

Empirical Inference with Equilibrium Search Models of the Labor Market – A Survey

Gerard J. van den Berg *

August 14, 1998

Abstract

This paper examines the use of equilibrium search models in the empirical analysis of labor markets. We survey the literature on structural estimation of these models with micro data on wages and durations, and we discuss the advantages of the equilibrium approach, for policy analysis and for understanding a number of stylized facts that are hard to explain otherwise. During the past years, substantial progress has been made in terms of the explanatory power of these models. We finish with a critical examination of the extent to which the approach can be fruitfully applied to (matched worker-)firm data.

*Department of Economics, Free University Amsterdam, De Boelelaan 1105, 1081 HV Amsterdam, The Netherlands, CEPR, and Tinbergen Institute. Email: gberg@econ.vu.nl.

Keywords: wages, durations, productivity, job search, unemployment, frictions, monopsony.
JEL codes: J6 J3 D83 C51.

Thanks to Audra Bowlus and Bas van der Klaauw for their comments.

1 Introduction

During the past 10 years, the structural empirical analysis of equilibrium search models of the labor market has become rather popular (see references below). In equilibrium search models, supply, demand and wage determination in the labor market are jointly modeled. The supply side is represented by a partial job search model with sequential search. Wage determination is affected by the presence of informational frictions or search frictions.

In this paper we survey the literature on the structural empirical analysis of these models with micro data. For a good understanding of the relevance of this literature, it is useful to start with an examination of the motivations given in the earlier contributions (Eckstein and Wolpin (1990), Kiefer and Neumann (1993), Van den Berg and Ridder (1993a, 1993b)). An important motivation concerns the limitations of the analysis of the so-called partial job search models. The latter models deal with job search behavior of unemployed or employed workers. The worker's optimal strategy is derived for a given distribution of wage offers and for given job offer arrival rates. The literature on partial job search models has made considerable progress in explaining supply-side behavior, unemployment durations, and job durations of workers looking for a (new) job.¹ Structural empirical inference allows one to estimate the underlying parameters of the search process, to formally test the adequacy of the theory, and to estimate the effects of policy changes.

In this partial approach, the distribution of wage offers is assumed to be fixed and is treated as exogenous. The most serious drawback of this is that the model is not able to deal with changes in the employers' behavior in response to policy changes or other structural changes. Most partial studies focus on the effects of changing the unemployment benefits level on the reservation wage and the duration of unemployment, and some studies examine changes in the job offer arrival rate as well. If in reality employers respond to such changes in a way such that the wage offer distribution changes as well, then the results based on partial models may be invalid.

There are of course good reasons to suspect that, in setting wages, firms do take the strategy of workers into account. The optimal strategy of the workers has the reservation wage property. If an employer knows this, then his optimal wage

¹Some examples of this approach are Yoon (1981), Flinn and Heckman (1982), Lancaster and Chesher (1983), Narendranathan and Nickell (1985), Blau and Robins (1986), Wolpin (1987), Van den Berg (1990a, 1990b, 1992), Engberg (1991), Blau (1991), Bonnal and Fougère (1991); for surveys see Devine and Kiefer (1991), Layard, Nickell and Jackman (1991) and Wolpin (1995).

offer will equal the reservation wage of some (group of) worker(s). Intuitively, this is because otherwise a firm can reduce its wage offer without loss of potential workers. As a result, employers can be expected to respond to changes in the behavior of job seekers that are induced by policy changes. The same applies to policies directly aimed at changing the wage distribution (for example, by changing the level of a mandatory minimum wage). An increase in the minimum wage affects the wage offers of firms that made offers below the old level, and this in turn affects the wage offers of the other firms.

Another major problem with structural inference of partial models has been that the wage offer distribution can not be identified from the type of data that are usually collected. These data are from labor force surveys and consist of individual labor market histories (i.e. durations spent in labor market states like unemployment and jobs, and the corresponding income flows). If all job seekers have a common reservation wage and a common wage offer distribution, then data on accepted wages amount to a sample from a truncated distribution where the reservation wage is the lower boundary of the support. One cannot identify (“recover”) the complete wage offer distribution from this truncated distribution (Flinn and Heckman (1982)). In particular, one cannot identify the probability mass below the reservation wage. This is a serious problem, because that probability mass is the probability that a job seeker rejects a wage offer, and therefore is a measure of the choosiness of the job seeker. One of the questions that empirical studies try to answer is whether length of a search spell is determined by the availability of job offers or by the rejection of offers. A common procedure in applications is to restrict the wage offer distribution to a parametric family of densities that is recoverable, so the untruncated density can be uniquely recovered from a truncated density. Usually, a family is chosen that is known to be able to give a good representation of wage distributions (like the lognormal family; see Van den Berg (1994) for a survey). Of course, if data on rejected offers or external information on the wage offer distribution are available, then this distribution may be identified, and the question above may be answered without the need to rely on untestable distributional assumptions. However, such data are rare. Equilibrium search models enable a different approach. Because the wage offer distribution is endogenous in those models, they provide guidance on the choice of the wage offer distribution.

Another problem with partial models concerns the assumption that wage offers are dispersed. A non-trivial model of labor market search requires the existence of a disperse wage offer distribution. However, as Diamond (1971) observed, the simple sequential search model, in which identical unemployed workers who

face identical firms accept/reject wage offers one at a time, is not consistent with a disperse wage equilibrium. The argument is simple: the optimal strategy with sequential search is a reservation wage strategy; for the employers it is suboptimal to offer a wage that is higher than the common reservation wage (a higher wage means that profits per worker are lower while the steady state number of workers is not higher), and hence, the equilibrium wage offer distribution is concentrated at this reservation wage; finally, this common reservation wage must be equal to the lowest wage that is acceptable to the unemployed, *i.e.* their value of leisure. Obviously, this equilibrium outcome is rather uninteresting. This challenges any empirical study with search models to address the issue of the origin of wage dispersion.

Finally, a number of important issues can not be analyzed with partial job search models. This of course includes all research issues related to wage determination, firm behavior, the interaction between worker and firm behavior, and the effects of policies that directly affect wages.

In response to all this, a literature on inference with equilibrium search models has emerged. In equilibrium search models, the wage offer distribution is endogenous. It results from optimal wage setting by firms that take account of the behavior by job seekers and other firms. As a result, it is affected by all structural parameters, including policy parameters.

It should be stressed that there have been additional motivations for inference with equilibrium search models. In particular, the equilibrium search models that have been derived in the theoretical literature are able to explain a large number of stylized facts on labor market behavior (*i.e.* properties of unemployment and job duration distributions and wage distributions, the relations between these variables, policy effects, etc.). For expositional reasons it is better to examine these after a discussion of the main models, so we defer this to the end of Section 2.

This survey focuses on the literature in which equilibrium search models are estimated with micro data, using formal econometric techniques. We thus abstract from studies in which models are estimated or calibrated with time-series macro data. In addition, we focus on models in which firms set wages, as opposed to models in which workers and firms bargain over the wage. This is primarily because most empirical inference has been with the former type of models. As we will see, the assumption that firms set wages is a natural complement to the assumption that firms are units that pay the same wage to each employee. In Section 6 we pay some attention to studies using wage bargaining models. In those models, a firm is effectively equal to a single job or vacancy. Finally, in this

survey we do not discuss equilibrium search models that have not been used in the empirical literature (see e.g. Davidson (1990), Burdett (1990) and Mortensen and Pissarides (1998) for surveys of the theoretical literature).

The outline of the paper is as follows. In Section 2 we briefly examine some explanations for wage dispersion, with the corresponding models. We then list the stylized facts that can be explained by these models. Sections 3-6 are the main sections of the paper, as they contain a discussion of the specifications of empirical equilibrium search models and their estimation. We examine identification with labor force survey data, and a number of different empirical approaches that have been used to obtain a good fit to the wage and duration data. It should be noted from the outset that the empirical models discussed in these sections are more realistic and more complex than the models of Section 2. In Section 7 we discuss a number of important topics for further research, as inspired by the availability of firm data. Section 8 concludes. Throughout the survey, the discussion of the literature is organized along thematic rather than chronological lines. At times we explicitly mention the pioneering role of certain contributions.

