

**The Effect of Skill Level on the Timing of
Childbearing and Number of Children**

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Abstract

This paper examines the effect of differences in ability on the timing and number of children. Higher skilled women have less disutility of labor and have relatively less utility of raising children. Motherhood has a negative effect on the accumulation of human capital by learning-by-doing and thereby on career perspectives. A decline in fertility is explained without a quantity-quality shift.

Keywords: heterogeneity in skill level, number of children, timing childbearing.

JEL Classification: J13, J24, J22

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1. Introduction

Changes in family life in developed countries have been enormous since the fifties. Whereas at the time it was rare that women worked after giving birth, now it is becoming rare that women do not work. The number of children declined dramatically, and the remaining children received more schooling. Decisions about labor participation of women, number and timing of children are made simultaneously and are affected by labor market conditions and government policies. This paper explains why childbearing is delayed, and why the number of children dropped simultaneously. Moreover, it explains why higher skilled women have less children and have their first child when they are older than lower skilled women.

The demographic shift coincided with an improvement of the position of women in the labor force. Career perspectives have grown for working women over time and investment in human capital has increased. The change in skill level of women resulted in a higher wage growth over time and increased psychological rewards. Akerlof (1988) et al. argue that there are clear nonpecuniary rewards to working. It affects one's self-fulfilment, social status and the people one meets.

Because children are time consuming, number and timing of children interfere with pursuing a career and 'enjoying' work. The main motive for having children in developed countries is as a form of 'consumption goods'¹: raising children and companionship of children increase utility. When work is of more importance or when working hours give less disutility, spending time with children will be of less concern. Women with a higher level of education can choose more jobs which suit their psychological needs. They can achieve jobs with a higher status level and jobs which give less disutility. As a result higher educated women prefer to spend less time with children.

Job characteristics and career perspectives affect the timing of childbearing as well. There is a trade-off between having children and pursuing a career. Because women still primarily provide the care for children, children affect their career perspectives and wage profile. Ambition levels might decline: it is hard to combine care tasks and pursuing a career. As a result women often prefer a part-time job or a less demanding job, which decreases training and career perspectives. Loss of career opportunities already exist when the employer has the perception that the woman will pay less attention to the company. Because the care for children affects career perspectives, women have an incentive to delay childbearing and find an optimum between the moment of childbearing, and the level of training. The opportunity costs of an earlier timing of childbearing

in the form of foregone wage growth are dependent on the skill level of women. High skilled workers typically have a much steeper wage profile than low skilled workers (see for instance Murphy and Welch (1992)). As a result loss of career opportunities is likely to be higher when the skill level of women rises.

The relation between number and timing of children and education of the mother is observed in various papers. For instance, Wolpin (1984) found that the higher the mother's education level, the lower was the utility of additional children. Heckman and Walker (1990) investigate the effects of income on fertility decisions and find that a higher level of female wages decreases the number of children and delays the timing of the first birth. A higher male income reduces time to conception and increases the quantity of children. Tasiran (1993) finds weaker effects of income levels and finds that female schooling and working experience significantly delays the first birth. Moreover, work has become a more central interest for women in fulfilling life-time goals. Olsen (1994) presents evidence that an increased fraction of women's occupations reveal a more permanent attachment to the labor force.

In the literature on fertility and childbearing the focus is primarily on the shift in the quantity of children. The explanation for a decline in fertility presented in the model is complementary to the theory of Becker and Lewis (1973) which claims that the difference in income elasticities of quantity and quality of children result in a quantity-quality shift. The model contrasts with the theory of Willis (1973), which stresses that consumption of goods is less time-intensive than consumption of children.

Although a decline in fertility has the same effect on population growth as a delay of bearing of children, the delay of giving birth is seldom modelled. Happel et al. (1984) and Razin (1980) present a theoretical model of the timing of fertility, but they do not treat human capital in their analysis and neglect the timing issue in the utility function of the decision maker. Blackburn et al. (1993) do treat human capital accumulation, but women only differ in preferences toward child-bearing; ability does not play a role and human capital investment does not have a time-cost component. Cigno and Ermisch (1988) focus on the timing of children, and use learning-by-doing as the process of human capital accumulation. They do not treat the effect of skill level on pleasure in working.

In this paper determinants of the timing of giving birth and the quantity of children are examined. Human capital is accumulated by learning-by-doing, which depends on the ability level of a woman. A higher skill level increases the number of jobs a woman can choose and the higher the

skill level, the lower the disutility of working will be or the lower the pleasure of raising children will be relatively. The effect of different economic policy instruments on family decisions are investigated.

The model stresses the importance of intrinsic values of jobs in fertility decisions and is able to explain a decline in fertility without a quality-quantity shift. It is found that changes in ability affect decisions of timing and number of children. Learning efficiency has been increased by an increase in public provision of schooling. With the increased learning efficiency over time and the differences in ability between women in a cross section of the population I am able to explain the trends in the number of children and the timing of childbearing, which are observed in time- and cross section series. The results depend especially on the elasticity of disutility of working with respect to ability and the effect on the level of additional (potential) earnings.

I first present the model and then I explain changes in the timing of childbearing and the number of children with comparative dynamics. I investigate differences in ability and years of schooling. Furthermore, I examine the effect of changes in government policies and their effect on family decisions.

