Equilibrium airfares, frequencies and airport taxes in a multiple airport region: an application of the nested logit demand model.

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Abstract

In this paper a model, based on the nested multinomial logit model, is used to analyze airport competition and airline competition in a multiple airport region. It is shown that if the frequency elasticity of demand is smaller than 1, airfare-frequency and airport tax equilibria exist and are unique. Symmetric equilibria are derived analytically and their properties are discussed. Asymmetric equilibria are obtained numerically, and their properties are discussed.

Keywords: discrete choice model, optimal airfares, frequencies and airport taxes.
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1 Introduction

In this paper we investigate whether airfare-frequency equilibria between all airlines in a multiple airport region and global equilibria between all airlines and airports exist, and if so, are unique. To be able to derive the desired equilibria some simplifying assumptions are needed. We also comment on the effect of a relaxation of these assumptions.

The model is based on a nested logit demand model. Multinomial logit models (MNL) have been used extensively in the literature to analyze passenger preferences concerning airports, airport access modes and direct/indirect routes (see e.g. Caves et al. (1991) and Brooke et al. (1994) for UK studies, Hansen (1990) and Harvey (1987) for US studies and Bondzio (1996) for German studies). In most studies, access time, the airfare and the frequency of service are identified as important determinants of airport choice. A notable exception is Harvey (1987) who does not include the airfare because there appeared to be more variation among fare classes on a given flight to a particular destination than among different flights to that destination. Frequency is treated differently in the different studies mentioned above. It is included directly, in some quadratic form or logarithmic form for various reasons. Ndoh et al. (1990) found that a nested multinomial logit model (NMNL) is statistically preferable to a MNL to analyze the combined route-departure airport choice. Pels et al. (1998) found a NMNL to be preferred to a MNL to explain the combined airport-airline choice in the San Francisco Bay Area, while Bondzio (1996) found that both the MNL and NMNL can be preferred, depending on the passenger type (business, leisure) and choice (access mode, airport).

Anderson et al. (1996) present various (theoretical) applications of discrete choice models. These include quality choice models, in which the consumers' utility is determined by both the quality and price of a product. A two-level NMNL is used to model a multi-product firm. In the upper level, a firm is selected by the consumer, in the lower level, a product is selected. For both models, existence and uniqueness of equilibria is proven.

2 The passenger discrete choice model

Assume a passenger first chooses an airport $i \in D$, $D = \{1,\ldots,d\}$and then an airline j, $j \in L = \{1,\ldots,l\}$. Furthermore, assume $L(i) \subset L$, $L(i) \cap L(i') = \emptyset$ for $i \neq i'$, and $\bigcup_{i \in D} L(i) = L$; $L(i)$ is the subset of airlines operating from airport i. The cross section with subsets of airlines operating from other airports i' is empty: airlines are restricted to operate from one airport only.

The attractiveness of an airline j is determined by the number of flights f_j it offers to a particular destination and the airfare p_j it charges. A common approach in (spatial) economics is to model the discrete choice for an airline as: $\mathrm{P}(j) = A_j \exp\left(-a_p p_j\right) \Big/ \sum_{j \in L} A_j \exp\left(-a_p p_j\right)$, where A_j represents the attractivity of the alternative and a_p is a non-negative parameter. Then, if $A_j = A_j(f_j)$, this probability is rewritten as $\mathrm{P}(j) = \exp\left(-a_p p_j + a_f \ln(f_j)\right) \Big/ \sum_{j \in L} \exp\left(-a_p p_j + a_f \ln(f_j)\right)$. Jaïbi and ten Raa (1998) show this specification fits into the maximum utility framework of the logit model.

The utility of using airport i depends on the airport tax tax_i, the access time to the airport t_i and the maximum expected utility $\tilde{V}_i$ of the alternatives in the choice set (of airlines) available from each departure airport i. Then the probability that a combination (departure airport i, airline j) is chosen can be expressed as[3]:

$$\mathrm{P}(j,i) = \mathrm{P}(j|i)\mathrm{P}(i) \tag{1}$$

$$\mathrm{P}(j|i) = \frac{\exp\left(\dfrac{a_j - a_p p_j + a_f \ln(f_j)}{m}\right)}{\sum_{j \in L(i)} \exp\left(\dfrac{a_{j'} - a_p p_j + a_j \ln(f_j)}{m}\right)} \tag{2}$$

[3] We assume the random utility component derived from using the *combination (j,i)* is (independent and identically) Gumbel distributed with scale parameter m. The random utility derived from using departure airport i is distributed so that $\max_i U_{j,i}$ is Gumbel distributed with scale q, where $U_{j,i}$ is the total utility of using the combination *(j,i)*.

$$\mathrm{P}(i) = \frac{\exp(\frac{b_i - b_t tax_i + b_a \ln(t_i) + m \ln \tilde{V}_i}{q})}{\sum_{i \in D} \exp(\frac{b_i - b_t tax_{i'} + b_a \ln(t_i) m \ln \tilde{V}_i}{q})} \tag{3}$$

with

$$\tilde{V}_i = \sum_{j \in L(i)} \left(\frac{a_j - a_p p_j + a_f \ln(f_j)}{m} \right) \tag{4}$$

We assume the parameters a_p, a_f , b_t and b_a are larger than zero. The parameter m represents the degree of heterogeneity of airlines (flights) within (from) an airport. The closer m is to 0, the higher the degree of substitutability between airlines. q is a measure of heterogeneity between airports. It is necessary that $q > m$ (see e.g. Anderson et al., 1996). This means that airlines (flights) operating (originating) from the same departure airports are closer substitutes than airlines (flights) operating (originating) from different departure airports.

In the next Sections the nested logit model, as specified in equations 1,2,3 and 4, will be used to model airport and airline competition in a multiple airport region.

