Diagnostic Tools for Nonlinearity in Spatial Models

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Abstract
There is an increasing awareness of the potential of nonlinear modeling in regional science, which can partly be explained by the recognition of the limitations of conventional equilibrium models in complex situations and partly by the easy availability and accessibility of sophisticated computational techniques. Among the class of nonlinear models chaos theory stands out as an interesting approach, in which notable progress has been made in economic research. However, the operational significance – in terms of solid empirical tests on chaos – has still been rather limited and a rigorous statistical-econometric treatment of chaos phenomena is still lacking. This paper is concerned with a methodological and empirical analysis of chaos in spatial systems. It seeks to break new research ground by linking the classical diagnostic tools developed in spatial econometrics to nonlinearity tests for empirical data series, in particular the so-called *BDS* (Brock, Dechert, Scheinkman) test. A spatial variant of the *BDS* test is developed, and subsequently applied in the context of a shift-share model for Dutch regional labor markets.

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1. Life can be so nonlinear[1]

Following the tradition of Newton and Laplace, many scientists have argued that perfect foresight based on predictability of dynamic systems is possible, if the necessary data and proper specifications of such systems are available. Nowadays, with the emerging interest in complexity theory and evolutionary theory – based on nonlinear dynamics – the idea of perfect prediction has largely vanished. Also, there is at present a wave of interest in modeling complex phenomena through simply structured systems, which may nevertheless exhibit a wide variety of dynamic behavior. Examples are catastrophe, chaos, bio-evolutionary, niche and synergetics theory and the like.[2]

Despite the sometimes fascinating images and appealing features of such nonlinear dynamic systems, it ought to be recognized that both the economic underpinnings and the empirical validity of several of such models are often feeble. Consequently, economists have tried to explore mainly two new research departures for coping with these shortcomings, viz. to extend the scope of such models by including more behavioral parameters, and to resort to new bio-computing paradigms (e.g., neural networks) to show the empirical relevance of these research tools.

A prominent class of models in the nonlinear dynamic (complex) domain is formed by chaos models. These models have generated a wealth of interesting research in economics, but thus far their operational significance has still been rather limited. In the regional, urban and transportation literature several studies on chaos modeling can be identified, but a rigorous statistical-econometric approach has not yet been developed in a spatial context. Furthermore, the concept of chaos may have a specific and pronounced appearance in an interactive spatial system leading to spatial or spatio-temporal chaos, but with a few exceptions (De Graaff, 1997; Nijkamp and Reggiani, 1998) these dimensions of chaos have never been investigated.

This paper seeks to offer a statistical-econometric analysis of spatial chaos phenomena from both a methodological and an empirical perspective. It starts with a concise overview of the state of affairs in chaos theory in section 2. Next, in section 3, attention is focussed on classical diagnostic tools to identify regularities in spatial data, as developed in spatial econometrics. Moreover, nonlinearity tests for the indentification of chaos in empirical data series are discussed, in particular the so-called *BDS* (Brock, Dechert and Scheinkman) test for which in the present paper a specific spatial variant is suggested. Section 4 then sets out to present an empirical application on the basis of a shift-share model for Dutch regional labor markets. In the final section the achievements so far are reviewed and an agenda for further research is presented.

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[1] Title adopted from Kovach (1960).

[2] See Nijkamp and Reggiani (1998) for a broad survey.

2. Modeling chaos: an overview

Already a century ago the great economist Alfred Marshall argued that the element of time is the chief difficulty at the center of almost every economic problem.[3] In recent years a new interest in the dynamics of economic phenomena has emerged, with a particular view on nonlinear evolution of complex systems allowing for the occurrence of (sudden) changes in the solution trajectory, e.g., bifurcations and catastrophes (Gandolfo, 1996). The analysis of nonlinear evolutionary systems has convincingly demonstrated that the instability of solution trajectories is an intrinsic feature of complex systems, in the sense that even marginal shifts in initial conditions can generate significant amplifications in final results. The increasing recognition that seemingly simple dynamic models may exhibit complex behavior in conditions of non-equilibrium, explains the current interest in non-equilibrated evolutionary systems (Nicolis and Prigogine, 1989). The current popularity of evolutionary economics also has to be seen in this context (Nelson, 1995). Standard (textbook) economics, on the contrary, takes for granted that only incremental changes can be envisaged in a stable economic system governed by steady-state trajectories. Hence, in this analytical framework small changes in initial conditions are not able to generate bifurcations or shocks, but only non-cumulative marginal shifts in process trajectories (Baumol and Benhabib, 1989).

The theory of complex systems has recently gained increasing interest among economists (Puu, 1989; Zhang, 1991). The awareness that dynamic interactions between the components of an evolutionary system may exhibit dissipative phenomena and lead to a large spectrum of possible evolutionary patterns has induced much research on the range of parameter values, for which in a nonlinear dynamic system multiple solution trajectories may emerge. Clearly, as soon as parameters of an otherwise linear system are time dependent with respect to the endogenous variables, we may observe a situation of endogenously determined structural change reflecting nonlinear dynamics (e.g., Fujita and Ogawa, 1982).

It is noteworthy that for a long time the mathematical-statistical difficulties inherent in nonlinear dynamic systems have discouraged the use of such models in economics. Rapid computational advances (including accessible computer software) have nowadays popularized the use of nonlinear dynamic models in economic research.

Chaos theory takes the above observations a step further. It does not only deal with the conditions under which nonlinear dynamic models may generate unexpected irregularities of a time-irreversible nature, but it also takes for granted that such singularities are of a deterministic nature. Economists are faced here with a difficult methodological problem. There is a need, on the one hand, to properly analyze dynamic economic phenomena that cannot be explained by conventional linear models, whereas on the other hand, the current knowledge about the behavior of systems in disequili-

[3] The working of the market depends '… chiefly on variations in the area of space, and the period of time over which the market in question extends; the influence of time being more fundamental than that of space' (Marshall, 1890, p. 496). See, for an intriguing overview of the economics of time, also O'Driscoll and Rizzo (1985).

brium and about the appropriate functional specification of systems models is too scanty. Thus, it can plausibly be argued that a necessary condition for chaos theory to make a substantive contribution to economic theory is, that it should be rooted in behavioral-economic frameworks and be confirmed by real-world data. The first condition has extensively been discussed in the literature, and it has been convincingly shown that even standard analytical economic frameworks can rather easily be cast in a chaos formulation, as soon as nonlinear resolution is introduced in a system's model (*inter alia* Brock et al., 1991; Lorenz, 1993; Mannermaa, 1995; Nijkamp and Reggiani, 1993; Peters, 1994; Scheinkman, 1990). The second question, on the empirical validity of chaos models, is more difficult to answer. Brock (1986) has claimed that experiments tend to show, that the parameter values used are often inconsistent with empirical studies.

In the light of these sometimes contradicting views on the principle of chaos, a few observations on chaos theory and modeling are in order here. In particular, it is necessary to highlight a few more characteristics of chaos theory (Rosser, 1991). Chaos is a mathematical concept and essentially a feature of a nonlinear dynamic model. It refers to an evolutionary system which, given an infinite set of initial conditions, displays trajectories that do not repeat themselves, do not attract other trajectories and are extremely sensitive to the initial state. Chaos systems are essentially governed by strange attractors which cause an exponential divergence of critically stable solutions leading to non-periodic equilibrium trajectories.