2 Equilibrium search models and stylized facts

2.1 Wage dispersion

For an equilibrium search model to have sufficiently rich empirical contents, it is necessary that it is able to generate wage dispersion for searching individuals. As we have seen, if all job searchers have a common reservation wage, then the equilibrium wage offer distribution is degenerate. Moreover, more in general, equilibrium wage offers are equal to the reservation wage of some (group of) worker(s). Thus, a model in which potential workers at a firm differ in their reservation wage values (i.e., the firm faces an upward sloping labor supply curve) may generate wage dispersion. Basically, two approaches can be distinguished in the literature, depending on the source of this reservation wage heterogeneity. In both cases, the Diamond (1971) model serves as the point of departure.

In the first approach (MacMinn (1980), Albrecht and Axell (1984)), workers are heterogeneous by nature. In particular, they differ in their opportunity cost of employment. The latter may be the result of heterogeneity in their value of leisure, their unemployment benefits, their search intensity, etc.. This implies heterogeneity of the unemployed workers' reservation wages. Allowing for such heterogeneity may generate wage dispersion. For a given firm, there is a trade-off between the profit per worker and the number of workers who are willing

to work at the firm. In equilibrium, a firm may be indifferent between offering a high or a low wage. It can be shown that, as expected, the support of the equilibrium wage offer distribution is a subset of (or coincides with) the set of unemployed workers' reservation wages. In fact, whether wages are dispersed or not seems to be very sensitive to small changes in the model assumptions (see e.g. Burdett (1990) and Van den Berg (1998)). This is dealt with to some extent by introducing heterogeneity of firms' productivities (see MacMinn (1980) and Albrecht and Axell (1984) for theoretical analyses). Then higher-productivity firms tend to offer higher wages and are therefore able to attract more workers.

In the second approach (Mortensen (1990), Burdett and Mortensen (1998)), *ex ante* identical workers are allowed to search for another job while working. In equilibrium, working individuals only change jobs if the wage offer exceeds a reservation wage value that equals (or is increasing in) their current wage. A firm that sets a high wage is thus able to attract workers from firms offering lower wages. So, if individuals work at different wages then, from the point of view of an employer, the labor supply curve is upward sloping, and there is again a trade-off between the wage and the labor force of the firm, which in turn generates equilibrium wage dispersion. In this type of model, the degree of wage dispersion is directly related to the degree of search frictions in the labor market. Wage dispersion is absent only in the polar cases of no on-the-job search (i.e. maximum friction) and instantaneous arrival of job offers (no friction). In the former case the Diamond (1971) solution applies, whereas in the latter case the wage is competitive, i.e. equal to the marginal value product. In all other case, wages are dispersed between these two extremes. Note that in both approaches discussed above, parameter changes that affect the reservation wages of job searchers also affect the wage offer distribution they face.

For the sequel of this survey it is useful to examine the model specifications in somewhat more detail. For expositional reasons we start with a description of the basic Burdett and Mortensen (1998) (BM) model. The basic model considers a labor market consisting of fixed continuums of homogeneous workers and firms. The supply-side is fully equivalent to a standard partial job search model with on-the-job search (see Mortensen (1986)). Workers obtain wage offers, which are random drawings from the wage offer distribution $F(w)$, at an exogenous rate λ_0 when unemployed and λ_1 when employed. Whenever an offer arrives, the decision has to be made whether to accept it or to reject it and search further for a better offer. Layoffs accrue at the constant exogenous rate δ . The opportunity cost of employment is denoted by b and is assumed to be constant across individuals and to be inclusive of unemployment benefits. The optimal acceptance strategy

for the unemployed is characterized by a reservation wage ϕ which can be solved using a Bellman equation,

$$\phi = \phi(b, \lambda_0, \lambda_1, F, \delta) \tag{1}$$

Note that F is endogenous here. Employed workers accept any wage offer that exceeds their current wage. In sum, workers climb the job ladder to obtain higher wages, but this effort may be frustrated by a temporary spell of frictional unemployment.

Now consider optimal wage setting by firms. We assume that the marginal value product (or “productivity”) p does not depend on the number of employees, *i.e.* we assume that the production function is linear in employment. Let w_{min} denote the mandatory minimum wage, and let $p > \max\{b, w_{min}\}$, so that profitable production is possible. Let $L(w)$ denote the steady-state labor supply to a firm setting a wage w , given the behavior of the workers (as summarized in their reservation wages), and given the behavior of all the other firms (as summarized in the distribution F of their wage offers). The steady-state profit flow π of this firm equals

$$\pi = (p - w)L(w) \tag{2}$$

It is intuitively clear that $L(w)$ increases in w for a given F . If w is high then many workers would prefer such a job over their current job, whereas the workers who are already employed at w would find it difficult to find a job with an even higher wage. In fact, what matters is the wage level in comparison to what other firms offer, so $L(w)$ depends on $F(w)$.

The wage offer of the firm is chosen such that it maximizes the profit flow π , given F and given the behavior of workers. If all firms offer the same wage then that induces each of these firms to offer a slightly higher wage, because then its steady-state labor force will be substantially larger at the cost of only a second-order decrease in the profit per worker. Thus, in equilibrium, wages are dispersed. Specifically², they are dispersed between $\underline{w} \equiv \max\{\phi, w_{min}\}$ and a value \bar{w} which is smaller than p . The whole shape of F depends on all structural parameters $\lambda_0, \lambda_1, \delta, p, b$, and w_{min} . Note that $\bar{w} < p$ implies that firms have monopsony power. Also note that unemployed workers accept any job.

It is important to distinguish between the wage offer distribution F and the distribution G of wages among currently employed workers. The latter dominates

²Since all firms are equal, the equilibrium steady-state profit flow π must be equal for a range of different wages. The lowest wage in the market \underline{w} is $\max\{\phi, w_{min}\}$, and the condition $(p - w)L(w) = (p - \underline{w})L(\underline{w})$ can be solved for F (see the literature for details).

the former due to the flow of employees to higher paying jobs. By assuming that the flows into and out of jobs and unemployment are equal, one can express G in terms of F , λ_1 , and δ .

Now consider the model by Albrecht and Axell (1984) (AA). In terms of the above framework, we obtain the basic version of this model by imposing that $\lambda_1 = 0$ and that individuals are heterogeneous with respect to their value of b . For a given b , equation (1) still holds, with $\lambda_1 = 0$. The distributions F and G are again different from each other, but now this is not because of job-to-job transitions but because unemployed individuals with high b only flow to jobs with high wages. For the same reason, $L(w)$ still increases in w . The expression for $L(w)$ in terms of F is different from above because it depends on the distribution of b . However, the profit flow of a firm with productivity p still satisfies (2). In equilibrium, wages can be dispersed, but wages only attain values that are equal to the reservation wage of a (group of) worker(s). Qualitatively, this description does not change if it is allowed that firms are heterogeneous in p .

2.2 Stylized facts

The equilibrium search models that have been derived in the theoretical literature are able to explain a number of stylized facts of the labor market that are hard to explain by any other single model. In this subsection we start with an examination of the stylized facts that are explained by the basic BM model. Here we rely on Kiefer and Neumann (1993) and Ridder and Van den Berg (1997), who discuss the empirical performance of the basic BM model. After that we return to the AA model, and we rely on Eckstein and Wolpin (1990), who estimate this model.

The first set of stylized facts concerns job-to-job transitions by workers. Wages are dispersed, and all workers face a non-degenerate wage offer distribution. As a result, job-to-job transitions do occur, at the rate $\lambda_1(1 - F(w))$ for a worker earning a wage w . These transitions are important in their own right, for understanding search behavior as well as wage growth. Job-to-job transitions represent a substantial fraction of all labor market transitions, and they provide an option for workers to increase their income in the long run. The possibility of on-the-job search changes the optimal search strategy of unemployed job seekers. Moreover, there is evidence that job-to-job transitions are an important source of wage growth (see e.g. Topel and Ward (1992)), and this points at the importance of wage setting for maintaining the workforce of a firm.

