letting Δs approach zero. This procedure justifies Proposition II:

Proposition II:

The function $g(s,r)$ is fully characterized by the following:

II-1) $g(s,r) = g(r,s)$;

II-2) the function $g(s,r)$, keeping r constant, is continuous but non-differentiable at $g(r,r)$;

II-3) apart from this non-differentiability, this function satisfies the following:

$$\frac{1}{v(s)w''(s)}g_{11}(s,r)-\frac{w''(s)v'(s)+w'''(s)v(s)}{[v(s)w''(s)]^2}g_1(s,r)+$$
$$\frac{w'''(s)v'(s)-v''(s)w''(s)+v'(s)^2w''(s)/v(s)}{[v(s)w''(s)]^2}g(s,r)-\frac{\eta}{v(s)}g(s,r)+v(r)=0 \quad \textbf{(11)}$$

II-4) its first derivatives at the boundaries of its domain satisfy

$$g_1(s^-,r)-\frac{v'(s^-)}{v(s^-)}g(s^-,r)=0 \quad \textbf{(12)}$$

and a similar equation for s^+;

II-5)

$$\int_{s^-}^{s^+} g(s,r)ds=0 \quad \textbf{(13)}$$

The proof of the proposition is given in Appendix A.

The locus $g(s,r)$ for s going from s^- to s^+ and for $r \in [s^-,s^+]$ is therefore described by two branches, which both satisfy the second-order differential equation (11) and which connect at $s = r$. Solving this second-order differential equation for both branches yields four constants of integration, which are determined by: i) the equality of $g(s,r)$ at $s = r$; ii) and iii) the initial conditions for the first derivatives of both branches at boundaries of the domain $g_1(s^-,r)$ (see equation (12)), and $g_1(s^+,r)$; and iv) the restriction that substitution effects sum to zero, which is equation (13). By the symmetry of $\mathbf{G}^+$, the same relations also apply when we fix s and let r vary.

Equation (11) reveals again the crucial importance of the second derivative of $w(s)$ referred to in Section 1. Within-job/between-worker-type substitution becomes more costly the higher is $w''(.)$. At the limit, there will be only between-job-type substitution, measured by the CES-elasticity η. Loosely speaking: $\lim_{w''(s)\to\infty} g_{ij} = v_i v_j/\eta$, which is the standard result for a CES production function.

Differential equation (11) will be solved to yield an explicit expression for $g(.)$ for the case when there is no between-job-type substitution:

Proposition III:

For $\eta = 0$, the elasticities of complementarity $e(s,r)$ satisfy:

$$s<r: \quad e(s,r) = -\int_{s^-}^{s} \frac{w''(x)V(x)}{v(x)}dx - \int_{r}^{s^+} \frac{w''(x)[1-V(x)]}{v(x)}dx + \int_{s^-}^{s^+} v(y) \int_{s^-}^{y} \frac{w''(x)V(x)}{v(x)}d \qquad \textbf{(14)}$$

where:

$V(s) \equiv \int_{s-}^{s} v(x)\mathrm{d}x$.

The proof of Proposition 3 can be found in Appendix B.

$V(s)$ can be interpreted as the distribution function of value over s. Hence: $V(s^+) = 1$. The expression for $s > r$ follows from symmetry, by exchanging s and r.

The first term in equation (14) varies with s only, the second term with r only, while the third term is a constant. Hence, the first derivative of a row of elasticities of complementarity has a simple structure:

for $s < r$: $\quad e_1(s,r) = -w''(s)V(s)/v(s)$.

Hence, elasticities decline monotonically until they reach a minimum value at the main diagonal. Since that $V(s^-) = 0$, and $V'(s) > 0$, there is a force that unequivocally tends to make the trajectory steeper the more it approaches the main diagonal. There might be offsetting forces in $w''(s)$ and $v(s)$. When the latter two are constant, $e_1(s,r)$ decreases linearly starting from $e_1(s^-,r) = 0$. Hence, $e(.)$ reduces to a parabola, with its top at $s = s^-$. At the main diagonal, the trajectory crosses the upward-sloping branch from a similar parabola, with its top at $s = s^+$. The minimum of the trajectory is therefore a rather sharp trough, since it coincides with the non-differentiability.[6]

There is a clear economic intuition for this pattern. Suppose that the supply of worker type s_i is to be increased by some amount. Then, the wages of worker type i will go down, pushing $c_i(s_{i-1},s_i)$ down and $c_{i+1}(s_i,s_{i+1})$ up. A larger interval of job types will be available for type s_i to accommodate the additional supply that is available. Hence, there will be fewer jobs available for both of the neighboring skill types. Their wages will go down as well, but by less than for type i, because

[6]This feature explains the minimum wage paradox in Teulings (1999).

otherwise the change in $c_i(.)$ and $c_{i+1}(.)$ would be exactly undone. However, the wage reduction for the neighboring skill types induces a fall of $c_{i-1}(.)$ and a rise of $c_{i+2}(.)$, thereby shifting the assignment intervals of type i-1 downward and of type i+1 upward in the job hierarchy. This will invoke wage reductions for the subsequent skill types on both sides, s_{i-2} and s_{i+2}, but by less than for the directly neighboring types, and so on and so forth. The substitution process materializes by spill-over effects from the one segment to the other, both in the upward and the downward direction of the job hierarchy.

Note that the cross-derivatives of $e(s,r)$ within the above and below diagonal triangles of the matrix are zero due to the additive structure of $e(s,r)$. This feature has important practical implications, which will be treated in Section 6.

6 The complexity dispersion parameter and the distributive effects of human capital

While equation (14) provides a useful characterization of elasticities of complementarity for theoretical work, the concepts introduced by the equation are not easily interpreted empirically. We have no direct observations on the skill level of a worker and therefore of the distribution of value added across skill groups. This makes it hard to get a feel for the elasticities of complementarity implied by the model. Equation (14) will therefore be specified in terms of the distribution of value across log wage levels instead of across skill groups. Define:

$F[w(s)] \equiv V(s)$ (hence: $v(s) = f[w(s)]\, w'(s)$),

$ew[w(s),w(r)] \equiv e(s,r)$,

$cd[w(s)] \equiv w''(s)/[w'(s)]^2$.