3 The airlines' maximization problem

An airline will operate a route to a particular destination from departure airport i, if it can generate non-negative profits (which implies the airfare p_j exceeds the marginal costs c_j). The airline's profits obtained from operations (out) of airport i on a certain route are:

$$p_j = (p_j - c_j) N \mathrm{P}(j,i) - k_j f_j - K_j, \ i \in D, j \in L(i) \tag{5}$$

where N is the total number of passengers in the system, P(j,i) is defined in equation (1), p_j is the price (airfare on that route), c_j is the (constant) marginal cost per passenger, f_j is frequency on that route, k_j is the constant marginal cost per flight and K_j is the fixed cost. The profit function is maximized with respect to the optimal fare and the frequency. Note that we do not assume the airlines play a two-stage game in which, for example, first the frequency and then the airfare is determined. Airfare and frequency are determined in a one stage game[4].

Maximizing profits with respect to the airfare yields:

$$\frac{\partial p_j}{\partial p_j} = N\mathrm{P}(j,i) + (p_j - c_j)\frac{\partial N\mathrm{P}(j,i)}{\partial p_j} = 0 \Leftrightarrow$$
$$p_j - c_j = \frac{qm}{a_p\left(m\mathrm{P}(j|i)(1-\mathrm{P}(i)) + q(1-\mathrm{P}(j|i))\right)} \tag{6}$$

Maximizing profits with respect to the frequency of service yields:

$$\frac{\partial p_j}{\partial f_j} = (p_j - c_j)\frac{\partial N\mathrm{P}(j,i)}{\partial f_j} = k \Leftrightarrow$$
$$f_j = \frac{a_f N\mathrm{P}(j,i)(p_j - c_j)}{k}\frac{\left(m\mathrm{P}(j|i)(1-\mathrm{P}(i)) + q(1-\mathrm{P}(j|i))\right)}{mq} \tag{7}$$

Equation 7 gives an expression for the optimum frequency at any given airfare; there is a whole range of airfares for which optimal frequencies can be determined. These airfares should satisfy equation (6) to be optimal. Substituting for the optimal airfare yields:

$$f_j = \frac{a_f}{a_p}\frac{\mathrm{P}(i)\mathrm{P}(j|i)}{k}N \tag{8}$$

Equations (6) and (7) give the optimal airfare and frequency for an airline *j*. At any other combination (f_j, p_j) the optimum will not be reached (i.e. only for one particular value of *p* the solution to the airline's best response frequency function (equation 7) maximizes profits).

Before deriving airline *j*'s optimal airfare and frequency, we first consider the frequency elasticity of demand. One desirable property of the demand function is that the frequency elasticity of demand, $e_{f_l}^{\mathrm{P}(l,d)}$, is smaller than 1; otherwise a 1 percent

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[4] In a "standard" two-stage game, the first order necessary conditions become much more complex.

change in the frequency would lead to a more than 1 percent change in the demand. As, in this model, the airline is assumed to have a constant load factor, the only way to accommodate this demand is by expanding its size (frequency), which results in a more than proportional increase in demand. Hence airline *j*'s demand exceeds its capacity. It will try to increase its capacity to generate more profits, and so will airline *j*'s competitors, but each increase in frequency leads to a bigger increase in demand; an equilibrium cannot exist. In Appendix A it is shown that a sufficient condition for an airfare-frequency optimum to exist is that the frequency elasticity of demand is smaller than 1. Furthermore, it is shown the airfare-frequency equilibrium, if it exists, is unique: for every possible (optimal) airfare-frequency combination of a competitor *j'*, there exists a unique response from airline *j*. In the remainder of the analysis, we assume $e_{f_j}^{P(j,i)} < 1$.

For a further analysis of the properties of the equilibrium at the airline market, we analyze a symmetrical equilibrium for mathematical convenience. First, we derive an equilibrium for airlines operating from the same airport, *given* the possible response from competitors operating from alternate airports. Let there be l_i airlines in $L(i)$, with $c_j=c_{j'}$ and $k_j=k_{j'}$, $j,j' \in L(i)$, $j \neq j'$. Then the following symmetric equilibrium is found.

$$p_i^* = c + \frac{l_i q m}{a_p\left(q\left(l_i - 1\right) + m\left(1 - P(i)\right)\right)} \tag{9}$$

$$f_i^* = \frac{1}{l_i}\frac{a_f}{a_p}\frac{P(i)N}{k} \tag{10}$$

As the heterogeneity between both airports and airlines increases (q,m), so does the airfare. Furthermore, as the marginal probability P(i) increases, so does the equilibrium airfare. Finally, the equilibrium airfare is decreasing in l_i. These equilibrium expressions for p_i^* and f_i^* only show how they are determined at airport i, *given* P(i). As P(i) changes (e.g. due to a change in $p_{i'}$ or $f_{i'}$, p_i^*) and f_i^* change accordingly. What is important, is that all airlines operating from airport i will still have equal prices and frequencies. This allows for a simplification of $\tilde{V}_i$ to $m\ln\left(l_i\right) - a_p p_i^* + a_f \ln\left(f_i^*\right)$. For airport(s) i' similar expressions for $p^*_{i'}$, $f^*_{i'}$ and $\tilde{V}_{i'}$ can be derived. Then, if d is the number of elements in D and $l_i = l_{i'}$, we find the following system wide symmetric equilibrium.

$$p^* = c + \frac{l_i dqm}{a_p\left(qd(l-1) + m(d-1)\right)} \tag{11}$$

$$f_d^* = \frac{1}{d}\frac{a_f}{a_p}\frac{N}{k} \tag{12}$$

Only the airfare is dependent on the heterogeneity between airlines operating from both the same airport and different airports. Both the equilibrium airfare and frequency are decreasing in d (and l_i). This is an equilibrium between all the airlines in the system, taking the airport authorities' behavior as given. Note that although airlines charge the same prices and offer the same frequencies, airports do not necessarily have the same airport taxes and access times. As is shown in Appendix B, for the symmetric equilibrium to exist it is necessary that $\Delta \equiv -b_t\left(tax_{i'} - tax_i\right) - b_a\left(\ln\left(t_{i'}\right) - \ln\left(t_i\right)\right)=0$. This relationship may be satisfied for $tax_{i'} \neq tax_i$ *and* $\ln(t_{i'}) \neq \ln(t_i)$. Hence an airport with excellent accessibility may charge higher airport taxes than an airport with lesser accessibility. The relation $\Delta=0$ says nothing about whether the airport taxes are optimal from the airports point of view. Therefore, in the next Subsection airports will be introduced as independent agents.