Now let us turn to wages. In general, wages exceed the reservation wage of

the unemployed (contrary to the AA model). The rate at which job spells end decreases with the wage. This is consistent with abundant empirical evidence (Lindeboom and Theeuwes (1991)). In equilibrium there is a positive association between the size of the firm and the wage paid by the firm. Hence, the model is consistent with the employer-size wage effect as well. Now suppose that the labor markets for different occupations are separate, and that each of them is described by a BM model with its own parameter values. This would predict that a certain fraction of all wage dispersion cannot be explained by any characteristic of the worker or the firm (this dispersion is due the presence of search frictions). This prediction is in accordance to the facts that there are persistent wage differences across firms and industries, and that it is almost impossible to explain more than a moderate part of total wage variation in a regression of wages on many worker-related characteristics.³

Now let us examine unemployment and policy effects on it. In equilibrium, all job offers are acceptable to the unemployed, and the re-employment hazard is equal to the offer arrival rate λ_0 . This is consistent with the empirical evidence surveyed in *e.g.* Devine and Kiefer (1991) and Van den Berg (1990b). Although job search models were originally introduced with the idea that job rejection is a potential explanation for unemployment, most empirical studies based on partial search models with European data conclude that rejection of job offers is rare. The basic BM model predicts that the level of unemployment benefits does not affect unemployment, and this is also broadly in agreement with structural and reduced-form empirical evidence for Europe⁴. When b increases and $\phi > w_{min}$, then, as in partial job search models, the unemployed individual's reservation wage ϕ increases. However, the employers modify their wage offers in response to this, and the increase in b merely leads to a redistribution of the rents of the match. For a similar reason, an increase in the minimum wage may lead to a redistribution. However, this does not affect unemployment as long as the minimum wage is smaller than the productivity level p . This prediction is in

³It should be noted that the basic model has been extended to account for facts such as transitions into and out of nonparticipation, transitions to jobs with lower wages, or a separation rate that depends on the current wage (see Van den Berg and Ridder (1993a) and Ridder and Van den Berg (1997) for theoretical analyses and Bowlus (1997) for a structural empirical analysis with nonparticipation). Mortensen and Vishwanath (1994) extend the model by allowing workers to use contacts with employed workers as a second job search channel; they predict that the wages of jobs found along this channel are higher than the wages of jobs found otherwise. Koning, Van den Berg and Ridder (1997) test and reject this hypothesis.

⁴See *e.g.* Van den Berg (1990b) and Jackman, Layard and Nickell (1991). The empirical evidence is however subject to some criticism; see Atkinson and Micklewright (1991).

agreement with recent evidence on minimum wage effects in Card and Krueger (1995).

Machin and Manning (1996) provide a very different type of stylized facts. They examine detailed information on a labor market for a very specific kind of labor in a very specific location. This information is in agreement to the stylized facts above. Moreover, it provides some direct evidence for the existence of monopsony power of the employers, and for unexplainable wage dispersion across firms.

Despite all this, there are a number of important stylized facts that cannot be explained by the basic model. First of all, individual exit rates out of states like unemployment often display duration dependence, even if one corrects for observed explanatory variables (see e.g. Devine and Kiefer (1991)). The model, however, predicts that all exit rates are constant. Secondly, and perhaps even more importantly, the actual solutions for the equilibrium wage (offer) distributions F and G have increasing densities. This is at odds with the data. In fact, it turns out that for reasonable degrees of search frictions, these densities are highly skewed towards the competitive wage. This is not *per se* a problem. The competitive model without search frictions predicts a unique equilibrium wage equal to the marginal productivity. It only means that the shape of the wage distributions is not explained by the model and additional heterogeneity among workers or firms is necessary to account for it.

Now let us turn to the AA model. This model does not explain job-to-job transitions. However, like the basic BM model, the model does explain that wages are dispersed and that larger firms pay higher wages. Also, the minimum wage does not necessarily have a negative effect on employment. Contrary to the basic BM model, the observable exit rate out of unemployment displays negative duration dependence. The latter is an important stylized fact which accounts for the substantial fraction of the unemployed in Europe with very long durations. In the model, the negative duration dependence is due to the unobserved heterogeneity in the unemployed workers' opportunity cost of employment. The model thus also predicts that at least some unemployed workers reject at least some of their job offers some of the time, and that unemployment benefits affect the unemployment duration distribution. The model has difficulty explaining the shape of the wage (offer) distribution. This is due to the fact that each point of support of the wage offer distribution necessarily equals the reservation wage of an unemployed worker type.

In the next sections we show that the model versions that have been estimated are able to deal with the unaccounted stylized facts, by way of incorporating

(additional) heterogeneity.

3 Structural estimation of the homogeneous Burdett-Mortensen model

For expositional reasons it is useful to start with an examination of the estimation of the basic BM model (Van den Berg and Ridder (1993b, 1998), Kiefer and Neumann (1993), and Bunzel et al. (1997) estimate this model before proceeding towards more complicated models). This model has five unknown parameters: λ_0 , λ_1 , δ , p , and b (or, equivalently, ϕ). The data that are typically used for the estimation are from longitudinal labor-force panel surveys. By following a sample of individuals during some period of time, such surveys provide individual labor market histories. These specify the dates at which transitions between labor market positions occur as well as other information pertaining to these positions, notably the income level (unemployment benefits in unemployment and wages in jobs).

In this context, sufficient data for structural estimation of the model are, for each respondent, an unemployment spell or a job spell with a wage, and in case of a job spell the type of state occupied after the current spell, i.e. either another job or unemployment. The corresponding duration, wage and exit destination variables are all endogenous according to the model, and their distributions depend on the model parameters. First, the duration of unemployment has an exponential distribution with parameter λ_0 (i.e., the density of this duration t equals $\lambda_0 \exp(-\lambda_0 t)$). Conditional on the wage w , the duration of a job has an exponential distribution with parameter $\delta + \lambda_1(1 - F(w))$. Exit from this job into unemployment occurs with probability $\delta/(\delta + \lambda_1(1 - F(w)))$, and exit into another job with one minus that probability. If the job spells concern the first job after a spell of unemployment, then the corresponding wages are random drawings from the wage offer distribution F . If the job spells concern jobs occupied by a random sample of all workers who are employed at a certain point in time, then the wages are random drawings from G , which can be expressed in terms of F , λ_1 and δ . Thus, G as well as F are completely specified as functions of the model parameters.

Mortensen (1990), Kiefer and Neumann (1993) and Ridder and Van den Berg (1993a) show that these data identify the parameters. Identification is most easily established by examining an estimation method that links the observables as directly as possible to the model primitives. Suppose one would estimate the

model in two steps. In the first step, F is treated as an unknown distribution to be estimated nonparametrically (e.g. by way of kernel estimation) along with the arrival rate parameters. Further, ϕ is estimated as the lowest observed wage (unless the latter equals w_{min} : then ϕ and b are unidentified). Clearly, the parameter λ_0 is identified from the unemployment durations, and δ and λ_1 are identified from the job durations ending in transitions to unemployment and to another job, respectively. In the second step, p is then identified by equating e.g. the mean of the theoretical F to the mean of the estimated F . In fact, the model is heavily overidentified with these data. For example, F is identified both from the wage sample and from the way in which the elasticity of the job-to-job transition rate with respect to the wage varies with the wage. Moreover, p is also identified from other moments of F .

In practice, λ_0 , λ_1 , δ , and p are estimated simultaneously with Maximum Likelihood (ML)-type procedures. There are two non-standard estimation issues here, and these also play a role in the estimation of partial job search models as well as in the estimation of any extension of the basic BM model. First of all, the support of F and G depends on the unknown parameters. This means that ML estimators have non-standard properties. In fact, the estimators of the bounds of the support converge at a faster rate than is usual for ML estimators. Because of this, one can treat the corresponding estimates (which are basically relations between the observed bounds and the corresponding model expressions) as known when estimating the remaining parameters with ML. The latter estimates then have the usual asymptotic distribution. Kiefer and Neumann (1993) and Bunzel et al. (1997) use this approach.