Hence, $F(w)$ is the distribution of value added across log wage levels. This distribution differs from the log wage distributions usually applied, which refer to hours worked or persons employed instead of value added. The interpretation of the parameter $cd(w)$ will be discussed below. Equation (15) can be respecified by a transform of variable, $w = w(s)$, as:

$$w_i<w_j: \quad ew(w_i,w_j) = -\int_{w^-}^{w_i} \frac{F(w)}{f(w)} cd(w)dw \; - \int_{w_j}^{w^+} \frac{[1-F(w)]}{f(w)} cd(w)dw \; + \int_{w^-}^{w^+} f(x) \int_{w^-}^{x} \qquad \textbf{(15)}$$

where: $w^- \equiv w(s^-)$ and: $w^+ \equiv w(s^+)$. The parameter $cd(.) = w''/[w']^2$ is dubbed the *complexity dispersion parameter* by Teulings and Vieira (1998), which refers to the fact that $w'' =$

$\lim_{\Delta s \to 0} \Delta c / \Delta s$ measures the dispersion of job complexity. However, *w*" is not fully appropriate as an empirical measure of the dispersion of job complexity. The metric of *s* can only be identified up to a linear transformation (see Teulings (1995: 301)). Thus, suppose analyst 1 uses *s* as his skill measure, while analyst 2 uses $z = \alpha_0 + \alpha_1 s$ as his skill measure. The second derivative of $w(s)$ established by analyst 1 will be a factor α_1^2 higher than the one established by analyst 2. Since there is no way to establish the values of α_0 and α_1 empirically, a measure of complexity dispersion should not depend on these parameters. The complexity dispersion parameter satisfies this criterium.

It should be stressed that the complexity dispersion parameter is not a structural technological parameter (as is the elasticity of substitution of a CES production function). The complexity dispersion parameter applies locally, in a particular market equilibrium, with a particular distribution of skill among labor supply (analogous to a value share in a CES function). Changes in the skill distribution will affect the value of the complexity dispersion parameter.

In general, the effects of a change in the distribution of skill types among labor supply on wages can be calculated by

$$d[w_i]_{|\text{skill constant}} = \int_{w^-}^{w^+} ew[w_i, w] f(w) d[\log f(w)]_{|\text{wages constant}} dw \qquad \textbf{(16)}$$

The quotes "skill constant" for $d[w_i]$ and "wages constant" for d[log $f(w)$] remind us that we refer to the change in $w(s_i)$ for a constant skill level, and to the change in $f[w(s)]$ due to a change in the number of workers with skill level *s* - and not due to a change in the number of workers earning a log wage $w(s)$. In the special case of human capital acquisition, the increase in the supply of the skill level to which a group of workers has been trained (the "destination" skill type) will be exactly offset by the decrease in supply of the skill level which these workers had before the training (the "source" skill type). Let $h[w(s)]$ be the increase in the human capital of all workers with skill type *s* before the acquisition of additional human capital Δ; $h(w)$ is measured by the wage increase that would be generated if the log wage schedule $w(s)$ would be unaffected: $h[w(s)] = w(s+\Delta) - w(s)$. Hence, d[log $f(w)$] is equal to minus unity, and d{log $f[w+h(w)]$} is equal to plus unity. The effect on log wages can therefore be calculated using equation (16):

$$d[w_i]_{|\text{skill constant}} = \int_{w^-}^{w^+} \left[ew\left(w_i, w+h(w)\right) - ew(w_i, w) \right] f(w)dw$$

$$\approx \int_{w^-}^{w^+} e_2(w_i, w) f(w) h(w) dw \qquad \textbf{(17)}$$

$$= \int_{w^-}^{w_i} F(w) cd(w) h(w) dw - \int_{w_i}^{w^+} [1 - F(w)] cd(w) h(w) dw$$

where the approximation follows from a Taylor expansion that applies to small values of $h(w)$. $\mathrm{d}[w_i]_{|\text{skill constant}}$ measures the general equilibrium effect for the wage of the skill type earning w_i before the increase in human capital, were her skill level not affected by the increase in human capital $h(w_i)$; were $\mathrm{d}w_i$ to include the direct effect of human capital acquisition, then we should add $h(w_i)$.

Equation (17) is particularly useful for the calculation of relative wage effects of the acquisition of human capital. Such calculations were virtually impossible using the standard CES framework or, more generally, production functions with a limited number of inputs. These models were able to calculate the effect of particular types of human capital acquisition, namely the transfer of a number of workers from one type to another (e.g. from low to high skilled), but were unable to access the effect of a marginal increase in human capital of a group of workers.

The general expression for wage effects of the acquisition of additional human capital in equation (17) yields transparent expressions in two special cases: i) an increase in human capital only for a single type of worker, and: ii) an equal increase for all worker types. Since these cases provide nice insights into the mechanisms at work in this type of economy, they will be considered in Propositions IV and V, respectively.

<u>Proposition IV:</u>

Consider the economy described in Section 2 with $\eta = 0$ (no between-job substitution) and consider in this economy the increase in the stock of human capital characterized by:

i) $h(w) = h$, for: $w_j < w < w_j + \Delta$;

ii) $h(w) = 0$, everywhere else,

where both h and Δ are small numbers. Then:

$$\text{i)}\quad d[w_i]_{|\text{skill constant}} = [1 - F(w_j)]cd(w_j)\Delta h, \quad \text{for: } w_i < w_j$$
$$\text{ii)}\quad d[w_i]_{|\text{skill constant}} = -F(w_j)cd(w_j)\Delta h, \quad \text{for: } w_i > w_j \qquad \textbf{(18)}$$

Proposition IV follows from a simple Taylor expansion of the integrals in equation (17). The proposition considers the case where all workers earning between w_j and $w_j + \Delta$ are provided with h units of additional human capital (measured in relative wage gain).

Given the generality of the production structure, proposition IV is a very strong implication. All workers types that are skilled less than the type earning w_j see their wages increased by some *equal* percentage, while all worker types that are skilled better than type w_j see their wage reduced by some *equal* percentage. Proposition IV implies, for example, that any increase in the stock of human capital at a particular wage level between the 10-th and 90-th percentile of the wage distribution will decrease the 10-90 log wage differential. The predictions of the comparative advantage model are therefore widely different from a CES model with, say, four skill categories. There, an increase in human capital, which transfers workers from the second to the third category, will not affect the wage differential between the first and the fourth category. Note that a value share $F(w_j)$ of the workers gets the wage increase, while a share $1\text{-}F(w_j)$ gets the decrease, so that the sum of positive and negative wage effects is equal to zero, as is required for substitution effects.