From a research perspective, an intriguing analytical question is whether and how chaos can be identified in nonlinear dynamic models. Chaos means essentially that a solution trajectory of a spatial or dynamic system exhibits an exponential separation on initially adjacent points. In the literature the occurrence of such phenomena can be empirically measured by investigating the Liapunov exponent.[4] A positive value of a Liapunov exponent would then imply a sensitive dependence on initial state conditions. In this framework, it appears crucial to characterize chaos by measuring the degree to which a spatial or dynamic system is chaotic, e.g., by using the Kolmogorov entropy. Such tests are based on the assumption of the existence of a well-specified nonlinear spatial or dynamic economic model, which can be tested for chaos properties on a feasible range of variables.

Instead of investigating the properties of spatial or dynamic nonlinear models, a new departure would be to inspect an empirical data base on the presence of chaos properties (Brock, 1986). Brock et al. (1987) proposed a residual test for economic time series on the basis of the so-called correlation dimension. This standard approach proposed by Brock, Dechert and Scheinkman (1987) is known as the *BDS* test statistic, which is able to identify whether a certain time series has chaotic properties by analyzing deviations from white noise in fluctuations.[5] Clearly, this approach reduces chaos notions to an empirical question related to a given data series.

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[4] See Schuster (1988) for an analytical formulation, and Wolf et al. (1985) for a calculus of algorithms.

[5] See for details also Ornstein and Weiss (1991).

In the recent regional science literature quite some attention has been given to chaos models in such fields as urban, transport, migration and industrial production systems.[6] Thus far, however, statistical tests on the emergence of spatial chaos have not been developed. Although much work has been undertaken in the area of spatial econometrics, e.g., on spatial autocorrelation, issues of spatial chaos have not yet been properly addressed from an econometric-statistical viewpoint. Therefore, this paper aims to deal with this new research challenge.

By way of introduction, the next section will present some highlights from misspecification testing in spatial econometrics, followed by the presentation of an adjusted *BDS* test for nonlinearity testing in spatial models.

3. Diagnostic checking for spatial dependence and a spatial *BDS* test

From the very outset of the development of spatial econometrics as a self-contained discipline in regional science, the analysis of spatial dependence has been predominant (Cliff and Ord, 1973; Hordijk, 1974; Paelinck and Klaassen, 1979). Although dependence over space bears a close resemblance to the more familiar serial or time-wise dependence, it has been rigorously demonstrated that both adapted tests and estimators are needed in order to appropriately deal with the specific features of spatial data. Initially, measures for spatial dependence were focussed on univariate series of nominal, ordinal or interval scaled data, but later the field extended to the analysis of regression residuals from spatial process models (Cliff and Ord, 1981). Currently, a wide array of misspecification tests is available, and both their asymptotic properties and small sample distributions have been extensively investigated and documented.[7]

The characteristic nature of various causation processes in the space and time domain are easily demonstrated. For a random variable *X*, which has been measured over various areas *i* in a spatial system, the value in region *i* is given by x_i. A simple model of spatial interdependence among the $\{x_i\}$ is given by $x_i = r \sum_j w_{ij} x_j + e_j$, for *i* and *j* = 1, 2, …, *n* and the $\{e_i\}$ being independent and identically distributed variates with common variance s^2. The fundamental difference between spatial and temporal correlation is the multidirectional nature: in the above model x_i depends on all x_j for which $w_{ij} \neq 0$. Anselin (1988, p. 57) also explicitly denotes the two-dimensional or reciprocal nature of spatial dependence.[8] Temporal dependence is much more restricted, as temporal or serial dependence among, for instance, $\{x_t\}$ implies that an observation x_t in a typical time series model, such as $x_t = r\, x_{t-k} + e_t$, can only be influenced by a prior observation x_{t-k}, with $k > 0$ (Cliff and Ord, 1981).

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[6] See Nijkamp and Reggiani (1988) for a broad review.

[7] See Anselin and Florax (1995b) and Anselin et al. (1996) for recent overviews.

[8] However, in a strict sense, this is already implied by the multidirectional nature of spatial dependence.

The multidirectional nature of spatial causation processes has serious implications in a regression context, as simple spatial analogues of well-known tests (e.g., Durbin-Watson) as well as estimators for autoregressive models (e.g., generalized least squares and maximum likelihood) are not appropriate (Anselin, 1988).

In a dynamic spatial process model, such as $x_{it} = f\left(x_{i,t-k}, x_{j,t-k}\right)$ with k = 1, 2, … and $j \neq i$, the dependencies within and between the spatial and temporal domains become even more complex, as autocorrelation can occurr over time as well as over space, and the model may also exhibit covariation in the space-time domain. This is illustrated in Figure 1. It should be observed, that in the field of spatial econometrics space-time models have received substantially less attention than purely spatial models. The sequel of this paper will, in the first instance, be limited to the formulation of a *BDS* test for spatial process models.[9]

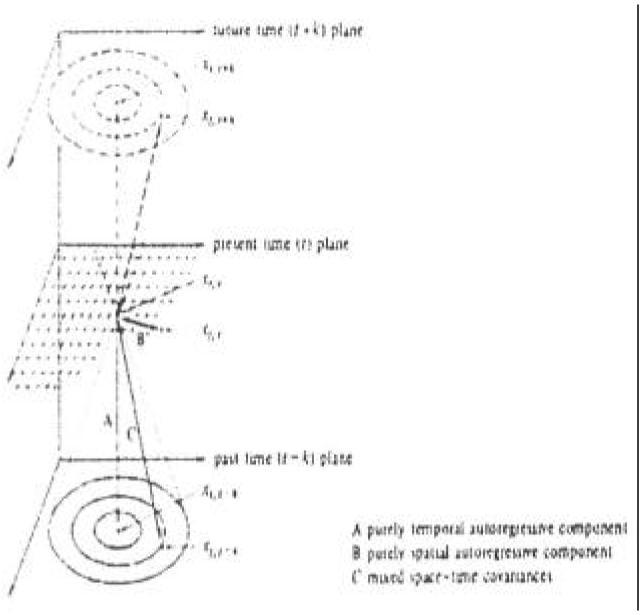

Figure 1: Patterns of dependencies between regions in time and space.
Source: Cliff and Ord (1981, p. 10)

The specification of spatial dependence crucially depends on the specification of the topology of the spatial system. In most cases spatial econometricians turn to the use of a so-called spatial weights matrix of size ($n \times n$), with zero's on the main diagonal and with off-diagonal elements specified in a binary form with unity or zero entries determined on the basis of adjacency, inverted distance, length

[9] It is straightforward to envisage the application of an adjusted *BDS* test for dynamic spatial models, which could likewise be illustrated with the shift-share technique. This is, however, left for future work.

of the common border or the like. Typically, the spatial weights matrix is determined exogenously, which may induce severe misspecification problems (Florax and Rey, 1995; Griffith, 1995). Moreover, although it is common practice to resort to a symmetric weights matrix, this may be inappropriate in a large number of cases, in particular if hierarchical diffusion patterns or core-periphery relations are dominant.[10]

Dependence in spatial process models may take the form of either a substantive process or a nuisance in the error term (Anselin and Rey, 1991; Anselin and Florax, 1995a). In the first case, an explicit interest in the spatial interaction of a certain variable (the dependent or the exogenous explanatory variables) should prevail, and a theoretical explanation for the specification is warranted. In the latter, spatial dependence occurs because of (erroneously) omitted spatially correlated variables, and is reflected in the error term. The different types of spatial dependence necessitate solutions of a different order and exhibit differing implications for estimation and statistical inference. Substantive dependence in the form of spatially lagged variables not only requires the development of a spatial process model, but also affects the properties of the estimator if the set of spatially lagged variables comprises the spatially lagged dependent variable. In contrast to the time series case, where OLS remains consistent if the errors are not serially correlated and its use is asymptotically warranted, OLS is biased as well as inconsistent, irrespective of the properties of the error term, in spatial models. Nuisance dependence just involves adjustments to statistical inference, as OLS is unbiased but inefficient. As estimated generalized least squares on the basis of an OLS estimator for the spatial autoregressive parameter is inconsistent, various maximum likelihood estimators have been developed (Anselin, 1988, pp. 58-9).