The resulting estimates are obviously sensitive to outliers (i.e. measurement errors) in the wage data. This has also been a problem in the structural estimation of partial search models, because there the reservation wage of the unemployed acts as a lower bound on the observed wages. There is evidence that wage measurement errors can be quite large (see e.g. Bound et al. (1994) and Hartog and Van Ophem (1991)). One may deal with this to a certain extent by trimming the wage data, e.g. by discarding the respondents with the highest and lowest, say 3%, of the wages. An alternative approach is to allow for wage measurement errors in the empirical model specification. This has been applied by Van den Berg and Ridder (1998) and, again, by Bunzel et al. (1997) (it has previously been applied by Wolpin (1987) in the estimation of a partial model). Because the measurement error in the wages makes the support of the distributions of observed wages independent of the parameters, the ML estimation of the model

is standard.⁵

The second noteworthy issue concerns the fact that some job-to-job transitions result in a job with a lower wage. Of course, one can ignore this by not using any data on accepted wages after a job-to-job transition, when estimating the model (Kiefer and Neumann (1993), Bunzel et al. (1997)). Another approach is, again, to allow for measurement errors in the wage data.⁶ This approach is adopted by Van den Berg and Ridder (1998) (see Flinn (1996) for an application in the estimation of a partial on-the-job search model). It should be noted that observations on wages in n consecutive jobs result in a likelihood contribution that contains an n -dimensional integral that has to be evaluated numerically, which is computationally demanding.

We end this section by noting that the estimation of even the simplest equilibrium search model entails the joint use of duration and wage data. This is not surprising in the light of the fact that this model explains wage variation in terms of the degree of frictions on the labor market. Attempts to estimate the basic model from just wage data or just duration data have not been very successful (Van den Berg and Ridder (1993b))⁷. Wage data are not sufficiently informative on the degree of frictions, and duration data are not sufficiently informative on productivity.

4 Structural estimation of models with heterogeneity between markets

Recall from Subsection 2.2 that the basic BM model cannot explain the shape of the observed wage and wage offer distributions G and F . It is intuitively plausible that heterogeneity in p can improve on this. One may distinguish between two different empirical approaches here, characterized by whether heterogeneity is “between markets” or “within the market”. In the first case, the labor market is segmented and consists of a large number of separate different submarkets within which workers and employers are homogeneous. In the second case, there is one

⁵A third estimation strategy is based on moments; see Van den Berg and Ridder (1993b) for an application.

⁶Obviously, the most ambitious approach would be to extend the theoretical model to explain wage decreases, e.g. by allowing for non-wage job characteristics. In certain cases, such a model is observationally equivalent to a model with wage measurement errors (Van den Berg and Ridder (1998)).

⁷This is also true for more general models; in fact, they may not be identified from either data.

labor market within which heterogeneous firms interact. In general, the equilibrium solution for a market with heterogeneous agents differs from the solution for a market with identical agents. We therefore start with the between-market approach (see Van den Berg and Ridder (1998))⁸, and we return to the other approach in the next section.

As a rule, a set of randomly sampled workers does not consist of workers who all compete on the same labor market. This is particularly true if one draws from the population of all workers, like in labor force surveys. To proceed, assume that an individual is attached to one submarket only, that each of these submarkets or segments is a separate labor market on its own, and that workers and employers within any particular segment are homogeneous. All structural parameters ($\lambda_0, \lambda_1, \delta, p$, etc.) may vary across segments.

Segments can be defined by observed characteristics x like occupation, education and age (e.g. each combination of occupation, education and age category defines a segment) as well as by unobserved characteristics. In the first case, the data generally do not contain sufficiently large subsamples per segment to enable separate estimation. In the second case, the data do not even enable an unequivocal determination of the segment to which a respondent belongs. An obvious way to proceed is to assume that the structural parameters are additive functions of the elements of x (e.g. $p = x'\beta$; this rules out interactions), and, in case of unobserved characteristics, assume a distribution for the structural parameters across the segments (like a log-normal or discrete distribution for p).⁹ In the latter case, the data provide observations on mixtures of equilibrium outcomes of homogeneous markets.

Allowing for between-market heterogeneity in p enriches the model by adding the possibility of structural unemployment. Suppose that the minimum wage w_{min} increases to a level that exceeds the productivity level in a certain segment. Then the firms in that segment will close, and all workers become permanently (structurally) unemployed. The same occurs if $b > p$ (but this turns out to be empirically less relevant). As an example, let p be distributed across workers in different segments according to the distribution function H , and assume that λ_0, δ, b and w_{min} do not vary across segments. Then the unemployment rate is

⁸See also Van den Berg and Ridder (1993b), Koning, Ridder and Van den Berg (1995) and Koning, Van den Berg and Ridder (1998) for applications.

⁹Van den Berg and Ridder (1998), who were the first to apply this, specify $\log p = x'\beta + v$, where v has a discrete distribution with three points of support.

equal to

$$\frac{\delta}{\delta + \lambda_0}(1 - H(\max\{w_{min}, b\})) + H(\max\{w_{min}, b\}) \quad (3)$$

The first term on the right-hand side of this equation reflects frictional unemployment and the second-term structural unemployment.

Suppose that segments are at least partly defined by unobserved characteristics. The distributions of the structural parameters across workers in different segments are then estimated along with any other model parameters. The likelihood is obtained by integrating (“mixing”) the likelihood associated with a homogeneous model with respect to the distributions of the parameters. Note that in this case the distributions of wage offers and wages among employed workers are mixtures of the corresponding distributions in the basic model. It turns out that, as expected, mixtures over the productivity parameter p provide an accurate fit to the wage data (see the empirical studies cited above). The model also predicts negative duration dependence of the observed exit rate out of unemployment: some individuals have a positive exit rate λ_0 whereas the structurally unemployed have a zero exit rate. This unobserved heterogeneity in the true exit rate causes the observed exit rate to decrease as a function of duration¹⁰, and the latter is a stylized fact. Negative duration dependence of this exit rate is also established by mixtures over λ_0 itself, and negative duration dependence of the exit rate out of jobs is established by mixtures over any parameter (as they all affect this exit rate by way of F).

Note that the distribution $H(p)$ is not fully identified. We only observe economic activity in segments where p exceeds the wage floor, and from this we can identify the shape of the distribution above this floor, but it is not possible to infer the distribution of p among “latent” segments that would come into existence without a wage floor. In a way, this is nothing but a more fundamental version of the non-recoverability problem that has haunted applied research with partial job search models (see Section 1). Instead of the left-hand tail of the wage distribution it is now the left-hand tail of the productivity distribution that cannot be identified. This problem comes up in each model that we examine in Sections 4-6. Its main implication concerns the analysis of policy effects that affect the wage floor (like changes in the minimum wage). Although the effect of a small increase in the wage floor is identified, the effect of a small decrease is not, unless one is prepared to make functional-form assumptions on the shape of H .

¹⁰See e.g. Ridder and Van den Berg (1998). For this result it is necessary that structurally unemployed individuals are recorded as unemployed and not as being out of the labor force.

However, a crucial difference with the non-recoverability problem in the partial approach is that we are now able to identify the *amount* of probability mass below the wage floor. In particular, this amount is identified from the unemployment duration data, since it corresponds to the fraction of permanently unemployed individuals (this is exploited by Koning, Van den Berg and Ridder (1998) and by Ridder and Van den Berg (1998)). This result neatly illustrates the interrelations between wage and duration variables in equilibrium search models, as well as the potential they offer for fruitful exploitation in empirical inference.

The model estimates can be used to decompose wage variation into variation due to search frictions (i.e., the variation present in the homogeneous model) and the additional variation due to heterogeneity across segments. Typically, at least 50% of wage variation is due to variation in productivity across segments, and at most 25% is due to search frictions.