Summarizing the discussion of the airline model so far, if the frequency elasticity of demand is smaller than 1 there exists a unique equilibrium. A special case is the symmetric equilibrium. Airports may have different airport taxes and access times; but for this equilibrium to exist it is necessary that $-b_t\left(tax_{i'} - tax_i\right) - b_a\left(\ln\left(t_{i'}\right) - \ln\left(t_i\right)\right) = 0$

4 The airport's maximization problem

In this Subsection we will describe how airports in a multiple airport system can compete for passengers. Airport i has a market share P(i), defined as in Section 3. Clearly, this probability also depends on the airline's optimal fares and frequencies.

Passenger demand for flights out of an airport i is given by P(i). Let the airport's optimization problem be (see also Oum et al., 1996):

$$\max_{tax_i} \int_{tax_i} \mathrm{P}(i)\mathrm{dtax} + \left(tax_i - m\mathrm{c}_i\right) N\mathrm{P}(i) - rK_i - g\left(\frac{N\mathrm{P}(i)}{K_i}\right) \qquad (13)$$
$$s.t.\ \Pi_i \equiv \left(tax_i - m\mathrm{c}_i\right) N\mathrm{P}(i) - rK_i \geq 0$$

the airport maximizes social welfare with respect to the airport tax tax_i under a cost recovery constraint. We assume the marginal costs per passenger mc_i are constant. rK_i is the capital cost of airport i (where K_i is the airport's capital stock) and g(..) is an external cost function; $\frac{\partial \mathrm{g}(..)}{\partial P(\mathrm{i})} > 0$. The first order conditions are:

$$\begin{cases} \left(tax_i - m\mathrm{c}_i\right) - \dfrac{\partial g\left(\dfrac{N\mathrm{P}(i)}{K_i}\right)}{\partial tax_d} - \beta_{tax}\mathrm{P}(i)\left(1 - \mathrm{P}(i)\right) + \lambda \dfrac{\partial \Pi_i}{\partial tax_i} = 0 \\ \lambda \Pi_i = 0; \Pi_i \geq 0, \lambda \geq 0. \end{cases} \qquad (14)$$

where $\frac{\partial \Pi_i}{\partial tax_i} = \mathrm{P}(i) - \left(tax_i - mc_i\right)\frac{\beta_{tax}}{\theta}\mathrm{P}(i)\left(1 - \mathrm{P}(i)\right)$. Then, if the cost recovery constraint is binding (λ>0):

$$tax_i = mc_i + \frac{rK_i}{N\mathrm{P}(i)} \qquad (15)$$

if $\frac{rK_i}{N\mathrm{P}(i)} > \frac{\partial \left(\frac{N\mathrm{P}(i)}{K_i}\right)}{\partial tax_i}$.

Hence, if the capital costs exceed the marginal external costs, the airport sets its tax at average cost level. If the same holds true for (all) the other airports in the system, the equilibrium taxes (15) are unique if airport i's profit function is non-decreasing in the airport tax; this is shown in Appendix D. It is a standard assumption that the profit function is non-decreasing in the output price.

For mathematical convenience, again we derive a symmetric equilibrium. Let l_i=2, $Kap_i = Kap_{i'}$, $mc_i = mc_{i'}$ and $t_i = t_{i'}$. Then

$$tax_i = mc_i + \frac{rK_i \delta}{N} \tag{16}$$

$$p^* = c + \frac{\delta \theta}{\alpha_p (\delta - 1)} \tag{17}$$

$$f^* = \frac{1}{\delta} \frac{\alpha_f}{\alpha_p} \frac{N}{k} \tag{18}$$

see Appendix D for details. The equilibrium airport tax is increasing in the number of airports; as the number of airports increases the market share of a particular airport will decrease (as by assumption all airports all equal in all aspects). Hence, to break even the airport will have to increase its tax as the capital cost has not changed. For the symmetric equilibrium it is necessary that airports are equal in all aspects. If e.g. $t_i \neq t_{i'}$, the symmetric equilibrium as described in Section 3 is no longer valid.

When the cost recovery constraint is not binding ($\lambda = 0$), then the optimal airport tax will be

$$tax_i = mc_i + \beta \left(1 - \frac{\partial g(..)}{\partial \Pi(i)} \frac{N}{K_i} \frac{1}{\theta} \right) \Pi(i) \left(1 - \Pi(i) \right) \tag{15'}$$

which implies $\frac{\partial g(..)}{\partial \Pi(i)} \frac{N}{K_i} \frac{1}{\theta} < 1$. Without knowing the functional form of $g(..)$ the optimal taxes cannot be derived.

5 Comparative statics

From equation (6) we can compute airline *j*'s responses to changes in airport taxes and changes in accessibility of the (alternate) airports. The reaction to a change in the airport tax of airport *i* is:

$$\frac{\partial p_j}{\partial t_i} = \frac{b_a q^2 \mathrm{P}(j|i)\mathrm{P}(i)(\mathrm{P}(i)-1)}{a_p\left(m\mathrm{P}(j|i)(1-\mathrm{P}(i)) + q\left(1-\mathrm{P}(j|i)\right)\right)^2} < 0 \qquad (19)$$

as the numerator is negative. Hence the airfare charged by an airline *j* operating out of airport *i* increases as airport *i*'s accessibility (airport tax) increases (decreases). If the airport attempts to increase its market share by investing in the accessibility, part of the increase in demand will be offset by the price increase with which the airline(s) respond (see (15)). The airport's change in tax due to the increase in accessibility is

$$\frac{\partial tax_i}{\partial t_i} = \frac{rK_i b_a (1-\mathrm{P}(i))}{qN\mathrm{P}(i)} > 0 \qquad (20)$$

