SPATIAL DIMENSIONS OF ENVIRONMENTAL POLICIES FOR TRANS-BOUNDARY EXTERNALITIES

A Spatial Price Equilibrium Approach

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Abstract
This paper presents a framework for analysing spatial aspects of environmental policies in the regulation of trans-boundary externalities. A spatial price equilibrium model for two regions is constructed, where interactions between these regions can occur via trade and transport, via mutual environmental spill-overs due to the externality that arises from production, and via tax competition when the regions do not behave cooperatively. Also the additional complications arising from emissions caused by the endogenous transport flows are considered explicitly.

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1. Introduction

Trans-boundary externalities in general, and global externalities like greenhouse gas emissions in particular, have recently received much attention; both in international negotiations on environmental protection and in the environmental economics literature. Apart from various barriers that may often frustrate environmental policy making in general, additional impediments to the effective and efficient regulation of trans-boundary externalities are the partly public character of the 'bad' in question, and the resulting incentives for individual countries to free-ride on other countries' achievements (Carraro, 1997). This is so much the more serious because of the potentially far-reaching, global consequences of excessive levels of these emissions in case of insufficiently stringent policies.

Apart from free-riding problems for trans-boundary externalities, there are more factors that may complicate the design of environmental policies as soon as the international dimension is considered explicitly. A particularly important matter concerns the potential negative effects of local environmental policies on the international competitiveness, and hence the profitability, of local producers. The relations between competitiveness, trade and environment, and therefore also between trade policies and environmental policies, have consequently received growing attention in the literature (see, for instance, Jaffe, Peterson and Portney, 1995, for an assessment of empirical evidence; Van Beers and Van den Bergh, 1996, and Ulph, 1997, for general surveys; Krutilla, 1998, for a survey on partial equilibrium models; and Steininger, 1998, for a discussion of general equilibrium models). To complicate matters further, trade usually requires transportation, which in itself can also give rise to emissions affecting the global environment. At the same time, trade often indirectly affects foreign production and hence emission levels. Consequently, a country should not ignore the derived environmental effects of local policies through the induced impacts on international transportation flows as well as on foreign production, in addition to the direct impact on local production and emission levels. Various types of interactions can thus be distinguished that will simultaneously affect individual countries' behaviour in terms of environmental policies when considering the international dimension.

This paper aims to offer a framework in which these interactions can be studied simultaneously. The central question concerns the first-best and second-best tax rules for open economies concerned with a trans-boundary environmental externality, and the relative efficiency of various policy alternatives, taking account of spatio-economic interdependencies between economies. The analysis explicitly pays attention to (1) the partly public character of trans-boundary environmental externalities, (2) the effect of environmental policies on the country's or region's competitiveness and the implied direct relation between trade policies and environmental policies, (3) the possibility of tax competition between countries or regions when behaving in a non-cooperative manner, and (4) the role of emissions caused by transport in evaluating environmental policies from a spatial perspective. Given these features, the model setting is, for instance, applicable to the case of regional or national environmental policies for the regulation of trans-boundary emissions from energy use in an international context.

The analysis is based on the spatial price equilibrium (SPE) methodology, and can therefore be characterized as a partial equilibrium model (see Van Beers and Van den Bergh, 1996, for a discussion on the use of partial versus general equilibrium models in this context). Among the growing number of contributions in this field, ours is probably most closely related to Krutilla (1991): we also present a partial equilibrium model allowing for the simultaneous consideration of terms-of-trade and environmental effects of tax policies for a competitive sector, and ignore the effects of local environmental policies on firm relocation (see for instance Markusen, Morey and Olewiler, 1993), or on the spatial diffusion of environmental technologies (Ulph, 1996). The analysis is nevertheless quite different from the one in Krutilla (1991), because of the explicit consideration of the simultaneous behaviour of two regions, so that issues of tax competition and policy coordination play a central role (Krutilla, 1991, focuses on one single open economy); by explicitly including transport and transport emissions in the analysis; and by the consideration of a trans-boundary externality, so that also induced effects on emissions by transport and by the 'foreign' suppliers affect the efficiency of environmental policies.

Without claiming to offer an exhaustive overview, we would like to briefly mention some more related studies here. Some studies explicitly focusing on market power of producers and strategic behaviour of producers and/or governments, typically using game-theoretic approaches, are Barret (1994), Conrad (1993), Kennedy (1994), Markusen (1975b), Rauscher (1994) and Ulph (1996ab). Some recent studies based on general equilibrium approaches are Copeland (1994), Frederikson (1997), Markusen *et al.* (1993) and Rauscher (1994). To the best of our knowledge, the present setting in which a partial equilibrium analysis is given of a situation where emissions from local and foreign production as well as from transport affect local welfare has no precedent in the literature.

The use of a partial equilibrium approach without a trade balance expression in a model with trade of course presupposes that factor income effects are not of interest, and that trade imbalances on a sectoral level may persist over the relevant planning period. Krutilla (1998) observes in this respect that "[i]n the trade-and environment context, partial equilibrium models are particularly useful for studying the consequences of terms-of-trade effects, and for indicating how such factors as a country's commodity trade balance, and the type of externality problem, affect the normative properties of environmental policy actions". These are exactly the type of questions addressed in the present paper.

The plan of this paper is as follows. Section 2 starts with a presentation of the general model. Section 3 presents general analytical solutions for first-best and second-best ('spatially non-cooperative') policies. Section 4 discusses the overall system effects of such non-cooperative policies. Finally, Section 5 offers some concluding remarks.

2. The spatial model of the firms, the markets, and the externalities

It is clear that, when one aims to study a topic as broad as sketched above in the introduction, a large variety of modelling approaches can be followed (see Van Beers and Van den Bergh, 1996, for an overview). A clear trade-off, for instance, exists between the level of generality of the model, and the extent to which it is capable of producing tractable analytical results. Our aim is to keep the model as general as possible, but to economize on complexity wherever this seems not overly restrictive, so that interpretable analytical results can still be obtained.

For that reason, we confine ourselves in the first place to a setting with only two independent regions, for instance countries. Furthermore, because of the relative complexity of the issue at hand itself, any other market or government failures, apart from the public environmental externality, and failures to coordinate policies optimally, are assumed away. This secures that second-best elements resulting from, for instance, market power of firms or consumers, inefficient labour markets, dynamics and uncertainty, or imperfect information will not affect the analyses to follow. Although we recognize that such other market failures may undoubtedly play an important role in reality, we leave their treatment in the current context to future research, and wish to concentrate solely on the economic principles underlying the efficiency and effectiveness of various types of environmental regulation of a global externality in an otherwise 'first-best' spatial world. Finally, as many 'other things' as possible are held constant, for the same reason. For instance, the technologies used are assumed to be fixed; although we intend to consider endogenous technologies in the present model in the near future (a non-spatial version of the type of model we plan to use can be found in Verhoef and Nijkamp, 1997a). These considerations have led us to the following model.

First, firms are assumed to be profit maximizing price-takers on both the output and the input markets. Firms are assumed to be identical within a region, but may be different between regions. Verhoef and Nijkamp (1997ab) have emphasized and demonstrated the importance of considering heterogeneity of firms in evaluating environmental policies; in the present model, this heterogeneity is caused by spatial factors. The assumption of identical price-taking firms within a region allows a simple derivation of regional supply functions and locally optimal environmental policies (Verhoef and Nijkamp, 1997b, discuss complications that arise with heterogeneity of firms within a single closed economy).

One homogeneous good is considered, which is produced and consumed in both regions (A and B). It is assumed that the production of the good in either region, as well as transport between the regions, gives rise to environmental emissions that affect welfare in both regions. The externality is therefore public in nature. The case of a purely public externality, where the extent to which welfare in a region is affected depends on the unweighted sum of all emissions, is a limiting case of the present specification. When the externality is not purely public, heavier weights for local emissions reflect localized externalities. The average and marginal emissions of production may vary between the regions, and also the regional valuation of the total emissions may vary.

For the modelling of the spatial market, we will use the spatial price equilibrium approach (SPE), first presented in a seminal paper by Samuelson (1952) and later on extensively studied and further developed by Takayama and others (see Takayama and Judge, 1971; Takayama and Labys, 1986; and Labys and Yang, 1997). The SPE-framework already proved to be a useful tool in investigating environmental policies in spatial systems in earlier papers which focused primarily on transport (Verhoef and Van den Bergh, 1996; Verhoef, Van den Bergh and

Button, 1997). Krutilla (1989) used an SPE-setting when analysing a different type of market failure in a comparable context, namely market power by the transportation sector.

The two regions themselves are modelled as spaceless nodes, in the sense that trade within a region requires no transport (or at least, this transport can safely be ignored)[1]. Trade between the regions, however, does require transport. Transport services are assumed to be offered by 'footloose outside transport suppliers', who reside in neither region A nor B. Average and marginal transport costs per unit of good traded are constant and given by t (there is no congestion); and, apart from its energy use (see below), transport is efficiently priced at price t. Hence, the transport supplier(s) make zero profits, so that their footloose status does not lead to a 'welfare surplus leak' in the model.