5 Structural estimation of models with heterogeneity within a market

Now let us consider the alternative case of *within market* heterogeneity in p . In reality, different firms active in the same labor market employ different production technologies. In line with this, one may extend the basic BM model by allowing firms to have different labor productivity levels p . As a result, workers are more productive in one firm than in another. In general, the equilibrium solution for such a market with heterogeneous agents differs from the solution for the homogeneous model. Mortensen (1990) contains the first theoretical analysis of the BM model with heterogeneous firms. He assumes a discrete distribution with a finite number of points of support for p (he also allows for heterogeneity in the individuals' value of b). Bontemps, Robin and Van den Berg (1997) provide a comprehensive theoretical analysis of the model with a general continuous distribution for p .

The supply side of the model is equivalent to that of the basic BM model. The expressions for ϕ and π are exactly the same as in (1) and (2). However, p in (2) now varies across firms, and in equilibrium more productive firms offer higher wages than less productive firms. In general, the mapping from productivities to wages is nonlinear and dependent on λ_1 and δ , and the densities of p and w need not be similar. The more productive firms are larger and they have a higher profit rate, so the model is in agreement with the stylized fact that wages, firm sizes and profits are positively correlated. Note that the less efficient firms survive

because of the search frictions. Productivity dispersion affects the distribution of the monopsony power across firms.

Bontemps, Robin and Van den Berg (1997) examine the set of wage and wage offer distributions that can be generated by varying the productivity distribution in the model over all possible continuous distributions. Interestingly, this set can be characterized by the restriction that the wage (offer) density does not increase as fast as the density in the basic BM model. This is obviously good news from an empirical point of view (recall the discussion in Subsection 2.2).

If a parametric functional form for the distribution of p is adopted then the model can be estimated with ML-type techniques. It turns out that if simple functional forms like a Pareto or lognormal distribution are adopted for p then this does not always give a very good fit to the whole wage density.¹¹ In response to this, Bontemps, Robin and Van den Berg (1997) develop and apply an estimation method that provides a nonparametric estimate of the productivity distribution. In the first step, the “transition” parameters λ_0, λ_1 and δ are estimated along the lines of the identification argument¹² in Section 3. In the second step, the productivity distribution is estimated from the nonparametric wage data distribution, using the relationship between p and w that follows from firm behavior. A couple of comments are in order. First of all, this estimation method provides a *perfect* fit to the wage data (if the model is correct). Given that the homogeneous model cannot explain a substantial amount of wage variation, it follows that productivity dispersion among firms is an important determinant of wage variation. Secondly, the estimates from the first step are only based on those parts of the model that describe worker behavior. The second step then exploits the first-order condition of the firms to estimate the productivity distribution. The estimates of the transition parameters can thus be expected to be consistent under a wide range of models of firm behavior and wage determination.¹³

Bowlus, Kiefer and Neumann (1995) develop and apply an estimation method for the model with a discrete distribution of p , and as such they were the first to

¹¹This is true even within segments based on occupation or industry. A Pareto distribution for p gives a better fit than (log)normal distributions or other popular distributions with a few parameters.

¹²In fact, G is estimated nonparametrically from cross-sectional wage data, and this is used as input in the estimation of the transition parameters. Standard errors are obtained by bootstrapping.

¹³Ridder and Van den Berg (1998) use aggregate data on marginal duration distributions to estimate the transition parameters without any need to be explicit on firm behavior or wage determination. Such estimates are also valid in case of (additional) between-market heterogeneity in p , provided that the transition parameters do not vary across segments.

estimate a model with between-market heterogeneity. The estimation method¹⁴ is a non-trivial extension of the method developed by Kiefer and Neumann (1993) for the estimation of a homogeneous model (see Section 3). In the case of a discrete distribution of p , the wage density is discontinuous at some points, so the asymptotics are non-standard, and numerical algorithms which do not require differentiation of the optimization criterion (simulated annealing) are needed. The method fixes the number of mass points of p by penalizing the likelihood value with a certain amount for each additional mass point. This typically results in about 4 to 9 mass points for p . The fit to the wage data density is not perfect. However, the main quantiles (median, quartiles) of the wage data are well fitted, and, as Bowlus, Kiefer and Neumann (1998) show, the estimates of λ_0 , λ_1 and δ are about the same as those obtained with a procedure similar to the first estimation step in Bontemps, Robin and Van den Berg (1997). This reinforces the conclusion that the distribution of wages is not very informative on the transition parameters.¹⁵

The results above have quite subtle implications for the importance of the determinants of wage variation. The basic BM has great intellectual appeal: it satisfies the applied researcher's desire for wage dispersion in search models by *explaining* wage dispersion completely out of the search frictions (that is, out of the positive finite job offer arrival rates). Nevertheless, it is not able to explain the empirical shape of the wage density. This can be remedied by allowing for productivity heterogeneity. However, productivity heterogeneity all by itself cannot explain wage dispersion either. So, even though search frictions by themselves only explain a small fraction of wage variation, it is the joint occurrence of search frictions and productivity heterogeneity that produces a fitting explanation.

We finish this section by briefly examining the estimation of models in which workers are heterogeneous by nature in terms of their value of b . First, recall that Eckstein and Wolpin (1990) estimate the AA model, which includes heterogeneity both of firms' p and of workers' b (but no job-to-job transitions). We discussed this model and its empirical performance in some detail in Subsections 2.1 and

¹⁴The model and estimation method have subsequently been applied in studies on e.g. male-female wage differences (Bowlus (1997)) and differences between youth labor markets in Canada and the U.S. (Bowlus (1998)). See Bunzel et al. (1997), Bowlus, Kiefer and Neumann (1998), and Bowlus and Seitz (1998) for other applications.

¹⁵This does not mean that any parametric specification for F will do, if one wants to estimate the transition parameters simultaneously with the parameters of F . In particular, if F is specified as in the homogeneous model then typically the estimate of λ_1 is biased toward zero. The latter improves the fit of the misspecified model to the wage data.

2.2, respectively. It is important to point out that the structural estimation of the AA model by Eckstein and Wolpin (1990) was the first ever structural empirical analysis of an equilibrium search model. This achievement is an important landmark in the history of labor economics, and it has had a clear impact on later work. Below we proceed by examining the incorporation of heterogeneity in b into the BM-type of models.

Burdett and Mortensen (1998) contains a theoretical analysis of a BM model with continuously distributed heterogeneity of workers' values of b , and homogeneous firms. Note from Subsection 2.1 that such a model integrates both possible sources of wage dispersion into account (that is, worker heterogeneity in b and on-the-job search). Bontemps, Robin and Van den Berg (1998) extend this theoretical analysis for the special case in which $\lambda_0 = \lambda_1$. In that case, being unemployed with benefits (or instantaneous utility) b is equivalent to being employed at a wage b , so the reservation wage of an unemployed individual with benefits b is simply equal to b . Bontemps, Robin and Van den Berg (1998) show that in this case the wage (offer) distributions are always even more skewed to the left than in the homogeneous model. The corresponding empirical distributions are however skewed to the right and have decreasing densities on most of their support (recall the discussion in Subsection 2.2), so this BM model with only worker heterogeneity does not give a satisfactory fit to the wage data. On the other hand, as we have seen, it can be expected to give a better fit to unemployment duration data than a homogeneous model.

Bontemps, Robin and Van den Berg (1998) subsequently estimate a more general BM-type model, in which both b and p are heterogeneous, each according to a continuous distribution, and with the restriction that $\lambda_0 = \lambda_1$. The estimation method is an extension of the method developed in Bontemps, Robin and Van den Berg (1997) discussed above. The distribution of b is estimated along with the transition parameters. It turns out that the vast majority of workers accept almost all job offers when unemployed. This is in line with results based on partial models with European data (see Subsection 2.2), and it has two important implications. First, a shift in the distribution of b (e.g. because of a shift in the average unemployment benefits level) does not have a sizeable effect on equilibrium unemployment or wages. Secondly, the dispersion in b is not an important determinant of wage variation.

6 Structural estimation of models with production sharing

In this section we briefly discuss structural estimation of a type of models that is fundamentally different from the models elsewhere in the survey even though they are equilibrium models where search frictions are essential. In these models, a firm is essentially equal to a single job or vacancy, and wages are determined after the realization of a stochastic *match-specific* productivity. The basic theoretical model is laid out in the seminal article by Flinn and Heckman (1982).