The total increase in human capital as a share of the total stock in Proposition IV is equal to $f(w_j)$ x Δ x h.

The reduction in the relative wage differential between all less-than-type-w_j skilled workers and all better skilled workers is therefore equal to [$cd(w_j)/f(w_j)$] x the relative increase in the total stock of human capital.

Proposition V:

Consider the economy described in Section 2 with $\eta = 0$, and consider in this economy an equal relative increase in human capital measured in value terms for all worker types:

$h(w) = h,$ for all w,

where h is a small number. Then,

$$\{d[w_i]-d[w_j]\}_{|\text{skill constant}} = -h\int_{w_j}^{w_i} cd(w)dw \tag{19}$$

Proposition V follows directly from equation (17). The proposition states that the compression of a log wage differential w_i - w_j (e.g. 10-90 log wage differential) due to a general increase of the stock of human capital by h x 100% is equal to the integral over the complexity parameter x the increase. Proposition V highlights the compressing effect of increasing the stock of human capital in this type of model, is similar to Tinbergen's (1975) race between education and technology. Propositions IV and V both suggest an alternative interpretation of the complexity dispersion parameter as being the *compression elasticity*: the percentage compression of log wage differentials per percent investment in the stock of human capital. The next section will elaborate on this theme.

7 Empirical applications: the accumulation of human capital

The analysis in Section 6 was quite general in the sense that no structure was imposed on the crucial factors in equation (15), the trajectory of the complexity dispersion parameter, or, the value distribution. In this section, we will consider a simple specification for both ingredients that allows the numerical evaluation of some policy experiments for realistic parameter values. First, assume that the complexity dispersion parameter is independent of the wage level:

$cd(w) = \gamma$.

Though the constancy of $cd(.)$ is of course a strong restriction, it provides a useful benchmark. Second, assume that the log wage distribution is log normal. This more structured version of the model allows a number of further insights into the mechanism at work in the model, in particular the relationship between human capital acquisition and wage dispersion.

Teulings and Vieira (1998) work out a simple methodology for estimating the complexity dispersion parameter for the case in which this parameter is constant (thus requiring only a couple of OLS-regressions). They estimate γ to be 2.40 for Portugal (comparing rates of return to schooling in Lisbon and in the rest of the country). They argue that the estimation results for the Netherlands in Teulings (1995) and for the United States in Teulings (1999) imply values for γ of the same order of magnitude (3.50 and 3.80, respectively). I shall apply a value for γ of 2.50 in the subsequent calculations.

When the distribution of log wages weighted by hours is normal, then the distribution weighted by value added is also normal, with the same variance. That is, when[7]

$w_{\text{weighted by hours}} \sim N(\mu,\sigma^2)$,

then:

$w_{\text{weighted by value added}} \sim N(\sigma^2+\mu,\sigma^2)$.

σ varies typically from 0.85 for Portugal, via 0.60 for the United States and 0.40 for the Netherlands, to 0.30 for the Scandinavian countries. It is convenient to normalize the level of log wages w such that the median log wage weighted by value added equals zero: $\mu = -\sigma^2$. Substituting these expressions in (15), and applying a transform of variable for the integrals, $x = w/\sigma$, and using $\Phi(x) = 1\text{-}\Phi(-x)$ yields:

$$\text{for } w_i < w_j\text{:}\quad ew(w_i,w_j) = -\gamma\sigma^2\left[\psi(w_i/\sigma) + \psi(-w_j/\sigma) - \int_{-\omega/\sigma}^{\omega/\sigma} \phi(x)\psi(x)dx\right] \tag{20}$$

where:

$\psi(y) = \int_{-\omega/\sigma}^{y} \Phi(x)/\varphi(x)\, dx$,

and where ω is the (absolute value) of the upper and lower support of w.[8] In this simple model, the elasticities of complementarity are proportional to the complexity dispersion parameter and the variance of log wages. No further parameters enter the model (apart from ω; see footnote 8). Table 1 gives an overview of the relevant values. The first two columns apply simultaneously to

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[7] $f(w) = A_1 \exp(w)\, \varphi[(w-\mu)/\sigma]/\sigma = A_2 \exp[-½(w-\sigma^2-\mu)^2\sigma^{-2}]/\sigma = \varphi[(w-\sigma^2-\mu)/\sigma]/\sigma$,
where $\varphi(.)$ is the standard normal density function and where A_i are appropriately chosen constants of integration.

[8] A more convenient choice would be to use an unbounded support for w. One would expect $e(s,r)$ to converge for large values ω, since the probability mass in the tails becomes very small. Inspection of the separate integrals in (20) reveals that they do not converge. However, numerical calculations suggest that the effect of changes in ω on the various terms cancel, as is to be expected. To avoid this complexity, we apply a bounded support. Subsequent calculations will be based on $\omega = 3\,\sigma$.

both values of σ, the next three columns apply for σ = 0.60, while the next three columns provide the same information, but then for σ = 0.30. The subsequent explication refers to the columns for σ = 0.60, but applies likewise to the final three columns.

Column 1 lists the cumulative value share, $F(w)$. Column 3 lists the corresponding wage level, while column 4 gives the cumulative hours share. Since the mean of the log wage distribution weighted by value has been normalized to zero, $F(0.00_{\text{column 3}}) = 50\%_{\text{column 1}}$. Obviously, the cumulative share weighted by hours exceeds the cumulative share weighted by value - since the least-paid hours count less in value-weighted distribution. The larger the dispersion of the wage distribution is, the more this applies - as can be seen by comparing the results for σ = 0.60 and σ = 0.30. Column 2 lists the relevant values of ψ(.). Finally, the integral in the third term of equation (20) is listed at the bottom of Table 1. The meaning of column 5 regarding the introduction of a minimum wage will be explained below. Though only results for σ = 0.60 and σ = 0.30 are presented, the numbers for ψ(.) and the simple proportionality of the elasticities in the complexity dispersion parameter and the variance of log earnings allow the reader to easily calculate himself elasticities for other values.

A. Minimum wages

The employment effect of minimum wages is one of the most researched areas in labor economics. Minimum wages are therefore a useful test for the comparative advantage model, the more so where the model explains some stylized facts that have not been predicted by the more standard CES type approach. We shall perform this test before turning to the accumulation of human capital.