Figure 1 gives a diagrammatic representation of SPE. The left panel shows the local demand and industry supply curves $D_A(Y_A)$ and $S_A(Q_A)$ for the good in region A (note that Figure 1 is a back-to-back diagram, so that Y_A and Q_A increase when moving leftwards from the origin). In autarky (denoted with superscripts A), the equilibrium is given by a production and consumption level $Q_A^A=Y_A^A$, and a local market price P_A^A. The right panel shows the same for region B, where the autarky equilibrium is given by Y_B^A, Q_B^A and P_B^A.

Now suppose that the per-unit-of-good transport cost between the regions is equal to t, which is smaller than the autarky price difference between the two regions. It is then profitable and efficient to transport some goods from the lower price region to the higher price region. In Figure 1, it is assumed that $P_A^A>P_B^A$ and that $P_A^A–P_B^A>t$. In order to determine the after-trade equilibrium (denoted with superscripts T), for both regions R an excess demand/supply curve $X_R(T_R)$ is constructed by horizontal subtraction of the industry supply curve from the demand curve. Hence, for each after-trade local price $P_A^T>P_A^A$, $P_A^T=X_A$ implies the equilibrium net export T_A that region A would supply to region B. For $P_A^T<P_A^A$, negative values of T_A imply positive net imports, showing that region A would be a net demander. A comparable $X_B(T_B)$ curve can be constructed for region B.

The after-trade equilibrium in a closed system is then given by $T_A=–T_B$ and $|P_A^T–P_B^T|=t$. In Figure 1, we find $P_A^T–P_B^T=t$, and $Q_B^T–Y_B^T=T_B=–T_A=Y_A^T–Q_A^T$: region B is the net exporter. Observing that D_R and S_R give the marginal benefits of consumption (D_R) and marginal costs of production (S_R) in region R, the two shaded areas can be identified as the net benefits of trade and transport. Both regions gain, as they should with voluntary trade. Moreover, assuming price-taking behaviour and ignoring the environmental externality, the after-trade equilibrium is Pareto efficient.[2] However, it can also be noted that in node A the consumers benefit and the producers lose, while the opposite occurs in node B.

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[1] That is to say, emissions from intra-regional transport are not considered explicitly, but could of course be assumed to be included in the emissions from local production, in which case the emissions from transport for trade would actually reflect the emissions due to the *additional* kilometres driven for inter-nodal trade compared to intra-nodal trade.

[2] This can be verified by solving the total social welfare problem (for both regions jointly), as given in (4) below for the case where the valuation of emissions is zero. All regulatory taxes then become equal to zero; see (14a-d).

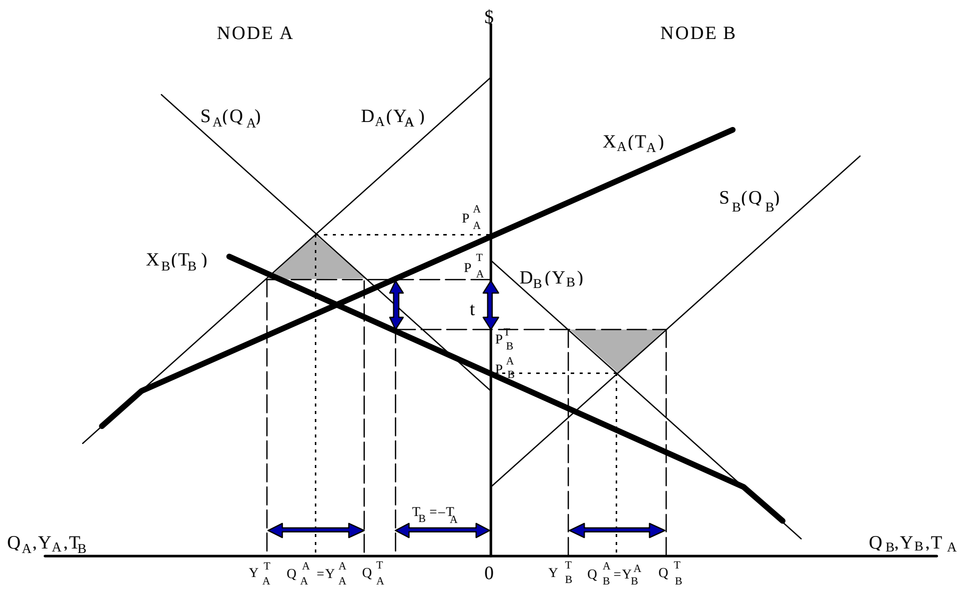

Figure 1. A graphical representation of the SPE model

Evidently, as soon as environmental externalities enter the picture, the unregulated spatial market is unlikely to produce a Pareto optimal outcome. For the evaluation of various policy alternatives in that case, the formal model consistent with Figure 1 has to be solved. For this purpose we define the following additional variables: $Z_{O,D}$ is the net delivery from region O (origin) to D (destination); $T=|T_A|=|T_B|$ is the total level of transport; $E_R(Q_A,Q_B,T)$ denotes the valuation of the total emissions by region R; and τ_{PR} and τ_{CR} give local taxes, in region R, on production and consumption respectively. Note that neither import/export tariffs nor taxes on transport – which are actually indistinguishable in the present model with constant technologies – will be considered. There's a number of reasons for this. First of all, these were already considered in great depth in Verhoef and Van den Bergh (1996) and Verhoef, Van den Bergh and Button (1997), where the attention was focused explicitly on transport, and on second-best transportation taxes. Secondly, with local production and consumption taxes present, the model would actually have an excessive number of regulatory taxes if also local transportation taxes were allowed. No unique solutions for the three tax expressions could then be derived. Therefore, the implicit transportation taxes or tariffs can in the present case be found as the difference between local production and local consumption taxes. Thirdly, we wish to place consumption taxes in the spotlight. Such an instrument may have the advantage that the international competitiveness of local firms is not affected as strongly as in case of taxes on production, because the relative terms of competition between local and domestic suppliers remain equal on both the local market and on the foreign market. In contrast, the local producer would suffer a comparative disadvantage on both markets from an environmental tax on production. Because the effects on international competitiveness are often put forward as an important objection against producer oriented taxes, the consumption tax may in some cases offer an attractive alternative, and therefore certainly deserves attention.

Continuing with the definition of variables, δ_{AB} is used as a dummy variable that takes on the value of 1 if $Z_{AB}>0$, and $\delta_{AB}=0$ otherwise. Likewise, $\delta_{BA}=1$ ($\delta_{AA}=1$, $\delta_{BB}=1$) if $Z_{BA}>0$ ($Z_{AA}>0$, $Z_{BB}>0$) and 0 otherwise. Observe that in the SPE model, $\delta_{AB}+\delta_{BA}\leq 1$: cross-hauling does not occur in the standard SPE model (see also Labys and Yang, 1997). This may seem a weakness, in particular since cross-hauling often seems to occur in reality – at least if the definition of a 'homogeneous good' is taken to include a sufficiently large number of product variants and brands. However, under the standard 'bench-mark' assumptions of a completely homogeneous good and perfect information, cross-hauling can of course not be consistent with utility and profit maximizing behaviour. Nevertheless, the consideration of heterogeneous products, possibly close substitutes, could offer an interesting question for follow-up research. In particular, it would be interesting to see how such cross-elasticities would affect the second-best tax rules to be presented below.

Throughout the paper, it is assumed that all relevant functions are differentiable; that marginal benefits are non-increasing in consumption; that marginal costs as well as marginal emissions are non-decreasing in production; and that the marginal valuation of total emissions is non-decreasing in the total level of emissions.

We can now define social welfare in region R as the sum of the Marshallian consumers' surplus (total benefits minus expenditures) *plus* the producers' surplus (total revenues minus total costs) *plus* the tax revenues *minus* the valuation of the environmental effect:

$$\begin{aligned} W_R = \int_0^{Y_R} D_R(Y_R)dy - D_R(Y_R)\cdot Y_R + t_{CR}\cdot Y_R - \int_0^{Q_R} S_R(Q_R)dq + S_R(Q_R)\cdot Q_R + t_{PR}\cdot Q_R \\ - E_R(Q_A, Q_B, T) \end{aligned} \qquad (1)$$

The following equalities and inequalities will hold in any decentralized market equilibrium (with or without taxes: each τ_{ij} could of course be equal to zero):

$$Y_A = Z_{AA} + Z_{BA} \qquad (2a)$$

$$Y_B = Z_{AB} + Z_{BB} \qquad (2b)$$

$$Q_A = Z_{AA} + Z_{AB} \qquad (2c)$$

$$Q_B = Z_{BA} + Z_{BB} \qquad (2d)$$

$$Z_{AA} \geq 0; \quad S_A + t_{PA} + t_{CA} - D_A \geq 0; \quad Z_{AA}\cdot\left(S_A + t_{PA} + t_{CA} - D_A\right) = 0 \qquad (2e)$$

$$Z_{AB} \geq 0; \quad S_A + t_{PA} + t + t_{CB} - D_B \geq 0; \quad Z_{AB}\cdot\left(S_A + t_{PA} + t + t_{CB} - D_B\right) = 0 \qquad (2f)$$

$$Z_{BA} \geq 0; \quad S_B + t_{PB} + t + t_{CA} - D_A \geq 0; \quad Z_{BA}\cdot\left(S_B + t_{PB} + t + t_{CA} - D_A\right) = 0 \qquad (2g)$$

$$Z_{BB} \geq 0; \quad S_B + t_{PB} + t_{CB} - D_B \geq 0; \quad Z_{BB}\cdot\left(S_B + t_{PB} + t_{CB} - D_B\right) = 0 \qquad (2h)$$

Equations (2a-h) give a straightforward description of a decentralized equilibrium with price-takers, where all markets clear and where positive deliveries imply that the marginal benefits must be equal to the marginal cost.