The taxes are set at average cost level. As P(*i*) increases due to the decrease in access times, the tax also decreases. However, as the airline increases its price, so will the airport as it needs to break even when the cost recovery constraint is active (P(*i*) decreases):

$$\frac{\partial tax_i}{\partial p_j} = \frac{a_p rK_i \mathrm{P}(j|i)(1-\mathrm{P}(i))}{qN\mathrm{P}(i)} > 0 \qquad (21)$$

Hence there are two opposite movements. As the airport increases its market share, airport taxes decrease as they are set at average costs. Then the airlines respond by increasing the airfares to cash in on the increased attractiveness of airport *i*. As a result, airport *i* is forced to adjust its taxes downward. The final effect on the airport tax of an increase in accessibility depends on the parameter values of a_p, b_a, r and K_i and is indeterminate. Only if these parameters are known and the probabilities (for an asymmetric equilibrium) can be calculated equations (19) - (21) can be evaluated. In the next Section, a numerical solution for an asymmetric equilibrium will be obtained to illustrate the functioning of the model.

7 The asymmetric case: a numerical solution

As analytical expressions for the asymmetric equilibrium are very complex it is difficult to arrive at definite results for a comparative static analysis. Therefore, in this Section, a numerical procedure is carried out.

Let there be two (origin) airports (A and B) and a single destination. At each of the two airports, there are two airlines (A_1, A_2, B_1 and B_2). In order to use "realistic" values of the parameters, we make use of some estimates we recently carried out in the San Francisco Bay Area (Pels et al., 1998), with some parameters added from other studies. The average travel time to the airport are: t_A = 38 and t_B = 44 (these are the average access times to San Francisco International Airport and San Jose International Airport, obtained from the 1995 Metropolitan Transportation Commission Airline Passenger Survey). Let the capital costs be: rK_A = 12000000 and rK_B = 10000000. The airline costs are: c_{A1} = 100, c_{A2} = 105, c_{B1} = 105 and c_{B2} = 95. k_{A1} = 115000, k_{A2} = 100000, k_{B1} = 90000 and k_{B2} = 110000. The parameter values used are: a_p = 0.04, which is an average of various parameter estimates encountered in the literature. a_f = 1.15, which is somewhat lower than the estimates by Pels et al. (1998). As argued in Section 4, a sufficient condition for the existence of an equilibrium is $e_{f_{l,d}}^{\mathrm{P}(l,d)} < 1$. This corresponds with values of a_f that are "not too high". b_a = 0.04, which is an average of the estimates by Pels et al. b_t was not encountered in the literature (and could not be estimated as taxes in the San Francisco Bay Area were constant across passenger types). Hence b_{tax} was assigned the same value as a_p. q was fixed at 1 and m was set at 0.85 (Pels et al., 1998). Finally, N = 1000000. With these inputs, the following optimal frequencies, airfares and taxes are found, see Table 1. In row I the airports set their taxes at marginal cost (which, for simplicity, are fixed at 0). In row II, the airports set their cost at marginal cost and there is a 2.5% reduction in access times. The latter may be assumed to be the consequence of an investment in roads by the regional government. By comparing I and II, we can analyze the consequence of such an investment for the optimal airfares, frequencies and taxes. In row III, airports again set their (optimal) taxes are set at average costs. In row IV, taxes are set at average costs and again there is a 2.5% reduction in access times.

Table 1 *Optimal airfares, frequencies and airport taxes*[1]

	p_{A1}	p_{A2}	$freq_{A1}$	$freq_{A2}$	p_{B1}	p_{B2}	$freq_{B1}$	$freq_{B2}$	tax_A	tax_B
I	136.77	128.02	87	18	132.85	131.27	69	100	0	0
II	137.95	127.76	91	16	132.61	131.23	66	98	0	0
III	135.03	128.52	80	23	133.14	131.33	71	99	3.00	1.67
IV	136.39	128.12	85	19	133.92	131.28	69	98	2.94	1.69

1) I: airport taxes at marginal costs (= 0) and t_A = 38, II: airport taxes at marginal costs (= 0) and t_A = 37, III: airport taxes at average costs and t_A = 38, II: airport taxes at average and t_A = 37.

Form Table 1 it can be seen that under both pricing regimes an improvement of airport *A*'s accessibility leads to an increase of both airline *A1*'s airfare and frequency and decrease of both airline *A2*'s airfare and frequency. At airport *B*, all airfares and frequencies go down. Although airline *A1*'s airfare increases and airline *A2*'s frequency decreases, the increase in accessibility, the increase of airline *A1*'s frequency and the decrease of airline *A2*'s airfare are more than enough to accommodate an increase in the demand at airport *A*; under the average cost pricing scheme airport *A*'s tax decreases. A 2.5% decrease in access times result in a 2% decrease in airport taxes. In Table 2 the marginal and conditional probabilities are given for both regimes of airport taxation.

Table 2 *marginal and conditional probabilities at the optimum*[1]

	I	II	III	IV
P(*A*)	0.41	0.42	0.40	0.41
P(1|*A*)	0.85	0.87	0.80	0.84
P(2|*A*)	0.15	0.13	0.20	0.16
P(*B*)	0.59	0.58	0.60	0.59
P(1|*B*)	0.38	0.36	0.37	0.37
P(2|*B*)	0.62	0.64	0.63	0.63

1) I: airport taxes at marginal costs (= 0) and t_A = 38, II: airport taxes at marginal costs (= 0) and t_A = 37, III: airport taxes at average costs and t_A = 38, II: airport taxes at average and t_A = 37.

When airport *A*'s accessibility improves (columns II and IV), the marginal probability airport *A* is chosen increases (a little). At both airports there is also a redistribution of traffic: at airport *A*, airline *A1*'s market share increases, at airport *B*, airline *B2*'s market share increases.