2. The model

A simple micro-economic model is used to analyze the change in number and timing of children over time. Moreover, the model is used to investigate the effect of heterogeneity in education levels² on timing and quantity of children. A household derives utility from consumption, number of children and from the length of the period parents live when their children are alive. I ignore issues of spacing: I assume that all children are born at T_* ³. It is assumed that lower skilled women find work of less importance for fulfilling life goals or have a higher disutility of working. As a result they enjoy rearing and raising children relatively more than higher skilled women. This is in line with psychological theories, like the motivation theory of Maslow (1943) and the Central Life Interest (CLI) theory of Dubin (1956), Dubin et al. (1975). Maslow claims that when physiological and security needs are fulfilled, esteem needs (prestige and status) and self-actualization become important. In the CLI theory work is one of the institutions for personal development. Dubin et al. (1975) finds that the organization is more important for people with a higher occupational level. Landy and Trumbo (1976) give different studies which find that job satisfaction increases with occupational level. Wolpin (1984) found that the higher the mother's

education level, the lower was the utility of additional children.

The utility function of the household is

$$V = \frac{\tilde{a}_1}{\delta} \ln p_n n T_o + \int_{T_*}^T (1 - \tilde{a}_1 - \tilde{a}_2) u(t) dt + \int_0^T \tilde{a}_2 \ln c(t) e^{-\delta t} dt \quad (1)$$

where

- $V(t)$ = utility function of household
- $c(t)$ = consumption at time t
- T_* = moment of birth children
- $T_r - T_* = T_o$ = the period of time in which children grow up
- T_r = end of rearing period children
- T = total lifetime
- n = number of children
- $p_n(t)$ = fixed time costs of rearing children
- \hat{a} = rate of time preference

with $n > 0$. I define $\delta = f(\hat{a})$, with $f' > 0$, where \hat{a} is the level of ability. $u(t)$ is the utility of the period that the parents and children are both alive, with $u > 0$ and $u' < 0$. Because I focus on birth decisions, I use a general utility function for the timing of children. For reasons of tractability I assume a log function for consumption and number of children.⁴

The household budget contains the income of the woman and her spouse, and initial assets. I treat the income of the spouse as exogenous and let the woman decide about the birth decisions. The woman has access to a perfect capital market. I assume that the cost of children grows with the same level as the growth of net wage. The cost of children are majorly the time costs of rearing children, i.e. the costs of forgone earnings (Espenshade, 1977).

The woman accumulates human capital by learning-by-doing. The rate of learning depends on the ability of the woman. I assume that the wages grow linearly till the moment of childbearing. From the moment of motherhood, the wage rate is flat. As a result, the opportunity costs of children contain the time costs of children and the foregone wage growth. After childbearing learning ends, independent of the number of children: "being a mother" results in a flat wage rate. The assumption is that after the birth of the first child, the ambition level of the mother declines

or that in the perception of the employer, being a mother and pursuing a career is hard to combine⁵. Women primarily provide the child care and nonmarket services (Genevie and Margolies, 1987). Fuchs (1989) gives three reasons why children depress woman's wages. First, women often leave the labor market during pregnancy. The moment of childbearing for most women coincides with the time that men are gaining the training and experience that lead to higher wages later in life. Second, even when women stay in the labor force they accept jobs with lower wages to have more flexible hours, location near home etc. Third, the disproportionate responsibility for child care and housework often result in sacrifices in their market work.

The budget constraint of the household is

$$\int_0^T c(t)e^{-rt}dt = \int_{T_s}^{T_*} wH(T_s)(1 + \hat{\alpha}t)e^{-rt}dt \quad (2)$$

$$I(T_s)(1 + \hat{\alpha}T_*)(1 - p_n n)e^{-rt}dt + \int_{T_r}^{T_p} wH(T_s)(1 + \hat{\alpha}T$$

- T_p = date of retirement
- T_s = end schooling period
- $H(T_s)$ = human capital level after the schooling period
- w = $(1 - \hat{\alpha}(t))w$ = net wage
- $\hat{\alpha}(t)$ = tax rate at time t
- r = interest rate
- $A(0)$ = assets at time 0, plus exogenous income of spouse

with $T_* + T_o \leq T$ and $T_* \geq T_s$. I assume that when a woman still gets schooling during pregnancy, schooling ends after giving birth. This assumption ignores teenage birth problems and childbearing of unplanned children. There are several reasons for delaying parenthood after finishing schooling. First of all, the woman might be constrained in her budget during study and be dependent on her parents. Secondly, there might exist normative expectations that young people that attend school do not enter parenthood (Blossfeld and Huinink (1990)). Thirdly, to follow a study successfully and be a mother is difficult to combine (Rindfuss et al (1988)). And opportunity costs of dropping out of school have increased severely (Oppenheimer (1988)).

Note that when the wage level is not subject to a decline after giving birth or when the wage

growth does not decline, the woman would choose to give birth immediately after the schooling period.

3. Comparative Dynamics

The level of provision and the quality of public and private schooling increased severely this century and the investment in education of women has boosted. Changing conditions on the labor market for women and the increased human capital affected the preference for the number of children and when to have them. Differences in learning efficiency (and preferences) can be observed in a cross-section of the population as well. I analyze the effect of heterogeneity in ability of women and the effect of government policies on the number and timing of children. I primarily focus the analysis on women that supply labor on the labor market. Women working at home would prefer children as early as possible in the model, unless the process of human capital accumulation for home production would be negatively affected. But the extra child care tasks make this very unlikely: the woman has to perform more household activities and the usage of her human capital will be more efficient. Decisions of the number of children and timing of childbearing change, however, when government policies or changing working conditions persuade her to supply labor on the labor market.

3.1 Effect of a shock in ability

Differences in ability affect the wage growth by the rate of learning-by-doing, and the (dis)utility in working. Higher skilled women have a higher growth of wages and can pick jobs that provide more self esteem and have more challenges. Hence, policies which affect the schooling level of women, affect birth decisions as well. As a result, the investment in public schooling in western societies might be an important explanatory variable for trends in fertility in birth statistics.