This completes the discussion of the general model. Section 3 proceeds by the derivation of policy rules under various circumstances.

3. First-best and second-best energy taxes

This section provides the analytical solutions to the model outlined above for first-best and second-best energy taxes. Section 3.1 presents the intuitive result that in order to obtain the first-best spatio-economic configuration, Pigouvian taxes should apply throughout. The subsequent sections provide the solutions for various second-best situations, which have in common that the two regions do not coordinate their policies, but instead set taxes so as to optimize regional welfare only. Section 3.2 considers uncoordinated second-best taxes for a region using both a production and a consumption tax. Sections 3.3 and 3.4 subsequently consider the use of a production tax and a consumption tax in isolation.

3.1. First-best taxes

In this section we consider the solution to the problem of maximizing total welfare in the two economies considered. This gives us the Pareto optimal configuration, and hence the first-best policy rules that would apply if either a supra-regional regulator could control the entire system, or if the two regions would coordinate their policies optimally. The solution can be found by maximizing total welfare for both regions as given in (1), subject to constraints posed by individual optimizing behaviour as given by (2e-h), under the assumption that the regulator can use all taxes he wishes to use, and therefore is not constrained to use imperfect second-best taxes. Using the dummy variables δ introduced above, the associate Lagrangian can be written as:

$$\begin{aligned}
\Lambda = & \int_0^{Y_A} D_A(Y_A)dy_A - D_A(Y_A)\cdot Y_A + t_{CA}\cdot Y_A - \int_0^{Q_A} S_A(Q_A)dq_A + S_A(Q_A)\cdot Q_A + t_{PA}\cdot Q_A \\
& - E_A(Q_A, Q_B, T) \\
& + \int_0^{Y_B} D_B(Y_B)dy_B - D_B(Y_B)\cdot Y_B + t_{CB}\cdot Y_B - \int_0^{Q_B} S_B(Q_B)dq_B + S_B(Q_B)\cdot Q_B + t_{PB}\cdot Q_B \\
& - E_B(Q_A, Q_B, T) \\
& + d_{AA}\cdot l_{AA}\cdot\left(S_A(Q_A) + t_{PA} + t_{CA} - D_A(Y_A)\right) \\
& + d_{AB}\cdot l_{AB}\cdot\left(S_A(Q_A) + t_{PA} + t + t_{CB} - D_B(Y_B)\right) \\
& + d_{BA}\cdot l_{BA}\cdot\left(S_B(Q_B) + t_{PB} + t + t_{CA} - D_A(Y_A)\right) \\
& + d_{BB}\cdot l_{BB}\cdot\left(S_B(Q_B) + t_{PB} + t_{CB} - D_B(Y_B)\right)
\end{aligned} \qquad (3)$$

The specification in (3) reveals that the structure of the optimization problem depends on the exact trade regime that applies (that is, the questions of whether the various deliveries Z_{ij} are either zero or positive), and hence that different first-order conditions will apply depending on the prevailing trade regime. The problem could be written in a Kuhn-Tucker form to capture these complications, but for the present purpose, where we wish to present optimal tax rules for each trade regime, this is not necessary. Moreover, we want to avoid complications due to non-negativity restrictions on Kuhn-Tucker multipliers (see also Verhoef and Van den Bergh, 1996).

The Lagrangian is solved by taking partial derivatives with respect to the variables Z, using equations (2a-d), τ and λ (primes denote derivatives for functions in one single argument, in particular the demand and supply functions):

$$\frac{\partial \Lambda}{\partial Z_{AA}} = -D'_A \cdot Y_A + t_{CA} + S'_A \cdot Q_A + t_{PA} - \frac{\partial E_A}{\partial Q_A} - \frac{\partial E_B}{\partial Q_A} + d_{AA} \cdot l_{AA} \cdot \left(S'_A - D'_A\right) + d_{AB} \cdot l_{AB} \cdot S'_A - d_{BA} \cdot l_{BA} \cdot D'_A = 0 \tag{4}$$

$$\frac{\partial \Lambda}{\partial Z_{AB}} = S'_A \cdot Q_A + t_{PA} - D'_B \cdot Y_B + t_{CB} - \frac{\partial E_A}{\partial Q_A} - \frac{\partial E_B}{\partial Q_A} - \frac{\partial E_A}{\partial T} - \frac{\partial E_B}{\partial T} + d_{AA} \cdot l_{AA} \cdot S'_A + d_{AB} \cdot l_{AB} \cdot \left(S'_A - D'_B\right) - d_{BB} \cdot l_{BB} \cdot D'_B = 0 \tag{5a}$$

$$\frac{\partial \Lambda}{\partial Z_{BA}} = -D'_A \cdot Y_A + t_{CA} + S'_B \cdot Q_B + t_{PB} - \frac{\partial E_A}{\partial Q_B} - \frac{\partial E_B}{\partial Q_B} - \frac{\partial E_A}{\partial T} - \frac{\partial E_B}{\partial T} - d_{AA} \cdot l_{AA} \cdot D'_A + d_{BA} \cdot l_{BA} \cdot \left(S'_B - D'_A\right) + d_{BB} \cdot l_{BB} \cdot S'_B = 0 \tag{5b}$$

$$\frac{\partial \Lambda}{\partial Z_{BB}} = -D'_B \cdot Y_B + t_{CB} + S'_B \cdot Q_B + t_{PB} - \frac{\partial E_A}{\partial Q_B} - \frac{\partial E_B}{\partial Q_B} - d_{AB} \cdot l_{AB} \cdot D'_B + d_{BA} \cdot l_{BA} \cdot S'_B + d_{BB} \cdot l_{BB} \cdot \left(S'_B - D'_B\right) = 0 \tag{6}$$

$$\frac{\partial \Lambda}{\partial t_{CA}} = Y_A + d_{AA} \cdot l_{AA} + d_{BA} \cdot l_{BA} = 0 \tag{7}$$

$$\frac{\partial \Lambda}{\partial t_{PA}} = Q_A + d_{AA} \cdot l_{AA} + d_{AB} \cdot l_{AB} = 0 \tag{8}$$

$$\frac{\partial \Lambda}{\partial t_{CB}} = Y_B + d_{AB} \cdot l_{AB} + d_{BB} \cdot l_{BB} = 0 \tag{9}$$

$$\frac{\partial \Lambda}{\partial t_{PB}} = Q_B + d_{BA} \cdot l_{BA} + d_{BB} \cdot l_{BB} = 0 \tag{10}$$

$$\frac{\partial \Lambda}{\partial l_{AA}} = d_{AA} \cdot \left(S_A + t_{PA} + t_{CA} - D_A\right) = 0 \tag{11}$$

$$\frac{\partial \Lambda}{\partial l_{AB}} = d_{AB} \cdot \left(S_A + t_{PA} + t + t_{CB} - D_B\right) = 0 \tag{12a}$$

$$\frac{\partial \Lambda}{\partial l_{BA}} = d_{BA} \cdot \left(S_B + t_{PB} + t + t_{CA} - D_A\right) = 0 \tag{12b}$$

$$\frac{\partial \Lambda}{\partial l_{BB}} = d_{BB} \cdot \left(S_B + t_{PB} + t_{CB} - D_B\right) = 0 \tag{13}$$

Because of the absence of cross-hauling, we know that at most either one of the equations (5a) and (5b) will apply, and that (4) and (6) only apply if $Z_{AA}>0$ and $Z_{BB}>0$, respectively. We solve the system of equations (4)-(13) by substituting (7) and (8) into (4); (8) and (9) into (5a); (7) and (10) into (5b); and (9) and (10) into (6). This yields:

$$t_{CA} + t_{PA} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_B}{\partial Q_A} \tag{14a}$$

$$t_{PA} + t_{CB} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_B}{\partial Q_A} + \frac{\partial E_A}{\partial T} + \frac{\partial E_B}{\partial T} \tag{14b}$$

$$t_{CA} + t_{PB} = \frac{\partial E_A}{\partial Q_B} + \frac{\partial E_B}{\partial Q_B} + \frac{\partial E_A}{\partial T} + \frac{\partial E_B}{\partial T} \tag{14c}$$

$$t_{CB} + t_{PB} = \frac{\partial E_A}{\partial Q_B} + \frac{\partial E_B}{\partial Q_B} \tag{14d}$$

The general message of (14a-d) is that each delivery, intra-regional or inter-regional, should be taxed at a rate equal to the marginal external cost associated with that particular type of delivery, which is the standard Pigouvian tax rule. Note that each Pigouvian tax is divided into a tax on the producers and a tax on the consumers involved. Because at most 3 equations from the set (14a-d) apply simultaneously, while 4 taxes are available, there is always at least one redundant tax. Therefore, the 4 taxes can always be set such that the relevant conditions among (14a-d) are satisfied.