Among the class of spatial process models the spatial autoregressive moving average (SARMA) process is general and resembles the corresponding specification for time series. It is given by:

$$(I - j\,W)y = Xb + (I - qV)e \qquad (3.1)$$

where y is a $(n \times 1)$ vector of observations on the dependent variable, X a $(n \times k)$ matrix of explanatory variables, b a $(k \times 1)$ vector of parameters, j and q are scalar spatial parameters, W and V are $(n \times n)$ exogenously determined weight matrices, and e is a vector of well-behaved innovations distributed $N(0, s^2 I)$. In many cases the specification is further restricted by assuming W is V, as there are no *a priori* reasons to suppose that the spatial interaction patterns are different (e.g., Huang, 1994).[11]

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[10] Because of the principle of standardization, asymmetry is induced in the estimation procedure, but this is for mathematical rather than theoretical reasons. Standardization implies the scaling of the elements of the weight matrix such that all row sums are equal to one.

[11] A slightly different general model is the model with a substantive and a nuisance AR process, for which, however, it is required that the *X* matrix contains at least one exogenous variable in addition to the constant term, in order to be identified (Anselin et al., 1996).

Sneek and Rietveld (1997) rightly observe that a SARMA type of model is not often used by spatial analysts, although a distinct advantage of the MA term in the specification is that in the variance matrix a nonzero value of q leads to correlation between first and second order neighbors only and does not extend to the whole spatial system as would be the case for an AR term. Computational difficulties provoked by the unboundedness of the parameter space may be an explanation for the lack of popularity of this specification among practitioners (Sneek and Rietveld, 1997). Moreover, it should be noted that an MA process cannot be distinguished from an AR process on the basis of the Lagrange Multiplier tests developed so far, as they are identical (Anselin, 1994).

The most common specification used, is the spatial autoregressive error model, or AR(1) model, which is defined as:

$$y = Xb + \left(I - lW\right)^{-1} e \tag{3.2}$$

with l denoting the spatial autoregressive parameter. As model specification issues have become an integral part of spatial econometrics nowadays, an extensive toolbox of diagnostic tests – comprising unidirectional, multidirectional and robust tests for OLS residuals as well as alternative tests for maximum likelihood residuals – has been developed (Anselin et al., 1996, for a review). In the current paper, however, two relatively simple tests are used, as this paper is not primarily concerned with the specification of spatial dependence. One is Moran's *I*, which reads as:

$$I = \frac{n}{S_0} \cdot \frac{e'We}{e'e} \tag{3.3}$$

with *n* for the number of observations, S_0 the sum of the elements of the spatial weight matrix and *e* the ($n \times 1$) vector of OLS residuals. The first term at the right hand side of equation (3.3) may be omitted when the weight matrix is standardized. Statistical inference follows from the asymptotical normal distribution of the standardized *z*-value of Moran’s *I*, obtained by substracting the expected value and dividing by the standard deviation. Moments and estimation details are given in e.g., Cliff and Ord (1973, 1981) and Anselin (1988).

Although Moran’s *I* has a correct size and shows considerable power in small samples, a distinct disadvantage of Moran's *I* is that the alternative hypothesis is not clear, as it merely establishes spatial correlation among the residuals. Tests developed in a maximum likelihood framework, such as *LMERR* referring to spatial dependence of the AR(1) type among the residuals, are of greater practical use in this respect. The *LMERR* test statistic is in fact identical to a scaled squared Moran coefficient, and is given by:

$$LMERR = \frac{1}{T_1} \cdot \left(\frac{e'We}{s^2} \right)^2 \quad (3.4)$$

with $s^2 = e'e/n$, and $T_1 = \mathrm{tr}\left(W'W + W^2\right)$ with tr as the matrix trace operator. The statistic is asymptotically distributed as c^2 with one degree of freedom.

The spatial AR(1) specification can be estimated straightforwardly by maximizing the following likelihood:

$$L = -\frac{N}{2} \cdot \ln p - \frac{N}{2} \cdot \ln s^2 + \ln|B| - \frac{1}{2s^2} (y - Xb)' B'B (y - Xb) \quad (3.5)$$

with $B = I - lW$. Details about the estimation procedure are not reproduced here, as they can easily be found in e.g., Anselin (1988) and Anselin and Hudak (1992).

Because of the multidirectional nature of spatial data it should not be surprising to frequently find nonlinearities, or even chaos, in spatial data. However, even though nonlinearities are likely to show up as heteroscedasticity, for which spatial dependence tests are very sensitive (Anselin and Griffith, 1988), an explicit treatment of this possible misspecification through diagnostic testing seldom occurs in applied work.[12] In Florax (1992) it was shown that the so-called *RESET* test is robust against any form of spatial dependence in the data, which is positive as the test should merely have power against the correct alternative. The *RESET* test boils down to an *F* test on g in an auxiliary regression $y = Xb + Zg + e$, with *Z* consisting of the squares, cubes and fourth powers of the *x* variables, and it may thus not be very adequate in detecting nonlinearities in a spatial system given its non-spatial formulation. Therefore, a spatially adjusted *BDS* test is presented and empirically illustrated below.

Brock et al. (1987) developed a non-parametric test on nonlinearity for time series data, which reads as:[13]

$$W_m(e, N) \equiv \sqrt{N} \, \frac{D_m(e, N)}{b_m(e, N)}, \qquad D_m \equiv C_m - [C_1]^m \quad (3.6)$$

where C_m is the correlation dimension,[14] defined as:

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[12] Also the increasingly popular statistical software package for spatial data, SpaceStat (Anselin, 1992), does not contain a test for nonlinearity.

[13] The test is generally referred to as W_m; in the present context the reader should therefore not confuse the *BDS* test with the weight matrix, referred to as *W*.

[14] A detailed description of the principles behind the use of the correlation dimension is left out here. See Grassberger and Procaccia (1983) for details.

$$C_m(e) \equiv \lim_{e\to 0,\, N\to\infty} \frac{\#\left\{(i,j): \left\| w_i^m - w_j^m \right\| < e, 1 \le i \le N_m, 1 \le j \le N_m \right\}}{N_m^2} \tag{3.7}$$

with $N_m \equiv N-(m-1)$. *N* represents the number of observations, *i* and *j* moments in time, and w_i^m, given by:

$$w_i^m \equiv \left(w_i, w_{i+1}, \cdots, w_{i+m-1}\right) \tag{3.8}$$

is referred to as the *m*-history, representing a series of succeeding temporal observation points.