Proposition 1:

a. Suppose that

$$\frac{d\acute{o}}{d\acute{a}} \frac{\acute{a}}{\acute{o}} < \frac{\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*)|_{T_* = T - T_O}^{-r}}{\frac{\acute{a}^2 (T - T_O - T_S)}{1 + \acute{a} (T - T_O)} + r(1 + \acute{a} T_S)} \quad (3)$$

and

$$\left. \frac{-\tilde{a}_2}{\acute{o} u(T_*)|_{T_* = T - T_O}^{-r}} \right) + \frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u'(T_*)|_{T_* = T} \quad (4)$$

Then women with higher ability have children when they are relatively older.

b. Suppose that

$$\frac{d\acute{o}}{d\acute{a}} \frac{\acute{a}}{\acute{o}} > \frac{\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*)|_{T_* = T_S}^{-r}}{\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*)|_{T_* = T_S} (1 + \acute{a} T_*)} \quad (5)$$

Then women with higher ability have fewer children.

Proof. See Appendix.

Whether the first condition is satisfied depends on the total effect of a change in ability on the price of timing of children and the effect on full income. The family receives full income when it abandons to have children. The cost of children can be divided in foregone earnings, because of time spent on rearing children, and foregone wage growth, because of earlier childbearing. A steeper wage profile makes the period adults and children live together more expensive.

The numerator of the RHS gives the effect of ability on the price of timing of children. The effect can be divided in an income- and a substitution effect. The higher price of the period of companionship, results in substitution of consumption and number of children for the length of the period parents and children live together. This effect dominates the effect of the increase in full income.

The LHS and denominator of the RHS represent the income effect of ability on spending time with one's children. Higher skilled women prefer to spend relatively less time with children. A

decline in the time spent with children implies that more income can be allocated for earlier childbearing. The size of the effect depends on the height of the interest rate. With a high interest rate a career is less beneficial: the profits of a career are foremost in the future and a high r implies that the value of higher income in the future is lower.

By reformulating the RHS of (3) (see (23) in Appendix) we can show that the total effect of a change in ability depends strongly on the marginal rate of substitution between number of children and the period of companionship of children and parents, and the elasticity of utility of raising children with respect to ability. We can assume that $d\phi/\phi < d\alpha/\alpha$: a doubling of the wage growth is unlikely to double the disutility of raising children. When the MRS of period of companionship for number of children is not approximately equal to r , the condition will be satisfied.

The second condition is satisfied when the marginal utility of an earlier childbearing is low relatively to the total utility of the period that parents and children are both alive. With a concave utility function, and when the period parents and children are alive is already long relatively to life-expectancy, we can expect this condition will be satisfied easily. In the Appendix this is illustrated with a common utility function.

The third sufficient condition shows that an increase in the level of ability results in a decline in the number of children, when the negative effect on the utility of raising children dominates the effect on the increased full income. The condition holds when the elasticity of utility of raising children with respect to ability is not very low.

In terms of empirical evidence, Wolpin (1984) found that expectations of rising incomes tended to delay childbearing. Tasiran (1993) found that a higher level of working experience delayed the first birth. When we proxy income levels for skill levels, the results of Heckman and Walker (1990) show that higher female wages decrease the number of children and delays time to all conceptions. Cigno and Ermisch (1988) found that women in occupations characterised by steeper earnings profiles will tend to have their children later and will have fewer children. Blossfeld and Huinink (1991) found that the level of career resources has a negative effect on the rate of entry into motherhood.

3.2 Effect of a longer schooling period

A longer period of schooling is correlated with the level of ability: higher skilled people tend to have a longer period of schooling. I analyze the effect of the length of the schooling period separately from the effect of differences in ability.

Proposition 2:

Let (4) from Proposition 1 be satisfied. Women with a longer schooling period have

- a. children when they are relatively older;*
- b. fewer children.*

Proof. See Appendix.

A longer period of schooling implies that the working period is shorter. Compared to a woman with the same wage growth, a woman with a longer schooling period has less income. The lower income leads to fewer children and a delay in childbearing.

There is strong empirical evidence that a longer period of schooling delays childbearing and results in a decline in the quantity of children. Blackburn et al. (1993) note that late childbearers are not only delaying because they spend more time on school. Women with higher education delay childbearing more than the extra years spent in school. Newman and McCulloch (1984) found that education had a negative effect on the timing of the first birth. Tasiran (1993) reports as well that years of education tends to delay childbearing. Wolpin (1984) found a strong negative effect between the years of schooling and number of children.

3.3 Effect of initial assets and income of spouse

Proposition 3:

Let (4) from Proposition 1 be satisfied. A woman with higher initial assets or with a spouse with a higher income has

- a. children when she is relatively younger;*
- b. a higher number of children;*

Proof. See Appendix.

A higher income of the spouse (or higher initial assets) increases the budget of the women and she can afford to have more children and to have them earlier.

Ermisch (1987) reports positive effects of male after-tax earnings on the number of children. Heckman and Walker (1990) found that a higher male income reduces times to conception and raises the total conceptions. Wolpin (1984) found that a rise in income of the husband had a large effect on the number of births in Malaysia.

3.4 Change in the tax system

The effects of tax reforms on fertility are often neglected. However, increasing tax brackets, tax rates, income exemption rules affect the progressivity of the tax system and implicitly affect career- and fertility decisions. For instance, the individual exemption in Sweden and the possibility of carrying over income exemption to a partner in the Netherlands, has coincided with lower labor participation rates of women in the Netherlands. I examine the effects of a change in the flat wage rate and the progressivity of the income tax. Future empirical research is needed to investigate the hypotheses.