3.2. Second-best taxes (I): optimizing regional welfare using both production and consumption taxes

The policy rules derived above will result in the Pareto optimal spatio-economic configuration where the joint welfare of the two regions is maximized. Hence, if the regions would fully cooperate by coordinating their policies optimally and sharing tax revenues according to rules such that both regions gain from marginal total welfare improvements, tax rules according to (14a-d) would apply. Note also that 'partial' welfare losses due to optimal taxation, in terms of reduced consumers' or producers' surpluses in either region, compared with any other feasible configuration, could be more than compensated for. This follows from the tautological observation that first-best taxes lead to a maximization of total welfare.

Nevertheless, regulatory taxes as given in (14a-d) do not normally constitute a Nash equilibrium. In particular, there will often be incentives for a regional government to deviate from the tax rules implied by (14a-d). In this sub-section, we will investigate this matter by deriving the tax rules to be adopted by a regional government that is solely concerned with local regional welfare when setting consumption and production taxes. Such a regulator fully ignores welfare in the other region, and thus may even 'exploit' foreign consumers or producers whenever this would improve the welfare in the home region.

Let us consider the relevant maximization problem for the regulator in region A. The associated Lagrangian can be written as:

$$\begin{aligned}\Lambda = \int_0^{Y_A} D_A(Y_A)dy_A - D_A(Y_A)\cdot Y_A + t_{CA}\cdot Y_A - \int_0^{Q_A} S_A(Q_A)dq_A + S_A(Q_A)\cdot Q_A + t_{PA}\cdot Q_A \\ - E_A(Q_A, Q_B, T) \\ + d_{AA}\cdot l_{AA}\cdot\left(S_A(Q_A) + t_{PA} + t_{CA} - D_A(Y_A)\right) \\ + d_{AB}\cdot l_{AB}\cdot\left(S_A(Q_A) + t_{PA} + t + t_{CB} - D_B(Y_B)\right) \\ + d_{BA}\cdot l_{BA}\cdot\left(S_B(Q_B) + t_{PB} + t + t_{CA} - D_A(Y_A)\right) \\ + d_{BB}\cdot l_{BB}\cdot\left(S_B(Q_B) + t_{PB} + t_{CB} - D_B(Y_B)\right)\end{aligned} \tag{15}$$

The following first-order conditions can now be derived:

$$\begin{aligned}\frac{\P\Lambda}{\P Z_{AA}} = -D'_A\cdot Y_A + t_{CA} + S'_A\cdot Q_A + t_{PA} - \frac{\P E_A}{\P Q_A} \\ + d_{AA}\cdot l_{AA}\cdot\left(S'_A - D'_A\right) + d_{AB}\cdot l_{AB}\cdot S'_A - d_{BA}\cdot l_{BA}\cdot D'_A = 0\end{aligned} \tag{16}$$

$$\begin{aligned}\frac{\P\Lambda}{\P Z_{AB}} = S'_A\cdot Q_A + t_{PA} - \frac{\P E_A}{\P Q_A} - \frac{\P E_A}{\P T} \\ + d_{AA}\cdot l_{AA}\cdot S'_A + d_{AB}\cdot l_{AB}\cdot\left(S'_A - D'_B\right) - d_{BB}\cdot l_{BB}\cdot D'_B = 0\end{aligned} \tag{17a}$$

$$\begin{aligned}\frac{\P\Lambda}{\P Z_{BA}} = -D'_A\cdot Y_A + t_{CA} - \frac{\P E_A}{\P Q_B} - \frac{\P E_A}{\P T} \\ - d_{AA}\cdot l_{AA}\cdot D'_A + d_{BA}\cdot l_{BA}\cdot\left(S'_B - D'_A\right) + d_{BB}\cdot l_{BB}\cdot S'_B = 0\end{aligned} \tag{17b}$$

$$\frac{\P\Lambda}{\P Z_{BB}} = -\frac{\P E_A}{\P Q_B} - d_{AB}\cdot l_{AB}\cdot D'_B + d_{BA}\cdot l_{BA}\cdot S'_B + d_{BB}\cdot l_{BB}\cdot\left(S'_B - D'_B\right) = 0 \tag{18}$$

$$\frac{\P\Lambda}{\P t_{CA}} = Y_A + d_{AA}\cdot l_{AA} + d_{BA}\cdot l_{BA} = 0 \tag{19}$$

$$\frac{\P\Lambda}{\P t_{PA}} = Q_A + d_{AA}\cdot l_{AA} + d_{AB}\cdot l_{AB} = 0 \tag{20}$$

$$\frac{\P\Lambda}{\P l_{AA}} = d_{AA}\cdot\left(S_A + t_{PA} + t_{CA} - D_A\right) = 0 \tag{21}$$

$$\frac{\P\Lambda}{\P l_{AB}} = d_{AB}\cdot\left(S_A + t_{PA} + t + t_{CB} - D_B\right) = 0 \tag{22a}$$

$$\frac{\P\Lambda}{\P l_{BA}} = d_{BA}\cdot\left(S_B + t_{PB} + t + t_{CA} - D_A\right) = 0 \tag{22b}$$

$$\frac{\P\Lambda}{\P l_{BB}} = d_{BB}\cdot\left(S_B + t_{PB} + t_{CB} - D_B\right) = 0 \tag{23}$$

Again at most either one of the equations (17a) and (17b) will apply; and (16) and (18) only apply if $Z_{AA}>0$ and $Z_{BB}>0$. However, in order to restrict the number of cases to be considered, it will be assumed from now on that $Z_{AA}>0$ and $Z_{BB}>0$, and hence $\delta_{AA}=1$ and $\delta_{BB}=1$. Second-best tax rules for the more restrictive cases where either of the two regions lacks operative producers or consumers can of course easily be derived in a similar manner as the tax rules to be derived below, and can usually be expected to be straightforward variations on these rules, with terms relating to the absent consumers or producers either removed or amended in an appropriate way. Under this assumption that δ_{AA}=1 and δ_{BB}=1, three trade regimes remain to be considered: A is a net exporter (regime I: δ_{AB}=1 and δ_{BA}=0); A is a net importer (II: δ_{AB}=0 and δ_{BA}=1); and the no-trade regime (III: δ_{AB}= δ_{BA}=0). The second-best taxes for these three regimes will be presented below. A last general remark is that the inefficiency due to the public nature of the externality is already reflected in (16)-(18) by the absence of terms related to E_B: the regulator in region A is, without policy coordination, of course not concerned with the valuation of the externality in region B.

I: Region A is a net-exporter

In case I, equations (17b) and (22b) become irrelevant. The remaining equations can then be solved for the following second-best optimal taxes:

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} + \frac{\partial E_A}{\partial T} - \frac{\frac{\partial E_A}{\partial Q_B}}{1 + \frac{S'_B}{-D'_B}} + \frac{Q_A - Y_A}{\frac{1}{-D'_B} + \frac{1}{S'_B}} \qquad (24)$$

$$\tau_{CA} + \tau_{PA} = \frac{\partial E_A}{\partial Q_A} \qquad (25)$$

First note that (25) could easily be solved for τ_{CA} alone by substitution of (24); the present representation merely facilitates the interpretation. In particular, (25) shows that the sum of τ_{CA} and τ_{PA} should be such that local deliveries are taxed according to the standard Pigouvian rule based on the local valuation of marginal emissions from local production. Hence, local production is regulated according to the 'quasi-first-best' tax rule, which only ignores the trans-boundary effects of local emissions are ignored. Equation (24) shows the remaining net tax on exports. Apart from the same quasi-first-best tax rule on marginal emissions from local production, three additional terms appear. These reflect various second-best considerations that are relevant in the present setting.

The second term in (24), for instance, captures the local impact of emissions from transport, and has the same quasi-first-best structure as the right-hand side of (25). As one might suspect, these external costs are treated in exactly the same manner as the external costs due to production are treated.