Though there is no agreement on the precise conclusions from the research on minimum wages, there seems to be a *communis opinio* that the effects on employment are small. In their review paper for the United States, Brown, Gilroy and Kohen (1982) report an elasticity of -0.1 until -0.3 for the employment of teenagers. More recent work by Card and Krueger (1994) suggests even smaller effects. However, the effects on the wage distribution seem to be substantial (see DiNardo, Fortin, and Lemieux (1996), Lee (1999) and Teulings (1998)). The latter two authors claim that the reduction in minimum wages can explain most of the increase in wage inequality in the United States during the eighties, in particular for the lower half of the distribution. The present model is particularly useful for understanding the effect on the wage distribution.

Consider the consequences of the introduction of a minimum wage that drives 1% (in value added) of the workforce out of employment. What will be the consequences of this policy for the wages of the other 99% of the workforce? To calculate these effects, we use equation (16). A minimum wage drives out the least skilled workers from employment. Hence, $d[\log f(w)]_{|\text{wages constant}} = 0$, except for the 1 (value)% next to $\bar{w}$, where:

$f(w)\, d[\log f(w)]_{|\text{wages constant}} = -1\%$.

Substituting equation (20) for *ew*(.) in equation (16), and noting that its first term vanishes (since $w_i = \bar{w}$), the effect on the wages for the skill types that remain employed are:

$$- \gamma\, \sigma^2 \left(\psi[-w_j/\sigma] - \int \varphi(y)\psi(y)dy \right) \times 1\%.$$

Table 1 lists the wage effects by percentile for $\sigma = 0.60$ and $\gamma = 2.50$ in column 5 (and for $\sigma = 0.30$ in column 8). The employment loss of 1 (value)% corresponds to a loss of approximately 4% of the hours worked (compare columns 1 and 4). The wage of the least skilled worker that remains employed goes up by some 17% for $\sigma = 0.60$ (somewhere between line 1 and 2 in Table 1, 4.2% for $\sigma = 0.30$), which squares well with the results of Teulings (1999).[9] Wages go up for all workers in the first 27 (value)% in hours of the distribution, or even the first 50% measured in hours for $\sigma = 0.60$ (or: 38% for $\sigma = 0.30$). They gain from the increase in the minimum. Because substitution effects must sum to zero, wages must go down in the upper 70 (value)%. Hence, an increase in the minimum wage reduces wage dispersion along two mechanisms: the truncation of low-skilled workers from employment and the compression of wage differentials for those who remain employed. These results are consistent with the findings of Lee (1999) and Teulings (1998) that the reduction in the minimum wage by about 30% in the United States during the eighties is the most important explanation for the rise in wage inequality in the lower half of the wage distribution.

B. The returns to training programs

Heckman, Lochner and Taber (1998a,b) have argued that standard methods for analyzing the returns to training programs are biased since they ignore general equilibrium effects. As long as the program is experimental, and only a limited number of people can actually apply, the shift in

[9] The 1% loss of employment referred to in the text is a value share, where the number in Teulings (1998a) is weighted by hours. The best comparison is the year 1990, for there the initial level of the minimum wage is low: Table 1 suggests a 1% reduction in hours to yield a wage increase for the least skilled worker that remains employed of about 7% for $\sigma = 0.60$, while Teulings (1998a) reports a number of approximately 8%.

the skill distribution will be too small to affect skill prices. However, when the program is open to all members of the workforce, skill prices will be affected. The wage of a group of workers with a skill level that is equivalent to that of those who enter the program will go up because they are in more limited supply, while the wage for the skill level that the program is aiming for will go down due to the increase in supply.

The comparative advantage model provides strong support for this notion. Proposition IV states that an increase in human capital at a particular skill level is going to raise the wages of all less skilled workers by an equal percentage and is going to reduce the wages of all better skilled, also by an equal percentage. Table 2 provides some calculations for this policy intervention for various percentiles of the wage distribution. Since all better skilled workers experience an equal wage loss and all less skilled workers profit from an equal wage gain, only two numbers have to be presented for each policy that affects the skill level of a particular percentile of the skill distribution. The higher up in the skill distribution is the group that receives additional training, the smaller will be the positive wage effects for the less skilled workers, and the higher will be the negative wage effects for the better skilled workers.

C. A general rise in the level of human capital

The rapid economic growth of the Asian tigers in the decades preceeding their recent collapse was fuelled to a large extent by a substantial investment in human capital for the new generations entering the labor market (see Young (1995)). The comparative advantage model developed here implies that a general increase in the level of human capital reduces wage inequality. The relative supply of low-skilled workers goes down, increasing their wages, while the relative supply of highly skilled workers goes up, reducing their relative wages. This mechanism offers an explanation for the rapid decline in income dispersion in some of the Asian tigers (see Birdsall, Ross, and Sabot (1995)). Proposition VI provides some insights into the relationship between the accumulation of human capital and wage dispersion.

Proposition VI:

Consider the economy described in Section 2 with $\eta = 0$, and consider in this economy a sequence of investments in human capital such that for all worker types in each investment round $h(w) = h$, for all w.

Let H be the log of the accumulated stock of human capital (such that $dH = h$) and let the initial equilibrium of the economy be characterized by

i) $H = 0$,

ii) $cd(w) = \gamma(0)$,

iii) $w \sim N[\mu(0),\sigma(0)^2]$;

where the parameters $\gamma(H)$, $\mu(H)$ and $\sigma(H)$ are viewed as functions of the stock of accumulated human capital, H. Define $w'(s,H)$ as the return on human capital for skill level s at a level of accumulated human capital H.

Then:

VI-1) the characteristics of equal complexity dispersion for all w and log normality remain preserved during the accumulation process,

VI-2) $\gamma(H) = \gamma(0)/[1 - \gamma(0)\, H]$,

VI-3) $w'(s,H) = w'(s,0)\, [1 - \gamma(0)\, H]$,

VI-4) $\sigma(H) = \sigma(0)\, [1 - \gamma(0)\, H]$.

Proposition VI follows from Proposition V. The formal proof has been relegated to Appendix C. The crucial ingredient in the proof is equation (19) of Proposition V, which under an equal complexity dispersion parameter for all wage levels, and using the notation introduced in Propostion VI, reduces to the following:

$d[w(s,H) - w(r,H)]/dH = - \gamma(H)\, [w(s,H) - w(r,H)]$.