3.4.1 Changes in the flat tax rate

Proposition 4:

An increase in the flat tax rate does not affect the decisions of timing of childbearing and number of children of women that supply labor on the labor market. However, it affects birth decisions of women that substitute working on the labor market for working at home. The probability that women stop working in the market increases when the taxes are increased.

Because time costs are the only costs of children in the model, opportunity costs of children and income rise in the same proportion. As a result, the change in the tax level does not change the budget share of children of working women and does not affect birth or labor supply decisions. When bequests, initial assets and fixed costs of children are introduced, the budget share of the costs of children changes with a shock in the tax level, and the total effect on birth decisions is

ambiguous.

Women that do not provide labor on the labor market might be stimulated to work when taxes are decreased or when the probability of finding a job is increased. The opportunity costs of working at home are increased.

Note that proportional taxes might affect the accumulation of human capital when time is not the only input in human capital accumulation, and other services and goods are not tax deductible. Trostel (1993) finds a negative effect of income taxation on human capital accumulation.

3.4.2. An increase in the progressivity of taxes

Proposition 5:

Let (4) from Proposition 1 be satisfied. An increase in the progressivity of the tax system results in

- a. earlier childbearing;*
 - b. an increase of the number of children;*
- for women that supply labor on the labor market.*

Proof. See Appendix.

When higher levels of income are relatively higher taxed, additional income of higher skill levels declines. Hence, increased progressivity makes having earlier childbearing less expensive. The effect on the number of children can be seen when we disentangle a change in progressivity of the tax system in a change in the tax level and a change in the level of tax exemption. Because increases in a flat tax rate do not affect fertility decisions, an increase in the progressivity of the tax system has the same effect as an increase in the exemption of taxes. A higher tax exemption has a similar effect as an increase in income and the number of children increases.

3.5 Change in the cost of children

The cost of children is affected by government policies. Subsidization of child care, tax exemptions on dependents and child allowances make children relatively less expensive.

Proposition 6:

Let (4) from Proposition 1 be satisfied. An increase in the cost of children

a. decreases the number of children;

b. does not affect the decisions of the timing of children.

Proof. See Appendix.

A decrease in the fixed time cost of children makes children less expensive and increases the number of children. The total time on raising children will not change.⁶ The time cost of children does not affect the costs of the timing of children and the moment of childbearing is not altered.

There is a difference between child care in the form of pecuniary support and the provision of child care that reduces the fixed time cost. When subsidization of children results in a decline of time spent on rearing children, it results in an increase in the number of children. But when subsidization of children results in a pure income effect, the higher income leads to an increase in the number of children and to earlier childbearing as well.

Oppenheim Mason and Kuhltau (1992) report that one-third of mothers of preschool-aged children are constrained in their labor participation by child care problems. Only one-tenth of the sample reported a child care constraint on fertility. Cigno and Ermisch (1988) find that higher child benefits raises the number of children. Whittington (1992) and Whittington et al. (1990) report positive effects of a higher tax exemption of dependants on the number of children in the United States. Zhang et al. (1994) find similar results for Canada and find that child tax credit and family allowances have significant positive effects on fertility as well.

3.6 Endogenous Quality Time and Quantity of Children

How does this model relate to the quantity-quality shift in the theory of Becker and Lewis (1973)? In Becker and Lewis the number of children declined because parents substituted quality of children for quantity of children, when income rose. When the income elasticity of quality of children is higher than of quantity of children, a higher income results in an increase in the demand for quality. Because quality of children is an important determinant of the price of quantity of children and vice versa, an increase in income can result in a decline in the demand for children, even when the demand for children behaves like the demand for a normal good.

The effects of price changes in the theory of Becker and Lewis (1973) can be applied when we

extend the model to quality of children. However, the source of a change in demand for children is different. Proposition 1 stresses the effect of ability on the utility of working time and demand for children. Higher skilled women do not want to have less children as a result of a change in income, but because they want to allocate less time for rearing children. The decrease in time for rearing children has to be divided between quantity and quality of children, and the change in the respective time-shares will be dependent on the income elasticities of quality and quantity of children.

To extend the model to quality of children we have to divide the time cost of children in 'fixed' time cost and in investment in the quality of children. The different components are hard to disentangle: very young children need supervision, but during time of parental guidance the child learns new abilities. Moreover, child care is done partly by parents, but child care centres and grandparents relieve parents of parts of child care tasks as well. Older children need less parental supervision, but it might be more difficult for other institutions to relieve parents. Teenagers are harder to control for grandparents.

When quality time is a choice variable, parents can divide their time between working, $h(t)$, and spending time to quantity and quality of children. When I assume that time invested in quality of children is equally divided between children then

$$1 - h(t) = (p_n(t) + q(t))n(t) \quad (6)$$

where

$q(t)$ = time invested in the quality of children

Parents with a higher learning efficiency have a preference to spend less time to children (see Proposition 1). They face the dilemma to have less children or to spend less quality time on children.

$$- dh(t) = (p_n(t) + q(t))dn + n(t)dq \quad (7)$$

We can consider the decision of time spent on children as a two-stage decision. In the first stage the total time for children is allocated and in the second stage the family decides about number of children and quality time for children. Parents with a higher learning efficiency spend less time to children and have a higher income, because of higher wage growth and because of more working hours. If we assume that the demand for quantity of children is less elastic to income than the demand for quality of children, higher skilled families choose to have less children. The

decline in children can be large enough for an increase in the quality of children. Moreover, we have to take into account that children come in integers, a decrease in the number of children allows a family to increase time spent per child.