In the test statistic above, b_m is an estimate of the standard deviation under the IID null. Brock et al. (1987) show that under the null of IID, $W_m \to N(0,1)$ as $N \to \infty$. Hence, if the estimated residuals of best fit linear models are indeed asymptotically IID, then the W_m-statistics will be asymptotically *N*(0,1), which indicates linearity. The standard deviation $b_m(e,N)$ is in this case equal to $\sqrt{V}$ (Brock and Dechert, 1990), with:

$$V = 4\left\{2\sum_{j=1}^{m-1} Q^{m-j} C^{2j} + Q^m + (m-1)^2 C^{2m} - m^2 Q C^{2(m-1)}\right\} \tag{3.9}$$

and

$$C = E\left\{I_e\left(X_i, X_j\right)\right\} \equiv C_1(e), \quad Q = E\left\{I_e\left(X_i, X_j\right)\ I_e\left(X_j, X_k\right)\right\} \tag{3.10}$$

The expression $I_e\left(X_i, X_j\right)$ is the indicator function of the event $\left\{\left|X-Y\right| < e\right\}$, and is set to one if the expression holds and to zero otherwise. *C* is the correlation dimension with an *m*-history of one, and *Q* is the expectation of the possibility that the triplet $\left(X_i, X_j, X_k\right)$ satisfies:

$$\left\{\left|X_i - X_j\right| < e, \left|X_j - X_k\right| < e\right\} \tag{3.11}$$

This nonlinearity specification test has been shown to perform well in Monte Carlo studies on the size and power distribution of the statistic, in particular when ε is chosen between 0.5-1 and 1-1.5 times the standard deviation of the data set being tested. However, it should be noted that the *BDS* test has power against a wide class of alternatives, and it may thus be sensitive to common misspecifications, such as (serial or spatial) dependence among the elements of the error term (Hsieh and LeBaron, 1988).

The *BDS* test is tailored to a temporal setting, as the *m*-history is defined as a series of *m* sequential observations over time. Where the consecutive numbering of temporal observations corresponds to a 'real' process, the numbering of spatial observations merely corresponds to the topology of a geographical map of a spatial system that inevitably exhibits discontinuities that do not represent 'real' phenomena. The multidirectional nature of spatial causation processes calls for an adjusted specification of the concept of *m*-histories in the spatial dimension, which will be referred to as '*m*-surroundings', defined as:

$$w_r^m = \left\{ w_r, w_{r'_1}, \cdots, w_{r'_{m-1}} \right\} \tag{3.12}$$

with $r \neq r'_i$ and i = 1, 2, …, m-1. The concept *m*-surroundings can be straightforwardly applied to spatial systems represented as a regular or irregular lattice structure. However, the relations among the areas should be defined in terms of a continuous measure (e.g., inverted distance), as otherwise the number of elements in an *m*-surrounding may differ among the areas considered. In an irregular lattice structure this may be due to differing numbers of e.g., first order contiguity neighbors; in a regular lattice structure the incompatibility is caused by edge effects. Hence, the r'-elements in equation (3.12) can be easily chosen on the basis of maximizing the inverted distance, given a fixed number of elements *m*.

In order to get a first impression of the performance of the spatial BDS test it will be applied to a residual empirical data series. This will be further dicussed in the next section.

4. An empirical example: A shift-share model for Dutch regional labor markets

For the illustration of the spatial BDS test the stochastic variant of the well-known shift-share model will be used. Over the last two or three decades shift-share analysis has become a very popular analytical tool in regional science. Although the technique has been criticized for various reasons (see e.g., Armstrong and Taylor, 1993), the use of the technique is widespread, *inter alia* in migration analysis and neoclassical growth models. The essentials of shift-share analysis are concisely as follows.

The technique essentially boils down to a decomposition of regional growth figures (e.g., employment, value-added or income) relative to a base economy, mostly the national economy. Although the terminology varies, regional growth c_{ij} is laid out in a national growth component n_{ij}, an industry-mix effect m_{ij}, and a regional effect r_{ij}:[15]

$$c_{ij} = n_{ij} + m_{ij} + r_{ij} \tag{4.1}$$

where the subscript *i* refers to an industry or sector, and *j* to a spatial unit. Defined in terms of employment *E*, the different components are given as:

$$c_{ij} = E_{ij}^{t} - E_{ij}^{t-t} \tag{4.2}$$

$$n_{ij} = E_{ij}^{t}\, g \tag{4.3}$$

$$m_{ij} = E_{ij}^{t}\left(g_i - g\right) \tag{4.4}$$

$$r_{ij} = E_{ij}^{t}\left(g_{ij} - g_i\right) \tag{4.5}$$

with the superscript *t* as the current year and $t-t$ referring to the base year. The national, sectoral and regional sectoral growth rates (g, g_i and g_{ij}, respectively), are straightforwardly given by:

$$g = \frac{1}{\sum_{i,j} E_{ij}^{t-t}}\left(\sum_{i,j} E_{ij}^{t} - \sum_{i,j} E_{ij}^{t-t}\right) \tag{4.6}$$

$$g_i = \frac{1}{\sum_{j} E_{ij}^{t-t}}\left(\sum_{j} E_{ij}^{t} - \sum_{j} E_{ij}^{t-t}\right) \tag{4.7}$$

$$g_{ij} = \frac{1}{E_{ij}^{t-t}}\left(E_{ij}^{t} - E_{ij}^{t-t}\right) \tag{4.8}$$

Various drawbacks of this type of analysis have been pointed out in the literature. The lack of an underlying theoretical context has been pointed out, although Casler (1989) develops a link with a regional input growth model in a standard microeconomic framework. Moreover, the problem of aggregation over space, time as well as industries has been intensively discussed (e.g., Barff and Knight, 1988; Holden et al., 1989). Finally, the forecasting accuracy of the technique has been scrutinized, and put in the context of standard time series analysis (e.g., Kurre and Weller, 1989).

[15] The terminology used is somewhat diffuse: the national growth component is also referred to as the share effect or total share; the industry mix and the regional effect are also called the proportional shift and the differen-

In order to overcome some of these drawbacks and in order to put the shift-share technique in a stochastic framework, allowing to derive statistical significance levels, econometric models have been developed (Emmerson et al., 1975; Berzeg, 1978, 1984; Knudsen and Barff, 1991).[16] It should be observed though, that spatial interdependencies have not yet been taken into account.

In order to arrive at data series for the industry-mix effect and regional effect per region, equations (4.4) and (4.5) are redefined by scaling the effects with the regional employment in the end year:

$$m_j = \frac{\sum_i \left(E_{ij}^t \left(g_i - g\right)\right)}{\sum_i E_{ij}^t} \tag{4.9}$$

$$r_j = \frac{\sum_i \left(E_{ij}^t \left(g_{ij} - g_i\right)\right)}{\sum_i E_{ij}^t} \tag{4.10}$$

The net shift can then be evaluated as:

$$s_j = m_j + r_j \tag{4.11}$$

where m_j refers to the regional industry-mix effect or proportional shift, r_j to the regional effect or differential shift, and s_j is the net shift.

The data set which has been compiled, consists of observations on regional employment in the Netherlands in 1987 and in 1992 in the so-called COROP-regions.[17] The data were desaggregated into 6 sectors (agriculture, manufacturing, building, trade, services and government) and 40 regions. The two figures presented below may give the reader a feel for the regional labor market situation in the Netherlands, although it should be noted that the level of sectoral desaggregation is lower than in the actual empirical analyses carried out.