Dividing by $(s\text{-}r)$ and taking the limit $(s\text{-}r) \to 0$ yields:

$dw'(s,H)/dH = - \gamma(H)\, w'(s,H)$,

where the superscript ' denotes the partial derivative with respect to the first argument (to maintain consistency with the notation used before). Hence, the rate of compression of the return to human capital is equal to $\gamma(H)$, which contributes to the interpretation of the complexity dispersion parameter as the compression elasticity. This rate of compression is equal across skill levels. Therefore, actual log wage schemes $w(s,H)$ are a linear transformation of log wage schemes prevailing during previous stages of the accumulation process. This explains why the complexity dispersion parameter remains equal across skill levels and why log wages remain distributed normally in the course of the accumulation process. Note that the normality of the distribution of log wages is only required for proposition VI-4) and VI-5). The other parts of Proposition VI) require only the independence of the complexity dispersion parameter of the wage level.

The contribution of subsequent investment rounds in human capital to the compression of wage dispersion is governed by three forces. First, the complexity dispersion parameter goes up, increasing the rate of compression. Second, compression is proportional to the level of dispersion, decreasing the rate of further compression. The first and second forces exactly cancel, yielding a constant absolute rate of compression. Finally, the return to human capital is reduced in line with wage dispersion. Each new round of investment in the value stock of human capital will therefore require a greater extension of the "physical" stock of human capital (e.g. years of education or experience) than previous rounds, since the investments yield lower returns.

Note that changes in the stock of human capital are valued at their contribution to productivity, not at the cost of investment. Proposition VI-2/3) imply that the return to human capital and the standard deviation of the log wage distribution vanish when: $H = \gamma(0)^{-1}$. From then on, further investment in human capital is impossible (since human capital is valued at its return and this return is equal to zero). Hence, the log stock of human capital, and therefore the log output gain, cannot exceed $\gamma(0)^{-1}$, which is equivalent to a 40% output gain under the assumption that the complexity dispersion parameter is equal to 2.50.

The intuition for this result is that each increase in human capital will raise productivity in all job types, but mostly in the most complex job types, because the marginal return of an additional skill is the highest in these occupations. Since there is no between-job substitution, output ratios remain constant. Hence, an ever increasing part of the workers will be needed in the least complex job types, since the investment in human capital yields no productivity gain in that job type. In the end, everybody works in the least complex job type, where there is no productivity gain.

The maximum to productivity gains that can be achieved by investment in human capital typically apply when technology is kept constant. When the demand for labor shifts to more complex functions as a result of skill-biased technological progress, then the return to human capital increases again. At that point, new rounds of investment in human capital become profitable. Technology might also be endogenous, which can be covered in the model by letting η be positive. A fall in the return to human capital will then reduce the wages of highly skilled workers and thereby also output prices for complex products. With η positive, this will induce a shift of product demand towards more complex products. The endogeneity of technology will therefore increase the maximum output gain that can be achieved by investment in human capital. Similarly, the assumption that the complexity dispersion parameter is constant along the wage schedule is a condition for this maximum to productivity gains. Other patterns for the complexity dispersion parameter might yield different outcomes.

Propositions VI-2) and VI-3) imply that there is an inverse relation between the complexity dispersion parameter and wage dispersion in the course of the accumulation process. Hence, the maximum productivity gain achieved by investment in human capital depends on the initial level of wage dispersion: in Portugal, where the stock of human capital is still at a low level and where the standard deviation of log wages is high, a greater productivity gain can be realized by investment in human capital than can be realized in a country with a highly educated workforce and a compressed wage distribution, such as the Netherlands. The inverse relation between complexity dispersion and the standard deviation of log wages implies that their product is a "natural" constant (Portugal: 2.0, United States: 2.3, the Netherlands: 1.4).

Considering the two economies described in Table 1 (γ = 2.50, σ = 0.60 or 0.30), each percent increase in the stock of human capital will reduce all log wage differentials by 2.5%. Taking as a point of reference the 10-90% log wage differential (or equivalently: 2.56 σ), each extra percent human capital will yield a reduction of the 10-90% log wage differential of 2.5 x 2.56 x 0.60% = 0.038 points (or half as much for σ = 0.30).

Kim and Topel (1995) present some evidence for the case of Korea during the seventies and eighties. The 10-90% log wage differential declined from 1.68 in 1971 to 1.22 in 1989, a trend that is due mainly to the compression of rewards for educational groups. The mean number of years education of the workforce rose by approximately 1.6 years.[10] Using a 10% return to a year of education (this high number is consistent with Kim and Topel's, p. 250, fig. 7.13), we find that this increase is equivalent to a 16% increase in the value of human capital. Where the level of wage dispersion is consistent with σ = 0.60, the comparative advantage model would predict a 16 x 3.8 = 61% point reduction in the 10-90% log wage differential, which squares reasonably well with the actual decrease.

The same mechanism explains the large compression in wage differentials in the United States during the forties and fifties (Goldin and Margo, 1992), following the high school revolution between 1910 and 1940 (Goldin and Katz, 1998).

[10]Kim and Topel (1995, p. 249, Fig. 7.12): a 20% point increase in the share of high school x 4 additional years of education compared to elementary education, 10% point increase in college x 8 additional years.

The effects of the policy interventions discussed above are conditional on the assumption of η being equal to zero. Without that assumption, we would not be capable of applying the closed-form solution equation (14). However, we can use the characterization of **H** in Section 4 and set I at a high value to calculate the inverse numerically. Similar calculations in Teulings (1999) suggest that setting η equal to unity reduces the elasticities of complementarity along the main diagonal by some 40%.[11] A comparison of the empirically observed spill-over effects of an increase in the minimum wage in Lee (1999) and Teulings (1998) and the calculated effects discussed here suggests that η must be close to zero.

7 Some final remarks

Our analysis has revealed a number of peculiarities in the structure of substitution and complementarity of the comparative advantage model. The substitution matrix **H** converges to the constant between-job elasticity η everywhere, except in the entries directly adjacent to the main diagonal. When the number of factors of production tends towards infinity, the surface of this area adjacent to the main diagonal relative to the total surface of the matrix tends toward zero. Hence, if one believes the comparative advantage model, then it makes little sense to look at the substitution matrix.

However, the complementarity matrix is very informative. The second derivative of the log wage function has been shown to be crucial for its shape. The higher the curvature of the log wage function, the higher are the elasticities of complementarity. The intuition for this result is that whereas the first derivative shows up in the first-order condition for optimal assignment, the second derivative measures the cost of shifts in the assignment.