Consider a labor market with workers as well as firms that are atomistic (i.e., consisting of either a vacancy or a filled job). Unemployed workers and firms with a vacancy search for each other. On-the-job search is ruled out by assumption. An unemployed worker encounters firms with vacancies at the rate λ_0 . Before contact, the worker and firm do not know the productivity of the match they can form. However, upon the encounter, the productivity p of the match is perfectly revealed. The gains of production are assumed to be split evenly among the worker and the firm, so if a match is formed then the wage equals $p/2$. The distribution of p over all possible matches is exogenous, identical for all workers and firms, and known to all agents in the market. Upon each contact, the agents must decide whether to form the match or to search further for a better match. The optimal strategy of workers can then be summarized by a reservation productivity level ϕ_p which is the minimum acceptable productivity level for a match. The associated reservation wage ϕ equals $\phi_p/2$. If the search environment is exactly the same for firms as for workers then firms use the same reservation productivity level ϕ_p , so there is mutual consent between the worker and the firm.

The way in which the rents of the match are split can be justified by an underlying wage bargaining model (see Pissarides (1984) and Eckstein and Wolpin (1995a)). Suppose that the wage is determined by decentralized bargaining between the job applicant and the employer. Each possible match has a surplus defined as the excess gain over continued search. According to the Nash Axiomatic solution to this bargaining game, a fixed fraction β of the surplus of the match goes to the worker in the form of a wage. In equilibrium, only matches with positive surplus are realized. As Eckstein and Wolpin (1995a) show, this underlying model reduces to the specific model described above if (i) $\beta = 1/2$ and (ii) the search environment is exactly the same for firms as for workers.

Note that wage dispersion in this model is a result of match-specific productivity dispersion in combination with search frictions. In equilibrium, the

supply-side of this model is observationally equivalent to a standard partial job search model. Unemployed workers do from time to time reject job opportunities. As a consequence, the mean productivity among accepted jobs of a specific group of workers over-estimates its underlying mean productivity.

The empirical analysis of this model faces two identification problems. First, as pointed out by Flinn and Heckman (1982), the left-tail of the distribution of p is not identified without functional-form assumptions ensuring recoverability. Secondly, as pointed out by Eckstein and Wolpin (1995a), if one would not fix β at e.g. $1/2$ then this parameter is not identified from the productivity distribution. The underlying intuition is that the wage is a fixed share of productivity, and the “share parameter” β does not affect other observable worker behavior. In the other equilibrium search models, the surplus is shared implicitly according to parameters which do turn up elsewhere (for the BM model in the job duration distribution and for the AA model in the unemployment duration and wage distribution).

These models are estimated by Flinn and Heckman (1982) and Eckstein and Wolpin (1995a, 1995b) using ML. Flinn and Heckman (1982) do not allow for measurement errors in wages, so their estimate of ϕ equals the lowest observed wage. Eckstein and Wolpin (1995a) do allow for measurement errors. They also allow for “between-market” heterogeneity in a number of model parameters, and this heterogeneity is partly unobserved. As a result, their model fits both the wage data and the unemployment duration data well. They apply their method in Eckstein and Wolpin (1995b) to study discrimination of certain groups on the labor market by comparing all structural parameters corresponding to these groups.

Note that here firm behavior is not modeled as elaborately as in the BM or AA models. The model essentially specifies a productivity distribution as a layer below the wage offer distribution, and the link between the two is rather simple. In return, the model allows for a richer specification of search behavior on the supply side. For example, contrary to BM or AA models, one may readily allow for endogenous search intensities (Eckstein and Wolpin (1995a)).

7 New avenues

Recently, detailed firm-level data on productivity, wages, firm size, capital, job and worker flows etc. have become available in a large number of countries.¹⁶

¹⁶For sake of convenience, we use the word “firm” to denote any type of firm-type unit (company, establishment, etc.).

In some cases, these firm data can be linked to individual records of workers, and the data may even be longitudinal both for workers and for firms. It is obvious that such data enable the estimation of richer equilibrium search models that deal with firm behavior in a more elaborate way. The availability of such data has spurred a large number of descriptive empirical studies, and these have established stylized facts that are of importance for analyses with equilibrium search models.

In sum, the firm data provide a valuable input for future empirical research with equilibrium search models. In this section we examine some directions for future research in some detail. In Subsection 7.1 we show how firm data enable novel specification tests of the existing models. These tests target the wage setting behavior of firms. In Subsection 7.2 we examine how firm data can be used to analyze richer models.

7.1 Specification tests with firm data

The labor force survey data used to estimate equilibrium search models provide ample opportunities for specification tests. Such tests focus on properties of distributions of endogenous variables across individuals. As we have seen, these tests have been applied in the literature, and the models that have recently been estimated generally provide a good fit to the worker data. In this subsection we examine specification tests based on firm data.

A very straightforward type of test is based on a direct comparison of (i) the wage offer distribution and productivity distribution as estimated from worker data, and (ii) the observed distributions of wages and productivities across firms, from the firm data. Such tests are performed in Bontemps, Robin and Van den Berg (1997). One may also compare the distribution of any function of w and p , for example the monopsony power index $(p - w)/p$.

Another type of specification test is based on a check of whether comparative-statics model implications are in line with differences in observed outcomes of labor markets with different fundamental parameters. Such tests have the advantage that there is no need to structurally estimate the whole model. As an example, consider the model from Bontemps, Robin and Van den Berg (1997) with heterogeneous firms. The hypothesis is that each separate labor market segment that we consider can be described by this model. The model predicts that an increase in the ease with which employed workers can climb the job ladder (i.e., an increase in λ_1/δ) shifts the wage (offer) density to the right. This makes sense, as it increases the market power of workers. So, a market with a higher

value of λ_1/δ has higher wages. However, markets may also differ in terms of their within-market productivity distribution, and we have to “correct” for this when examining the relation between wages and λ_1/δ across segments. Indeed, the effect of the productivity variance on wages may be interesting by itself. Koning et al. (1998) show that the comparative-statics effects can be neatly translated into a wage-regression representation amenable to estimation with worker-firm data. For each segment, we can estimate λ_1 and δ from worker data. Perhaps these can be estimated from firm data as well, considering the worker flows into and out of firms. The firm data provide observations on p and on w for every firm. This gives estimates of $E_F(w)$, $E(p)$ and $\text{var}(p)$ for every segment. These can be used in the estimation of the regression equation. We may include additional observables to control for other firm characteristics like capital equipment of the firm (though formally this would have to be analyzed in a structural model first, in order to examine the equilibrium properties in such cases).

7.2 Heterogeneity of worker productivity

The equilibrium search models with wage setting that have been used in the empirical literature always assume that there is no dispersion of worker-specific productivity in a given labor market. It is obvious that any real-life firm contains workers with a wide range of productivities, and that there are synergy effects of employing different worker types. Recent empirical decompositions of wage variation, based on matched worker-firm data, clearly show that the wage has a large worker-specific component (see Abowd, Kramarz and Margolis (1998) and Bingley and Westergård-Nielsen (1996)). This strongly suggests that there is a worker-specific component in productivity.

Models in which a firm is essentially equivalent to a single job or vacancy are relatively amenable to the inclusion of individual-specific or match-specific productivity dispersion, and to the inclusion of shocks in these productivity components. Such models have become very popular in macro labor economics (see e.g. the survey by Mortensen and Pissarides (1998)), for the analysis of the relations between skills, wages, employment, and business cycles. However, these models are less attractive for empirical analysis with micro data on firms consisting of many workers. In this subsection we examine the extension of the BM-type models towards individual-specific productivity heterogeneity. Note that traditional labor demand models can easily accommodate skill differences across workers at a given firm. However, these models do not take account of informational frictions. In a world with frictions and skill differences, the fact that it takes time

and effort to hire workers plays a crucial role in the optimal skill mix at a firm. The firms' recruiting possibilities are constrained by the supply of (un)skilled workers as well as by the degree of frictions per skill level. Moreover, the steady state labor force of skilled labor for a particular firm may be affected by the wage setting policy of the firm in comparison to other firms. Wage dispersion across skills is then affected not just by technology but also by informational frictions.