Figure 2 represents the spatial partitioning of the Netherlands in COROP-regions and shows the labor market volume in terms of the number of jobs for employees. A first glance at the map leads to the conclusion that there is a concentration of labor volume in the western part of the Netherlands, i.e., the so-called Randstad. It is also noteworthy that in the Randstad the share of the service sector is strikingly large as compared to agriculture and industry. Finally, the exceptionally small share of agriculture in all regions stands out; as sectors with a labor volume smaller than 5 percent are not displayed

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tial shift, respectively.

[16] The stochastic shift-share approach based on information theory (Theil and Gosh, 1980) is ignored in this paper (see e.g., Knudsen and Barff, 1991, for an overview).

[17] The division in COROP-regions is one of the standard statistical divisions used by the Central Bureau of Statistics (CBS). The spatial aggregation level corresponds to the situation that every region comprises one major city and its immediate hinterland. The employment data are taken from CBS files.

on the map, there is only one region (in the western part, the so-called Westland) with a sufficiently high percentage of labor volume in the agricultural sector.

Figure 3 disaggregates the service sector into trade, transport, finances and other services. The absolute values of the labor volume are again the largest in the western part of the Netherlands. Trade is the largest service sector in all regions. The transport sector, relatively evenly divided over all regions, comes on a second place. The financial sector is concentrated in the western part of the Netherlands.

On the basis of the CBS labor volume data for 1987 and 1992, growth rates may be calculated along the lines of equations (4.1)-(4.11) in order to arrive at shift and share proportions in a conventional manner. Following the earlier work of Berzeg (1978, 1984) and Knudsen and Barff (1991), the latter may also be derived in a stochastic framework. As suggested by Arcelus (1984) a two way fixed effects specification can be used to estimate the share, the proportional and the differential shift. In a succinct notation (Judge et al., 1985, p. 531) this specification can be presented as follows:

$$g = \left[j_{SR} \quad \begin{pmatrix} I_{S-1} \\ 0' \end{pmatrix} \otimes j_R \quad j_S \otimes \begin{pmatrix} I_{R-1} \\ 0' \end{pmatrix} \right]' \begin{pmatrix} a \\ b_{S-1} \\ b_{R-1} \end{pmatrix} + e \qquad (5.1)$$

where, with S as the number of sectors and R the number of regions, j is a column vector of units, and I the identity matrix (the dimensions are given by the subscripts), $0'$ is a corresponding vector of zero's and e is a vector of innovations.

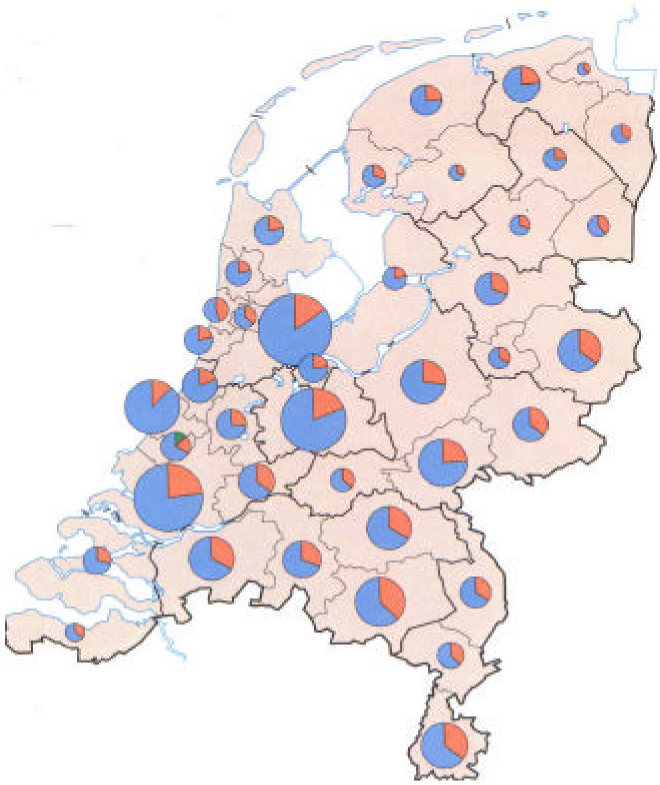

Figure 2: Labor volume in the Netherlands in agriculture, industry and the service sector (presented clock-wise). The larger the circle, the larger the volume of labor, with the smallest circle indicating 20,000 jobs. Source: Bosatlas (1995, p. 48).

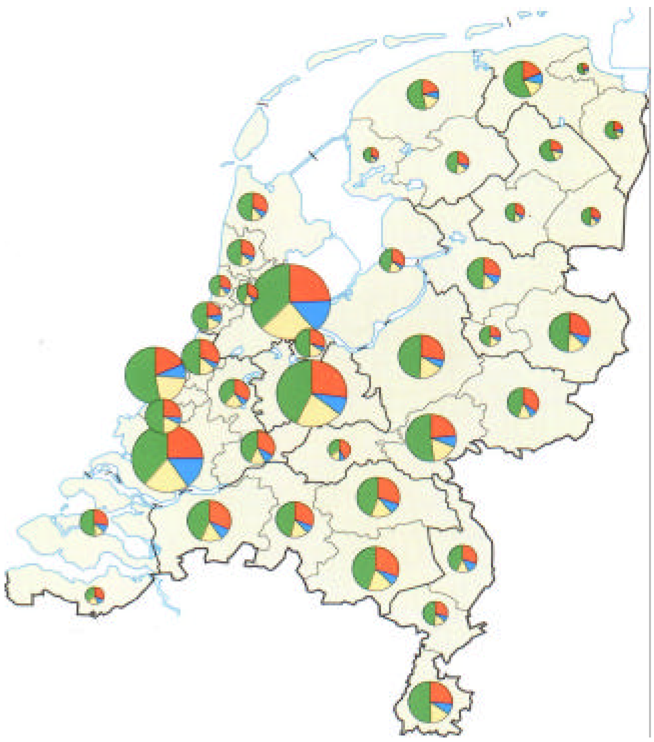

Figure 3: Labor volume in the Netherlands in the service sector for trade, transport, banking and insurance, and other services (again clock-wise and with the smallest sector representing 20,000 jobs). Source: Bosatlas (1995, p. 51).

Consequently, a is a scalar estimate for the national share plus the proportional and differential shift of the reference sector and region, b_{S-1} is a vector with estimates for the proportional shift for all but the reference sector, and b_{R-1} a vector containing the estimates for the differential shift for all but the reference region.

The disturbance term is, however, not homoscedastic. The dependent as well as the independent (dummy) variables, including the constant term, are therefore weighted by multiplying them with:

$$\frac{E_{ij}^{t-t}}{\sum_{i,j} E_{ij}^{t-t}}$$

so that the weighted least squares estimator for the parameter vector is best linear unbiased.
The adopted procedure is the following. First, equation (5.1) is estimated by means of weighted least squares. Subsequently, the estimated disturbances are checked for spatial dependence by using Moran's *I* and the *LMERR* test, and for chaotic patterns by means of the adjusted *BDS* test. If spatial dependence is present, the specification in (5.1) is re-estimated by means of the maximum likelihood estimator. Again the error term may then be inspected on the presence of chaos. The results of the succesive steps are compared below.