Next comes a term that is related to the local valuation, in region A, of marginal emissions from production in region B. This term has an additional second-best weighting factor w=1/(1+(S_B'/–D_B')). For the interpretation of this third term, first observe that due to the weighting factor, its overall sign is non-positive and generally negative (D_R' denotes the slope of the demand curve in region R and is therefore non-positive; S_R' gives the slope of the supply curve and is therefore non-negative). This captures the derived impact that local taxation of exports will have on emissions by foreign producers: a higher tax will lead to a comparative disadvantage of the local suppliers on the foreign market, and will therefore generally lead to an increase in foreign production for the foreign market, and hence to increased emissions. The weighting factor (denoted w hereafter) may run from 0 to 1, which indicates that in the two limiting cases the local valuation of emissions by foreign producers can be completely ignored (w=0), or should be considered equally important as the direct effect of the tax on emissions by local producers and the transport sector (w=1). It is instructive to consider these limiting cases somewhat further.

The case where w=0 can occur in the first place when the foreign demand is perfectly elastic in the second-best optimum: $D_B'=0$. This reflects that in that case, the local taxes in region A will not affect the size of the intra-regional delivery in region B, due to the insensitivity of the market price in region B. Secondly, w=0 can occur when the foreign supply is perfectly inelastic: $S_B'=\infty$. Also then, the size of the intra-regional delivery in region B will not be affected by policies in region A, simply because the foreign production level is given. In either case, the policies in A will leave the local production in B and therewith the emissions unaffected, so that the potential interaction can in fact be ignored completely in the second-best tax rule.

In contrast, w=1 occurs in the opposite cases where either the foreign demand is perfectly inelastic ($D_B'=-\infty$), or its supply is perfectly elastic ($S_B'=0$) in the second-best optimum. In both cases, on the market in B, every unit of good produced by A that is 'priced off the market' due to the regulation in A itself will be replaced by a unit of good produced in B. With $D_B'=-\infty$, this results directly from the inelasticity of the demand. With $S_B'=0$, the market price in B cannot be influenced by region A, and hence neither can the total consumption in B. Recalling that intra-regional deliveries in region A itself are fine-tuned using (25), the result that in these cases where w=1, the tax on exports should be corrected by fully subtracting the marginal external costs caused by production in region B then follows directly. Due to the complete substitution, the 'net' marginal external cost of production in A for the foreign market is $\partial E_A/\partial Q_A + \partial E_A/\partial T - \partial E_A/\partial Q_B$. Indeed, if production in B for the local market is more polluting than production in A and transporting goods to B, the sum of the first three terms in (24) becomes negative, showing that subsidization of exports is in such cases beneficial from an environmental point of view.

For intermediate cases where $0<w<1$, the various processes indicated above are traded off according to the expression for w. Having discussed the polar cases, the observation that w is increasing in (absolute value of) the slope of the foreign demand curve and decreasing in the slope of the foreign supply curve is sufficient to trace the eventual impact of these slopes on the extent to which emissions from production abroad should be considered in setting the second-best taxes, and to understand why this should be the case.

The fourth and last term in (24) represents the so-called 'terms-of-trade' effect. In the present case where A is a net exporter, this term represents the revenues that can be extracted from the foreign market by taxing local production for the purpose of driving up prices in region B. The numerator shows that the advantage of doing so increases linearly in the exports to region B. The denominator shows that if either the demand or the supply in the foreign market is perfectly elastic ($D_B'=0$ or $S_B'=0$), this term vanishes in the second-best tax rule. In that case, taxation for this purpose of affecting the terms of trade becomes useless: prices are given in region B, and taxes would only distort the essentially efficient market process in A (note that for the terms-of-trade effect *per se*, inefficiencies due to environmental externalities do not play a role). The more inelastic the excess demand in region B, the larger the ‘market power’ enjoyed by the regulator in A, and hence the more strongly he would be inclined to exploit this power.

II: Region A is a net-importer

In case II, equations (17a) and (22a) become irrelevant among the first-order conditions (16)-(23). Solving the remaining set of equations then yields the following second-best optimal taxes:

$$t_{CA} = \frac{\partial E_A}{\partial T} + \frac{\dfrac{\partial E_A}{\partial Q_B}}{1 + \dfrac{S'_B}{-D'_B}} + \frac{Y_A - Q_A}{\dfrac{1}{-D'_B} + \dfrac{1}{S'_B}} \tag{26}$$

$$t_{CA} + t_{PA} = \frac{\partial E_A}{\partial Q_A} \tag{27}$$

Because (27) is identical to (25), intra-regional deliveries in region A are again taxed according to the ‘quasi-first-best’ tax rule. The difference with case I is that now the local consumption tax (26) is adapted to deal with second-best aspects due to the openness of the economy. This is what one could expect for a net-importer.

The first term in (26) shows how the local valuation of emissions from transport should according to the quasi-first-best principle enter the tax rule for imports. The second term shows how the local valuation of emissions from foreign production should be added to this. Apart from differing in sign for the obvious reason that now a local consumption tax instead of a local production tax is used to influence emissions abroad, this term, including its weighting factor, is identical to the third term in (24). The weight with which the factor $\partial E_A/\partial Q_B$ should be included in the second-best pricing rule again varies between 0 and 1, for exactly the same reasons and according to the same principles as outlined above. Finally, the third term again represents the terms-of-trade effect. In this case, a higher tax on imports will have a depressing effect on the market price in region B, and hence will improve region A’s terms of trade. This term is now linearly increasing in the net imports in the second-best optimum, and is weighted by the same factor encountered in the fourth term in (25). Hence, this terms-of-trade effect again becomes more important in the second-best tax rule as the excess supply of region B becomes more inelastic, so that region A has more control over the price in region B.

III: No trade

Finally, in the no-trade equilibrium, both (17ab) and (22ab) become irrelevant. The first-order conditions can then be solved to yield:

$$t_{CA} + t_{PA} = \frac{\partial E_A}{\partial Q_A} \tag{28}$$

Local deliveries should be taxed according to the quasi-first-best Pigouvian principle. Because no direct interactions exist between the two regions, other than environmental spill-overs, there is obviously no ground for adopting local tax policies for the purpose of affecting emissions from transport or from production in the other region, or for the purpose of improving the terms of trade.

3.3. Second-best taxes (II): optimizing regional welfare using production taxes only

The tax scheme presented in the foregoing sub-section involves the joint usage of two different taxes. Under the assumption of non-cooperation, this yields the optimal outcome for a region, since taxes can be differentiated for the two commodity flows (intra-regional deliveries and inter-regional trade) that the region can directly affect. In practice, however, such a scheme may be considered as too complicated, and regulators may rely on simpler solutions, involving the use of one single tax only. We will therefore now present the second-best tax rules for production taxes (this sub-section) and consumption taxes (Section 3.4).

For finding the second-best production taxes without inter-regional policy coordination, the Lagrangian in (15) and the first-order conditions (16)-(23) should be adapted by setting the term τ_{CA} equal to zero and removing (19). The implied system of equations can then be solved for the three possible trade regimes. Again assuming that for both regions, intra-regional deliveries are positive in all second-best optima, the following tax rules can then be found:

I: Region A is a net-exporter

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} + \frac{\frac{\partial E_A}{\partial T}}{1+\frac{1}{\frac{-D'_A}{-D'_B}+\frac{-D'_A}{S'_B}}} - \frac{\frac{\partial E_A}{\partial Q_B}}{1+\frac{S'_B}{-D'_B}+\frac{S'_B}{-D'_A}} + \frac{Q_A - Y_A}{\frac{1}{-D'_A}+\frac{1}{-D'_B}+\frac{1}{S'_B}} \qquad (29)$$

II: Region A is a net-importer

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} - \frac{\frac{\partial E_A}{\partial T}}{1+\frac{1}{\frac{-D'_A}{-D'_B}+\frac{-D'_A}{S'_B}}} - \frac{\frac{\partial E_A}{\partial Q_B}}{1+\frac{S'_B}{-D'_B}+\frac{S'_B}{-D'_A}} - \frac{Y_A - Q_A}{\frac{1}{-D'_A}+\frac{1}{-D'_B}+\frac{1}{S'_B}} \qquad (30)$$

III: No trade

$$\tau_{PA} = \frac{\partial E_A}{\partial Q_A} \qquad (31)$$

First of all, comparing (31) with (28) reveals that in a second-best optimum without trade, the unavailability of the second tax – the consumption tax in this case – creates no additional constraints, and hence no additional welfare losses compared to case discussed in Section 3.2. This is consistent with the redundancy of one of the two taxes in (28). The interpretation of (31) is therefore identical to that of (28).

Equations (29) and (30) show that in second-best equilibria with trade, however, things will change when the consumption tax can no longer be used. Evidently, the local welfare level achieved will then normally be below, and never above, the level obtained with the combinations (24) and (25) for a net-exporter, and (26) and (27) for a net-importer. This follows from the fact that the second-best optimal consumption taxes (25) and (27) can always be set at zero.