Optimal airfares and frequencies are determined by the parameter values and marginal costs, but from Tables 1 and 2 it is not clear whether the marginal cost per flight or the marginal cost per passenger plays a dominant role (if at all). Therefore, in Table 3 pricing regime I (airport taxes fixed at marginal cost (which are fixed at 0)) is compared with the same regime with equal marginal costs per flight or equal marginal cost per passenger. In Table 4 the corresponding marginal and conditional probabilities are given.

Table 3 Optimal airfares, frequencies[1]

	p_{A1}	p_{A2}	$freq_{A1}$	$freq_{A2}$	p_{B1}	p_{B2}	$freq_{B1}$	$freq_{B2}$	tax_A	tax_B
I	136.77	128.02	87	18	132.85	131.27	69	100	0	0
I’	134.18	139.75	91	129	122.43	128.47	10	40	0	0
I’’	152.10	126.25	150	0	126.25	143.06	0	125	0	0

1) I: airport taxes at marginal costs (= 0), I’: airport taxes at marginal costs and $c_{A1} = c_{A2} = c_{B1} = c_{B2} = 100$, II’: airport taxes at marginal costs and $k_{A1} = k_{A2} = k_{B1} = k_{B2} = 100000$.

Table 4 marginal and conditional probabilities at the optimum[1]

	I	I’	I’’
P(*A*)	0.41	0.82	0.52
P(1|*A*)	0.85	0.45	1
P(2|*A*)	0.15	0.55	0
P(*B*)	0.59	0.18	0.48
P(1|*B*)	0.38	0.17	0
P(2|*B*)	0.62	0.83	1

1) see comment made under Table 3.

Form Table 4 it can be seen that under regime I’ (equal marginal costs per passenger) airport *A*’s position improves substantially. This is reflected by a substantial increase in the frequency offered from airport 1. From Table 3 it appears that at airport *A* the airline *A2* (with the lowest marginal cost per flight) is the winner; it increases both the airfare and frequency. Airline *A2* also increases its frequency, probably as a response to the more favorable position compared to the airlines operating from airport *B*. Compared to airline *A1*, it looses ground (and reduces its airfare to compensate). At airport *B*, both players “loose”. Under regime II’ (equal marginal costs per flight), at both airports the players with the highest marginal cost per passenger disappear if the marginal costs per flight are equal; differences in marginal cost per flight are apparently necessary to justify the existence of more than one player at each airport. This observation is not extended to the case of equal marginal cost per passenger. As a result of the increased market power, the surviving airlines are able to increase their airfares. Airport *A*’s position also improves under this regime, though less dramatic than under regime I’.

8 Conclusion

In this paper optimal airfares, frequencies and airport taxes in a multiple airport region were analyzed using a nested logit demand model. It was found that if the frequency elasticity of demand is smaller than 1 there exists an equilibrium. Moreover, it was found that if an equilibrium exists, it is unique.

The model can be used to determine optimal airport taxes, given the (optimal) behavior of the airlines. If a regional government plans to invest in improving accessibility, the model can be used to determine the new optimal airport taxes. Because the equilibrium, if it exists, is unique, one knows that, like in the old situation, in the new situation there will be only one optimal airport tax. Although in the paper symmetric equilibria were derived to show the workings of the model, the conclusion also holds true for (more realistic) asymmetric equilibria.

In a numerical solution for the asymmetric case, it was concluded that an increase in the accessibility of an airport leads to an decrease in the airport tax. However, the increase in accessibility also led to a redistribution of traffic between the airlines. If the increase in the airfare would outweigh the increase in frequency and decrease in the competitor's airfare, an increase in an airport's accessibility may even lead to a decrease of the airport's market share (the "winning" airline cashes in on the improved accessibility). Whether this happens or not depends on the parameter values and the relative cost positions. It was also concluded that differences in marginal costs per flight are necessary to justify the existence of more than 1 competitor at each airport. Translated into real life, a "low cost" carrier competing with one or more larger carriers, then will have low marginal costs per passenger but high marginal costs per flight. There seems to be some truth to that, after the deregulation of the aviation market some "low cost" carriers entered the market using older aircraft, which are generally more expensive to operate.

The research agenda that follows from this paper is the following. First, more research should be done to see whether the frequency elasticity of demand is in fact smaller than 1. Caves et al. (1991), using a MNL with both the frequency and fare as explanatory variables, found elasticities for the UK that were in fact smaller than 1. Pels et al. (1998), using a NMNL with the frequency but without the fare as explanatory variables, found elasticities smaller than 1 for the San Francisco Bay Area. Estimates using NMNL with both the fare and frequency as explanatory variables were not encountered in the literature. Second, the cost functions were kept very simple and these will have to be verified empirically. Finally, the model could be restated so that competition between airports for a hub position in an airline network can be examined.

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Appendices

A Existence of the airfare-frequency equilibrium.

To show there exists an equilibrium we show the profit functions π_j, $j \in L$, are quasi-concave in (p_j^*, f_j^*).

The second order derivative of π_j with respect to p_j is:

$$\frac{\partial^2 \pi_j}{\partial p_j^2} = 2N\frac{\partial P(j,i)}{\partial p_j} + \left(p_j - c_j\right)\frac{N}{P(j,i)}\left(\frac{\partial P(j,i)}{\partial p_j}\right)^2 + \left(p_j - c_j\right)\alpha_f P(j,i) N\left(\frac{P(j,i)\left(P(i)-1\right)}{\theta^2} + \frac{\left(P(j|i)-1\right)\left(P(i)-1\right)}{\theta\mu} + \frac{\left(P(j|i)-1\right)}{\mu^2}\right) \tag{A1}$$

which, for every p_j which solves the first order necessary condition for profit maximization, equals

$$\frac{\partial^2 \pi_j}{\partial p_j^2} = \frac{\alpha_p P(j,i) N}{\theta\mu} \times \left(\frac{\theta\mu P(j|i)\left(P(j|i)-1\right)\left(P(i)-1\right) - \theta^2\left(P(j|i)-1\right) - \mu^2 P(j|i)^2\left(P(i)-1\right)}{\theta\left(P(j|i)-1\right) + \mu P(j|i)\left(P(i)-1\right)}\right) < 0 \tag{A2}$$

as the numerator of the seconds RHS term is positive and the denominator of the second RHS term is negative.