The trajectory of a column vector of the complementarity matrix is governed by a second-order differential equation, with a negative second derivative and a non-differentiability at the main diagonal. Due to these features, the complementarity matrix has a deep trough at the main diagonal, while there is relatively little action in the regions further from the diagonal. The relative wage effects of an increase in the supply of a particular skill type are therefore heavily concentrated at skill types within a shorter distance: the DIstance Dependent Elasticity of

[11] The characterization of **H** applied in Teulings (1998) is based on a more complicated and less robust approach, where the within-job/between-worker-type elasticity of substitution is set at a large but finite value. The present paper uses an infinite elasticity.

Substitution (DIDES) structure. This feature leads to the aggregation bias in traditional estimates of elasticities of complementarity discussed in Teulings (1999).

The comparative advantage model, thus, shed new light on the relation between human capital accumulation and wage dispersion, in particular when the elasticity of between-job substitution is equal to zero. For that case, a simple closed-form solution for the differential equation is available. There is a nice correspondence between the determinants of supply and demand for human capital. On the supply side, $w'(s)$ measures the return to human capital (or: skill) to the worker. On the margin, the cost of the acquisition of an additional unit will be set equal to this return. On the demand side, $w'(s)$ measures the cost of an additional unit of skill to the firm. In equilibrium, it will be set equal to the productivity gain of an extra unit of skill, that is, to the level of complexity of the job. This is equivalent to the first-order condition for optimal assignment: $w'(s) = c$. This double role of $w'(s)$ can be applied fruitfully.

A first attractive feature of the closed-form solution for the elasticities of complementarity is that the cross derivative of the row and column index equals zero. The implication of this characteristic is that a training program that raises the skill level of all workers of skill type s_0 by an amount Δ will yield an equal relative wage gain for all less skilled workers and an equal loss for all better skilled workers. It is remarkable that this quite general class of production functions yields this strong prediction.

A second feature arises when the complexity dispersion parameter is assumed to be constant. The complexity dispersion parameter is introduced as an alternative statistic for the second derivative of the log wage function. Whereas this second derivative can be identified empirically only up to a multiplicative factor, the related concept of the complexity dispersion parameter does not depend upon a non-identifiable linear transformation. This parameter can therefore be estimated empirically. Three estimates are available from previous work (for Portugal, and indirectly, for the United States and the Netherlands), varying between 2.4 and 3.8. This parameter and the shape of the wage distribution are sufficient statistics for the calculation of the complementarity matrix.

In principle, the complexity dispersion parameter varies along the wage schedule. However, as a first-order approximation, it can be assumed to be constant. For this case, the accumulation process of human capital can be characterized. Suppose that the human capital of all workers (measured by its marginal productivity) is increased by an equal percentage for each type. Then, the complexity dispersion parameter measures the percentage decline in the return to human capital per percent increase in the stock of human capital (again measured by its marginal productivity).

Given the estimates that are available of the complexity dispersion parameter, each percent increase in the value of the stock of human capital reduces the return on further investments by 2.4 - 3.8%. This result motivates the interpretation of the complexity dispersion parameter as the compression elasticity. At the same time, each percent increase in the stock of human capital will also increase the complexity dispersion parameter itself. As the accumulation process continues long, the return to human capital and therefore wage dispersion will decrease. In the end, both converge to zero. From that moment on, further investment in human capital yields no return. The maximum productivity gain from investment in human capital can therefore be calculated as the inverse of the complexity dispersion parameter, which is in the range of 26 - 42% for the estimates of the complexity dispersion parameter that are available.

These results allow empirical inference on the relation between human capital accumulation and wage dispersion. The redistributive effect of a general increase in human capital is substantial. This might explain the strong compression of the wage differential in the United States in the fifties and the sixties following the high school *revolution* between both World Wars and the strong compression in some Asian tigers during the seventies, again following heavy investment in human capital in the preceeding period. In fact, the predictions of the model match the actual compression remarkably well for the case of South Korea.

The quality of the schooling system might therefore be an important explanation for cross country differences in wage dispersion. Leuven, Oosterbeek, and Van Ophem (1997) present evidence in favor of this idea, using a new multi-country OECD dataset with test scores that are comparable across countries.

These results have important positive and normative implications for economic policy. They point to a set of both positive and negative externalities that the schooling decision of one worker type might present to the value of human capital of others. On the positive side, these externalities explain why many democracies subsidize higher education. Each dollar of tax money that is spent on the education of the upper half of the skill distribution has a positive external effect on the wage of the median voter. Hence, the median voter will find it optimal to subsidize higher education - at least to some extent.

On the normative side, there are implications for the targetting of training programs for the relief of the low-skilled. It is not necessary to target these programs tightly to the left tail of the skill distribution to let only the least skilled workers gain from the policy. Schooling programs aimed at worker types somewhat higher in the skill distribution still have a positive impact for the least skilled, due to their general equilibrium effects. When training policies for the least skilled are costly due the inefficiency of the education production function for this group, it might be a worthwhile alternative to aim the policy at a somewhat higher skill level and to let the least skilled benefit from the general equilibrium effects. The policy maker then faces a trade-off between the general equilibrium effect, on the one hand (the higher up in the skill distribution is the focus of the training program, the smaller is its general equilibrium effect on the left tail), and the efficiency of the education production function, on the other hand.

Appendix A The proof of Proposition II

Proposition II-1) follows from the symmetry of $\mathbf{H}^+$.

The proof of the other propositions requires two steps: first, the characterization of $\mathbf{G}^+$, and then taking the limit: $\lim_{\Delta s \to 0}$.