The quantity-quality shift in my model is similar to Willis (1973). Willis argues that children are relatively more time intensive than consumption. Higher income might result in more consumption and parents have less time for rearing children. The remaining time for children will be especially used for quality of children.

4. Conclusion

In this paper effects of heterogeneity in women's skill level on birth decisions are examined. The paper emphasizes the importance of satisfaction in work in birth decisions. The model explains why higher skilled women tend to have less children and delay their childbearing more than lower skilled women. Motherhood decreases career perspectives and because women with a higher skill level have better career perspectives, they have an incentive to delay childbearing. The effects of a higher ability on birth decisions depends on the discounted value of additional income of a career and the elasticity of utility of raising children with respect to ability.

Women with a higher ability level have jobs with less disutility of labor, which makes spending time to raising children less attractive compared to lower skilled women. When the effect of ability on joy in working dominates the effect of the increase in full income, the number of children declines.

We might observe a quantity-quality shift as in Becker and Lewis (1973). Becker and Lewis emphasize the increase in income and differences in income elasticities of quality and quantity of children to explain a decline in fertility. In contrast to Becker and Lewis I stress the effect of ability on the demand for spending time with children, which leads to a decline in the time allocated for children. Changes in quantity and quality of children are made along the lines of Becker and Lewis, in a second stage.

The effect on birth decisions can be observed in a cross section of the population, but in time series as well. Increased human capital investment and changed labor market conditions have increased the rate of learning-by-doing over time. The increased schooling of women and the increased career perspectives over time adds to the explanation of the decline in fertility and the delay of childbearing over time. Future research should pay more attention to the effect of endogenous schooling and changes in risks of divorce on birth decisions.

Endnotes

1. The other basic motive to have children is a means of providing material benefits. In developed countries this motive is less important.
2. To focus on birth decisions, we abstract from uncertainty of fecundity and fertility of women over time to focus on the birth decision.
3. Skill level, ability, learning efficiency and education level are used interchangeably.
4. The model could be easily extended with a term in the utility function for the status and prestige of having children. The results with this extension are similar to the model's conclusions. However, when the status of children is more important, the probability increases that the family prefers to have more children, when skill level increases.
5. To keep the model simple, I abstract from the effect of number of children on career perspectives.
6. Note that we have abstracted from spacing of children and fecundity of women.

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Appendix

A woman optimizes the Lagrangian

$$\begin{aligned} \max_{T_*, \ddot{e}} \mathcal{L}(t) = & \frac{\tilde{a}_1}{\delta} \ln p_n n T_o + \int_0^{T_s} \left[\tilde{a}_2 \ln c e^{-\hat{a}t} - \ddot{e}(\hat{a} + c - r) \right. \\ & \left. + \int_{T_s}^{T_*} \left[\tilde{a}_2 \ln c e^{-\hat{a}t} - \ddot{e}(\hat{a} + c - wH(T_s)(1 + \hat{a}t) - ra) \right] dt \right. \\ & \left. - e^{-\hat{a}t} + (1 - \tilde{a}_1 - \tilde{a}_2)u(t) - \ddot{e}(\hat{a} + c - wH(T_s)(1 + \hat{a}T_*)(1 - p_n) \right. \end{aligned} \quad (8)$$

$$\begin{aligned} & \left. - ce^{-\hat{a}t} + (1 - \tilde{a}_1 - \tilde{a}_2)u(t) - \ddot{e}(\hat{a} + c - wH(T_s)(1 + \hat{a}T_*) \right. \\ & \left. + \int_{T_r}^T \left[\tilde{a}_2 \ln c e^{-\hat{a}t} + (1 - \tilde{a}_1 - \tilde{a}_2)u(t) - \ddot{e}(\hat{a} + c - ra) \right] dt \right. \end{aligned}$$

First order conditions:

$$\dot{\ddot{e}} = -r\ddot{e} \quad (9)$$

$$\tilde{a}_2 \frac{1}{c} e^{-\hat{a}t} - \ddot{e} = 0 \quad (10)$$

$$\begin{aligned} & -\tilde{a}_2)u(T_*) + \ddot{e}(0)wH(T_s)p_n n(1 + \hat{a}T_*) \left[e^{-rT_*} - e^{-r(T_*+T_o)} \right. \\ & \left. - rT_* - \hat{a}^{-r(T_*+T_o)} \right] (1 - p_n n) + \frac{\hat{a}}{r} \ddot{e}(0)wH(T_s) \left[e^{-r(T_*+T_o)} \right. \end{aligned} \quad (11)$$

$$\tilde{a}_1 = \acute{o} \frac{1 + \acute{a}T_*}{r} \ddot{e}(0) wH(T_S) p_n n \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] \quad (12)$$

With the budget constraint

$$\begin{aligned} & \frac{1}{r} wH(T_S) \left[(1 + \acute{a}T_S) e^{-rT_S} - (1 + \acute{a}T_*) e^{-rT_*} \right] + \frac{\acute{a}}{r} \left[e^{-rT_*} \right. \\ & \left. - p_n n \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] \right] + \frac{1 + \acute{a}T_*}{r} wH(T_S) \left[e^{-r(T_*+T_O)} \right. \end{aligned} \quad (13)$$

Combination of the f.o.c. to T_* and n gives

$$\begin{aligned} & -p_n n \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] + \acute{a} \left[e^{-r(T_*+T_O)} - e^{-rT_*} \right] \\ & n \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] (1 + \acute{a}T_*) \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - \right. \end{aligned} \quad (14)$$

Which leads to

$$\left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] \left[(1 + \acute{a}T_*) \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - \right. \right] = p_n n \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] \quad (15)$$