If we assume that $(p_j - c_j) > 0$[5], then the second order condition for profit maximization with respect to the frequency is:

$$\frac{\partial^2 \pi_j'}{\partial f_j^2} = \frac{\alpha_f N}{f^2}\left(\frac{\alpha_f P(j|i)\left(1-P(i)\right)}{\theta} + \frac{\alpha_f P(j,i)\left(1-P(j|i)\right)}{\mu} - P(j,i)\right) < 0 \tag{A3}$$

A sufficient condition for (A3) to hold is that the frequency elasticity of demand

$$\varepsilon_{f_j}^{P(j,i)} = \frac{\partial \ln\left(P(j,i)\right)}{\partial \ln\left(f_j\right)} = \frac{\alpha_f P(j|i)\left(1-P(i)\right)}{\theta} + \frac{\alpha_f\left(1-P(j|i)\right)}{\mu} < 1.$$

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[5] Then as $p_j^* - c_j$ is invariably positive, $\frac{\partial^2 \pi_j}{\partial f_j^2} = \left(p_j^* - c_j\right)\frac{\partial \pi_j'}{\partial f_j}$ will be invariably negative; hence the profit function is quasi concave in the frequency if $\varepsilon_{f_j}^{P(j,i)} < 1$.

Finally, for an optimum to exist, it is necessary that $\mathrm{H} = \frac{\P^2 p_j}{\P p_j{}^2}\frac{\P^2 p_j}{\P f_j{}^2} - \left(\frac{\P^2 p_j}{\P p_j \P f_j}\right)^2 > 0$.

The analytical expression is rather involved, but simulations have shown that if $e_{f_j}^{\mathrm{P}(j,i)} < 1$, which is sufficient for $\frac{\P^2 p_j}{\P f_{j,i}{}^2} < 0$, $\mathrm{H} > 0$ near the optimum (f^*_j, p^*_j)[6]. For the simulations, q was standardized to 1, which implies $m < 1$. Values for m were drawn form a uniform distribution between 0 and 1. For the simulation, values for P(i) and P($j|i$) were drawn from a uniform distribution between 0 and 1. The parameter a_f and a_p were drawn from a uniform distribution between 0 and twice the maximum estimate encountered in the literature[7]. Simulation results are presented in Table A1.

Table A1 Simulation results

	$e_{\mathrm{f_j}}^{P(\mathrm{j,i})} < 1$	$e_{\mathrm{f_j}}^{P(\mathrm{j,i})} > 1$
minimum H	0.00	-7031877
average H	0.19	-621.07
maximum H	295.72	73.62
lower bound H[1]	0.08	-1701.65
upper bound H[1]	0.30	459.51

1) For a 95% confidence interval around the average.

Hence a sufficient condition for airfare-frequency optimum to exist is that the frequency elasticity of demand is smaller than 1; then the profit function is quasi-concave in (f_j,p_j). If all profit functions j, $j \in L$ are quasi-concave in (f_j, p_j) (and the strategy spaces are convex and compact) then there exists an equilibrium (Anderson et al. (1992, page 161)).

[6] Note that the reverse is not necessarily true, it *can* also be that $\mathrm{H} > 0$ if the frequency elasticity of demand is larger than 1.

[7] Estimates for a_f were taken from studies including the frequency in *logarithmic* form. Maximum values were: 1.29 (0.17) (Hansen, 1990, MNL) and 1.47 (0.06) (Pels et al., 1998b, NMNL). Fare parameter estimates encountered were (in absolute values): 0.0045 (0.0010), (Hansen, 1990), 0.09 (not reported), (Thompson and Caves, 1993). Standard errors between parentheses.

B Uniqueness of the airfare-frequency equilibrium.

To show that the equilibrium between airlines operating from the same airport is unique, we have to show that airline *j*'s best response function is a contraction[8]. First we show that $\sum_{j' \in L(i), j' \neq j} \left| \frac{\partial p_j^{br}}{\partial p_{j'}} \right| < 1$, where p_j^{br} is airline *j*'s best (airfare) response function$\left(= \frac{\partial p_j}{\partial p_j} = 0 \right)$; if this condition is satisfied the airlines' best airfare functions, *given* the frequencies, converge to a single point. $\left| \partial p_j^{br} / \partial p_{j'} \right|$ is an expression in P(*i*), P(*j*|*i*) and P(*j'*|*i*). For simplicity we check the condition for l_i=2; then P(*j'*|*i*) = 1 - P(*j*|*i*). Substituting for P(*j'*|*i*) and standardizing *q* to 1:

$$\left| \partial p_j^{br} / \partial p_{j'} \right| = \left| - \frac{\mathrm{P}(j|i)\left(\mathrm{P}(jl|i) - 1\right)\left[\mathrm{P}(i)\left(\mathrm{P}(i) - 1\right) + \left(\mathrm{P}(i) - 1\right)m + 1\right]}{\mathrm{P}(j|i)^2 \left(\mathrm{P}(i) - 1\right) m^2 - \mathrm{P}(j|i)\left(\mathrm{P}(j|i) - 1\right)\left(\mathrm{P}(i) - 1\right)m + \mathrm{P}(j|i) - 1} \right| < 1$$

which was verified by simulation; after 99,999 simulations, with *m*, P(*i*) and P(*j*|*i*) drawn from uniform distributions between 0 and 1, it turned out that $\left| \partial p_j^{br} / \partial p_{j'} \right|$ lies within the interval $\langle 0,1 \rangle$. Hence, *given* the frequency, the local airfare equilibrium is unique.