Let $\underline{g}_i$ and $\underline{h}_i$ denote the i-th vector of $\mathbf{G}^+$ and $\mathbf{H}^+$ respectively. It is easy to see that: $\underline{g}_{I+1} = [\underline{V} \mid 0]$, since: $\mathbf{H}^+ [\underline{V} \mid 0] = [\underline{0} \mid 1]$ by the definition of the diagonal elements of $\mathbf{H}$. Furthermore, the identity: $\mathbf{G}^+ \mathbf{H}^+ = \mathbf{I}$ and the symmetry of $\mathbf{H}^+$ define the following relations:

A1) $\underline{g}_j' \underline{h}_{I+1} = 0 \quad \Rightarrow$

$\sum_{i=1}^{I} g_{ij} = 0;$

A2) $\underline{g}_j' \underline{h}_i = 0, \quad$ for $i \neq 1,j,I \quad \Rightarrow$

$\sum_{k=1}^{I} \eta\, g_{kj} - \eta\, g_{ij}/v_i + q_i g_{i-1,j} - 2\, qq_i g_{ij} + q_{i+1} g_{i+1,j} + v_j = 0;$

A3) $\underline{g}_i' \underline{h}_i = 1 \quad$ for $i \neq 1,I \Rightarrow$

$\sum_{k=1}^{I} \eta\, g_{ki} - \eta\, g_{ii}/v_i + q_i g_{i-1,i} - 2\, qq_i g_{ii} + q_{i+1} g_{i+1,i} + v_i = 1;$

A4) $\underline{g}_j' \underline{h}_1 = 0, \quad \Rightarrow$

$\sum_{k=1}^{I} \eta\, g_{kj} - \eta\, g_{1j}/v_1 - (v_2/v_1) q_2 g_{1j} + q_2 g_{2j} + v_j = 0$

(a similar equation is available for $\underline{g}_j' \underline{h}_I$);

where: $qq_i \equiv {}^1/_2\, [(v_{i-1}/v_i) q_i + (v_{i+1}/v_i) q_{i+1}]$.

The term $\sum_{k=1}^{I} \eta\, g_{kj}$ in equation A2)-A4) drops out due to equation A1). Define the first difference operator: $\Delta g_{ij} = g_{ij} - g_{i-1,j}$. Hence, this operator refers to the first suffix of g_{ij}. Likewise, a second difference operator is defined. Then, equation A2) can be written as:

$$qq_i \Delta^2 g_{i+1,j} + (qq_i - q_i)\Delta g_{ij} + (q_{i+1} - qq_i)\Delta g_{i+1,j} + (q_i - 2qq_i + q_{i+1}) g_{ij} - \eta g_{ij}/v_i + v_j = 0 \qquad \textbf{(21)}$$

This completes the first step, the characterization of $\mathbf{G}^+$. We now turn to the second step, taking the limit for Δs as it approaches zero. Since: $g(s_i,s_j) \equiv \lim_{\Delta s \to 0} g_{ij}\, (\Delta s)^{-2}$, we have:

$\lim_{\Delta s \to 0} \Delta g_{ij}\, (\Delta s)^{-3} = g_1(s_i,s_j);$

$\lim_{\Delta s \to 0} \Delta^2 g_{ij}\, (\Delta s)^{-4} = g_{11}(s_i,s_j),$

where the $g_i(.)$ refers to the partial derivative of $g(.)$ to its i-th argument. Furthermore:

$\lim_{\Delta s \to 0} qq_i\,(\Delta s)^3 = [v(s_i)w''(s_i)]^{-1}$;

$\lim_{\Delta s \to 0} (qq_i - q_i)(\Delta s)^2 = -{}^1/_2\,[w''(s_i)v'(s_i)+w'''(s_i)v(s_i)] / [v(s_i)w''(s_i)]^2$;

$\lim_{\Delta s \to 0} (q_i - 2qq_i + q_{i+1})\Delta s = [v'(s_i)/v(s_i)]\,[w''(s_i)v'(s_i)+w'''(s_i)v(s_i)] / [v(s_i)w''(s_i)]^2 -$

$[v''(s_i)w''(s_i)] / [v(s_i)w''(s_i)]^2$.

Multiplying equation (21) by Δs and applying these relations yields equation (11) in proposition II-3).

When: $\lim_{\Delta s \to 0} \Delta g_{ij}\,(\Delta s)^{-3} \neq \lim_{\Delta s \to 0} \Delta g_{i+1,j}\,(\Delta s)^{-3}$, then:

$\lim_{\Delta s \to 0} \Delta g_{ij}\,(\Delta s)^{-3} = g_{1\uparrow}(s_i,s_j)$;

$\lim_{\Delta s \to 0} \Delta g_{i+1,j}\,(\Delta s)^{-3} = g_{1\downarrow}(s_i,s_j)$,

where the arrows denote the left and right partial derivatives of $g(.)$ respectively. Hence, relation A3) implies:

$$\frac{1}{v(s)w''(s)}[g_{1\uparrow}(s,s) - g_{1\downarrow}(s,s)] = 1 \qquad \textbf{(22)}$$

$g(s,r)$ is therefore continuous but non-differentiable at $s = r$ as it approaches zero, proving proposition II-2). Furthermore:

$\lim_{\Delta s \to 0} q_i\,(\Delta s)^3 = [v(s_i)w''(s_i)]^{-1}$.

The limit for Δs of equation A4) yields equation (12) in proposition II-4). A similar equation applies for $s = s^+$. Finally, the limit of equation A1) yields equation (13) in proposition II-5). Q.E.D.

Appendix B The proof of proposition III

$g(s,r)$ can be calculated from equation (14) by multiplying both sides by $v(s)v(r)$ (see equation (5)). Differential equation (11) can be recovered from there by dividing by $v(s)$, differentiating once, multiplying the result by $v(s)/w''(s)$, differentiating a second time, and dividing the result by $v(s)$. The four initial conditions are satisfied. The equality of $g(s,r)$ for $s = r$ for both branches of the locus follows from the symmetry of equation (14). The initial condition for the first differential equation for $s = s^-$ given in equation (12) is satisfied because the derivative of the first term vanishes since $V(s^-) = 0$. Likewise, the initial condition for s^+ is satisfied since $V(s^+) = 1$ (for being $V(.)$ the distribution function of value). Equation (13) can be checked by first evaluating it for $r = s^+$. Then: $s < r$ for the full support of s and hence, we have only to apply equation (14) and not its symmetric counterpart for $s > r$. The second term drops out. It then follows immediately that equation (13) is satisfied. Next, the equation has to be verified for $r < s^+$. Since equation (13) applies identically for all r, it can be differentiated with respect to r. Substitution of equation (14) yields the following:

$$\int_{s^-}^{s^+} g_2(s,r)ds = \frac{v'(r)}{v(r)}\int_{s^-}^{s^+} g(s,r)ds - w''(r)[1-V(r)]\int_{s^-}^{r} v(s)ds + w''(r)V(r)\int_{r}^{s^+} v(s)ds \qquad \textbf{(23)}$$

The first term accounts for the derivative of the factor $v(r)$, which comes in due to the transfer from $e(s,r)$ to $g(s,r)$. The second term accounts for the derivative of $e(s,r)$ for $s < r$ and the third term for $s > r$. The second and third terms cancel. The first term vanishes when equation (13) is satisfied. Since equation (13) is satisfied for $r = s^+$, by induction it will also be satisfied for $r < s^+$. Q.E.D.