Our starting point is the basic BM model with homogeneous workers and firms. The most simple extension towards individual-specific productivity is a between-market heterogeneity model (see Section 4) in which each skill group has its own separate labor market, with its own firms. A slightly less simple approach would be to assume that the firms' production function is additive in the inputs of its workers. For example, suppose that firms have the following profit function,

$$\pi = (p_s - w_s)L(w_s) + (p_u - w_u)L(w_u) \quad (4)$$

in which p_i denotes the productivity level associated with employing a worker of type i (skilled or unskilled), while w_i and $L(w_i)$ are the corresponding wage and steady state labor force conditional on the behavior of all other agents in the labor market. If firms are homogeneous in p_s, p_u then an equilibrium exists in which the wage distribution for either skill type equals a wage distribution in the homogeneous equilibrium search model.¹⁷ This model may not be sufficiently rich either for an analysis of skills, wages, and production. The production technology is such that there is no synergy of employing both types of workers.¹⁸

Now let us allow for a production function with two types of labor input (skilled and unskilled), excluding the additive specification (4) above. To fix

¹⁷Note that this extension and the extension in which different skills have different labor markets are observationally equivalent if only labor force survey data are available. The model estimated by Van den Berg and Ridder (1998) can be interpreted either way.

¹⁸An interesting alternative specification is proposed by Manning (1996). He assumes that workers are inherently heterogeneous in that their *worker-specific* productivity levels p are dispersed. Concerning wage formation, the central assumption is that a firm pays the same wage to all of its workers, whatever their productivity. This is justified by way of a within-firm fairness constraint. A firm only accepts a worker if the worker's productivity level p exceeds the wage w set by that firm. The wage offer distribution then depends on the distribution of p . It is not difficult to see that this model is able to explain many of the stylized facts on worker data. Moreover, even though the production technology has the same additive structure as in (4), there is now a relationship between the different types of workers, because a firm pays only one wage to all of its employees. In equilibrium, skilled workers earn higher wages because they are the only acceptable type of workers for firms paying high wages. However, the model is not able to explain a worker-specific component in the wage at a given firm.

thoughts, take a constant returns to scale Cobb-Douglas specification. Firms are allowed to pay different wages to skilled and unskilled workers, and different skill types may have different transition parameters λ_0, λ_1 and δ . A complication in the formal analysis of such a model concerns the fact that production displays decreasing returns to scale for a single production factor (given the amount of the other production factor). Given the wages set by the firm, there is a certain maximum number of workers of each type beyond which marginal returns fall below the wage, and the firm is not willing to expand.¹⁹ Whether the latter occurs in equilibrium depends on the parameters of the model. If the labor supply constraints (that is, the constraints on the firm's labor force that are imposed by the presence of frictions and the limited supply) are not binding for any wage offer exceeding the wage floor, then search frictions are basically irrelevant for the firms. This case is rather uninteresting. In the opposite case, the labor supply constraints are binding at the going wages for both skill types. The expressions for the first-order conditions of the firm's optimization problem can be used for an empirical specification. Typically, firm data provide information on marginal revenue product, firm size, the fractions of skilled and unskilled workers, and the mean wage. The transition parameters like λ_1 (which affect the attainable steady-state labor force of the firm) could be estimated from worker data, but firm data may contain information on this as well. The estimation results would enable e.g. a study of the effects of the frictional parameters for skilled and unskilled workers on the distribution of wages of unskilled workers. Subsequently, such models may be useful for the analysis of the effects of training for unskilled workers. A major advantage of such an analysis would be that it takes account of equilibrium effects on wages.

It should be noted that the importance of an individual-specific productivity component can also be investigated to a certain extent by using worker data on wages in consecutive job durations. Such data have not been used to date in the empirical analysis of equilibrium search models, with the exception of Van den Berg and Ridder (1998) who use them as overidentifying information. Intuitively, if workers with similar observed characteristics have diverging wage paths in their subsequent jobs then this is evidence that they have different productivities.

We end this section by briefly examining some other topics for further research that are inspired by firm data. A major issue concerns the absence in equilibrium search models of nonignorable fluctuations in firm size. The existing models are not very useful for the analysis of job creation and job destruction as

¹⁹For a model with a single production factor, decreasing returns to scale have been analyzed by Mortensen and Vishwanath (1994), Manning (1993), and Ridder and Van den Berg (1997).

responses to external shocks, or to the analysis of endogenous layoffs as responses to dissatisfaction with an employee's performance. This limitation is due to the fact that the firms' optimal strategies and the equilibrium outcomes are rather difficult to analyze if the firm cares about short-term fluctuations in its labor force. The same holds regarding non-constant wage profiles in a job.

Of course, with high-quality firm data, model extensions in other directions can be estimated. Robin and Roux (1998) structurally estimate a model with capital as an additional production factor and endogenous recruitment effort of firms, using firm data. Their analysis provides interesting insights into the relations between investment in capital, investment in hiring, and wages. The estimated model enables an examination of the effects of various worker and firm taxation policies.

8 Conclusion

In this paper we have surveyed the literature on the structural estimation of equilibrium search models. During the past decade, substantial progress has been made in terms of improving the fit of the model specification. Models with on-the-job search, heterogeneity of firms' productivity levels, and heterogeneity of workers' value of leisure provide a good fit to duration and wage data from labor force surveys. Recently, the availability of high-quality firm data has generated a large number of descriptive empirical studies with results that are of importance for the equilibrium search literature. A particularly important topic for the future seems to be the inclusion of worker-specific productivity heterogeneity in a structural empirical analysis.

References

- Abowd, J.M., F. Kramarz, and D.N. Margolis (1998), “High wage workers and high wage firms”, *Econometrica*, forthcoming.
- Albrecht, J.W. and B. Axell (1984), “An equilibrium model of search unemployment”, *Journal of Political Economy*, 92, 824–840.
- Atkinson, A.B. and J. Micklewright (1991), “Unemployment compensation and labor market transitions: A critical review”, *Journal of Economic Literature*, 29, 1679–1727.
- Bingley, P. and N. Westergård-Nielsen (1996), “Worker and plant wages – estimates from a multi-level model”, Working paper, University of Aarhus, Aarhus.
- Blau, D.M. (1991), “Search for nonwage job characteristics: a test of the reservation wage hypothesis”, *Journal of Labor Economics*, 9, 186–205.
- Blau, D.M. and P.K. Robins (1986), “Job search, wage offers and unemployment insurance”, *Journal of Public Economics*, 29, 173–197.
- Bonnal, L. and D. Fougère (1991), “Estimating the structural effect of the unemployment benefit on job search costs”, Working paper, Université des Sciences Sociales, Toulouse.
- Bontemps, C., J.M. Robin, and G.J. van den Berg (1997), “Equilibrium search with continuous productivity dispersion: theory and non-parametric estimation”, Working paper, Free University and Tinbergen Institute, Amsterdam.
- Bontemps, C., J.M. Robin, and G.J. van den Berg (1998), “An empirical equilibrium job search model with continuously distributed heterogeneity of workers’ opportunity costs of employment and firms’ productivities, and search on the job”, Working paper, Tinbergen Institute, Amsterdam.
- Bound, J., C. Brown, G.J. Duncan, and W.L. Rodgers (1994), “Evidence on the validity of cross-sectional and longitudinal labor-market data”, *Journal of Labor Economics*, 12, 345–368.
- Bowlus, A.J. (1997), “A search interpretation of male-female wage differentials”, *Journal of Labor Economics*, 15, 625–657.
- Bowlus, A.J. (1998), “U.S.-Canadian unemployment rate and wage differences among young, low-skilled males in the 1980s”, *Canadian Journal of Economics*, 31, 437–464.