Likewise, we have to show that $\sum_{j' \in L(i), j' \neq j} \left| \frac{\partial f_j^{br}}{\partial f_{j'}} \right| < 1$. Substituting for the optimal airfare in equation 10 (i.e. evaluating the best response frequency function at the unique optimal airfare) results in $f_j^{br} \equiv \frac{\alpha_f}{\alpha_p} \frac{\mathrm{P}(i)\mathrm{P}(j|i)}{f_j} N - k = 0$. Then:

$$\left| - \frac{\partial f_j^{br}}{\partial f_{j'}} \right| = \left| - \frac{\alpha_f \left(\mathrm{P}(j|i) - 1\right)}{\alpha_f \left(\mathrm{P}(j|i) - 1\right) + m} \right| < 1 \qquad \text{A(6)}$$

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[8] *f(x)* is a contraction if ||*f(x)*-*f(x')*|| < λ||*x*-*x'*||, 0 λ < 1; the function values of two points are closer than the two points themselves. For a differentiable function *f(x)* this is the same as $\sum_j \left| \partial f_i(x) / \partial x_j \right| < \lambda$. (Friedman, 1989).

if $\alpha_f < \dfrac{\mu}{1-\mathrm{P}(j|i)}$, as in equilibrium $f_{j,i} = f_{j',i}$. The condition $\alpha_f < \dfrac{\mu}{1-\mathrm{P}(j|i)}$ ensures that the frequency elasticity of the (local) demand is smaller than 1; $\varepsilon_{f_j}^{\mathrm{P}(j|i)} = \dfrac{\partial \ln(\mathrm{P}(j|i))}{\partial \ln(f_j)} = \dfrac{\alpha_f}{\mu}(1-\mathrm{P}(j|i)) < 1$. This will be the case if $\varepsilon_{f_j}^{\mathrm{P}(j,i)} < 1$, which, as shown, is a necessary and sufficient condition for an equilibrium to exist. Hence, *given* the (optimal) airfare, the frequency equilibrium is unique.

As shown in equations (7) and (8), for each optimal p_j there is a single f_j which maximizes p_j; as p^*_j changes, so does f^*_j. As $p_{j'}$ changes, so does $p^*_{j'}$; and from equation A(5) it follows that the new airfare equilibrium is unique. f^*_j also changes, and from equations (7) and (8), again there is a single f^*_j that maximizes p_j. From equation A(6) we know that the frequency equilibrium is unique. Hence, we conclude the *local* airfare-frequency equilibrium is unique if the frequency elasticity of demand is smaller than 1.

To show that the equilibrium between airlines operating from different airports is unique, we show that $\sum_{i' \in D, i' \neq i, j \in L(i)} \left| \dfrac{\partial p_j^{br}}{\partial p_{i'}} \right| < 1$ and $\sum_{i' \in D, i' \neq i, j \in L(i)} \left| \dfrac{\partial f_j^{br}}{\partial p_{i'}} \right| < 1$. Again, for simplicity, we check this for d=2. Then

$$\left| \frac{\partial p_j^{br}}{\partial p_{i'}} \right| = \left| -\frac{\mu^2 \mathrm{P}(i)(\mathrm{P}(i)-1)\mathrm{P}(j|i')}{\mathrm{P}(j|i)^2(\mathrm{P}(i)-1)\mu^2 - \mathrm{P}(j|i)(\mathrm{P}(j|i)-1)(\mathrm{P}(i)-1)\mu + \mathrm{P}(j|i) - 1} \right| < 1 \qquad \text{A(7)}$$

Likewise,

$$\left| -\frac{\dfrac{\partial \mathrm{P}(j|i)}{\partial f_{i'}}}{\dfrac{\partial \mathrm{P}(j|i)}{\partial f_j} - \dfrac{\mathrm{P}(j|i)}{f_j}} \right| = \left| -\frac{-\dfrac{\alpha_f}{\mu f_{i'}}\mathrm{P}(j|i)(1-\mathrm{P}(i))}{\dfrac{\alpha_f}{\mu f_j}\mathrm{P}(j|i)(1-\mathrm{P}(i)) - \dfrac{\mathrm{P}(j|i)}{f_j}} \right| < 1 \qquad \text{A(8)}$$

if $a_f < \dfrac{m}{1-\mathrm{P}(i)}$. This will be the case if the frequency elasticity of demand is smaller than 1, which is necessary for an equilibrium to exist. Hence, the *global* airfare-frequency equilibrium, if it exists, is unique.

C Derivation of the symmetric airfare-frequency equilibrium.

Of the l_i airlines operating out of airport i, let there be l_i-1 airlines (charging $p_{j'}$ and (each) offering a frequency $f_{j'}$) already in equilibrium. Hence we need to find an equilibrium between the l_i-1 airlines already in equilibrium and the remaining airline j. P($j|i$) can be rewritten as:

$$\mathrm{P}(l|d) = \frac{\exp(\dfrac{a_0 - a_p p_j + a_f \ln(f_j)}{m})}{\exp(\dfrac{a_0 - a_p p_j + a_f \ln(f_j)}{m}) + (l_i - 1)\exp(\dfrac{a_0 - a_p p_{j'} + a_f \ln(f_{j'})}{m})} \qquad \text{(A9)}$$

Then, if k_j=$k_{j'}$ and c_j=$c_{j'}$=c, j≠j, the symmetric equilibrium as in equations (9) and (10) is found. Hence at the local equilibrium all l_i airlines have the same frequency and airfare.