The results of estimating equation (5.1) are presented in Table 1. The spatial *BDS* test is calculated on the basis of the *m*-surroundings, as has been explained in the foregoing section. Only *m*-surroundings of 2 and 3 are used, since the topography of the Netherlands is such that a major area is covered by a specific region and this number of neighbors. The surroundings are calculated for error margins *e*, which are set around the standard deviations of the spatial residuals of equation (5.1). Almost all parameters, representing the share and the proportional and differential shifts, appear to be statistically significant. Because of the use of a zero dummy for the government sector and the Flevoland region, the parameter values should be recalculated in order to obtain the conventional shift-share results. The parameters can be assessed by calculating the sectoral or regional coefficient of the conventional shift-share minus the proportional or differential shift of the omitted sector or region. Moran’s *I* and the *LMERR* test indicate the presence of spatial dependence, while the *BDS* test strongly suggests the presence of nonlinearity. However, because the latter test has power against several alternatives, it may falsely detect nonlinearity, due to the erroneously omitted spatial dependence.

Therefore, the model given in equation (5.1) is respecified so as to incorporate a spatial AR(1) process in the error term, and subsequently re-estimated with the appropriate maximum likelihood estimator given in equation (3.5). It should be noted that, given the dimension of the model, the spatial weight matrix *W* is a block-diagonal matrix with the original inverted distance weights matrix on the main diagonal.

Table 1: The shifts and share estimated with the weighted least squares estimator.[a]

Share		**Differential shift**	
Intercept	0.191 (4.36)	Oost-Groningen	-0.216 (-3.10)
		Delfzijl	-0.224 (-2.38)
Proportional shift		Overig-Groningen	-0.267 (-5.05)
		Noord-Friesland	-0.207 (-3.76)
Agriculture	0.178 (5.94)	Zuidwest-Friesland	-0.147 (-1.88)
Manufacturing	0.054 (3.99)	Zuidoost-Friesland	-0.163 (-2.63)
Building	0.054 (3.15)	Noord-Drenthe	-0.245 (-3.57)
Trade	0.234 (17.66)	Zuidoost-Drenthe	-0.260 (-4.03)
Services	0.170 (10.00)	Zuidwest-Drenthe	-0.188 (-2.83)
Government	0.000	Noord-Overijssel	-0.144 (-2.72)
		Zuidwest-Overijssel	-0.223 (-3.49)
Tests		Twente	-0.187 (-3.83)
R^2	0.750	Veluwe	-0.203 (-4.16)
Adjusted R^2	0.694	Achterhoek	-0.140 (-2.72)
F-test	133.240 (0.00)	Arnhem-Nijmegen	-0.220 (-4.55)
Moran's *I*	0.065 (3.90)	Zuidwest-Gelderland	-0.067 (-1.16)
LMERR	5.788 (0.02)	Utrecht	-0.143 (-3.14)
BDS (m = 2)	16.466 (0.00)	Kop van N.-Holland	-0.123 (-2.20)
BDS (m = 3)	34.365 (0.00)	Alkmaar e.o.	-0.202 (-3.35)
		IJmond	-0.283 (-4.91)
		Agglom. Haarlem	-0.281 (-4.81)
		Zaanstreek	-0.184 (-2.98)
		Groot Amsterdam	-0.232 (-5.17)
		Gooi en Vechtstreek	-0.341 (-6.06)
		Agglom. Leiden	-0.213 (-4.07)
		Agglom. Den Haag	-0.267 (-5.60)
		Delft en Westland	-0.194 (-3.52)
		Oostelijk Z.-Holland	-0.334 (-6.25)
		Groot Rijnmond	-0.260 (-5.76)
		Zuidoost Z.-Holland	-0.223 (-4.42)
		Zeeuwsch-Vlaanderen	-0.246 (-3.65)
		Overig Zeeland	-0.209 (-3.66)
		West N.-Brabant	-0.168 (-3.50)
		Midden N.-Brabant	-0.194 (-3.85)
		Noordoost N.-Brabant	-0.171 (-3.56)
		Zuidoost N.-Brabant	-0.244 (-5.17)
		Noord-Limburg	-0.155 (-2.91)
		Midden-Limburg	-0.194 (-3.41)
		Zuid-Limburg	-0.190 (-3.92)
		Flevoland	0.000

[a] Two-sided *t*-values are given in parentheses. For the diagnostic tests significance levels are based on the appropriate distribution (see the main text for details). For Moran's *I* the standardized *z*-value is given in parentheses.

Maximum likelihood results for the parameter vector and the spatial dependence parameter l can be found in Table 2. Nearly all parameters are statistically significant, except for the autoregressive parameter l. Again the *BDS* test appears to detect nonlinearity, but not as strongly as in the model without spatial dependence. Thus, it seems that the *BDS* test is also somewhat sensitive for the alternative hypothesis of spatial dependence.

Table 2: The shifts and share estimated with the maximum likelihood AR(1) estimator.[a]

Share		**Differential shift**	
Intercept	0.229 (3.64)	Oost-Groningen	-0.378 (-4.96)
		Delfzijl	-0.382 (-4.45)
Proportional shift		Overig-Groningen	-0.350 (-4.11)
		Noord-Friesland	-0.317 (-3.67)
Agriculture	0.190 (7.56)	Zuidwest-Friesland	-0.166 (-1.96)
Manufacturing	0.084 (3.35)	Zuidoost-Friesland	-0.236 (-2.76)
Building	0.110 (4.37)	Noord-Drenthe	-0.207 (-2.39)
Trade	0.264 (10.53)	Zuidoost-Drenthe	-0.335 (-3.91)
Services	0.571 (22.50)	Zuidwest-Drenthe	-0.270 (-3.16)
Government	0.000	Noord-Overijssel	-0.222 (-2.58)
		Zuidwest-Overijssel	-0.205 (-2.37)
Spatial dependence		Twente	-0.254 (-2.94)
parameter λ		Veluwe	-0.245 (-2.83)
		Achterhoek	-0.218 (-2.52)
Inverse distance matrix	-0.342 (-1.31)	Arnhem-Nijmegen	-0.310 (-3.56)
		Zuidwest-Gelderland	-0.127 (-1.47)
Tests		Utrecht	-0.204 (-2.36)
BDS (*m*=2)	12.535 (0.00)	Kop van N.-Holland	-0.214 (-2.46)
BDS (*m*=3)	5.012 (0.00)	Alkmaar e.o.	-0.258 (-2.99)
		IJmond	-0.407 (-4.76)
		Agglom. Haarlem	-0.395 (-4.65)
		Zaanstreek	-0.150 (-1.75)
		Groot Amsterdam	-0.329 (-3.88)
		Gooi en Vechtstreek	-0.464 (-5.38)
		Agglom. Leiden	-0.289 (-3.32)
		Agglom. Den Haag	-0.413 (-4.83)
		Delft en Westland	-0.253 (-2.98)
		Oostelijk Z.-Holland	-0.419 (-4.87)
		Groot Rijnmond	-0.372 (-4.33)
		Zuidoost Z.-Holland	-0.297 (-3.47)
		Zeeuwsch-Vlaanderen	-0.312 (-3.61)
		Overig Zeeland	-0.310 (-3.65)
		West N.-Brabant	-0.184 (-2.12)
		Midden N.-Brabant	-0.144 (-1.67)
		Noordoost N.-Brabant	-0.209 (-2.45)
		Zuidoost N.-Brabant	-0.214 (-2.49)
		Noord-Limburg	-0.204 (-2.36)
		Midden-Limburg	-0.270 (-3.13)
		Zuid-Limburg	-0.279 (-2.36)
		Flevoland	0.000

[a] See the footnote of Table 1.