Combination of the budget constraint and the f.o.c. to c and n gives

$$\begin{aligned} & \frac{\tilde{a}_2}{\tilde{a}_1} \acute{o} (1 + \acute{a}T_*) \frac{1}{\acute{a}} \left[1 - e^{-\acute{a}T} \right] p_n n \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] = \\ & (1 + \acute{a}T_S) e^{-rT_S} - (1 + \acute{a}T_*) e^{-rT_*} + \frac{\acute{a}}{r} \left[e^{-rT_S} - e^{-rT_*} \right] \\ & (t)n(t) \left[e^{-rT_*} - e^{-r(T_*+T_O)} \right] + (1 + \acute{a}T_*) \left[e^{-r(T_*+T_O)} - \right. \end{aligned} \quad (16)$$

Substitution of (15) gives

$$\begin{aligned}
& \frac{(1 + \hat{a} T_*) \left(\frac{\tilde{a}_2}{\tilde{a}_1 \hat{a}} [1 - e^{-\hat{a} T}] \acute{o} + 1 \right) \hat{a} \left[e^{-r T_*} - e^{-r T_p} \right]}{(1 + \hat{a} T_*) \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right) + \hat{a}} = \\
& \hat{a} T_s e^{-r T_s} - (1 + \hat{a} T_*) e^{-r T_p} + \frac{\hat{a}}{r} \left[e^{-r T_s} - e^{-r T_*} \right] + \frac{r A}{w \hat{H}}
\end{aligned} \tag{17}$$

Which leads to

$$\begin{aligned}
& \left(\frac{\tilde{a}_2}{\tilde{a}_1 \hat{a}} [1 - e^{-\hat{a} T}] \acute{o} + 1 \right) \left[e^{-r T_*} - e^{-r T_p} \right] = \\
& \frac{2}{\acute{o} u(T_*) - r} \left[\frac{1 + \hat{a} T_s}{\hat{a}} e^{-r T_s} - \frac{1 + \hat{a} T_*}{\hat{a}} e^{-r T_p} + \frac{1}{r} \left[e^{-r T_s} \right. \right. \\
& \left. \left. + \frac{1 + \hat{a} T_s}{1 + \hat{a} T_*} e^{-r T_s} - e^{-r T_p} + \frac{\hat{a}}{r(1 + \hat{a} T_*)} \left[e^{-r T_s} - e^{-r T_*} \right] \right. \right. \\
& \left. \left. + \left[\frac{1}{\hat{a}} \left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right] + \frac{1}{1 + \hat{a} T_*} \right] \frac{r A(0)}{w H(T_s)} \right. \right.
\end{aligned} \tag{18}$$

Comparative dynamics

For reasons of tractability I assume that income of the spouse and the initial assets are 0, except when I examine the effect of an increase in these assets.

Effect of a shock in ability

Taking the total differential of (18) gives

$$\begin{aligned}
& \frac{\tilde{a}_2}{\tilde{a}_1 \hat{a}} \left[1 - e^{-\hat{a}T} \right] \left[e^{-rT_*} - e^{-rTp} \right] d\acute{o} - \\
& \frac{\tilde{a}_2}{\tilde{a}_1} u(T_*) \left[\frac{1 + \acute{a}T_s}{\acute{a}} e^{-rT_s} - \frac{1 + \acute{a}T_*}{\acute{a}} e^{-rTp} + \frac{1}{r} \left[e^{-rT_s} - e^{-rT_*} \right] \right. \\
& \quad \left. + \left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right] \frac{1}{\acute{a}^2} \left[e^{-rT_s} - e^{-rTp} \right] d\acute{a} \right. \\
& \quad \left. \frac{(T_* - T_s)}{(1 + \acute{a}T_*)^2} e^{-rT_s} d\acute{a} - \frac{1}{r(1 + \acute{a}T_*)^2} \left[e^{-rT_s} - e^{-rT_*} \right] d\acute{a} \right. \\
& \quad \left. r \left[\frac{\tilde{a}_2}{\tilde{a}_1 \hat{a}} \left[1 - e^{-\hat{a}T} \right] \acute{o} + 1 \right] e^{-rT_*} dT_* \right. \\
& \quad \left. \frac{\tilde{a}_2}{\tilde{a}_1} \acute{o} u \left[\frac{1 + \acute{a}T_s}{\acute{a}} e^{-rT_s} - \frac{1 + \acute{a}T_*}{\acute{a}} e^{-rTp} + \frac{1}{r} \left[e^{-rT_s} - e^{-rT_*} \right] \right. \right. \\
& \quad \left. \left. + \left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right] \left[e^{-rT_*} - e^{-rTp} \right] dT_* \right. \right. \\
& \quad \left. \left. - e^{-rT_s} dT_* - \frac{\acute{a}^2}{r(1 + \acute{a}T_*)^2} \left[e^{-rT_s} - e^{-rT_*} \right] dT_* + \frac{\acute{a}}{1 + \acute{a}} \right. \right. \\
& \quad \left. \left. \right. \right.
\end{aligned} \tag{19}$$

First I will proof that there is a high probability that LHS>0, with a positive shock in ability. Using (18) the LHS>0 when

$$\begin{aligned}
& s + \frac{\acute{a}}{r(1 + \acute{a}T_*)} \left[e^{-rT_s} - e^{-rT_*} \right] - \frac{r(1 + \acute{a}T_s)}{\acute{a}} e^{-rT_s} + \frac{r(1 + \acute{a}T_*)}{\acute{a}} \\
& \quad + \left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right] \left[e^{-rT_s} - e^{-rTp} \right] \\
& \quad + \frac{T_* - T_s}{(1 + \acute{a}T_*)^2} e^{-rT_s} - \frac{1}{r(1 + \acute{a}T_*)^2} \left[e^{-rT_s} - e^{-rT_*} \right] > 0
\end{aligned} \tag{20}$$

Whether this condition holds depends strongly on the elasticity of the disutility of labor with respect to ability and the marginal rate of substitution between number of children and the period of companionship of children and parents.