For the interpretation of (29) and (30), first note their similar structures. The only differences are given by the sign of the terms related to the emissions from transport and the terms-of-trade effect. Both are positive when region A is a net exporter, which reflects that in that case taxation of local production is beneficial for these two sub-goals: it will reduce transport emissions, and it will lead to a higher price in region B from which the exporting region A can benefit. The opposite holds when A is a net importer. A higher local production tax is then counter-productive: it would then lead to more transport because the comparative advantage of the exporting region B is increased, while the higher price in region B then implies a negative terms of trade effect for the importing region A.

A further interpretation of (29) becomes possible after observing that this tax rule can actually be seen as a 'weighted average' of the tax rules (24) and (25), that apply when also a separate consumption tax is available. One has to be careful with the term 'weighted average', because it should of course not be ignored that the terms involved in the optimal tax rules may differ for the respective second-best equilibria, due to the possible variability of the terms involved. Still, it can be observed that when the local demand is perfectly inelastic ($D_A'=-\infty$), (29) becomes identical to (24), which is the net tax on exports when both taxes are available. In contrast, when the local demand is perfectly elastic ($D_A'=0$), the last three terms vanish and (29) becomes identical to (25), which is the net tax on intra-regional deliveries when both taxes are available. In that way, the two sub-goals of regulating intra-regional deliveries and exports are traded off when only one tax is available, and the relative weights given to these sub-goals depends on the elasticity of the local demand. The further structure of (29) can therefore be explained along the same lines as (24) and (25) were interpreted.

The same reasoning actually holds for the tax rule (30), applying in case II where region A is a net-importer. This can be verified by substituting (26) in (27). It then also follows immediately that when the local demand is perfectly inelastic ($D_A'=-\infty$), (30) becomes identical to τ_{PA} implied by (26) and (27), and when the local demand is perfectly inelastic ($D_A'=0$), (30) becomes identical to the net tax on intra-nodal deliveries (27).

3.4. Second-best taxes (III): optimizing regional welfare using consumption taxes only

Finally, for finding the second-best consumption tax without inter-regional policy coordination, the Lagrangian (15) and the first-order conditions (16)-(23) should be adapted by setting τ_{PA} equal to zero and removing (20). The implied system of equations can then again be solved for the three possible trade regimes. Maintaining the assumption that for both regions, intra-regional deliveries are positive in all second-best optima, the following tax rules can then be found:

I: Region A is a net-exporter

$$\tau_{CA} = \frac{\frac{\partial E_A}{\partial Q_A}}{1+\frac{S'_A}{-D'_B}+\frac{S'_A}{S'_B}} - \frac{\frac{\partial E_A}{\partial T}}{1+\frac{1}{\frac{S'_A}{-D'_B}+\frac{S'_A}{S'_B}}} + \frac{\frac{\partial E_A}{\partial Q_B}}{1+\frac{S'_B}{-D'_B}+\frac{S'_B}{S'_A}} - \frac{Q_A - Y_A}{\frac{1}{-D'_B}+\frac{1}{S'_A}+\frac{1}{S'_B}} \tag{32}$$

II: Region A is a net-importer

$$\tau_{CA} = \frac{\frac{\partial E_A}{\partial Q_A}}{1+\frac{S'_A}{-D'_B}+\frac{S'_A}{S'_B}} + \frac{\frac{\partial E_A}{\partial T}}{1+\frac{1}{\frac{S'_A}{-D'_B}+\frac{S'_A}{S'_B}}} + \frac{\frac{\partial E_A}{\partial Q_B}}{1+\frac{S'_B}{-D'_B}+\frac{S'_B}{S'_A}} + \frac{Y_A - Q_A}{\frac{1}{-D'_B}+\frac{1}{S'_A}+\frac{1}{S'_B}} \tag{33}$$

III: No trade

$$\tau_{CA} = \frac{\partial E_A}{\partial Q_A} \tag{34}$$

The interpretation of (32)-(34) can be given along the same lines as that of (29)-(31). Now the important additional factor, compared to the case where both taxes are available, is the sensitivity of local supply in the second-best optimum, reflected by the slope of the supply curve S_A'. With perfectly inelastic local supply ($S_A'=\infty$), (32) becomes identical to the expression for τ_{CA} according to (24) and (25), and (33) becomes identical to (26); with perfectly elastic supply ($S_A'=0$), both (32) and (33) become identical to (25) and (27). Finally, it is again found that in a no-trade second-best equilibrium, local deliveries should be taxed according to the quasi-first-best rule (34). Because of the similarity of the present case with what was found for the previous case where only production taxes could be used, we abstain from a further interpretation of (32)-(34).

3.5. One tax only: production tax or consumption tax?

The last question we want to address in this section concerns the question of which of the two above taxes – that is: production versus consumption – would be preferable in case the regulator is, for whatever reason, restricted to making this choice. Evidently, if regulation were costless, it is always preferable to use both: as argued above, one of the two can always be set at a zero level, and hence higher levels of regional welfare generally result when (24) and (25), or (26) and (27), are applied simultaneously. However, when for instance administrative costs are considered, which are not modelled explicitly here, things may become different, and especially when the raising of an additional tax involves some fixed costs, it may be preferable to use only one tax. In such cases, this question becomes relevant.

A general answer to this question does not exist. That is, it is not the case that either production taxes or consumption taxes by definition always allow the regulator to obtain a higher level of regional welfare than would be achievable with the other tax.

In general, however, the tendency will be that net-exporters prefer to use production taxes, and net-importers prefer consumption taxes. The intuition is that they can then directly affect a larger share of the total production in the entire system. Moreover, the various sub-goals reflected in the second-best tax rules then require, at least to a larger degree, equally signed tax terms. For a net-importer, this is evident because (33) consists of four positive terms, while (30) has one positive term and three negative ones. For a net-exporter, the difference is somewhat more subtle: (29) has three positive terms and one negative one, and (32) two positive terms and two negative ones. In general, different signs in a second-best tax rule indicate counteracting forces in the determination of the second-best optimal tax rule: the various sub-goals require opposite incentives. The more this can be avoided, the more efficiently the tax can be used. In the extreme case, where the various sub-goals would exactly cancel and sum up to a second-best tax equal to zero, the instrument is entirely ineffective and inefficient. The best strategy is then not to use the tax at all, since any value different from zero – be it positive or negative – then implies a lower welfare level than obtained when the tax in fact is not used, and is set equal to zero.

Along this same line of reasoning, it is also possible to demonstrate that it is not generally true that a net-exporter would prefer production taxes, and a net-importer consumption taxes. To do so, it is sufficient to construct a counter-example in terms of a set of parameter values, for which the second-best production tax for the net-exporter would be equal to zero whereas the second-best consumption tax would not, and reversely, the second-best consumption tax for the net-importer would be equal to zero whereas the second-best production tax would not.

For the net-exporter, this would be the case when (29) is equal to zero while (32) is not, for instance if the net-exporter's local demand is perfectly elastic (D'_A=0), and the local producers do not emit any local externalities ($\partial E_A/\partial Q_A$=0). The production tax then indeed is an entirely inefficient instrument. Because of the perfectly elastic local demand, the after-production-tax-producers'-price in the exporting region cannot be affected by the production tax. This implies that also the trade and transport flow cannot be affected. Hence, the only delivery that can be affected is the local intra-regional delivery. However, because local production causes no local externalities at all, it is actually welfare reducing to affect this intra-regional delivery. In particular, any deviation from the market outcome with a zero production tax would imply a welfare deterioration, because a production tax would drive a (welfare-reducing) wedge between regional marginal benefits and regional marginal costs of local deliveries. Since (32) certainly is not necessarily equal to zero when D'_A=0 and $\partial E_A/\partial Q_A$=0, it follows that a local consumption tax may yield regional welfare improvements, and hence would be preferable to a local consumption tax under the described circumstances.

Likewise, one can construct a comparable situation where for a net-importer, the tax rule in (33) would produce a second-best optimal consumption tax equal to zero, implying no possible welfare gain whatsoever, whereas the second-best production tax rule in (30) would be unequal to zero, indicating that welfare improvements can be achieved. This would involve the situation where the net importer's local supply is perfectly elastic (S'_A=0), and the local producers do not emit any local externalities ($\partial E_A/\partial Q_A$=0). A consumption tax would then be unable to affect the (after-tax) producers' price, implying that the trade and transport volume is given. The same sort of reasoning as given above therefore again applies: only the intra-regional delivery – not the trade and transport volume – can be affected. However, this delivery is optimized already under unregulated free-market conditions.

We therefore conclude that if, for whatever reason, a regulator is restricted to using one tax instrument only, the tendency will be that net-exporters prefer to use production taxes, and net-importers to use consumption taxes. However, counter-examples where the opposite holds can easily be constructed, and hence this inference is not generally true.