Let there be d airports, each of which accommodates l_i airlines (i.e. $l_i = l_{i'}$). Furthermore, define $\Delta \equiv -b_t(tax_{i'} - tax_i) - b_a(\ln(t_{i'}) - \ln(t_i))$, where the subscript i' is used for all alternate airports; i.e. all airports other than i have equal covariates. Of the d airports in the system, let there be d-1 already in equilibrium. Then $\mathrm{P}(i) = \dfrac{\exp(V_i)}{\exp(V_i) + (d-1)\exp(V_{i'})}$, where V_i is the utility derived from using airport i (as specified in equation (3)). Then, if $l_i = l_{i'}$, solving the system of equations (9) and (11) for both airport i and the alternate airports i', we find a general solution:

$$p_i^* = p_{i'}^* = c + \frac{a_f q m l_i N}{a_p\left(a_p k \exp(Z_0)(d-1) + m(l_i - 1)N\right)}$$
$$f_i^* = \frac{a_f N}{a_p k} - \exp(Z_0) \qquad \text{(A10)}$$
$$f_{i'}^* = \exp(Z_0)$$

where Z_0 is a root of exp(Ω)[9];

$$\Omega\Big|_{l_i>1} = \left[\begin{array}{c} a_p^2 \exp(Z)k\left(\frac{1}{2}\Psi\left(q-a_f\right)+\Delta\right)+a_f^2\frac{1}{2}\Psi N + \\ \left(\left(\frac{1}{2}\Psi+\frac{d}{d-1}\right)q-\Delta\right)Na_f a_p \end{array}\right](d-1)\exp(Z)km^2 - a_f^2 qm^2 N^2 \qquad \text{(A11)}$$

where ψ =-2Z-2ln(k) + 2ln(exp(Z)$a_p k$(1-D) + $\alpha_f N$) - 2ln(a_p). Solving ψ = 0 results in

$$Z_0 = \ln\left(\frac{a_f N}{a_p dk}\right) \qquad \text{(A12)}$$

Substituting equation (A8) for Z and ψ = 0 in Ω yields Ω = 0 if Δ =0. Substituting for Z_0 in the general solution results in equations (11) and (12).

D Derivation of the symmetric airfare-frequency-airport tax equilibrium.

Let the airport’s cost recovery condition be binding (i.e. λ>0). Then the airport's optimal tax is given by equation 18 and the airport's best response function is the airport's profit function. Furthermore, let $Kap_i=Kap_{i'}$ and $\tau \equiv t_{i'}-t_i$. For $L_i=L_{i'}=2$, the general solution to the system of equations 9, 10 and 15 is

$$\begin{aligned} tax_i &= \frac{a_f Kap_i}{a_f N - a_p kZ_0(d-1)} \\ tax_{i'} &= \frac{a_f Kap_{i'}}{a_p kZ_0} \\ p_i &= \frac{a_p^2 ckZ_0(d-1)+a_f Nq}{a_p^2 kZ_0(d-1)} \\ p_{i'} &= -\frac{a_p c\left(a_f N - a_p kZ_0\right)-a_f Nq}{a_p\left(a_f N - a_p kZ_0\right)} \\ f_i &= -\frac{-a_f N + a_p kZ_0(d-1)}{a_p k} \\ f_{d'} &= Z_0 \end{aligned} \qquad \text{(A13)}$$

where Z_0 is a root of exp(Ω);

[9] For convenience we give here the expression for L=1. When L>1, $\Omega|_{L>1}$ would be far more complicated. However, Z_0 will attain the same value. It is crucial that $L_i=L_{i'}$.

$$\begin{aligned}\Omega = & \begin{bmatrix} \left(\left(y f_1 + (2q + b_a t)\exp(\mathrm{Z}) a_f^2 a_p\right)k + b_t a_f^3 rKap_d\right)d + \\ -\left(y f_1 + (q + b_a t)\exp(\mathrm{Z}) a_f^2 a_p\right)k - b_t a_f^3 rKap_d \end{bmatrix} N^2 + \\ & \begin{bmatrix} \left(\left(y f_2 + (q + b_a t)\exp(\mathrm{Z}) a_f a_p^2\right)k^2 + b_t a_p a_f^2 rKap_d \exp(\mathrm{Z})\right)d^2 + \\ -\left(y f_1 + (q + b_a t)\exp(\mathrm{Z}) a_f a_p^2\right)k - b_t a_p a_f^2 rKap_d \exp(\mathrm{Z}) \end{bmatrix} N + \\ & \left(\left(y f_3 + a_p^3 b_t t \left(\exp(Z)\right)^3\right)k^3 + a_p^2 a_f b_t k^2 rKap_d \left(\exp(Z)\right)^2\right)d^2 - \\ & \left(2\left(y f_3 + a_p^3 b_t t \left(\exp(Z)\right)^3\right)k^3 + a_p^2 a_f b_t k^2 rKap_d \left(\exp(Z)\right)^2\right)d + \\ & \left(y f_3 + a_p^3 b_t t \left(\exp(Z)\right)^3\right)k - a_f^3 N^3 q \end{aligned} \qquad \text{(A17)}$$

where y =-2Z-2ln(k) + 2ln(exp(Z)$a_p k$(1-d)+$a_f N$)-2ln(a_p),

f_1=½$a_p^3 a_f^2$exp(Z)(a_f-m),

f_2=-½$a_p^2 a_f$(exp(Z))^2(a_f-m) and

f_3=½a_p^3(exp(Z))^3(a_f-m).

Solving y=0 yields $Z_0 = \ln\left(\frac{1}{2}\frac{a_f N}{a_p dk}\right)$. Substituting 0 for y in W and solving W = 0 yields $Z_0 = \ln\left(\frac{1}{2}\frac{a_f N}{a_p dk}\right)$ if t =0. Finally, substituting for Z in the general solution yields equations 16, 17 and 18. To show that the symmetric equilibrium is unique we again look at the best response functions; we show this for d=2. We have already shown that the airlines' equilibrium airfares and frequencies are unique; hence we only look at the airports' best response functions:

$$\begin{aligned} & \left| -\frac{¶\Pi_i / ¶tax_{i'}}{¶\Pi_i / ¶tax_i} \right| = \\ & \left| -\frac{\left(tax_i - mc_i\right)\frac{b_t}{q}\mathrm{P}(i)\left(1-\mathrm{P}(i)\right)}{\mathrm{P}(i) - \left(tax_i - mc_i\right)\frac{b_t}{q}\mathrm{P}(i)\left(1-\mathrm{P}(i)\right)} \right| < 1 \end{aligned} \qquad \text{(A14)}$$

if (tax_i-mc_i)b_t < 2q. This condition implies that the airport's profit function is non-decreasing in the airport tax. The argument runs along similar lines if we look at the airport's best response to changes in airfare or frequency. Hence the equilibrium as given by equations 16, 17 and 18 is unique.