While these results suggest the presence of nonlinearity in the data set, other factors, such as correlations between sectors and regions, may also be a potential cause for this behavior. Therefore, also a model in which only the industry mix effect and the national share are estimated, will be analyzed. The specification boils down to a one way fixed effects model which, again in the notation of Judge et al. (1985, p. 520), reads as:

$$g = \left[j_R \quad \begin{pmatrix} I_{S-1} \\ 0' \end{pmatrix} \otimes j_R \right]' \begin{pmatrix} a \\ b_{S-1} \end{pmatrix} + m \tag{5.2}$$

with all notation as before, and m for the IID error term. Consequently, the model comprises 40 observations and can be estimated by the weighted least squares estimator.

The results of equation (5.2) can be found in Table 3, which shows that some of the parameters for the coefficients are not statistically significant, and that again both Moran's *I* and the *LMERR* test suggest spatial dependence. The large value of R^2 shows that a significant part of the regional growth figures is in fact explained by the industry mix. The numerical value of the *BDS* test is considerably lower than in the extended model (see Table 1), although it still rejects the null-hypothesis of linearity.

Table 3: Proportional shifts and share estimated with the weighted least squares estimator.[a]

Share		**Tests**		
Intercept	-0.023 (-1.24)			
		R^2	0.549	
Proportional shift		Adjusted R^2	0.539	
Agriculture	0.199 (3.62)	*F*-test	56.856	(0.00)
Manufacturing	0.061 (2.53)	Moran's *I*	0.124	(94.58)
Building	0.073 (2.33)	*LMERR*	38.440	(0.00)
Trade	0.242 (9.81)			
Services	0.453 (14.37)	*BDS* (m = 2)	6.350	(0.00)
Government	0.000	*BDS* (m = 3)	5.038	(0.00)

[a] See the footnote of Table 1.

As the spatial dependence tests are significant, the specification in equation (5.2) is respecified to contain a spatial AR(1) process in the error term, and estimated with the corresponding ML estimator. The results for this specification are given in Table 4.

It should be noted that the parameter value for the intercept represents the proportional shift of the government sector in addition to the the national growth rate, and its value changes only slightly. The other coefficients do not exhibit significant changes in the respecified model. It may also be observed that the parameter value of the spatial dependence parameter in the model with both sectoral and regional dummy variables in comparison to the value in the model with exclusively sectoral dummy variables included, is only slighty different. The *BDS* test still rejects the null hypothesis of linearity and its numerical value is even slightly higher compared to the specification without a spatial AR(1) process.

Table 4: Proportional shifts and share estimated with the maximum likelihood estimator.[a]

Share		**Spatial dependence parameter λ**	
Intercept	-0.043 (-1.86)		
Proportional shift		Inverse distance matrix	-0.214 (-0.89)
Agriculture	0.190 (5.82)		
Manufacturing	0.085 (2.59)	**Tests**	
Building	0.110 (3.37)	*BDS*-test (m = 2)	10.724 (0.00)
Trade	0.265 (8.11)	*BDS*-test (m = 3)	5.438 (0.00)
Services	0.573 (17.44)		
Government	0.000		

[a] See the footnote of Table 1.

One may speculate that these results show that the *BDS* test is not sensitive for spatial dependence. However, it should be noted that the results of the *BDS* test are only asymptotically warranted and its small sample behavior is in fact unknown.

5. Conclusions

The potentials of nonlinear modeling are increasingly recognized in regional science, as the limitations of equilibrium models in explaining real world phenomena stand out. Wheras conventional equilibrium models can only explain marginal departures from 'golden rule' development trajectories, nonlinear models are capable of describing and forecasting dissipative evolutionary patterns, with bifurcations and catastrophes as sudden changes in a solution trajectory.

Although mathematical and statistical difficulties have long stood in the way of operationalizing nonlinear models, such as those developed in chaos theory, the rapid development of computer technology and appropriate software now enables us to apply nonlinear modeling to a wide array of topics in regional science. Recent advances are made in the analysis of urban, transport, migration and industrial production systems. However, a rigorous statistical-econometric treatment of chaos phenomena has not been developed to date. This paper sets out to start to fill this gap in the literature.

In particular, a linkage between the toolbox of diagnostic misspecification tests developed in spatial econometrics and the nonlinearity test suggested by Brock, Dechert and Scheinkman (the so-called *BDS* test) was established. The multidirectional nature of spatial causation processes prohibits a straightforward application of the *BDS* test to spatial process models, as it is tailored to time series analysis. This paper therefore has aimed to develop a spatial variant of the *BDS* test and has subsequently illustrated the use of this test by means of a shift-share model for regional labor market performance in the Netherlands, over the period 1987-1992.

In the empirical analyses it was easily demonstrated that the spatial residual series clearly exhibit nonlinearity. It is, however, crucial to observe that the *BDS* test is likely to have power against various alternative hypotheses. For instance, in the first part of the empirical analyses there is preliminary evidence that the *BDS* test is sensitive to spatial autocorrelation among a series of (residual) observations. In the second part of the analyses this assertion is not corroborated, although it demonstrates that the sample size may also be a factor of crucial importance. Moreover, as in the time series case further investigations are needed regarding the impact of the length of the *m*-surrounding as well as the value of the error margins.

Consequently, further research is warranted, in particular into the comparison of the performance of the standard *BDS* test and the spatial variant. Insight should also be attained regarding the small sample size and distribution of the *BDS* test in a spatial setting. This can only be achieved in an experimental setting. Finally, the diagnostic tool box for chaos testing should be extended to include dynamic spatial process models, which adds another dimension as well as complexity.

References

Anselin, L., *Spatial Econometrics: Methods and Models,* Dordrecht: Kluwer Academic Publishers, 1988.

Anselin, L., *SpaceStat: A Program for the analysis of spatial data*, Santa Barbara, CA: National Center for Geographic Information and Analysis, University of California, 1992 (see also: *Spacestat Version 1.50: Revision Notes*, Morgantown, WV: West Virginia University, Regional Research Institute, Research Paper 9428, 1994).

Anselin, L., *Testing for Spatial Dependence in Linear Regression Models: A Review*, Morgantown, WV: West Virginia University, Regional Research Institute, Research Paper 9416, 1994.

Anselin, L. and R.J.G.M. Florax, *New Directions in Spatial Econometrics,* Berlin: Springer-Verlag, 1995a.

Anselin, L. and R.J.G.M. Florax, Small Sample Properties of Tests for Spatial Dependence in Regression Models: Some Further Results, in: L. Anselin and R.J.G.M. Florax, *New Directions in Spatial Econometrics,* Berlin: Springer-Verlag,1995b.

Anselin, L., A.K. Bera, R.J.G.M. Florax and M.J. Yoon, Simple Diagnostic Tests for Spatial Dependence, *Regional Science and Urban Economics*, 26, 1, 77-104, 1996.

Anselin, L. and D.A. Griffith, Do Spatial Effects Really matter in Regression Analysis ?, *Papers of the Regional Science Association*, 65, 11-34, 1988.

Anselin, L. and S. Hudak, Spatial Econometrics in Practice: A Review of Software Options, *Regional Science and Urban Economics*, 22, 3, 509-36, 1992.

Anselin, L. and S. Rey, Properties of Tests for Spatial Dependence in Linear Regression Models, *Geographical Analysis*, 23, 112-31, 1991.