It can be shown that the sum of the last two terms is positive. At $T_* = T_s$ the sum of the two terms is 0. By taking the derivative to T_* , we know that for $T_* > T_s$

$$(T_* - T_s)e^{-rT_s} - \frac{1}{r} \left[e^{rT_s} - e^{-rT_*} \right] > 0 \quad (21)$$

Decomposing the first term of (20):

$$\begin{aligned} & \frac{\dot{a}(T_s - T_*)}{1 + \dot{a}T_*} \left[e^{-rT_s} - e^{-rT_p} \right] - \frac{r(1 + \dot{a}T_s)}{\dot{a}} \left[e^{-rT_s} - e^{-rT_p} \right] \\ & \frac{(T_s - T_*)}{1 + \dot{a}T_*} e^{-rT_p} + \frac{\dot{a}}{r(1 + \dot{a}T_*)} \left[e^{-rT_s} - e^{-rT_*} \right] + r(T_* - T_s)\epsilon \end{aligned} \quad (22)$$

The sum of the last three terms of (22) is positive. At $T_* = T_s$ the sum is positive and the derivative to T_* is positive as well. Sufficient condition for a positive LHS of (19), with an increase in the level of ability, is

$$\frac{\dot{a}}{\dot{o}} < \frac{\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \dot{o} u(T_*) - r}{\frac{\dot{a}^2(T_* - T_s)}{1 + \dot{a}T_*} + r(1 + \dot{a}T_s)} = \frac{-\frac{V_{T_*}}{V_n} \frac{1}{n} - r}{\frac{\dot{a}^2(T_* - T_s)}{1 + \dot{a}T_*} + r(1 + \dot{a}T_s)} \quad (23)$$

where the minimum of the RHS is at $T_* = T_o$. The MRS between number of children and the period of companionship of children and parents depend largely on the rate of wage growth (see (15)).

By using (18) it can be shown that when dT_* is positive, the $RHS > 0$.

$$\begin{aligned}
& \left. \left(u'(T_*) - r \right) \right] \left[\frac{1 + \acute{a} T_S}{\acute{a}} e^{-rT_S} - \frac{1 + \acute{a} T_*}{\acute{a}} e^{-rT_P} + \frac{1}{r} \left[e^{-rT_*} - e^{-rT_P} \right] \right] dT_* \\
& + \frac{1 + \acute{a} T_S}{1 + \acute{a} T_*} \left[r - \frac{\acute{a}}{1 + \acute{a} T} \right] \left[e^{-rT_*} - e^{-rT_P} \right] dT_* \\
& + \left[\frac{\acute{a}}{1 + \acute{a} T_*} - \frac{\acute{a}^2}{r(1 + \acute{a} T_*)^2} \right] \left[e^{-rT_S} - e^{-rT_*} \right] dT_* \\
& + \frac{\acute{a}}{1 + \acute{a} T_*} e^{-rT_*} dT_* + r \frac{\tilde{a}_2}{\tilde{a}_1} \left[1 - e^{-rT} \right] \acute{o} e^{-rT_P} dT_*
\end{aligned} \tag{24}$$

The first term of (24) is positive when

$$r \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u'(T_*) - r \right) + \frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u'(T_*) > 0 \tag{25}$$

The condition shows that when the marginal utility of being alive when your children are living relatively to the utility of living together is low, the condition will be satisfied. With a concave utility function and when the period parents and children are alive is already long compared to life-expectancy, this condition is met normally. This can be illustrated with a common utility function and reasonable parameters

$$u(T_*) = \ln(T - T_*) \tag{26}$$

Taking the maximum of $T_* = T - T_o$. The condition will be

$$r \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} \ln(T - T_*) - r \right) - \frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} \frac{1}{T - T_*} > 0 \tag{27}$$

using (14) we see that the condition is satisfied already for very low values for r and \acute{a} .

To prove that the sum of the third, fourth and fifth term of (24) is positive or equal to 0 we have to prove that

$$\frac{1 + \acute{a} T_S + \acute{a}}{r(1 + \acute{a} T_*)^2} \left[r(1 + \acute{a} T_*) - \acute{a} \right] e^{-rT_S} + \frac{\acute{a}^2}{r(1 + \acute{a} T_*)^2} e^{-rT_*} \tag{28}$$

At $T_s=T_*$, the sum is >0 . Taking the derivative of the numerator to T_* proves that when $T_s < T_*$ the sum is >0 , because

$$\acute{a} r^2 (1 + \acute{a} T_s) + \acute{a}^2 r \left[e^{-rT_s} - e^{-rT_*} \right] > 0 \quad (29)$$

Effect of a shock in ability on n

We can write (16) as

$$p_n \left[1 - e^{-rT_o} \right] = \frac{\left[1 - e^{-r(T_* - T_p)} \right]}{\frac{1 + \acute{a} T_*}{\acute{a}} \left(\frac{(1 - \tilde{a}_1 - \tilde{a}_2)}{\tilde{a}_1} \acute{o} u(T_*) - r \right) + 1} \quad (30)$$