Appendix C The proof of proposition VI

Under the assumption of the complexity dispersion parameter being constant, and using the notation developed in Proposition VI, equation (19) in Proposition V reduces to the following:

$\mathrm{d}[w(s,H) - w(r,H)] = - \gamma(H)\,[w(s,H) - w(r,H)]\,\mathrm{d}H$.

Dividing by $(s-r)\mathrm{d}H$ and taking the limit $(s-r)\mathrm{d}H \rightarrow 0$ yields

$w_2'(s,H) = - \gamma(H)\, w'(s,H)$,

where the suffix 2 refers to the partial derivative to the second argument. The superscript ' is used to denote the partial derivative with respect to the first argument to maintain consistency with the notation used in previous sections. Differentiating with respect to s yields:

$w_2''(s,H) = - \gamma(H)\, w''(s,H)$.

By the definition of the complexity dispersion parameter we have

$\gamma'(H) = \gamma(H)\,[w_2''/w'' - 2\, w_2'/w'] = \gamma(H)^2$.

Hence,

$\gamma(H) = \gamma(0)/[1 - \gamma(0)H]$.

which proves proposition VI-2). Note that $w'(s,H)$ and $w''(s,H)$ drop out of the equation for $\gamma'(H)$, so that the complexity dispersion parameter remains independent of w in the course of the accumulation process, which proves the first part of proposition VI-1). Substituting the relation for $\gamma(H)$ in that for

$w_2'(s,H)$ yields VI-3). Furthermore,

$w_2(s,H) = - \gamma(H)\, w(s,H) + M(H)$,

where $M(H)$ is a proper constant of integration. Two events cause the distribution of *w* to change in the course of the accumulation process. First, consider the change due to the accumulation of human capital. Were relative wages unaffected, the change would be

$w \sim N[\mu(H)+dH,\sigma(H)^2]$,

since each worker type gets an equal increase in her human capital. The increase in human capital itself does therefore not disturb the normality of *w*. Next, consider the change in relative wages. Note that the new log wage schedule is a linear function of the old schedule. Hence, the change in the relative wage does not affect the normality of *w* either, proving the second part of proposition VI-1). Since it applies in general that: d std.dev.$[g(H)w]/dH = g'(H)$ std.dev.$[g(H)w]$, we have

$\sigma'(H) = \gamma(H)\, \sigma(H)$.

This relation can be integrated, yielding proposition VI-4).

Q.E.D.

Table 1 Percentiles (value and hours), $\Psi(w/\sigma)$, w, the effect of an increase in the minimum wage yielding a 1 (value)% employment loss, and the effect of an increase in human capital of 1 (value)%

(value)%	Ψ	$\sigma = 0.60$			$\sigma = 0.30$		
		w	(hrs)%	min.w[1)]	w	(hrs)%	min.w[1)]
0.008	0.209	-1.440	0.036	0.194	-0.720	0.018	0.048
0.014	0.285	-1.320	0.055	0.128	-0.660	0.029	0.032
0.023	0.366	-1.200	0.081	0.087	-0.600	0.045	0.022
0.036	0.454	-1.080	0.115	0.059	-0.540	0.067	0.015
0.055	0.550	-0.960	0.159	0.040	-0.480	0.097	0.010
0.081	0.653	-0.840	0.212	0.027	-0.420	0.136	0.007
0.115	0.767	-0.720	0.274	0.017	-0.360	0.184	0.004
0.159	0.892	-0.600	0.345	0.010	-0.300	0.242	0.003
0.212	1.032	-0.480	0.421	0.004	-0.240	0.309	0.001
0.274	1.188	-0.360	0.500	0.000	-0.180	0.382	0.000
0.345	1.365	-0.240	0.579	-0.004	-0.120	0.460	-0.001
0.421	1.567	-0.120	0.655	-0.006	-0.060	0.540	-0.002
0.500	1.802	0.000	0.726	-0.009	0.000	0.618	-0.002
0.579	2.077	0.120	0.788	-0.011	0.060	0.691	-0.003
0.655	2.406	0.240	0.841	-0.013	0.120	0.758	-0.003
0.726	2.804	0.360	0.885	-0.014	0.180	0.816	-0.004
0.788	3.298	0.480	0.919	-0.016	0.240	0.864	-0.004
0.841	3.923	0.600	0.945	-0.017	0.300	0.903	-0.004
0.885	4.734	0.720	0.964	-0.018	0.360	0.933	-0.005
0.919	5.813	0.840	0.977	-0.019	0.420	0.955	-0.005
0.945	7.292	0.960	0.986	-0.020	0.480	0.971	-0.005
0.964	9.384	1.080	0.992	-0.021	0.540	0.982	-0.005
0.977	12.443	1.200	0.995	-0.022	0.600	0.989	-0.005
0.986	17.072	1.320	0.997	-0.023	0.660	0.994	-0.006
0.992	24.340	1.440	0.999	-0.023	0.720	0.997	-0.006

[1)] effect on w due to the introduction of a minimum wage causing a 1 (value)% loss of employment.

$\int[\varphi(x)\ \psi(x)]\delta x$: 2.799
γ: 2.500
ω/σ: 3.000

Table 2 Effect of an increase in human capital of 1(value)% for 1(value)% wage level and (value) percentile of the workers who receive the training

σ = 0.60

		wage effects for:	
w	%	less skilled	better skilled
-0.96	0.05	12.78	-0.74
-0.72	0.12	6.84	-0.89
-0.48	0.21	4.08	-1.10
-0.24	0.34	2.67	-1.40
0.00	0.50	1.88	-1.88
0.24	0.66	1.40	-2.67
0.48	0.79	1.10	-4.08
0.72	0.88	0.89	-6.84
0.96	0.95	0.74	-12.78

σ = 0.30

		wage effects for:	
w	n%	less skilled	better skilled
-0.48	0.05	6.39	-0.37
-0.36	0.12	3.42	-0.44
-0.24	0.21	2.04	-0.55
-0.12	0.34	1.33	-0.70
0.00	0.50	0.94	-0.94
0.12	0.66	0.70	-1.33
0.24	0.79	0.55	-2.04
0.36	0.88	0.44	-3.42
0.48	0.95	0.37	-6.39

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