Arcelus, F.J., An Extension of Shift-Share Analysis, *Growth and Change*, 15, 3-8, 1984.

Armstrong, H. and J. Taylor, *Regional Economics & Policy*, Oxford: Philip Allan,1993.

Barff, R.A. and P.L. Knight III, Dynamic Shift-Share Analysis, *Growth and Change*, 19, 1-10, 1988.

Baumol, W.J. and J. Benhabib, Chaos: Significance, Mechanism and Economic Applications, *Journal of Economic Perspectives*, 3, 1, 77-105, 1988.

Berzeg, K. The Empirical Content of Shift-Share Analysis, *Journal of Regional Science*, 18, 463-9, 1978.

Berzeg, K. A Note on Statistical Approaches to Shift-Share Analysis, *Journal of Regional Science*, 24, 277-85, 1984.

Bosatlas, Groningen: Wolters-Noordhoff, 1995.

Brock, W.A., Distinguishing Random and Deterministic Systems, *Journal of Economic Theory*, 40, 1, 168-95, 1986.

Brock, W.A. and W.D. Dechert, *A General Class of Specification Tests: The scalar case*, Wisconsin: Social Systems Research Institute, 1990.

Brock, W.A., W.D. Dechert and J. Scheinkman, *A Test for Independence Based on the Correlation Dimension*, Madison: Department of Economics, University of Wisconsin, SSRI Working Paper 8702, 1987.

Brock, W.A., D. Hsieh and B. Le Baron, *Nonlinear Dyncamics, Chaos and Instability: Statistical Theory and Economic Evidence*, Cambridge, Mass.: MIT Press, 1991.

Casler, S.D., A theoretical Context for Shift and Share Analysis, *Regional Studies*, 23, 43-8, 1989.

Cliff, A.D. and J.K. Ord, *Spatial Autocorrelation*, London: Pion, 1973.

Cliff, A.D. and J.K. Ord, *Spatial Processes, Models & Applications*, London: Pion, 1981.

Emmerson, R., R. Ramanathan and W. Ramm, On the Analysis of Regional Growth Patterns, *Journal of Regional Science*, 15, 17-28, 1975.

Fischer, M.M. and A. Getis**,** *Recent Developments in Spatial Analysis: Spatial Statistics, Behaviour Modelling and Computational Intelligence*, Berlin: Springer-Verlag, 1997.

Florax, R., *The University: A Regional Booster?, Economic Impacts of Academic Knowledge Infrastructure*, Aldershot: Avebury, 1992.

Florax, R.J.G.M. and S. Rey, The Impacts of Misspecified Spatial Interaction in Linear Regression Models, in: L. Anselin and R.J.G.M. Florax, *New Directions in Spatial Econometrics*, Berlin: Springer-Verlag, 1995.

Fujita, M. and H. Ogawa, Multiple Equilibria and Structural Transformation of Non-Mono-Centric Urban Configurations, *Regional Science and Urban Economics*, 12, 2, 161-96, 1982.

Gandolfo, G., *Economic Dynamics*, Berlin: Springer-Verlag,1996.

Graaff, T. de, *Nonlinearity in Spatial Structures*, Amsterdam: Free University, 1997 (mimeo).

Grassberger, P. and I. Procaccia, Measuring the Strangeness of Strange Attractors, *Physica 9D*, 1983.

Griffith, D., *Some Guidelines for Specifying the Geographic Weights Matrix Contained in Spatial Statistical Models*, Rome: Universitá degli Studi di Roma “La Sapienza”, 1995 (mimeo).

Holden, D.R., A.G.M. Nairn and J.K. Swales, Shift-share Analysis of Regional Growth and Policy: A Critique, *Oxford Bulletin of Economics and Statistics*, 51, 15-34, 1989.

Hordijk, L. Spatial Correlation in the Disturbances of a Linear Interregional Model, *Regional and Urban Economics*, 4, 117-40, 1974.

Hsieh, D. and B. LeBaron, *Small Sample Properties of the BDS-Statistic I"*, Chicago: Graduate School of Business, University of Chicago, 1988.

Huang, J.S. The Autoregressive Moving Average Model for Spatial Analysis, *Australian Journal of Statistics*, 26, 169-87, 1984.

Judge, G.G., W.E. Griffiths, R. Carter Hill, H. Lütkepohl and T.C. Lee, *The Theory and Practice of Econometrics*, New York: John Wiley & Sons, 1985.

Knudsen, D.C. and R. Barff, Shift-share Analysis as a Linear Model, *Environment and Planning A*, 23, 421-31, 1991.

Kovach, L.D., Life Can Be So Nonlinear, *American Scientist*, 48, 1960.

Kurre J.A. and B.R. Weller, Forecasting the Local Economy, Using Time-Series and Shift-Share Techniques, *Environment And Planning A*, 21, 753-70, 1989.

Lorenz, H.-W., *Non-Linear Dynamical Economics and Chaotic Motion*, Berlin: Springer-Verlag, 1993.

Mannermaa, M., Alternative Futures Perspectives on Sustainability, Coherence and Chaos, *Journal of Contingencies and Crisis Management*, 3, 1, 27-34, 1995.

Marshall, A., *Principles of Economics: An Introductory Volume*, London: Macmillan, 1890 (eight edition, 1930).

Nelson, R.R., Recent Evolutionary Theorizing About Economic Change, *Journal of Economic Literature*, 33, 48-90, 1995.

Nicolis, G. and I. Prigogine, *Exploring Complexity: An Introduction*, New York: Freeman, 1989.

Nijkamp, P. and A. Reggiani, *Nonlinear Evolution of Spatial Economic Systems*, Berlin: Springer-Verlag, 1993.

Nijkamp, P. and A. Reggiani, *The Economics of Complex Systems*, Amsterdam: Elsevier, 1998 (forthcoming).

O'Driscoll, G.P. and M.J. Rizzo, *The Economics of Time and Ignorance*, New York: Basil Blackwell, 1985.

Ornstein, D.S. and B. Weiss, Statistical Properties of Chaotic Systems**,** *Bulletin of the American Mathematical Society*, 24, 1, 11-116, 1991.

Paelinck, J. and L. Klaassen, *Spatial Econometrics*, Farnborough: Saxon House, 1979.

Peters, E.E., *Fractal Market Analysis: Applying Chaos Theory to Investment and Economics*, New York: John Wiley, 1994.

Puu, T., *Non-Linear Economic Dynamics*, Berlin: Springer-Verlag, 1989.

Rosser, J.B., *From Catastrophe to Chaos: A General Theory of Economic Discontinuities*, Dordrecht: Kluwer Academic Publishers, 1991.

Scheinkman, J.A., Nonlinearities in Economic Dynamics, *The Economic Journal*, 150, 33-48, 1990.

Schuster, H.G., *Deterministic Chaos*, Veinheim: VCH, 1988.

Sneek, J.M. and P. Rietveld, *Higher Order Spatial ARMA Models*, Amsterdam: Tinbergen Institute, discussion paper 97-643/3, 1997.

Theil, H. and R. Gosh, A Comparison of Shift-Share and the RAS Adjustment, *Regional Science and Urban Economics*, 10, 175-80, 1980.

Wolf, A., J. Swift, H. Swinney, and J. Vastano, Determination of Lyapunov Exponents from a Time-series, *Physica 16D*, 1985.

Zhang, W.B., *Synergetic Economics: Time and Change in Non-linear Economics*, Berlin: Springer-Verlag, 1991.