- Bowlus, A.J., N.M. Kiefer, and G.R. Neumann (1995), “Estimation of equilibrium wage distributions with heterogeneity”, *Journal of Applied Econometrics*, 10, S119–S131.
- Bowlus, A.J., N.M. Kiefer, and G.R. Neumann (1998), “Equilibrium search models and the transition from school to work”, Working paper, University of Western Ontario, London, Ontario.
- Bowlus, A.J. and S. Seitz (1998), “Search friction in the U.S. labor market: equilibrium estimates from the PSID”, Working paper, University of Western Ontario, London, Ontario.
- Bunzel, H., B.J. Christensen, P. Jensen, N.M. Kiefer, L. Korsholm, L. Muus, G.R. Neumann, and M. Rosholm (1997), “Specification and estimation of equilibrium search models”, Working paper, University of Aarhus, Aarhus.
- Burdett, K. (1990), “Search market models: a survey”, Working paper, University of Essex, Colchester.
- Burdett, K. and D.T. Mortensen (1998), “Wage differentials, employer size, and unemployment”, *International Economic Review*, forthcoming.
- Card, D. and A.B. Krueger (1995), *Myth and measurement*, Princeton University Press, Princeton.
- Davidson, C. (1990), *Recent developments in the theory of involuntary unemployment*, W.E. Upjohn Institute, Kalamazoo.
- Devine, T.J. and N.M. Kiefer (1991), *Empirical Labor Economics*, Oxford University Press, Oxford.
- Diamond, P.A. (1971), “A model of price adjustment”, *Journal of Economic Theory*, 3, 156–168.
- Eckstein, Z. and K.I. Wolpin (1990), “Estimating a market equilibrium search model from panel data on individuals”, *Econometrica*, 58, 783–808.
- Eckstein, Z. and K.I. Wolpin (1995a), “Duration to first job and return to schooling: estimates from a search-matching model”, *Review of Economic Studies*, 62, 263–286.
- Eckstein, Z. and K.I. Wolpin (1995b), “Estimating the effect of labor market discrimination on black-white differences in wage offers using a search-matching-bargaining model”, Working paper, Tel Aviv University, Tel Aviv.
- Engberg, J. (1991), “The impact of unemployment benefits on job search: structural unobserved heterogeneity and spurious spikes”, Working paper, Carnegie-Mellon University, Pittsburgh.

- Flinn, C.J. (1996), “Labor market structure and welfare: a comparison of Italy and the U.S.”, Working paper, New York University, New York.
- Flinn, C.J. and J.J. Heckman (1982), “New methods for analyzing structural models of labor force dynamics”, *Journal of Econometrics*, 18, 115–168.
- Hartog, J. and H. van Ophem (1991), “Wages and measurement errors”, *Annales d’Économie et de Statistique*, 20, 243–256.
- Kiefer, N.M. and G.R. Neumann (1993), “Wage dispersion with homogeneity: the empirical equilibrium search model”, in H. Bunzel et al., editors, *Panel data and labor market analysis*, North Holland, Amsterdam.
- Koning, P., G. Ridder, and G.J. van den Berg (1995), “Structural and frictional unemployment in an equilibrium search model with heterogeneous agents”, *Journal of Applied Econometrics*, 10, S133–S151.
- Koning, P., G.J. van den Berg, and G. Ridder (1997), “A structural analysis of job search methods and subsequent wages”, Working paper, Tinbergen Institute, Amsterdam.
- Koning, P., G.J. van den Berg, and G. Ridder (1998), “Semi–nonparametric estimation of an equilibrium search model”, Working paper, Tinbergen Institute, Amsterdam.
- Koning, P., G.J. van den Berg, G. Ridder, and K. Albæk (1998), “The relation between wages and labor market frictions: An empirical analysis based on matched worker–establishment data”, Working paper, Free University Amsterdam, Amsterdam.
- Lancaster, T. and A.D. Chesher (1983), “An econometric analysis of reservation wages”, *Econometrica*, 51, 1661–1676.
- Layard, R., S. Nickell, and R. Jackman (1991), *Unemployment; Macroeconomic Performance and the Labour Market*, Oxford University Press, Oxford.
- Lindeboom, M. and J. Theeuwes (1991), “Job duration in The Netherlands”, *Oxford Bulletin of Economics and Statistics*, 53, 243–264.
- Machin, S. and A. Manning (1996), “The structure of wages in what should be a competitive labour market”, Working paper, University College London, London.
- MacMinn, R.D. (1980), “Job search and the labor dropout problem reconsidered”, *Quarterly Journal of Economics*, 95, 69–87.
- Manning, A. (1993), “Endogenous labour market segmentation in a matching model”, Working paper, Centre for Economic Performance, London.

- Manning, A. (1996), “Labour markets with company wage policies”, Working paper, London School of Economics, London.
- Mortensen, D.T. (1986), “Job search and labor market analysis”, in O. Ashenfelter and R. Layard, editors, *Handbook of Labor Economics*, North-Holland, Amsterdam.
- Mortensen, D.T. (1990), “Equilibrium wage distributions: a synthesis”, in J. Harthog et al., editors, *Panel data and labour market studies*, North-Holland, Amsterdam.
- Mortensen, D.T. and C.A. Pissarides (1998), “New developments in models of search in the labor market”, in *Handbook of Labor Economics*, North-Holland, Amsterdam, forthcoming.
- Mortensen, D.T. and T. Vishwanath (1994), “Personal contacts and earnings: it is who you know!”, *Labour Economics*, 1, 187–201.
- Narendranathan, W. and S.J. Nickell (1985), “Modelling the process of job search”, *Journal of Econometrics*, 28, 28–49.
- Pissarides, C.A. (1984), “Efficient job rejection”, *Economic Journal*, 94, 97–108.
- Ridder, G. and G.J. van den Berg (1997), “Empirical equilibrium search models”, in D.M. Kreps and K.F. Wallis, editors, *Advances in Economics and Econometrics: Theory and Applications*, Cambridge University Press, Cambridge.
- Ridder, G. and G.J. van den Berg (1998), “Estimating measures of labor market imperfection for five OECD countries, using aggregate data in an equilibrium search framework”, Working paper, Free University Amsterdam, Amsterdam.
- Robin, J.M. and S. Roux (1998), “Random matching or balanced matching: an equilibrium search model with endogenous capital and matching”, Working paper, LEA-INRA, Paris.
- Topel, R.H. and M.P. Ward (1992), “Job mobility and the careers of young men”, *Quarterly Journal of Economics*, 107, 439–479.
- Van den Berg, G.J. (1990a), “Nonstationarity in job search theory”, *Review of Economic Studies*, 57, 255–277.
- Van den Berg, G.J. (1990b), “Search behaviour, transitions to nonparticipation and the duration of unemployment”, *Economic Journal*, 100, 842–865.
- Van den Berg, G.J. (1992), “A structural dynamic analysis of job turnover and the costs associated with moving to another job”, *Economic Journal*, 102, 1116–1133.

- Van den Berg, G.J. (1994), “The effects of changes of the job offer arrival rate on the duration of unemployment”, *Journal of Labor Economics*, 12, 478–498.
- Van den Berg, G.J. (1998), “Multiple equilibria and minimum wages in labor markets with informational frictions and heterogeneous production technologies”, Working paper, Free University Amsterdam, Amsterdam.
- Van den Berg, G.J. and G. Ridder (1993a), “On the estimation of equilibrium search models from panel data”, in J.C. van Ours et al., editors, *Labor Demand and Equilibrium Wage Formation*, North Holland, Amsterdam.
- Van den Berg, G.J. and G. Ridder (1993b), “Estimating an equilibrium search model from wage data”, in H. Bunzel et al., editors, *Panel data and labor market analysis*, North Holland, Amsterdam.
- Van den Berg, G.J. and G. Ridder (1998), “An empirical equilibrium search model of the labor market”, *Econometrica*, forthcoming.
- Wolpin, K.I. (1987), “Estimating a structural job search model: the transition from school to work”, *Econometrica*, 55, 801–818.
- Wolpin, K.I. (1995), *Empirical methods for the study of labor force dynamics*, Harwood Academic Publishers, Luxembourg.
- Yoon, B.J. (1981), “A model of unemployment duration with variable search intensity”, *Review of Economics and Statistics*, 63, 599–609.