4. The overall system effects of second-best policies

If the two regions cooperate, they would, of course, be best off applying the first-best tax scheme (14a-d). Provided the regions also negotiate on the distribution of the tax revenues, compensation schemes then by definition must exist that also maximize the individual regional welfare levels. Hence, both from an overall system's perspective and from the regional perspective, such a tax scheme could be welfare maximizing. On the other hand, the schemes discussed in Sections 3.2-3.4 describe situations where regions, under various restrictions, aim to maximize regional welfare only. The question then of course arises how these policies compare, from overall system's perspective, to the two bench-marks one could distinguish, namely the non-intervention outcome and the first-best situation. In other words, the question is what implicit price is paid for the failure of the two governments to cooperate in formulating joint welfare maximising policies.

Based on the general specifications used in this paper, no conclusive answers can be given to this question, since comparative static analyses can only be performed once an explicit model is specified. This, in turn, would have the obvious disadvantage that the level of generality is brought down considerably.[3] However, based on the tax rules presented in Section 3, some first inferences still can be made. For that purpose, it can in the first place be noted that, even if we restrict ourselves to equilibria with trade, no fewer than 17 relevant regimes can be considered: the optimal one presented in Section 3.1, and 4×4 regimes where both regions can employ no tax at all, both second-best taxes as in Section 3.2, a production tax as in Section 3.3, or a consumption tax as in Section 3.4. Another 5 additional cases become relevant when we would also consider equilibria without trade: optimal taxes, and 4 non-coordinated tax regimes, where both regions can choose whether or not to apply the second-best non-coordinated tax.

Among the resulting 22 possible schemes, we limit ourselves in the first place only to equilibria with trade. Table A.1 in the Appendix then shows the possible taxes that both regions (the net-exporter E and the net-importer I) can apply. The tax rules shown in Table A.1 follow directly from those derived in Section 3. The table shows the marginal effect of a tax on the three types of delivery (intra-regional deliveries within both regions, and the uni-directional inter-regional trade flow), and thus enables one to assess the marginal tax rates that in each of the 17 regimes with trade would apply for each of these deliveries (rows

[3] A paper presenting a simulation model based on the analyses presented here, however, is in preparation.

representing ‘zero-taxes’ are of course left out of the table). Note that for the determination of the marginal tax on the inter-regional delivery, one should sum two expressions, from the two columns, because both regions will normally tax (or subsidize) this delivery.

Next, we limit ourselves further by considering only 4 tax regimes among these 17. These are in the first place the two bench-marks, where either first optimal taxes or no taxes at all apply. We concentrate, however, on two second-best regimes, namely the case where both regions apply both taxes, and the case where the net-exporter uses a production tax and the net-importer a consumption tax. Using Table A.1, one can then derive the marginal tax rates, for each of the three deliveries Z_{EE}, Z_{II} and Z_{EI}, in each of the three regimes with taxes. These are presented in Table A.2. Therefore, the tax rates shown in Table A.2 indicate the differences between these three tax regimes, and the situation with no taxes at all.

In particular for the interpretation of the second-best tax rules, it is of course tempting to compare these rules directly to those applying in the first-best situation. It is important to emphasize, however, that this exercise has only a limited comparative static relevance, because many of the terms involved in the various tax rules may vary between equilibria (for instance, the first-best optimum compared to one of the second-best equilibria). Therefore, differences between expressions in Table A.1 and A.2, shown in Table A.3, would straightforwardly give the difference in actual equilibrium tax rates only if all terms involved were constants. While this could still be true for the various marginal external costs and the slopes of the relevant supply and demand curves, it is hard to see how this could be the case for the trade balance T. Consequently, the expressions presented in Tables A.1 and A.2 should be compared only with sufficient care, and differences should be interpreted only as listings of relevant terms lacking from (or ‘falsely’ included in) second-best marginal tax rules, compared to optimal rules.

Now if we first consider the regime with both taxes (regime BB), and focus on taxes on the both intra-regional deliveries, it appears that the relevant taxes expressions suggest tax rates between those applying in the no-tax situation, and first-best taxes. Although the cross-effects in the optimal taxes, accounting for spatial externality spill-overs, are lacking, the remaining terms still indicate positive taxes. For the tax on intra-regional deliveries, however, thing are more complex. The resulting rule may imply a negative tax (when all externalities except $\partial E_E/\partial Q_i$ are zero and $S'_E=S'_I=0$), may take on a value below the optimal level, or may suggest a tax exceeding the first-best tax (for instance if $S'_E=0$ and $S'_I=\infty$). As long as the tax rule suggests a positive tax below the optimal level, one would therefore expect the non-cooperative equilibrium where both regions apply both taxes to result in a below optimal welfare level, but still an improvement compared to the situation where no taxes at all apply. However, if the intra-regional delivery is relatively important, and if the relevant parameters dictate an overall tax on this delivery that is either negative or exceeds the optimal rule, it could in theory be possible that the total welfare in the system even falls below the level with no taxes. Hence, a general conclusion is not possible.

If we next look at the marginal tax rules applying in the situation where the net-exporter uses production taxes and the net-importer consumption taxes (regime PC), it can be noted that the variability increases compared to the situation where both regions apply both taxes. The tax on the intra-regional delivery in the exporting region may become negative, may be between the tax in regime BB and the optimal tax, and may exceed the optimal tax. The tax on the intra-regional deliver in the importing region, although strictly non-negative, may be below the tax in region BB or may exceed it, and could also exceed the optimal tax. Finally, if we compare the tax on the inter-regional delivery in regimes PC and BB, it can be seen that in the former contains one extra positive term, but that the denominators in the subsequent terms are usually larger. Again, therefore, no general conclusion on the relative size of this tax, compared to other regimes, can be given.

Clearly, therefore, if we do not consider a full equilibrium model, and compare the marginal tax rules in the various regimes, no general conclusion on the relative performance of the two-second-best regimes can be given. The question of how the BB and PC-regime compare to each other is absolutely open. However, it can be noted that for most parameter combinations, one would expect a welfare level exceeding the level resulting in the no-tax regime for both the BB and the PC-regime. Nevertheless, given the relative complexity of the various tax rules, however, and given the neglect of the endogeneity of the relevant terms present in these rules, it seems worthwhile to consider these questions further using a numerical simulation model.

5. Conclusion

The international and spatial dimension of environmental pollution and environmental policy making has recently received ample attention in international negotiations on environmental protection as well as in the environmental economics literature. In this paper, we presented an analytical framework for analysing spatial aspects of environmental policies in the regulation of trans-boundary externalities. A spatial price equilibrium model for two regions was discussed, where interactions between the regions can occur via trade and transport, via mutual environmental spill-overs due to the externality that arises from production, and via tax competition when the regions do not behave cooperatively. Also the additional complications arising from emissions caused by the endogenous transport flows are considered explicitly in the model presented. First-best and second-best tax rules were derived and interpreted. First-best, here, refers to the situation where both regions coordinate their policies optimally so as to maximize their joint welfare, whereas second-best taxes apply in those situations where policy coordination is lacking and regions aim to maximize their own welfare only.

The analyses showed that, whereas first-best taxes still take on the standard Pigouvian form, second-best tax expressions can become quite complex, even in the relatively simple setting chosen. The general forms clearly reflect the various sub-goals that a region has to trade off in the maximization of the local welfare. Because local consumers and producers are price-takers and therefore maximize the consumers' and producers' surpluses through their market behaviour, the remaining sub-goals for a regulator to consider are the local valuation of emissions by local producers, by transport, and by foreign producers; and the region's terms-of-trade. These goals are then traded-off in different manners in the various second-best tax rules discussed, depending of course on how the tax intervenes in the economic process.

Generally, the tax rules are weighted averages of terms reflecting the four sub-goals mentioned, where the weights are made up of terms related to the elasticities of the various relevant demand and supply functions. The exact interpretation of the various tax rules was given in Section 3.

A main conclusion arising from the analyses is, paradoxically, that general conclusions can hardly be drawn. For instance, although it was argued that (and why) in case a region can apply one tax only, a net-exporter will generally prefer a production tax and a net-importer a consumption tax, also counter examples could easily be given where the opposite holds. And, to give another example, although it was argued that (and why) uncoordinated second-best policies will often lead to a welfare improvement compared to the non-intervention situation, again this was seen to be not generally true. Moreover, when comparing two relatively likely second-best tax configurations, namely one where both regions use both taxes and one where the net-exporter uses a production tax and the net-importer a consumption tax, it appeared that from an overall system's perspective, no general welfare ranking can be given (although it was shown that from the perspective of a single region, the use of both taxes is strictly preferable when there are no administrative costs of regulation).

Notwithstanding the ambiguities, however, these findings are of course valuable, as they demonstrate and unravel the complexity of the issues at hand. Moreover, the various tax rules presented are of course generally valid, and enhance our insight into the general problems studied. These tax rules, as well as the general modelling framework, will proof to be useful in further research into this important area. Two important extensions of the present analyses we intend to address in the near future include the construction of a numerical simulation model in which the full equilibrium effects – as opposed to marginal tax rules – of first-best and various second-best regulatory schemes can be investigated, and the consideration of endogenous environmental technologies in the present framework.