Taking the total differential gives

$$\begin{aligned} p_n \left[1 - e^{-rT_o} \right] dn = & \\ \frac{\left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} - r \right) d\acute{a} - \frac{1 + \acute{a} T_*}{\acute{a}} \frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} u(T_*) d\acute{o} \left[1 - e^{-r(T_* - T_p)} \right]}{\left[\frac{1 + \acute{a} T_*}{\acute{a}} \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right) + 1 \right]^2} & \\ - \frac{r e^{-r(T_* - T_p)} dT_*}{\frac{1 + \acute{a} T_*}{\acute{a}} \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right) + 1} & \\ \frac{\left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} \left(u(T_*) + \frac{1 + \acute{a} T_*}{\acute{a}} u' \right) - r \right) \left[1 - e^{-r(T_* - T_p)} \right]}{\left[\frac{1 + \acute{a} T_*}{\acute{a}} \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} u(T_*) - r \right) + 1 \right]^2} d & \end{aligned} \quad (31)$$

When $dT_*/d\acute{a} > 0$ a sufficient condition for a $dn/d\acute{a} < 0$ is

$$\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \phi u(T_*) \left[d\dot{a} - (1 + \dot{a}T_*) \frac{\dot{a}}{\dot{\phi}} d\dot{\phi} \right] - r d\dot{a} < 0 \quad (32)$$

Or

$$\frac{d\dot{\phi}}{d\dot{a}} \frac{\dot{a}}{\dot{\phi}} > \frac{\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \phi u(T_*) - r}{\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \phi u(T_*) (1 + \dot{a}T_*)} = \frac{-\frac{V_{T_*}}{V_n} \frac{1}{n} - r}{-\frac{V_{T_*}}{V_n} \frac{1}{n} (1 + \dot{a}T_*)} \quad (33)$$

with the maximum of the RHS at $T_* = T_s$.

When $dT_*/d\dot{a} < 0$, $dn/d\dot{a}$ is negative normally, because the first term of (31) is normally much bigger than the sum of the last two terms. For $dn/d\dot{a} > 0$, $dT_*/d\dot{a}$ must be unrealistically high (in the order of $1/\dot{a}^2$).

Shock in p_n

(18) shows that $dT_*/dp_n = 0$; from (30) we get

$$p_n dn = -n dp_n \quad (34)$$

Shock in T_s

Using (18), comparative dynamics with a shock in T_s gives

$$\begin{aligned}
 & \left(\frac{(1 - \tilde{a}_1 - \tilde{a}_2)}{\tilde{a}_1} \acute{o}u(T_*) - r \right) \left[\frac{r(1 + \acute{a}T_s)}{\acute{a}} e^{-rT_s} \right] dT_s \\
 & \frac{r(1 + \acute{a}T_s)}{1 + \acute{a}T_*} e^{-rT_s} dT_s = r \left[\frac{\tilde{a}_2}{\tilde{a}_1} [1 - e^{-rT}] \acute{o} + 1 \right] e^{-rT_*} dT \\
 & \frac{\tilde{a}_2 \acute{o}u'}{\acute{a}} \left[\frac{1 + \acute{a}T_s}{\acute{a}} e^{-rT_s} - \frac{1 + \acute{a}T_*}{\acute{a}} e^{-rT_p} + \frac{1}{r} \left[e^{-rT_s} - e^{-rT_*} \right] \right. \\
 & \quad \left. + \left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o}u(T_*) - r \right] \left[e^{-rT_*} - e^{-rT_p} \right] \right] dT_* \\
 & \quad - e^{-rT_s} dT_* - \frac{\acute{a}^2}{r(1 + \acute{a}T_*)^2} \left[e^{-rT_s} - e^{-rT_*} \right] dT_* + \frac{\acute{a}}{1 + \acute{a}}
 \end{aligned} \tag{35}$$

and hence $dT_*/dT_s > 0$ when condition (4) is satisfied. From (30) we get

$$\begin{aligned}
 - e^{-rT_o} \Big] dn &= - \frac{r e^{r(T_* - T_p)}}{\frac{1 + \acute{a}T_*}{\acute{a}} \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o}u(T_*) - r \right) + 1} \frac{\partial \acute{o}}{\partial \acute{a}} \\
 & \frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o} \left(u(T_*) + \frac{1 + \acute{a}T_*}{\acute{a}} u' \right) - r \Big] \left[1 - e^{r(T_* - T_p)} \right] \frac{\partial T}{\partial T} \\
 & \frac{\left[\frac{1 + \acute{a}T_*}{\acute{a}} \left(\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \acute{o}u(T_*) - r \right) + 1 \right]^2}{\partial T}
 \end{aligned} \tag{36}$$

$dn/dT_s < 0$ when $dT_*/dT_s > 0$.

Shock in assets

Using (18) we observe that the effect of a shock in initial assets or income of spouse is

$$\frac{dA(0)}{dn} = \hat{a} \left[e^{-rT_s} - e^{-rT_p} \right] + \frac{\left(\frac{\tilde{a}_2}{\tilde{a}_1 \hat{a}} [1 - e^{-\hat{a}T}] \hat{o} + 1 \right)}{\left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \hat{o} u(T_*) - r + \frac{1}{1 + \hat{a}T_*} \right]} \quad (37)$$

$$\left(\hat{o} u(T_*) - r + \frac{\hat{a}}{1 + \hat{a}T_*} \right) + \left[\frac{1 - \tilde{a}_1 - \tilde{a}_2}{\tilde{a}_1} \hat{o} u'(T_*) - \frac{\hat{a}^2}{(1 + \hat{a}T_*)^2} \right] \left[\epsilon \right]$$

Hence, when condition (4) is satisfied, $dA(0)/dT_* < 0$ and $dA(0)/dn > 0$.