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Appendix: Tables

Table A.1 Second-best tax rules for net-exporters (region E) and net-importers (region I)

	Net-Exporter	**Net-Importer**
	Optimum	
Z_{EE}	$\frac{\partial E_E}{\partial Q_E}$	$\frac{\partial E_I}{\partial Q_E}$
Z_{II}	$\frac{\partial E_E}{\partial Q_I}$	$\frac{\partial E_I}{\partial Q_I}$
Z_{EI}	$\frac{\partial E_E}{\partial Q_E}+\frac{\partial E_E}{\partial T}$	$\frac{\partial E_I}{\partial Q_E}+\frac{\partial E_I}{\partial T}$
	Non-coordination	
	Both taxes	
Z_{EE}	$\frac{\partial E_E}{\partial Q_E}$	0
Z_{II}	0	$\frac{\partial E_I}{\partial Q_I}$
Z_{EI}	$\frac{\partial E_E}{\partial Q_E}+\frac{\partial E_E}{\partial T}-\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}}+\frac{T}{\frac{1}{-D_I'}+\frac{1}{S_I'}}$	$\frac{\partial E_I}{\partial T}+\frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}}+\frac{T}{\frac{1}{-D_E'}+\frac{1}{S_E'}}$
	Production taxes	
Z_{EE}	$\frac{\partial E_E}{\partial Q_E}+\frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{-D_E'}{-D_I'}+\frac{-D_E'}{S_I'}}}-\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}+\frac{S_I'}{-D_E'}}+\frac{T}{\frac{1}{-D_E'}+\frac{1}{-D_I'}+\frac{1}{S_I'}}$	0

Z_{II}	0	$\frac{\partial E_I}{\partial Q_I} - \frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{-D_I'}{-D_E'}+\frac{-D_I'}{S_E'}}} - \frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}+\frac{S_E'}{-D_I'}} - \frac{T}{\frac{1}{-D_I'}+\frac{1}{-D_E'}+\frac{1}{S_E'}}$
Z_{EI}	$\frac{\partial E_E}{\partial Q_E} + \frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{-D_E'}{-D_I'}+\frac{-D_E'}{S_I'}}} - \frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}+\frac{S_I'}{-D_E'}} + \frac{T}{\frac{1}{-D_E'}+\frac{1}{-D_I'}+\frac{1}{S_I'}}$	0
	Consumption taxes	
Z_{EE}	$\frac{\frac{\partial E_E}{\partial Q_E}}{1+\frac{S_E'}{-D_I'}+\frac{S_E'}{S_I'}} - \frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{S_E'}{-D_I'}+\frac{S_E'}{S_I'}}} + \frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}+\frac{S_I'}{S_E'}} - \frac{T}{\frac{1}{-D_I'}+\frac{1}{S_E'}+\frac{1}{S_I'}}$	0
Z_{II}	0	$\frac{\frac{\partial E_I}{\partial Q_I}}{1+\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}} + \frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}} + \frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}+\frac{S_E'}{S_I'}} + \frac{T}{\frac{1}{-D_E'}+\frac{1}{S_I'}+\frac{1}{S_E'}}$
Z_{EI}	0	$\frac{\frac{\partial E_I}{\partial Q_I}}{1+\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}} + \frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}} + \frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}+\frac{S_E'}{S_I'}} + \frac{T}{\frac{1}{-D_E'}+\frac{1}{S_I'}+\frac{1}{S_E'}}$

Table A.1 Second-best tax rules for net-exporters (region E) and net-importers (region I) (con'd)

Table A.2 Marginal tax rates in three regimes

	Optimum
Z_{EE}	$\frac{\partial E_E}{\partial Q_E}+\frac{\partial E_I}{\partial Q_E}$
Z_{II}	$\frac{\partial E_E}{\partial Q_I}+\frac{\partial E_I}{\partial Q_I}$
Z_{EI}	$\frac{\partial E_E}{\partial Q_E}+\frac{\partial E_E}{\partial T}+\frac{\partial E_I}{\partial Q_E}+\frac{\partial E_I}{\partial T}$
	Both regions use both taxes
Z_{EE}	$\frac{\partial E_E}{\partial Q_E}$
Z_{II}	$\frac{\partial E_I}{\partial Q_I}$
Z_{EI}	$\frac{\partial E_E}{\partial Q_E}+\frac{\partial E_E}{\partial T}-\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}}+\frac{T}{\frac{1}{-D_I'}+\frac{1}{S_I'}}+\frac{\partial E_I}{\partial T}+\frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}}+\frac{T}{\frac{1}{-D_E'}+\frac{1}{S_E'}}$
	Net-exporter uses production taxes / Net-importer uses consumption taxes
Z_{EE}	$\frac{\partial E_E}{\partial Q_E}+\frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{-D_E'}{-D_I'}+\frac{-D_E'}{S_I'}}}-\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}+\frac{S_I'}{-D_E'}}+\frac{T}{\frac{1}{-D_E'}+\frac{1}{-D_I'}+\frac{1}{S_I'}}$
Z_{II}	$\frac{\frac{\partial E_I}{\partial Q_I}}{1+\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}+\frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}}+\frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}+\frac{S_E'}{S_I'}}+\frac{T}{\frac{1}{-D_E'}+\frac{1}{S_I'}+\frac{1}{S_E'}}$

Z$\mathbf{Z_{EI}}$	$\frac{\partial E_E}{\partial Q_E} + \frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{-D'_E}{-D'_I}+\frac{-D'_E}{S'_I}}} - \frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S'_I}{-D'_I}+\frac{S'_I}{-D'_E}} + \frac{T}{\frac{1}{-D'_E}+\frac{1}{-D'_I}+\frac{1}{S'_I}} + \frac{\frac{\partial E_I}{\partial Q_I}}{1+\frac{S'_I}{-D'_E}+\frac{S'_I}{S'_E}} + \frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{S'_I}{-D'_E}+\frac{S'_I}{S'_E}}} + \frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S'_E}{-D'_E}+\frac{S'_E}{S'_I}} + \frac{T}{\frac{1}{-D'_E}+\frac{1}{S'_I}+\frac{1}{S'_E}}$

Table A.2 Marginal tax rates in three regimes (con'd)

Table A.3 Differences in tax rules, optimum minus second-best in two regimes

	Both regions use both taxes
Z_{EE}	$\frac{\partial E_I}{\partial Q_E}$
Z_{II}	$\frac{\partial E_E}{\partial Q_I}$
Z_{EI}	$\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}}+\frac{\partial E_I}{\partial Q_E}-\frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}}-\frac{T}{\frac{1}{-D_E'}+\frac{1}{S_E'}}-\frac{T}{\frac{1}{-D_I'}+\frac{1}{S_I'}}$
	Net-Exporter uses production taxes / Net-Importer uses consumption taxes
Z_{EE}	$\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_I'}+\frac{S_I'}{-D_E'}}-\frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{-D_E'}{-D_I'}+\frac{-D_E'}{S_I'}}}+\frac{\partial E_I}{\partial Q_E}-\frac{T}{\frac{1}{-D_E'}+\frac{1}{-D_I'}+\frac{1}{S_I'}}$
Z_{II}	$\frac{\partial E_E}{\partial Q_I}+\frac{\partial E_I}{\partial Q_I}-\frac{\frac{\partial E_I}{\partial Q_I}}{1+\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}-\frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}+\frac{S_E'}{S_I'}}-\frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}}-\frac{T}{\frac{1}{-D_E'}+\frac{1}{S_I'}+\frac{1}{S_E'}}$
Z_{EI}	$\frac{\frac{\partial E_E}{\partial Q_I}}{1+\frac{S_I'}{-D_E'}+\frac{S_I'}{-D_I'}}+\frac{\partial E_E}{\partial T}-\frac{\frac{\partial E_E}{\partial T}}{1+\frac{1}{\frac{-D_E'}{-D_I'}+\frac{-D_E'}{S_I'}}}-\frac{\frac{\partial E_I}{\partial Q_I}}{1+\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}+\frac{\partial E_I}{\partial Q_E}-\frac{\frac{\partial E_I}{\partial Q_E}}{1+\frac{S_E'}{-D_E'}+\frac{S_E'}{S_I'}}+\frac{\partial E_I}{\partial T}-\frac{\frac{\partial E_I}{\partial T}}{1+\frac{1}{\frac{S_I'}{-D_E'}+\frac{S_I'}{S_E'}}}$ $-\frac{T}{\frac{1}{-D_E'}+\frac{1}{S_E'}+\frac{1}{S_I'}}-\frac{T}{\frac{1}{-D_E'}+\frac{1}{D_I'}+\frac{1}{S_I'}}$